

Forward Guidance and the State of the Economy*

Benjamin D. Keen

Alexander W. Richter

Nathaniel A. Throckmorton

First Draft: December 18, 2013

This Draft: December 31, 2015

ABSTRACT

This paper examines the efficacy of forward guidance using a New Keynesian model with a zero lower bound (ZLB) constraint on the short-term nominal interest rate. Forward guidance is modeled with news shocks to the monetary policy rule, which one can think of as a modeling device for generating innovations in expected future interest rates. There are four key findings: (1) The stimulative effect of forward guidance falls as the economy deteriorates or as households expect a slower recovery from a recession, which suggests plans to keep future rates low should be communicated early in an economic downturn to maximize their stimulative effect; (2) When the ZLB binds, less uncertainty about future economic conditions or an expectation of a stronger monetary response to inflation reduces the stimulative effect of forward guidance; (3) In steady state, an unanticipated shock is more stimulative on impact than a news shock, but a news shock is more stimulative near the ZLB and always has a larger cumulative effect; (4) At the ZLB, the cumulative effect on output from lengthening the forward guidance horizon increases over short horizons but decreases thereafter, which indicates the central bank faces limits on how far forward guidance can extend into the future and continue to add stimulus.

Keywords: Forward Guidance; Zero Lower Bound; News Shocks; Global Solution Method

JEL Classifications: E43; E58; E61

*Keen, Department of Economics, University of Oklahoma, 308 Cate Center Drive, 437 Cate Center One, Norman, OK (ben.keen@ou.edu); Richter, Department of Economics, Auburn University, 0332 Haley Center, Auburn, AL (arichter@auburn.edu); Throckmorton, Department of Economics, College of William & Mary, Morton Hall 131, Williamsburg, VA (nathrockmorton@wm.edu). We are especially grateful to Bill Gavin for his contributions to earlier versions of this paper. We thank Klaus Adam, Marco Del Negro, Evan Koenig, Mike Plante, Daniel Rees, Chris Vickers, and Mark Wynne for useful suggestions. The paper also benefited from comments by conference and seminar participants at the Federal Reserve Bank of Dallas, the Federal Reserve Bank of St. Louis, the Bundesbank, Wits Business School, Vanderbilt University, the University of Kansas, the 2014 MEA meeting, the 2014 Midwest Macro meeting, the 2014 CEF conference, the 2014 WEAI meeting, the 2014 Dynare Conference, the 2014 SEA meeting, the 2014 ECB workshop on Non-Standard Monetary Policy Measures, and the 2015 RBA Quantitative Macroeconomics Workshop. The authors appreciate the research support the Federal Reserve Bank of St. Louis provided on this project.

1 INTRODUCTION

The global economic slowdown in 2008 led many central banks to sharply reduce their policy rates. When rates could not be reduced further, some central banks resorted to unconventional policies, such as forward guidance. Forward guidance refers to central bank communication about future policy, which has many forms including announcements about objectives, contingencies, policy actions, and speeches. This paper focuses on communication about the path of future policy rates.

We examine the effects of forward guidance using a New Keynesian model with a zero lower bound (ZLB) constraint on current and future policy rates. Forward guidance is modeled with news shocks to the monetary policy rule similar to Laséen and Svensson (2011).¹ The central bank implements forward guidance by communicating the news over a specific forward guidance horizon. The news is the difference between the expected policy rates before and after the central bank's announcement, so one can think of the news as a modeling device for generating innovations in future expectations. News that the central bank intends to keep future policy rates lower than previously expected generates higher inflation, lowers real interest rates, and raises real GDP.

We show how the ZLB constraint, the state of the economy, the size of the news shocks, the degree of economic uncertainty, the monetary response to inflation, the speed of the recovery, and the forward guidance horizon nonlinearly impact the efficacy of forward guidance. We then use our results to interpret the effects of recent forward guidance by the Fed. There are four key findings:

1. The stimulative effect of forward guidance falls as the economy deteriorates or as households expect a slower recovery from a recession, which suggests plans to keep future rates low should be communicated early in an economic downturn to maximize their stimulative effect.
2. When the ZLB binds, less uncertainty about future economic conditions or an expectation of a stronger monetary response to inflation reduces the stimulative effect of forward guidance.
3. In steady state, an unanticipated shock is more stimulative on impact than a news shock, but a news shock is more stimulative near the ZLB and always has a larger cumulative effect.
4. At the ZLB, the cumulative effect on output from lengthening the forward guidance horizon increases over short horizons but decreases thereafter, which indicates the central bank faces limits on how far forward guidance can extend into the future and continue to add stimulus.

To our knowledge, this paper is the first to study forward guidance with news shocks using a global solution method. This solution method enhances our analysis of forward guidance in several ways. One, it enables ZLB events to endogenously reoccur, which impacts households' expectations of future policy rates and the central bank's ability to provide economic stimulus. Two, we can assess the impact of forward guidance at the ZLB, near the ZLB, or at any other state of the economy. Three, it allows us to evaluate forward guidance in a setting where changes in economic conditions affect both the probability and expected duration of a ZLB event. For example, a negative demand shock while the ZLB binds reduces a central bank's margin to lower expected policy rates by decreasing the probability of exiting the ZLB. Four, we are able to analyze forward guidance across all possible realizations of shocks, which nonlinearly impact the economy.

Campbell et al. (2012) introduce two terms to separate the types of forward guidance: Delphic and Odyssean. Delphic forward guidance is a central bank's forecast of its own policy, which is

¹Gomes et al. (2013) and Milani and Treadwell (2012) estimate unconstrained New Keynesian models that include news shocks in the monetary policy rule. They find news shocks play an important role in matching data. Ben Zeev et al. (2015) and Campbell et al. (2012) develop methods to identify anticipated monetary policy shocks in the data.

based on its projections for inflation and real GDP as well as an established policy rule.² Odyssean forward guidance is a promise to deviate from the policy rule in the future by setting the policy rate lower than the rule recommends. News shocks are one way to model Odyssean forward guidance.

Central banks have recently used both date-based and threshold-based forward guidance. Date-based forward guidance provides information on the intended policy rate path over a fixed period and is often modeled using an interest rate peg. To a modeler, an interest rate peg is a special case of our news shock approach, where the central bank provides news that it intends to fix the policy rate for a set number of periods. We compare the results of our approach to modeling forward guidance to those of an interest rate peg. We find the peg generates increasingly larger impact effects on output because it gives the central bank a growing ability to affect expected future interest rates.

With threshold-based forward guidance, the central bank agrees to maintain a policy rate until a specific event occurs. For example, the central bank might announce it intends to keep its policy rate at zero until the unemployment rate falls below a certain value. Our news shock approach is similar to threshold-based forward guidance because it allows the policy rate to endogenously respond to economic conditions once the objectives for output and inflation have been met. While the news shocks are Odyssean, the endogenous response of monetary policy to economic conditions is Delphic because households know the central bank's rule and use it to forecast future policy rates.

There are four reasons why we advocate using news shocks instead of an interest rate peg to model forward guidance. One, news shocks are more flexible since an interest rate peg corresponds to a specific sequence of anticipated shocks. Two, an interest rate peg produces a degenerate distribution for the policy rate that contradicts recent survey and options data. In our model, the distribution for every future nominal interest rate depends on the distribution of future economic outcomes. Three, households never expect the central bank will adjust its forward guidance policies to economic conditions under an interest rate peg, which is inconsistent with the threshold-based nature of recent forward guidance. With news shocks, households' expectations incorporate the possibility that the policy rate could rise due to improving economic conditions. Four, an interest rate peg does not separate the effects of additional news from a longer horizon because extending a peg is analogous to providing increasingly large news shocks. For those reasons, we believe news shocks provide the sophistication necessary to accurately assess the economic effects of forward guidance.

Other papers examine the effects of forward guidance in an economy with a binding ZLB constraint through the perspective of optimal monetary policy under commitment (i.e., a promise to implement a specific policy regardless of changes in future economic conditions). Eggertsson and Woodford (2003) and Jung et al. (2005) solve for the optimal commitment policy assuming the policy rate initially equals zero and cannot return to its ZLB. They find the optimal policy is to maintain a policy rate equal to zero even after the natural real interest rate rises. Such a policy generates higher future inflation and lowers the real interest rate, which moderates the declines in output and inflation that occur at the ZLB. Levin et al. (2010) show the optimal policy stabilizes the economy after small shocks but not after large and persistent shocks. In that situation, they argue that a central bank must employ other unconventional policies, such as large-scale asset purchases, to stabilize the economy. Adam and Billi (2006) relax the assumption that the policy rate initially equals zero by allowing the ZLB to occasionally bind. They find the optimal commitment policy

²See the Bank of England (2013) for a discussion of how forward guidance helps the public form more accurate expectations about future central bank policy. See den Haan (2013) for a collection of essays about forward guidance and the International Monetary Fund (2013) for a detailed account of recent unconventional monetary policies.

is to respond more aggressively to adverse shocks that cause output and inflation to decline.³

There is also research on forward guidance outside the optimal policy literature. Del Negro et al. (2012) use a log-linear New Keynesian model to show that extending the forward guidance horizon causes the model to overpredict the actual increases in output and inflation. They call that result the “forward guidance puzzle.” Several papers offer explanations for the puzzle: McKay et al. (2015) introduce uninsurable income risk and borrowing constraints; Kiley (2014) considers a model with sticky information rather than sticky prices; De Graeve et al. (2014) and Haberis et al. (2014) account for imperfect credibility; and Caballero and Farhi (2014) develop a model where the ZLB binds due to a safety trap—a shortage of safe assets—instead of a demand-side shock.⁴

We emphasize the ZLB constraint on current and future policy rates and the state of the economy as a way of explaining the forward guidance puzzle. In our model, demand shocks push the policy rate to its ZLB. The size of those shocks and whether news shocks occur determine how long the policy rate remains at zero. As demand falls, the ZLB constraint further limits the stimulative effect of forward guidance by preventing future policy rates from declining. Although most New Keynesian models overpredict the stimulative effect of forward guidance, our results are consistent with the estimates in D’Amico and King (2015). They find anticipated reductions in the policy rate boost output over horizons up to four quarters but have much weaker effects over longer horizons.

The rest of the paper is organized as follows. [Section 2](#) provides a post-financial crisis account of Federal Open Market Committee (FOMC) forward guidance in its policy statements. [Section 3](#) describes our theoretical model. [Sections 4](#) and [5](#) show the stimulative effects of forward guidance across horizons up to 10 quarters. [Section 6](#) conducts case studies of recent FOMC forward guidance and uses our key findings to explain the effects of that communication. [Section 7](#) concludes.

2 RECENT FEDERAL RESERVE FORWARD GUIDANCE

There are two ways the Fed communicates information about future policy rates. One, it releases the individual forecasts of the FOMC members every quarter. Two, it provides forward guidance about the future federal funds rate in its policy statements and has consistently done so since 2008.

At the December 16, 2008 meeting, the FOMC decided to target a range for the federal funds rate of 0% to 0.25% and announced it would likely remain at that low level for “some time.” The FOMC continued to use similar language until its August 9, 2011 statement, which said that low range was likely warranted “at least through mid-2013.” The announcement was the FOMC’s first use of date-based forward guidance, and it had a modest effect on expected future interest rates.

The next change in forward guidance occurred in the statement released after the January 25, 2012 FOMC meeting. That statement was different in two ways. One, the time the federal funds rate was expected to remain at zero was updated to read “at least through late 2014,” which was an increase of six quarters. Two, the FOMC expressed a more pessimistic economic outlook

³There are several other optimal policy papers related to forward guidance. Krugman (1998) was one of the first to argue that the central bank can mitigate the effects of the ZLB by promising to allow prices to rise. Reifschneider and Williams (2000) develop the merits of that argument in a dynamic model. Werning (2011) shows it is also optimal to commit to higher future inflation when the ZLB binds in a continuous-time model. Adam and Billi (2007) find discretionary policy is unable to generate the higher inflation that is necessary to offset the adverse effects of the ZLB. English et al. (2015) show that introducing threshold-based forward guidance into the monetary policy rule generates outcomes that are closer to the optimal commitment policy. Coenen and Warne (2014) find that date-based forward guidance increases the risk of price instability, but introducing a threshold on inflation can help mitigate that risk.

⁴See Moessner et al. (2015) for a detailed summary and analysis of the recent research on forward guidance.

and indicated the projected path for the federal funds rate was conditional on that outlook, which suggests the FOMC's policy rule was already projecting a much later date for raising its policy rate. Therefore, the forward guidance provided in the January statement was likely viewed as Delphic.

Despite the forward guidance extension, the economy continued to disappoint policymakers, which motivated the FOMC to amend its statement after the September 13, 2012 meeting to read:

To support continued progress toward maximum employment and price stability, . . . *a highly accommodative stance of monetary policy will remain appropriate for a considerable time after the economic recovery strengthens*. . . the Committee also decided today to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that exceptionally low levels for the federal funds rate are likely to be warranted *at least through mid-2015*.

The statement included a pledge to increase asset purchases and a 2-quarter extension to the time the FOMC promised to keep its policy rate at zero. The language "for a considerable time after the economic recovery strengthens" conveys Odyssean forward guidance. Without that language, it suggests the FOMC would raise its policy rate as the economy improves. On the other hand, the FOMC statement also included information about business spending that likely lowered real GDP growth forecasts, which suggests the change in forward guidance may have been viewed as Delphic.

On December 12, 2012, the FOMC switched its forward guidance from the date-based language "at least through mid-2015" to threshold-based language. The policy statement read:

. . . this exceptionally low range for the federal funds rate will be appropriate *at least as long as the unemployment rate remains above 6-1/2 percent*, inflation between one and two years ahead is projected to be no more than a half percentage point above the Committee's 2 percent longer-run goal, and longer-term inflation expectations continue to be well anchored.

FOMC participants' forecasts indicated the unemployment rate would likely hit 6.5% in mid-2015. Therefore, the statement was not intended to change expectations about when the policy rate would rise, but rather to emphasize that any change in the policy rate is conditional on inflation expectations and labor market conditions. The phrase "at least as long as" suggests the unemployment rate threshold was not a trigger for when the FOMC would automatically raise its policy rate.

Over the next year, the labor market continued to improve, and it was evident the unemployment rate might cross the 6.5% threshold. On December 18, 2013, the FOMC began tapering its monthly asset purchases and redrafted its forward guidance to explain how it intended to react to future economic conditions. The statement said ". . . it likely will be appropriate to maintain the current target range for the federal funds rate *well past the time that the unemployment rate declines below 6-1/2 percent*." The change in language from "at least as long as" to "well past" may have been viewed as Odyssean because it implied that the policy rate would remain near zero even though stronger economic conditions would normally cause the FOMC to raise its policy rate.

In 2014, the FOMC continued to reduce its asset purchases and communicate state-contingent forward guidance. For example, the March 19, 2014 statement said the Committee would likely target a low range for the federal funds rate for a "considerable time after the asset purchase program ends." In its January statement, the FOMC changed its forward guidance to simply say "it can be patient in beginning to normalize" rates. By June 17, 2015, future rate increases appeared imminent as 15 of the 17 committee members were forecasting a rate increase in 2015. The FOMC decided to increase the federal funds rate by 25 basis points on December 16, 2015, which was the first increase since June 2006. The high likelihood of remaining in a low interest rate environment

emphasizes the importance of analyzing forward guidance not only at the ZLB, but also at states near the ZLB, especially since the FOMC has repeatedly said “economic conditions may, for some time, warrant keeping the target federal funds rate below levels the Committee views as normal.”⁵

3 ECONOMIC MODEL

We use a theoretical model to analyze the stimulative effect of forward guidance. The model has three sectors: a representative household that maximizes its utility, firms that produce intermediate inputs that are bundled together into a final good, and a central bank that sets the short-term nominal interest rate. Forward guidance enters our model through news shocks to the monetary policy rule.

3.1 HOUSEHOLDS A representative household chooses $\{c_t, n_t, b_t\}_{t=0}^{\infty}$ to maximize expected lifetime utility, $E_0 \sum_{t=0}^{\infty} \tilde{\beta}_t [\log c_t - \chi n_t^{1+\eta}/(1+\eta)]$, where c is consumption, n is labor hours, b is the real value of a 1-period nominal bond, $1/\eta$ is the Frisch elasticity of labor supply, E_0 is an expectation operator conditional on information available in period 0, $\beta_0 \equiv 1$, and $\tilde{\beta}_t = \prod_{i=1}^{t>0} \beta_i$. Following Eggertsson and Woodford (2003), β is a time-varying discount factor that follows $\beta_t = \bar{\beta}(\beta_{t-1}/\bar{\beta})^{\rho_\beta} \exp(v_t)$, where $\bar{\beta}$ is the steady-state discount factor, $0 \leq \rho_\beta < 1$, and $v_t \sim \mathbb{N}(0, \sigma_v^2)$.

The household’s choices are constrained by $c_t + b_t = w_t n_t + i_{t-1} b_{t-1}/\pi_t + d_t$, where π_t is the gross inflation rate, w_t is the real wage rate, i_t is the gross nominal interest rate, and d_t are the dividends from intermediate firms. The optimality conditions to the household’s problem imply

$$w_t = \chi n_t^\eta c_t, \quad (1)$$

$$1 = i_t E_t [\beta_{t+1} (c_t/c_{t+1})/\pi_{t+1}]. \quad (2)$$

3.2 FIRMS The production sector consists of monopolistically competitive intermediate goods firms and a final goods firm. Intermediate firm $f \in [0, 1]$ produces a differentiated good, $y_t(f)$, according to $y_t(f) = n_t(f)$, where $n_t(f)$ is the labor used by firm f . Each intermediate firm chooses its labor input to minimize operating costs, $w_t n_t(f)$, subject to its production function. The final goods firm purchases $y_t(f)$ from each intermediate firm to produce the final good, $y_t \equiv [\int_0^1 y_t(f)^{(\theta-1)/\theta} df]^{\theta/(\theta-1)}$, according to a Dixit and Stiglitz (1977) aggregator, where $\theta > 1$ is the elasticity of substitution between the intermediate goods. The demand function for intermediate inputs is $y_t(f) = (p_t(f)/p_t)^{-\theta} y_t$, where $p_t = [\int_0^1 p_t(f)^{1-\theta} df]^{1/(1-\theta)}$ is the price of the final good.

Following Rotemberg (1982), each firm faces a price adjustment cost, $adj_t(f)$. Using the functional form in Ireland (1997), $adj_t(f) = \varphi [p_t(f)/(\bar{\pi} p_{t-1}(f)) - 1]^2 y_t/2$, where $\varphi \geq 0$ scales the size of the adjustment costs and $\bar{\pi}$ is the steady-state gross inflation rate. Real dividends are then given by $d_t(f) = (p_t(f)/p_t) y_t(f) - w_t n_t(f) - adj_t(f)$. Firm f chooses its price, $p_t(f)$, to maximize the expected discounted present value of real dividends, $E_0 \sum_{t=0}^{\infty} \tilde{\beta}_t (c_0/c_t) d_t(f)$. In a symmetric equilibrium, all firms make identical decisions and the optimality condition implies

$$\varphi \left(\frac{\pi_t}{\bar{\pi}} - 1 \right) \frac{\pi_t}{\bar{\pi}} = (1 - \theta) + \theta w_t + \varphi E_t \left[\beta_{t+1} \frac{c_t}{c_{t+1}} \left(\frac{\pi_{t+1}}{\bar{\pi}} - 1 \right) \frac{\pi_{t+1} y_{t+1}}{\bar{\pi} y_t} \right]. \quad (3)$$

Without price adjustment costs, the gross markup of price over marginal cost equals $\theta/(\theta - 1)$.

⁵Forward guidance has also been used by the Bank of Canada, Bank of England, European Central Bank, Bank of Japan, Reserve Bank of New Zealand, Norges Bank, and the Riksbank. See Andersson and Hofmann (2010), Filardo and Hofmann (2014), Kool and Thornton (2015), Moessner and Nelson (2008), Svensson (2011, 2015), and Swanson and Williams (2014) for an overview of the various policies and econometric analysis of their economic impacts.

3.3 CENTRAL BANK AND FORWARD GUIDANCE The policy rate is set according to

$$\begin{aligned} i_t &= \max\{\underline{i}, i_t^*\}, & i_t^* &= \bar{i}(\pi_t/\bar{\pi})^{\phi_\pi} (y_t/\bar{y})^{\phi_y} \exp(x_t), \\ x_t &\equiv \sum_{j=0}^q \alpha_j \varepsilon_{t-j}, & \sum_{j=0}^q \alpha_j &= 1, \end{aligned} \quad (4)$$

where \underline{i} is the lower bound on the nominal interest rate, i_t^* is the notional interest rate (i.e., the rate the central bank would set if it was unconstrained), $\bar{\pi}$ and \bar{i} are the steady-state inflation and nominal interest rates, ϕ_π and ϕ_y are the policy responses to the inflation and output gaps, $\varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ is a monetary policy shock, α_j is the weight on the shock to the nominal interest rate j periods ahead, and $q \geq 0$ is the forward guidance horizon. For example, when $(\alpha_0, \alpha_1, \dots, \alpha_q) = (1, 0, \dots, 0)$, the shock is unanticipated (no forward guidance) and when $(\alpha_0, \alpha_1, \dots, \alpha_q) = (0, 0, \dots, 1)$, the shock is anticipated in q periods (q -period forward guidance).

The constraint on the α 's holds the total weight on the news shocks constant across various forward guidance horizons, which is crucial for two main reasons. One, it allows us to isolate the effect of a longer horizon from additional news. Without the restriction, it would be impossible to identify the effect of an increase in q , because the forward guidance extension would also increase the total amount of news and stimulate the economy. Two, it places a restriction on the total amount that the central bank can affect expected future interest rates, otherwise the central bank would have a growing ability create innovations in expectations just like with an interest rate peg.

3.4 EQUILIBRIUM The resource constraint is $c_t = y_t - adj_t \equiv y_t^{gdp}$, where y_t^{gdp} includes the value added by intermediate firms, which is their output minus price adjustment costs. Thus, y_t^{gdp} represents real GDP in the model. A competitive equilibrium consists of sequences of quantities, $\{c_t, n_t, y_t, b_t\}_{t=0}^\infty$, prices, $\{w_t, i_t, \pi_t\}_{t=0}^\infty$, and discount factors, $\{\beta_t\}_{t=0}^\infty$, that satisfy the household's and firms' optimality conditions, (1)-(3), the monetary policy rule, (4), the production function, $y_t = n_t$, the bond market clearing condition, $b_t = 0$, the discount factor process, and the resource constraint, given the initial conditions, β_{-1} and $\{\varepsilon_{-j}\}_{j=1}^q$, and sequences of shocks, $\{\varepsilon_t, v_t\}_{t=0}^\infty$.

3.5 CALIBRATION We calibrate our model at a quarterly frequency to match moments in U.S. data from 1983Q1 to 2014Q4. The parameters are summarized in [table 1](#). The steady-state discount factor, $\bar{\beta}$, is set to 0.9957, which equals the average ratio of the quarter-over-quarter percentage change in the GDP implicit price deflator to the 3-month T-bill rate. The Frisch elasticity of labor supply, $1/\eta$, is set to 3, which is consistent with Peterman (2016). The leisure preference parameter, χ , is calibrated so that steady-state labor equals 1/3 of the available time. The elasticity of substitution between intermediate goods, θ , is calibrated to 6, which corresponds to a 20% average markup of price over marginal cost. The price adjustment cost parameter, φ , is set to 160, which matches the estimate in Ireland (2003). The lower bound on the nominal interest rate, \underline{i} , is calibrated to 1.00022, which equals the average 3-month T-bill rate from 2009Q1 to 2014Q4.

The steady-state inflation rate, $\bar{\pi}$, is calibrated to 1.0057 to match the average GDP deflator inflation rate. Using the estimates in Smets and Wouters (2007), we set the monetary response to changes in inflation, ϕ_π , equal to 2 and the response to fluctuations in output, ϕ_y , equal to 0.08.

The persistence of the discount factor, ρ_β , equals 0.87 and the standard deviation of the shock, σ_v , equals 0.00225, which are close to the estimates in Gust et al. (2013). The standard deviation of the monetary policy shock, σ_ε , is set to 0.003. We chose these parameters to match volatilities in the data and the length of time people *expected* the ZLB to bind, rather than the duration of

Steady-State Discount Factor	$\bar{\beta}$	0.9957	Nominal Interest Rate Lower Bound	\underline{r}	1.00022
Frisch Elasticity of Labor Supply	$1/\eta$	3	Monetary Policy Response to Inflation	ϕ_π	2
Elasticity of Substitution between Goods	θ	6	Monetary Policy Response to Output	ϕ_y	0.08
Rotemberg Adjustment Cost Coefficient	φ	160	Discount Factor Persistence	ρ_β	0.87
Steady-State Labor	\bar{n}	0.33	Discount Factor Standard Deviation	σ_v	0.00225
Steady-State Inflation Rate	$\bar{\pi}$	1.0057	Monetary Policy Shock Standard Deviation	σ_ε	0.003

Table 1: Calibrated parameters.

the current ZLB episode. In the data, the annualized standard deviations of quarter-over-quarter percent changes in real GDP, the GDP deflator inflation rate, and the 3-month T-bill rate are 2.58%, 0.99%, and 2.79%, respectively, per year. To compare our model to those values, we ran 10,000 simulations that are each 128 quarters long (i.e., the same length as our data). We then computed the median standard deviations of real GDP growth, the inflation rate, and the nominal interest rate. Those values and their 95% credible intervals are 2.45% (1.92%, 3.67%), 1.07% (0.74%, 1.63%), and 2.29% (1.83%, 2.90%), respectively, per year. The median standard deviations in the model are near their historical averages, and all three credible intervals contain the values in the data.

Prior to the FOMC’s August 2011 date-based forward guidance, survey data indicated the 3-month T-bill rate was not expected to remain near zero for very long. Blue Chip consensus forecasts from 2009 and 2010 reveal that the 3-month T-bill rate was expected to exceed 0.5% within three quarters. In our model, a ZLB event lasts an average of 2.12 quarters when the economy is initialized at its steady state but rises to 3.10 quarters when it is initialized at a notional interest rate that is consistent with estimates during and immediately after the Great Recession. Therefore, our calibration produces ZLB events with a similar average duration to what was expected prior to the FOMC’s forward guidance. It is also possible for the model to generate much longer ZLB events.

3.6 SOLUTION METHOD The model is solved using the policy function iteration algorithm described in Richter et al. (2014), which is based on the theoretical work on monotone operators in Coleman (1991). This method discretizes the state space and iteratively solves for updated policy functions until the tolerance criterion is met. We use linear interpolation to approximate future variables, since it accurately captures the kink in the policy functions, and Gauss-Hermite quadrature to numerically integrate. See [appendix C](#) for a formal description of the algorithm.⁶

4 ONE-QUARTER HORIZON RESULTS

This section first quantifies the stimulative effect of forward guidance over a 1-quarter horizon. We then show the importance of the ZLB constraint, the level of uncertainty, the monetary response to inflation, the state of the economy, the size of the news shock, and the speed of the recovery.

4.1 EFFECTS OF FORWARD GUIDANCE [Figure 1](#) plots the decision rules for real GDP, the inflation rate, and the current and expected future nominal interest rates as a function of the monetary policy shock, $\hat{\varepsilon}_t$.⁷ The time subscript is the period households learn about the shock and not necessarily the period the shock impacts the economy. If the central bank provides no forward guidance, then $\hat{\varepsilon}_t$ is an unanticipated monetary policy shock that impacts the economy in period t . When the

⁶Benhabib et al. (2001) show that models with a ZLB constraint have two steady-state equilibria. See Gavin et al. (2015) for a discussion of the equilibrium that our algorithm converges to in both a deterministic and stochastic model.

⁷In our results, a hat denotes a percent change and a tilde denotes a percentage point difference between net rates.

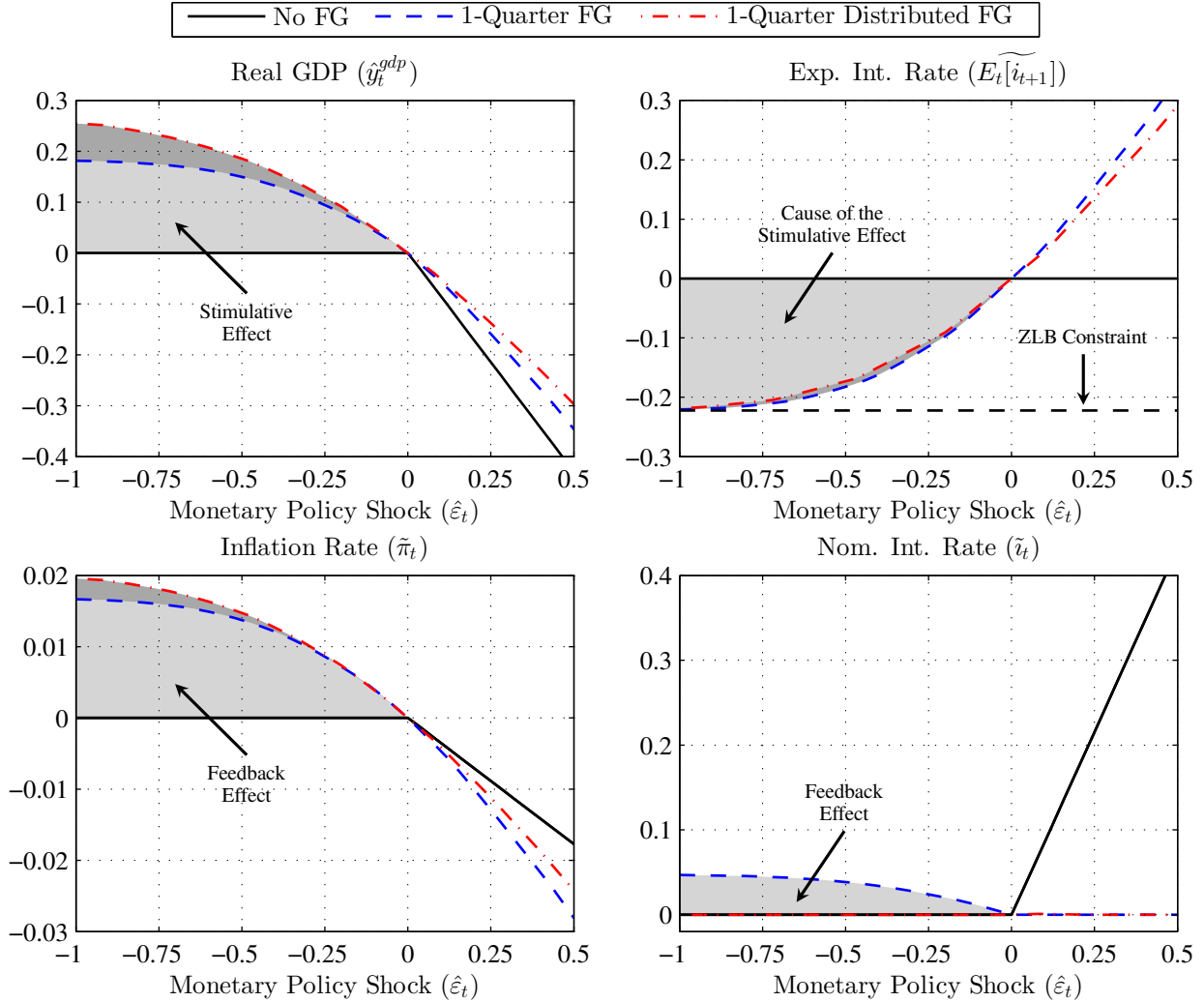


Figure 1: Decision rules as a function of the monetary policy shock with no forward guidance, $(\alpha_0, \alpha_1) = (1, 0)$ (solid line); 1-quarter forward guidance, $(\alpha_0, \alpha_1) = (0, 1)$ (dashed line); and 1-quarter distributed forward guidance, $(\alpha_0, \alpha_1) = (0.13, 0.87)$ (dash-dotted line). In this cross section, the initial notional interest rate equals zero.

central bank provides 1-quarter forward guidance, $\hat{\varepsilon}_t$ is a news shock that households learn about in period t but does not impact the economy until period $t + 1$. Thus, a news shock creates an innovation in the expected nominal interest rate, which can be directly mapped to changes in forecasts that occur after an FOMC statement is released. We quantify the effects of 1-quarter forward guidance by comparing the differences in forecasts before and after the policy announcement. The vertical axis displays the marginal effect of a monetary policy shock relative to when there is no shock. For example, a 1-quarter news shock of $\hat{\varepsilon}_t = -0.25$ lowers the expected nominal interest rate by over 0.1 percentage points and raises real GDP by about 0.1% relative to when $\hat{\varepsilon}_t = 0$.

We focus on a cross section of the decision rules where the initial notional interest rate equals zero because it produces the largest stimulative effect of forward guidance when the central bank is constrained by the ZLB. The notional rate equals zero when the discount factor is 0.61% above its steady state. The elevated discount factor signifies an increased desire by households to save, which lowers inflation and real GDP. Households, however, expect the discount factor to decline

over time. If no forward guidance is provided, that belief raises the expected nominal interest rate.

When $(\alpha_0, \alpha_1) = (1, 0)$ (solid line), the central bank provides no forward guidance, so $\hat{\varepsilon}_t$ represents an unanticipated policy shock. If $\hat{\varepsilon}_t > 0$, then the shock contracts economic activity by raising the current nominal interest rate and lowering inflation and real GDP. The expected nominal interest rate is unaffected since the shock is serially uncorrelated. If, however, $\hat{\varepsilon}_t < 0$, then monetary policy has no impact on the nominal interest rate since it is already at its ZLB. Thus, the decision rules remain at zero when $\hat{\varepsilon}_t < 0$ since conventional monetary policy is ineffective.

When $(\alpha_0, \alpha_1) = (0, 1)$ (dashed line), the central bank provides households with 1-quarter forward guidance. The light-shaded regions represent the marginal effects of that policy. The news in period t that an expansionary shock will occur in period $t + 1$ leads to a downward revision in the expected nominal interest rate. That expectational effect stimulates real GDP, which raises both the inflation and nominal interest rates—what we refer to as *feedback effects*—even though the discount factor remains at the minimum value necessary for the ZLB to bind. The maximum amount the expected nominal interest rate can decline is the difference between the expected rate in the absence of forward guidance and the ZLB, which is represented by a horizontal dashed line.

The feedback effect on the current nominal interest rate from 1-quarter forward guidance is counterfactual to recent FOMC forward guidance, and it would show up in expected nominal rates over longer horizons. In reality, the FOMC did not communicate an increase in either current or future nominal interest rates. In our model, the central bank can eliminate the feedback effect by redistributing the weights on the policy shock, while holding the total weight fixed. An example of that policy is $(\alpha_0, \alpha_1) = (0.13, 0.87)$ (dash-dotted line), which we refer to as 1-quarter distributed forward guidance. In that case, just enough of the weight is taken from the 1-quarter ahead news shock, α_1 , and placed on the unanticipated shock, α_0 , so the current nominal rate remains at zero. Note, however, that the feedback effect on the policy rate would be much smaller if we initialized the economy at a negative notional rate, and it would be nonexistent given a deep enough recession.

Expansionary news shocks under both types of 1-quarter forward guidance have diminishing positive impacts on real GDP as the shock size increases. For example, a -0.5% news shock under 1-quarter forward guidance increases real GDP by 0.15 percentage points, whereas a -1% news shock raises real GDP by 0.18 percentage points. Thus, doubling the size of the news shock only leads to a small additional increase in real GDP. The small marginal effect occurs because a larger expansionary policy shock increases the likelihood that next period's nominal interest rate will fall to its ZLB, which is evident from the decision rule for the expected nominal interest rate.

Another way to examine forward guidance is with generalized impulse response functions (GIRFs) following Koop et al. (1996). GIRFs are based on simulations that are consistent with households' expectations. The benefit of GIRFs is they show the dynamic effects of a shock, whereas decision rules show the impact effects for a range of shocks. [Figure 2](#) plots the responses to a -0.5% monetary policy shock at the ZLB with no forward guidance (solid line), 1-quarter forward guidance (dashed line), and 1-quarter distributed forward guidance (dash-dotted line). To compute the GIRFs, we calculate the mean of 10,000 simulations conditional on random shocks. We then calculate a second mean from a new set of 10,000 simulations, but this time the random policy shock in the first quarter of each simulation is replaced with a -0.5% shock. The GIRFs are the percentage change (or difference in rates) between the two means. Each simulation is initialized at a notional rate equal to zero. See [appendix D](#) for details on how the GIRFs are calculated.

In each simulation, households learn about the monetary policy shock in period 1. With no forward guidance, the shock is unanticipated and occurs in period 1. With 1-quarter forward guidance,

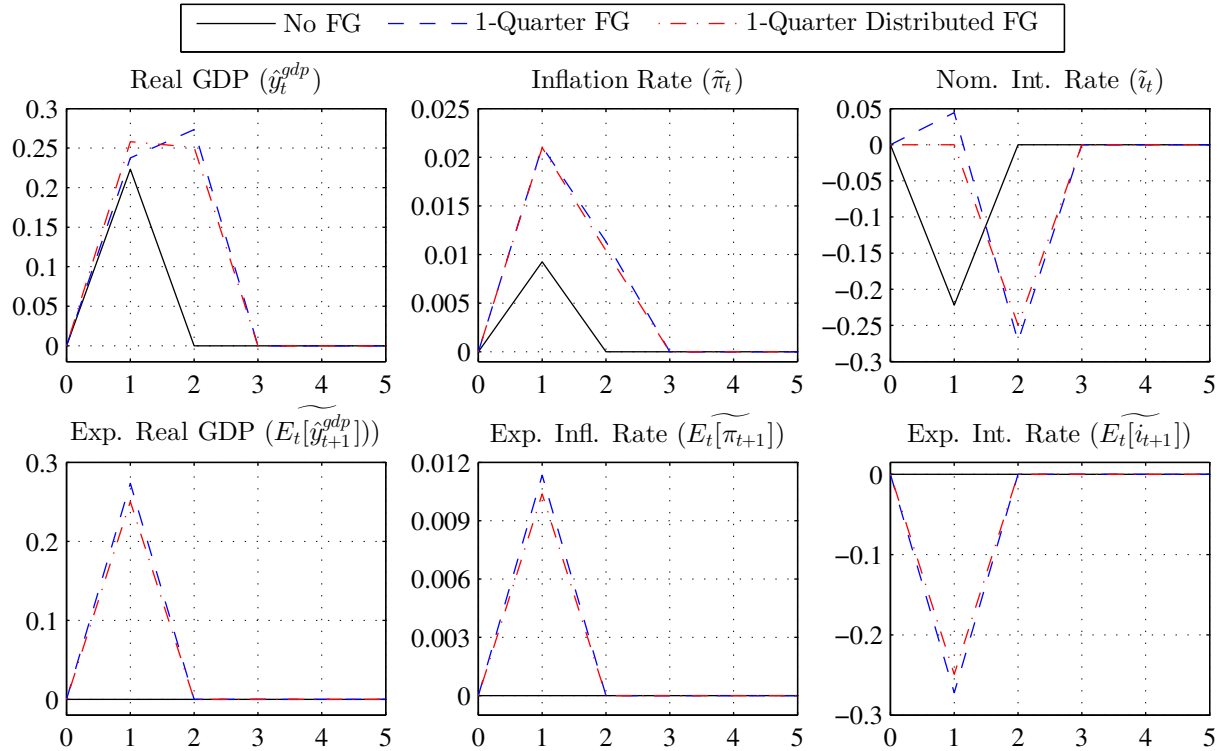


Figure 2: Generalized impulse responses to a -0.5% monetary policy shock with no forward guidance, $(\alpha_0, \alpha_1) = (1, 0)$ (solid line); 1-quarter forward guidance, $(\alpha_0, \alpha_1) = (0, 1)$ (dashed line); and 1-quarter distributed forward guidance, $(\alpha_0, \alpha_1) = (0.13, 0.87)$ (dash-dotted line). Each simulation is initialized at a notional rate equal to zero.

households receive news in period 1 about a policy shock that will hit in period 2. The combination of a zero notional interest rate in period 0 and a mean reverting discount factor causes the period 1 nominal interest rate to rise above its ZLB in 59% of the simulations without a monetary policy shock. Therefore, an unanticipated expansionary policy shock $[(\alpha_0, \alpha_1) = (1, 0)]$ in period 1 reduces the nominal rate in most simulations, so the shock on average is stimulative.

A -0.5% 1-quarter forward guidance shock $[(\alpha_0, \alpha_1) = (0, 1)]$, dashed line] lowers the expected nominal interest rate and raises expected real GDP and expected inflation in period 2. That change boosts real GDP in period 1. Therefore, 1-quarter forward guidance stimulates the economy over the entire forward guidance horizon. The feedback effect increases the nominal interest rate by 0.04% in period 1. Our specification of 1-quarter distributed forward guidance $[(\alpha_0, \alpha_1) = (0.13, 0.87)]$, dash-dotted line] shifts just enough weight to the unanticipated shock to completely offset the feedback effect from the period 1 news shock, so the shock has no effect on the nominal interest rate in period 1. As a result, real GDP rises 0.02 percentage points more on impact with distributed forward guidance, while the response in period 2 is only slightly smaller.

4.2 IMPORTANCE OF THE ZLB CONSTRAINT The previous section shows forward guidance becomes progressively less stimulative as the expected nominal interest rate approaches zero. Essentially, the ZLB constraint truncates the distribution for the future nominal interest rate at zero, which limits the central bank's ability to lower its expected value. Figure 3 compares the effects of 1-quarter forward guidance with (light-shaded area) and without (dark-shaded area) a ZLB constraint under the assumption that the initial notional interest rate equals zero. That assumption

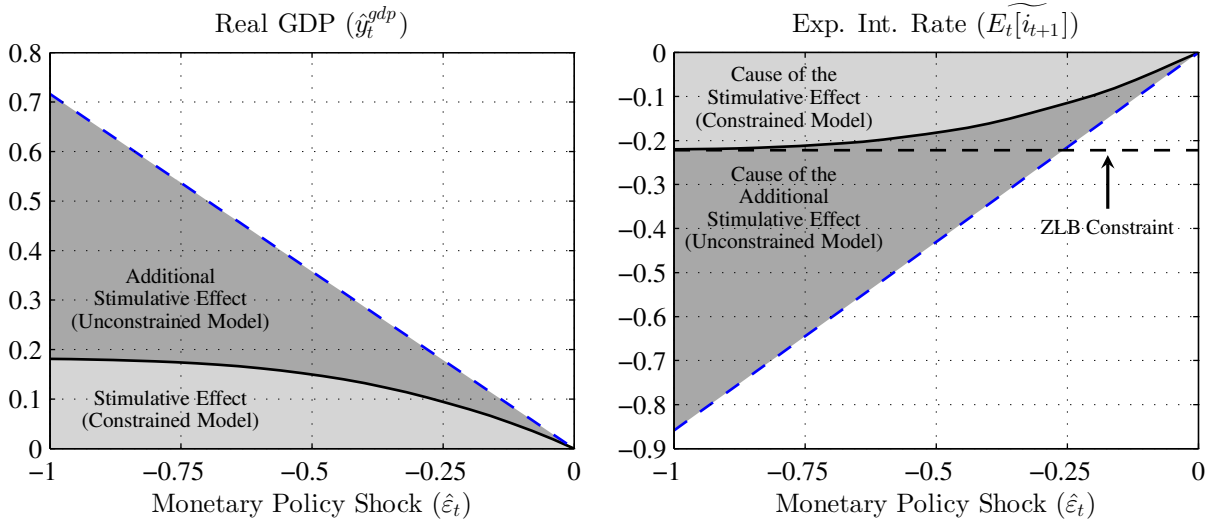


Figure 3: Comparison of decision rules with (solid line) and without (dashed line) a ZLB constraint given 1-quarter forward guidance, $(\alpha_0, \alpha_1) = (0, 1)$. In this cross section of the decision rules, the initial notional rate equals zero.

enables us to analyze the effects of the ZLB constraint when the expected nominal rate is near zero. We show the effects of 1-quarter forward guidance rather than distributed forward guidance, so the stimulative effect is only due to changes in the expected nominal interest rate. As in figure 1, the vertical axis measures the marginal effect of the news shock relative to when there is no shock.

Figure 3 reveals the stimulative effect of forward guidance is overstated when the model does not contain a ZLB constraint and the expected nominal interest rate is near or below zero. For example, a -0.5% (-1%) news shock in the constrained model reduces the expected nominal interest rate by 18 (22) basis points and increases real GDP by 0.15 (0.18) percentage points. The same shock in the unconstrained model pushes down the expected nominal rate by 43 (86) basis points and raises real GDP by 0.36 (0.72) percentage points. In that example, the expected nominal rate is below its ZLB, but an overstatement of real GDP also transpires when the expected rate is positive but near zero because part of the distribution for the future nominal rate is negative. The same overstatement would occur if the ZLB constraint is imposed when simulating the model but not when solving it. Since the constraint only affects the current nominal rate when simulating the model and the stimulative effect is entirely driven by the change in the expected nominal rate, it is essential to include the constraint when solving the model to constrain all expected future rates.

4.3 STATE OF THE ECONOMY This section shows how a weak economy can render forward guidance less effective by examining different initial states of the economy. Figure 4 plots histograms of the simulated values of next quarter's nominal interest rate without forward guidance. The dashed lines represent the expected nominal interest rates. The simulations are initialized at two alternative notional interest rates: $\tilde{v}_t^* = 0$ (left panel) and $\tilde{v}_t^* = -0.5$ (right panel). These histograms reveal the distribution for the future nominal interest rate becomes more skewed toward zero as the initial notional rate becomes more negative. For example, 37% (69%) of simulations for \tilde{v}_{t+1} are constrained by the ZLB when $\tilde{v}_t^* = 0$ ($\tilde{v}_t^* = -0.5$), which causes the expected nominal rate to equal 0.23% (0.10%). That is, a weaker economy skews a larger fraction of the future nominal interest rate distribution towards the ZLB, which dampens the expected nominal rate. The lower

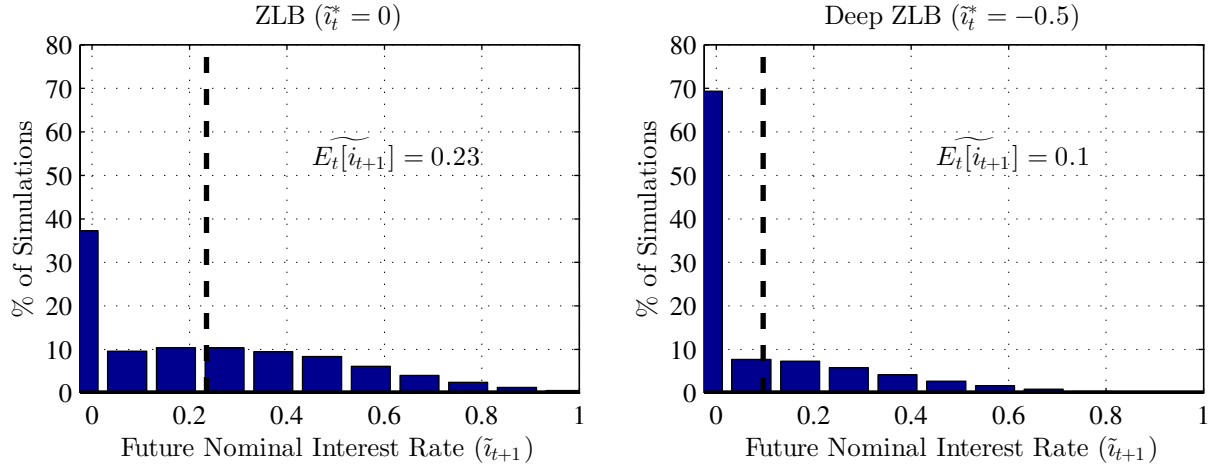


Figure 4: Histograms of the simulated values of next quarter’s nominal interest rate without forward guidance. The simulations are initialized at two alternative notional interest rates: $\tilde{i}_t^* = 0$ (left panel) and $\tilde{i}_t^* = -0.5$ (right panel).

expected value means forward guidance has a smaller margin to stimulate demand. Since estimates of the notional rate were well below zero during and immediately after the Great Recession, those results provide one reason why recent forward guidance likely had a limited economic effect.⁸

GIRFs are a practical tool to show how the stimulative effect of forward guidance is influenced by the state of the economy. Figure 5 displays generalized impulse responses to two different types of -0.5% monetary policy shocks: an unanticipated shock (left panels) and a 1-quarter distributed forward guidance shock (right panels). The effect of each shock is examined given four alternative initial notional interest rates: (1) $\tilde{i}_0^* = 1$ (solid line) represents an economy at its steady state; (2) $\tilde{i}_0^* = 0.25$ (dashed line) is a low policy rate that is consistent with the FOMC’s June 2015 forecast for 2016; (3) $\tilde{i}_0^* = 0$ (circle markers) denotes an economy that is just weak enough, so the ZLB binds (i.e., the same value used in earlier figures); and (4) $\tilde{i}_0^* = -0.5$ (triangle markers) represents an economy in a severe recession where the policy rate is constrained by the ZLB, which is based on its estimated value during the Great Recession. In each case, the weights on the 1-quarter distributed forward guidance shock (i.e., α_0 and α_1) are set so that monetary policy does not affect the nominal interest rate in period 1 (i.e., the feedback effect is eliminated). A policy that does not generate feedback effects on the nominal rate is consistent with recent FOMC forward guidance.

There are two key takeaways from our simulations. One, monetary policy shocks are less stimulative when the initial notional interest rate is near or below zero. In steady state ($\tilde{i}_0^* = 1$), a -0.5% shock (unanticipated or anticipated) generates the largest decline in the nominal interest rate and has the greatest stimulative effect on real GDP because the policy rate rarely falls by enough to hit its ZLB. As the initial notional interest rate declines, the effects of the policy shock become more limited. For example, a -0.5% shock has a smaller effect on real GDP when $\tilde{i}_0^* = 0.25$ because the current and expected nominal interest rates are closer to zero and, as a result, have less room to fall after the shock. Those effects are further reduced when \tilde{i}_0^* equals 0% and -0.5% .

Two, an unanticipated shock is more stimulative on impact than a news shock when the economy is at steady state, while a news shock becomes relatively more stimulative as the policy rate approaches its ZLB. At steady state ($\tilde{i}_0^* = 1$), a -0.5% unanticipated shock initially increases real

⁸See Bauer and Rudebusch (2014), Krippner (2013), and Wu and Xia (2014) for estimates of the notional rate.

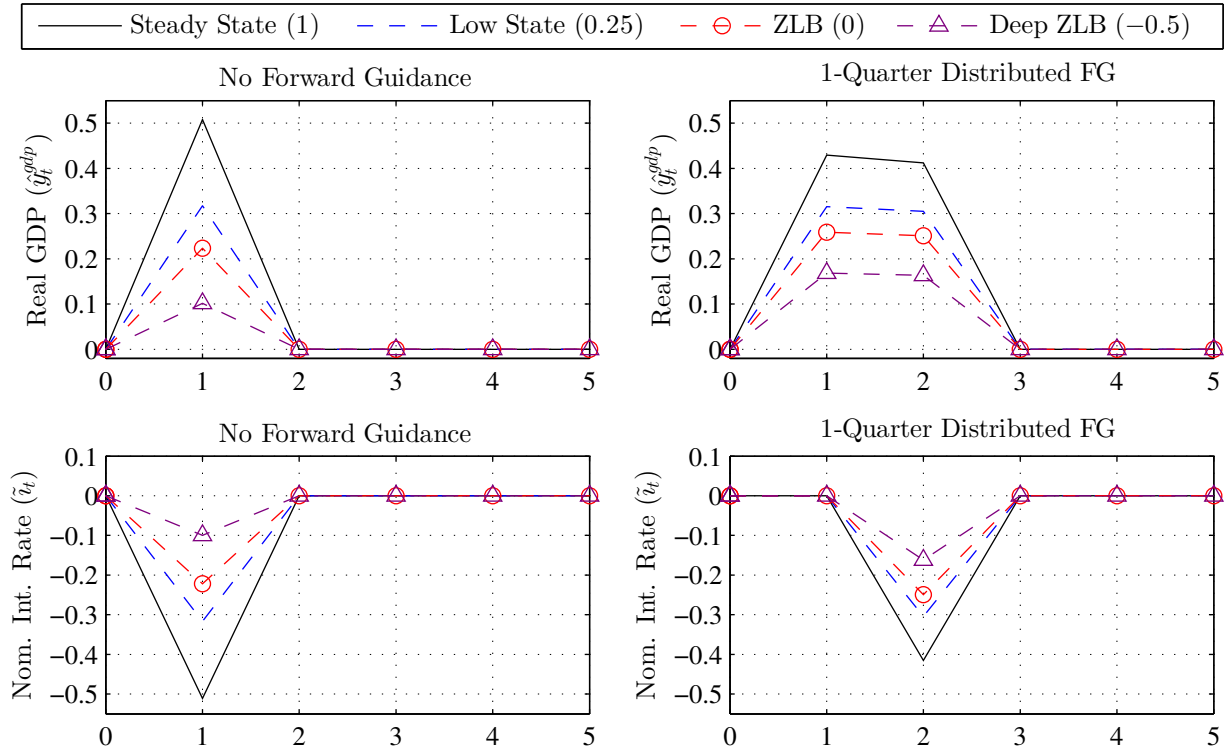


Figure 5: Generalized impulse responses to a -0.5% monetary policy shock. Two types of monetary policy are examined: No forward guidance, $(\alpha_0, \alpha_1) = (1, 0)$, (left panels) and 1-quarter distributed forward guidance (right panels). Each line represents a simulation initialized at a specific notional interest rate. In each case, the weights on the 1-quarter distributed forward guidance shock are set to eliminate any feedback effects on the nominal interest rate.

GDP by 0.51% , whereas a 1-quarter distributed forward guidance shock pushes up real GDP by 0.43% . That same shock raises real GDP by only 0.10% in a severe recession ($\tilde{i}_0^* = -0.5$), while the distributed shock increases real GDP by 0.17% . The relative effectiveness of unanticipated shocks versus news shocks depends on how far the current and expected nominal interest rates are from the ZLB. When $\tilde{i}_0^* = 1$, the initial notional rate is high enough that the ZLB binds only 1% of the time. The low probability enables the entire unanticipated shock to stimulate the economy most of the time. That result changes when $\tilde{i}_0^* = -0.5$. At that state, the ZLB binds 67% of the time, so unanticipated shocks hardly have any effect. The stimulative effect of the distributed shock also declines as the policy rate approaches zero. Its economic effects, however, depend on how close the expected nominal rate, as opposed to the current nominal rate, is to the ZLB. Therefore, if the economy is expected to improve, then the expected nominal rate will be higher than the current rate, which gives news shocks a larger margin to stimulate the economy than unanticipated shocks.

A key policy implication of these results is that forward guidance is more beneficial when used proactively. That is, forward guidance is more stimulative early in an economic downturn when the policy rate is still above its ZLB. In early 2008, however, the Fed only started to use forward guidance after the policy rate fell to its ZLB. Our theory suggests that the Fed's sluggish response resulted in its forward guidance announcements having a more limited effect on real GDP.

The recent experience of the Bank of Canada provides further support for using proactive forward guidance. The Bank of Canada was the first to adopt date-based forward guidance when it promised in April 2009 to keep its policy rate at 0.25% until mid 2010. Data indicate the news

lowered future interest rates and likely helped the Canadian economy recover faster than the U.S. economy. For example, the unemployment rate declined quicker and real GDP growth was higher in Canada from 2009-2012, even though both countries were equally impacted by the recession.

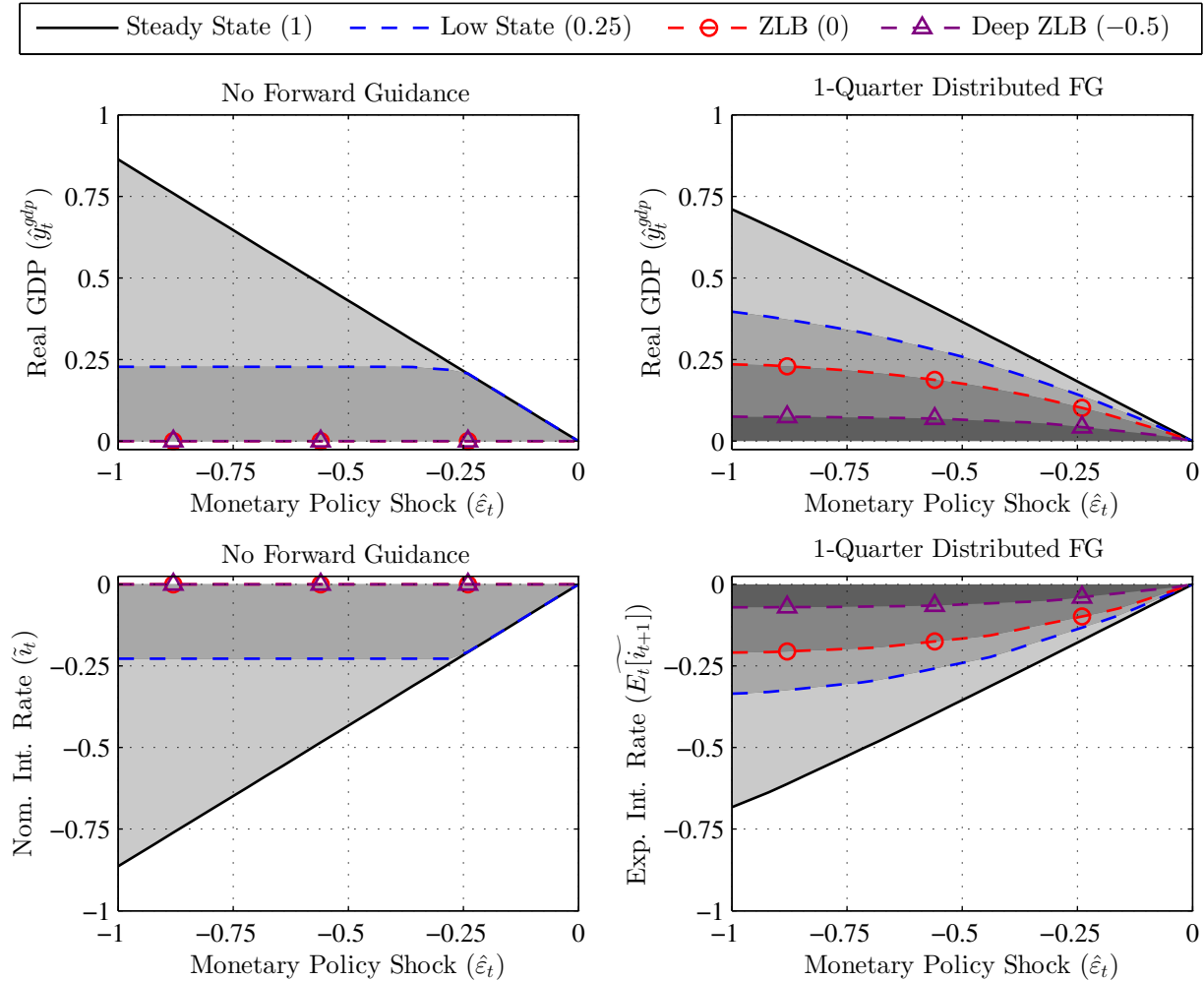


Figure 6: Decision rules with no forward guidance, $(\alpha_0, \alpha_1) = (1, 0)$, (left panels) and distributed forward guidance (right panels). Each line represents a cross section of the decision rules. In each cross section, the weights on the distributed forward guidance shock (α_0 and α_1) are set to eliminate any feedback effects on the nominal interest rate.

4.4 SIZE OF THE SHOCK The size of the monetary policy shock is another factor that determines whether an unanticipated or distributed news shock is more stimulative on impact. Figure 6 plots the decision rules as a function of the entire distribution of policy shocks with no forward guidance (left panels) and 1-quarter distributed forward guidance (right panels) for the same four initial notional interest rates examined in figure 5. In each cross section, the distributed forward guidance weights (α_0 and α_1) are set so the news shock has no feedback effects on the nominal interest rate.

A comparison of the right and left panels of figure 6 enables us to determine whether an unanticipated shock or news shock is more stimulative in each state without having the analysis distorted by the feedback effect on the current nominal interest rate. When the economy is at steady state ($i_t^* = 1$), an unanticipated shock (solid line, left panel) always raises real GDP more on impact than

a 1-quarter distributed forward guidance shock (solid line, right panel). The economic effects of an unanticipated shock, however, are more limited when the initial notional interest rate is low enough that the shock causes the ZLB to bind. If the economy is expected to improve, situations exist in which a promise to lower future nominal interest rates generates a larger increase in real GDP than an equivalent shock to the current nominal rate, which cannot fall below the ZLB. Consider the case where $\tilde{i}_t^* = 0.25$. A small unanticipated shock, $\hat{\varepsilon}_t > -0.26$, does not drive the nominal interest rate to its ZLB, so the jump in real GDP is higher than with a 1-quarter distributed shock. A moderate-sized unanticipated shock, $-0.42 < \hat{\varepsilon}_t < -0.26$, reduces the nominal interest rate to zero, but the initial stimulative effect is still stronger than the effect of distributed forward guidance. A large unanticipated shock, $\hat{\varepsilon}_t < -0.42$, causes an increasingly smaller rise in real GDP than the same distributed news shock. The upshot is that any forward guidance communicated when the policy rate is close to zero can generate a larger boost in real GDP than conventional open market operations as long as the news produces a meaningful revision in expected future interest rates.

When a recession is severe enough to cause the ZLB to bind ($\tilde{i}_t^* = 0$), distributed forward guidance is always more stimulative because an unanticipated shock cannot reduce the nominal interest rate. In a deeper recession ($\tilde{i}_t^* = -0.5$), the probability of exiting the ZLB next period becomes smaller, which reduces the expected nominal rate and limits the stimulative effect of forward guidance. In fact, it is possible that forward guidance will not have any stimulative effect if the initial notional rate is sufficiently low. These results reinforce our finding from [figure 5](#) that the stimulative effect of forward guidance is much more limited in a severely depressed economy, which provides further support for communicating forward guidance early in an economic downturn.

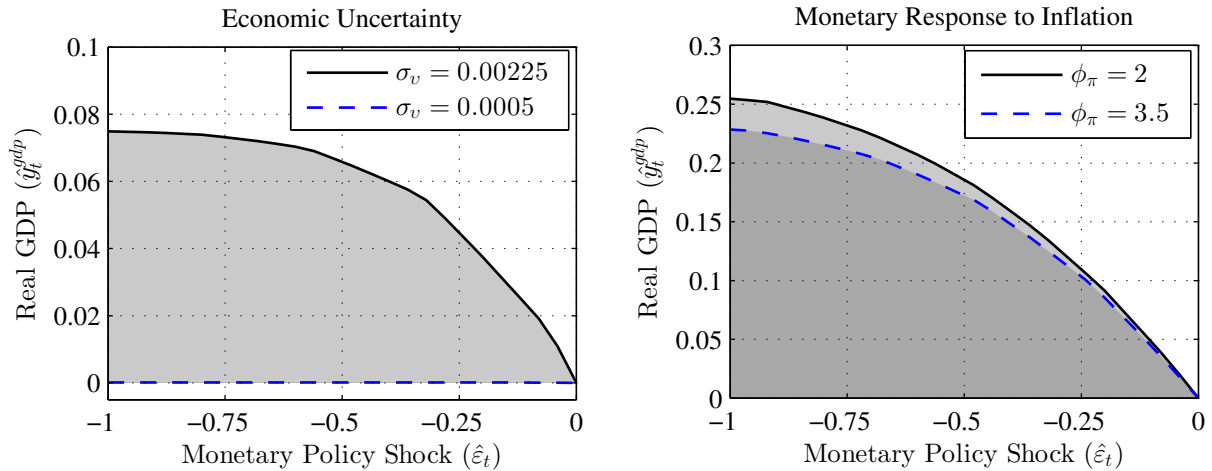
Uncertainty	Initial Notional Interest Rate			
	Steady State (1)	Low State (0.25)	ZLB (0)	Deep ZLB (-0.5)
High ($\sigma_v = 0.00225$)	0.43	0.32	0.26	0.17
Low ($\sigma_v = 0.0005$)	0.44	0.41	0.26	0.09

Table 2: Impact effect on real GDP in response to a -0.5% 1-quarter distributed forward guidance shock.

4.5 MONETARY POLICY AND ECONOMIC UNCERTAINTY The degree of economic uncertainty and the expected stance of monetary policy when the ZLB does not bind also influence the effectiveness of forward guidance. [Table 2](#) shows the impact effect on real GDP from a -0.5% 1-quarter distributed forward guidance shock under high and low levels of uncertainty about the future path of the discount factor. The high calibration represents the degree of uncertainty in our baseline model, while the low calibration approximates the behavior of our model under perfect foresight. The consequences of economic uncertainty are state dependent. When the economy is in a deep recession ($\tilde{i}_0^* = -0.5$), higher uncertainty increases the stimulative effect of forward guidance, whereas the stimulative effect is smaller when the economy is in a low state ($\tilde{i}_0^* = 0.25$). In steady state ($\tilde{i}_0^* = 1$) and when the interest rate is right at the ZLB ($\tilde{i}_0^* = 0$), uncertainty has little effect.

To further illustrate how economic uncertainty affects forward guidance, [figure 7a](#) plots the 1-quarter distributed forward guidance decision rules when the economy is in a deep recession. In this state, lower uncertainty about the discount factor makes households more confident that the nominal interest rate will remain at or near the ZLB. That is, positive discount factor shocks are less likely to warrant and increase increase in the policy rate. Therefore, the central bank has a

smaller margin to reduce the expected interest rate, which limits the stimulative effect of forward guidance. Right at the ZLB, the degree of economic uncertainty has no effect on the probability of leaving the ZLB. When economic conditions warrant a low policy rate, less uncertainty causes the future nominal interest rate distribution to be less constrained, which generates a larger margin for the news to stimulate the economy. In steady state, the short-term probability of hitting the ZLB is low, so the degree of uncertainty has little influence on the effectiveness of forward guidance.



(a) Decision rules with high uncertainty, $\sigma_v = 0.00225$, (solid line) and low uncertainty, $\sigma_v = 0.0005$, (dashed line). In this cross section of the decision rules, the initial notional rate equals -0.5% .

(b) Decision rules with a low inflation response, $\phi_\pi = 2$, (solid line) and a high inflation response, $\phi_\pi = 3.5$, (dashed line). In this cross section of the decision rules, the initial notional rate equals zero.

Figure 7: Effect of economic uncertainty (panel a) and the expected monetary response to inflation (panel b).

When the Fed lowered its policy rate to zero in December 2008, there was a high degree of uncertainty about future economic conditions that persisted for several years. Our results suggest the Fed could have taken advantage of the high uncertainty by communicating its intention to keep the federal funds rate low for several years. For example, the FOMC did not use specific language about its future policy rate path until it began using date-based forward guidance in August 2011. By that time, however, forecasters had become much more pessimistic about future economic conditions. Thus, the date-based language likely would have been much more effective at boosting real GDP if it had been used in 2008 or 2009 when economic forecasts were more uncertain.

Figure 7b shows a larger inflation coefficient in the monetary policy rule reduces the stimulative effect of forward guidance when the ZLB binds. In that state, inflation is well below its target rate. A larger ϕ_π implies that inflation must rise more and be closer to its target for the policy rule to call for an increase in the interest rate above the ZLB. As a consequence, households expect lower future nominal interest rates, which reduces the margin for forward guidance to stimulate the economy. When communicating forward guidance, central banks may be tempted to affirm their commitment to fighting inflation to contain the inflationary pressures generated by that policy. Our results, however, suggest that such a statement would reduce the effectiveness of forward guidance.

4.6 SPEED OF THE RECOVERY Another important determinant of the stimulative effect of forward guidance is how quickly households expect the economy to recover from a recession where the ZLB binds. Unfortunately, the continuous process for the discount factor makes it impossible

to change the probability of leaving the ZLB (i.e., the speed of the recovery) without simultaneously changing the probability of going to the ZLB (i.e., the likelihood of a recession). To avoid that problem, we assume the discount factor follows a 2-state Markov chain with transition matrix $\Pr\{s_t = j | s_{t-1} = i\} = p_{ij}$ for $i, j \in \{1, 2\}$. The discount factor is at its steady state in state 1, whereas the discount factor is high enough for the ZLB to bind in state 2. We set p_{12} equal to 1% and then conduct sensitivity analysis on p_{21} , which determines the expected speed of the recovery.

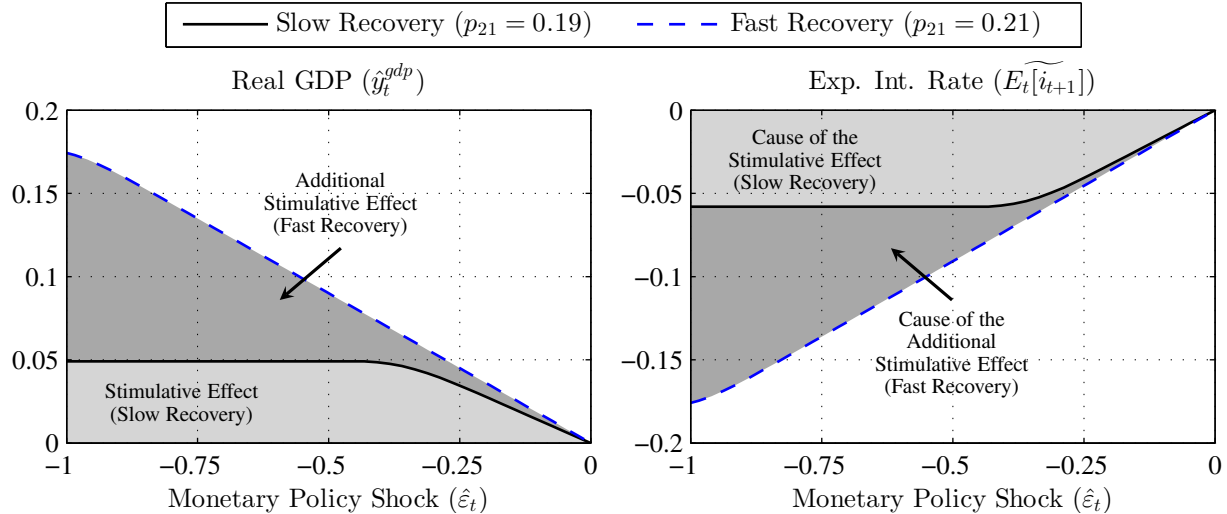


Figure 8: The stimulative effect of 1-quarter forward guidance, $(\alpha_0, \alpha_1) = (0, 1)$, given a slow recovery (solid line) and a fast recovery (dashed line). In this cross section of the decision rules, the initial notional interest rate equals zero.

Figure 8 shows decision rules with 1-quarter forward guidance as a function of the monetary policy shock, given a slow recovery ($p_{21} = 0.19$, solid line) and a fast recovery ($p_{21} = 0.21$, dashed line). The light-shaded region represents the stimulative effect of forward guidance when the economy recovers slowly and the dark-shaded region is the marginal effect of a faster recovery. As in figure 1, the initial notional interest rate equals zero in this cross section of the decision rules.

The stimulative effect of forward guidance is dampened when households expect a slower economic recovery. A less rapid return to steady state reduces demand and lowers the expected nominal interest rate. The smaller jump in the expected nominal rate implies that a promise to maintain a low policy rate in the future will have a weaker effect on real GDP because there is a smaller margin for policy to push down the expected nominal rate in order to stimulate real GDP.⁹

The decision rules under the slow recovery exhibit a kink due to the lower expected nominal interest rate. For small news shocks, $\hat{\epsilon}_t > -0.25\%$, the expected nominal rate decreases linearly since expectations are a convex combination of the future interest rates across the two states. For large news shocks, $\hat{\epsilon}_t < -0.50\%$, the expected nominal rate is at the ZLB in both states, so its decision rule is flat. With a fast recovery, however, large news shocks do not push the expected nominal rate to its ZLB, so they generate a larger increase in real GDP that grows with the size of the news shock. For example, news this period that the policy rate will be cut by 0.5% (1%) next period causes real GDP to rise by 0.05% (0.05%) when $p_{21} = 0.19$ and by 0.09% (0.17%)

⁹Levin et al. (2010) also show a slower expected recovery hinders forward guidance. They assume a real rate shock hits the economy, decays at a constant rate for four periods, and then switches to a slower rate of decay. Eggertsson and Mehrotra (2014) argue that forward guidance is less effective when the economy is in a near-permanent slump.

when $p_{21} = 0.21$. Those results demonstrate that forward guidance has a more limited stimulative effect if the policy causes households to revise their expectations about the future economy or it is communicated at the same time they learn about a weaker economic outlook from other sources.

4.7 SUMMARY Our findings demonstrate that in order to accurately assess the stimulative effect of forward guidance, it is essential to account for the state of the economy, the degree of economic uncertainty, the stance of monetary policy when the ZLB does not bind, and the speed of the recovery while respecting the ZLB constraint on current and future policy rates. Each of these factors is even more important when analyzing forward guidance horizons beyond one quarter.

5 LONGER HORIZON RESULTS

This section first compares the stimulative effects of forward guidance over horizons up to 10 quarters. We then show how a simultaneous demand shock can obscure the impact of forward guidance. It concludes by showing how an interest rate peg overstates the stimulative effect of forward guidance and argues that news shocks better reflect actual policy and its observed effects.

5.1 METHODOLOGY Our results in [section 4](#) use Gauss-Hermite quadrature to evaluate expectations. That approach allows us to obtain an accurate approximation of the decision rules and to quantify the stimulative effect of forward guidance for many different monetary policy shocks, which is important because the responses of key economic variables are nonlinear functions of the shock size. Using that technique, [appendix A](#) presents the economic effects of 2-quarter forward guidance across all policy shocks. That solution method, however, is numerically infeasible with longer forward guidance horizons because the state space grows exponentially with the horizon.

We reduce the dimensionality of the state space when analyzing horizons beyond 2 quarters by discretizing the news process using the method in Tauchen (1986). Specifically, we assign three values for each monetary policy shock, $(-0.6, 0, 0.6)$, and then calculate the probabilities of the transitional events. Tauchen's (1986) method is particularly useful for examining longer forward guidance horizons because it enables us to analyze the effects of specific shocks to the news process without having to solve the model for several other possible realizations of the shocks. See [appendix E](#) for more details on how this solution procedure differs from the previous method.

5.2 FORWARD GUIDANCE HORIZON [Figure 9](#) shows the generalized impulse responses to a -0.6% monetary policy shock distributed over 1-, 4-, 8-, and 10-quarter forward guidance horizons. For each horizon, we set the weights on the shocks so there are no feedback effects on the policy rate. Removing those effects better reflects actual policy and allows us to obtain an accurate comparison across the various horizons. In the top, middle, and bottom panels, the simulations are initialized at steady state ($\tilde{i}_0^* = 1$), the ZLB ($\tilde{i}_0^* = 0$), and a severe recession ($\tilde{i}_0^* = -0.5$). In each case, households learn about the shock in period 1, which is either unanticipated or anticipated.

When the economy is initialized at steady state ($\tilde{i}_0^* = 1$, top row), the unanticipated monetary policy shock raises real GDP more on impact than the distributed news shock, regardless of the forward guidance horizon. Unlike the effects of an unanticipated shock, which disappear after period 1, the impact of a q -quarter distributed forward guidance shock persists for q more quarters. To prevent the policy rate from changing over the horizon, any future deviation from the Taylor rule at the end of the horizon necessitates a deviation from the rule over the whole horizon. Therefore, distributed forward guidance shifts some of the weight on the policy shock from period $q + 1$ to

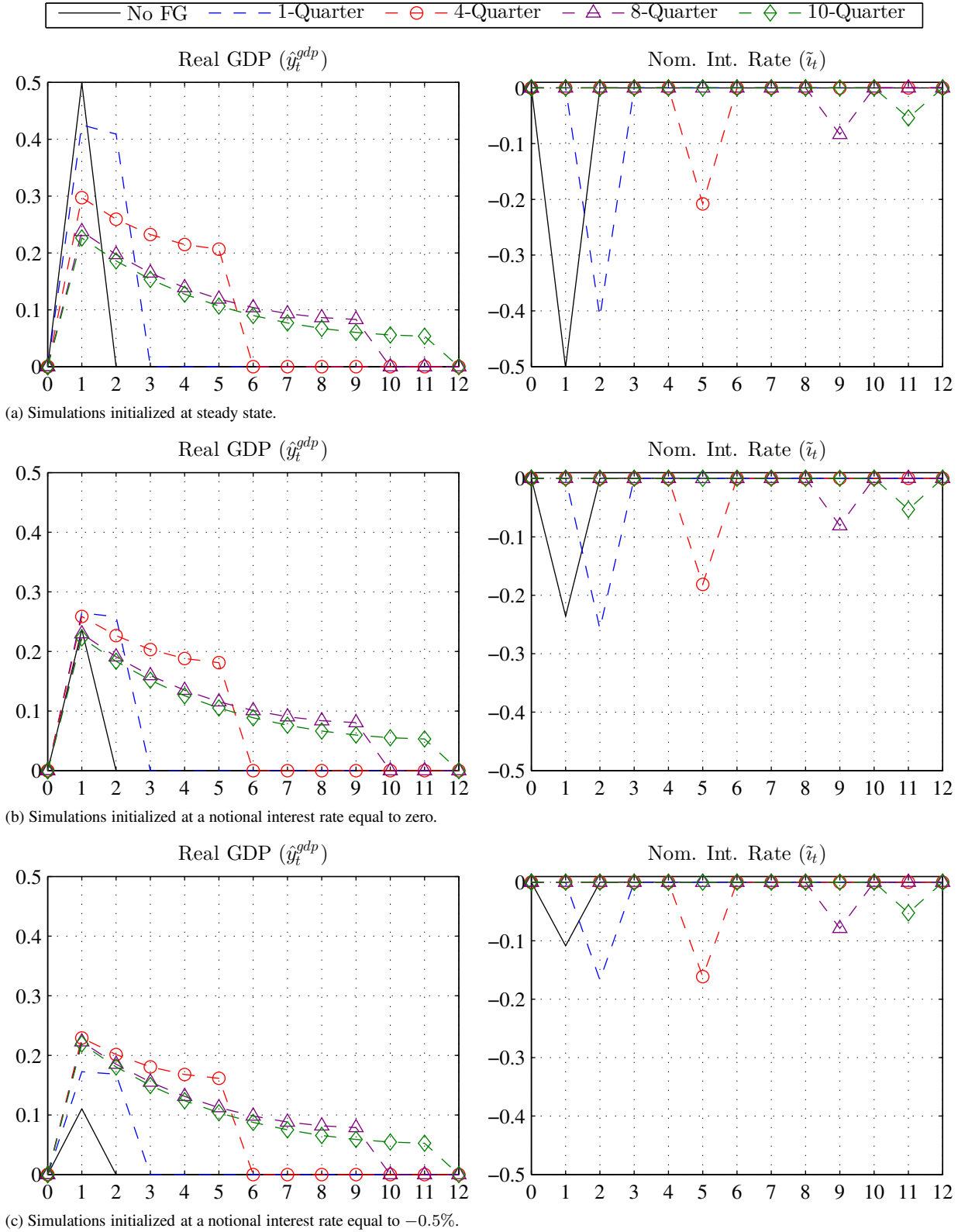


Figure 9: Generalized impulse responses to a -0.6% monetary policy shock with no forward guidance, $(\alpha_0, \alpha_1) = (1, 0)$, and distributed forward guidance at various states of the economy. In each simulation, the weights on the distributed forward guidance shock $(\alpha_j, j = 0, 1, \dots, q)$ are set to eliminate any feedback effects on the policy rate.

periods 1 to q to eliminate the feedback effect on the nominal interest rate. The result is that the size of the shock in period $q + 1$ becomes smaller as q increases (i.e., α_q declines as q rises). The smaller shock dampens the initial rise in real GDP, but the increase persists over the entire forward guidance horizon. Beyond period $q + 1$, the news shocks do not have any effect on the economy.¹⁰

When the economy begins in a recession that is just severe enough for the ZLB to bind ($\tilde{i}_0^* = 0$, middle row), the initial rise in real GDP is similar across all forward guidance horizons. The boost in real GDP, however, is smaller in every period over the forward guidance horizon than occurs when the economy is initialized at steady state. The reduced stimulative effect is due to the smaller margin that the central bank has to lower expected nominal interest rates over the next few periods. In an economic downturn similar to the Great Recession ($\tilde{i}_0^* = -0.5$, bottom row), the stimulative effect of forward guidance is even more limited, especially over short horizons. At longer horizons, the response of real GDP in every quarter is mostly unaffected by the initial state of the economy.

There are two key takeaways from these simulations. One, longer horizons do not generate increasingly larger impact effects on real GDP when the total amount of news is fixed, unlike with an interest rate peg. Two, forward guidance can have a larger cumulative effect on real GDP without increasing the total weight on the shocks if the news is distributed across a longer horizon.

To quantify the cumulative effect of the forward guidance policies shown in [figure 9](#) across the entire horizon, we calculate the present value of the percent change in real GDP in every period:

$$\text{Cumulative Effect } \hat{y}(q) = \frac{1}{N} \sum_{j=1}^N \sum_{t=1}^{q+1} \frac{100(y_{j,t}^\varepsilon / y_{j,t}^{no\varepsilon} - 1)}{\prod_{k=2}^t r_{j,k}},$$

where $y_{j,t}^{no\varepsilon}$ is real GDP conditional on draw j of the shocks, $y_{j,t}^\varepsilon$ is real GDP conditional on the same draw of shocks except $\hat{\varepsilon}_1 = -0.6\%$, $r_{j,t}$ is the gross real interest rate from draw j , and N is the number of simulations. [Table 3](#) shows the present value of the cumulative percent change in real GDP over various forward guidance horizons in response to a -0.6% monetary policy shock.

Initial State of the Economy	Forward Guidance Horizon				
	0	1	4	8	10
Steady State ($\tilde{i}_0^* = 1$)	0.50	0.83	1.19	1.20	1.17
Recession ($\tilde{i}_0^* = 0$)	0.23	0.51	1.00	1.09	1.09
Deep Recession ($\tilde{i}_0^* = -0.5$)	0.11	0.33	0.87	1.03	1.04

Table 3: Present value of the cumulative percent change in real GDP in response to a -0.6% monetary policy shock.

For all states of the economy, q -quarter distributed forward guidance always has a larger cumulative effect on real GDP than an unanticipated shock. The size of the cumulative effect, however, depends on both the state of the economy and the forward guidance horizon. In steady state ($\tilde{i}_0^* = 1$), extending the forward guidance horizon to 4 quarters increases the cumulative effect on real GDP, but provides little effect thereafter. At the ZLB ($\tilde{i}_0^* = 0$), increasing the horizon from 4 to 8 quarters only raises the present value of real GDP by 0.09%, while increasing the horizon beyond 8 quarters has no additional effect. In a deep recession ($\tilde{i}_0^* = -0.5$), an increase in the

¹⁰De Graeve et al. (2014) show that if the model contains backward-looking endogenous state variables, such as habit formation or inflation indexation, then the effects of the policy will persist beyond the forward guidance horizon.

horizon from 4 to 8 quarters increases the present value of real GDP by 0.16%, but no meaningful increase beyond 8 quarters. Those results indicate it is more beneficial to extend the horizon when the economy is facing worse economic conditions, but in any state of the economy the central bank faces limits on how far forward guidance can extend into the future and continue to add stimulus.

Carlstrom et al. (2015) and De Graeve et al. (2014) show that endogenous state variables can affect the dynamics generated by forward guidance. To test the robustness of our results, [appendix B](#) extends our model in [section 3](#) to include habit formation. That feature dampens, delays, and extends the stimulative effect of forward guidance, but all of our key findings continue to hold. We separately examined inflation indexation, but that feature had a much smaller quantitative effect.

5.3 FORWARD GUIDANCE AND LOWER DEMAND Despite the Fed’s use of forward guidance and other unconventional policy measures since late 2008, professional forecasts of real GDP remained low and some even fell in response to recent FOMC statements. One plausible explanation for the weak real GDP forecasts is that the forward guidance announcements were accompanied by weak economic assessments by the Fed. Using simulations, this section reconciles the apparent contradiction between the effects of news shocks in our model and forecasts observed in the data.

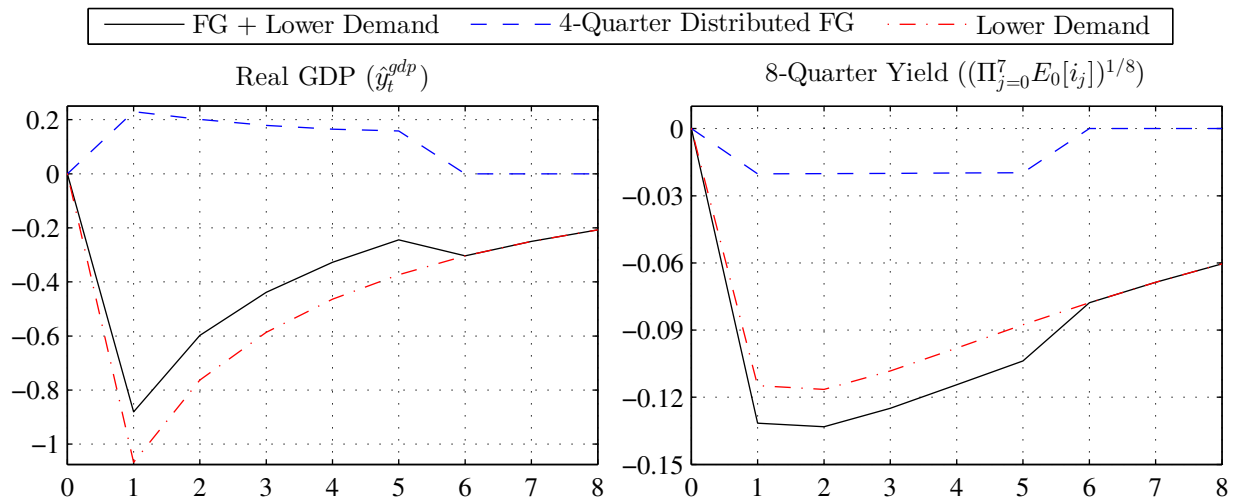


Figure 10: Generalized impulse responses to a 1 standard deviation positive discount factor shock and a -0.6% monetary policy shock with 4-quarter distributed forward guidance (solid line). The combined responses are compared to responses with only the monetary policy shock (dashed line) and only the discount factor shock (dash-dotted line). Each simulation is initialized at a notional rate equal to -0.5% to reflect the environment of recent forward guidance.

Figure 10 compares the economic effects of a decline in demand and an announcement of 4-quarter distributed forward guidance. To assess their combined effects, we compute generalized impulse responses to a simultaneous 1 standard deviation positive discount factor shock that reduces demand and a -0.6% forward guidance shock distributed over 4 quarters (solid line). Those responses are then compared to the responses with only the forward guidance shock (dashed line) and the responses with only the discount factor shock (dash-dotted line). The simulations are initialized at a notional interest rate equal to -0.5% . The distance between the dashed line and the solid line measures the effect of the negative demand shock, whereas the distance between the dash-dotted line and the solid line is the marginal benefit of 4-quarter distributed forward guidance. Since the simulations are consistent with households’ expectations, we interpret the GIRFs

as consensus forecasts made by households in period zero. In [figure 10](#), the forecast incorporates forward guidance about a policy rate cut and/or an expected increase in the discount factor.

An announcement in period 1 of 4-quarter distributed forward guidance reduces the 8-quarter yield and raises the real GDP forecast, whereas a forecast of a negative demand shock in period 1 also pushes down the yield curve but at the expense of lower expected real GDP. When the two shocks simultaneously hit the economy, the yield curve shifts down, and the forecast for the path of real GDP depends on which of the two shocks dominate. In [figure 10](#), the discount factor shock dominates the forward guidance announcement, so the real GDP forecast falls. Those findings illustrate two important points. One, identifying the source of empirically-observed changes in interest rate forecasts is challenging because households often receive forward guidance and information about current and future economic conditions at the same time. Two, forward guidance is stimulative in the absence of any other shocks, but the observed effect on real GDP forecasts is smaller or even negative if another shock is expected to simultaneously reduce current demand.

In [Figure 10](#), households forecast a specific discount factor shock, but we can also simulate the model over a selected range of shocks. That approach is useful for analyzing forward guidance in the aftermath of the Great Recession because it was when the economy recovered slower than households expected. Specifically, we restrict our sample of shocks to values of the discount factor that keep the policy rate at its ZLB in the absence of forward guidance. That set of shocks then is used to generate a distribution of real GDP outcomes with and without forward guidance. The differences between those two distributions indicate the effectiveness of recent forward guidance.

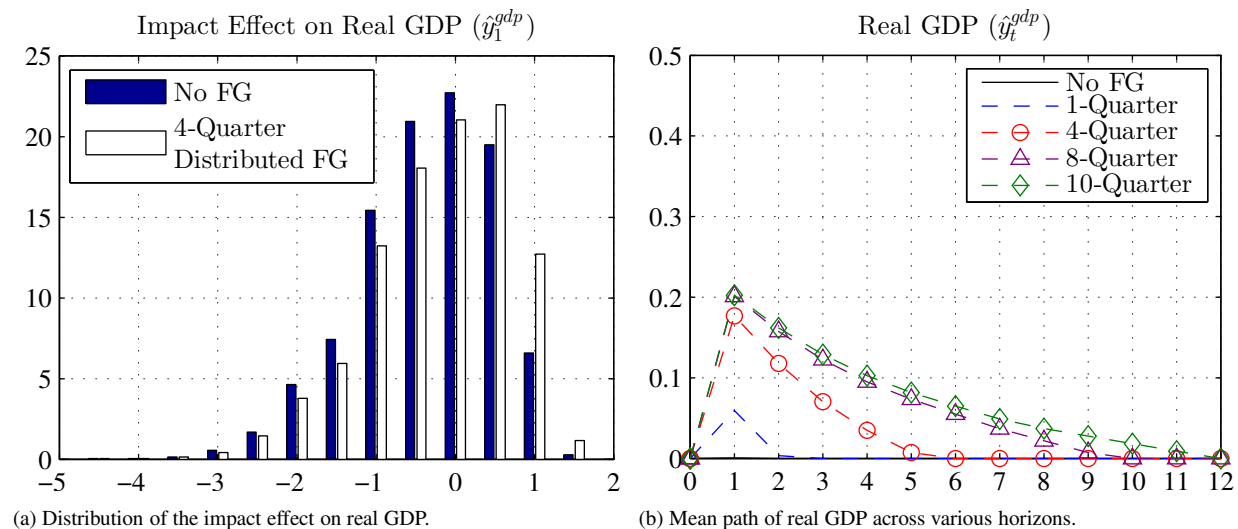


Figure 11: Generalized impulse responses to a -0.6% policy shock when the economy recovers slower than expected. Each simulation is initialized at a notional rate equal to -0.5% and the policy rate remains at zero for 11 quarters.

[Figure 11a](#) shows the distribution of the forecasted impact effect on real GDP from generalized impulse responses with no forward guidance (dark bars) and a -0.6% 4-quarter distributed forward guidance shock (light bars). The simulations used to produce both distributions are initialized at the deep ZLB state ($\tilde{i}_0^* = -0.5$) and are based on sequences of discount factor shocks that keep the nominal interest rate at zero for a minimum of five periods, so the economy does not recover as fast as households expect. Those expectations, however, are strong enough to provide the central bank with a small margin to lower expected nominal interest rates even though the actual nominal rate

remains at its ZLB over the entire forward guidance horizon. A comparison of both distributions for real GDP reveals that forward guidance shifts the distribution to the right, so real GDP falls less often on impact. That is, the impact effect on real GDP is negative in 62% of the simulations without forward guidance and drops to 53% with forward guidance. In both cases, the declines in real GDP are caused by a negative demand shock, but the forward guidance shock is strong enough in some cases to prevent real GDP from falling. In the remaining portion of the distribution where real GDP declines with forward guidance, the demand shocks are large enough to mask the stimulative effect of the news shock. Those findings reinforce our contention that forward guidance boosts real GDP, even though the evidence from recent forward guidance might suggest otherwise.

We also use this technique to regenerate the generalized impulse responses in [figure 9c](#) based on sequences of discount factor shocks that keep the policy rate at zero for the next 11 quarters, so they are comparable to the results with an interest rate peg. [Figure 11b](#) shows forward guidance continues to stimulate demand because households expect the policy rate to rise. As in [figure 9c](#), lengthening the forward guidance horizon increases the cumulative effect of real GDP to a certain point, even though the policy rate remains at zero for the entire horizon. The cumulative effect on real GDP, however, is only about half as large as when the economy recovers at the expected rate.

We do not take a position on why demand shifts in our model. In reality, there are several reasons why the discount factor may change when forward guidance is announced. One, households may interpret news of lower future policy rates as a signal of a weaker economy or a slower economic recovery. Two, policy statements may also provide a forecast of economic conditions that is worse than private forecasts, which leads households to revise their forecasts downward. Three, other sources may provide information that the economy is not performing as well as previously expected at the same time as the forward guidance announcement. Any of those scenarios could decrease real GDP, even if the forward guidance is successful at reducing expected policy rates.

5.4 INTEREST RATE PEG Modeling forward guidance with an interest rate peg is a special case of the news shock approach. This section uses the same model as in [section 3](#), but modifies (4) with a Markov process that governs whether the policy rate is pegged. The monetary policy rule is

$$i_t = \begin{cases} \max\{\underline{l}, i_t^*\} & \text{for } e_t = 0 \\ \underline{l} & \text{for } e_t = 1 \end{cases}.$$

In period t , the gross nominal interest rate is determined endogenously when $e_t = 0$ and is exogenously pegged at its ZLB when $e_t = 1$. Central bank forward guidance is characterized by a vector of nominal interest rate policies, $[e_t, e_{t+1}, \dots, e_{t+q}]$, communicated to households in period t over horizon q . The state of forward guidance is s_t and a particular forward guidance policy is given by $\mathbf{f}(s_t, q) = [s_t \bmod 2, \lfloor s_t/2 \rfloor \bmod 2, \dots, \lfloor s_t/2^q \rfloor \bmod 2]$, where $s_t \in \{0, \dots, 2^{q+1} - 1\}$. The matrix of all forward guidance policies is defined by $F(q) \equiv [\mathbf{f}(s_t, q)]$. The forward guidance state, s_t , evolves according to a 2^{q+1} -state Markov chain with a transition matrix given by

$$P(q) \equiv \underbrace{[I_{2^q} \otimes [p, p]']}_{2^{q+1} \times 2^q}, \underbrace{[I_{2^q} \otimes [1 - p, 1 - p]']}_{2^{q+1} \times 2^q}.$$

Our approach to modeling an interest rate peg is unique because it allows the ZLB to bind without any forward guidance. In other words, when the central bank does *not* peg the policy rate,

households still form expectations about the possibility that economic conditions could deteriorate and cause the ZLB to bind.¹¹ For example, with 1-quarter forward guidance

$$F(1) = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}, \quad P(1) = \begin{bmatrix} p & 0 & 1-p & 0 \\ p & 0 & 1-p & 0 \\ 0 & p & 0 & 1-p \\ 0 & p & 0 & 1-p \end{bmatrix},$$

where the first (second) column of F corresponds to the possible realizations of e_t (e_{t+1}). In state 0, the nominal interest rate is endogenous in periods t and $t + 1$. Thus, economic conditions and not an exogenous interest rate peg determine whether the ZLB binds. The probability that forward guidance remains in state 0 is p , whereas $1 - p$ is the probability that forward guidance will enter state 2. In state 2, the period t nominal interest rate is still set endogenously, but the central bank credibly announces the period $t + 1$ nominal rate will be pegged to \underline{i} regardless of economic conditions. That promise exogenously sets i_{t+1} , whereas news shocks allow for the possibility that $i_{t+1} > \underline{i}$. Forward guidance then transitions from state 2 to state 1 with probability p such that the nominal interest rate is pegged in period t but is endogenously set in period $t + 1$. Alternatively, a $1 - p$ probability exists that forward guidance moves from state 2 to state 3, which extends the interest rate peg by one quarter. In that case, households only know with certainty that the peg will last one quarter, although it may actually last for several quarters. With a longer forward guidance horizon, households expect the central bank will peg the nominal interest rate for more periods.

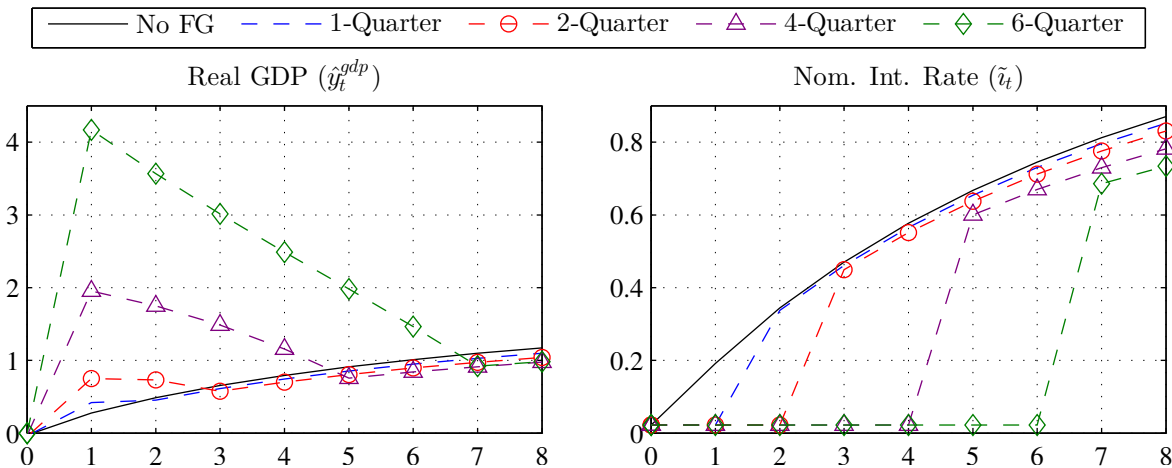


Figure 12: Economic effects of alternative interest rate pegs when the initial notional interest rate equals zero.

Figure 12 shows the effects of various interest rate pegs when the initial notional rate equals zero. With a 1-quarter peg, forward guidance begins in state 3, where the central bank promises to keep the nominal interest rate at zero until next period. Forward guidance then transitions to state 1 in the first period, where the central bank holds the current nominal rate at its ZLB but allows the peg to lapse in period 2. Without a peg in period 2 and beyond, households expect the nominal rate to rise as the economy recovers. With a 2-quarter peg, forward guidance begins in state 7, which guarantees the nominal rate will remain at zero for two periods. Forward guidance then transitions to states that reflect the number of periods remaining in the peg. Longer pegs evolve similarly.

¹¹Blake (2012) examines alternate ways to peg the policy rate in a model with an endogenous monetary policy rule.

With an interest rate peg, longer forward guidance horizons generate increasingly larger impact effects on real GDP because every additional quarter the policy rate is pegged at zero is equivalent to a news shock that is large enough to drive the expected nominal rate to its ZLB. Therefore, an interest rate peg gives the central bank a much stronger ability to affect expected nominal rates than is observed in the data. The peg also ignores the possibility the policy rate could rise before or after the target date, which is inconsistent with the threshold-based forward guidance used from December 2012-January 2014 and the state-contingent nature of earlier statements. In other words, the peg implies there is no uncertainty about future interest rates, which is unlikely and at odds with recent data. After all, if households knew the policy rate would remain at zero, no one would spend time looking at job reports and other data to figure out when the central bank will raise rates.

Instead of pegging the nominal rate at its ZLB, it is possible to peg a specific path for the interest rate so it is lower than the path that would occur without any forward guidance. That specification would be closer to our news shock approach, but it would still create a degenerate distribution for the future nominal interest rate because the peg would not depend on future economic conditions.

6 CASE STUDIES OF FEDERAL RESERVE FORWARD GUIDANCE

This section uses the qualitative predictions of our theoretical model to help explain the economic effects of three recent FOMC policy statements that communicated date-based forward guidance.

6.1 2011 POLICY STATEMENT On August 9, 2011, the FOMC announced it “anticipates that economic conditions. . . are likely to warrant exceptionally low levels for the federal funds rate at least through mid-2013,” which was the Committee’s first use of date-based forward guidance. It also said, “The Committee now expects a somewhat slower pace of recovery over coming quarters,” but the Fed’s quantitative easing policy was unchanged, which makes this statement ideal to study.

Date	2011Q4	2012Q1	2012Q2	2012Q3	2012Q4
BCFF (7/20-21)	0.14	0.26	0.43	0.75	1.08
BCEI (8/4-5)	0.13	0.19	0.29	0.50	0.77
BCFF (8/24-25)	0.07	0.09	0.12	0.14	0.20
Total Change	-0.07	-0.17	-0.31	-0.61	-0.88
Change following GDP	-0.01	-0.07	-0.13	-0.25	-0.31
Change following FOMC	-0.06	-0.10	-0.18	-0.36	-0.57

Table 4: Blue Chip consensus forecasts of the 3-month T-Bill rate. All values are annualized net rates.

Blue Chip forecasts of interest rates and real GDP changed after the August 9th FOMC statement was released. Assessing the effect of that statement on economic forecasts is complicated by a downward revision of GDP on July 29, 2011, which reduced real GDP growth in most quarters since the Great Recession. For example, real GDP growth in 2011Q1 (2008Q4) declined from 1.9% (7.0%) to 0.4% (9.2%). To separate the impact of the two events, we follow Crump et al. (2013) and use forecasts from the July and August 2011 Blue Chip Financial Forecasts (BCFF) survey and similar forecasts from the August 2011 Blue Chip Economic Indicators (BCEI) survey.

Table 4 shows the consensus BCFF and BCEI forecasts of the 3-month T-bill rate from 2011Q4 to 2012Q4. The BCFF forecasts were made on July 20-21 before the GDP revision was released on

July 29th, while the BCEI forecasts were made on August 4-5 just prior to the August 9th FOMC statement. The difference between the late-July and early-August forecasts is an implicit measure of the impact that the GDP revision had on the forecasts. The next BCFF forecasts were made on August 24-25. The difference between the BCEI's August 4-5 forecasts and the BCFF's August 24-25 forecasts indirectly measures the effect of the FOMC statement, which communicated date-based forward guidance and provided an assessment of current and expected economic conditions.

Data indicate the July 29th GDP revision led to a 13 basis point decline in the consensus forecast of the 2012Q2 3-month T-bill rate and a 31 basis point decline in the 2012Q4 rate. After the FOMC statement, there were even larger decreases in expected interest rates, with the 2012Q2 and 2012Q4 rates falling by an additional 18 and 57 basis points, respectively. In fact, 3-month T-bill forecasts for all of 2012 declined more after the FOMC statement than after the GDP revision. Following both events, the 2012Q4 rate was only 13 basis points higher than the 2011Q4 rate, which means forecasters believed the policy rate was unlikely to rise until 2013. Moessner (2013) and Raskin (2013) find the FOMC statement had similar effects on future interest rates.

While many forecasters expected the federal funds rate to remain near zero throughout 2012, some forecasters believed rates would rise in 2012, despite the FOMC's forward guidance in August 2011. For example, the average of the top 10% of the 8/24-25 BCFF forecasts for 2012Q4 was 0.54% and the highest forecast was 1.17%. A surprising 20% of the FOMC members also thought the federal funds rate would rise in 2012. Using options data, Swanson and Williams (2014) report that there was a 15% chance in late 2011 that the federal funds rate would rise above 0.5% by the end of 2012. Our news shock approach to modeling forward guidance accounts for the tails of the interest rate distribution, whereas there is no expectation of higher rates during an interest rate peg.

Date	2011Q4	2012Q1	2012Q2	2012Q3	2012Q4
BCFF (7/20-21)	3.09	2.75	2.97	3.07	3.17
BCEI (8/4-5)	2.53	2.38	2.59	2.81	2.88
BCFF (8/24-25)	2.17	2.13	2.44	2.69	2.90
Total Change	-0.92	-0.62	-0.53	-0.38	-0.27
Change following GDP	-0.56	-0.37	-0.38	-0.26	-0.29
Change following FOMC	-0.36	-0.25	-0.15	-0.12	0.02

Table 5: Blue Chip consensus forecasts of quarter-over-quarter real GDP growth. All values are annualized net rates.

Table 5 displays consensus forecasts of real GDP growth from 2011Q4 to 2012Q4. The forecast for 2011Q4 dropped 0.56 percentage points after the GDP revision but only 0.36 percentage points after the FOMC statement. The 2012Q4 forecast of real GDP growth declined by almost 0.3 percentage points after the GDP revision but slightly increased after the FOMC statement. A comparison of all forecast horizons through 2012 reveals that the decline in the forecasts of real GDP after the GDP revision is larger than the change that was observed after the FOMC statement.

Our model predicts forward guidance will reduce expected interest rates and push up real GDP when it is communicated without conflicting information. Data following the FOMC statement, however, indicate that near-term real GDP forecasts declined. Our theory provides two potential explanations. One, the GDP revision before the FOMC statement lowered expected interest rates and limited the Fed's ability to stimulate the economy. Two, the forward guidance was communicated at the same time households received information about a weaker economic outlook. A

comparison of the forecasts following the GDP revision and the FOMC statement shows there was a larger decline in expected interest rates and a smaller decline in forecasts of real GDP after the FOMC statement. [Figure 10](#) demonstrates that when forward guidance is accompanied by a negative demand shock, it lowers expected interest rates and may cause real GDP to fall. Furthermore, real GDP is higher and expected nominal rates are lower than without forward guidance. Those theoretical results are consistent with the changes in the forecasts following the FOMC statement.¹²

6.2 2012 POLICY STATEMENTS The January 25, 2012 and September 13, 2012 statements lengthened the forward guidance horizon for the federal funds rate. The January statement extended the horizon by six quarters (from mid-2013 to late-2014), but it was announced five quarters before the end of the August 2011 horizon. The September statement extended the horizon by two quarters (from late-2014 to mid-2015), six quarters before the January forward guidance ended.

Statement	1	2	3	4	6	8	10
08/09/2011	-0.09	-0.18	-0.25	-0.29	-0.28	-0.20	-0.12
01/25/2012	-0.04	-0.10	-0.13	-0.14	-0.11	-0.05	0.01
09/13/2012	-0.01	-0.03	-0.05	-0.07	-0.06	-0.01	0.05

Table 6: Expected changes in forward rates j -years ahead on the date of the statement. Values are annualized net rates.

The January and September 2012 FOMC statements only contained news that was intended to lower expected nominal rates beyond five quarters because the August 2011 statement already said the policy rate was likely to remain at its ZLB at least until mid-2013. Blue Chip forecasts, however, do not extend that far into the future. Thus, we use daily term structure data from Gürkaynak et al. (2007), which is regularly updated by the Board of Governors. [Table 6](#) shows changes in instantaneous forward rates j -years ahead on the date of the FOMC statements. Following the January 2012 statement, the decline in the forward rates at 1 to 4 years ahead was about half the decline that occurred after the August 2011 statement. At longer horizons, the response is smaller and at 10 years ahead it is near zero. The September 2012 statement had an even smaller effect on future interest rates. Similarly, Raskin (2013) argues the August 2011 and January 2012 statements had different effects since the market was surprised by the first FOMC statement but not the second statement. Those results provide evidence that the central bank has a limited ability to affect future interest rates and stimulate the economy by extending the horizon, just like our theory predicts.

[Table 7](#) displays survey data analogous to what is shown in [table 5](#). The data indicate the January and September FOMC statements had little effect on real GDP forecasts. The small marginal effect is consistent with our theory for two reasons. One, the August 2011 policy change reduced expected interest rates so much that the modest extension of the forward guidance horizon had a smaller margin to lower expected rates in order to stimulate real GDP. Two, the extension of the

¹²Walsh (2009) cautions that aggressively reducing the policy rate in response to adverse shocks may cause a downward revision in people's economic outlook when their information set differs from the central bank. Campbell et al. (2012) suggest that real GDP declined in response to recent forward guidance because forecasters believed the Fed's communication was based on information about future economic conditions that was not available to the public. Bullard (2012) and Woodford (2012) argue date-based forward guidance may cause people to expect worse economic conditions over its horizon, whereas threshold-based forward guidance alleviates that problem by linking policy rate changes to economic conditions. Yellen (2013, 2014) refers to that type of communication as an "automatic stabilizer."

Date	2012Q2	2012Q3	2012Q4	2013Q1	2013Q2
BCFF (1/4-5)	2.14	2.30	2.66	2.59	2.72
BCEI (2/6-7)	2.21	2.40	2.59	2.51	2.68
Change	0.06	0.10	-0.07	-0.08	-0.04

(a) January 25, 2012 FOMC statement release.

Date	2012Q4	2013Q1	2013Q2	2013Q3	2013Q4
BCFF (9/5-6)	1.90	1.78	2.34	2.66	2.83
BCEI (9/24-25)	1.84	1.91	2.23	2.61	2.80
Change	-0.06	0.12	-0.12	-0.06	-0.03

(b) September 13, 2012 FOMC statement release.

Table 7: Blue Chip consensus forecasts of quarter-over-quarter real GDP growth. All values are annualized net rates.

existing forward guidance horizon was less likely to be interpreted as news than the August 2011 announcement. Our results in [figure 9](#) show modest extensions to the horizon can lead to a larger cumulative effect on real GDP, but most of that increase occurs at the end of the horizon. It is also possible that concurrent information about a weak economy dampened real GDP forecasts, just like in [figures 10](#) and [11a](#). Interestingly, a headline in the *New York Times* on the day of the January statement read, “Fed Signals That a Full Recovery Is Years Away.” Such a reaction illustrates the challenge central banks face in achieving the desired effect of their forward guidance policies.

7 CONCLUSION

This paper examines the stimulative effects of forward guidance at and away from the ZLB. The central bank conducts forward guidance by promising to keep future nominal interest rates lower than its policy rule suggests. That policy can stimulate economic activity if households believe the economy will recover and exit the ZLB. If, however, households do not expect any meaningful recovery, then expected future nominal rates will remain near zero and forward guidance will have little effect on the economy. Therefore, the ability of forward guidance to stimulate demand is limited when the economy is in a deep recession or households expect a slow economic recovery. We also find lower uncertainty about the future economy or an expectation of a stronger monetary response to inflation reduces the stimulative effect of forward guidance when the ZLB binds.

Distributed forward guidance has a smaller impact effect than a conventional monetary policy shock at steady state, but it has a larger cumulative effect in every state of the economy. At the ZLB, the cumulative effect on real GDP from lengthening the forward guidance horizon increases until the horizon reaches eight quarters but not thereafter, which suggests that the central bank faces limits on how far forward guidance can extend into the future and continue to add stimulus. More intense news in the current period or news in future periods may generate a larger cumulative effect, but that effect is limited by the central bank’s influence over expected policy rates.

Empirical estimates indicate that recent FOMC forward guidance reduced expected interest rates. It is unclear, however, how much of that decline was due to forward guidance and how much was due to changes in current and expected economic conditions. Overall, we find that recent

forward guidance was often associated with declines in real GDP. Those outcomes were likely due to the fact that news was often accompanied by weak economic assessments and prior expectations of a weak economy gave policymakers a very small margin to lower expected policy rates.

This paper demonstrates that the forward guidance horizon, the state of the economy, the speed of the recovery, the degree of economic uncertainty, the expected stance of monetary policy away from the ZLB, and the size of monetary policy shocks all nonlinearly impact the economic effects of forward guidance due to the ZLB constraint on current and future policy rates. Future research could generalize our model to examine other features that might influence the effects of forward guidance. For example, households could learn about the policy rule over time, instead of knowing the rule with certainty, in order to examine how much forward guidance increases welfare by reducing uncertainty. Another possibility is to assume the news process depends on future discount factor shocks. In that case, central bank communication would depend directly on the future state of the economy, which would provide a new way to model threshold-based forward guidance. It would also be interesting to examine various forms of communication about exiting the ZLB. Overall, we believe our findings provide a solid foundation for future research on forward guidance.

REFERENCES

- ADAM, K. AND R. M. BILLI (2006): “Optimal Monetary Policy under Commitment with a Zero Bound on Nominal Interest Rates,” *Journal of Money, Credit and Banking*, 38, 1877–1905.
- (2007): “Discretionary Monetary Policy and the Zero Lower Bound on Nominal Interest Rates,” *Journal of Monetary Economics*, 54, 728–752.
- ANDERSSON, M. AND B. HOFMANN (2010): “Gauging the Effectiveness of Quantitative Forward Guidance: Evidence from Three Inflation Targeters,” in *Twenty Years of Inflation Targeting: Lessons Learned and Future Prospects*, ed. by D. Cobham, Ø. Eitrheim, S. Gerlach, and J. F. Qvigstad, Cambridge University Press, chap. 15, 368–397.
- BANK OF ENGLAND (2013): “Monetary Policy Trade-offs and Forward Guidance,” London, UK.
- BAUER, M. D. AND G. D. RUDEBUSCH (2014): “Monetary Policy Expectations at the Zero Lower Bound,” FRB San Francisco Working Paper 2013-18.
- BEN ZEEV, N., C. M. GUNN, AND H. U. KHAN (2015): “Monetary News Shocks,” Carleton Economic Paper 15-02.
- BENHABIB, J., S. SCHMITT-GROHE, AND M. URIBE (2001): “The Perils of Taylor Rules,” *Journal of Economic Theory*, 96, 40–69.
- BLAKE, A. (2012): “Fixed Interest Rates Over Finite Horizons,” BOE Working Paper 454.
- BULLARD, J. (2012): “Making Sense of Thresholds, Triggers, Twists, and Timelines,” 147th Annual Meeting of the Little Rock Regional Chamber of Commerce, Little Rock, Arkansas.
- CABALLERO, R. J. AND E. FARHI (2014): “The Safety Trap,” NBER Working Paper 19927.
- CAMPBELL, J. R., C. EVANS, J. D. M. FISHER, AND A. JUSTINIANO (2012): “Macroeconomic Effects of Federal Reserve Forward Guidance,” *Brookings Papers on Economic Activity*, Spring.

- CARLSTROM, C. T., T. S. FUERST, AND M. PAUSTIAN (2015): “Inflation and Output in New Keynesian Models with a Transient Interest Rate Peg,” *Journal of Monetary Economics*, 76, 230–243.
- CHRISTIANO, L. J., M. EICHENBAUM, AND C. L. EVANS (2005): “Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy,” *Journal of Political Economy*, 113, 1–45.
- COENEN, G. AND A. WARNE (2014): “Risks to Price Stability, the Zero Lower Bound, and Forward Guidance: A Real-Time Assessment,” *International Journal of Central Banking*, 10, 7–54.
- COLEMAN, II, W. J. (1991): “Equilibrium in a Production Economy with an Income Tax,” *Econometrica*, 59, 1091–1104.
- CRUMP, R., S. EUSEPI, AND E. MOENCH (2013): “Making a Statement: How Did Professional Forecasters React to the August 2011 FOMC Statement?” Liberty Street Economics.
- D’AMICO, S. AND T. KING (2015): “What Does Anticipated Monetary Policy Do?” FRB Chicago Working Paper 2015-10.
- DE GRAEVE, F., P. ILBAS, AND R. WOUTERS (2014): “Forward Guidance and Long Term Interest Rates: Inspecting the Mechanism,” Sveriges Riksbank Working Paper 292.
- DEL NEGRO, M., M. GIANNONI, AND C. PATTERSON (2012): “The Forward Guidance Puzzle,” FRB New York Staff Report 574.
- DEN HAAN, W., ed. (2013): *Forward Guidance: Perspectives from Central Bankers, Scholars and Market Participants*, Center for Economic Policy Research.
- DIXIT, A. K. AND J. E. STIGLITZ (1977): “Monopolistic Competition and Optimum Product Diversity,” *American Economic Review*, 67, 297–308.
- EGGERTSSON, G. B. AND N. R. MEHROTRA (2014): “A Model of Secular Stagnation,” NBER Working Paper 20574.
- EGGERTSSON, G. B. AND M. WOODFORD (2003): “The Zero Bound on Interest Rates and Optimal Monetary Policy,” *Brookings Papers on Economic Activity*, 34(1), 139–235.
- ENGLISH, W. B., J. D. LÓPEZ-SALIDO, AND R. J. TETLOW (2015): “The Federal Reserve’s Framework for Monetary Policy: Recent Changes and New Questions,” *IMF Economic Review*, 63, 22–70.
- FILARDO, A. AND B. HOFMANN (2014): “Forward Guidance at the Zero Lower Bound,” *BIS Quarterly Review*.
- GAVIN, W. T., B. D. KEEN, A. W. RICHTER, AND N. A. THROCKMORTON (2015): “The Zero Lower Bound, the Dual Mandate, and Unconventional Dynamics,” *Journal of Economic Dynamics and Control*, 55, 14–38.

- GOMES, S., N. ISKREV, AND C. MENDICINO (2013): “Monetary Policy Shocks: We Got News!” Banco de Portugal Working Paper 07-2013.
- GÜRKAYNAK, R. S., B. SACK, AND J. H. WRIGHT (2007): “The U.S. Treasury Yield Curve: 1961 to the Present,” *Journal of Monetary Economics*, 54, 2291–2304.
- GUST, C., D. LÓPEZ-SALIDO, AND M. E. SMITH (2013): “The Empirical Implications of the Interest-Rate Lower Bound,” CEPR Discussion Paper 9214.
- HABERIS, A., R. HARRISON, AND M. WALDRON (2014): “Transitory Interest-Rate Pegs under Imperfect Credibility,” CFM Discussion Paper 2014-22.
- INTERNATIONAL MONETARY FUND (2013): “Unconventional Monetary Policies—Recent Experience and Prospects,” Washington, D.C.
- IRELAND, P. N. (1997): “A Small, Structural, Quarterly Model for Monetary Policy Evaluation,” *Carnegie-Rochester Conference Series on Public Policy*, 47, 83–108.
- (2003): “Endogenous Money or Sticky Prices?” *Journal of Monetary Economics*, 50, 1623–1648.
- JUNG, T., Y. TERANISHI, AND T. WATANABE (2005): “Optimal Monetary Policy at the Zero-Interest-Rate Bound,” *Journal of Money, Credit and Banking*, 37, 813–835.
- KILEY, M. (2014): “Policy Paradoxes in the New-Keynesian Model,” Finance and Economics Discussion Series 2014-29.
- KOOL, C. J. AND D. L. THORNTON (2015): “How Effective is Central Bank Forward Guidance?” Federal Reserve Bank of St. Louis *Review*, 97, 303–322.
- KOOP, G., M. H. PESARAN, AND S. M. POTTER (1996): “Impulse Response Analysis in Non-linear Multivariate Models,” *Journal of Econometrics*, 74, 119–147.
- KRIPPNER, L. (2013): “A Tractable Framework for Zero-Lower-Bound Gaussian Term Structure Models,” Australian National University CAMA Working Paper 2013-49.
- KRUGMAN, P. R. (1998): “It’s Baaack: Japan’s Slump and the Return of the Liquidity Trap,” *Brookings Papers on Economic Activity*, 29, 137–206.
- LASÉEN, S. AND L. E. SVENSSON (2011): “Anticipated Alternative Policy Rate Paths in Policy Simulations,” *International Journal of Central Banking*, 7, 1–35.
- LEVIN, A., D. LÓPEZ-SALIDO, E. NELSON, AND T. YUN (2010): “Limitations on the Effectiveness of Forward Guidance at the Zero Lower Bound,” *International Journal of Central Banking*, 6, 143–189.
- MCKAY, A., E. NAKAMURA, AND J. STEINSSON (2015): “The Power of Forward Guidance Revisited,” NBER Working Paper 20882.

- MILANI, F. AND J. TREADWELL (2012): “The Effects of Monetary Policy “News” and “Surprises”,” *Journal of Money, Credit and Banking*, 44, 1667–1692.
- MOESSNER, R. (2013): “Effects of Explicit FOMC Policy Rate Guidance on Market Interest Rates,” DNB Working Paper 384.
- MOESSNER, R., D. JANSEN, AND J. DE HAAN (2015): “Communication about Future Policy Rates in Theory and Practice: A Survey,” DNB Working Paper 475.
- MOESSNER, R. AND W. R. NELSON (2008): “Central Bank Policy Rate Guidance and Financial Market Functioning,” *International Journal of Central Banking*, 4, 193–226.
- PETERMAN, W. B. (2016): “Reconciling Micro and Macro Estimates of the Frisch Labor Supply Elasticity,” *Economic Inquiry*, 54, 100–120.
- RASKIN, M. D. (2013): “The Effects of the Federal Reserve’s Date-based Forward Guidance,” Finance and Economics Discussion Series, 2013-37.
- REIFSCHNEIDER, D. AND J. C. WILLIAMS (2000): “Three Lessons for Monetary Policy in a Low-Inflation Era,” *Journal of Money, Credit and Banking*, 32, 936–966.
- RICHTER, A. W., N. A. THROCKMORTON, AND T. B. WALKER (2014): “Accuracy, Speed and Robustness of Policy Function Iteration,” *Computational Economics*, 44, 445–476.
- ROTEMBERG, J. J. (1982): “Sticky Prices in the United States,” *Journal of Political Economy*, 90, 1187–1211.
- SMETS, F. AND R. WOUTERS (2007): “Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach,” *American Economic Review*, 97, 586–606.
- SVENSSON, L. E. O. (2011): “Practical Monetary Policy: Examples from Sweden and the United States,” *Brookings Papers on Economic Activity*, 42, 289–352.
- (2015): “Forward Guidance,” *International Journal of Central Banking, Special Supplemental Issue: Reflecting on 25 Years of Inflation Targeting*, 11, 19–64.
- SWANSON, E. T. AND J. C. WILLIAMS (2014): “Measuring the Effect of the Zero Lower Bound on Medium- and Longer-Term Interest Rates,” *American Economic Review*, 104, 3154–3185.
- TAUCHEN, G. (1986): “Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions,” *Economics Letters*, 20, 177–181.
- WALSH, C. E. (2009): “Using Monetary Policy to Stabilize Economic Activity,” in *Financial Stability and Macroeconomic Policy*, FRB Kansas City Jackson Hole Symposium, 245–296.
- WERNING, I. (2011): “Managing a Liquidity Trap: Monetary and Fiscal Policy,” NBER Working Paper 17344.
- WOODFORD, M. (2012): “Methods of Policy Accommodation at the Interest-Rate Lower Bound,” in *The Changing Policy Landscape*, FRB Kansas City Jackson Hole Symposium, 185–288.

WU, J. C. AND F. D. XIA (2014): “Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound,” Manuscript, Chicago Booth.

YELLEN, J. (2013): “Communication in Monetary Policy,” Society of American Business Editors and Writers 50th Anniversary Conference, Washington D.C.

——— (2014): “Monetary Policy and the Economic Recovery,” Economic Club of New York.

A TWO-QUARTER HORIZON RESULTS

This section examines how the results in [figure 1](#) change when the forward guidance horizon is extended by one quarter. [Figure 13](#) plots the 2-quarter forward guidance [$(\alpha_0, \alpha_1, \alpha_2) = (0, 0, 1)$, dashed line] decision rules for real GDP and the current and expected nominal interest rates as a function of the monetary policy shock, $\hat{\varepsilon}_t$. With 2-quarter forward guidance, households receive news about a policy shock two periods before the shock hits the economy. As a reference, we also show the decision rules without forward guidance [$(\alpha_0, \alpha_1, \alpha_2) = (1, 0, 0)$, solid line]. Once again, we focus on a cross section of the decision rules where the initial notional interest rate equals zero.

When households receive news in period t about an expansionary monetary policy shock that will occur in period $t + 2$, the impact on real GDP is similar to the impact with 1-quarter forward guidance. Given households prefer a smooth consumption path, the expectation of monetary stimulus in period $t + 2$ encourages households to raise their consumption not only in period $t + 2$, but also in periods t and $t + 1$. The higher consumption in those periods stimulates current real GDP.

Central banks, in practice, offset the feedback effects on current and expected future nominal interest rates by promising to keep the nominal rate at zero over the entire forward guidance horizon. Thus, [figure 13](#) also shows the decision rules when households receive 2-quarter distributed forward guidance [$(\alpha_0, \alpha_1, \alpha_2) = (0.16, 0.125, 0.715)$, dash-dotted line]. Substantial differences exist between the two types of 2-quarter forward guidance. With 2-quarter distributed forward guidance, the central bank announces in period t that an expansionary monetary policy shock will occur in periods t , $t + 1$, and $t + 2$. The shocks in periods t and $t + 1$, which are not present with 2-quarter forward guidance, hold the current nominal interest rate at zero and lower the expected rate in period $t + 1$. Those two additional policy shocks more than compensate for the smaller weight on the period $t + 2$ news shock, so 2-quarter distributed forward guidance produces a slightly larger stimulative effect than the more heavily weighted news shock that occurs in period $t + 2$. For example, a -0.5% (-1%) shock announced in period t raises real GDP by 0.05 (0.12) percentage points more with 2-quarter distributed forward guidance than with 2-quarter forward guidance.

Extending the horizon from 1 to 2 quarters less than doubles the stimulative effect on real GDP. For example, a -0.5% (-1%) policy shock increases real GDP by 0.18 (0.25) percentage points with 1-quarter distributed forward guidance and by 0.20 (0.33) percentage points with 2-quarter distributed forward guidance. The extra quarter of news only raises real GDP by an additional 0.02 (0.08) percentage points, since the total amount of news is held constant across the two horizons.

B MODEL WITH HABIT FORMATION

This section shows how the effects of forward guidance change when we extend the model in [section 3](#) to allow for habit formation in the household’s preferences—a feature many economists

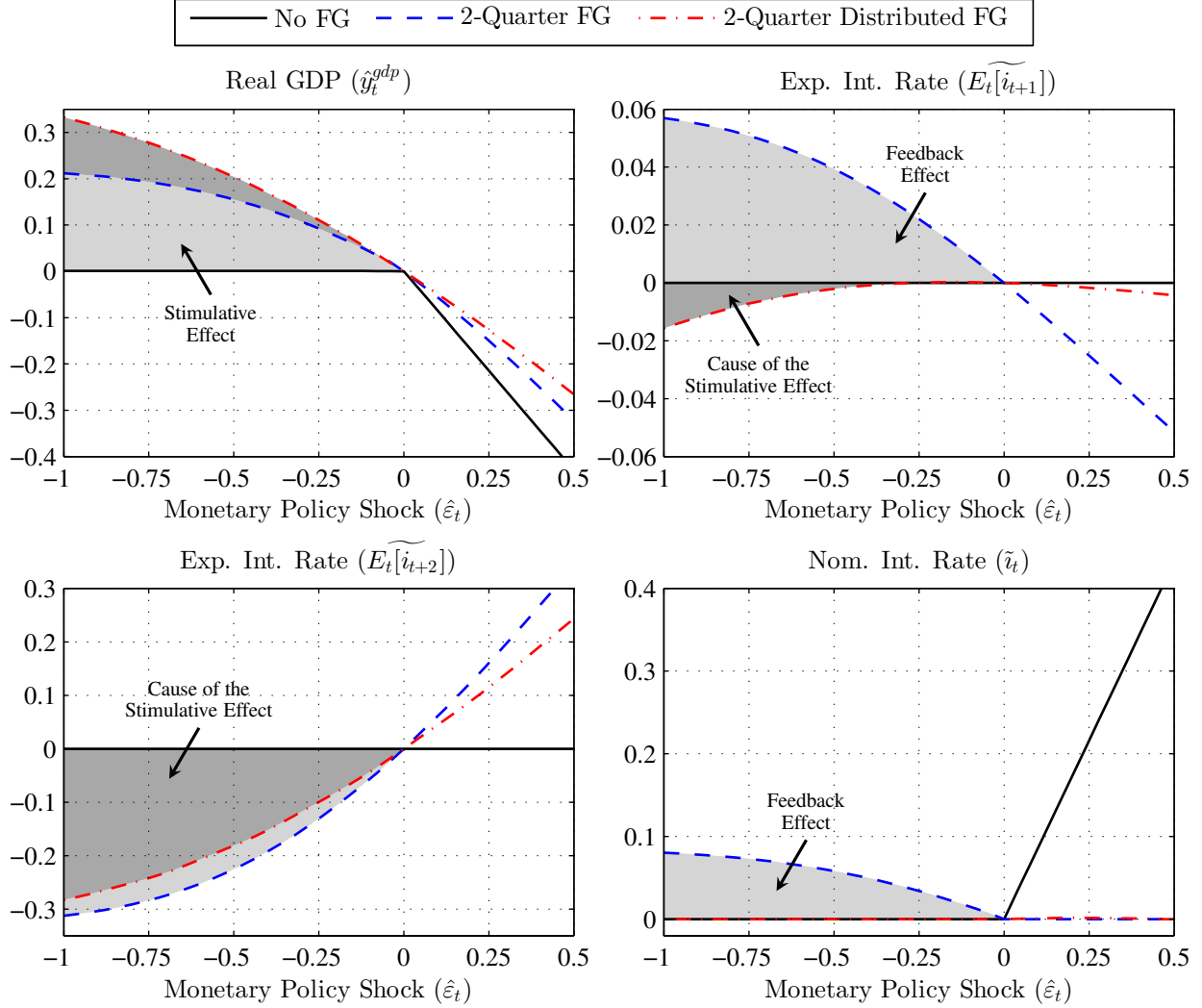


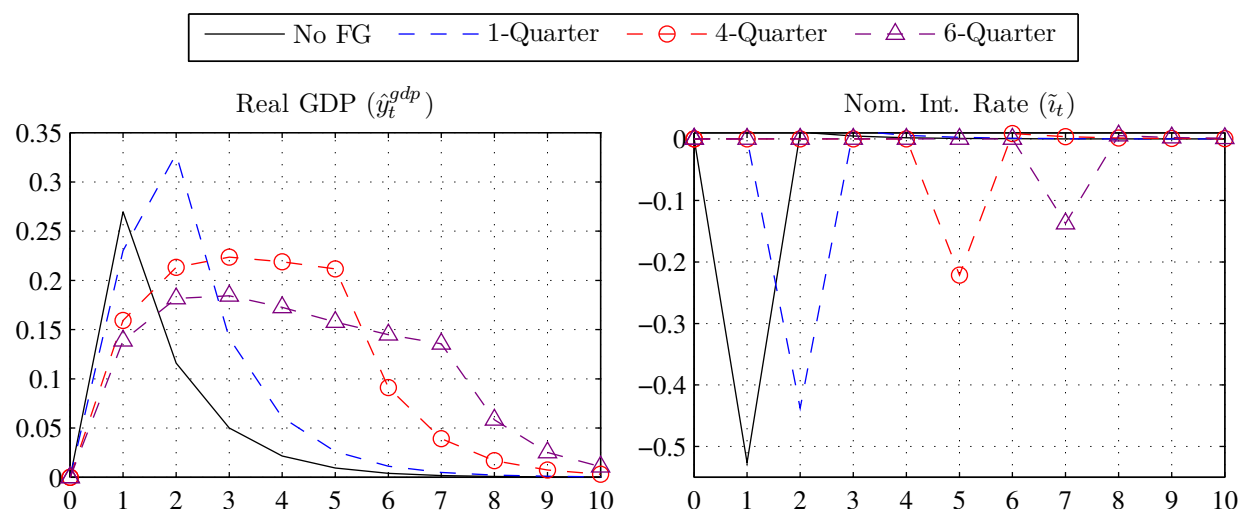
Figure 13: Decision rules as a function of the policy shock with no forward guidance, $(\alpha_0, \alpha_1, \alpha_2) = (1, 0, 0)$ (solid line); 2-quarter forward guidance, $(\alpha_0, \alpha_1, \alpha_2) = (0, 0, 1)$ (dashed line); and 2-quarter distributed forward guidance, $(\alpha_0, \alpha_1, \alpha_2) = (0.16, 0.125, 0.715)$ (dash-dotted line). In this cross section, the initial notional rate equals zero.

argue improves the model's empirical fit [e.g., Christiano et al. (2005) and Smets and Wouters (2007)]. A representative household chooses $\{c_t, n_t, b_t\}_{t=0}^{\infty}$ to maximize $E_0 \sum_{t=0}^{\infty} \tilde{\beta}_t [\log(c_t - hc_{t-1}^a) - \chi n_t^{1+\eta} / (1+\eta)]$, where c^a is aggregate consumption, which is taken as given by the household, and h is the degree of external habit formation. The household's choices are constrained by $c_t + b_t = w_t n_t + i_{t-1} b_{t-1} / \pi_t + d_t$. The optimality conditions to the household's problem imply

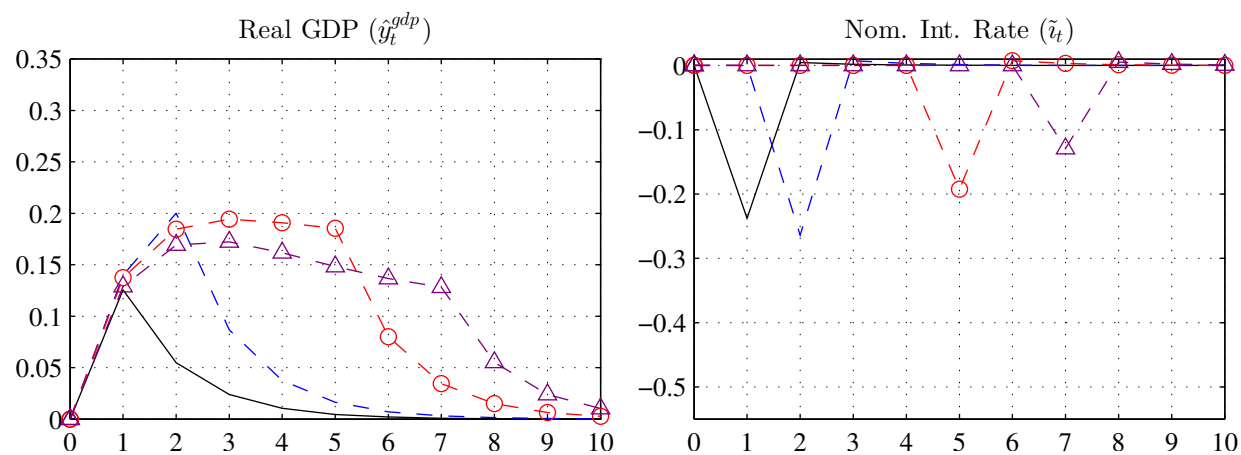
$$w_t = \chi n_t^\eta (c_t - hc_{t-1}^a), \quad (5)$$

$$1 = i_t E_t[q_{t,t+1} / \pi_{t+1}], \quad (6)$$

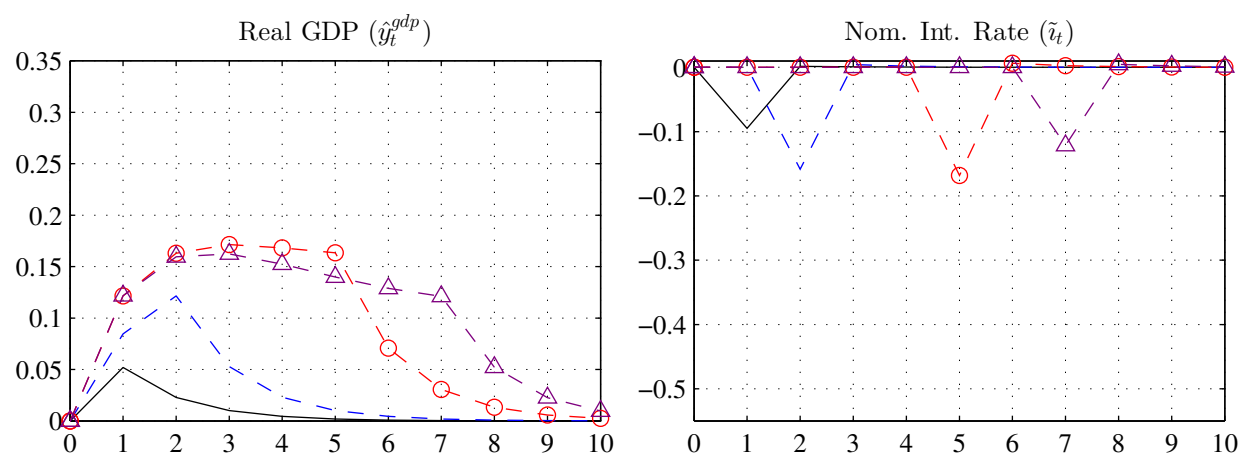
where $q_{t,t+1} \equiv \beta_{t+1} (c_t - hc_{t-1}^a) / (c_{t+1} - hc_t^a)$ is the pricing kernel between periods t and $t+1$ and $c_t = c_t^a$ in equilibrium. The production sector is unchanged, except firms now discount future dividends by $q_{t,k} \equiv \prod_{j=t+1}^{k>t} q_{j-1,j}$. When $h = 0$, the model is identical to the one in section 3. Gust et al. (2013) use a particle filter to estimate a constrained nonlinear model similar to this model. Thus, we set the habit formation parameter, h , to their mean posterior estimate of 0.46629.



(a) Simulations initialized at steady state.



(b) Simulations initialized at a notional interest rate equal to zero.



(c) Simulations initialized at a notional interest rate equal to -0.5% .

Figure 14: Generalized impulse responses to a -0.6% monetary policy shock with no forward guidance, $(\alpha_0, \alpha_1) = (1, 0)$, (left panels) and distributed forward guidance (right panels). In each simulation, the weights on the distributed forward guidance shock $(\alpha_j, j = 0, 1, \dots, q)$ are set to eliminate any feedback effects on the nominal interest rate.

Habit formation in consumption influences both the impact effect and duration of real GDP’s response to forward guidance. Nevertheless, this paper’s five key findings are unaffected by habit formation. [Figure 14](#) shows generalized impulse responses to a -0.6% monetary policy shock distributed over 1-, 4-, and 6-quarter forward guidance horizons in the model with habit formation. The assumptions underlying the GIRFs are identical to [figure 9](#) and are thus directly comparable.

Initial State of the Economy	Forward Guidance Horizon							
	Without Habit Formation				With Habit Formation			
	0	1	4	6	0	1	4	6
Steady State ($\tilde{i}_0^* = 1$)	0.50	0.83	1.19	1.21	0.47	0.80	1.17	1.19
Recession ($\tilde{i}_0^* = 0$)	0.23	0.51	1.00	1.08	0.22	0.48	0.97	1.06
Deep Recession ($\tilde{i}_0^* = -0.5$)	0.11	0.33	0.87	0.99	0.09	0.29	0.84	0.97

Table 8: Present value of the cumulative percent change in real GDP in response to a -0.6% monetary policy shock.

There are four important differences in the responses compared to the model without habit formation. One, the impact effect of forward guidance is much smaller. Two, the peak response of real GDP is delayed such that real GDP increases gradually until about half way through the forward guidance horizon. Three, the stimulative effect of forward guidance lasts beyond the forward guidance horizon, although its post-horizon effect is small relative to real GDP’s response in each quarter over the horizon. Four, despite being more persistent, [table 8](#) shows the cumulative effect of forward guidance at each horizon and each initial notional interest rate is slightly smaller.

Essentially, the presence of habit formation breaks the link between consumption growth and the real interest rate, so current real GDP is less sensitive to changes in current and expected future real interest rates. Therefore, a distributed news shock, which simultaneously eliminates the feedback effect on the nominal interest rate and pushes up inflation, causes real GDP to peak on impact in our model without habit formation but is delayed in our model with habit formation.

C NUMERICAL ALGORITHM

A formal description of the numerical algorithm begins by writing the model compactly as

$$\mathbb{E}[f(\mathbf{v}_{t+1}, \mathbf{w}_{t+1}, \mathbf{v}_t, \mathbf{w}_t) | \Omega_t] = 0,$$

where f is a vector-valued function that contains the equilibrium system, $\mathbf{v} = \beta$ is a vector of exogenous variables, $\mathbf{w} = (c, m, y, w, \pi, i)$ is a vector of endogenous variables, and $\Omega_t = \{S, P, \mathbf{z}_t\}$ is the household’s information set in period t , which contains the structural model, S , its parameters, P , and the state vector, \mathbf{z} . For example, with 1-quarter distributed forward guidance, $\mathbf{z}_t = (\varepsilon_{t-1}, \varepsilon_t, \beta_t)$. Each state variable is discretized into 61 points, so the state space contains 226,981 nodes. The bounds of each state variable are ± 4 standard deviations of their processes.

The following steps outline our policy function iteration algorithm:

1. Obtain initial conjectures for the approximating functions, \hat{c}_0 and $\hat{\pi}_0$, on each node from the log-linear model without the ZLB imposed. We use `gensys.m` to obtain those conjectures.
2. For iteration $i \in \{1, \dots, I\}$ and node $n \in \{1, \dots, N\}$, implement the following steps:
 - (a) On each node, solve for $\{y_t, i_t, w_t\}$ given $\hat{c}_{i-1}(\mathbf{z}_t^n)$ and $\hat{\pi}_{i-1}(\mathbf{z}_t^n)$ with the ZLB imposed.

- (b) Linearly interpolate $\{c_{t+1}, \pi_{t+1}\}$ given the state, $\{\varepsilon_t, \varepsilon_{t+1}^m, \beta_{t+1}^m\}_{m=1}^M$ (1-quarter forward guidance). Each of the M pairs of $\{\varepsilon_{t+1}^m, \beta_{t+1}^m\}$ are Gauss-Hermite quadrature nodes. In a constrained model, the accuracy of expectations is crucial, so we use 31 nodes on each shock ($M = 31^2$). We use Gauss-Hermite quadrature, since it is accurate for normally distributed shocks. We use piecewise linear interpolation to approximate future variables that show up in expectation, since that approach more accurately captures the kink in the decision rules than continuous functions such as Chebyshev polynomials.
- (c) On each node, solve for time $t + 1$ variables, $\{y_{t+1}^m, c_{t+1}^m\}_{m=1}^M$, that enter the expectation operators. Then, numerically integrate to approximate the expectations by computing

$$\mathbb{E} [f(\mathbf{x}_{t+1}^m, \mathbf{x}_t^n) | \Omega_t] \approx \frac{1}{\pi} \sum_{m=1}^M f(\mathbf{x}_{t+1}^m, \mathbf{x}_t^n) \phi(\varepsilon_{t+1}^m, \beta_{t+1}^m),$$

where $\mathbf{x} \equiv (\mathbf{z}, \mathbf{w})$, and ϕ are the respective Gauss-Hermite weights. The superscripts on \mathbf{x} indicate which realizations of the state variables are used to compute expectations. Finally, use the nonlinear solver, `csolve.m`, to minimize the Euler equation errors.

3. Define $\text{maxdist}_i \equiv \max\{|\hat{c}_i - \hat{c}_{i-1}|, |\hat{\pi}_i - \hat{\pi}_{i-1}|\}$. Repeat step 2 until $\text{maxdist}_i < 10^{-9}$ on every node for 10 consecutive iterations. At that point, the algorithm converged to a solution.

Richter et al. (2014) demonstrate the accuracy of this algorithm in a model with a ZLB constraint.

D GENERALIZED IMPULSE RESPONSE FUNCTIONS

The general procedure for calculating GIRFs is described in Koop et al. (1996). The GIRFs are based on the average path from repeated simulations of our model and generated by following:

1. Initialize each simulation by solving for the constant discount factor shock that yields the desired notional interest rate. Define the corresponding state vector as \mathbf{z}_0 .
2. Draw random monetary policy and discount factor shocks, $\{\varepsilon_t, v_t\}_{t=0}^N$, for each simulation, where N is the number of quarters in the simulation. Beginning at the initial state vector, \mathbf{z}_0 , simulate R equilibrium paths, $\{\mathbf{x}_t^j(\mathbf{z}_0)\}_{t=0}^N$, where $j \in \{1, 2, \dots, R\}$ and $R = 100,000$.
3. Using the same R draws of shocks from step 2, replace the policy rate shock in period one with a -0.5% shock (i.e., set $\varepsilon_1 = -0.5$ for all $j \in \{1, 2, \dots, R\}$). Simulate the model with these alternate sequences of shocks. This yields R equilibrium paths, $\{\mathbf{x}_t^j(\mathbf{z}_0, \varepsilon_{z,1})\}_{t=0}^N$.
4. Average across the R simulations from step 2 and step 3 to obtain average paths given by

$$\bar{\mathbf{x}}_t(\mathbf{z}_0) = R^{-1} \sum_{j=1}^R \mathbf{x}_t^j(\mathbf{z}_0), \quad \bar{\mathbf{x}}_t(\mathbf{z}_0, \varepsilon_{z,1}) = R^{-1} \sum_{j=1}^R \mathbf{x}_t^j(\mathbf{z}_0, \varepsilon_{z,1}).$$

5. The difference between the two paths is a GIRF. In our figures, a variable with a hat equals $100(\hat{\mathbf{x}}_t(\mathbf{z}_0, \varepsilon_{z,1})/\bar{\mathbf{x}}_t(\mathbf{z}_0) - 1)$, and a variable with a tilde is $100(\tilde{\mathbf{x}}_t(\mathbf{z}_0, \varepsilon_{z,1}) - \bar{\mathbf{x}}_t(\mathbf{z}_0))$.

E COMPUTING LONGER HORIZONS

To make our numerical algorithm tractable across forward guidance horizons up to 10 quarters, we discretize each monetary policy shock with 3 points by following the procedure in Tauchen

(1986). The state vector, $\mathbf{z}_t = (\beta_t, s_{0,t}, s_{1,t})$, is independent of the horizon. The monetary policy state, $s_{0,t} \in \{0, 1, 2\}$, determines the realization of the monetary policy shock, ε_t , according to

$$\varepsilon_t = \begin{cases} -0.006 & \text{for } s_{0,t} = 0 \\ 0 & \text{for } s_{0,t} = 1 \\ 0.006 & \text{for } s_{0,t} = 2 \end{cases}.$$

A particular realization of the lagged monetary policy states in the news process is given by $\mathbf{e}(s_{1,t}, q) = [s_{1,t} \bmod 3, \lfloor s_{1,t}/3 \rfloor \bmod 3, \dots, \lfloor s_{1,t}/3^{q-1} \rfloor \bmod 3]$, where $s_1 \in \{0, 1, \dots, 3^q - 1\}$. The matrix of all realizations of lagged states is $E(q) \equiv [\mathbf{e}(s_1, q)]$. For example, when $q = 2$

$$E(2) = \begin{bmatrix} 0 & 1 & 2 & 0 & 1 & 2 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 & 1 & 1 & 2 & 2 & 2 \end{bmatrix}',$$

where the first (second) column of E corresponds to the state underlying the realization of ε_{t-1} (ε_{t-2}). The evolution of the state of lagged policy shocks is given by $s_{1,t+1} = s_{0,t} + 3(s_{1,t} \bmod 3^{q-1})$. If we further suppose $s_{0,t} = 2$ and $s_{1,t} = 3$, then $(\varepsilon_t, \varepsilon_{t-1}, \varepsilon_{t-2}) = (0.006, -0.006, 0)$. In order for $s_{1,t+1}$ to be consistent with the history of shocks, it must equal 2, which is given by $2 + 3(3 \bmod 3)$, so $(\varepsilon_t, \varepsilon_{t-1}) = (0.006, -0.006)$ (i.e., the third row of E).

The transition matrix for $s_{0,t}$ is ergodic and is characterized by a single vector of probabilities,

$$P = (\lambda_1, \lambda_2, \lambda_3) = (0.1587, 0.6827, 0.1587),$$

where $\lambda_k = \Pr(s_{0,t+1} = k)$. We discretize the initial discount factor, β_t , into 61 points, so the state space contains $N = 61 \times 3 \times 3^q$ nodes. We approximate the expectation operators by computing

$$\mathbb{E} [f(\mathbf{x}_{t+1}^k, \mathbf{x}_t^n) | \Omega_t] \approx \frac{1}{\pi} \sum_{k=1}^3 \lambda_k \sum_{m=1}^M f(\mathbf{x}_{t+1}^{m,k}, \mathbf{x}_t^n) \phi(\beta_{t+1}^m),$$

where k is the realization of $s_{0,t+1}$. In all other aspects, the algorithm is the same as in [appendix C](#).