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Optimal life cycle portfolio choice with variable annuities offering liquidity and investment downside protection

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This paper assesses optimal life cycle consumption and portfolio allocations when households have access to Guaranteed Minimum Withdrawal Benefit (GMWB) variable annuities over their adult lifetimes. Our contribution is to evaluate demand for these products which provide access to equity investments with money-back guarantees, longevity risk hedging, and partially-refundable premiums, in a realistic world with uncertain labor and capital market income as well as mortality risk. Others have predicted that consumers will only purchase such annuities late in life, but we show that they will optimally purchase GMWBs prior to retirement, consistent with their recent rapid uptick in sales. Additionally, many individuals optimally adjust their portfolios and consumption streams along the way by taking cash withdrawals from the products. These products can substantially enhance consumption, by up to 10% for those who experience highly unfavorable experiences in the stock market.

1. Introduction

Defined contribution pensions are the most rapidly growing form of retirement saving product around the world. Yet participants in such self-directed pension plans often fail to understand the risks associated with investment and spending decisions at older ages (Lusardi and Mitchell, 2007, 2008), which exposes them to the potential of severe retirement shortfalls (Poterba et al., 2007). Accordingly, many households could benefit from enhancements to their financial literacy (van Rooij et al., 2011), as well as products which incorporate income and return guarantees into defined contribution pension plans (Feldstein, 2009). Indeed, in July 2014, the US Department of Treasury issued final rules making it possible for workplace and individual retirement plans to include instruments with guaranteed lifetime income and longevity risk protection in the menu of funds offered to plan participants (US Department of the Treasury, 2014). Such products are offered to consumers in the form of investment-linked variable annuities (VAs) with guaranteed living benefits, which also provide downside asset protection from capital market shocks. Based on a realistically calibrated life cycle consumption and portfolio choice model, this paper shows how VAs with guarantees can be used by households to enhance retirement security.

Variable annuities have been one of the most rapidly growing financial products over the past few decades. As of 2014, some \$1.9 trillion of assets were invested in VA contracts in the US (IRI, 2014a). By contrast, assets invested in fixed annuities were only \$722 billion.[1] In 2013, annuity sales in the US totaled \$220.9 billion, nearly two-thirds (65%) of which were variable annuities. The specific and popular innovation we examine in this paper is a variable annuity with living benefits using a Guaranteed Minimum Withdrawal Benefit (GMWB) rider. These products were introduced at the beginning of the new century and constitute a major portion of recent variable annuity sales (Geneva Association, 2013; IRI, 2014a).

GMWBs are financial products with both investment and guarantee components. During the accumulation phase, the policyholder pays premiums to a life insurer, which (after expenses) are invested in separate mutual fund-style subaccounts.

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[1] Income annuity contract owners only have claim to the income streams, so this figure excludes insurance company reserves estimated at about \$90 billion held for income annuity contracts.

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The product provider guarantees that the policyholder may elect to take back her entire premium (or another guarantee base) in small portions (i.e. a "money-back" guarantee) over a certain time frame, regardless of the actual investment performance of her underlying portfolio (Milevsky and Salisbury, 2006). Typically, the consumer may withdraw up to a certain percentage of her premium per year until the premium is completely recouped. Any remaining capital in the investment subaccounts or, if greater, the guarantee amount at the end of the deferral period, can be converted either into a lifelong annuity or paid to the policyholder in the form of a lump sum. In this way, the GMWB offers access to equity investments, asset downside protection against market risk, and the possibility of hedging longevity risk via annuitization. Because of the withdrawal option, premiums are at least in part refundable, so that GMWBs offer also liquidity which can help overcome consumer reluctance to voluntarily annuitize retirement wealth.[2]

Prior literature on dynamic portfolio choice has examined household demand for life annuities and its welfare implications.[3] While several authors explored immediate and fixed annuities,[4] only a few considered variable annuities with deferred benefits in a realistically-calibrated life cycle portfolio choice model.[5] For instance, Horneff et al. (2009) investigated gradual annuitization with immediate variable annuities. Brown et al. (2001) used the utility equivalence concept introduced by Mitchell et al. (1999) to evaluate the welfare implications of having access to variable annuities during the decumulation phase. Feldstein and Ranguelova (2001) studied variable annuities in an investment-linked Social Security system where households could take individual investment risk. Using a continuous time framework, Milevsky and Young (2007) allowed the retiree to defer annuitization and move his entire savings into an equity-linked annuity as of a single switching date. Recent work by Maurer et al. (2013) examined variable deferred annuities where payments begin only when the deferring period is over and continue for life. In that context, benefits depend on the performance of the underlying asset portfolio (stocks, bonds, or some combination). Nevertheless, these studies did not account for the guarantees typically included in VAs actually offered in the market.

Several authors have investigated how to price the complex option features embedded in these contracts.[6] Milevsky and Salisbury (2006) argued that this valuation problem may be traced back to the optimal stopping problem akin to pricing an American put option. Bauer et al. (2008) developed a generalized pricing formula assuming optimal behavior of policyholders, using risk neutral valuation techniques and efficient numerical procedures. By contrast, the research literature on the optimal demand for GMWBs by risk-averse households within a realistically calibrated dynamic consumption and portfolio choice setting is sparse.[7] Steinorth and Mitchell (2012) evaluated variable annuities with guarantee riders and fees typical of the US annuity market. They concluded that these products were welfare-enhancing for risk-averse consumers, despite the fact that their money's worth ratios were substantially below 100%. Yet that study focused only on the decumulation phase.

The present paper extends previous work by incorporating fairly priced GMWBs with an annuitization option into the investment opportunity set of a utility-maximizing risk averse investor who faces an uncertain lifetime, along with risky labor income and equity returns as well as mortality risk. We examine how such an investor will optimally buy these products at various points in time during her work life, given that she can take withdrawals until the end of the deferral period (up to a limit). At retirement, the policyholder can take any remaining assets as a lump sum, or she can convert the remainder into a lifetime income stream. The model setup allows us to derive her optimal consumption and portfolio allocation across risky stocks, bonds, and variable annuities of the sort of interest here. We solve our realistically calibrated optimization problem using stochastic dynamic programming in discrete time. Sensitivity analysis shows how demand for the product would respond to alternate preferences, interest rate risk, volatility risk, taxes, and deferred payouts.

We find that investors will optimally purchase variable annuities prior to retirement because of their flexibility and access to the stock market. Moreover, many consumers will also optimally adjust their portfolios and consumption streams along the way by taking cash withdrawals from the products. We also show that the cashout option at retirement is exercised when equity returns are higher. We show that differences in households' cashout vs. annuitization patterns optimally result from variations in realized cumulative equity market return and labor income trajectories, whereas Chalmers and Reuter (2012) suggested that such withdrawals could be explained by financial illiteracy. The GMWB can substantially enhance consumption, by up to 10% for those who experience highly unfavorable experiences in the stock market, and welfare increases measured in terms of lifetime utility are meaningful.

In what follows, we first discuss the mechanics and pricing of GMWB annuities. Next, we introduce our life cycle model and use it to study how a household will optimally consume, save, invest, gradually buy annuities and cash them out, over its lifetime. Scenario analysis evaluates how demand for and welfare gains due to GMWB products vary given alternative formulations for preferences, uninsurable labor income uncertainty, and interest rate risk. We also examine how results vary with the economic environment including taxes, deferred annuity payouts, and Social Security replacement rates. A final section concludes.

2. Mechanics and pricing of GMWBs

A single premium deferred variable annuity with a GMWB rider and an annuitization option (termed here GMWB for short) is a contract between a life insurance company and an individual consumer who pays the firm an initial amount A at time $t = 1$ when signing the contract. The claims of the policyholder consist of the following elements: the *Fund Account*, the *Guarantee Account*, and an option (but not an obligation) to annuitize at time K, the end of the deferral period. The insurance company invests the premium into a *Fund Account* F which earns an uncertain growth rate R_{t+1}

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2 The discrepancy between the theoretical dominance of annuitization predicted by many economists (see originally Yaari, 1965, and more recently Davidoff et al., 2005) and the low annuity take-up rates of older households around the world is known as the 'annuity puzzle' (see, e.g., Inkmann et al., 2011).

3 Other innovative retirement and saving products recently introduced into life cycle models include longevity bonds to hedge systematic mortality risk (Cocco and Gomes, 2012).

4 See for instance Blake et al. (2014), Milevsky and Young (2007), Horneff et al. (2008), and Huang and Milevsky (2008) who studied annuities with immediate fixed payouts within a dynamic portfolio choice model. Horneff et al. (2010) explored deferred annuities with flat benefits.

5 Xiong et al. (2010) undertook a simulation analysis of GMWB products with various asset allocations, but that analysis did not embed the product in a life cycle portfolio context.

6 Early research in a no-arbitrage framework by Brennan and Schwartz (1976) showed how to price equity-linked variable life insurance policies that provide an asset value guarantee. More recent work includes, among others, Mahayni and Schneider (2012); the products they analyze did not, however, provide the withdrawal option characteristic of GMWB annuities.

7 For an optimal consumption and asset allocation model for VA with death benefits, see Gao and Ulm (2012).

from time t to $t+1$ and from which the annual fees are deducted. Typically the policyholder can influence the risk-return profile of the investments in the Fund Account by selecting from a menu of mutual funds (e.g. equity, fixed-income, real estate). The insurer holds the assets of the Fund Account separate from the general account to protect the policyholder in the event of insolvency (Geneva Association, 2013). As with all life-contingent annuity products, if the policyholder were to die during the deferral period, any remaining cash value in the Fund Account would be transferred to the insurance company.[8] But in contrast to traditional deferred life annuities, as long as she is alive, the policyholder may request a return of premium paid to the insurance company, within some limits. If the GMWB variable annuity also offers a death benefit, the remaining cash value may be transferred to the policyholder's heirs.

Formally, the development of the Fund Account value F_t $(t = 1, \ldots, K)$ until the end of the deferral period (K) is given by:

$$F_{t+1} = \max\left(F_t - E_t,\ 0\right) \cdot (1-\varphi) \cdot R_{t+1}, \tag{1}$$

where $F_1 = A$. Here E_t denotes the withdrawal from the Fund Account at time $t (E_1 = 0)$, and φ is the annual expense factor charged by the insurer as a constant percentage of the Fund Account value. A key element of the GMWB is that the insurance company guarantees the policyholder that the sum of all possible (life-contingent) withdrawals until the end of the deferral period is at least a certain amount, $\sum_{t=1}^{K} E_t \geq E_{1,K}^{min}$. In our case, this amount is equal to the policyholder's premium paid, $E_{1,K}^{min} = A$. At the same time, prior to age K, periodic withdrawals from the Fund Account are limited to a maximum amount $E_t \leq E^{max}$, typically a fraction of the premium.

The value of the guaranteed benefit payments is tracked in a special *Guarantee Account* G_t, which evolves according to:

$$G_{t+1} = G_t - E_t. \tag{2}$$

The initial value of this notional account is defined as $G_1 = E_{1,K}^{\min}$. Eq. (2) shows that the remaining value of the Guarantee Account is given by the value in the previous period minus any withdrawals during the current period. Hence the policyholder can decide in each period $t = 2, \ldots, K-1$ to withdraw an amount that satisfies the following limits:

$$0 \leq E_t \leq \min\left(G_t,\ E^{\max}\right). \tag{3}$$

The value of the Guarantee Account may differ from the Fund Account as a result of investment returns and expenses. At the end of the deferral period, at time $t = K$, the remaining cash value of the GMWB is given by the value of the Fund Account, F_K, or the remaining value of the Guarantee Account G_K, whichever is greater:

$$L_K = \max\left(G_K,\ F_K\right). \tag{4}$$

At the end of the deferral period at time K, the policyholder may take a final lump sum, $0 \leq LS_K \leq L_K$, with the remainder $L_K - LS_K$ transformed into a payout annuity with lifelong fixed benefits.[9]

The insurer is at risk under this contract, since it takes a short position on a (complex) option. If the Fund Account becomes depleted $(F_t = 0)$ during the deferral period, or, if at the end of the deferral period the remaining Guarantee Account exceeds the Fund Account $(F_K < G_K)$, the insurer must pay the shortfall using own resources. Hence, the insurer must levy an appropriate risk charge in exchange for providing the guarantee. Based on standard arguments from option pricing theory and assuming a rational withdrawal strategy by the policyholder, Milevsky and Salisbury (2006) and Bauer et al. (2008) have developed efficient numerical solutions to price the GMWB option (see Appendix A for details).

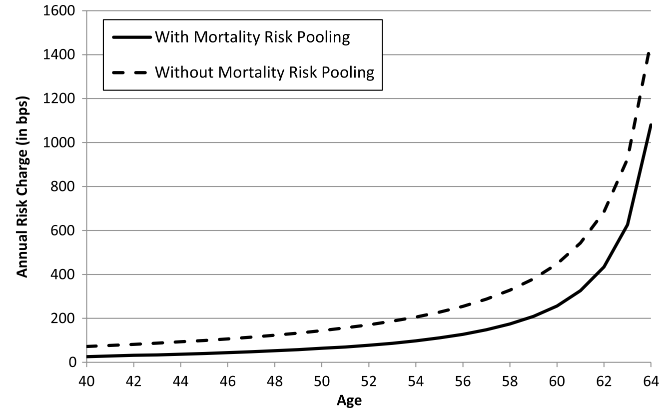

Fig. 1. Annual risk charges for a single premium GMWB at alternative purchase ages. Notes: Annual risk charges reported in basis points (bps) of the current Fund Account value. The Fund Account is assumed to be fully invested in equities with a volatility of 18%, a risk-free rate of 2%, and a deferral period ending at age 65. The solid (dashed) line represents the situation when the risk charge is calculated with (without) mortality risk pooling. Details are provided in Appendix A.
Source: Authors' calculations.

Using this approach, we generate the annual risk charge expressed as a percentage of the Fund Account. This fee must depend on the policyholder's age when the contract is signed (in all cases here, the deferral period ends at age 65), as well on her asset allocation within the product. Here and in what follows, the pricing approach assumes that the participant's Fund Account is fully invested in equities, since that allocation maximizes the value of the individual's guarantee inside the GMWB. Equity returns are assumed independently log-normally distributed with a standard deviation of 18%; additionally the assumed risk-free interest rate is 2%. Later, in sensitivity analyses, we allow for uncertain interest rates and non-normally distributed log returns with time varying volatility. The maximum guaranteed yearly withdrawal from a GMWB purchased at time $t (t = 1, \ldots, K-1)$ for a premium of A_t is given by $E^{max} = \frac{A_t}{K-t}$. At time K, $L_K = \max(F_K,\ G_K)$. For example, a policyholder age 40 (45) buying a GMWB with a deferral period to age 65 may withdraw no more than 4% (5%) of her initial premium per year.

As is evident from the solid line in Fig. 1, the insurer's annual risk charge for the GMWB rises with age. A policyholder who purchases the contract at age 40 would have to pay an annual fee of 26 basis points (bps) of her Fund Account per year until the deferral period of age 65. If the buyer were instead age 50, her yearly fee would rise to 64 bps (assuming the same deferral period). And someone who purchased the same policy at age 64 would pay 1080 bps, but for only a single year.

The risk charge increases with the purchaser's age because it is paid annually and not as a one-time lump sum; clearly, the younger buyer pays the charge over more years. Additionally, when a policyholder dies, remaining wealth in the Guarantee and Fund Accounts transfers to the insurer. This generates the well-known mortality credit due to mortality risk pooling, which the insurance company incorporates in calculating the product risk charge. If, instead, the insurer permitted the policyholder to receive a death benefit prior to time K equal to the cash value remaining, this would rule out the mortality credit, making the product more

8 In the United States and in most European countries, the regulatory framework only allows insurance companies and pension funds to provide annuities with life-contingent payouts. Other important regulated financial institutions such as banks or mutual funds are not allowed to underwrite life-contingent annuities. See Dellinger (2006, p. 18).

9 This is similar (but not identical) to a Guaranteed Lifelong Withdrawal Benefit (GLWB) annuity. Yet in our case, the annuitization decision is unrecoverable and provides no death benefits.

expensive. The dashed line in Fig. 1 illustrates the additional risk charge to cover this death benefit. For example, the life-contingent GMWB purchased at age 40 would involve a risk charge of 26 bps vs. 72 bps for the death benefit. At age 60, the annual risk charges are 257 bps vs. 448 bps.

It is also worth emphasizing that, in the context of a life cycle portfolio choice model with incremental annuity purchases, GMWBs bought at different ages must be tracked individually. This is because the risk charges and guaranteed withdrawal amounts permitted from the Fund Accounts vary with the buyer's purchase age. We analyze this process in the next section.

3. The life cycle model with guaranteed minimum withdrawal benefit annuities

In this section, we integrate GMWBs into a consumption and portfolio choice life cycle model for a utility-maximizing representative individual having an uncertain lifespan. We work in discrete time, starting at the end of age 39 ($t = 0$), and we assume that the individual's decision period runs from age 40 ($t = 1$) to 100 ($T = 61$). Until retirement, she earns an exogenously-determined labor income $Y_t = f(t-1) \cdot P_t \cdot U_t$, consisting of a deterministic trend $f(t)$ as well as permanent and transitory income components ($P_t = P_{t-1} \cdot N_t$ and U_t, respectively). After retirement, the individual receives a constant fraction of her last permanent salary as an annuitized lifelong benefit stream. She may purchase GMWBs from age 40 to 64, and retirement starts at age 65. For all GMWBs regardless of when purchased, the deferral age is 65. After that, annuity benefits are paid to the retiree for life. Hence, we assume an incomplete annuity market. In each period, the goal is to decide how much to consume, to save in stocks and bonds, and (prior to retirement) how much to spend on new GMWBs or withdraw from existing GMWBs.

Preferences: The individual's subjective probability of survival from t until $t+1$ is denoted by p_t^s. For our base case, we assume that preferences at time t can be specified by a time-separable *CRRA* utility function defined over current consumption C_t. (In the next section we extend our treatment of preferences.) The variable Q_t denotes the level of bequest at time $t+1$ if the decision maker dies between t and $t+1$, and the strength of her bequest motive is controlled by the parameter b. The term ρ refers to the coefficient of relative risk aversion, and β is the time preference rate. Then, the recursive definition of the corresponding value function is given by:

$$J_t = \frac{(C_t)^{1-\rho}}{1-\rho} + \beta E_t \left(p_t^s J_{t+1} + \left(1 - p_t^s\right) b \frac{(Q_{t+1})^{1-\rho}}{1-\rho} \right). \tag{5}$$

As $p_T^s = 0$, terminal utility is given by $J_T = \frac{(C_T)^{1-\rho}}{1-\rho} + \beta E_T \left(b \frac{(Q_{T+1})^{1-\rho}}{1-\rho} \right)$. From the final value, we work backwards to find the optimal policies for consumption, saving, and portfolio allocation over the life cycle.

GMWBs in a life cycle setting: In our life cycle model, the individual has the opportunity to incrementally purchase her GMWB contracts between age 40 and 64.[10] As noted above, for each age at which she buys a GMWB policy, a new Fund Account is defined wherein returns (net of fees) minus withdrawals are accumulated until retirement at age 65. Additionally, for each annuity purchase, a specific Guarantee Account as well as the specific annual maximum withdrawal amount must be tracked. Accordingly, modeling all possible purchases between age 40 and 64 requires following 25 different Fund Accounts, Guarantee Accounts, and withdrawal limits, or 75 accounts in total. Moreover, this also requires deciding how to optimally distribute withdrawals over the (potentially) 25 Fund Accounts. Explicitly keeping track of so many state and decision variables in a dynamic optimization model is (currently) infeasible due to the computational burden. In what follows, therefore, we describe a more efficient and novel approximation strategy which we devise for resolving this issue.

Let $F_{i,t}$ denote the value of the Fund Account at time t from a GMWB purchased at time $i \le t$ for a premium of $A_i = F_{i,i}$. Any withdrawal from this account is given by $E_{i,t}$. The yearly fee charged by the insurance company to cover the guaranteed minimum benefits promised to the policyholder is represented by φ_i. Analogous to Eq. (1), the Fund Account at time t for this GMWB purchased at time i develops as follows:

$$F_{i,t+1} = \max\left(F_{i,t} - E_{i,t},\ 0\right) \cdot (1 - \varphi_i) \cdot R_{t+1}. \tag{6}$$

The corresponding development of the Guarantee Account is given by:

$$G_{i,t+1} = G_{i,t} - E_{i,t}. \tag{7}$$

At each specific age, the overall values of the various Fund and Guarantee Accounts from all previously-purchased GMWBs as well as the sum of withdrawals from these accounts are given by:

$$F_t = \sum_{i=1}^{t-1} F_{i,t}, \qquad G_t = \sum_{i=1}^{t-1} G_{i,t}, \quad \text{and} \quad E_t = \sum_{i=1}^{t-1} E_{i,t}. \tag{8}$$

Additionally, we define an expense factor Φ_t that is applied to the total Fund Account to calculate the total risk charge levied by the insurer for all previously-purchased annuities. Accordingly, the value of the total Fund Account evolves as follows (where $E_1 = F_1 = 0$):

$$F_{t+1} = [\max(F_t - E_t,\ 0) \cdot (1 - \Phi_t) + A_t \cdot (1 - \varphi_t)] \cdot R_{t+1}. \tag{9}$$

The total Fund Account next period, at time $t+1$, has two parts. The first component represents the individual's current Fund Account value F_t, accumulated from previous annuity purchases. This value is reduced by withdrawals E_t and by annual fees using the expense factor Φ_t. The second component consists of her additional annuity purchases A_t in the current period, minus the specific risk charge φ_t at time t. Both components grow according to the gross return R_{t+1} earned by the assets backing the annuity.[11]

The expense factor Φ_t is defined as a weighted average of the risk charges applied to previous GMWB purchases and the fee levied on any new purchase at time t:

$$\Phi_{t+1} = \Phi_t \cdot x_{t+1} + (1 - x_{t+1}) \cdot \varphi_t. \tag{10}$$

The weight x_{t+1} on the previous period's expense factor refers to the current value of previously-purchased annuities divided by the current value of the total Fund Account including new purchases at time t. Formally (where $F_{t+1} > 0$):

$$x_{t+1} = \frac{\max(F_t - E_t,\ 0) \cdot (1 - \Phi_t) \cdot R_{t+1}}{\max(F_t - E_t, 0) \cdot (1 - \Phi_t) \cdot R_{t+1} + A_t \cdot (1 - \varphi_t) \cdot R_{t+1}}. \tag{11}$$

10 Previous work on optimal gradual annuitization with fixed and variable deferred annuities over the life cycle showed that annuity purchases are negligible before age 40 (Horneff et al., 2010; Maurer et al., 2013).

11 In our simulation analysis using optimal feedback controls, we track the specific subaccounts for the GMWB purchased at different ages; this allows us to apply the specific risk charges. The overall optimal withdrawal amount E_t at time t is allocated to specific subaccounts as a percentage calculated according to $E_{i,t} = E_t \cdot E_i^{max} / \sum_{i=1}^{t-1} E_i^{max}$. Here, E_i^{max} is the maximum allowable withdrawal for subaccount i.

In the first period, the expense factor is $\Phi_1 = 0$; if GMWBs are purchased at $t = 1$, the second-period expense factor Φ_2 is equal to φ_1. Consequently, in a two-period situation, the formula is exact. In a more general case, our approach represents a very close approximation to the cost structure of a portfolio of gradually-purchased GMWBs with different age-specific risk charges. This permits us to reduce the number of state variables for the GMWB in our backward optimization, from 75 to three.[12]

Finally, the development of the total Guarantee Account is given by the last period's Guarantee Account value, reduced by withdrawals and increased by new GMWB purchases. Before retirement, i.e. $t < K$, the total Guarantee Account evolves as follows:

$$G_{t+1} = G_t - E_t + A_t. \tag{12}$$

At retirement, the individual must decide how much of her GMWB value, i.e. $\max(F_K, G_K)$, she takes out as a lump sum LS_K. The remaining amount is converted into a payout annuity with constant lifelong benefits (PA_K):

$$PA_K = \frac{\max(F_K, G_K) - LS_K}{\ddot{a}_K}, \tag{13}$$

where $\ddot{a}_K = 1 + \sum_{s=1}^{\omega-K} \left(\prod_{u=K}^{K+s-1} p_u^a\right) R_f^{-s}$ denotes an actuarial annuity factor at retirement. The year-to-year survival probabilities p_u^a used to price the annuity are specified by a mortality table with assumed last age ω, and R_f is the interest rate used by the insurance company to discount future benefit payments. After the remaining fund value has been converted into a fixed payout annuity with no additional access to guaranteed withdrawals, no future risk charges are levied, i.e. $\Phi_t = 0\,(t > K)$. Accordingly, the dynamic portfolio choice problem during the retirement period can be solved using only one state variable to represent the annuity.

Budget constraints: In each period, the individual may allocate cash on hand (W_t) to consumption (C_t), liquid saving in stocks (S_t) and bonds (B_t), and – prior to retirement – additional GMWB purchases (A_t). At retirement, cash on hand increases by final lump-sum distributions from the GMWBs (LS_K) and, additionally, by the GMWB annuity payouts (PA_K). Formally, the resulting budget constraints are:

$$W_t = \begin{cases} C_t + S_t + B_t + A_t & t < K \\ C_t + S_t + B_t - PA_K - LS_K & t = K \\ C_t + S_t + B_t - PA_K & t > K. \end{cases} \tag{14}$$

During the work life, cash on hand next period is given by the value of the stock and bond investments including any returns, labor income (Y_t), and any withdrawals from the previously-purchased GMWBs (E_t). At retirement, labor income is replaced by a constant Social Security benefit stream (Y_K). Formally, the development of cash on hand is given by:

$$W_{t+1} = \begin{cases} S_t R_{t+1} + B_t R_f + Y_{t+1} + E_t & t < K \\ S_t R_{t+1} + B_t R_f + Y_K & t \geq K. \end{cases} \tag{15}$$

Finally, the policies must satisfy the following constraints:

$$\begin{aligned} & C_t, A_t, S_t, B_t \geq 0 (t < K) \\ & 0 \leq E_t \leq \frac{G_t}{K - t} \quad (t < K); \\ & LS_K \leq \max(F_K, G_K); \qquad E_t = 0 \quad (t > K) \\ & A_t = 0 \quad (t \geq K). \end{aligned} \tag{16}$$

Model calibration and numerical strategy: For calibration of our base case parameters, we use standard values in the literature (Cocco et al., 2005; Cocco and Gomes, 2012; Blake et al., 2014); in the next section, we compare results for various alternative values.

For this purpose, the coefficient of relative risk aversion is set at 5 and the time discount rate at 0.96. We also abstract from an intentional bequest motive[13] and set b to 0. The risk-free interest rate is set at 2% and stock returns are serially independent and log-normally distributed with a mean of 6% and volatility of 18%. Survival probabilities which enter in the utility function are specified by the US 2000 Basic Population Table. GMWB annuities are priced (as described in Appendix A) using the same mortality table and assuming that the Fund Account is fully invested in equities. All labor income parameters and initial wealth values for a US single female college graduate are derived from Love (2010). Accordingly, in our base case, the 40 year old single female has liquid wealth of \$120,000 and earns \$29,600 per year. Thereafter her labor earnings profile follows the typical hump-shaped pattern, and she is not exposed to labor income shocks. After retirement at age 65, she receives a combined pension from a defined benefit (DB) pension plan and Social Security totaling 73.6% of her last permanent labor income. Our model calibration and therefore the results therefrom are intended to represent the US economic and financial setting, as this nation comprises a large share of the global insurance market place. Other countries will have different mortality tables, labor income profiles, capital market environments, and tax rules; future work can investigate results in other such regimes.

The consumer's optimization problem is solved using dynamic stochastic programming. In the base case, we have five state variables prior to retirement: wealth (W_t), the total Fund Account (F_t), the value of the total Guarantee Account (G_t), the expense factor (Φ_t), and time (t). Up to the retirement age, we discretize the five-dimensional state space using a grid of size $30(W) \times 20(F) \times 20(G) \times 6(\Phi) \times 25(t)$ with equal spacing for Φ and logarithmic spacing for W, F, and G.[14] After retirement, we do not need to track the expense factor, i.e. we work on a $30\,(W) \times 20\,(F) \times 20\,(G) \times 36(t)$ state space. For each grid point, we calculate the optimal policies and value function using quadrature integration and spline interpolation.[15] Subsequently we simulate

12 The approximation results from the fact that, for two annuities purchased at different points in time with risk charges $\varphi_1 < \varphi_2$, the relative importance of the second risk charges decreases over time, while Eq. (10) assumes it is constant. Accordingly, Φ_t slightly overestimates the cost. To assess the potential approximation error we proceed as follows. We generate 10,000 life cycle profiles based on the optimal feedback controls using Monte Carlo simulations for the uncertain equity returns and labor income profiles. For each of these 10,000 paths, we calculate the total expense ratios at each age for two cases: (i) using Eqs. (10) and (11), and (ii) using the exact risk charges for the specific accounts. Calculating the differences between the two total expense ratios for each of the 10,000 life cycle paths provides a distribution of the approximation error. Even at age 64, when the approximation error is at its maximum, it merely amounts to 0.29 (0.92) bps at the 50% (95%) quantile in the base case. In the sensitivity analysis, the approximation errors are comparably small.

13 Empirical evidence regarding the existence and the strength of intentional bequest motives is mixed. Hurd (1989) estimates an almost zero intentional bequest preference and concludes that most households have only accidental bequests. By contrast, Bernheim et al. (1985) report that many US older persons indicate they have a significant bequest motive. Recent empirical work by Ameriks et al. (2011, p. 554) for US-households shows that "bequest motives are more prevalent than previously thought".

14 Overall this requires optimization of the multivariate objective function over 72,000 nodes for each of the 25 years up to retirement and 12.000 nodes for each of the 36 years after retirement. We use a high performance cluster in a MATLAB environment to evaluate the multi-dimensional integral, relying on cubic spline interpolation to derive the continuation value (= next period's utility) at integral supporting points that do not coincide with nodes of the state space.

15 An example of a result from the optimization is provided in Fig. A.1, which depicts a 3-dimensional excerpt of the 6-dimensional solution for the investment policies by holding constant the variables fund value, guarantee value, and risk charge. Obviously, analyzing and discussing in detail the 6-dimensional optimization results is infeasible, so to help summarize the optimal policies, we run an additional Monte-Carlo simulation assuming that the household acts according to its optimal policies, and we present average household behaviors. These show that our results are numerically stable.

10,000 independent life cycles using the optimal policy controls, permitting us to evaluate the expected life cycle outcomes for consumption, investment, and annuitization profiles, and we also examine the distributions of cashout patterns over the lifetime.

4. Results for the base case

This section describes the consumer's optimal demand for GMWBs over the life cycle. Specifically, we analyze when she will optimally purchase annuities during her work life, and how much she buys. In addition, we investigate how she exercises the GMWB withdrawal options, in reaction to the stock market's uncertain development.

Fig. 2 displays our results from the base case. The top panel reports the paths of expected consumption, labor income, wealth in liquid assets (stocks and bonds) and GMWBs, along with annuity purchases over time and withdrawals from existing GMWB accounts. Consistent with the real-world product, the GMWB value is stipulated as equal to the greater of either the Fund or the Guarantee Account prior to age 65, and thereafter the (actuarial) present value of the lifetime annuity payments. We define financial wealth as the sum of stocks, bonds, and GMWB values. In this setting, at age 40, the individual optimally allocates a substantial portion of her financial wealth to the GMWB, about $48,000 or 42%. The value of the Fund Account continues to rise during her work life, peaking at age 64 when it amounts to about $210,000 in expectation. At age 65, she takes a lump-sum of about $17,000, around 7.4% of her GMWB value, which is reflected in the sharp increase in liquid wealth. All remaining GMWB assets are converted into a lifelong annuity paying fixed benefits of about $14,000 per year, or 49% of her last labor income. Since, in our model, no further annuity purchases are possible after age 65, the present value of the annuitized financial wealth continuously declines with age during retirement.

The fine dotted line in the top panel of Fig. 2 indicates that average consumption is quite smooth over the life cycle. During her work life, the individual's consumption exceeds her labor income, with the gap mostly financed by a gradual depletion of her liquid wealth. In retirement, the individual consumes more than her Social Security benefit, with the excess financed mostly by the GMWB annuity which amounts to about two-thirds of the Social Security benefit. Precautionary saving is low, since there is no labor income risk during the work life in this base case. As is typical in such models, liquid assets are fully invested in equities at the outset (not shown in the graph), with the equity share declining around half by retirement age. During retirement, since the Fund Account has been converted into a bond-like fixed payout annuity, the remaining liquid assets are fully invested in stocks. Liquid assets are fully depleted around age 85, on average, due to the lack of a bequest motive.

The bottom panel of Fig. 2 shows average annuity purchases (black bars) and withdrawals (clear bars) from existing accounts, as well as the values of the Guarantee (solid line) and Fund Accounts (dashed line). Account values correspond to the left axis, while purchases and withdrawals refer to the right axis. Focusing first on purchases and withdrawals, we note that when a household buys additional annuities, its corresponding withdrawals amount to zero and vice versa. Purchases (withdrawals) depicted in the figure are generated by averaging over the 10,000 simulated realizations of A_t and E_t, respectively. Hence, at any given age, some people are purchasing new annuities, while others are withdrawing funds.

As noted above, at age 40, individuals on average devote a substantial amount of their financial wealth ($48,000) to the GMWB, which is relatively inexpensive due to the low annual guarantee risk charge of about 26 bps. This high initial annuity purchase, combined with rising fees for additional purchases, produces negligible additional GMWB purchases until just prior to retirement. Then, at age 64, some individuals take advantage of the final annuitization opportunity and shift a small amount of their liquid wealth into the annuity product despite its relatively high fee (around 10.80%). Other policyholders find it optimal to take small withdrawals from their GMWB Fund Accounts through age 64. At age 65, unlimited withdrawals are permitted, and on average people withdraw $17,000.

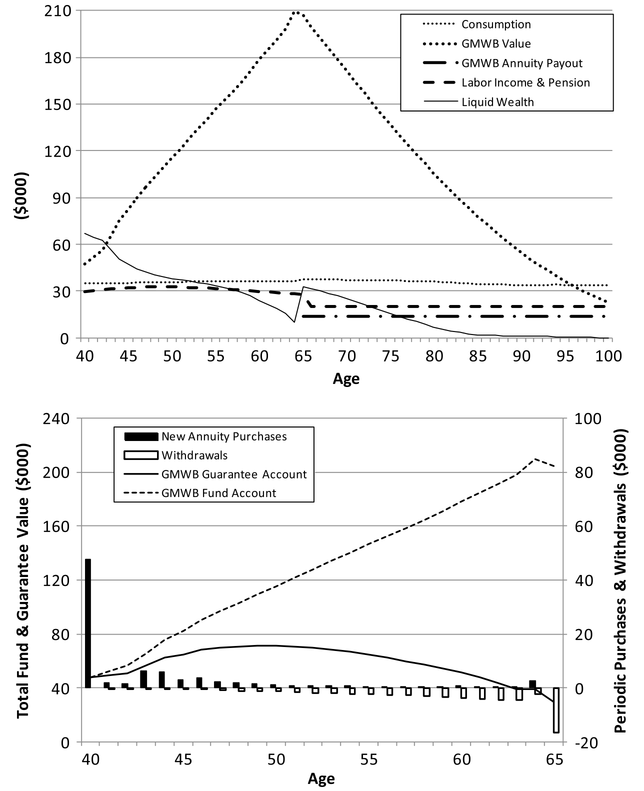

Fig. 2. Expected life cycle profiles in the GMWB environment. Notes: The top panel shows the development of labor/pension income, liquid and GMWB wealth, GMWB annuity income, and optimal consumption as explained in the text. The bottom panel displays the development of the fund and the guarantee account, as well as the optimal purchases and withdrawals from the GMWB account. Expected values (in $000) based on 10,000 simulated life cycles using the base case calibration: risk aversion $\rho = 5$; time preference $\beta = 0.96$; no bequest motive ($b = 0$); initial liquid wealth (labor income) of $120,000 ($29,600 p.a.) at age 40; no labor income risk ($\sigma_u^2 = \sigma_n^2 = 0$); retirement age: 65; pension replacement rate 73.6% no taxes; risk-free interest rate 2%; mean stock return 6%; stock return volatility 18%. *Source:* Authors' calculations.

Turning to the development of the Fund and Guarantee Accounts, despite small withdrawals from the GMWB, the Fund Account nevertheless continues to grow due to the expected return on stocks in which the assets are invested. The Guarantee Account increases to around age 50 because GMWB purchases exceed withdrawals. Thereafter, it declines indicating that withdrawals from existing GMWBs supersede additional purchases.

Optimal behavior with respect to withdrawing funds from existing GMWBs as well as purchasing additional GMWBs is complex: it depends on the interactions between stock market returns and their impact on the Fund Account, between the Fund and the Guarantee Account values, and the age-dependent fees for the GMWB rider. Fig. 3 sheds some light on these interactions by analyzing the optimal behavior for two specific scenarios.

Fig. 3(top panel) presents the case where equity markets perform well. Here, stocks usually generate substantially positive returns; in only a few periods do equities exhibit moderately negative earnings. In this setting, the Fund Account rises strongly with age and its value comes to substantially exceed the Guarantee Account. Consequently, the put option embedded in the GMWB is

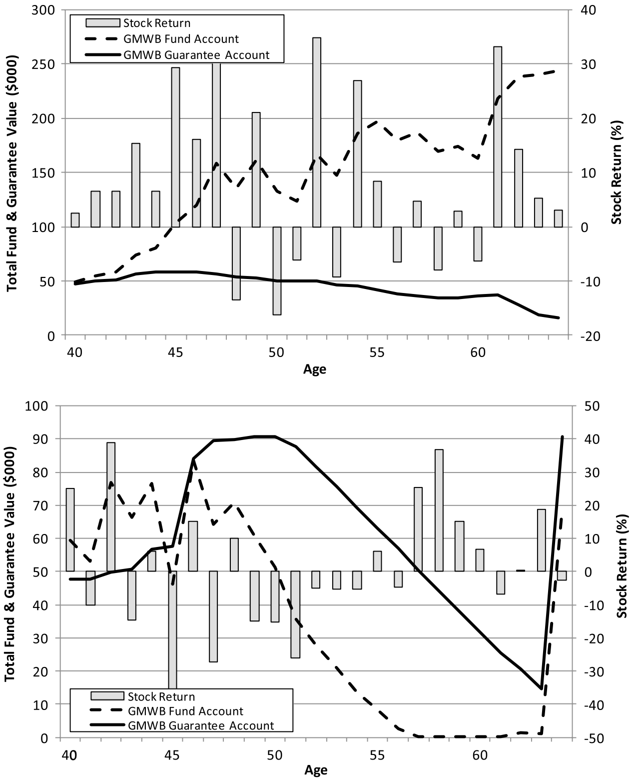

Fig. 3. Illustrative examples of paths of equity returns and GMWB account values to age 65. Notes: Each panel presents the development of the GMWB fund and guarantee accounts (in $000, left axis) as well as the realized stock returns (in %, right axis) for one selected simulated life cycle trajectory using the base case calibration: risk aversion $\rho = 5$; time preference $\beta = 0.96$; no bequest motive ($b = 0$); initial liquid wealth (labor income) of $120,000 ($29,600) at age 40; no labor income risk ($\sigma_u^2 = \sigma_n^2 = 0$); retirement age 65; pension replacement rate 73.6%; no taxes; risk-free interest rate 2%; mean stock return 6%; stock return volatility 18%.
Source: Authors' calculations.

well out of the money, and it is unlikely that the option will ever be exercised. Since the annual risk charge is based on the total value of the now-large Fund Account, the investor pays an increasing risk charge (in absolute terms) despite the low value of the guarantee. Seeking to reduce the costs of this portfolio insurance now rendered unnecessary by the bullish equity market, the investor continuously withdraws the maximum feasible amount from the annuity. Accordingly, the Guarantee Account declines monotonically until it is minimal at age 64. Since the maximum withdrawals are small compared to the overall Fund Account value, the latter continues to increase. Finally at age 65, the Fund Account is worth a great deal, which can be converted into a life annuity or taken as a lump sum.

By contrast, Fig. 3(bottom panel) depicts the investor's behavior in an unfavorable equity market scenario. As before, high stock returns earned early in life imply that the Fund Account will exceed the Guarantee level; again, the policyholder then withdraws funds from the GMWB to reduce her costs. Yet these withdrawals together with strongly negative equity returns thereafter subsequently reduce her Fund Account value to around the level of the Guarantee Account. As soon as the embedded put option is in the money (age 45 in bottom panel), the product once again offers the possibility of downside protection. Moreover, fees are still relatively low, so the individual will undertake additional annuity purchases. Unfortunately, a bear market experienced from her late 40s to her mid-50s dramatically erodes her Fund Account, making it unlikely that it will again reach or exceed the guarantee level. So despite the bull market during her late 50s and early 60s, the maximum payable from the GMWB is capped at the guarantee. This leads the investor to withdraw the maximum amount possible from the product and invest it in liquid assets, inasmuch as leaving the funds in the GMWB simply incurs risk charges and involves an opportunity cost (in the form of lost interest earnings). Only immediately prior to retirement will this individual move a substantial fraction of her liquid wealth into the GMWB, to benefit from the lifelong income provided by the annuity.

Table 1
Distribution of cashouts at age 65 and GMWB annuity income in the base case.

	Quantile				
	5%	25%	Median	75%	95%
Cashout amount ($000)	0	1.63	17.17	23.62	41.49
Annuity amount ($000)	3.99	6.59	10.27	16.41	33.63

Notes: Distributions are based on 10,000 simulated life cycles using the base case calibration: risk aversion $\rho = 5$; time preference $\beta = 0.96$; no bequest motive ($b = 0$); initial liquid wealth (labor income) of $120,000 ($29,600 p.a.) at age 40; no labor income risk ($\sigma_u^2 = \sigma_n^2 = 0$); retirement age 65; pension replacement rate 73.6%; no taxes; risk-free interest rate 2%; mean stock return 6%; stock return volatility 18%. *Source:* Authors' calculations.

To further illustrate the heterogeneity of outcomes with respect to how people deploy their GMWB accounts at retirement, we report next the distribution of amounts cashed out as well as the annuity income streams purchased at age 65.[16] We recall that this is the age at which policyholders can access their entire GMWB accounts, should they wish, or be defaulted into payout life annuities. Table 1 presents the quantiles of amounts cashed out as well as annuity income chosen, calculated over the 10,000 simulated life cycle paths. At the median, an individual cashes out around $17,000 at retirement, with the remainder of her GMWB wealth converted into a lifelong benefit stream of about $10,300 per year.

All individuals take some periodic annuity income, with the minimum yearly payout of about $1,350; as many as 12% withdraw no cash from their GMWBs. This raises the question as to what drives this decision. To investigate this, Fig. 4 relates the levels of GMWB wealth (horizontal axis) and liquid wealth (vertical axis) to the amounts cashed out at retirement (top panel) and the cumulative stock market returns (bottom panel) for all simulation paths. In the top panel, the color of the individual points indicates the amounts withdrawn at age 65, with turquoise (magenta) representing low (high) values as depicted in the color bar to the right of the panel. Those with higher GMWB values ($500,000 and more) hold virtually no liquid wealth. To diversify their post-retirement portfolios, they take large cashouts, thus retaining access to the stock market while receiving their bond-like payout annuities. While not shown here, the fraction of GMWB wealth cashed out is fairly constant at 8% for these individuals. At lower GMWB values, we see much more dispersion in liquid wealth and cashout patterns. Here, those who hold relatively high liquid wealth compared to their GMWBs already have adequate capital market exposure and, hence, they take little cash at retirement. Conversely, cashouts rise substantially for those having a larger share of their financial wealth in GMWBs. On this side of the GMWB wealth distribution, cashout ratios range from 0%–25%.

We investigate the role of their lifetime stock market experiences, to understand why some people arrive at retirement with

[16] As this is a high-dimensional problem in terms of state and decision variables, we find it helpful to conduct a statistical analysis instead of providing the usual discussion of policy functions.

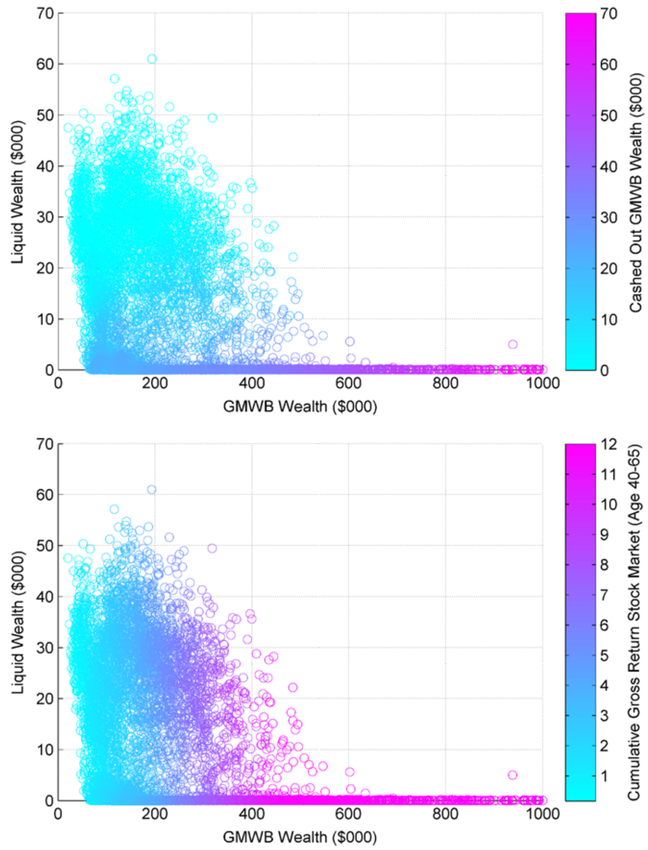

Fig. 4. Heterogeneity analysis: Cashout pattern at age 65. Notes: The top panel shows the amount cashed out of the GMWB at age 65 in relation to the total GMWB (x-axis) and liquid (y-axis) wealth available. The bottom panel shows the realized cumulative gross stock market return from age 40 to age 65 in relation to the total GMWB (x-axis) and liquid (y-axis) wealth available at age 65. Each circle represents one of the 10,000 simulated life cycles, and its color represents the amount cashed out (the cumulative stock return) — from low (turquoise) to high (magenta). Base case calibration: risk aversion $\rho = 5$; time preference $\beta = 0.96$; no bequest motive ($b = 0$); initial liquid wealth (labor income) of $120,000 ($29,600 p.a.) at age 40; no labor income risk ($\sigma_u^2 = \sigma_n^2 = 0$); retirement age 65; pension replacement rate 73.6%; no taxes; risk-free interest rate 2%; mean stock return 6%; stock return volatility 18%.
Source: Authors' calculations.

different fractions of liquid wealth in their portfolios. Peoples' different equity return paths are summarized using each individual's realized cumulative gross return on stocks between age 40 and 65; formally $\prod_{t=1}^{25} R_{i,t}$ $(i = 1, \ldots, 10000)$.[17] In the bottom panel of Fig. 4, the color of the individual points indicates the individual's cumulative stock return, with turquoise (magenta) representing low (high) values (see color bar to the right). Those who experienced superior cumulative stock market returns, substantially in excess of expectation ($4.3 = 1.06^{25}$), clearly end up with higher GMWB values ($500,000 and more). Those with the lowest cumulative stock market return are naturally those with lower total financial wealth. Their low GMWB account values and larger fractions of liquid assets can be explained by the fact that they took advantage of the GMWB guarantee and cashed out more prior to retirement, as illustrated in the bottom panel of Fig. 3. For those with mid-range GMWBs, the fraction of financial wealth held in liquid form decreases with increasing realized stock returns. Overall, our measure of stock market experience underscores the

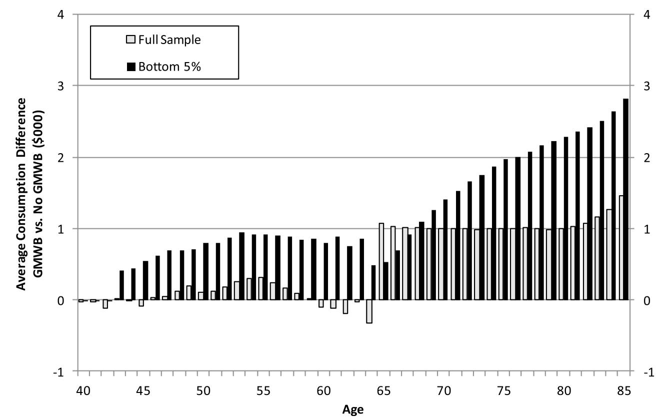

Fig. 5. Impact of GMWB access on average consumption. Notes: Differences in average consumption of (otherwise identical) individuals in a world with access to GMWBs compared to a non-GMWB world over time. Full Sample (gray bars): average consumption in both worlds calculated over all 10,000 simulated life cycles. Bottom 5% (black bars): average consumption in both worlds calculated over those 500 simulated life cycles that – at the respective age – exhibit the lowest consumption. Base case calibration: risk aversion $\rho = 5$; time preference $\beta = 0.96$; no bequest motive ($b = 0$); initial liquid wealth (labor income) of $120,000 ($29,600) at age 40; no labor income risk ($\sigma_u^2 = \sigma_n^2 = 0$); retirement age 65; pension replacement rate 73.6%; no taxes; risk-free interest rate 2%; mean stock return 6%; stock return volatility: 18%.
Source: Authors' calculations.

importance of equity returns as a key driver of cashouts at retirement.[18]

Finally, we analyze how GMWB access affects policyholders' optimal lifetime consumption. To this end, Fig. 5 presents the differences between average consumption of individuals in the GMWB world vs. that of otherwise identical households without access to this annuity product. Results are presented for the full sample of 10,000 simulated households (clear bars), as well as for a subset including only those with the 5% lowest consumption (black bars). First, looking at the full sample, we see that consumption differences are rather low prior to retirement, ranging between ±$300 (or up to 1% of the consumer's consumption level if he had no annuity available). During retirement individuals having access to GMWBs benefit from the lifelong income provided by the annuity. Accordingly, at around age 80, they can afford to persistently consume $1,000 per year (or about 3%) more than their counterparts in the non-GMWB world. Second, for those having experienced the worst shocks and are found in the lowest 5% consumption quantile, having access to GMWBs is even more important in protecting their consumption. For example, at age 50, the poorest people in the GMWB world can consume around $800 more per year or 3% of what they could have afforded in a non-GMWB world. The reason is that these individuals have highly unfavorable experiences in the stock market, but GMWB holders are protected by the embedded money-back guarantees while those without GMWB access have their financial wealth erode during capital market downturns. The additional consumption is even more pronounced in retirement: around age 80, they can afford to persistently consume $2,300 per year (or about 10%) more than their counterparts in the non-GMWB world. In other words, holding GMWBs insures a steady lifelong income from the annuity product so the elderly are less exposed to downturns in the stock market. To evaluate the total welfare gain associated with the discussed extension of consumption opportunities over the complete life cycle, we

17 For example, a cumulative gross return of 2 indicates that an investment of $1 at age 40 doubles by age 65.

18 In a descriptive linear regression of GMWB cashouts on cumulative stock returns, the estimated constant term is $5,139 and the slope is $2,624 (all coefficients are highly significant), and the R^2 is 0.54. Accordingly, our cumulative returns measure accounts for much but not all of the variance in cashouts.

Table 2
GMWB scenario analysis I: Alternative preference specifications.

	Base case	Less risk averse	More risk averse	Bequest motive	EZ-Preferences (high EIS)	EZ-Preferences (low EIS)
A: GMWB fund account ($000)						
Age 40–44	59.4	52.1	69.6	26.6	92.0	29.0
Age 45–49	96.2	73.0	109.9	70.0	164.4	104.5
Age 50–54	127.9	102.5	139.8	97.8	225.1	147.2
Age 55–59	158.7	135.7	170.3	126.9	292.5	179.2
Age 60–64	192.5	175.7	203.8	159.5	370.8	211.4
B: GMWB guarantee account ($000)						
Age 40–44	53.5	46.6	63.4	25.2	83.5	27.5
Age 45–49	69.0	50.3	78.5	55.0	120.7	85.8
Age 50–54	69.3	55.4	73.4	59.9	127.5	93.4
Age 55–59	59.7	55.2	60.1	57.7	122.8	79.9
Age 60–64	44.1	51.7	40.7	50.7	110.0	57.5
C: Stock investment ($000)						
Age 40–44	60.1	67.6	49.7	94.1	41.8	94.0
Age 45–49	41.9	66.0	24.1	71.3	14.5	42.7
Age 50–54	31.8	62.6	13.6	65.3	8.1	25.7
Age 55–59	19.4	55.2	7.1	49.2	6.5	15.8
Age 60–64	8.3	33.3	5.2	26.6	5.5	7.4
D: Bond investment ($000)						
Age 40–44	0.0	0.0	0.6	0.0	0.1	0.0
Age 45–49	0.8	0.0	5.0	1.6	2.2	0.8
Age 50–54	4.6	0.1	11.2	9.1	8.2	3.9
Age 55–59	11.3	0.7	13.7	25.1	9.9	8.8
Age 60–64	9.6	2.1	7.9	35.6	5.3	6.7
E: Summary statistics						
Cashout Ratio (%)	7.4	13.2	4.3	5.0	<0.01	7.1
Ann. Repl. Rate (%)	49.1	42.9	53.6	42.3	105.7	53.7
Welfare gain (%)	1.7	0.9	2.5	1.5	7.3	2.3

Notes: Expected values are based on 10,000 simulated life cycles; we report average values over 5-year age bands. Base case calibration: risk aversion $\rho = 5$; time preference $\beta = 0.96$; no bequest motive ($b = 0$); initial liquid wealth (labor income) of $120,000 ($29,600 p.a.) at age 40; no labor income risk ($\sigma_u^2 = \sigma_n^2 = 0$); retirement age 65; pension replacement rate 73.6%; no taxes, risk-free interest rate 2%; mean stock return 6%; stock return volatility 18%. Comparative static alternative calibrations alter one parameter at a time, with the remaining parameters equal to those in the base case. For alternative calibrations: less risk averse $\rho = 3$; more risk averse $\rho = 7$; bequest motive $b = 2$; high/low EIS (elasticity of intertemporal substitution): $\psi = 0.5$ ($\psi = 0.1$). EZ refers to the Epstein–Zin utility function; see text. *Source:* Authors' calculations.

compute the individual's certainty equivalent wealth at age 40 by inverting the value function according to $CE_t(W_t, F_t, G_t, \Phi_t) = ((1-\rho) \cdot J_t(W_t, F_t, G_t, \Phi_t))^{\frac{1}{1-\rho}}$. Then we calculate the relative change in certainty equivalent wealth when moving from a world without to a world with GMWBs. For our base case, this welfare gain amounts to a meaningful 1.7%.

5. Scenario analyses

The sensitivity analyses in this section assess the impacts of changes in preferences and risk factors. We also explore some scenarios permitting us to assess how the demand for GMWBs would change if Social Security benefits were reduced, and when the GMWB can be purchased within a tax-qualified pension account.

5.1. Alternative preference specifications

Table 2 illustrates how our results respond to alternative assumptions regarding the household's preferences. The first column replicates the base case findings where the coefficient of relative risk aversion was $\rho = 5$ and where there was no bequest motive ($b = 0$). Columns 2 and 3 report the impact of assuming lower and higher risk aversion values ($\rho = 3$ and $\rho = 7$ respectively). Column 4 provides results with a bequest motive of $b = 2$. Finally, Columns 5 and 6 present the outcomes for high and low elasticity of intertemporal substitution (*EIS*) under Epstein–Zin preferences. The table reports the impact of each experiment on the expected values of the Fund and Guarantee Accounts, as well as equity and bond holdings in liquid wealth. Values are presented as averages over 5-year age bands. We also provide the cashout ratio (the percentage of the GMWB value taken as a lump sum at age 65), the annuity replacement rate (the payout annuity as a percentage of last labor income), and the welfare gain from having access to the GMWB vs. no access.

Variations in risk aversion. An extremely risk averse individual will naturally invest less in stock and much more in GMWBs early on, compared to the base case. During her late 40s, she reduces her stock holdings considerably. This is partly to diversify her portfolio into bonds; partly to acquire more GMWBs (as seen from the increment in the Guarantee Account of $63,400 to $78,500); and partly to finance consumption in excess of labor income. This pattern continues until her mid-50s. When her labor income profile turns down, she withdraws from all accounts to smooth consumption. In particular, she exercises the liquidity option in the GMWB for consumption but not for investment purposes. This pre-retirement behavior is similar to, but more pronounced than, the base case. At retirement, she takes a smaller lump sum and preserves more in the GMWB annuity than the base case individual. Not surprisingly, according to our welfare measure, this highly risk-averse individual is better off having access to the GMWB than in the base case (2.5% vs. 1.7%).

Conversely, a less risk-averse individual in her early 40s invests more in stocks ($67,000 vs. $60,000), and less of her wealth in GMWBs ($47,000 vs. $54,000). This is because the guarantee feature of the annuity product provides a bond-like asset, which is less attractive to such a person. Subsequently, her equity holdings decrease, but by less than in the base case (50% vs. 80%). These withdrawals mainly finance excess consumption and not portfolio reallocations, as she makes virtually no additional bond or GMWB purchases. At age 65, she cashes out a larger share of her GMWB account (13.2% vs. 7.4%), producing a lower payout annuity. This is a result of her desire to hold more stocks in her post-retirement portfolio. As one might anticipate, the welfare gain from having access to this GMWB product is smaller than in the base case, under 1%.

Introduction of bequest motives. Next, we turn to the case where the consumer has a bequest motive of $b = 2$. Not surprisingly, this individual will optimally hold much more liquid wealth ($94,100 vs. $60,100) and less in GMWBs ($26,600 vs. $59,400) during her early 40s than in the base case, so as to leave an inheritance in case of early death. Initially, her liquid wealth is mostly held in stocks. As she ages, she gradually shifts her wealth into bonds to diversify her allocation of financial resources available for bequests. Despite the need to have liquid wealth for bequest purposes, her GMWB purchases rise strongly in her late 40s and early 50s, as can be seen from the rising value of the Guarantee Accounts. Subsequent withdrawals for consumption purposes are comparably low. At retirement, only 5% of the GMWB assets are cashed out, and the bulk of the money is converted into the payout annuity amounting to 42.3% of the last labor income, only 7% below the base case. Hence, although annuitized assets are no longer available for bequest, even individuals interested in transferring wealth to their heirs value the income certainty and the mortality credit associated with the annuity product.[19] This is also reflected in welfare gains similar to those in the base case (1.5% vs. 1.7%).

Epstein–Zin preferences. Next we study the sensitivity of our results with respect to the household's utility function. Specifically, we follow Blake et al. (2014), Cocco et al. (2005), and Gomes and Michaelides (2005) in assuming that the individual has Epstein–Zin preferences (see Epstein and Zin, 1989):

$$J_t = \left\{ \left(1 - \beta p_t^s\right) C_t^{1-1/\psi} + \beta E_t \left(p_t^s J_{t+1}^{1-\rho} \right)^{\frac{1-1/\psi}{1-\rho}} \right\}^{\frac{1}{1-1/\psi}}, \tag{17}$$

where ψ represents the elasticity of intertemporal substitution (*EIS*). Epstein–Zin preferences allow us to disentangle the household's risk aversion, which determines the utility of risky consumption at one point in time, from the household's *EIS*, which determines the utility of deterministic consumption at different points in time. By contrast, under CRRA preferences the relation of between risk aversion and *EIS* is fixed at $\psi = 1/\rho$, which means that Epstein–Zin preferences nest CRRA utility.

In line with Cocco et al. (2005), we study both a high-*EIS* scenario ($\psi = 0.5$) and a low-*EIS* scenario ($\psi = 0.1$), while holding risk aversion constant at our baseline level ($\rho = 5$). As investment opportunities improve, the high-*EIS* individual grows increasingly interested in intertemporal consumption substitution. That is, she is willing to markedly reduce consumption today in exchange for higher future consumption. She is also prepared to hold more illiquid assets if they can generate higher returns. Accordingly, she generally holds more assets than our baseline individual, with the lion's share being invested in the GMWB product. In her early 40s, GMWB (stock) holdings average $92,000 ($41,800), compared to $59,400 ($60,100) in the base case. Bond holdings are also higher than in the base case, though still negligible ($100 vs. $0 at the mean). Thereafter, the asset allocation pattern over time is generally comparable to the base case. GMWB (stock) holdings increase (decrease) continuously, while bond holdings increase until the late 50s and decrease in the early 60s.

The high-*EIS* individual also draws down liquid assets faster than the baseline household. By contrast, GMWB withdrawals are lower, as is clear from a comparison of the development of the GMWB Guarantee Accounts. The value of the Guarantee Account peaks in her early 50s at $127,500 in the high-*EIS* case, vs. $69,300 in the base case. Until her early 60s, the high-*EIS* individual on average withdraws $17,500 or 14% of the maximum guaranteed amount, compared to $25,200 or 36% in the base case. The high-*EIS* individual also refrains from cashing out accumulated GMWB funds at retirement. Instead, she rolls over the money into the life annuity, which – over the long run – generates higher returns than liquid assets due to the mortality credit. Consequently, her additional life annuity income from the GMWB product averages 105.7% of her last labor income, more than double the base case amount. This produces substantially higher retirement consumption compared to the baseline model. Overall, having access to the GMWB product improves household welfare in the high-*EIS* case by 7.3% in terms of certainty equivalent wealth, substantially above that of the base case.

By contrast, low-*EIS* households are more interested in smoothing consumption over time. Hence, compared to the high-*EIS* case, asset build-ups are lower throughout the accumulation phase, while consumption is higher. Total financial assets, i.e. the sum of stock and bond investments, and the GMWB Fund Account, average $123,000 ($225,500) in the early 40s (60s), compared to $133,900 ($381,600) in the high-*EIS* case. At the same time, more of the accumulated funds are held in stocks ($94,000 vs. $41,800 in the early 40s; $7,400 vs. $5,500 in the early 60s). The annuity product is less highly valued by the low-*EIS* individual. GMWB holdings are low in the early 40s ($29,000 on average) and only rise later in life. In the early 60s, the GMWB Fund Account averages $211,400, or 43% below the high-*EIS* case. This difference is driven by lower purchases and higher withdrawals from the GMWB. The Guarantee Account again peaks in the early 50s, averaging $93,400, i.e. $34,100 less than in the high-*EIS* case. Until the early 60s, withdrawals amount to 38% of that maximum guaranteed amount (vs. 14% under high *EIS*). Last-minute withdrawals just before retirement average 7.1% of the accumulated GMWB value, resulting in additional life annuity income of 53.7% of the last labor income. The smaller valuation of GMWBs in the low-*EIS* case is corroborated by a smaller welfare gain of 2.3%, less than one-third of that for the high-*EIS* case.

5.2. Additional risk factors

Next we evaluate how our results change when incorporating additional risks that households may face. Results are provided in Table 3, where Column 1 again repeats the base case, Column 2 reports results for a household exposed to labor income risk, Column 3 shows the impact of including interest rate risk, and Column 4 presents results assuming stock price dynamics follow a stochastic volatility process. Again, the table reports 5-year average Fund and Guarantee Account values, liquid equity and bond holdings, cashout ratios, annuity replacement rates, and welfare gains.

Labor income risk. Instead of assuming that the household has a deterministic hump-shaped labor income profile over time, we next posit that the labor income process is driven both by permanent and transitory income shocks. Following Cocco et al. (2005), we assume that the logs of the permanent and transitory income shocks are uncorrelated and normally distributed according to $\ln(N_t) \sim N\left(-0.5\sigma_n^2,\ \sigma_n^2\right)$ and $\ln(U_t) \sim N\left(-0.5\sigma_u^2,\ \sigma_u^2\right)$. In line with Love (2010), we set $\sigma_n^2 = 0.0169$ and $\sigma_u^2 = 0.0418$.[20]

[19] This is in line with previous findings on optimal annuitization behavior in the presence of bequest motives. For example, Horneff et al. (2008) report optimal expected annuity holdings of over 60% of financial wealth for individuals with a comparable bequest motive to that assumed here.

[20] As usual in the life cycle literature, we normalize by the permanent income in the case with labor income risk which reduces the complexity of the numerical optimization problem by one state variable.

Table 3
GMWB scenario analysis II: Additional risk factors.

	Base case	Labor income risk	Interest rate risk	Volatility risk
A: GMWB fund account ($000)				
Age 40–44	59.4	6.9	69.3	53.1
Age 45–49	96.2	34.3	96.6	76.2
Age 50–54	127.9	115.6	123.6	103.1
Age 55–59	158.7	199.0	152.7	132.0
Age 60–64	192.5	247.2	185.6	164.8
B: GMWB guarantee account ($000)				
Age 40–44	53.5	6.5	62.2	47.8
Age 45–49	69.0	29.7	66.6	54.1
Age 50–54	69.3	93.6	63.1	57.7
Age 55–59	59.7	136.4	53.3	55.9
Age 60–64	44.1	126.8	39.7	49.4
C: Stock investment ($000)				
Age 40–44	60.1	105.2	48.6	67.8
Age 45–49	41.9	109.8	37.1	65.8
Age 50–54	31.8	75.7	30.7	61.0
Age 55–59	19.4	27.9	20.0	43.7
Age 60–64	8.3	12.9	5.8	23.0
D: Bond investment ($000)				
Age 40–44	0.0	32.8	0.0	0.0
Age 45–49	0.8	55.2	0.1	0.9
Age 50–54	4.6	58.8	1.2	7.9
Age 55–59	11.3	60.3	4.1	23.1
Age 60–64	9.6	39.0	2.5	28.4
E: Summary statistics				
Cashout Ratio (%)	7.4	2.0	13.4	1.1
Ann. Repl. Rate (%)	49.1	65.0	45.3	46.1
Welfare Gain (%)	1.7	4.3	6.5	1.7

Notes: Expected values are based on 10,000 simulated life cycles; we report average values over 5-year age bands. Base case calibration: risk aversion $\rho = 5$; time preference $\beta = 0.96$; no bequest motive ($b = 0$); initial liquid wealth (labor income) of $120,000 ($29,600 p.a.) at age 40; no labor income risk $\left(\sigma_u^2 = \sigma_n^2 = 0\right)$; retirement age 65; pension replacement rate 73.6%; no taxes, risk-free interest rate 2%; mean stock return 6%; stock return volatility 18%. Alternative calibration for labor income risk sets $\sigma_u^2 = 0.0418$ and $\sigma_n^2 = 0.0169$. For interest rate risk, we use a 1-factor CIR model with $\kappa_r = 0.008$, $\theta_r = 0.051$, $\sigma_r = 0.01$. For volatility risk we use a Heston-model with parameters: $\rho_{VSP} = -0.674$, $\sigma_V = 0.2720$, $\kappa_V = 0.797$ and $\theta_V = 0.0284$; see text. *Source:* Authors' calculations.

Column 2 of Table 3 indicates that, in the presence of labor income risk, the household's initial annuity purchases are substantially lower in her 40s compared to the base case. This is because a substantial level of liquid wealth is needed as a buffer to smooth consumption in the face of possible negative labor income shocks. While GMWBs provide the individual with some liquidity, her purchases early in life only allow for small withdrawals (4% of the guaranteed amount at age 40, as discussed above), which may be insufficient to offset temporary income drops. Nevertheless, in her late 40s, she has already accumulated around 17% of her financial wealth in this special deferred annuity product. This finding contrasts with previous findings in the literature, where individuals of that age were said to avoid substantial holdings of deferred annuities without a withdrawal feature (e.g. Horneff et al., 2010; Maurer et al., 2013). In other words, the cashout feature of the GMWB is quite appealing in an economic environment with labor income uncertainty.

Later in life, as labor income risk-related liquidity needs decrease and GMWB liquidity increases, the individual seeks to compensate for her low purchases early in life. In her 50s, she will optimally purchase more GMWBs than in the base case. This is evident when examining the changes in average levels of the GMWB Guarantee Accounts, which rise from $29,700 at ages 45–49 to $136,400 at ages 55–59. As a result, from the late-50s on, the level of the GMWB Fund Account is higher than that in the base case. To compensate for the additional uncertainty due to labor income risk, the individual holds far more bonds throughout her work life.[21]

At retirement, her cashout ratio is 2%, more than five percent below the base case. At the same time, the annuity replacement rate is 60.2% compared to 49.1% in the base case. Since the pension benefit is also a function of risky labor income, the possibility of annuitizing GMWB assets is used to enhance retirement security.[22] The welfare gain of 4.3% indicates that an individual exposed to both labor income and capital market shocks values the access to GMWBs much more than her counterpart who only faces equity risk.

Interest rate risk. Next we introduce an uncertain interest rate into our model, which influences the economic environment in three key ways. First, the product provider must take into account uncertain interest rates when pricing the annuity, which in turn affects risk charges. Second, in the life cycle model the consumer no longer has a risk-free investment – that is, bonds are risky now as well as stocks – which affects investors' portfolio choices. Third, the annuity factor that consumers will face at age 65 now also becomes uncertain.

21 This finding is well documented in the literature. For a theoretical discussion in the context of a realistically calibrated life cycle model see, e.g., Cocco et al. (2005), for an analysis of empirical data see, e.g., Betermier et al. (2012).

22 In contrast to the base case, annuity incomes are higher and cashouts are lower across the entire distribution of our simulated individuals. One-third takes no cashouts, and even at the median, cashouts only amount to around $120. As in Fig. 4, those with higher GMWB values hold less liquid wealth and cash out more, while those with measurable holdings of liquid wealth cash out little if any. Cumulative stock market returns, however, are less clearly the driver of retirement cashouts, due to the additional influence of labor income risk. In particular, even individuals benefiting from high cumulative stock returns might not take cashouts due to having more liquid wealth. Here, neither a linear regression of cashouts on cumulative stock returns alone nor on both stock returns and the sum of realized labor incomes exhibit measurable explanatory power (adjusted R^2 around 0.08). Regression coefficients of the cumulative stock return, however, are statistically significantly negative, while the coefficient of labor income is statistically significantly positive.

To implement this approach, we use the Cox et al. (1985) one-factor model as it is both computationally tractable and very conventional in the literature. This model posits that the dynamics of the interest rate are described by a mean-reverting square-root process:

$$dr_t = \kappa_r\,(\theta_r - r_t)\,dt + \sigma_r\sqrt{r_t}dW_t \tag{18}$$

where θ_r represents the long-run mean of the interest rate, σ_r is the volatility, and κ_r is the elasticity that governs the speed of reversion to the long-run mean. We employ this model for GMWB pricing (see Appendix A) and also for the simulation-based evaluation of the optimal consumption and portfolio choice policies.[23] We calibrate the *CIR* model to historical data on 3-months US Treasury Bills over the period 1952–2001[24], provided by the WRDS database, using the martingale estimators described in Fischer et al. (2003). This produced the following parameter estimates: $\kappa_r = 0.008$, $\theta_r = 0.051$, $\sigma_r = 0.01$, and a correlation between the innovations in stock prices and interest rates of zero. We adjust the long-term mean to 2%, matching our interest rate assumption in the base case. This allows us to isolate the impact of interest rate volatility on our results.[25]

Incorporating interest rate risk results in a substantial increase in GMWB risk charges. For purchases at age 40, the annual fee rises by about 20% to 31 bps, vs. 26 bps in the base case (see Table A.1 in Appendix A). Again, the charge is higher for later purchase ages. At age 64, it is 11.02%, 22 bps above the baseline level.

The life cycle results are presented in the last third of Table 3. In the interest rate risk scenario, the household experiences a much more hostile capital market environment. With the lack of a truly risk-free investment and higher fees for GMWB guarantees, overall volatility increases while expected returns decrease, rendering saving and investing less attractive and boosting consumption early in the life cycle. Despite higher GMWB fees, however, the investor holds more of her assets in the annuity product early in life relative to the base case: about 17% more in her early 40s. This is because the GMWB is relatively safer when bonds are no longer risk-free. On average, the cashout ratio is higher and the replacement rate is lower, because the household annuitizes less when interest rates are low and annuity prices rise. Nonetheless, the welfare advantage conveyed by the GMWB is three times as large as in the fixed interest rate environment, underscoring the appeal of GMWB products as a source of steady income in retirement, especially under volatile market conditions.

Stochastic volatility. The assumption of independent and identically-distributed yearly log returns is still standard in the discrete time optimal life cycle portfolio and consumption choice literature (c.f., Cocco and Gomes, 2012; Love, 2010; Blake et al., 2014). An important reason for this is that it makes the model more tractable, given the complexity and computational burden of numerically solving the dynamic optimization problem. Nevertheless the extreme stock price movements seen during several crashes (e.g., the October 1987 crash, the dot-com bubble burst in the early 2000s, and the 2008 financial crisis) raise concern about this assumption, especially for longer-term problems (see, e.g., Hardy, 2003, p. 26). Furthermore, there is empirical evidence indicating that the volatility of stock prices over time is not stable but instead might suggest an additional risk factor. To address this issue, we use the Heston (1993) model in an additional sensitivity analysis allowing the (local) volatility of the stock price dynamics to be stochastic.[26] In this framework, the randomness of the stock price variance $V_t \geq 0$ is given as a square root process according to $dV_t = \kappa_V\,(\theta_V - V_t)\,dt + \sigma_V\sqrt{V_t}dW_{1,t}$ (similar to the CIR model for interest rates). Here θ_V represents the long-term volatility, κ_V the mean-reverting speed parameter, and σ_V the volatility of the volatility process. The dynamics of the underlying stock price process are described by $dSP_t = \mu SP_t dt + SP_t\sqrt{V_t}dW_{2,t}$ with drift μ. Assuming that the innovation of volatility process $W_{1,t}$ has a correlation of ρ_{VSP} with the innovation of the underlying stock price process $W_{2,t}$, we get (under the real-world P-measure):

$$dSP_t = \mu SP_t dt + SP_t\sqrt{V_t}\left(\rho_{VSP}dW_{1,t} + \sqrt{1-\rho_{VSP}^2}\,dW_{2,t}\right). \tag{19}$$

Fitting a stochastic volatility model to data on stock index returns and prices on derivatives is a well-developed field of research[27]; in what follows we use the parameterization provided by Aït-Sahalia et al. (2014) who use time series data and quotes on variance swaps on the S&P500 index. These parameters are as follows: $\rho_{VSP} = -0.674$, $\sigma_V = 0.2720$, $\kappa_V = 0.797$; the long-term variance is set to $\theta_V = 0.0284$ (comparable with our base-case model). Such a model produces time-varying volatility and highly non-normal log returns. Cross-sectional information on 100,000 simulated trajectories of stock prices over the 61-year horizon between ages 40 and 100 indicates an average skewness (across years) for the corresponding log returns of −0.4706, with a range of −0.7216 to −0.3537. The kurtosis metric has a mean of 6.1649 and a range of 3.6882 to 15.6938. Compared to the normal assumption such a capital market environment is much more 'dangerous' because of the high chance of extreme negative events.

We use this model under the risk-neutral Q-measure to price the guarantees of the GMWB.[28] This generates much higher risk charges compared to the base case (reported in Table A.1). For purchases at age 40, the annual fee rises from 0.26% in the base case to 0.71%. Based on these risk charges, we repeat the optimization and simulate 100,000 optimal life cycle patterns using the stochastic volatility model for stock price dynamics; results are provided in the final column of Table 3. Given such an unfavorable capital market environment with higher guarantee fees, the household invests less in annuity products and more in liquid assets. Nevertheless, compared to the base case, the consumer's reduction is fairly moderate: about 11% (17%) less in her early 40s (60s). The risk charges are much higher, but the economic value (in terms of lifetime utility) of a product offering downside protection via the money back guarantee, participation in increasing stock returns, partial liquidity to smooth consumption, and the opportunity to convert assets into a safe lifelong income stream at retirement remains substantial. The lower cashout ratio further supports the argument that households need safe income in retirement, particularly when return volatility is extreme. Overall, the welfare advantage provided by the GMWB is comparable to that of the base case, again underscoring the appeal of this product despite volatile capital market conditions.

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23 In our dynamic optimization, however, the autoregressive structure of the model would require integrating an additional state variable, which would render infeasible solving the optimization problem in a timely manner. Here, we therefore approximate the interest rate dynamics by an i.i.d. process with equal mean and volatility.

24 Given the unprecedented decline in interest rates over the last decade, including more recent observations would have produced invalid parameter estimates that could result in negative interest rates.

25 Moreover, we take a conservative approach and posit a flat term structure, which enhances interest rate volatility and, hence, GMWB risk charges.

26 For additional models for long-term stock returns providing time-varying parameters and non-normal distributions (e.g. the regime-switching model) see, e.g., Hardy (2003, chapter 2).

27 For an overview see for example Hurn et al. (forthcoming).

28 The parameter for the market price of volatility risk is $\lambda_V = -1.322$ as reported by Aït-Sahalia et al. (2014). The parameters under the Q-measure are $\kappa_V^Q = (\kappa_V + \lambda_V\sigma_V) = 0.4374$ and $\theta_V^Q = (\theta_V\kappa_V)/(\kappa_V\ \lambda_V\sigma_V) = 0.0559$. Under the risk-neutral Q-measure, the log returns show a mean skewness (across years) of −1.3225 and mean kurtosis of 13.067.

Table 4
GMWB scenario analysis III: Variation in policy parameters.

	Base case	Deferral age 85	Low replacement rate	Taxes
A: GMWB fund account ($000)				
Age 40–44	59.4	23.2	92.1	49.2
Age 45–49	96.2	92.1	125.0	90.3
Age 50–54	127.9	128.4	165.8	137.8
Age 55–59	158.7	158.2	211.6	186.2
Age 60–64	192.5	186.6	262.0	225.7
B: GMWB guarantee account ($000)				
Age 40–44	53.5	22.1	82.4	44.4
Age 45–49	69.0	75.9	85.1	67.0
Age 50–54	69.3	81.9	85.3	83.3
Age 55–59	59.7	71.8	77.8	87.0
Age 60–64	44.1	51.6	62.0	69.5
C: Stock investment ($000)				
Age 40–44	60.1	97.8	33.0	68.3
Age 45–49	41.9	48.5	31.6	43.8
Age 50–54	31.8	33.3	29.5	23.1
Age 55–59	19.4	20.4	16.3	9.8
Age 60–64	8.3	12.2	8.0	3.2
D: Bond investment ($000)				
Age 40–44	0.0	0.0	0.0	0.4
Age 45–49	0.8	1.2	0.0	3.2
Age 50–54	4.6	5.9	3.4	7.5
Age 55–59	11.3	12.5	15.7	7.8
Age 60–64	9.6	10.5	15.1	4.3
E: Summary statistics				
Cashout Ratio (%)	7.4	54.1	2.7	4.8
Ann. Repl. Rate (%)	49.1	35.4	71.0	59.9
Welfare Gain (%)	1.7	1.0	4.0	4.0

Notes: Expected values are based on 10,000 simulated life cycles; we report average values over 5-year age bands. Base case calibration: risk aversion $\rho = 5$; time preference $\beta = 0.96$; no bequest motive ($b = 0$); initial liquid wealth (labor income) of $120,000 ($29,600 p.a.) at age 40; no labor income risk ($\sigma_u^2 = \sigma_n^2 = 0$); retirement age 65; pension replacement rate 73.6%; no taxes; risk-free interest rate 2%; mean stock return 6%; stock return volatility 18%. Comparative static alternative calibrations alter one parameter with the remaining parameters set equal to those in the base case. For alternative calibrations: low replacement rate 50%; for taxes, GMWB available in tax-qualified account, taxation as described in Appendix B; deferral age 85 means annuitized GMWBs assets pay lifelong annuity benefits from age 85 on. *Source:* Authors' calculations.

5.3. Alternative economic environments

We next explore alternative economic environments likely to be of interest in the context of an aging society. The first evaluates the demand for GMWBs if annuity payouts are deferred until age 85, also referred to as *longevity income annuity* (IRI, 2014b). This is compatible with a focus on pure longevity risk protection, to insure people against running out of money in old age.[29] J. Mark Iwry, Senior Advisor to the Secretary of the US Treasury and Deputy Assistant Secretary for Retirement and Health Policy, has recently noted that "longevity income annuities can be an important option to help Americans plan for retirement and ensure they have a regular stream of income for as long as they live" (Iwry, 2014). The second explores how the demand for GMWBs might change if replacement rates from Social Security and private DB pensions were to fall from three-quarters to half of preretirement income. Such reductions can be expected in the face of widespread pension terminations in the US and the EU.[30] Our final policy scenario incorporates taxes, allowing the GMWBs to be purchased in a tax-qualified account. For instance, in the US, workers are allowed to accumulate pre-tax funds in Individual Retirement Accounts, and pay income taxes only when the benefits are withdrawn. Results appear in Table 4.

Pure longevity insurance. If, at retirement age 65, any GMWB cash value remaining after lump-sum distributions would need to be fully converted into a deferred annuity with benefit payments commencing only at age 85, this would markedly reduce demand for the product early in the life cycle. Compared to the base case, however, the most striking difference in outcomes is the much higher cashout ratio (54.1% vs. 7.4%) and the lower annuity replacement rate (35.4% vs. 49.1%) during retirement. When annuities pay benefits only from age 85, to smooth consumption the individual must rely heavily on her liquid wealth to finance consumption needs over her first 20 years in retirement. Despite the fact that this pure longevity risk insurance is much more restrictive than the immediate annuity in the base case, the individual still enjoys a positive, albeit small, welfare gain of 1% compared to 1.7% in the base case.

Lower social security replacement rates. If pension replacement rates fall to 50%, the consumer will need to invest much more in GMWBs to generate private retirement income protection. Compared to the base case, she will optimally hold much less stock and invest about three-quarters of her financial wealth in GMWBs, vs. about half in the base case. Her early GMWB contributions are paired with smaller withdrawals later in life, generating a 71% annuity replacement rate, or around 40% greater than the base case. The fact that the welfare gain is so large, 4%, confirms the value of this product in a more DC-oriented world.

Taxes. To explore how taxes change results, we construct a relatively realistic parameterization following the structure of current US tax rules. In particular, we implement a progressive tax system on labor income and capital gains taxes on investments held outside the GMWB (see Table A.2). The household can also open a tax-qualified account for purchases of annuities (up to an

29 For instance, at age 55, participants in Singapore's Central Provident Fund must use a portion of their retirement savings to purchase a deferred annuity. The German Riester pension system also requires that retirees use some or all of their assets to buy deferred annuities payable from age 85.

30 The US Department of Labor (2012) reports that the number of defined benefit plans has fallen by 54% between 1975 and 2010, while the number of defined contribution pensions rose by over 200%. The number of active DB plan participants decreased by 37% and rose by 554% for DC plans over the same period.

annual maximum); withdrawals are then taxed as ordinary income and, when made prior to age 60, with an additional penalty tax of 10%. Compared to the base case, the household now purchases more equity and holds less in the GMWB account. This is because the liquidity option in the GMWB is now less appealing, due to the income and penalty tax on premature withdrawals. Her higher labor income in her 50s puts her in a higher income tax bracket, making tax-deductible GMWB contributions relatively more valuable. Moreover, her new GMWB purchases continue until her 60s, while in the base case she begins to withdraw as of age 55. Toward the end of her work life, her fund account has now accumulated $225,700. The cashout ratio at retirement is substantially lower than in the base case, as large lump sum withdrawals will be taxed immediately at a high rate due to tax progressivity. Consequently, the after-tax replacement rate from the GMWB is much higher than in the base, 59.9% vs. 49.1%. In this environment, having access to GMWB annuities increases welfare by about 4%, twice as much as in the base case.

In sum: our sensitivity analyses show that, as a rule, households enjoy higher wellbeing when they have access to the GMWB products, compared to the base case.[31] This supports our conclusion that these products will be appealing to a variety of consumer types and in a variety of real-world market environments.

6. Discussion and conclusions

In this paper we develop and solve a realistically-calibrated life cycle consumption and portfolio choice model in discrete time using dynamic programming for a utility-maximizing household which can purchase – in addition to stocks and bonds – fairly-priced deferred variable annuities with Guaranteed Minimum Withdrawal Benefit (GMWB) riders. Prior to retirement, GMWBs offer access to the stock market with investment downside protection and minimum withdrawal guarantees. At retirement, they allow the policyholder to completely cash out her remaining accumulated account or else to convert it into a fixed lifetime income stream. Our contribution is to solve for the investor's optimal consumption, saving, investment, and GMWB variable annuity purchases as well as cashout paths. In extensive sensitivity analyses, we show how GMWB demand responds to alternate preferences, interest rate and labor income risk, taxes, and deferred payouts.

We find that the key guarantee and liquidity features, as well as the access to the mortality credit in this investment-linked deferred annuity, make such a blended product quite attractive to the consumers examined here. GMWBs contribute to enhanced lifetime utility across a number of scenarios and policy alternatives, compared to an environment without them. Whereas other authors have suggested that consumers wait to buy deferred annuities late in life, here we show that investors will optimally purchase reasonable amounts of GMWBs well before retirement, because of their flexibility and access to the stock market. This finding is consistent with empirical evidence of the growth in variable annuity demand over time (Geneva Association, 2013; IRI, 2014a). Our results indicate that policyholders will exercise this flexibility by taking withdrawals to adjust their portfolios and consumption streams along the way. Nevertheless, at retirement, they also convert much of their accumulated amounts into retirement annuities. Moreover, heterogeneity analysis suggests that differences in individuals' cashout and annuitization patterns result from variations in realized cumulative equity market returns and labor income trajectories. For those experiencing particularly bad income and capital market draws, the GMWB offers especially valuable consumption insurance.

Sensitivity analyses show that having access to GMWBs is particularly welfare-enhancing in the presence of labor market and interest rate risk. The existence of volatility risk in the stock market makes the guarantees more expensive, but keeps the economic value for policyholders in terms of lifetime utility. We also show that if the GMWB annuitization option at retirement were offered *only* as longevity risk insurance – requiring the buyer to defer her payout until age 85 – she would optimally cash out more to finance consumption until that age. Nevertheless, she still enjoys a welfare gain from access to the product. If retirement income replacement rates from Social Security and private defined benefit pensions were to drop by a third, individuals would purchase far more GMWBs and cash out less. And a similar result applies when GMWBs are available in a tax-qualified retirement accounts.

Our research will be of clear relevance to all those concerned with retirement security. Financial institutions such as insurers and mutual funds which offer retirement products as well as financial planners will find our work useful in their efforts to design and market appropriate products for real-world customers. Policymakers can also learn from our analysis, given their expressed interest in retirement solutions which integrate lifetime income protection into defined contribution retirement plans, or in other words: "putting the pension back in our private pension system" (Iwry, 2014). Because of their many advantages, reasonably-priced GMWBs including a longevity income annuitization option to hedge longevity risk are likely to become important candidates for automating retirement saving and decumulation.

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Appendix A. Valuation of GMWB variable annuities

The valuation of the options incorporated within a GMWB contract is complex, since the policyholder can decide every period whether and how much to withdraw. In addition, mortality aspects as well as the annual payment of risk charges (instead

[31] In further analyses not reported here in detail, we also study alternative calibrations with respect to initial wealth — high ($240,000) or low ($0). Both poorer and richer individuals continue to value the GMWB, though both exhibit lower demand for the product than does the base case individual. While the less wealthy household lacks financial resources, her wealthier peer holds more bonds to de-risk the overall portfolio of financial and human capital. Welfare gains from GMWB access are greater for the wealthier individual than for the less-fortunate (2.7% vs. 0.8%). Moreover, we study the impact of incorporating death benefits prior to age 65, which makes the product more expensive (as in Fig. 1). Results are similar to the base case: GMWBs purchases are lower early in the work life and higher just before retirement, while welfare gains are slightly lower.

Table A.1
Age dependent annual risk charges for single premium GMWBs.

A. Base Case: risk charge per annum (in %) with mortality risk pooling													
Age	40	41	42	43	44	45	46	47	48	49	50	51	52
Charge	0.26	0.29	0.32	0.34	0.37	0.40	0.44	0.48	0.53	0.58	0.64	0.70	0.78
Age	53	54	55	56	57	58	59	60	61	62	63	64	
Charge	0.87	0.98	1.11	1.27	1.48	1.74	2.09	2.57	3.26	4.34	6.27	10.80	
B. Risk charge per annum (in %) without mortality risk pooling													
Age	40	41	42	43	44	45	46	47	48	49	50	51	52
Charge	0.72	0.77	0.82	0.88	0.94	1.00	1.06	1.14	1.23	1.33	1.44	1.57	1.71
Age	53	54	55	56	57	58	59	60	61	62	63	64	
Charge	1.87	2.06	2.28	2.55	2.88	3.28	3.80	4.48	5.43	6.84	9.25	14.60	
C. Risk charge per annum (in %) with interest rate risk													
Age	40	41	42	43	44	45	46	47	48	49	50	51	52
Charge	0.31	0.34	0.36	0.39	0.42	0.45	0.49	0.53	0.58	0.63	0.69	0.76	0.84
Age	53	54	55	56	57	58	59	60	61	62	63	64	
Charge	0.94	1.05	1.18	1.35	1.56	1.83	2.19	2.67	3.37	4.47	6.43	11.02	
D. Risk charge per annum (in %) with volatility risk													
Age	40	41	42	43	44	45	46	47	48	49	50	51	52
Charge	0.71	0.74	0.78	0.83	0.87	0.92	0.98	1.05	1.13	1.22	1.33	1.43	1.56
Age	53	54	55	56	57	58	59	60	61	62	63	64	
Charge	1.71	1.87	2.07	2.33	2.65	3.03	3.53	4.18	5.14	6.61	9.15	15.12	

Notes: Annual risk charges are given in basis points (bps) of the current Fund Account value. The Fund Account is assumed to be fully invested in equities with a volatility of 18%, the risk-free rate is 2%, and the deferral period ends at age 65. For interest rate risk (Panel C), we use a 1-factor CIR model with $\kappa_r = 0.008$, $\theta_r = 0.051$, $\sigma_r = 0.01$. Parameters for the stochastic volatility model (Panel D) are $\kappa_V^Q = 0.4374$ and $\theta_V^Q = 0.0559$. *Source:* Authors' calculations.

of lump sums) must be taken into consideration. This paper uses the general pricing model of Bauer et al. (2008) assuming a stochastic withdrawal strategy. Hereby a rational investor selects from all possible strategies the withdrawal tactic that maximizes the option-value of the contract. Thus the risk charges calculated here represent upper bounds for the insurer's costs of hedging GMWB guarantees.

The value of the contract for a withdrawal strategy X can be represented as follows:

$$\begin{aligned} VL_0(X) &= \sum_{\tau=1}^{\mathrm{T}} {}_{\tau-1}p_x \cdot q_{x+\tau-1} \\ &\cdot E_Q\left[e^{-\Pi_{\tau=1}^{\mathrm{T}} r(\tau)}\left(L_{\mathrm{T}}(X,\tau) + WA_{\mathrm{T}}(X,\tau)\right)\right] \\ &+ {}_{\mathrm{T}}p_x \cdot E_Q[e^{-\Pi_{\tau=1}^{\mathrm{T}} r(\tau)}\left(L_{\mathrm{T}}(X,\mathrm{T}) + WA_{\mathrm{T}}(X,\mathrm{T})\right)] \end{aligned} \tag{A.1}$$

where ${}_{\tau}p_x$ denotes the cumulative probability to survive from age x (the start of the contract) to $x+\tau$, and $q_{x+\tau}$ is the probability of an $x+\tau$ year old individual to die over the next year. Mortality risk and financial markets risks are assumed to be independent. Q is the risk-neutral measure, which implies an arbitrage free financial market. Let WA_τ be a general withdrawal account and L_τ be the performance account that is the maximum of the Guarantee and Fund Account at the end of the deferral period at time T. The accounts L and WA develop as follows:

$$\begin{aligned} &L_{\mathrm{T}} = \max\left(G_{\mathrm{T}}, A_{\mathrm{T}}\right), \\ &WA_{\tau+1} = (WA_\tau + E_\tau)\cdot \exp(r(\tau+1)), \\ &A_{\tau+1} = \max\left(A_\tau - E_\tau, 0\right)\cdot \frac{SP_{\tau+1}}{SP_\tau}\cdot \exp(-\varphi), \\ &\frac{SP_{\tau+1}}{SP_\tau} = \exp\left\{\left(r(\tau) - \frac{\sigma^2}{2}\right) + \sigma z_\tau\right\}, \quad z_\tau \sim N(0,1), \\ &G_{\tau+1} = G_\tau - E_\tau, \end{aligned} \tag{A.2}$$

where E_τ is the withdrawal at time τ, SP_τ is the stock price and $r(\tau)$ is the log interest rate at time τ. Under risk-free interest rates, we set $r(\tau) = r_f$, while given stochastic interest rates, $r(\tau)$ follows the *CIR* 1-factor model described in Eq. (18); in the stochastic volatility case, the dynamic of stock prices is given in Eq. (19).

To determine the optimal withdrawal strategy X we have implemented the numerical method of Bauer et al. (2008). In our case, where the policyholder is not permitted to lapse the contract, it is optimal for the investor to always take the full amount permitted. This is in line with the findings of Dai et al. (2008), who studied the optimal withdrawal rate when lapsing is permitted. Here too, the minimum optimal withdrawal was the full withdrawal amount; that is, it was never optimal to not withdraw.

Table A.2
Federal income tax brackets for singles in 2012.

Lower bound	Upper bound	Marginal tax rate
$0	$8,700	10%
$8,701	$35,350	15%
$35,351	$85,650	25%
$85,651	$178,650	28%
$178,651	$388,350	33%
$388,351	–	35%

Source: IRS (2012).

We use quasi-Monte Carlo simulation with 100,000 iterations to determine value of the contract for a withdrawal strategy X, $VL_0(X)$. To calculate the fee, we use cubic spline interpolation. The Table A.1 shows the numerical values of the annual risk charges during the accumulation phase for GMWBs with and without mortality risk pooling, with interest rate and volatility risks (see also Fig. 1 in the main text). Mortality rates are specified by the US 2000 Basic Population Table.

Appendix B. GMWB annuities within a tax-qualified pension account

We integrate a US-type progressive tax system in our model to explore the impact of having access to GMWBs within a qualified (tax-sheltered) pension account of the EET type. Here the household must pay taxes on labor income and on capital gains from investments in bonds and stocks. During the worklife, it buys GMWBs worth A_t in the tax-qualified account which reduce the taxable income up to an annual maximum amount D_t. For Individual Retirement Accounts (IRA) in the US, this maximum amount is $5,000 for purchases between age 40 and 50, and

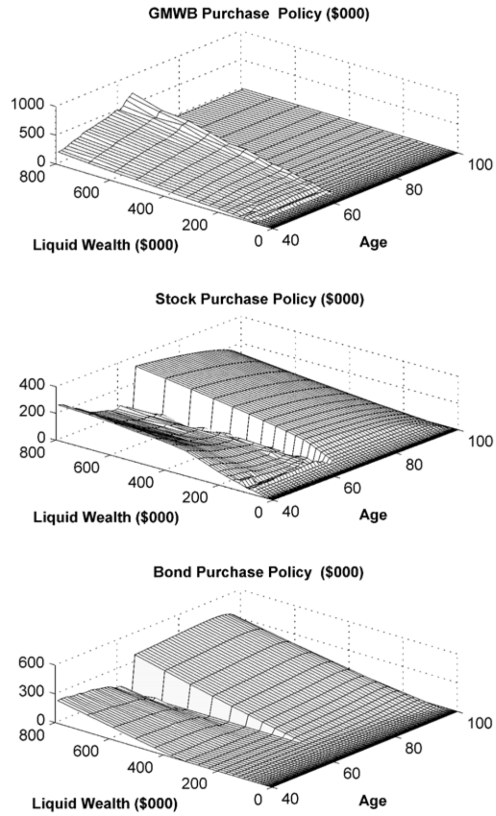

Fig. A.1. Asset allocation policy functions. Notes: Policy functions are depicted for allocations of wealth to GMWBs, stocks, and bonds, for alternative combinations of age and liquid wealth on hand assuming that no GMWBs have been purchased previously, i.e. $F = G = \Phi = 0$. Base case calibration: risk aversion $\rho = 5$; time preference $\beta = 0.96$; no bequest motive ($b = 0$); initial liquid wealth (labor income) of \$120,000 (\$29,600 p.a.) at age 40; no labor income risk ($\sigma_u^2 = \sigma_n^2 = 0$); retirement age 65; pension replacement rate 73.6%; no taxes; risk-free interest rate 2%; mean stock return 6%; stock return volatility 18%.
Source: Authors' calculations.

\$6,000 from age 51 onwards.[32] Correspondingly, withdrawals E_t from the GMWB in the tax-qualified account increase taxable income. During retirement, the individual can no longer purchase GMWBs. Instead, the household may cash out (some of) the accumulated wealth at retirement or convert the (remaining) assets into a lifelong stream of annuity benefits. Both cashouts as well as annuity benefits are taxed as ordinary income. Finally, the household's taxable income is reduced by a general standardized deduction *GD*. For a single household, this deduction amounts to \$5,950 per year. Consequently, taxable income is given by:

$$Y_{t+1}^{tax} = \begin{cases} \max\big[\max\left(S_t \cdot (R_{t+1} - 1) + B_t \cdot \left(R_f - 1\right);\ 0\right) \\ \quad + Y_{t+1} - \min\left(A_t; D_t\right) + E_t - GD; 0\big] \quad t < K \\ \max\big[\max\left(S_t \cdot (R_{t+1} - 1) + B_t \cdot \left(R_f - 1\right);\ 0\right) \\ \quad + Y_K + PA_K + LS_K - GD; 0\big] \quad t = K \\ \max\big[\max\left(S_t \cdot (R_{t+1} - 1) + B_t \cdot \left(R_f - 1\right);\ 0\right) \\ \quad + Y_K + PA_K - GD; 0\big] \quad t > K. \end{cases} \tag{B.1}$$

In line with US rules for federal income taxes, our progressive tax system has six income tax brackets. These brackets $i = 1, \dots, 6$ are defined by a lower and an upper bound of taxable income $Y_{t+1}^{tax} \in [lb_i, ub_i]$ and determine a marginal tax rate r_i^{tax}. For the year 2012, the marginal taxes rates for a single household are presented Table A.2.

Based on these tax brackets, the household's dollar amount of taxes payable is given by:[33]

$$\begin{aligned} Tax_{t+1}(Y_{t+1}^{tax}) = {} & \left(Y_{t+1}^{tax} - lb_6\right) \cdot 1_{\left\{Y_{t+1}^{tax} \geq lb_6\right\}} \cdot r_6^{tax} \\ & + \left(\left(Y_{t+1}^{tax} - lb_5\right) \cdot 1_{\left\{lb_6 > Y_{t+1}^{tax} \geq lb_5\right\}} \right. \\ & \left. + (ub_5 - lb_5) \cdot 1_{\left\{Y_{t+1}^{tax} \geq lb_6\right\}}\right) \cdot r_5^{tax} \\ & + \left(\left(Y_{t+1}^{tax} - lb_4\right) \cdot 1_{\left\{lb_5 > Y_{t+1}^{tax} \geq lb_4\right\}} \right. \\ & \left. + (ub_4 - lb_4) \cdot 1_{\left\{Y_{t+1}^{tax} \geq lb_5\right\}}\right) \cdot r_4^{tax} \\ & + \left(\left(Y_{t+1}^{tax} - lb_3\right) \cdot 1_{\left\{lb_4 > Y_{t+1}^{tax} \geq lb_3\right\}} \right. \\ & \left. + (ub_3 - lb_3) \cdot 1_{\left\{Y_{t+1}^{tax} \geq lb_4\right\}}\right) \cdot r_3^{tax} \\ & + \left(\left(Y_{t+1}^{tax} - lb_2\right) \cdot 1_{\left\{lb_3 > Y_{t+1}^{tax} \geq lb_2\right\}} \right. \\ & \left. + (ub_2 - lb_2) \cdot 1_{\left\{Y_{t+1}^{tax} \geq lb_3\right\}}\right) \cdot r_2^{tax} \\ & + \left(\left(Y_{t+1}^{tax} - lb_1\right) \cdot 1_{\left\{lb_2 > Y_{t+1}^{tax} \geq lb_1\right\}} \right. \\ & \left. + (ub_1 - lb_1) \cdot 1_{\left\{Y_{t+1}^{tax} \geq lb_2\right\}}\right) \cdot r_1^{tax}, \end{aligned} \tag{B.2}$$

where, for $A \subseteq X$, the indicator function $1_A \to \{0,\ 1\}$ is defined as:

$$1_A(x) = \begin{cases} 1 \mid x \in A \\ 0 \mid x \notin A. \end{cases} \tag{B.3}$$

In line with US regulation, the individual must pay an additional penalty tax of 10% on early withdrawals before age 60 ($t = 21$):

$$Tax_{t+1}(Y_{t+1}^{tax}) = \begin{cases} Tax_{t+1}(Y_{t+1}^{tax}) & t \geq 21 \\ Tax_{t+1}\left(Y_{t+1}^{tax}\right) + 0.1E_t & t < 21. \end{cases} \tag{B.4}$$

In order to incorporate progressive taxation into our life cycle model, we assume that all payments are made at the end of the period. Consequently, the new budget equations are given by:

$$W_t = \begin{cases} C_t + S_t + B_t + A_t & t < K \\ C_t + S_t + B_t & t \geq K \end{cases} \tag{B.5}$$

and

$$W_{t+1} = \begin{cases} S_t R_{t+1} + B_t R_f + Y_{t+1} + E_t - Tax_{t+1} & t < K \\ S_t R_{t+1} + B_t R_f + Y_K + PA_K + LS_K - Tax_{t+1} & t = K \\ S_t R_{t+1} + B_t R_f + Y_K + PA_K - Tax_{t+1} & t > K. \end{cases} \tag{B.6}$$

To make the results comparable to the base case, we adjust the labor income process such that the level of after-tax labor income corresponds to the labor income in the base case.

32 If the GMWB annuity is purchased in a company pension plan, e.g. 401(k) plan, the maximum contribution limit is higher. Also some employers make additional ('matching') contribution into the pension accounts of the employees.

33 Here we assume that capital gains are taxed at the same rate as labor income, so we abstract from the possibility that long-term investments may be taxed at a lower rate.

Appendix C. Asset allocation policies

Fig. A.1 displays three policy functions for the base case allocations of wealth to GMWBs, stocks, and bonds, for alternative combinations of age and liquid wealth on hand. These assume that no GMWBs have been purchased previously (i.e. $F = G = \Phi = 0$). All parameters are as reported in notes to Fig. 2.

References

Aït-Sahalia, Y., Karamen, M., Mancini, L., 2014. The term structure of variance swaps and risk premia. SSRN-Working Paper 2136820 (May, 10, 2014).

Ameriks, J., Caplin, A., Laufer, S., Van Nieuwerburgh, S., 2011. The joy of giving or assisted living? using strategic surveys to separate public care aversion from bequest motives. J. Finance 66, 519–561.

Bauer, D., Kling, A., Russ, J., 2008. A universal pricing framework for guaranteed minimum benefits in variable annuities. Astin Bull. 38, 621–651.

Bernheim, B.D., Shleifer, A., Summers, L.H., 1985. The strategic bequest motive. J. Polit. Econ. 93, 1045–1076.

Betermier, S., Jansson, T., Parlour, C., Walden, J., 2012. Hedging labor income risk. J. Financ. Econ. 105, 622–639.

Blake, D., Wright, D., Zhang, Y., 2014. Age-dependent investing: Optimal funding and investment strategies in defined contribution pension plans when members are rational life cycle financial planners. J. Econom. Dynam. Control 38, 105–124.

Brennan, M., Schwartz, E., 1976. The pricing of equity-linked life insurance policies with an asset value guarantee. J. Financ. Econ. 3, 195–213.

Brown, J.R., Mitchell, O.S., Poterba, J.M., Warshawsky, M.J., 2001. The Role of Annuity Markets in Financing Retirement. MIT Press, Cambridge, M.A.

Chalmers, J., Reuter, J., 2012. How do retirees value life annuities? evidence from public employees. Rev. Financ. Stud. 25, 2601–2634.

Cocco, J., Gomes, F., 2012. Longevity risk, retirement savings, and financial innovation. J. Financ. Econ. 103, 507–529.

Cocco, J., Gomes, F., Maenhout, P., 2005. Consumption and portfolio choice over the life cycle. Rev. Financ. Stud. 18, 491–533.

Cox, J.C., Ingersoll, J.E., Ross, S.A., 1985. A theory of the term structure of interest rates. Econometrica 53, 385–407.

Dai, M., Kwok, Y.K., Zong, J., 2008. Guaranteed minimum withdrawal benefit in variable annuities. Math. Finance 18, 595–611.

Davidoff, T., Brown, J., Diamond, P., 2005. Annuities and individual welfare. Am. Econ. Rev. 95, 1573–1590.

Dellinger, J.K., 2006. The Handbook of Variable Income Annuities. John Wiley & Sons, Inc., Hoboken, New Jersey.

Epstein, L., Zin, S., 1989. Substitution, risk aversion and temporal behaviour of consumption and asset returns: A theoretical framework. Econometrica 57, 937–969.

Feldstein, M., 2009. Reducing the risk of investment-based social security reform. In: Brown, J., Lieberman, J., Wise., D. (Eds.), Social Security Policy in a Changing Environment. Chicago University Press, Chicago, pp. 201–218.

Feldstein, M., Ranguelova, E., 2001. Individual risk in an investment-based social security system. Am. Econ. Rev. 91, 1116–1125.

Fischer, T., May, A., Walther, B., 2003. Anpassung eines CIR-1-Modells zur Simulation der Zinsstrukturkurve. Bl. DGVFM 26 (2), 193–206.

Gao, J., Ulm, E.R., 2012. Optimal consumption and allocation in variable annuities with guaranteed minimum death benefit. Insurance Math. Econom. 51, 586–598.

Geneva Association 2013. Variable Annuities — An Analysis of Financial Stability. The International Association for the Study of Insurance Economics, Geneva.

Gomes, F., Michaelides, A., 2005. Optimal life-cycle asset allocation: Understanding the empirical evidence. J. Finance 60, 869–904.

Hardy, M., 2003. Investment guarantees: Modeling and risk management for equity-linked life insurance. John Wiley & Sons, Inc., New Jersey.

Heston, S., 1993. A closed form solution for options with stochastic volatility with applications to bond and currency options. Rev. Financ. Stud. 6 (2), 327–344.

Horneff, W., Maurer, R., Mitchell, O.S., Stamos, M., 2009. Asset allocation and location over the life cycle with survival-contingent payouts. J. Bank. Financ. 33, 1688–1699.

Horneff, W., Maurer, R., Rogalla, R., 2010. Dynamic portfolio choice with deferred annuities. J. Bank. Financ. 34, 2652–2664.

Horneff, W., Maurer, R., Stamos, M., 2008. Life-cycle asset allocation with annuity markets. J. Econom. Dynam. Control 32, 3590–3612.

Huang, H., Milevsky, M.A., 2008. Portfolio choice and mortality-contingent claims: The general HARA case. J. Bank. Financ. 32, 2444–2452.

Hurd, M., 1989. Mortality risk and bequests. Econometrica 57, 779–813.

Hurn, A.S., Lindsay, K.A., McClelland, A.J., 2014. Estimating the parameters of stochastic volatility models using option price data. J. Bus. Econom. Statist. forthcoming, http://dx.doi.org/10.1080/07350015.2014.981634.

Inkmann, J., Lopez, P., Michaelides, A., 2011. How deep is the annuity market participation puzzle? Rev. Financ. Stud. 24, 279–319.

Insured Retirement Institute (IRI), 2014a. IRI Fact Book. Washington, D.C. Downloaded 9-24-14. www.myirionline.org/home.

Insured Retirement Institute (IRI) 2014b. Treasury makes longevity annuities more accessible in retirement plans. IRI Insight. 2 (6), September. Downloaded 9-30-14. myIRIonline.org.

Internal Revenue Service (IRS), 2012. Form 1040 (Tax Tables), Tax table and tax rate schedules. Downloaded 03/12/2013. www.irs.gov/pub/irs-pdf/i1040tt.pdf.

Iwry, M., 2014. Excerpted Remarks of J. Mark Iwry, Senior Advisor to the Secretary of the Treasury and Deputy Assistant Secretary for Retirement and Health Policy, to the Insured Retirement Institute. U.S. Treasury Department Office of Public Affairs. July 1, 2014, Washington, D.C..

Love, D.A., 2010. The effects of marital status and children on savings and portfolio choice. Rev. Financ. Stud. 23, 385–432.

Lusardi, A., Mitchell, O.S., 2007. Baby boomer retirement security: The roles of planning, financial literacy, and housing wealth. J. Monet. Econ. 54, 205–224.

Lusardi, A., Mitchell, O.S., 2008. Planning and financial literacy: How do women fare? Am. Econ. Rev. P&P 98, 413–417.

Mahayni, A., Schneider, J.C., 2012. Variable annuities and the option to seek risk: Why should you diversify? J. Bank. Financ. 36, 2417–2428.

Maurer, R., Mitchell, O.S., Rogalla, R., Kartashov, V., 2013. Lifecycle portfolio choice with systematic longevity risk and variable investment-linked deferred annuities. J. Risk Insur. 80, 649–676.

Milevsky, M.A., Salisbury, T.S., 2006. Financial valuation of guaranteed minimum withdrawal benefits. Insurance Math. Econom. 38, 21–38.

Milevsky, M.A., Young, V., 2007. Annuitization and asset allocation. J. Econom. Dynam. Control 31, 3138–3177.

Mitchell, O.S., Poterba, J., Warshawsky, M., Brown, J., 1999. New evidence on the money's worth of individual annuities. Am. Econ. Rev. 89, 1299–1318.

Poterba, J., Rauh, J., Venti, S., Wise, D., 2007. Defined contribution plans, defined benefit plans, and the accumulation of retirement wealth. J. Public Econ. 91, 2062–2086.

Steinorth, P., Mitchell, O.S., 2012. Valuing variable annuities with guaranteed minimum lifetime withdrawal benefits. Pension Research Council Working Paper 2012-04. The Wharton School.

US Department of Labor, 2012. Private pension plan bulletin historical tables and graphs. Employee Benefits Security Administration. November.

US Department of the Treasury, 2014. Treasury issues final rules regarding longevity annuities. Press Release. http://www.treasury.gov/press-center/press-releases/Pages/jl2448.aspx.

van Rooij, M., Lusardi, A., Alessie, R., 2011. Financial literacy and stock market participation. J. Financ. Econ. 101, 449–472.

Xiong, J.X., Idzorek, T., Chen, P., 2010. Allocation to deferred variable annuities with GMWB for life. J. Financ. Plann. 23, 42–50.

Yaari, M., 1965. Uncertain lifetime, life insurance, and the theory of the consumer. Rev. Econom. Stud. 32, 137–150.