

Double Round Auctions

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Abstract

This article analyzes auctions that can feature two bidding rounds for the sale of a single good. In the first round the seller, after analyzing the received bids, may elect to have k bidders rebid. The highest bidder in the second round receives the asset at the highest bid price. We use a sample of 67 properties that sold through this auction process. The 40 hotels in this sample are matched to a control group of 165 hotel properties that were sold in the conventional manner in order to develop a hedonic model for sale prices. From this model, we find that the double round auction mechanism increases the seller's revenue significantly: specifically, we estimate that the expected value of a double round auction sale (in either round) is 15.4% larger than if the property were sold using traditional methods. We further find (controlling for property characteristics) that the average bid increases by 3.7% from the first to the second round and that the expected value of a second round bid is 8.5% larger than the expected value of a first round bid.

Key words: auction theory, winner's curse.

1 Introduction

Small guys need not have applied. And even larger ones may not have been inclined. But for those big enough and brave enough, the ultimate reward was sufficient for several major developers to submit bids on Tuesday to construct a new 2.1 million square feet F.B.I. headquarters for 11,000 employees in return for the right to tear down the aging J. Edgar Hoover Building and redevelop its prime real estate on Pennsylvania Avenue ... the protracted process generated frenzied activity in recent days and weeks as developers scrambled to assemble teams with the resources and ability to do the job. They hoped their bids would lead to yet another and final round of competition, as the government chooses developers to submit specific proposals late this year.

New F.B.I. Headquarters Enters the Bidding Phase, *New York Times*, February 10, 2015.

The above quote provides an example of the type of multiple round auction analyzed in this paper. Auctions have played an important role in financial markets for centuries; for example, auctions for stamp collectors were common in the 1800s.¹ Many diverse forms of auctions are currently used for selling a wide variety of assets, including bonds, distressed securities, oil-bearing properties,² state owned enterprises,³ spectrum rights,⁴ art, wine and real estate.⁵ More recently, Internet auctions play an increasingly important role in modern commerce.⁶ They also represent a rich area for theoretical analysis, with Vickrey (1961) being a seminal contribution.

¹See Lucking-Reiley (2000) for an interesting history of Vickrey auctions in practice.

²Reece (1979).

³Atanasov (2005) provides an interesting example from Bulgaria.

⁴See, i.a., Bruner, Goeree, Holt and Ledyard (2010).

⁵A historical perspective is given in Cassady (1967). Surveys include Hansen (1985), McAfee and McMillan (1987), Milgrom (1987, 1989, 2004), Rothkopf and Harstad (1994), Quan (1994), Klemperer (1999, 2004) and Bergemann and Said (2011).

⁶See, for example, Bajan and Hortacsu (2003) and Ariely, Ockenfels and Roth (2005)

One of the most challenging problems in financial economics is the design of efficient auction mechanisms. However, a substantial gap exists between the theoretical and empirical aspects of this literature. As noted by Rothkopf and Harstad (1994):

“... we find gaps between bidding theory and bidding decision making which seriously limit the direct usefulness of much theory to those who decide how much to bid and those who design auctions.”

This study tries to address this gap between the theory and practice of bidding theory by theoretically and empirically analyzing a specific auction mechanism that is used in many types of large dollar transactions. In particular it is used in situations where it is costly to determine a bid or where there is significant uncertainty about the asset’s value. While we study real estate transactions, the applications are much broader; for example, in the auctioning of electrical generating assets, privatizations, and mergers and acquisitions.⁷ The approach commonly taken is called *indicative bidding*. This auction features two rounds of bidding. In the first round the auctioneer solicits a large group of potential bidders and asks them to submit bids indicating their interest in the asset being auctioned. In some cases the first round bids can be binding but in most examples the first round is used merely for information gathering. The auctioneer then selects a subset of k of these bidders to bid a second time, often using the k^{th} largest bid as a minimum for the second and final round.⁸ It can also be the case that further information is released before the second round bidding begins.

The special case of a multiple round auction mechanism that we analyze is a double round auction for the sale of a single good. The key issue we address is whether or not the double round auction selling process we observe for real estate auctions is superior to the more traditional methods of selling. This mechanism is often used when auctioning either

⁷See Kagel, Pevnitskaya and Ye (2004).

⁸Many variations exist, such as survival auctions (Kagel, Pevnitskaya and Ye 2006), ascending-price clock auctions (McAdams, Fujishima and Shoham 1999) or Ausubel auctions (Ausubel 2004).

an equity or debt contract on a unique real estate property.⁹ Prior to the first round of bids, the seller provides potential buyers with data on the asset being sold. Bids are then submitted and the seller selects a subset of the highest bidders from the first round to rebid in the second round. The highest bidder in round two typically, but not always, is declared the winner. This situation could occur if the highest bidder has potential credibility issues, for example with respect to the strength of its financing.¹⁰

In the practical and theoretical literature this auction has been largely ignored although for a number of major real estate brokers it is the primary manner in which they sell so-called “trophy properties.”¹¹ These are properties of high value (typically exceeding \$30 million) with an uncertain true value (due to lack of comparable assets) and relatively few buyers with the expertise and capital to purchase the asset.¹²

The first part of our analysis uses a unique data set of 67 transactions using this auction process matched to a much larger set of comparable properties that used a conventional selling approach. The primary focus of our empirical analysis is to determine the economic value of choosing the dual round process over more conventional methods.¹³ Our approach is relatively straightforward. We develop a hedonic model for hotel sale prices and find that the double round auction mechanism increases the seller’s revenue significantly: specifically, we estimate that the expected value of a double round auction sale in either round is 15.4% larger than if the property were sold using traditional methods. This figure is economically meaningful; it implies that assets using this auction mechanism, after controlling for observable property characteristics, sell for an average of \$5.2 million more than if sold in the

⁹It has other applications where there is a substantial cost in making an informed bid. An example would be in bidding on a new aircraft; see Hansen (1991).

¹⁰This situation is analogous to evaluating potential acquirers in a merger setting.

¹¹Hansen (1991) treats the case of a two-round auction where the seller sequentially releases information about the asset (in this application, a company) being sold. The motivation is that bidders can obtain valuable competitive information through the auction process. In contrast, in our setting all information is available before the first round.

¹²Foley (2003) provides some institutional data on real estate auctions. Ashenfelter and Genesove (1992) analyze the sale of apartment units. See also Mayer (1992, 1995) and Lusht (1992).

¹³Rolfe and Windle provide empirical support for the efficiency of multiple round auctions for conservation auctions.

conventional manner. We further analyze the bidding behavior in each round. We find that the average bid increases by 3.7% (again controlling for property characteristics) from the first to the second round. We also show that the expected value of a second round bid is 8.5% larger than the expected value of a first round bid.

To help motivate our empirical work, the second part of our analysis is a theoretical treatment of this double round auction, characterizing the optimal strategies of both buyers and sellers. The basic tension in our model arises from the seller’s desire to increase the number of bidders (in order to maximize the potential selling price) and the resulting more conservative bidding that comes from having a larger number of bidders: a reflection of the “winner’s curse” phenomenon.¹⁴

The study is organized as follows: Section 2 briefly surveys the relevant literature. Section 3 describes the data and empirical framework. Sections 4 and 5 present our empirical analysis and conclusions. The hedonic model variables are defined in Appendix 1. The theoretical model is presented in Appendix 2.

2 Literature Review

The relevant auction research is very extensive. We focus here solely on the literature that involves bidding over one or more rounds or auctions with sequential information release. An important type of auction is the survival auction; important sources include: McAdams, Fujishima and Shoham (1999); Perry, Wolfsetter and Zamir (2000); Ye (2007); Kagel, Pevnit-skaya and Ye (2007). A central aspect of this auction form is the nature of the information revealed in each round. For example, if a highly informed agent remains in the auction after others have dropped out, that can signal to the other remaining bidders that the item is of high quality.

While, as noted earlier, many variations exist, the key characteristic of a survival auction is that there are multiple rounds of bidding and a specific rule that sequentially reduces the

¹⁴Dyer and Kagel (1996) study the winner’s curse in the commercial construction industry.

number of bidders. For example, in one form of a survival auction, in each round, the bidder with the lowest bid is eliminated from the auction process. In the subsequent rounds, bidders place new bids where the minimum allowable bid is the lowest bid from the previous round. McAdams, Fujishima and Shoham (1999) show that the allocations from this survival bid auction are equivalent to ascending-bid auction allocations. Perry, Wolfsetter and Zamir (2000) examine a double round auction where only 2 of N bidders are invited to the second round and find that the expected revenue is identical to the ascending bid auction.

However the ascending bid auction has practical limitations. Most notably all bidders must be present during the auction process in order to drop out of the auction and the ascending bid auction can thus be a drawn out process and the length of the auction can be random. Instead, the survival auction as described above guarantees that a single good will always be allocated after $N - 1$ rounds, when N bidders are present.

A permutation of the survival auction is an auction with indicative bidding as described in Ye (2007). In this model, agents have noisy valuations and make (indicative) bids in the first round. The auctioneer uses these bids to invite a subset of the bidders to the second round. Bidders who remain in the second round pay a fixed cost in order to acquire another signal about the object's value. This cost is interpreted as the cost of due diligence on the bidders' part.

Ye (2007) provides two important results in the two-round survival auction: First, if there is a fixed entry-cost for bidding in the second round, no symmetric increasing equilibrium can exist when the indicative bids are non-binding. However, the auctioneer can provide a subsidy to the bidders that will ensure the existence of an equilibrium. Second, entry costs from all bidders are borne by the auctioneer. This suggests the auctioneer has an incentive to mitigate the total amount of information acquisition costs by limiting the number of bidders in the second round.

Despite these theoretical results, empirical work on survival auctions with indicative bid-

ding is sparse and confined to experimental settings with relatively minor economic rewards.¹⁵ Kagel, Pevnitskaya and Ye (2007) examine survival auctions using a laboratory experiment and find that survival auctions outperform Vickery auctions in terms of allocative efficiency. Kagel and Levin (2001) document the superior performance of ascending price auctions, noting the transparency and simplicity of these auctions compared to sealed-bid auctions.

3 Data and empirical framework

Our analysis is primarily based on a proprietary data set of double round auctions for hotel, office and retail properties. It includes 40 hotel properties, 14 retail properties and 13 office properties. Due to the small number of auctions for office and retail, our analysis using hedonic methods is confined to the hotel properties. However, we use the office and retail properties in our analysis of bidding behavior. The model presented in the appendix provides some motivation for our empirical testing. The intuition is quite simple. In round 1 the seller should invite as many potential buyers as possible in order to obtain an accurate estimate of the signals the bidders have of the value of the asset. Armed with this information, the seller then chooses an optimal number of bidders to re-bid in the second round by balancing the winner's curse effect against the potentially higher price resulting from having a large number of possible buyers.

3.1 Auction and control sample

Table 1 gives the characteristics of the properties in our double round auction sample. The 40 hotel properties are sold in 40 separate double round auctions. Although these properties were all subject to a potential second round of bidding, 11 auctions lasted only 1 round.¹⁶ A total of 226 bids are made over all first round auctions for an average of 6.5 first round

¹⁵Kagel and Levin (2002) survey the experimental research in this area.

¹⁶We have no data on the bidder's characteristics (such as experience or financial strength) which could potentially influence the decision to allow certain bidders to bid in the second round.

bids per property.¹⁷ A total of 105 bids are made over all second rounds for an average of 2.9 bids per property. The average bid in the first round is \$34.7 million. The average bid in the second round is \$32.9 million. This decrease is largely due to the number of properties that sold after the first round, presumably because the seller was satisfied by the price she received. Looking at the difference in bids between the first and second rounds we find that the average increase was 1.9%. We performed a *t*-test for the difference in the standard deviation of the bids in the second round and the standard deviation of the bids in the first round that were invited to the second round. Measurement error aside, the difference in the log bid standard deviation was 0.013 with a *t*-statistic of 1.4572.

These 40 hotel properties are of course a small subset of the entire population of hotel sales. In order to estimate the price premium associated with the first round bids, we collect 1,100 hotel sales from Costar and Real Capital Analytics. After cleaning, there were 940 remaining comparable sales. Of these 940 sales, there are 165 sales that are either off market sales or sales without a listing broker. Off market sales are sales that were not listed for sale using a listing broker. Sales without a listing broker are those sales where it is explicitly stated that no listing broker was used.¹⁸ Off market sales and sales without a listing broker are sales where we are certain that an auction of some type was not used to sell the property.

The remaining 165 properties are used as a comparable group in our estimation. By including only non-auction properties in the comparable group, we are able to mitigate the effect of any omitted variable bias. Omitted variable bias could impact our analysis in the following way. If we were to include all 1,100 hotel sales, it is possible that we could include properties that were auctions using a double round auction but were not identified as such in our data. By including these observations in a comparable group, we could not interpret any premium or discount associated with observations in the double round auction data as a premium or discount compared to a traditional, non-auction sale.

¹⁷Practitioners using this auction often noted that bidders in the first round were trying to find a price floor and made bids that they believed would be large enough to get them into the second round.

¹⁸This is different from the situation where a listing broker was used, but the listing broker information is missing in the data.

The variables used in this analysis are defined in Appendix 1 and the descriptive statistics for the comparison sample are reported in Table 2. The mean sale price is \$29.1 million. The average number of rooms was 193. Relatively few sales were luxury properties (5.6%) or economy hotels (5.3%). There are a significant number of sales where the property is REO (14.8%), where the seller is bankrupt (2.2%) or when the sale is distressed (7.7%).

The corresponding statistics for our double round auction sample are given in Table 3. The figures are quite similar, although this sample contains slightly larger and somewhat more sales under duress. The mean sale price is \$33.8 million. The average number of rooms was 237. Relatively few sales were luxury properties (5.6%) or economy hotels (5.3%). The sales where the property is REO (25.7%), where the seller is bankrupt (11.4%) or when the sale is distressed (2.9%).

Table 4 displays the least-squares estimates from the regression $variable = \alpha + \beta \times 1_{DoubleRound}$ using the 165 control group properties and the 40 hotels in the double round auction. Here $1_{DoubleRound}$ is set to 1 for the double round properties and to 0 for the control group properties. The table shows that the auction sample and the control group are very similar. There is no significant difference in the sale price, but higher quality properties were more likely to use the auction mechanism (based on the significant positive coefficient on the LUX dummy and the significant negative coefficient on the ECON dummy).

3.2 Double round auction premium

Each deal d has a total of $N_{d_1} \geq 1$ bids in the first round and $N_{d_2} \geq 0$ bids in the second round.¹⁹ We code the log bid as b_{drj} where $d = 1, \dots, D$ is the transaction, $r = 1, 2$ is the auction round and $j = 1, \dots, N_{d_r}$ is the order statistic of the bid in round r . We estimate the hedonic regression using sale prices, first-round bids and second-round bids:

¹⁹Of course there are 0 bids in the second round if the property is sold in the first round.

$$\begin{bmatrix} p \\ b^1 \\ b^2 \end{bmatrix} = \begin{bmatrix} X^0 \\ X^1 \\ X^2 \end{bmatrix} \beta + \begin{bmatrix} 0 \\ Z^1 \\ Z^2 \end{bmatrix} \gamma + \begin{bmatrix} \epsilon^0 \\ \epsilon^1 \\ \epsilon^2 \end{bmatrix} \quad (1)$$

with $p = y_i$ the log sale price for property i or a first or second round log bid for property i ; X^r is a matrix of controls for property characteristics where $r = 0$ corresponds to the comparable conventional sales; β is a vector of attribute loadings; Z^r is a vector of indicator variables derived from the double round auction observations; γ is an associated price differential; ϵ^r is a vector of error terms. The control variables in X^r are defined in Appendix 1; the variables in Z^r are described below.

Summary statistics were presented in Tables 2 and 3. As a check, we show estimated coefficients on β for the hedonic regression excluding Z^r . The estimated values for β are given in Table 5 and are similar to the estimates including the r variables.

We focus on the characteristics of the bids in the double round auctions. In order to do this, we estimate four versions of Z^r . In particular we allow Z^r to include indicator variables classifying the specific observation type. In Model 1, we include an indicator variable AUCTION equal to 1 if the observation is from the double round auction data set. In Model 2, we include 2 indicator variables. The first, FIRST_ROUND, equals 1 if the observation is a bid from the first round of the auction. The second variable, SECOND_ROUND, equals 1 if the observation is from the second round. In Model 3, we include both AUCTION and SECOND_ROUND.

The coefficient on AUCTION in Model 1 indicates the difference between the unconditional expected bid in any round of a double round auction and the expected sale price if a traditional sale is used, controlling for variables in X^r . It is possible that X^r does not contain all relevant variables related to pricing. These could include qualitative variables such as unobserved measures of quality correlated with the double round auctions. Because of this potential omitted variable bias, the coefficient on AUCTION in Model 1 should be interpreted with care.

In Model 2, we separately estimate coefficients for FIRST_ROUND and SECOND_ROUND. In this framework, the coefficient on SECOND_ROUND is interpreted as the difference between the expected value of a bid in the second round and the expected sale price were the property sold using traditional methods. Both coefficients in Model 2 are potentially biased due to any omitted quality variable common to properties in the double round auction data set.

The interpretation of the coefficient on SECOND_ROUND in Model 3 is different. By including AUCTION in Model 3, the coefficient on SECOND_ROUND is the difference in the expected bid between a first round bid and a second round bid, controlling for any unobserved quality common to properties in the double round auction data set. That is, the coefficient on AUCTION is still potentially biased due to omitted measures of quality, but the coefficient on SECOND_ROUND is not biased.

The bid can be expressed as

$$b_{drj} = \mu_d + f_r(s_{rj}) \quad (2)$$

where μ_d is a fixed effect for the property, f_r is the bid function that maps the signal in round r for bidder j , s_{rj} , into the bid. If bids have this representation, changes in the bids are given by

$$\Delta b_{drj} = f_2(s_{2j}) - f_1(s_{1j}) \quad (3)$$

In the following equation, the SECOND_ROUND indicator variable is interpreted as $E[f_2(s_{2j}) - f_1(s_{1j})]$. Thus, we can estimate the expected change in the bid from one round to the next, conditional on the first round bid. We estimate the hedonic model with the modification

$$\begin{bmatrix} p \\ b^1 \\ \Delta b^2 \end{bmatrix} = \begin{bmatrix} X^P \\ X^1 \\ 0 \end{bmatrix} \beta + \begin{bmatrix} 0 \\ Z^1 \\ \Delta Z^2 \end{bmatrix} \gamma + \begin{bmatrix} \epsilon^P \\ \epsilon^1 \\ \Delta \epsilon^2 \end{bmatrix} \quad (4)$$

The vector Δb^2 is the vector of second round bids minus the largest N_{d_2} bids from the first round. The components of Δb^2 are $b_{d_{2j}} - b_{d_{1j}}$. By differencing, we are able to remove the influence of any property attributes that are unobservable to the researcher, but observable to the bidders.

4 Empirical results

Our basic empirical approach tries to distinguish the characteristics of the dual round auctions from sales conducted in the traditional manner. The analysis begins with the hedonic models for sales price.

4.1 Hedonic models

The regression results are displayed in Tables 5 and 6. Table 5 presents the baseline coefficient estimates for the regression where X^r is excluded. All coefficients have the correct sign and a reasonable magnitude. As expected, luxury hotels fetch a significant premium and our set of 4 "distressed" sale dummies²⁰ are all significantly negative.

Table 6 is the most important segment of our empirical analysis. It displays the results where all 331 bids from the first and second round are used to estimate the coefficients. Interpreting the results in the first column, the expected value of a double round auction bid in either round is 15.4% larger²¹ than if the property were sold using traditional methods. Columns 2 and 3 indicate that second round bids are significantly different from traditional sale prices, but first round bids are not. Column 2 shows that the expected value of second round bids is 21.1% larger than the expected value of first round bids. Column 3 indicates that the expected value of second round bids is 8.5% larger than the expected value of first round bids. However, this is not surprising given that the second round bidders are a subset

²⁰These are dummies for real estate owned properties, properties where the owner is bankrupt, distressed sales and short sales.

²¹For our interpretation we use the approximation $e^x - 1 \approx x$.

of the first round bidders with the largest N_{d_2} bids.

Column 4 provides an estimate of the second round premium controlling for heterogeneous property effects and individual valuations. The results in Model 4 indicate that the second round bid, for those bidders that remain, increases by 3.7% from the first to the second round.

4.2 Office and retail bids

In this section we further investigate the relative size of and influences on the second round bids. In this analysis we use the entire sample of 67 auction sales. We address this issue by estimating the model

$$\begin{aligned}
 \Delta b_{d_2j} &= \alpha + \beta N_{d_2} + \epsilon_{drj} \\
 \Delta b_{d_2j} &= \alpha + \beta N_{d_2} + \epsilon_{drj} \\
 \Delta b_{d_2j} &= \alpha + \beta [N_{d_1} - N_{d_2}] + \epsilon_{drj} \\
 \Delta b_{d_2j} &= \alpha + \beta \sigma_{d_1} + \epsilon_{drj}
 \end{aligned} \tag{5}$$

In regression (5), the change in the log second round bids is modeled as functions of (i) the number of second round bidders, (ii) the number of first round bidders, (iii) the number of bidders dropped in the second round and (iv) the standard deviation of first round bids. These results are reported in Table 7 and indicate that none of the variables significantly increase or decrease the changes in the second round bids. This is important as the choice of N_{d_2} is in the hands of the auctioneer. These results are again not surprising. Table 2 in Ye (2007) finds that the optimal choice of bidders in the second round equals 2 for all configurations of simulations in the common values auction.

4.3 Early termination of the auction

In the double round auction, the auctioneer decides which bidders will be selected to participate in the second round. In the Ye (2007) model, the auctioneer preselects the number

of individuals, n^* , before the first round auction. In our setting, the auctioneer does not select n^* , nor does she post a reservation price. As previously discussed, some auctions do not go to a second round. In these instances, the auctioneer decides that a first round bid is adequate and that there can be no gain in expected revenue by continuing the auction.

In order to simultaneously determine which bids are sufficient to enter the second round and which bids are large enough to purchase the property in the first round, we estimate an ordered logit model. The response variable is $y_j = 0$ if the j^{th} bid was not large enough to be invited to the second round; $y_j = 1$ if the bid was large enough to be invited to the second round; $y_j = 2$ if the bid was large enough that the auctioneer ended the auction after one round.

In the ordered logit, the response variable is a function of an underlying latent variable. We model the latent variable as the log bid minus the average bid for the property, \bar{b}_d . We have the system

$$\begin{aligned}
 y_{d1j} = 0 &\Leftrightarrow y_{d1j}^* \leq \delta_1 \\
 y_{d1j} = 1 &\Leftrightarrow \delta_1 < y_{d1j}^* \leq \delta_2 \\
 y_{d1j} = 2 &\Leftrightarrow \delta_2 < y_{d1j}^* \\
 y_{d1j}^* &= \beta \times (b_{d1j} - \bar{b}_d) + \epsilon_{d1j}
 \end{aligned} \tag{6}$$

In the above equations, we demean the bids. The results are reported in Table 8. In this table, δ_1 is not statistically different from 0. By demeaning the bids, the latent variable evaluated at the average bid for property d is simply the standard normal error term: $y_{d1j}^* = \epsilon_{d1j}$. Thus, a bidder bidding at the mean first round bid has a fifty-fifty chance of being invited to the second round. We can then determine the probability that a bid will be large enough to secure entry to the second round with any given probability. Using the probit, the bid will have a 95% probability of entering the second round if $y_{d1j}^* = 1.645$. The estimated coefficient is $\hat{\beta} = 6.53$. Therefore, if $b_{d1j} - \bar{b}_d \approx 0.25$, the latent variable is $6.53 \times 0.25 \approx 1.645$.

A bid that is 25% above the average bid in the first round has a 95% chance of being invited to the second round.

A first round bid will win the property in the first round if $\delta_2 < y_{d_{1j}}^*$. The coefficient $\hat{\delta}_2 = 2.53$. If an individual bids 25% above the average bid in the first round, the latent variable is approximately 1.645. As mentioned above, there is a 95% chance the bid will be large enough to secure an invitation to the second round. The probability that the bid is large enough to win the property in the first round is given by

$$\Pr(2.53 < y_{d_{1j}}^*) = \Pr(2.53 < 1.645 + \epsilon_{d_{1j}}) = \Pr(0.885 < \epsilon_{d_{1j}}) \approx .18.$$

To illustrate our data, Table 9 shows the results of a sample auction. The first deal is for a hotel in the southeast with 217 rooms. The auction lasted two rounds with a winning bid of \$23,500,000.

5 Conclusions

This study has presented a theoretical and empirical analysis of a double round auction. Our theoretical model is used to develop some insight into the optimal behavior of the bidder and auctioneer, highlighting the interplay between the positive impact of having a large number of bidders (i.e., a higher sales price) and the negative impact (potential buyers bidding more conservatively because of the winner's curse). This model generally supports our empirical analysis, especially in its predictions of the number of bidders in each round.

Our empirical analysis is based on 67 double round auctions. It shows that the double round auction, on average, creates substantial value for the seller. The expected value of a double round auction sale is 15.4% larger than if the property were sold using traditional methods. This figure represents the average computed over the auctions that went to a second and those that, while having the possibility of a second round, were terminated after only a single round of bids. The latter group represent 11 of our sample of 67 double round auctions. This percentage increase is economically meaningful; it implies that assets sold

through this auction mechanism, after controlling for observable property characteristics, obtain an average additional revenue of \$5.2 million over properties sold in the conventional manner. We further analyze the bidding behavior in each round. We find that the average bid increases by 3.7% (controlling for property characteristics) from the first to the second round.

In summary our analysis shows that the double round auction is an economically efficient mechanism (at least from the seller's perspective) for selling valuable real estate properties with often unique characteristics. Further research should investigate how this mechanism could be used for selling other unique assets.

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7 Appendix 1: Hedonic model definitions

Log(Rooms) is log of the number of rooms

Log(EffectiveAge) is log of the effective age, which is the year of sale minus the year of the most recent renovation

LUX is an indicator variable for luxury hotels

ECON is an indicator variable for economy hotels

Dum_Full is an indicator variable for full service hotels

Dum_Main is an indicator variable for properties with deferred maintenance

Dum_BVal is an indicator variable for properties where the business value is included in the transaction

Dum_Brand is an indicator variable for properties where the brand is not sold with the property

Dum_1031 is an indicator variable for 1031 exchanges

Dum_REO is an indicator variable for real estate owned properties

Dum_BkrptS is an indicator variable for properties where the owner is bankrupt

Dum_Distress is an indicator variable for distressed sales

Dum_Short is an indicator variable for short sales

Dum_Fee is an indicator variable for fee simple exchanges

TwoRounds is an indicator variable where the transaction comes from the double round auction data set

ReBrand is an indicator variable for properties that will be re-branded

Renov0 is an indicator variable for properties renovated within 4 quarters of sale

Renov1 is an indicator variable for properties renovated between 4 and 8 quarters from the date of sale

S_BANK is an indicator variable when the seller is a bank

S_REIT is an indicator variable when the seller is a REIT

8 Appendix 2: Theoretical model

We assume that there are N bidders in round 1. There is a single seller auctioning an asset with uncertain value. The bidders are stochastically identical. Prior to round 1, bidder i receives a signal, v_i , of the value of the asset. The asset is assumed to have a true value of v , which is unknown to all bidders as well as the seller. At the end of round 2, the value of the asset is the same for each potential buyer; i.e., this is a common-value auction. We assume that the bidders' signals are independent draws from a common, known Weibull distribution.²² The following theoretical analysis is made significantly more tractable by using the Weibull distribution.²³

We restrict the bids to be a constant multiple, λ , of the observed signal. This buys us considerable gains in tractability. In addition, Paarsch (1992) has shown that models based on this assumption perform better empirically than more general models. We assume that there is a credible threat that the seller can accept a bid in the first round; as our data show, this possibility is indeed viable. This requirement prevents bidders from making extremely high (and infeasible) bids in round 1 merely to ensure a chance to bid in round 2.

Each bidder's signal is drawn independently from a common Weibull distribution with parameters c, α and m . We assume that each bidder knows the *ex ante* distribution of the other $(N - 1)$ bidders' signals. In addition, we assume the signals are an unbiased estimate of the asset's true value. Thus

$$v = m + \alpha \Gamma\left(\frac{1}{c} + 1\right)$$

where Γ is the gamma function

$$\Gamma(x) = \int_0^{\infty} t^{x-1} \exp(-t) dt$$

Since it is credible that the seller can accept a bid from the first round, the bidder treats the first round and second round independently, attempting to maximize expected profit.

²²The most complete reference for results used here is Johnson, Kotz and Balakrishnan (1994).

²³This model is in the spirit of Rothkopf (1969, 1980).

The seller observes the bids after round 1 and chooses a subset of the highest k bidders to re-bid in round 2. It is the selection of k based on the seller's assumptions of bidding behavior that is at the center of this model. The bidder receives no new signal of firm value after round 1, only whether or not she is permitted to bid in round 2. Note however that the expected bids will differ in round 1 and 2 because of the presumably different number of bidders in each round.

Bidder i 's expected profit is given by

$$E[\pi_i] = \int_0^{\infty} (v - \lambda_i v_i) f_i(v_i) \prod_{i \neq j} F_j(v_i) dx_i$$

This expression has a simple interpretation. The first term in the integrand, $v - \lambda_i v_i$, represents the realized return for acquiring an asset with value v by bidding the amount $\lambda_i v_i$. The second term represents the probability of receiving the signal v_i . The third term is the probability that bidder i 's bid is the highest of the N bidders, i.e., the probability that the return is attained.

Theorem: The expected highest bid is

$$\lambda_i m + \alpha \lambda_i N \Gamma\left(1 + \frac{1}{c}\right) \sum_{i=0}^{N-1} \frac{(-1)^i \binom{N-1}{i}}{(i+1)^{1+\frac{1}{c}}}$$

Proof: The distribution of the N^{th} order statistic from N independently distributed standard Weibull random variables is given by

$$\int_0^{\infty} \frac{N!}{(N-1)!} [1 - \exp(-(x)^{-c})]^{N-1} \times \exp[-(x)^{-c}] c(x)^{c-1} dx$$

Since we have assumed that the signals are distributed via $v_i \sim Weibull(m, \alpha, c)$, we know that the bid distribution is $\lambda_i v_i \sim Weibull(\lambda_i m, \lambda_i \alpha, c)$. Now using the transformation to take the standard Weibull into the three-parameter Weibull, we obtain a simpler expression for the N^{th} order statistic

$$= \lambda_i m + \int_0^{\infty} \lambda_i \alpha N [1 - \exp(-[m + \alpha x_i]^{-c})]^{N-1} \times \exp[-[m + \alpha x_i]^{-c}] c[m + \alpha x_i]^{c-1} dx_i$$

We integrate this expression to obtain

$$\lambda_i m + \alpha \lambda_i N \Gamma \left(1 + \frac{1}{c}\right) \sum_{i=0}^{N-1} \frac{(-1)^i \binom{N-1}{i}}{(i+1)^{1+\frac{1}{c}}}$$

which is the desired result.

Note that since

$$v = m + \alpha \Gamma \left(\frac{1}{c} + 1\right)$$

the return to the highest bidder is

$$m + \alpha \Gamma \left(\frac{1}{c} + 1\right) - \lambda_i m - \alpha \lambda_i N \Gamma \left(1 + \frac{1}{c}\right) \sum_{i=0}^{N-1} \frac{(-1)^i \binom{N-1}{i}}{(i+1)^{1+\frac{1}{c}}}$$

Since all bidders are stochastically identical, the probability of a bidder obtaining this return is $\frac{1}{N}$. Turning to the seller's problem, his objective is to minimize the bidder's expected return by an optimal selection of N . Thus, the seller's program is

$$\underset{n}{Min} \left[m + \alpha \Gamma \left(\frac{1}{c} + 1\right) - \lambda_i m - \alpha \lambda_i N \Gamma \left(1 + \frac{1}{c}\right) \sum_{i=0}^{N-1} \frac{(-1)^i \binom{N-1}{i}}{(i+1)^{1+\frac{1}{c}}} \right]$$

Since N is naturally constrained to be an integer this problem is not solvable by the usual first-order condition. In addition, for non-integral N , the summation term is undefined. However, it is instructive to analyze this program numerically. A sample analysis is presented in Figure 1.

Here we have assumed $c = 2; \alpha = 1; m = .114$.²⁴ Note that λ_i (the bid as a proportion of the signal) is usually called the shading function in auction theory since it represents the amount by which a bidder will under-bid her true expected value because of the winner's curse. Here we have graphed the bidder's profit for values of the shading function of .5, .6, .7, .8 and .9.

When the shading function $\lambda = .5$, bidders are bidding very conservatively: half of their signal of asset value. In this case, for up to 11 bidders the expected profit of the bidders is

²⁴The value of m is chosen so that the estimated value of the asset being sold is 1.

positive and decreasing as the number of bidders increases. This is because the conservative bids yield high profits to the bidder; the expected profit declines as the number of bidders increases, since the likelihood of winning decreases. On the other hand, when potential buyers are willing to bid closer to their signals of value, profits are negative and there exists an optimal number of bidders. With the given parameters, the optimal (from the seller's perspective) number of bidders is between 3 to 5. This is in general agreement with our empirical data.

This intuition can be expanded by the analysis presented in Figure 1: when potential buyers bid conservatively (i.e., a relatively small fraction of the unbiased signal of the asset's value) then the expected profits to the bidder are high and decrease with the number of bidders. However, when bidders are more aggressive, then the winner's curse effect becomes more significant and there is an optimal (from the seller's perspective) number of bidders in order to maximize profit. The theoretical data with reasonable estimates of the parameters²⁵ suggests that this optimum occurs with values of N from 3 to 5. This is in agreement with our empirical data, which shows that the expected number of bidders in the second round is 3.

²⁵Here from the Weibull distribution.

Table 1: Auction Bidding

This table displays the properties and bids by property type and auction round for the 66 properties in our dual round auction sample.

	<u>All Rounds</u>			<u>First Rounds</u>			<u>Second Round</u>		
	Deals	Bids	Average Bid	Deals	Bids	Average Bid	Deals	Bids	Average Bid
Hotel	40	361	34.003	40	254	34.722	29	107	32.297
Office	13	142	52.872	13	95	48.159	12	47	62.399
Retail	14	168	55.952	14	112	50.661	12	56	66.534

Table 2: Descriptive Statistics for Hotel Sales

Table 2 displays the descriptive statistics for the hotel comps used in estimating our hedonic model. The data set excludes off market transactions and sales without a selling broker. The variable definitions are given in Appendix 1.

Variable	Min	Mean	Max	Std Dev
Sale.Price	4	29.146	400	44.89
Number_Of_Rooms	5	192.89	2003	160.958
LUX	0	0.056	1	0.229
ECON	0	0.053	1	0.225
Dum_Full	0	0.573	1	0.495
Dum_Maint	0	0.075	1	0.264
Dum_BVal	0	0.06	1	0.237
Dum_Brand	0	0.235	1	0.424
Dum_1031	0	0.038	1	0.192
Dum_REO	0	0.148	1	0.355
Dum_BkrptS	0	0.022	1	0.146
Dum_Distress	0	0.077	1	0.267
Dum_Short	0	0.009	1	0.093
Dum_Fee	0	0.946	1	0.227
Renov0	0	0.059	1	0.235
Renov1	0	0.194	1	0.396

Table 3: Descriptive Statistics for Double Round Auction Bids

Table 3 displays the descriptive statistics for the bids from the double round auctions data set used in estimating our hedonic model. The variable definitions are given in Appendix 1. The sample size is 40.

Variable	Min	Mean	Max	Std Dev
Sale.Price	4.13	33.773	195.5	35.193
Number_Of_Rooms	41	236.8	650	129.315
LUX	0	0.229	1	0.426
ECON	0	0	0	0
Dum_Full	0	0.714	1	0.458
Dum_Maint	0	0.143	1	0.355
Dum_BVal	0	0.114	1	0.323
Dum_Brand	0	0.257	1	0.443
Dum_1031	0	0.029	1	0.169
Dum_REO	0	0.257	1	0.443
Dum_BkrptS	0	0.114	1	0.323
Dum_Distress	0	0.029	1	0.169
Dum_Short	0	0.029	1	0.169
Dum_Fee	0	0.971	1	0.169
Renov0	0	0.086	1	0.284
Renov1	0	0.171	1	0.382

Table 4: Difference in Control Variables

This table displays the least-squares estimates from the regression $Variable = \alpha + \beta \times 1(DoubleRound)$ using the control group properties and the hotels in the double round auction. $1(DoubleRound) = 1$ for the 40 double round properties and 0 for the 165 control group properties.

Variable	Estimate	Std Err	p-value
Sale.Price	4,626,479.535	6,214,908.813	0.457
Number_Of_Rooms	43.91	22.805	0.054
LUX	0.173	0.073	0.019
ECON	-0.053	0.007	0.000
Dum_Full	0.141	0.080	0.079
Dum_Maint	0.068	0.062	0.271
Dum_BVal	0.054	0.056	0.331
Dum_Brand	0.022	0.077	0.778
Dum_1031	-0.010	0.030	0.748
Dum_REO	0.109	0.077	0.157
Dum_BkrptS	0.092	0.056	0.096
Dum_Distress	-0.049	0.030	0.108
Dum_Short	0.020	0.029	0.496
Dum_Fee	0.026	0.030	0.387
Renov0	0.027	0.049	0.586
Renov1	-0.022	0.067	0.737

Table 5: Hedonic Regression Coefficients

This table displays the regression coefficients for the control variables using the specification in equation (1).

Variable	Estimate	Std. Error	t-statistic	p-value
(Intercept)	12.738	0.340	37.449	0.000
Log.Rooms	0.741	0.055	13.429	0.000
Log.EffectiveAge	0.029	0.035	0.831	0.406
LUX	0.576	0.076	7.591	0.000
ECON	-0.290	0.211	-1.374	0.170
Dum_Full	0.237	0.075	3.158	0.002
Dum_Maint	-0.601	0.103	-5.836	0.000
Dum_BVal	0.405	0.101	4.031	0.000
Dum_Brand	0.106	0.077	1.386	0.166
Dum_1031	-0.236	0.170	-1.384	0.167
Dum_REO	-0.605	0.078	-7.757	0.000
Dum_BkrptS	-0.919	0.104	-8.853	0.000
Dum_Distress	-0.353	0.133	-2.659	0.008
Dum_Short	-0.637	0.175	-3.645	0.000
Dum_Fee	0.033	0.147	0.228	0.820
Renov0	-0.291	0.129	-2.248	0.025
Renov1	0.138	0.105	1.308	0.191
Observations	496			
R-Squared	0.6216			
RMSE	0.5649			

Table 6: Double Round Auction Coefficients for Hotels Using All Bids

This table displays the coefficient estimates for the z_i variables using all first round bids and all second round bids. Table 4b displays the coefficient estimates using only the N_{d2} largest, first round bids for deal d . N_{d2} is equal to the number of second round bids for deal d .

Variable	Model 1	Model 2	Model 3	Model 4
AUCTION	0.154** (0.077)		0.126 (0.08)	
FIRST_ROUND		0.126 (0.08)		
SECOND_ROUND		0.211*** (0.082)	0.085* (0.048)	0.037*** (0.006)
N	496	496	496	496
R squared	0.621	0.622	0.622	0.999
RMSE	0.563	0.562	0.562	0.523
Differenced Second Round Bids	NO	NO	NO	YES

*, **, *** indicate 10%, 5% and 1% statistical significance, respectively. Standard errors are in parenthesis.

Table 7: Dual Round Regression

This table displays the regression coefficients for the DR equations. The intercept is a measure of the expected change in the second round bid conditional on the regressor being equal to 0. Standard errors are calculated using White (1980) heteroskedastic standard errors.

	Bidders Dropped	Original Bidders	Final Bidders	Sigma
Intercept	0.034*** (0.005)	0.029*** (0.007)	0.023*** (0.008)	0.029*** (0.004)
Variable	0 (0.001)	0.001 (0.001)	0.002* (0.001)	0.048 (0.037)
R Squared	0	0.004	0.013	0.007
RMSE	0.047	0.047	0.047	0.047
Observations	210	210	210	210

Table 8: Ordered Response Model

This table displays the ordered response results where the response variable indicates if the bid did not secure an invitation to the second round, the bid did secure an invitation to the second round, the bid was large enough to award the property to the bidder in the first round with no second round of bidding required. The latent variable is given by

$$y_{d1j}^* = \mu_d + \beta \times b_{d1j} + \epsilon_{d1j}.$$

	Probit	Logit
β	6.53 (0.638)	11.88 (1.254)
δ_1	0.1066 (0.0657)	0.1667 (0.1086)
δ_2	2.38 (0.1484)	4.4894 (0.3452)
Residual Deviance	596.0662	590.4941
AIC	596.4941	602.0662

Table 9. Sample Auction

This table shows a sample of the data for one property in the auction data set: a hotel in the southeast with 217 rooms. The auction lasted two rounds with a winning bid of \$23,500,000.

Keys	SF	Deal Type	Is Portfolio	Region	Market Type
217	-	Full Service Hotel	No	Southeast	Primary

Bid	First Round Offers	Second Round	Winning Bid
1	\$22,500,000	\$23,500,000	\$23,500,000
2	\$20,000,000	\$22,500,000	
3	\$19,000,000	\$22,000,000	
4	\$18,500,000	\$20,000,000	
5	\$18,000,000		
6	\$14,000,000		

Figure 1: Theoretical bidder profits

This figure shows the bidder's profit for values of the shading function (the bid as a proportion of the bidder's signal) of .5, .6, .7, .8 and .9 based on our theoretical model. It shows that for a plausible set of parameters, the optimal (from the seller's perspective) number of bidders is from 3 to 5.

