Welfare Costs of Informed Trade

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Abstract

We examine the welfare costs of informed trade in a new Glosten-Milgrom type model with elastic uninformed trade. Welfare losses occur when uninformed agents choose not to trade because their idiosyncratic valuation lies within the bid-ask spread. Informed trade causes wider spreads initially, but narrower spreads later because information is reflected in prices faster. For sufficiently long lived information, the benefits of narrow spreads later outweigh the wider initial spreads such that the average spread and total welfare loss are mainly decreasing in the amount of informed trade. For short-lived information, this tradeoff does not materialize and spreads and welfare losses are increasing in informed trade. Our findings suggest that regulation of information and informed trade should consider the horizon of private information.

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1. Introduction

A significant amount of capital market regulation around the world is concerned with maintaining a level playing field with respect to information. For example, in the US, Regulation FD ("Fair Disclosure") states that if a company provides material information to anyone, it must release it to everyone, and if material information is leaked, the information must be promptly made public. The intent is clear—companies cannot convey to anyone an informational advantage.[1] Rule 242.601 of Regulation NMS ("National Market System") that covers the "dissemination of transaction reports and last sale data" states, among other things, "…no national securities exchange or national securities association may…prohibit, condition or otherwise limit…the ability of any vendor to retransmit or display in moving tickers, transaction reports or last sale data…" The regulation does not control what an exchange may charge the vendor for the data, except to say that the charges must be "…reasonable [and] uniform…" At least in part, the regulation is aimed at leveling the playing field with respect to trade history information. Finally, Rule 10b-5 of the 1934 act has been construed as protecting against individuals trading on private information. In fact, insider trading cases are typically prosecuted under 10b-5 and the SEC enacted Rule 10b-5-1 in 2000 to clarify the use of 10b-5 in these prosecutions. Insider trading is prohibited in most jurisdictions and is often punishable with criminal sanctions.[2]

These examples suggest that regulators, among others, consider it important to have a level playing field with respect to information; that is, the cost of an un-level playing field is at least as large as the cost of regulation. We propose, however, that we really do not have much of an idea of the welfare cost of informed trade. The questions we seek to address in this paper are what, in fact, are the welfare consequences of

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[1] Corporate disclosure rules in some other countries are even more stringent. For example, in Australia, Continuous Disclosure Rules oblige listed companies to inform investors (through the stock exchange) of any information that could be reasonably expected to have a material effect on the price or value of the securities as soon as the company becomes aware of the information. Violations can attract civil and criminal sanctions.

[2] For example, in 2014 the European Union (EU) harmonized criminal sanctions for insider trading (Criminal Sanctions for Market Abuse Directive). All EU Member States agreed to introduce maximum prison sentences of at least four years for serious cases of insider trading, and at least two years for improper disclosure of insider information.

informed trade? What are the consequences of reducing but not eliminating informed trade?

For this analysis, we are not interested in the value (or cost) of the information that informed trade might convey, but rather the mechanism by which the information comes to be reflected in prices. One might imagine that informed trade provides signals to managers and thus leads to better real decisions (e.g., Fishman and Hagerty, 1992; Chen, Goldstein and Jiang, 2007). On the other hand, information that comes out before hedgers have an opportunity to hedge provides a welfare cost of information (e.g., Foucault and Cespa, 2008; Glosten, 1989). We provide no formal analysis of these questions here. Our paper analyzes the welfare created by gains from trade (the buyer and seller surpluses) and how this quantity is impacted by informed trade. In our model, the bid-ask spread is entirely due to information asymmetry. The bid-ask spread causes a welfare loss because uninformed agents choose not to trade when their idiosyncratic valuation lies within the spread and thus realized gains from trade are less than potential gains from trade. The welfare loss associated with informed trade is defined as the difference between the realized gains from trade and the potential gains from trade with zero bid-ask spread but the same information arrival process.

To see why the welfare consequences of informed trade are not obvious, consider the standard example of a Glosten-Milgrom model in which the future price will either be high or low. Informed traders know the future value and arrive at rate α. Uninformed randomly buy or sell and arrive at rate $1 - \alpha$. The higher is α, the higher is the *initial* spread, but the faster information is revealed through trade, and the lower is the spread after some time has elapsed. Thus, if we consider the spread to be a measure of welfare loss (not in general legitimately as we will show), we see that with higher α, the total welfare loss might be quite low since most of the time trading is at a very narrow spread. On the other hand, with low α, the initial spread is low, but this spread will persist for a longer time since information revelation is slower.

Market microstructure models are suggestive of a welfare cost of informed trade since they deliver a trading friction—a spread between the bid and offer. Yet, the models have not generally been useful for analyzing welfare since they typically involve noise traders with inelastic trading demands—traders that are insensitive to the terms of trade.

Both Kyle and Glosten-Milgrom models typically use noise traders. With noise traders, the spread merely results in transfers from the uninformed to the informed with the dealers as zero profit conduits. A few market microstructure papers have fully endogenized trade in order to make welfare statements (e.g., Glosten, 1989; Bhattacharya and Spiegel, 1991; Ausubel, 1990; Medrano and Vives, 2004). These models, however, consider only a point in time and do not address the dynamic issues considered here.

This paper presents a new Glosten-Milgrom type model with informed and uninformed traders. Informed traders receive signals about the future value of the security, which is either high or low. Unlike typical incarnations of this model, uninformed trade is modeled rather than specified exogenously. Risk averse uninformed traders have private endowments that correlate with the risky security, which they trade for hedging purposes. Their private valuations of the security are determined by their eagerness to hedge endowment risk, which in turn is determined by their wealth or degree of risk aversion. They buy (sell) if their private value is sufficiently high (low). They abstain from trade if their private valuation lies within the spread, resulting in unrealized gains from trade. Although we endogenize uninformed trade, the zero profit quotes are easily derived and the transaction prices have nice dynamic properties.

Given that welfare losses stem from the bid-ask spread, we start by analyzing the dynamics of spreads and how they are impacted by informed trade. Our analysis reveals a tradeoff: more informed trade causes wider spreads in early trading but also faster convergence of prices towards the fundamental value and therefore narrower spreads in later trading periods. For sufficiently long-lived information (and/or sufficiently precise information), a higher rate of informed trade can actually *decrease* expected spreads in contrast to familiar comparative statics. For short-lived information (information that is revealed within a relatively short period of time), the tradeoff does not materialize and expected spreads increase with the amount of informed trade.

Our characterization of spreads mirrors Roşu's (2014) result that in a dynamic limit order book model a higher share of informed traders *improves* liquidity because it reduces uncertainty about the fundamental value. Importantly, Roşu's results arise in a dynamic setting with long-lived information. We show that the same mechanism is at work in our simpler sequential trade model with sufficiently long-lived information when

accounting for the dynamics. Furthermore, Roşu (2014) conjectures that the 'adverse selection view' (that more informed trading leads to higher adverse selection risk) is more appropriate in circumstances with short-lived information and the 'dynamic efficiency view' (that more informed trading leads to better overall liquidity) is applicable with long-lived information. We formalize this intuition by showing that the relation between informed trading and liquidity depends on the horizon of the information.

The specification of uninformed preferences allows calculation of the expected total welfare gained from trading. We compare this welfare to the expected total welfare that would be earned in an environment with the same information process as that generated by the informed trade but with no spread. The latter minus the former is the welfare loss of informed trade, keeping the information the same. The expected welfare loss per period is related to the spread, but is not identical to it.

Our analysis shows that the welfare loss is single peaked in the amount of informed trade, reaching a maximum at an internal point. That is, after some point, the welfare loss is decreasing in the amount of informed trade because with more informed trade information gets into prices faster and average spreads decline. For short-term information, the maximum loss occurs at a very high probability of informed trade meaning that the welfare loss is mostly increasing in the amount of informed trade. For longer term information, the maximum occurs at a relatively small probability of informed trade suggesting that over some range, the welfare loss is actually *decreasing* in informed trade.

It is easy to imagine that the per capita profit to the informed traders declines with the number of informed traders. Thus the equilibrium number of informed traders (and hence the probability of informed trade) is determined by a break-even condition based on the cost of becoming informed. The analysis above suggests that if obtaining private information is relatively costly and therefore the amount of informed trade is low, then an increase in the cost of obtaining information will reduce the probability of informed trade, which will reduce the welfare loss and increase total welfare. On the other hand, if information is relatively cheap and the probability of informed trade is high, an increase in the cost and consequent decrease in the amount of informed trade will decrease the

welfare loss for short-lived information, but *increase* the welfare loss for long-lived private information.

Regulators can influence the expected costs of different forms of private information by imposing restrictions and penalties. An implication of our analysis is that regulation of information disclosure and informed trade should consider the horizon of private information in question. For example, Ding, Hanna and Hendershott (2014) find that with access to exchange data feeds, one can calculate the National Best Bid and Offer (NBBO) faster than the official NBBO and use this information to profit at the expense of slower market participants. Our analysis suggests that trading on such extremely short-lived private information is almost certainly detrimental to total welfare. Similarly, if corporate insiders were to trade immediately prior to a corporate announcement such as earning figures, their trading would likely be harmful to liquidity and welfare. Yet, if corporate insiders trade at other times, when their private information would not generally be revealed within a short amount of time, their trading may in fact increase total welfare and reduce average spreads. Thus, our model provides a rationale for a policy of allowing corporate insiders to trade most of the time, but not during 'blackout periods' preceding important corporate announcements.

The welfare costs and benefits of insider trading have been extensively debated in the law and economics literature as well as in the finance literature. In the finance literature, the discussion is typically framed within a noisy rational expectations equilibrium paradigm. Furthermore, most of these papers are of the normal-CARA type (except Ausubel (1990) and Bhattacharya and Nicodano (2001)), and investigate the effect on real investment of insider trading. Ausubel (1990), Bhattacharya and Nicodano (2001) and Medrano and Vives (2004) are notable in that they do not rely on noise trade (random supply) and hence can calculate expected utilities. Indeed Medrano and Vives (2004) provides numerical results that suggest it is not legitimate to do a welfare analysis in the presence of noise trade.

The normal-CARA model, is typified by the one in Medrano and Vives (2004). There are three types of traders—those with information, those who wish to hedge a random position correlated with the future security value and speculators. The future value of the security is an exogenous normal random variable. Three types of results

come out of this model. The fact that the price conveys information makes uninformed traders' demands less responsive to price than if there were no private information. That is, the market is thinner due to adverse selection. This reduces the amount of risk sharing and hence reduces total welfare.

The second effect is that the price revealing information implies that there is less unrevealed information and the risk premium goes down. This effect is also examined in Easley and O'Hara (2004). This is perhaps a less interesting result since it follows from the exogenous future payoff. An institutional change (insider trading) that makes the current price more informative will likely make the future price more informative as well leaving the uncertainty of the price change mostly unchanged.

The third effect is often referred to as the Hirschleifer effect—revealing more information before agents get a chance to hedge is welfare reducing. Informed trade then has a negative welfare effect, and the more informed there are, the more information gets into prices (unless the market closes down) and the worse is the welfare effect. This effect is probably stronger in the models than in reality and the model result is again due to the exogenous uncertainty. A change in an institution that increases the amount of information in prices today will also increase the amount of information in prices tomorrow, leaving the unresolved uncertainty roughly the same.

The next section defines the model and derives bid and ask quotes and their dynamic properties. We then characterize spreads and analyze welfare. Finally, we discuss some applications of the model and conclude.

2. The Model

We adopt a Glosten-Milgrom framework in which there is one risky security, three types of traders (informed, uninformed, and liquidity providers), and $t = 1, \dots, T$ trading rounds. The future value of the security at the end of the T trading rounds is one of two equally likely values, which (without loss of generality) we set to zero and one, $V \in \{0,1\}$. The structure of the model is common knowledge among all participants. Public information at time t consists of the sequence of past buys and sells and their transaction prices, which we denote by $History_t$.

Informed traders are risk neutral and serially receive a signal about the future value of the security. Upon receiving the information, they go to the market planning to trade once. The signal an informed trader receives is either H (high) or L (low) and the quality of the signal is measured by $q = \Pr\{Signal_t = H \mid V = 1, History_t\} = \Pr\{Signal_t = L \mid V = 0, History_t\}$, with $q \in (\frac{1}{2}, 1]$. When the signal quality is $q = 1$, the informed traders' information is perfect. When $q = \frac{1}{2}$, the signal is completely uninformative. An informed trader who sees the signal H when the current (time t) expected value of V is p_t will, through Bayesian updating, have revised private value $v_t^H = p_t q / [p_t q + (1 - p_t)(1 - q)]$. Similarly, an informed trader who sees signal L will have revised private value $v_t^L = p_t(1 - q) / [p_t(1 - q) + (1 - p_t)q]$.

Uninformed traders do not have private information about the security value; they trade for hedging purposes. They are homogenous in their utility function and risk aversion, but differ in wealth. Utility is an increasing, concave function of wealth at the future point in time T, $U(W_T) = -\frac{1}{W_T}$. This isoelastic utility function implies constant relative risk aversion and is depicted in Figure 1, Panel A.

< Figure 1 >

Uninformed traders serially receive endowment shocks that are correlated with the future value of the risky security and cause them to go to the market to hedge the shocks. With equal probability an endowment shock is either V or $1 - V$. The private endowment shocks can be interpreted as non-tradable assets such as labor income, or long/short positions in the security or a related security. Uninformed traders with positive (negative) exposure to the future value of the security can hedge their endowment risk by selling (buying) one unit of the security.

In addition to their endowment shock, uninformed traders have other assets that are uncorrelated with the future value of the risky security and have a certainty equivalent of $M \in (0, \infty)$, which is distributed according to:

$$F_M(m) = \Pr(M < m) = \frac{2m}{1 + 2m}, \text{ and}$$

$$f_M(m) = \begin{cases} \frac{2}{(1+2m)^2} & \text{for} \quad m > 0 \\ 0 & \text{otherwise} \end{cases}.$$

Figure 1 Panel B depicts this distribution. Variation in wealth across uninformed traders gives rise to a distribution of private valuations. Less wealthy investors are more anxious to hedge.

Result 1

Given the expectation of the future value of the security, $p_t = \mathbb{E}_t[V \mid History_t]$, and the uninformed traders' utility and endowment described above, the private valuation of an uninformed trader that arrives at the market at time t (v_t) has the following distribution:

$$f_{v_t}(v) = \begin{cases} \frac{p_t(1-p_t)}{(2p_t v - p_t - v)^2} & \text{for } 0 \le v \le 1 \\ 0 & \text{otherwise} \end{cases} \tag{1a}$$

$$F_{v_t}(v) = \Pr(v_t < v) = \begin{cases} 0 & \text{for } v < 0 \\ \frac{v(p_t-1)}{2p_t v - p_t - v} & \text{for } 0 \le v \le 1 \\ 1 & \text{for } v > 1 \end{cases} \tag{1b}$$

where $f_{v_t}(v)$ is the probability density function and $F_{v_t}(v)$ is the cumulative probability density function.

Proof: See Appendix.

The private valuations of uninformed traders (v_t) are simply certainty equivalents of the endowment shocks. The distribution of v_t is depicted in Figure 2 for three different values of p_t. When $p_t = 0.5$, the distribution is uniform. When $p_t < (>) \; 0.5$ the distribution has more mass near zero (one). Thus, private valuations cluster around the public information expected future value of the security, p_t. Private valuations less than p_t arise from uninformed traders that have positive exposure to the future value of the security and would like to sell the security to hedge their endowment risk. They are willing to sell at prices below the expected future value with the extent of price concession they are willing to tolerate determined by their wealth; private valuations near zero are from the least wealthy traders, and private valuations marginally less than p_t are

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from the wealthiest. Similarly, private valuations greater than p_t arise from uninformed traders that have negative exposure to the future value of the security and would like to buy the security to hedge their endowment risk. The premium they are willing to pay above the current expected future value is also determined by their wealth; the highest (lowest) valuations are held by the least wealthy (wealthiest) individuals. Because the endowments with positive/negative exposure to the future value of the security are equally likely, the distribution of private value has equal mass above and below p_t.

< Figure 2 >

Similar distributions of private valuations are obtained for different specifications of trading motivations. For example, suppose uninformed traders are homogenous with respect to wealth, but have different degrees of risk aversion. Particular forms of this setup give rise to the distribution of private valuations as in (1a-1b).[3] Traders that have an endowment that is positively (negatively) correlated with the risky security will want to hedge endowment risk by selling (buying) the security and they will be willing to sell (buy) at a concession (premium) to the security's expected value. The magnitude of the concession or premium is determined by the degree of risk aversion; marginally risk averse individuals will not tolerate a large concession or premium and will thus have private valuations close to p_t, whereas highly risk averse individuals will be very anxious to hedge and will thus have private valuations close to zero or one. Therefore, cross-sectional variation in risk aversion can lead to a similar distribution of private valuations.

Modeling uninformed traders as having a distribution of private valuations due to hedging desires provides a generalization of (rather than simply a departure from) models that specify inelastic exogenous trading demands for uninformed 'noise' traders. Inelastic uninformed trading demands can be obtained as a special case of our setup. Hedgers have inelastic demands if all traders have zero wealth other than the endowment

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[3] The same distribution of private valuations is obtained by using the same endowment shocks as above (V and $1 - V$), normalising other wealth to zero ($M = 0$), assuming a utility function of $U(W_T, r) = W_T(r - 2)/[2W_T(r - 1) - r]$, and assuming risk aversion, r, is uniformly distributed across individuals, $r \sim U(0,1)$.

shock that is perfectly correlated with the risky security, or if all hedgers are infinitely risk averse.[4]

Finally, the third type of trader is a representative risk-neutral competitive liquidity provider that has no private information. As is standard in Glosten-Milgrom type models, the liquidity provider posts bid and ask quotes (for a unit volume) that earn zero expected profits. At each time, t, a trader arrives at the market and can buy at the ask, sell at the bid, or abstain from trading. With probability α the trader arriving at the market is informed and with probability $1 - \alpha$ they are uninformed. After each trade, the liquidity provider Bayesian updates her beliefs about the future value of the security and posts new quotes before the next trader arrives.

An informed trader that arrives at the market having seen the high signal will buy if their private valuation is higher than the ask, $v_t^H > Ask_t$. Similarly, an informed trader that has seen the low signal will sell if their private valuation is lower than the bid, $v_t^L < Bid_t$. In equilibrium it will always be the case that $v_t^H > Ask_t$ and $v_t^L < Bid_t$, and therefore the informed traders will always trade in the direction of their information.[5]

Similarly, an uninformed trader with private valuation v_t will buy if $v_t \geq Ask_t$, sell if $v_t \leq Bid_t$, and choose not to trade if his private valuation lies within the spread, $Bid_t < v_t < Ask_t$. Uninformed traders that are sufficiently anxious to hedge (either sufficiently low wealth or sufficiently high risk aversion) will trade, buying (selling) if they have an endowment that is negatively (positively) correlated with the risky security. Those that are not particularly anxious to hedge will have private valuations within the spread and therefore choose not to trade. It is this lack of trade that creates a welfare cost.

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[4] The opposite scenarios also provide an interesting special case of the model. When all uninformed traders are either infinitely wealthy or entirely risk neutral, their private valuations are all equal to the current expectation of the future value of the security, p_t. In such a scenario the market closes down because uninformed traders are unwilling to trade at prices other than p_t (i.e., at a spread of zero) so the liquidity provider is unable to set quotes that recoup from uninformed traders the losses incurred from trading with informed traders.

[5] As will become clear later, if the Bid_t and Ask_t quotes are set wider than v_t^L and v_t^H (i.e., $Bid_t < v_t^L$ and $Ask_t > v_t^H$), then no informed traders would trade and all trades would arise from uninformed traders. The competitive, zero expected profit Bid_t and Ask_t quotes without any informed trades are equal to the current expected security value, $Bid_t = Ask_t = p_t$, with a spread of zero. Because signals are always informative ($q > 1/2$), $v_t^H > p_t$ and $v_t^L < p_t$ and therefore in a competitive equilibrium the Bid_t and Ask_t quotes cannot be wider than v_t^L and v_t^H.

The standard assumption that the liquidity supplier expects zero profits implies that the liquidity provider's bid (ask) quote is the expected future value of the security, conditional on receiving a sell (buy) order. That is, $Bid_t = \mathbb{E}_t [V \mid History_t, D_t = -1]$ and $Ask_t = \mathbb{E}_t [V \mid History_t, D_t = +1]$, where D_t is the trade direction (-1 for a sell, $+1$ for a buy and 0 for no trade). This leads to a characterization of equilibrium Ask_t and Bid_t, which is stated as Result 2.

Result 2

The zero expected profit ask and bid when $p_t = \mathbb{E}_t [V \mid History_t]$ and the quality of informed traders' signals is $q < 1$, are given by:

$$Ask_t = \frac{\delta p_t}{1 + p_t(\delta - 1)} \quad ; \tag{2a}$$

$$Bid_t = \frac{p_t}{\delta + p_t(1 - \delta)} \quad ; \tag{2b}$$

$$\delta = \frac{2\alpha q - 1 + \sqrt{1 - 4\alpha(1 - \alpha)(2q - 1)}}{2\alpha(1 - q)} . \tag{2c}$$

Proof: See Appendix.

As long as $\alpha < 1$, Ask_t is less than the informed valuation given a high signal (v_t^H) and Bid_t exceeds the informed valuation given a low signal (v_t^L) so the market is always open. If the informed traders' information is perfect ($q = 1$), then for $\alpha < \frac{1}{2}$, the above holds with $\delta = 1/(1 - 2\alpha)$. The market closes down if the informed traders' information is perfect and it is anticipated that $\alpha \geq \frac{1}{2}$.

For $q < 1$, it is clear why the market is open no matter what $\alpha < 1$ is. Since the informed valuation given a high signal is less than one ($v_t^H < 1$), a possible ask quote is that valuation itself ($Ask_t = v_t^H$). Since the probability of an uninformed valuation (v_t) in the neighborhood of one is positive ($\Pr\{v_t > v_t^H\} > 0$), there is a positive probability of an uninformed trade at this quote and this quote will yield positive expected profits to the liquidity provider. Thus, if Ask_t is only slightly less than v_t^H, it will yield positive profits. A lower Ask_t will yield zero profits.

When $q = 1$, the above argument fails because $\Pr\{v_t > v_t^H = 1\} = 0$. Consider what happens when $\alpha = \frac{1}{2}$. In this case, as Ask_t is increased, the losses to informed traders decrease. While the profits per uninformed trader increase, the probability of the uninformed trade decreases at a greater rate than the losses to the informed decrease. As a result there are no quotes that will allow trade and nonnegative expected profits to liquidity providers.

Recall that bids and asks are, respectively, updated expectations of V in response to a sell and buy. Thus,

$$p_{t+1} = \mathbb{E}_{t+1}\left[V \mid History_t, D_t\right] = \mathbb{E}_{t+1}\left[V \mid History_{t+1}\right] = \begin{cases} Ask_t & \text{if } D_t = +1 \\ Bid_t & \text{if } D_t = -1 \\ p_t & \text{if } D_t = 0 \end{cases}$$

Given the equilibrium bids and asks in (2a-2c), the dynamics of expectations are of a particularly convenient form, given in Result 3.

Result 3

Let $p_{t+1} = \mathbb{E}_{t+1}\left[V \mid History_t, D_t\right]$ indicate the updated expectation in response to a trade of direction $D_t \in \{-1 \text{ for a sell}, +1 \text{ for a buy}, 0 \text{ for no trade}\}$ at time t. Further, let $N_t = \sum_{\tau=1}^{\tau=t-1} D_\tau$ indicate the net number of buys received up to (but not including) the trade at time t (number of buys minus number of sells). Then the dynamics of expectations are given by:

$$\frac{p_{t+1}}{1 - p_{t+1}} = \frac{p_t}{1 - p_t} \delta^{D_t} ;$$ (3a)

and

$$p_t = \frac{\delta^{N_t}}{1 + \delta^{N_t}} .$$ (3b)

Proof: See Appendix.

Equation (3a) shows that the odds of a high future value are revised upward following a buy, downward following a sell and remain unchanged if no trade occurs (because only an uninformed trader may find it optimal not to trade). The amount by which expectations are revised following a trade is determined by δ, which is an

increasing function of the probability of informed trade, α, and the quality of the informed traders' information, q. Thus, δ measures of the informativeness of trades.

Equation (3b) shows that all of the information contained in past trades and trade prices can be represented by the number of net buys (buys minus sells), N_t. Thus, the expectation of V at any point in time can be expressed succinctly as a function of the number of net buys up to that point in time, and δ, the informativeness of trades.

For reasons that will become clear later, it is important to note that given V, the probability distribution of D_t depends only on the parameters (α and q) and not on the endogenous expectations of V. Specifically,

$$P_1 = \Pr\{D_t = +1 \mid V = 1\} = \Pr\{D_t = -1 \mid V = 0\} = \alpha q + \frac{(1 - \alpha)}{(\delta + 1)} \; ; \qquad (4a)$$

$$P_2 = \Pr\{D_t = +1 \mid V = 0\} = \Pr\{D_t = -1 \mid V = 1\} = \alpha(1 - q) + \frac{(1 - \alpha)}{(\delta + 1)} \; ; \qquad (4b)$$

$$P_3 = \Pr\{D_t = 0\} = (1 - \alpha)\frac{(\delta - 1)}{(\delta + 1)} \; . \qquad (4c)$$

Therefore, given V, the number of buys seen up to (but not including) time t ($b_t = \sum_{\tau=1}^{\tau=t-1} \mathbf{1}_{\{D_t=+1\}}$), the number of sells seen up to time t ($s_t = \sum_{\tau=1}^{\tau=t-1} \mathbf{1}_{\{D_t=-1\}}$), and the number of non-trades (when an uninformed trader chooses not to trade) up to time t ($n_t = t - 1 - b_t - s_t$), follow a trinomial distribution with fixed probabilities (4a-4c).

3. Analysis of the Spread

We now turn to a detailed examination of the bid-ask spread, given that the welfare costs of informed trade stem from the spread. In the model, the bid-ask spread arises entirely due to adverse selection risks faced by the liquidity providers who incur losses when they trade against informed traders. The spread allows liquidity providers to recoup from uninformed traders the losses they make to informed traders.

Together, the dynamics of expectations in (3a-3b), and the expressions for the bid and ask quotes in (2a-2c), define the dynamics of the bid-ask spread. The spread at a point in time t is given by,

$$Spread_t = Ask_t - Bid_t = \frac{(\delta^2 - 1)(p_t - 1)p_t}{(\delta(p_t - 1) - p_t)(1 + p_t(\delta - 1))} \; . \qquad (5)$$

Figure 3 illustrates the relation between the spread and other variables at a given point in time. Panel A plots $Spread_t$ against p_t, the contemporaneous expectation of V, for three different values of the trade informativeness measure, δ. Given the parameters that govern trade informativeness, the spread is at its maximum when $p_t = \frac{1}{2}$, i.e., when (based on public information) there is maximum uncertainty about the future value of the security. During such times, the expected loss to the liquidity provider per informed trade is at its maximum. As the uncertainty about V is resolved (either $p_t \to 0$ or $p_t \to 1$), the spread approaches zero.

Panel A also shows that the spread at a given point in time is increasing in the informativeness of trades, δ. Two parameters determine the informativeness of trades: the probability of informed trade, α, and the quality of the informed traders' information, q. Substituting (2c) into (5) to re-express the spread in terms of α and q and setting $p_t = \frac{1}{2}$ for illustration, Panel B shows the relation between spreads, α and q. At a given point in time and a given expectation of the future value, the spread is increasing in both the probability of informed trade and the quality of the informed traders' information. With a higher probability of informed trade, the liquidity providers encounter more informed traders per uninformed trader and therefore must earn a larger profit per uninformed trader to break even. When the informed traders have better quality information, the liquidity providers' expected loss per informed trade is higher, again necessitating a wider spread.

< Figure 3 >

The relations illustrated in Figure 3 mirror the familiar comparative statics obtained in most models of equilibrium bid-ask spreads under information asymmetry: more informed trade leads to wider spreads. Less explored, and perhaps more important for welfare analysis, are the dynamics of spreads and the average spreads through time. In contrast to the previous analysis of spreads at a point in time, it is not necessarily the case that average spreads through time are wider in the presence of more informed trading. This is because with more informed trading, prices converge faster to the true future value of the security, resolving the uncertainty and eliminating the adverse

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selection problem sooner. In the comparative statics of Figure 3, we see this tradeoff in Panel A. Starting at $p_t = \frac{1}{2}$, a higher δ implies a wider spread; however, a higher δ also results in sliding down the curve towards $p_t = 0$ or $p_t = 1$ (where the spread is at its minimum) at a faster rate.

To examine the expected spreads, accounting for the dynamics, we re-express the spread in (5) as a function of the parameters that govern informed trading (δ, which is a function of α and q) and the number of buys and sells received up to that point in time:

$$Spread_t = \frac{\delta^{b_t + s_t}(\delta^2 - 1)}{(\delta^{1+b_t} + \delta^{s_t})(\delta^{b_t} + \delta^{1+s_t})} \,. \tag{6}$$

Result 4

Recall that the number of buys and sells received up to a point in time, b_t and s_t, and the number of non-trades, follow a trinomial distribution with fixed probabilities. The expected spread through time (the average across the trading periods $t = 1, \dots, T$) with $q < 1$ (so the market is open irrespective of α) is,

$$\mathbb{E}\,[Spread\,] = \frac{1}{2T} \sum_{t=1}^{T} \sum_{b=0}^{t-1} \sum_{s=0}^{t-1-b} \frac{(t-1)!}{b!\, s!\, (t-1-b-s)!} P_3^{(t-1-b-s)} \left(P_1^b P_2^s + P_2^b P_1^s\right) Spread_t \tag{7}$$

where P_1, P_2 and P_3 are the conditional probabilities of different trade types defined in (4a)-(4c).

Proof: See Appendix.

Result 4 shows that the expected spread through time can be computed as a function of three underlying parameters, α and q, which govern the intensity of informed trade and quality of informed traders' information, and T, the number of trading rounds. The parameter T measures the horizon of the informed traders' information because after T trading rounds the information about the security value is revealed.[6]

Figure 4 plots the expected bid-ask spread against the probability of informed trade for four different values of the information horizon (the number of trading rounds,

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[6] Letting T be stochastic produces very similar results to fixed T and therefore we report results from the simpler model (fixed T).

T). In Panel A informed traders have perfect information, i.e., $q = 1$. In Panel B signals are noisy, with $q = 0.9$ (high quality information) for the left hand side plot and $q = 0.6$ (low quality information) for the right hand side plot. Starting with the case where $q = 0.9$, as the rate of informed trade increases from zero, expected spreads initially become wider reflecting an increase in average adverse selection risks. They reach a maximum at some level of informed trade and thereafter actually *decline* in the rate of informed trade. The decline occurs because after some point, the effect of faster convergence of prices to V and thus lower adverse selection in later trading periods exceeds the effect of higher initial adverse selection. The point at which expected spreads reach their maximum is dependent on the horizon of information, T. The longer the information horizon, the lower the rate of informed trade at which the expected spread is maximized. With longer-lived information, it is more likely that increasing the amount of informed trade will decrease average spreads because there is a longer period of time in which to accumulate the benefits of narrow spreads following convergence of prices towards V. For similar reasons, expected spreads are generally wider for shorter-lived information.

< Figure 4 >

Similar patterns emerge for different values of q; although lower quality information tends to increase the value of α at which expected spreads peak. For example, with low quality ($q = 0.6$) short-lived information ($T = 5$), expected spreads are increasing over the whole range of informed trading intensity, α. Only when the low quality information is sufficiently long-lived (e.g., $T = 100$ and $T = 1000$) do expected spreads initially increase and subsequently decrease in the rate of informed trade. The reason is that price discovery is generally slower when information is of poorer quality and thus the benefits from more informed trade take longer to materialize. In Panel A (perfect information) the α at which expected spreads are maximized is the lowest of the three plots (note the horizontal axis only goes to $\alpha = 0.5$ because with perfect information the market fails to open if $\alpha > 0.5$). Nevertheless, when the private

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information is sufficiently short-lived (e.g., $T = 5$) expected spreads are effectively increasing over the whole range of α for which the market is open.

In summary, the analysis of expected spreads through time reveals a tradeoff: higher intensity of informed trade causes wider spreads in early trading but also faster convergence of prices towards the fundamental value and therefore narrower spreads in later trading periods. For sufficiently long-lived information (and/or sufficiently precise information), a higher rate of informed trade can decrease expected spreads.[7] These observations suggest a somewhat more complex relation between spreads and informed trading than the familiar comparative static that is often presented in models of trading under information asymmetry. The relation that we have characterized is consistent with the results of Roşu's (2014) dynamic limit order book model. Roşu (2014) finds that a higher share of informed traders *improves* liquidity because it reduces uncertainty about the fundamental value. Importantly, Roşu's results arise in a dynamic setting with long-lived information. We show that the same mechanism is at work in our simpler sequential trade model with sufficiently long-lived information when accounting for the dynamics.

Furthermore, our analysis of expected spreads provides a way to understand the connection between what Roşu (2014) describes as two opposing views: the *adverse selection view* that more informed trading leads to higher adverse selection risk and the *dynamic efficiency view* (stemming from Roşu's model) that more informed trading leads to better overall liquidity. These two views arise from interpreting a comparative static describing the relation between informed trading and the spread *at a single point in time* compared to analysis of liquidity *through time*, accounting for the dynamics. Our analysis shows that a positive relation between informed trading and liquidity is not a unique feature of Roşu's model; it occurs in sequential trade models as long as the dynamics are taken into consideration. Roşu (2014) conjectures that the adverse selection view is more appropriate in circumstances with short-lived information and the dynamic efficiency view is applicable with long-lived information. Our analysis formalizes this intuition by showing that the relation between informed trading and

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[7] This result is not a consequence of the elastic uninformed demand; it also occurs under inelastic uninformed trading demands.

liquidity depends on the horizon of the information. Finally, our characterization of spreads and their dynamics suggests the welfare costs of informed trade are likely to depend on the horizon of information – we turn to this issue next.

4. Analysis of Welfare

The bid-ask spread causes a welfare loss because uninformed agents choose not to trade when their idiosyncratic valuation lies within the spread, resulting in lost gains from trade. Gains from a trade for an individual are measured as the difference between the individual's private valuation and the price at which the trade is executed. Thus, we focus on ex-ante welfare; ex-post there are no welfare costs, just transfers.

Assuming the market is open, the welfare gain of an informed trader that arrives at time t is:

$$(v_t^H - Ask_t)\mathbf{1}_{\{Signal_t=H\}} + (Bid_t - v_t^L)\mathbf{1}_{\{Signal_t=L\}},$$

where $\mathbf{1}_{\{\cdot\}}$ is an indicator function. Recall v_t^H (v_t^L) is an informed trader's expected value given a high (low) signal. Similarly, the welfare gain of an uninformed trader that arrives at time t given a private valuation v_t is:

$$(v_t - Ask_t)\mathbf{1}_{\{v_t>Ask_t\}} + (Bid_t - v_t)\mathbf{1}_{\{v_t<Bid_t\}}.$$

As indicated by the expression above, the uninformed trader has a positive welfare gain if their private valuation is outside the spread and thus their arrival to the market results in a trade and it is zero when the uninformed trader chooses not to trade because their valuation lies within the spread.

Liquidity suppliers lose when the arriving trader is informed (expected loss equal to the difference between the informed private valuation and the trade price), but gain when the arriving trader is uninformed (expected gain equal to the difference between the prevailing expected value and the trade price). Therefore, the liquidity supplier's welfare from trading round t is:

$$\mathbf{1}_{\{Trader_t=\text{Informed}\}}\left[(Ask_t - v_t^H)\mathbf{1}_{\{Signal_t=H\}} + (v_t^L - Bid_t)\mathbf{1}_{\{Signal_t=L\}}\right] +$$

$$\mathbf{1}_{\{Trader_t=\text{Uninformed}\}}\left[(Ask_t - p_t)\mathbf{1}_{\{v_t>Ask_t\}} + (p_t - Bid_t)\mathbf{1}_{\{v_t<Bid_t\}}\right],$$

where $\mathbf{1}_{\{Trader_t=\text{Informed}\}}$ and $\mathbf{1}_{\{Trader_t=\text{Uninformed}\}}$ are indicators for the arrival of an informed and an uninformed trader respectively at time t.

The total welfare from trading round t is the sum of the above components:

$$W_t = \mathbf{1}_{\{Trader_t=\text{Uninformed}\}}\left[(v_t - p_t)\mathbf{1}_{\{v_t>Ask_t\}} + (p_t - v_t)\mathbf{1}_{\{v_t<Bid_t\}}\right] \quad (8)$$

The expression above indicates that net welfare gains stem from uninformed trade; informed trades simply result in transfers. Furthermore, welfare depends on the amount of informed trade because informed trade determines the bid and ask quotes, which in turn determine whether an uninformed trader will choose to trade. The higher Ask_t and the lower Bid_t, i.e., the wider the spread, the more uninformed choose not to trade. These forgone gains from trade reduce overall welfare.

The benchmark welfare is the potential gains from trade that could be earned with a zero spread. Replacing Ask_t and Bid_t with p_t in (8) results in the maximum welfare that could be earned at a point in time:

$$W_t^{MAX} = \mathbf{1}_{\{Trader_t=\text{Uninformed}\}}\left[(v_t - p_t)\mathbf{1}_{\{v_t>p_t\}} + (p_t - v_t)\mathbf{1}_{\{v_t<p_t\}}\right] \quad (9)$$

Subtracting (8) from (9) leads to the per period welfare cost of informed trade:

$$WC_t = \mathbf{1}_{\{Trader_t=\text{Uninformed}\}}\left[(v_t - p_t)\mathbf{1}_{\{Ask_t>v_t>p_t\}} + (p_t - v_t)\mathbf{1}_{\{Bid_t<v_t<p_t\}}\right] \quad (10)$$

The welfare cost associated with informed trade stems from, but is different to, the welfare cost incurred by uninformed traders. Part of the welfare cost borne by uninformed traders is a transfer to informed traders via the liquidity provider, and part is a true welfare loss to society. Figure 5 illustrates this point. If the uninformed private valuation (v_t), lies outside the spread, a trade occurs. In such cases, the uninformed trader gains the difference between their private valuation (v_t) and the trade price (the bid or the ask quote). The gain to the uninformed trader is lower than it would be if the spread were zero; the uninformed trader incurs a welfare cost equal to the effective half-spread (the difference between the trade price and expected value, p_t). This welfare cost to the uninformed trader, however, is not a loss from society's perspective; rather, it is a transfer from the uninformed trader to the informed traders via the liquidity provider. The liquidity provider earns the half-spread on uninformed trades and uses these profits to offset the expected losses to informed traders. So when an uninformed agent arrives at the market and decides to trade, no welfare losses are incurred from society's perspective, only gains and transfers. In contrast, if the uninformed private valuation lies within the spread, no trade occurs and the foregone potential gains from trade (the difference

between the private value (v_t) and the public information expected value (p_t)) constitutes a loss to society.

<center>< Figure 5 ></center>

Using the distribution function in (1a) and the bids and asks reported in (2a)-(2b), we can calculate the expected welfare loss conditional on p_t (the market's expectation of V). The expected one-period welfare cost is given in the following result.

Result 5

Conditional on p_t, and $q < 1$ (so the market is open irrespective of α), the expected one-period welfare cost in a given trading round t is given by:

$$\mathbb{E}_t[WC_t] = \begin{cases} (1-\alpha)\dfrac{p_t(1-p_t)}{(1-2p_t)^2}\ln\left(\dfrac{(\delta+1)^2}{4(p_t+(1-p_t)\delta)(1-p_t+p_t\delta)}\right) & \text{for } p_t \neq \frac{1}{2} \\ (1-\alpha)\dfrac{(\delta-1)^2}{4(\delta+1)^2} & \text{for } p_t = \frac{1}{2} \end{cases} \tag{11}$$

For $q = 1$ (with $\alpha < \frac{1}{2}$ so that the market is open), the corresponding expressions are:

$$\mathbb{E}_t[WC_t] = \begin{cases} (1-\alpha)\dfrac{p_t(1-p_t)}{(1-2p_t)^2}\ln\left(\dfrac{(1-\alpha)^2}{1-2\alpha+4p_t(1-p_t)\alpha^2}\right) & \text{for } p_t \neq \frac{1}{2} \\ (1-\alpha)\dfrac{\alpha^2}{4(1-\alpha)} & \text{for } p_t = \frac{1}{2} \end{cases} \tag{12}$$

Proof: See Appendix.

The above welfare costs are expected welfare costs at a point in time. They are maximized at $p = \frac{1}{2}$, similar to the spread, and the costs are symmetrical about $p = \frac{1}{2}$. Furthermore, it is straightforward to verify that (11) is increasing in the informativeness of trades, δ. While not so obvious, it is possible to show that keeping q fixed, expected welfare costs at a point in time (both (11) and (12)) are increasing in the rate of informed trade, α. These observations are similar to the comparative statics for spreads, which, at a point in time, are also increasing in α.

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It is interesting to relate the welfare cost to the spread. This comparison is easiest at $p = \frac{1}{2}$, and $q = 1$. The spread is given by $\alpha/(1 - \alpha)$, while the welfare cost is $\alpha^2/4(1 - \alpha)$. One is proportional to the other, but this proportionality changes as the probability of informed trade changes. The spread is proportional to the welfare cost per informed trader.

The main aim of this analysis is to characterize the total welfare loss through time. The per period expected welfare costs in (11) and (12) are a function of p_t and the parameters that govern informed trading (δ, which is a function of α and q). The dynamics described in (3a) and (3b) show that p_t is a function of the number of buys and number of sells received up to time t, which follow a trinomial distribution, conditional on V. Therefore, we can write an expression for the expected total welfare cost over time, which is given in the following result.

Result 6

The expected total welfare cost over time is:

$$\mathbb{E}\left[TWC\right] = \frac{1}{2} \sum_{t=1}^{T} \sum_{b=0}^{t-1} \sum_{s=0}^{t-1-b} \frac{(t-1)!}{b!\,s!\,(t-1-b-s)!} P_3^{(t-1-b-s)} \left(P_1^b P_2^s + P_2^b P_1^s\right) EWCP_t(b, s, \alpha, q) \quad (13)$$

where P_1 to P_3 are given in (4a-4c), and $EWCP_t(b_t, s_t, \alpha, q)$ is the expected welfare cost per period defined in (11) replacing p_t with $\delta^{b_t-s_t}/(1 + \delta^{b_t-s_t})$ from (3b) and replacing δ with (2c) if $q < 1$ and with $1/(1 - 2\alpha)$ if $q = 1$.

Proof: See Appendix.

The expected welfare cost per period in (13), $EWCP_t(b_t, s_t, \alpha, q)$, is a function of the number of buys and sells received up to time t and the parameters that govern informed trading, α and q. Therefore, the expected total welfare loss in (13) is a function of only three parameters: the rate of informed trade, α, the quality of informed traders' signals, q, and the horizon of informed traders' information, T.

To show how the expected total welfare costs are impacted by informed trade, Figure 6 plots the relation between $\mathbb{E}\left[TWC\right]$ and α for different information horizons.

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Panel A shows that when informed traders' signals are perfect ($q = 1$) and information is short-lived ($T = 5$) the expected total welfare cost is essentially increasing for most of the range of α.[8] However, for longer lived information ($T = 30$, $T = 100$, and $T = 1000$), the figure indicates that there is indeed a trade-off between high initial per period welfare costs and faster price discovery. As the probability of informed trade increases, the expected total welfare cost initially increases because as the spread becomes wider more uninformed agents' private valuations fall within the spread. But, more informed trade implies that information gets into prices more rapidly, leading to narrower spreads in later trading periods. So, beyond a certain 'tipping' point, an increase in α actually *reduces* the expected total welfare costs. The longer the information horizon, the lower the tipping point beyond which increases in informed trade benefit total welfare. The reason that total welfare costs are largely increasing in α when information is short-lived is that the aforementioned tradeoff has little time to materialize. It is only for the longer term information that the tradeoff can be realized.

< Figure 6 >

Panel B of Figure 6 illustrates the relation between $\mathbb{E}[TWC]$ and α when information is imperfect. The left hand side plot uses $q = 0.9$ (relatively good quality information) and the right hand side has $q = 0.6$ (relatively noisy information). The same tradeoff holds with imperfect information. Expected total welfare costs are single-peaked in α, reaching a maximum at an internal point. For short-lived information, the maximum occurs at a very high probability of informed trade and thus the expected total welfare cost is mostly increasing in informed trade. For longer lived information the maximum occurs at a relatively small probability of informed trade and thus the expected total welfare cost is mostly decreasing in informed trade.

It is not surprising that as T increases, the total welfare cost increases. Once the information is made public, there is no further welfare cost and so if that occurs quickly, there will be a lower cost. The quality of information also influences the shapes of the

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[8] Although the total welfare cost is increasing for *most* of the range of α, it is possible to show that in the limit as α goes to ½, the derivative of the expected total welfare cost with respect to α is negative.

curves. The more precise the information, the earlier the tipping point in α after which increases in informed trade tend to reduce welfare costs.

Welfare losses are only incurred when uninformed traders arrive at the market. Therefore, some of the decline in expected total welfare costs as α increases is due to the declining proportion of uninformed traders. This is not, however, what causes expected total welfare costs to be single-peaked in α when T is sufficiently large; the decline in welfare costs beyond a point is due to information getting into prices faster leading to lower average adverse selection costs. Expected spreads, for example, have the same single-peaked pattern with respect to α and they are computed across all trades irrespective of trader type. Furthermore, expected total welfare costs *holding constant the expected number of uninformed traders* display similar single-peaked patterns as α increases. To illustrate, define the information horizon, H, as the expected number of uninformed arrivals before the information is released, $H = T(1 - \alpha)$. Holding H fixed, as we increase α we increase the total number of trading rounds, $T = \frac{H}{1-\alpha}$, i.e., we add more informed traders to the population leaving the number of uninformed traders unchanged. If informed and uninformed traders arrive at the market according to Poisson processes, holding H fixed and increasing α is like increasing the arrival rate of informed traders without changing the arrival rate of uninformed traders.[9]

< Figure 7 >

Figure 7 illustrates the relation between $\mathbb{E}[TWC]$ and α, holding fixed the expected number of uninformed traders, H. The pattern is similar to before: with sufficiently long-lived (and sufficiently precise) information, the welfare costs associated with informed trade are mainly decreasing in the amount of informed trade, whereas for short-lived information or very imprecise information, welfare costs are mainly increasing in the amount of informed trade.

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[9] In contrast, holding T fixed and increasing α is like increasing the arrival rate of informed traders and concurrently decreasing the arrival rate of uninformed traders to keep the sum of the arrival rates fixed.

5. Applications

Our analysis provides insights and predictions about a number of settings involving trading on private information. Many companies have self-imposed policies governing when and how company insiders (directors and specific employees) are allowed to trade shares in the company. For example, Bettis, Coles and Lemmon (2000) document that in their sample of listed US corporations in 1996 approximately 92% have policies restricting trading by insiders and 78% have explicit blackout periods. Blackout periods are typically windows of a few days prior to earnings announcements (but can also precede other corporate events such as dividend announcements, quarter-end, and M&A) during which insiders are prohibited from trading.

Our model provides a rationale for blackout periods. The private information that corporate insiders possess in the period immediately prior to an important company announcement such as earnings is relatively short-lived because it is due to become public in a matter of days. Our model predicts that for short-lived information, the bid-ask spread and the expected total welfare cost are mainly increasing in informed trade because the tradeoff involving faster price discovery and lower adverse selection in later periods has too little time to materialize. Thus, blackout periods may reduce average spreads and reduce total welfare losses. Consistent with this prediction, Bettis et al. (2000) find that during blackout periods, bid-ask spreads are approximately 8.5% or 2 bps narrower. In contrast, the information possessed by corporate insiders at other times is often relatively longer-lived, e.g., information on company performance between reporting periods, or a better sense of the company's long-run prospects. Allowing insiders to trade on such information can be beneficial because, although the insider trades will initially lead to wide spreads and considerable welfare losses, they will be more than offset by lower adverse selection risks and lower welfare losses after the private information is impounded into prices.

With trading fragmented across many markets and occurring with a high degree of automation and speed, determining the best available quotes at a point in time is no longer a trivial exercise. With traders increasingly using technology to compete on speed, timely access to market data and the ability to act sufficiently quickly on such information is also a type of informational advantage in today's markets. For example,

Ding, Hanna and Hendershott (2014) find that the official National Best Bid and Offer (NBBO) is slow: proprietary data feeds from exchanges can be used to construct a faster NBBO, which frequently experiences dislocations from the official NBBO. The dislocations occur several times a second in very active stocks and typically last one or two milliseconds. Ding et al. (2014) show that access to the exchange data feeds and quick calculation of the NBBO generates opportunities to profit at the expense of slower market participants.

Having access to proprietary data feeds and being able to calculate the NBBO before most other market participants is private information of perhaps the shortest possible horizon: within one or two milliseconds the private information becomes public. In this scenario, our model suggests spreads and expected total welfare costs are increasing in the amount of trade that is based on dislocations between a proprietary 'fast' NBBO and the official 'slow' NBBO. Thus, our model predicts that there are benefits to limiting (ideally, eliminating) the number of market participants that use the 'fast' NBBO to exploit slower market participants.

Another example of an extremely short-lived information advantage is documented by Hu, Pan and Wang (2013). The Michigan Index of Consumer Sentiment (ICS) is disseminated bi-monthly by Thomson Reuters and often moves financial markets. Between 2007 and 2013, Thomson Reuters (who has exclusive rights to disseminate the index) would send the ICS to a small group of fee-paying high-speed clients exactly two seconds before the official broad release of the ICS, giving them an "early peek advantage".[10] Hu et al. (2013) find that there is intense competition between the high-speed traders to exploit their short-lived information advantage: trading volume in the one-second interval after the "early peek" is more than ten times its normal level. In terms of our model, this implies a very high α during the seconds preceding the official ICS announcement.

The welfare implications of the private information in this instance are perhaps not particularly meaningful given the information advantage is restricted to a two-second period that, unlike the regular dislocations of the fast and slow NBBO, only occurs bi-monthly. However, a fascinating aspect of this example is the speed of price discovery.

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[10] In 2013 the practice attracted media attention and Thomson Reuters has since ceased this practice.

Hu et al. (2013) find that almost all of the price discovery takes place in the 15 milliseconds immediately after the early peek, or equivalently, the first 10% of trades in the first second after the peek. After that time, there is virtually no more drift in prices and the official ICS release no longer moves prices substantially. The extremely fast price discovery is consistent with our model, because in this setting the information is very precise and the α extremely high.

As a final example, Irvine, Lipson and Puckett (2007) find evidence of "tipping" in which Wall Street Analysts tell good clients when there will be an equity recommendation change (hold to buy or strong buy). This is not illegal, unless the Wall Street Firm has a policy of not doing it. Should it be illegal? This is relatively short-lived information as the authors suggest the tip could be received up to four days before the announcement, but often even closer to the announcement date. Thus, the spread and total welfare costs are likely to be increasing in the amount of trading on such tips, up to a fairly high rate of informed trade. Institutions doing unusual buying prior to an upgrade account for only 2.5% of volume suggesting that α is small and we are almost certainly in the region where total welfare is decreasing in the trading of institutions privy to the analysts' recommendations. This might call for a "Reg FD" type response.

6. Conclusion

Informed trade can cause welfare losses. Liquidity providers set a positive bid-ask spread to recoup from uninformed traders the losses they incur when trading against the informed. Thus, the bid-ask spread facilitates transfers from the uninformed to the informed. Additionally, the bid-ask spread causes welfare losses because the individuals with private valuations that fall within the spread will choose not to trade. These forgone gains from trade constitute a welfare loss to society.

Our analysis suggests that decreasing the amount of informed trade by increasing the costs of informed trade or by imposing restrictions is not necessarily welfare improving, nor does it necessarily lead to narrower spreads on average. If the economy lies on the downward face of the welfare cost hill, then increasing the cost of informed trade will reduce the amount of informed trade and actually *increase* welfare costs.

Similarly, decreasing the amount of informed trade can actually increase average bid-ask spreads.

These counterintuitive relations arise because of the following tradeoff. More informed trade causes higher adverse selection risks, wider spreads and greater welfare losses initially. But, through time, more informed trade also causes information to be impounded into prices faster, and thus narrower spreads and lower welfare losses later. If the private information is sufficiently long lived (a long time until it is due to be announced), the latter effect can dominate the former, causing average spreads and total welfare costs to be *decreasing* in the amount of informed trade above a certain amount. This result follows from analysis of allocative efficiency and gains from trade, and thus does not consider other ways in which informative prices create value, such as better corporate decisions. Additional sources of value would merely strengthen the result that with sufficiently long-lived information, beyond a point, total welfare costs are decreasing in informed trade.

In contrast, there do not appear to be similar benefits from increasing the amount of trade based on short-lived information. For short-lived information it might be argued that there is no value to information getting into prices via informed trade. At the same time, with short-lived information there is insufficient time for the aforementioned tradeoff to materialize and thus reducing the probability of informed trade is likely to be welfare enhancing.

The model in this paper is deliberately kept relatively simple, and therefore a few limitations are worth noting. First, the amount of informed trade is taken to be exogenous and constant through time. The first part is quite intentional. Our view is that information arrives to traders serially. When an individual becomes informed he or she trades on that information. Alternatively, a model might specify a cost of becoming informed which will determine the number of informed agents and the rate at which they arrive to trade.

The constancy of the rate of informed trade is somewhat more problematic, as it is inconsistent with the analysis of Back and Baruch (2004). They show that a Glosten-Milgrom type model converges to a Kyle type model as uninformed trades become small and frequent. But that implies that the intensity of informed trade increases as time

passes. Back and Baruch (2004) consider a monopolist informed trader, while we have in mind a number of distinct individuals becoming informed. Back and Baruch also show that a monopolist informed trader will often engage in bluffing (trading in the opposite direction to their information). The motivation for bluffing is reduced by the presence of other traders with private information because a bluff by one informed trader provides a potential profit opportunity for another informed trader. Bluffing does not occur in our model because each informed individual trades at most once.

Finally, the two point Bernoulli distribution of the future value is certainly unrealistic. However, we are not sure that it is any more unrealistic than the normal model, particularly because the model is designed to examine welfare costs over rather short periods of time.

Appendix: Proofs

Proof of Result 1

In the absence of hedging, W_T is the certainty equivalent of assets that are uncorrelated with the risky security, M, plus the endowment shock. Therefore, $W_T = M + V\mathbf{1}_{\{Long\}} + (1 - V)\mathbf{1}_{\{Short\}}$, where $\mathbf{1}_{\{Long\}}$ and $\mathbf{1}_{\{Short\}}$ are indicators for the long and short shocks respectively. The wealth and utility of an uninformed agent (conditional on the future value of the security) that arrives at the market at time t are as follows:

Future value of security	Long endowment shock (V)	Short endowment shock $(1 - V)$
$V = 0$	$W_T = M;\ U(M) = -\frac{1}{M}$	$W_T = M + 1;\ U(M + 1) = -\frac{1}{M+1}$
$V = 1$	$W_T = M + 1;\ U(M + 1) = -\frac{1}{M+1}$	$W_T = M;\ U(M) = -\frac{1}{M}$

Given that $V = 1$ with probability p_t and $V = 0$ with probability $1 - p_t$, the expected utility of the uninformed agent with endowment shock V is $\mathbb{E}_t[U] = \frac{p_t}{M(M+1)} - \frac{1}{M}$, and for the endowment shock $1 - V$ it is $\mathbb{E}_t[U] = \frac{-p_t}{M(M+1)} - \frac{1}{M+1}$.

An agent with endowment shock V can hedge by selling one unit of the risky security at the bid, giving them final wealth of $W_T = M + Bid_t$ and utility of $U(M + Bid_t) = -\frac{1}{M+Bid_t}$. Similarly, an agent with endowment shock $1 - V$ can hedge their risk by buying one unit of the risky security at the ask, giving them final wealth of $W_T = M + 1 - Ask_t$ and utility of $U(M + 1 - Ask_t) = -\frac{1}{M+1-Ask_t}$. If the bid-ask spread is zero, i.e., $Bid_t = Ask_t = p_t$, the utility of a hedged endowment exceeds the expected utility without hedging for both endowment shocks. As the ask increases and the bid decreases (as the spread widens), the expected benefits from hedging diminish, and at a certain point (which is a function of the certainty equivalent of the investor's other assets, M) the benefits from hedging become zero. Thus, uninformed agents will hedge their endowment risk if the spread is sufficiently narrow. Less wealthy investors are more anxious to hedge the endowment shock and are willing to hedge at wider spreads than wealthier investors.

The private valuation of an uninformed trader with long endowment shock V (who wants to sell the security to hedge endowment risk) is the value of the bid at which

the agent is indifferent between selling the security to hedge risk and not hedging, i.e., the point at which the expected benefits from hedging are zero, which is the certainty equivalent of the endowment shock. Similarly, for the short endowment shock $1 - V$, the private valuation is the ask at which the expected benefits from hedging are zero. Thus,

	Long endowment shock (V)	Short endowment shock $(1 - V)$
Private valuations:	$v_t^{Long} = \dfrac{Mp_t}{1 + M - p_t}$	$v_t^{Short} = \dfrac{p_t(1 + M)}{M + p_t}$
	v_t^{Long} is increasing in M and has the range, $0 \leq v_t^{Long} \leq p_t$	v_t^{Short} is decreasing in M and has the range, $p_t \leq v_t^{Short} \leq 1$
Rearranging:	$M = \dfrac{v_t^{Long}(p_t - 1)}{v_t^{Long} - p_t}$	$M = \dfrac{p_t(v_t^{Short} - 1)}{p_t - v_t^{Short}}$
Taking the derivative:	$\dfrac{dM}{dv_t^{Long}} = \dfrac{p_t(1 - p_t)}{\left(v_t^{Long} - p_t\right)^2}$	$\dfrac{dM}{dv_t^{Short}} = \dfrac{p_t(p_t - 1)}{\left(v_t^{Short} - p_t\right)^2}$

Given that M is distributed according to $F_M(m) = \Pr(M < m) = \frac{2m}{1 + 2m}$, we can apply a transform of variables to obtain the density of uninformed private valuations for both endowment shocks. Specifically, $f_{v_t}(v) = f_M(M(v_t, p_t)) \left| \frac{dM}{dv_t} \right|$, and $F_{v_t}(v) = F_M(M(v_t, p_t))$ if v_t is increasing in M and $F_{v_t}(v) = 1 - F_M(M(v_t, p_t))$ if v_t is decreasing in M. $f_{v_t}(v)$ is the probability density function of uninformed private valuations and $F_{v_t}(v)$ is the cumulative probability density function. Applying the transform in variables we obtain the following.

For the long endowment shock, V:

$$f_{v_t^{Long}}(v) = \begin{cases} \dfrac{2p_t(1 - p_t)}{(2p_t v - p_t - v)^2} & \text{for } 0 \leq v \leq p_t \\ 0 & \text{otherwise} \end{cases}$$

$$F_{v_t^{Long}}(v) = \Pr\left(v_t^{Long} < v\right) = \begin{cases} 0 & \text{for } v < 0 \\ \dfrac{2v(p_t - 1)}{2p_t v - p_t - v} & \text{for } 0 \leq v \leq p_t \\ 1 & \text{for } v > p_t \end{cases}$$

For the short endowment shock, $1 - V$:

$$f_{v_t^{Short}}(v) = \begin{cases} \dfrac{2p_t(1 - p_t)}{(2p_t v - p_t - v)^2} & \text{for } p_t \leq v \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

$$F_{v_t^{Short}}(v) = \Pr(v_t^{Short} < v) = \begin{cases} 0 & \text{for } v < p_t \\ \frac{(p_t - v)}{2p_t v - p_t - v} & \text{for } p_t \le v \le 1 \\ 1 & \text{for } v > 1 \end{cases}$$

Given that the endowment shocks V and $1 - V$ are equally likely, we have,

$$f_{v_t}(v) = (\tfrac{1}{2})f_{v_t^{Long}}(v) + (\tfrac{1}{2})f_{v_t^{Short}}(v) \ ,$$

$$F_{v_t}(v) = (\tfrac{1}{2})F_{v_t^{Long}}(v) + (\tfrac{1}{2})F_{v_t^{Short}}(v) \ ,$$

and therefore,

$$f_{v_t}(v) = \begin{cases} \frac{p_t(1-p_t)}{(2p_t v - p_t - v)^2} & \text{for } 0 \le v \le 1 \\ 0 & \text{otherwise} \end{cases}$$

$$F_{v_t}(v) = \Pr(v_t < v) = \begin{cases} 0 & \text{for } v < 0 \\ \frac{v(p_t - 1)}{2p_t v - p_t - v} & \text{for } 0 \le v \le 1 \\ 1 & \text{for } v > 1 \end{cases}$$

∎

Proof of Result 2

Recall that α is the probability that a trader arriving at the market is informed; $F_{v_t}(v)$ is the cumulative probability density of uninformed private values; D_t is the trade direction (-1 for a sell, $+1$ for a buy and 0 for no trade); p_t is the time t expectation of the future value of the security, which implies $p_t = \Pr\{V = 1 \,|\, History_t\}$; and q is the informed traders' signal quality, $q = \Pr\{Signal_t = H \,|\, V = 1, History_t\}$. The following expressions are immediate:

$$\Pr\{D_t = +1 \,|\, V = 1, History_t\} = \alpha q + (1 - \alpha)\big(1 - F_{v_t}(Ask_t)\big);$$
$$\Pr\{D_t = -1 \,|\, V = 1, History_t\} = \alpha(1 - q) + (1 - \alpha)F_{v_t}(Bid_t);$$
$$\Pr\{D_t = +1 \,|\, History_t\} = \alpha(p_t q + (1 - p_t)(1 - q)) + (1 - \alpha)\big(1 - F_{v_t}(Ask_t)\big);$$
$$\Pr\{D_t = -1 \,|\, History_t\} = \alpha(p_t(1 - q) + (1 - p_t)q) + (1 - \alpha)F_{v_t}(Bid_t).$$

Bayes' rule gives us:

$$Ask_t = \mathbb{E}\,[V = 1 \,|\, History_t, D_t = +1]$$

$$= \Pr\{V = 1 \,|\, History_t\}\frac{\Pr\{D_t = +1 \,|\, V = 1, History_t\}}{\Pr\{D_t = +1 \,|\, History_t\}}$$

$$Bid_t = \mathbb{E}\,[V = 1 \,|\, History_t, D_t = -1]$$

$$= \Pr\{V = 1 \mid History_t\}\frac{\Pr\{D_t = -1 \mid V = 1, History_t\}}{\Pr\{D_t = -1 \mid History_t\}}$$

With some substitution we obtain:

$$Ask_t = p_t\frac{\alpha q + (1-\alpha)\big(1 - F_{v_t}(Ask_t)\big)}{\alpha(p_t q + (1-p_t)(1-q)) + (1-\alpha)\big(1 - F_{v_t}(Ask_t)\big)}$$

$$Bid_t = p_t\frac{\alpha(1-q) + (1-\alpha)F_{v_t}(Bid_t)}{\alpha(p_t(1-q) + (1-p_t)q) + (1-\alpha)F_{v_t}(Bid_t)}$$

Substituting for $F_{v_t}(.)$ from (1b), solving the quadratic equations for Ask_t and Bid_t, and rearranging, we obtain (for $q < 1$):

$$Ask_t = \frac{\delta p_t}{1 + p_t(\delta - 1)} \quad;$$

$$Bid_t = \frac{p_t}{\delta + p_t(1 - \delta)} \quad;$$

$$\delta = \frac{2\alpha q - 1 + \sqrt{1 - 4\alpha(1-\alpha)(2q-1)}}{2\alpha(1-q)}$$

∎

Proof of Result 3

Rearranging the expressions for Ask_t and Bid_t in (2a) and (2b) gives:

$$\frac{Ask_t}{(1 - Ask_t)} = \delta\frac{p_t}{(1 - p_t)} \quad;$$

$$\frac{(1 - Bid_t)}{Bid_t} = \delta\frac{(1 - p_t)}{p_t} \quad.$$

Recall that:

$$p_{t+1} = \mathbb{E}_{t+1}[V \mid History_t, D_t] = \mathbb{E}_{t+1}[V \mid History_{t+1}] = \begin{cases} Ask_t & \text{if } D_t = +1 \\ Bid_t & \text{if } D_t = -1 \\ p_t & \text{if } D_t = 0 \end{cases}$$

Therefore:

$$\frac{p_{t+1}}{1 - p_{t+1}} = \begin{cases} \dfrac{Ask_t}{1 - Ask_t} & \text{if } D_t = +1 \\[2mm] \dfrac{Bid_t}{1 - Bid_t} & \text{if } D_t = -1 \\[2mm] \dfrac{p_t}{1 - p_t} & \text{if } D_t = 0 \end{cases}$$

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Substituting for $Ask_t/(1 - Ask_t)$ and $Bid_t/(1 - Bid_t)$ from above and rearranging we obtain,

$$\frac{p_{t+1}}{1 - p_{t+1}} = \frac{p_t}{1 - p_t} \delta^{D_t}$$

Iterating from the time of the first trade $t = 1$,

$$\frac{p_t}{1 - p_t} = \left(\frac{p_1}{1 - p_1}\right) \delta^{(D_1 + \cdots + D_{t-1})} = \left(\frac{p_1}{1 - p_1}\right) \delta^{(\#buys - \#sells \text{ up to time } t)}$$

Given that initially it is equally likely that V is high or low ($p_1 = \frac{1}{2}$), and $N_t = \sum_{\tau=1}^{\tau=t-1} D_\tau$ is the net number of buys received up to (but not including) the trade at time t, the above simplifies to:

$$\frac{p_t}{1 - p_t} = \delta^{N_t}$$

and therefore,

$$p_t = \frac{\delta^{N_t}}{1 + \delta^{N_t}}.$$

∎

Proof of Result 4

Equation (6) gives the spread at a point in time, $Spread_t$, as a function of δ, b_t and s_t. Recall that the number of buys, sells and non-trades received up to a point in time (b_t, s_t and $n_t = t - 1 - b_t - s_t$) follow a trinomial distribution with fixed probabilities given in (4a)-(4c). The probabilities are P_1, P_2, and P_3, respectively when $V = 1$, and P_2, P_1, and P_3, respectively when $V = 0$. Therefore, the expected spread at a point in time, conditional on $V = 1$ is,

$$\mathbb{E}_t [Spread_t \mid V = 1] = \sum_{b=0}^{t-1} \sum_{s=0}^{t-1-b} \frac{(t-1)!}{b! \, s! \, (t-1-b-s)!} P_1^b P_2^s P_3^{(t-1-b-s)} Spread_t$$

and similarly for $V = 0$,

$$\mathbb{E}_t [Spread_t \mid V = 0] = \sum_{b=0}^{t-1} \sum_{s=0}^{t-1-b} \frac{(t-1)!}{b! \, s! \, (t-1-b-s)!} P_2^b P_1^s P_3^{(t-1-b-s)} Spread_t$$

Given that $V = 1$ and $V = 0$ are equally likely,

$$\mathbb{E}_t [Spread_t] = \frac{1}{2} \mathbb{E}_t [Spread_t \mid V = 1] + \frac{1}{2} \mathbb{E}_t [Spread_t \mid V = 0]$$

Taking the time-series average of the expected spread at every trading round, $t = 1, \ldots, T$,

$$\mathbb{E}\left[Spread\right] = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{1}{2}\mathbb{E}_t\left[Spread_t \mid V = 1\right] + \frac{1}{2}\mathbb{E}_t\left[Spread_t \mid V = 0\right]\right)$$

Substituting and simplifying, we obtain,

$$\mathbb{E}\left[Spread\right] = \frac{1}{2T}\sum_{t=1}^{T}\sum_{b=0}^{t-1}\sum_{s=0}^{t-1-b}\frac{(t-1)!}{b!\,s!\,(t-1-b-s)!}P_3^{(t-1-b-s)}\left(P_1^b P_2^s + P_2^b P_1^s\right)Spread_t$$

∎

Proof of Result 5

Taking expectations of the realized welfare cost per trading round,

$$WC_t = \mathbf{1}_{\{Trader_t = \text{Uninformed}\}}\left[(v_t - p_t)\mathbf{1}_{\{Ask_t > v_t > p_t\}} + (p_t - v_t)\mathbf{1}_{\{Bid_t < v_t < p_t\}}\right]$$

gives:

$$\mathbb{E}_t\left[WC_t\right] = (1-\alpha)\left[\mathbb{E}_t\left[(v_t - p_t)\mathbf{1}_{\{Ask_t > v_t > p_t\}}\right] + \mathbb{E}_t\left[(p_t - v_t)\mathbf{1}_{\{Bid_t < v_t < p_t\}}\right]\right]$$

because $\mathbf{1}_{\{Trader_t = \text{Uninformed}\}}$ is independent of v_t, p_t, Bid_t and Ask_t. The two expectations in the expression above can be computed using integrals and therefore:

$$\mathbb{E}_t\left[WC_t\right] = (1-\alpha)\left[\int_{p_t}^{Ask_t}(v_t - p_t)\,f_{v_t}(v_t)dv_t + \int_{Bid_t}^{p_t}(p_t - v_t)\,f_{v_t}(v_t)dv_t\right]$$

Substituting in the distribution function from (1a) and the bids and asks from (2a)-(2b), evaluating the integrals and simplifying, with and $q < 1$ we obtain:

$$\mathbb{E}_t\left[WC_t\right] = \begin{cases} (1-\alpha)\dfrac{p_t(1-p_t)}{(1-2p_t)^2}\ln\left(\dfrac{(\delta+1)^2}{4(p_t + (1-p_t)\delta)(1-p_t + p_t\delta)}\right) & \text{for } p_t \neq \tfrac{1}{2} \\[4mm] (1-\alpha)\dfrac{(\delta-1)^2}{4(\delta+1)^2} & \text{for } p_t = \tfrac{1}{2} \end{cases}$$

When $q = 1$ (with $\alpha < \tfrac{1}{2}$ so that the market is open) the bids and asks in (2a)-(2b) hold with $\delta = 1/(1-2\alpha)$. Using this expression for δ, we obtain the expected welfare costs per trading round when informed have perfect information ($q = 1$):

$$\mathbb{E}_t\left[WC_t\right] = \begin{cases} (1-\alpha)\dfrac{p_t(1-p_t)}{(1-2p_t)^2}\ln\left(\dfrac{(1-\alpha)^2}{1-2\alpha + 4p_t(1-p_t)\alpha^2}\right) & \text{for } p_t \neq \tfrac{1}{2} \\[4mm] (1-\alpha)\dfrac{\alpha^2}{4(1-\alpha)} & \text{for } p_t = \tfrac{1}{2} \end{cases}$$

∎

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Proof of Result 6

Re-express the expected welfare cost per period in (11) by replacing p_t with $\delta^{b_t - s_t}/(1 + \delta^{b_t - s_t})$ from (3b) and δ with (2c) if $q < 1$ and with $1/(1 - 2\alpha)$ if $q = 1$. This gives the expected welfare cost per period as a function of the number of buys and sells received up to time t, b_t and s_t, and the parameters that govern informed trading, α and q. Denote this function by $EWCP_t(b_t, s_t, \alpha, q)$.

Recall that the number of buys, sells and non-trades received up to a point in time (b_t, s_t and $n_t = t - 1 - b_t - s_t$) follow a trinomial distribution with fixed probabilities given in (4a)-(4c). The probabilities are P_1, P_2, and P_3, respectively when $V = 1$, and P_2, P_1, and P_3, respectively when $V = 0$. Therefore, the expected welfare cost in a given period, conditional on $V = 1$ is,

$$\mathbb{E}_t\left[EWCP_t \mid V = 1\right] = \sum_{b=0}^{t-1} \sum_{s=0}^{t-1-b} \frac{(t-1)!}{b!\, s!\, (t-1-b-s)!} P_1^b P_2^s P_3^{(t-1-b-s)} EWCP_t(b_t, s_t, \alpha, q)$$

and similarly for $V = 0$,

$$\mathbb{E}_t\left[EWCP_t \mid V = 0\right] = \sum_{b=0}^{t-1} \sum_{s=0}^{t-1-b} \frac{(t-1)!}{b!\, s!\, (t-1-b-s)!} P_2^b P_1^s P_3^{(t-1-b-s)} EWCP_t(b_t, s_t, \alpha, q)$$

Given that $V = 1$ and $V = 0$ are equally likely,

$$\mathbb{E}_t\left[EWCP_t\right] = \frac{1}{2}\mathbb{E}_t\left[EWCP_t \mid V = 1\right] + \frac{1}{2}\mathbb{E}_t\left[EWCP_t \mid V = 0\right]$$

Summing the expected welfare costs at every trading round, $t = 1, \dots, T$, we get the expected total welfare cost,

$$\mathbb{E}\left[TWC\right] = \sum_{t=1}^{T}\left(\frac{1}{2}\mathbb{E}_t\left[EWCP_t \mid V = 1\right] + \frac{1}{2}\mathbb{E}_t\left[EWCP_t \mid V = 0\right]\right)$$

Substituting and simplifying, we obtain,

$$\mathbb{E}\left[TWC\right] = \frac{1}{2}\sum_{t=1}^{T}\sum_{b=0}^{t-1}\sum_{s=0}^{t-1-b} \frac{(t-1)!}{b!\, s!\, (t-1-b-s)!} P_3^{(t-1-b-s)}\left(P_1^b P_2^s + P_2^b P_1^s\right) EWCP_t(b, s, \alpha, q)$$

∎

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Panel A: Utility function of an uninformed trader

$U(W_T)$
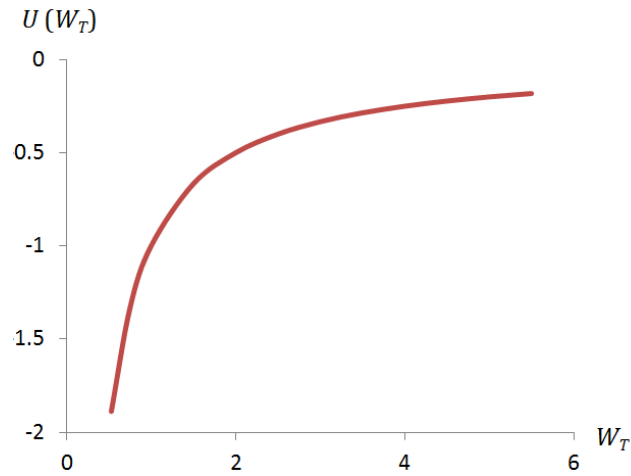
Panel B: Probability density function of uninformed traders' other wealth
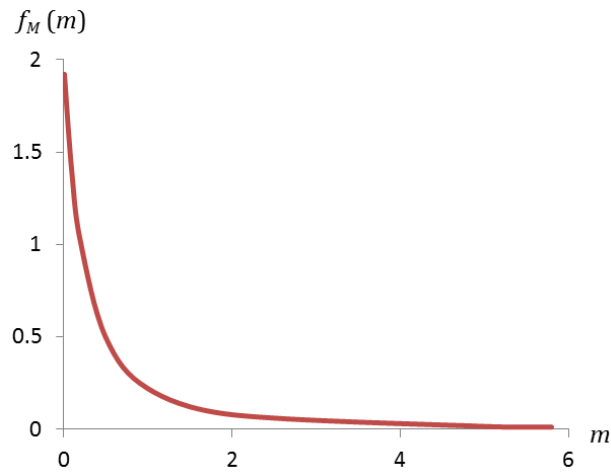
$f_M(m)$

Figure 1. Utility function and distribution of wealth across uninformed traders. Uninformed traders all have the same utility function depicted in Panel A. Utility is a function of final wealth, W_T, which is the sum of an endowment shock and the certainty equivalent of other assets, M. The probability density function of M is depicted in Panel B.
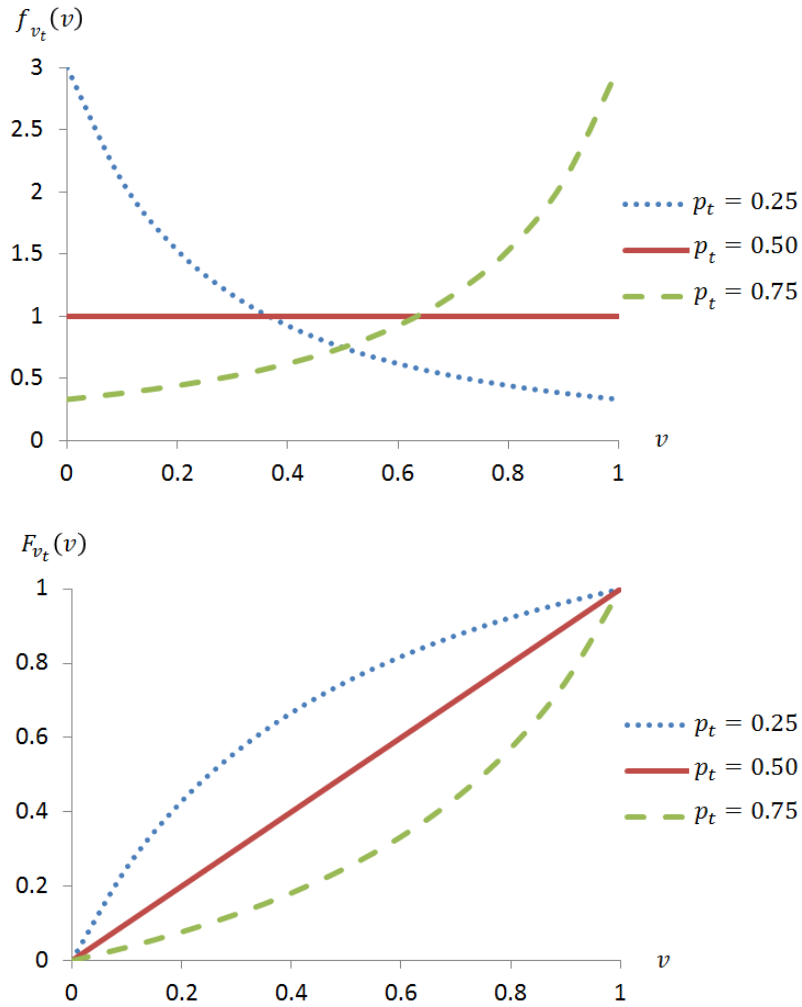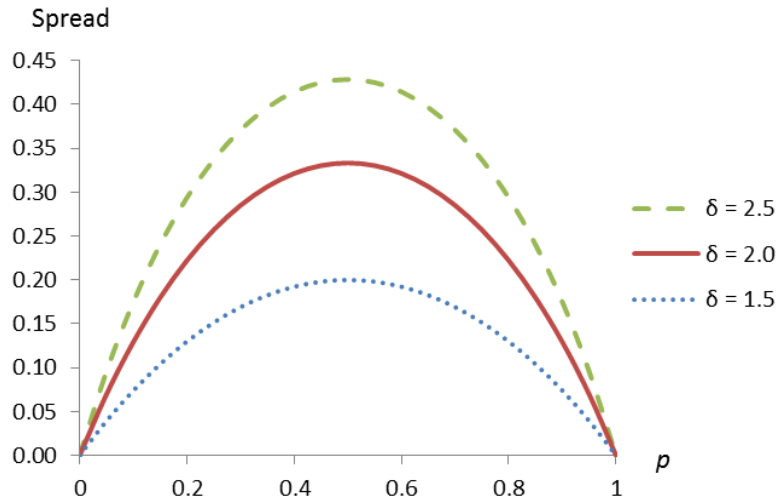
Figure 2. Probability density functions of uninformed traders' private valuations. The figure on the top (bottom) shows the (cumulative) probability density function of uninformed traders' private valuations, v_t. The densities are plotted for three different values of the probability that the security value is 1, p_t.

Panel A

Panel B
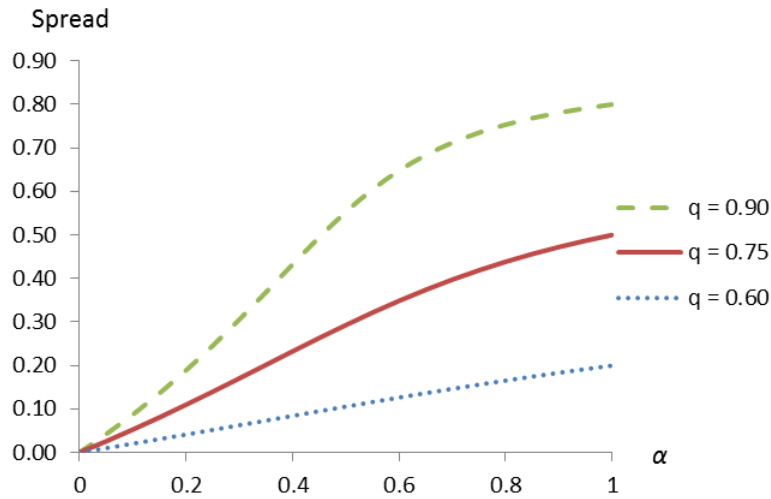
Figure 3. Relation between the spread at a given point in time and parameters governing informed trade. Panel A plots the bid-ask spread at a given point in time, against the public information expectation of the future value of the security (p) on the horizontal axis, for three different values of the informativeness of trades (δ). Panel B plots the bid-ask spread at a given point in time, against the probability of informed trade (α) on the horizontal axis, for three different values of the quality of informed traders' information (q).

Panel A: Perfect information ($q = 1$)
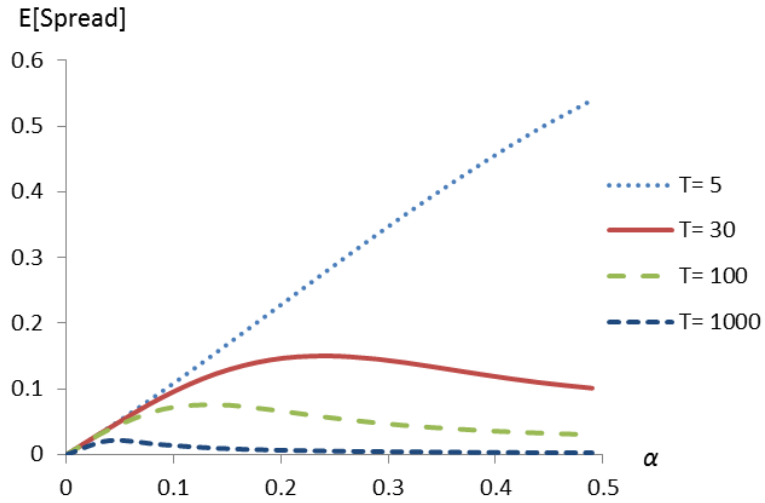
Panel B: Imperfect information ($q = 0.9$ on left and $q = 0.6$ on right)
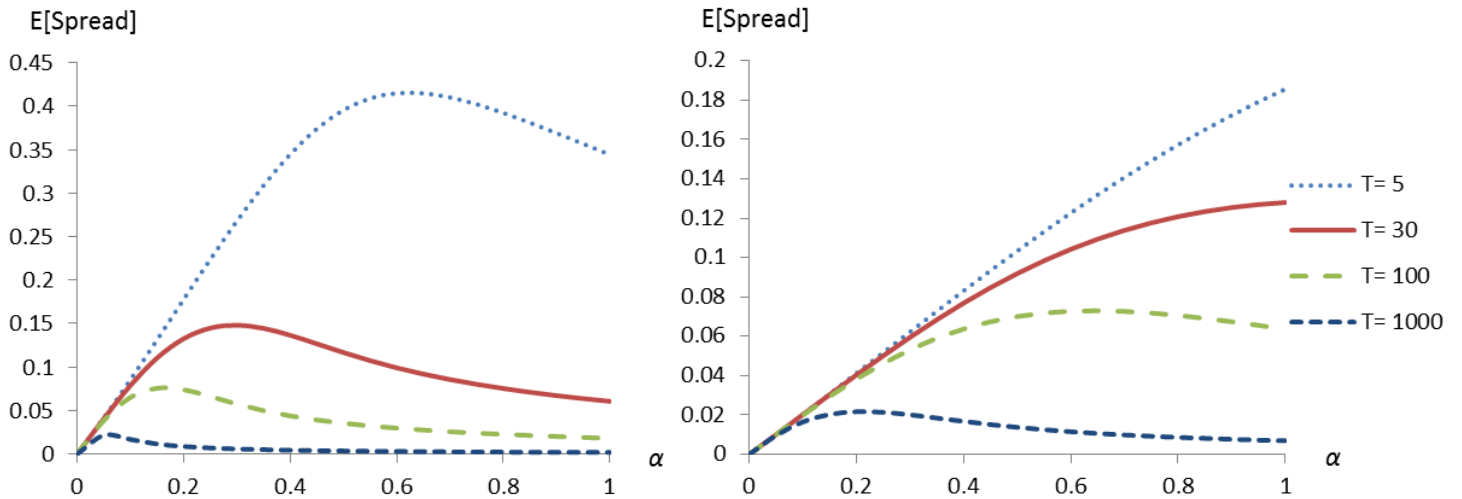
Figure 4. Relation between the expected spread (time-series average) and the rate of informed trade. The figure plots the expected bid-ask spread against the probability of informed trade (α) on the horizontal axis, for three different values of the information horizon (the number of trading rounds, T). In Panel A informed traders have perfect information ($q = 1$), and in Panel B informed traders have imperfect information ($q = 0.9$ and $q = 0.6$).
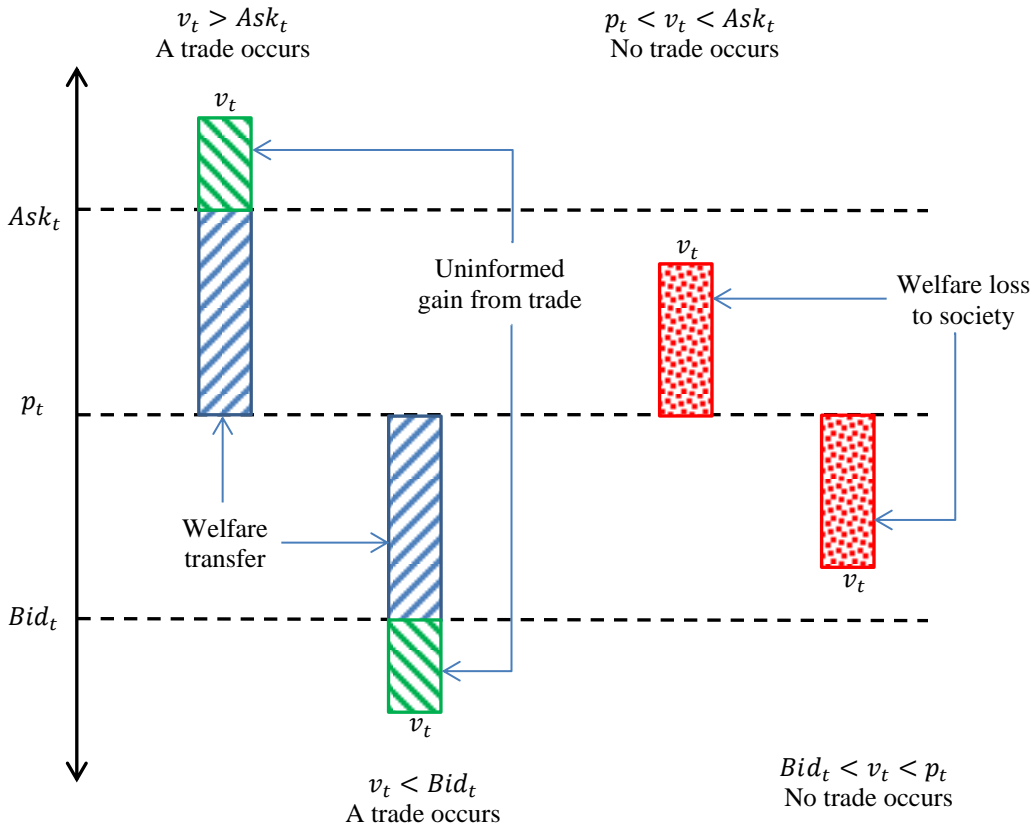
Figure 5. Gains from trade, transfers and welfare losses when the trader arriving at the market is uninformed. The figure illustrates welfare effects associated with an uninformed trader arriving at the market when the bid-ask spread is positive. If the uninformed private valuation (v_t), lies outside the spread, a trade occurs. In such cases, the uninformed trader gains the difference between their private valuation (v_t) and the trade price (the bid or the ask), and the difference between the trade price and the public information expected value (p_t) constitutes a transfer to the liquidity provider to cover losses incurred to informed traders. If the uninformed private valuation lies within the spread, no trade occurs and the foregone gains from trade (the difference between the private value (v_t) and the public information expected value (p_t)) constitutes a loss to society.

Panel A: Perfect information ($q = 1$)
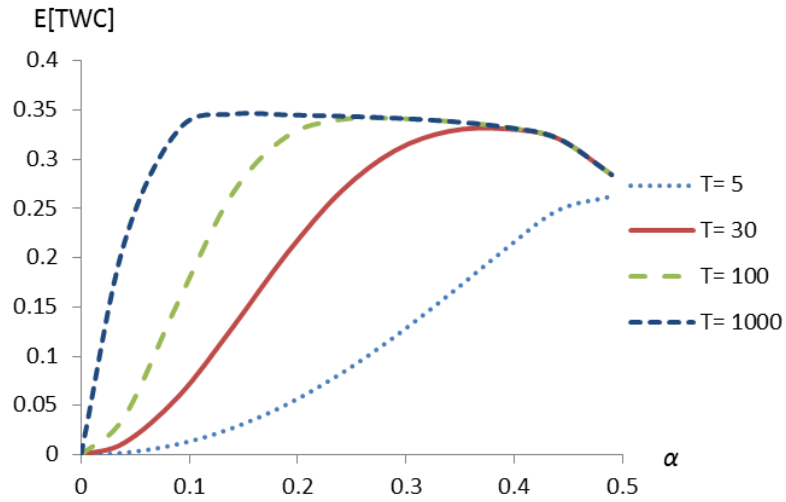
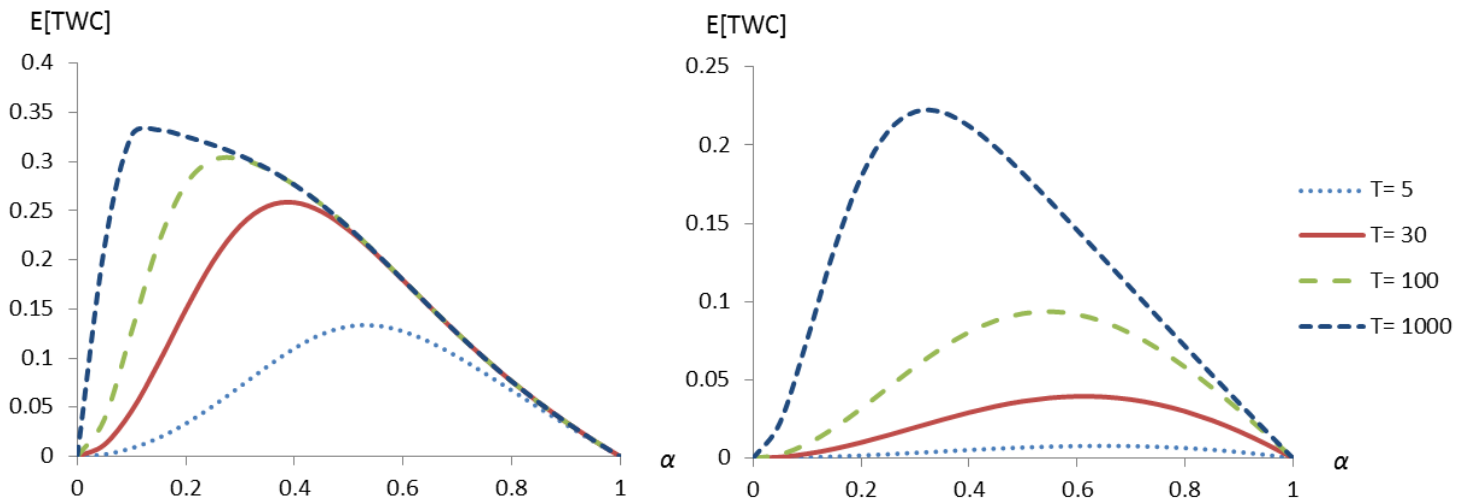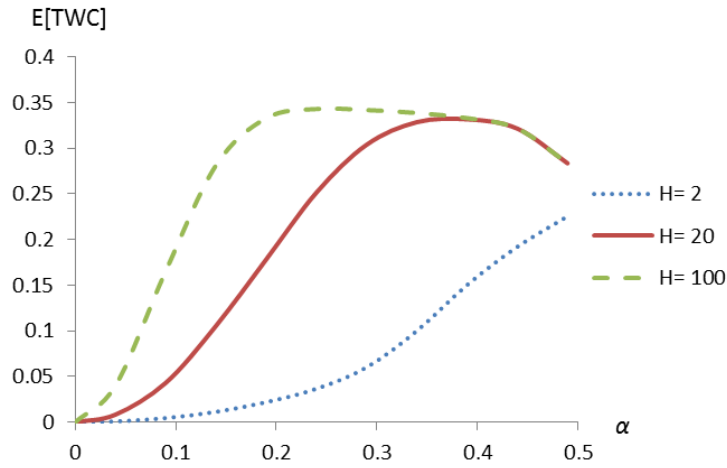**Panel B: Imperfect information** ($q = 0.9$ on left and $q = 0.6$ on right)

Figure 6. Relation between the expected total welfare cost and the rate of informed trade. The figure plots the expected total welfare cost against the probability of informed trade (α) on the horizontal axis, for three different values of the information horizon (the number of trading rounds, T). In Panel A informed traders have perfect information ($q = 1$), and in Panel B informed traders have imperfect information ($q = 0.9$ and $q = 0.6$).

Panel A: Perfect information ($q = 1$)

Panel B: Imperfect information ($q = 0.9$ on left and $q = 0.6$ on right)
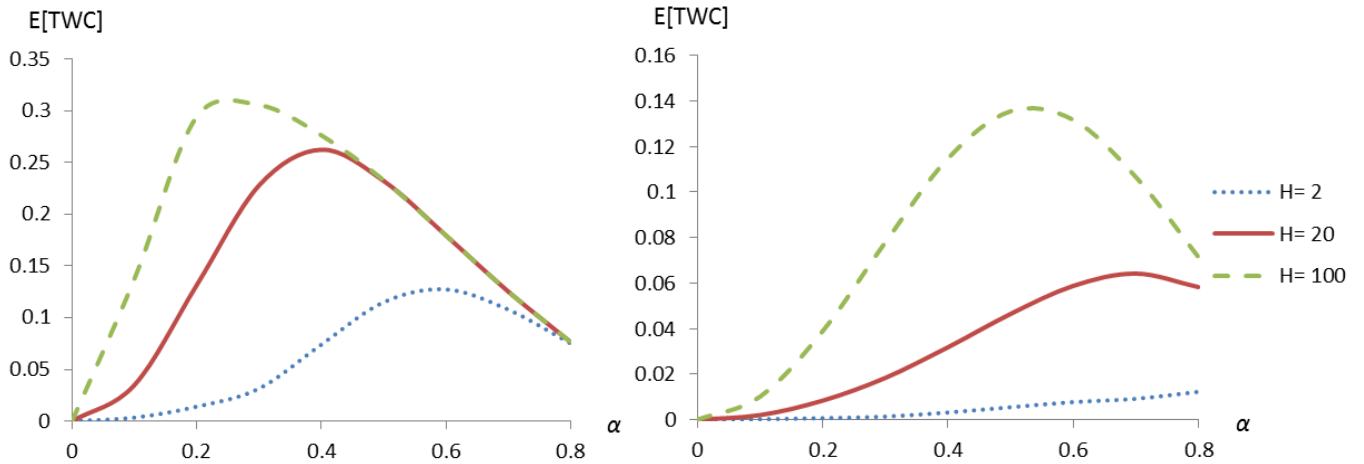

Figure 7. Relation between welfare costs and informed trade holding fixed the expected number of uninformed traders. The figure plots the expected total welfare cost against the probability of informed trade (α) on the horizontal axis holding fixed the expected number of uninformed traders, H. H is also a measure of the information horizon as it is the expected number of uninformed arrivals before the information is released. In Panel A informed traders have perfect information ($q = 1$), and in Panel B informed traders have imperfect information ($q = 0.9$ and $q = 0.6$).