Detail Disagreement and Innovation Booms [†]

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Abstract

We theoretically investigate how disagreement among investors shapes the innovative landscape. Disagreement about which project is likely to succeed creates a form of competition neglect: investors finance their favorite project, ignoring displacement from other firms. More generally, in a market where investors disagree, firms' interactions — positive or negative — are neglected, affecting the nature of the innovation process. Depending on what opinions differ about, disagreement can lead to the exploration of diverse innovative paths or the concentration on a single direction. Those results highlight how features of financing markets affect innovation. We revisit the impact of various interventions put forward to support healthier innovation in the context of markets with heterogenous beliefs.

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1 Introduction

In the face of a new discovery, it is natural for investors to disagree not only about its overall effect but also about the details of its implementation. In this paper, we study the implications of investors' disagreement at the firm level for firm interaction and the diversity of innovation. In particular, we focus on disagreement among investors about which firms will be successful in a specific sector. Understanding the effect of detail disagreement and the speculation it generates on financial markets provides an explanation for the dramatic boom accompanying technological innovations. For instance, between 1998 and 2001, the emergence of the Internet saw a large increase in the valuation of high-tech firms, large numbers of new firms being created and introduced on public markets and subsequently a large drop in asset prices. A similar pattern has been emerging again since about 2009. We argue detail disagreement plays an important role in shaping these events, distinct from aggregate disagreement.

The main contribution of this paper is to highlight the link between detail disagreement and innovation booms. We present an equilibrium model of firm creation and financial markets in the presence of firm level disagreement. This model clarifies the role of this type of disagreement in shaping firm interaction and ultimately innovation. Further, it highlights conditions under which firm level disagreement has a greater influence on innovation and asset prices. Our second contribution is to show disagreement not only impacts the quantity of innovation through but also the very diversity of innovation. We show disagreement gives rise to explorative episodes, where firms are created across more ideas and technologies. Finally, we consider the potential normative implications of the presence of detail disagreement. Other dimensions can generate innovation booms: aggregate disagreement or uncertainty for instance. However, we highlight unique predictions of our framework, mainly portfolio specialization in markets and the explorative dimension of innovation booms.

How does the presence of detail disagreement create an innovation boom? Let us take the example of social networking platforms like Facebook or Twitter. Even if all investors agree a large social network will emerge and dominate the market in the long run, they are likely to disagree on which particular firm will take over the market. Naturally, investors buy shares in firms they believe are likely to succeed and not those they expect to fail. This is the first step towards creating the boom: portfolio specialization. The matching of investors with firms they believe in directly impacts asset prices. Each firm is valued by investors who believe it will succeed, pushing its price above the average belief prevailing in the market. Adding up, this association implies that, even though all investors agree on the total value of the sector, the price of a portfolio of all firms in the sector will be higher than this value. Of course, in the long run, only one firm succeeds and takes over the market. As this information is revealed all other firms disappear and there is a massive asset price drop.

Now, take the perspective of an entrepreneur who considers entering this sector. The attractive prospect of large amounts of capital from investors encourages more entry, fueling the innovation boom. But the quantity of firms is not the only margin of adjustment. For instance, if the entrepreneur creates a firm very similar to Twitter, investors are not likely to develop a different opinion about the new firm. Moreover, the new firm will have to compete directly against Twitter. On the other hand, by creating a more radically different technology, the entrepreneur can ride the differences of opinions, and create a group of investors that specifically prefers to invest in her project. This effect transforms innovation booms into exploration booms. Firms not only make small improvements on existing technologies, but instead seek to create innovations that are greatly different from existing products.

To better understand what exactly about detail disagreement favors the emergence of these episodes, we investigate the role of the competitive structure of the industry. Our example of social network features a winner-takes-all structure: there is a finite amount of profits that will be captured by only one firm. We show such structures are particularly sensitive to detail disagreement. Indeed, once investors match with firms they particularly believe in, they behave as if the competitive pressure from other firms is absent. This effect of competition neglect has a large impact on industries where displacement across firms is strongest. We also show it is at the heart of generating the explorative dimension of innovation as investors are less sensitive to the knowledge externality of following mainstream technologies. This role of competition neglect distinguishes our approach from the standard Miller (1977) effect where disagreement increases prices.

Finally we consider the normative implications of the presence of detail disagreement. Detail disagreement generates larger amounts of entry, but speculative investors tend to find it socially optimal. Looking back at the social network industry, it might seem wasteful to have so many resources involved in a large number of firms where only one is needed in the long run. However, if all investors think that when firms are created they will be able to obtain that likely to success at an attractive price, they all favor firm creation. Naturally, we also show that if the planner imposes common beliefs to evaluate the economy, he will prevent the large amounts of firm creations. However, if one is willing to entertain social benefits of higher levels of exploration, for instance to generate disruptive innovations, the speculative episodes linked to detail disagreement might still be desirable.

Literature Review

Our paper is centered around the real effects of heterogeneous beliefs, particularly the role of disagreement on firm interaction and innovation diversity. There is a substantive literature on the role of heterogeneous beliefs on asset prices that traces back to Miller (1977). The role of heterogeneous beliefs and its role on the overvaluation of asset prices has been further studied in Harrison and Kreps (1978) and Scheinkman and Xiong (2003). Chen, Hong, and Stein (2002) confirm empirically how higher disagreement leads to overvaluations. On the other hand Varian (1985), Carlin, Longstaff, and Matoba (2014) and others argue that without short sale constraints, such disagreement leads to lower prices as investors demand compensation for the trading risk induced by disagreement. We follow the former strand of the literature; however we focus on the consequences of overvaluations, namely how it affects firms' interactions.

In that sense our work relates to studies the real effects of asset prices (see Edmans, Goldstein, and Jiang (2012) for a survey). Goldstein, Ozdenoren, and Yuan (2013) analyze how prices can feedback into investments' decisions. Albagli, Hellwig, and Tsyvinski (2014) show how distorted prices from market imperfections generate inefficient investments.

In our framework investors are overconfident in their investments, generating inefficiencies due to competition neglect. Gennaioli, Ma, and Shleifer (2015) survey CEO and show their extrapolative expectations also lead to inefficient investments.

Finally we think our model of disagreement and innovation sharpens our understanding of events such as "tech bubbles". Pastor and Veronesi (2009) study how the emergence of innovations generate ex-post "rational bubbles", that are unpredictable ex-ante. A growing literature explores the links between innovation and the stock market. Some of the studies emphasize the tension between new firms and incumbents not able to adapt to technological innovation; see for e.g. Boyan Jovanovic (1994), Greenwood and Jovanovic (1999), Hobijn and Jovanovic (2001), Kogan, Papanikolaou, and Stoffman (2015). However we are quite different from these studies as our interests lays on interactions between entrants and how it shapes the innovative landscape.

Outline

The paper is structured as follows. In Section 2, we describe the general role of disagreement at the firm level for innovation. Section 3 presents a first model with firm level disagreement and how it generates competition neglect. In Section 4 we expose a model with two types of disagreement, firm level and field level, and discuss the trade-off between competition neglect and diversity in innovation. We consider the role of government intervention in the context of both types of disagreement in Section 5. Section 6 concludes. All proofs are in the Appendix.

2 Disagreement and Technology Booms

We now expose informally the concept of detail disagreement, that is central to our paper. We coin this term to describe a type of disagreement about the prospects of firms within a given sector.

2.1 Detail disagreement

The premise of our analysis is that market participants often disagree on which firms or technologies are likely to be successful. Our disagreement is not caused by heterogenous information sets but by heterogenous priors; our agents agree to disagree. Morris (1995) provides some motivation to the assumption of heterogenous priors.

However detail disagreement differs from the general disagreement literature so far as we specify heterogeneity in terms of relative evaluation among firms. This type of disagreement has different implications than traditional forms of disagreement that focuses on aggregate states of the world.

To enlighten our discourse we work through a simple example: Let's consider social networking platforms on the internet, such as facebook or twitter. It is fair to say investors tend to agree about the importance of the role of social networks in the future. However it is hard to find the same agreement on the details through which their success will eventually materialise. Will the first one dominate, or the moder newcomer, could both coexist?

This kind of detail disagreement stands in sharp contrast to the classic concept of aggregate disagreement found for instances in Harrison and Kreps (1978) or Scheinkman and Xiong (2003). In the mid-1990s, the interrogation of investors about the future role of the world wide web and its importance for future businesses is an example of aggregate disagreement. Investors do not question individual successes but rather the aggregate final state.

We argue our concept of disagreement is not in opposition with the traditional view. More likely than not, both types will coexist. Our attention however is focused solely on detail disagreement, as we believe it is new in the literature and its implications are not well understood.

We argue our notion of disagreement has important consequences for (a) investor behavior and asset prices: disagreement alters portfolio choices; (b) firm creation due increases with detail disagreement due to valuation effect and a shutdown of competitive concerns. We expose mechanisms below before developping a first model of detail disagreement in Section (3)

3 A Simple Model

We have informally described the mechanism by which detail disagreement favors the emergence of innovation booms. To precise our approach, we now turn to a formal model of detail disagreement. We start by a simple setup in which all firms compete to capture a market. Then we analyze the role of the competitive environment in shaping the effect of detail disagreement. Beyond the points we have already put forward, the main takeaway is that the presence of detail disagreement among investors profoundly affects the competitive environment, creating a form of competition neglect.

3.1 Model setup

We study an economy populated by a unit mass of investors indexed by $j \in [0, 1]$, an infinity of firm creators that in equilibrium create a number n of firms indexed by $i \in \{1, \dots, n\}$. To simplify the exposition we replace n by a continuous variable in optimization problems in order to obtain simple first order conditions. This is without loss of generality in this setup. There are two types of decision makers in the economy: investors and firm creators. Firms are created and traded on financial markets at date 0 and output is realized at date 1.

Firms — Each firm i in the economy is of type θ_i, unknown at date 0. At date 1, the firm produces a cash-flow $y_i = \theta_i \cdot \mathbf{1}\{\theta_i = \max_{i'} \theta_{i'}\}$. This assumption corresponds to a winner-take-all setting. The strongest the winner is, the more profits are collected. We capture our intuitive description of internet social networks where a single actor is likely to capture all the market, however its identity is yet unknown.

Firm creators — Firm creators decide whether to create a firm. Their decision is entirely static. They sell the firm on capital markets as soon as they have created it. To start-up a new firm, firm creators hire one unit of labor. We assume both labor and capital markets are competitive and therefore that firm creators are price-takers. Noting p_i the price of firm i and w the wage prevalent on the labor market, each firm creator decides to create a firm if and only if they can generate positive profits:

$$\pi_i = p_i - w \geq 0,$$

with indifference in the case of equality.

Investors — All investors are ex-ante identical and are endowed with possession of all firm creators and one unit of date 0 consumption good. Their utility of consumption is linear across periods. Further, they can provide l_j units of labor at an increasing convex cost of effort $W(l_j)$ in exchange for the wage w. Investors buy shares s_i^j of firm i in order to obtain future consumption. We assume short-selling is not allowed, $s_i \geq 0$. Once firms are created, investors potentially differ in their beliefs as they have different priors on firm types. We write $\mathbb{E}_j$ the expectation operator correponding to the beliefs of agent j. The optimization problem of an agent is therefore:

$$\begin{aligned} &\max_{c_0, s_i, l} c_0 + \mathbb{E}_j \left[\sum_i s_i y_i \right] - W(l) \\ \text{s.t.} \qquad & c_0 + \sum s_i p_i \leq \pi + wl. \end{aligned}$$

Beliefs — To study the role of detail disagreement, we focus on two extreme cases: one with completely common beliefs, the second with tight heterogenous priors. To discipline the comparison, we focus on a case where the distribution of beliefs across agents under disagreement corresponds exactly to the common belief under agreement. Finally we include a common and idiosyncratic component to firm productivity to differentiate the role of aggregate and detail disagreement.

We assume $\theta_i = \bar{\theta} + \tilde{\theta}_i$. For the case of agreement, the common belief is that $\bar{\theta}$ is drawn from a cumulative distribution function $H(.)$ and that the specific productivity values $\tilde{\theta}_i$ are drawn independently from a cumulative distribution function $F(.)$. The distribution for each firm type θ_i is therefore

$H * F$, where the symbol $*$ represents the convolution product. In the case of disagreement, each investor receives a draw from the joint distribution of types. They each have a tight prior on their draw. The assumption of a completely tight prior is somehow extreme as investors are sure of their knowledge about the firms. We make this assumption as it greatly simplifies derivations, and some milder version of the results we derive hereafter hold without such tight prior. Notice however that uncertainty only plays a minor role in this setup as investors are risk neutral.

Equilibrium conditions — Finally, we need to insure all markets are in equilibrium. This corresponds to the fact that the labor market clears, the market for ownership of the firms, and the market for time 0 consumption. This corresponds to the conditions:

$$\begin{aligned} n &= l, \\ \forall i, \ \int_j s_i^j &= 1, \\ \int_j c_0^j &= 1. \end{aligned}$$

3.2 Equilibrium

First we solve our model in the case of a winner-takes-all market structure. We solve for both cases of common and heterogeneous priors, that is under agreement and disagreement. The dichotomy sheds light on the precise implications of detail disagreement for investor behavior and firm creation. We discuss a comparison of equilibria in the last section.

3.2.1 Agreement

Under agreement, all firms are equally likely to be the most productive:

$$\mathbb{E}_j \left[\sum_i s_i y_i \right] = \sum_i s_i \frac{1}{n} \mathbb{E} \left[\theta | \theta = \max_{i'} \theta_{i'} \right].$$

Because agents are risk-neutral, they take an interior position in asset i if and only if the price equals their expected valuation of the asset. Remarking the distribution of the maximum of n draws from the same distribution F is F^n leads to:

$$\forall i, \ p_i = \frac{1}{n} \mathbb{E} \left[X | X \sim H * F^n \right].$$

This first equation pins down the demand for firms. To obtain the supply of firms, we use the first order condition for labor supply and the zero profit condition for firm creators, to obtain:

$$W'(n) = w = p_i.$$

This concludes the determination of the equilibrium.

Proposition 3.1. *Under agreement, the price and quantity of firms in the unique equilibrium are given by the unique solution to*

$$\begin{aligned} \frac{1}{n}\mathbb{E}\left[X|X \sim H * F^n\right] &= W'(n), \\ p &= W'(n). \end{aligned}$$

3.2.2 Disagreement

Under disagreement, agent j has a tight belief about the productivity of firms. Therefore, the expected date 1 consumption of agent j is:

$$\mathbb{E}_j\left[\sum_i s_i y_i\right] = \sum_i s_i \theta_i^j \mathbf{1}\left\{\theta_i^j = \max_{i'} \theta_{i'}^j\right\}.$$

Clearly, investor j only invests in the stock she expects to succeed, and only does so if her belief about the productivity of this asset is larger than the price. In appendix, we show only symmetric equilibria of this model exist. A fraction $1/n$ of investors have firm i has their highest outcome where there valuation is distributed by $H * F^n$. All those that have a valuation larger than the price invest all their wealth (initial endowment and labor income). Therefore, market clearing for the asset implies

$$\frac{p}{1 + wl} = \frac{1}{n}\left(1 - H * F^n(p)\right)$$

On the left-hand-side figures the fraction of the investors wealth invested in a share of the firm: the ratio of the share price p to their total wealth $1 + wl$. On the right-hand-side The first term is the fraction of investors that prefer sepcifially firm i. The second term is the fraction of investors for which their prefered firm has value over p. Noticing that market clearing for labor implies $l = n$ and that the zero profit condition for firm producers implies $p = w$, we arrange this equation to obtain

$$1 = H * F^n(p)(1 + np).$$

To complete the equilibrium one needs to determine the labor supply as before. The marginal cost of effort the agent is willing to suffer is larger than the price of the asset as investors buying shares perceive they will receive an extra return on their investment. This corresponds to

$$W'(n) = \mathbb{E}\left[\max\left(p, X\right)|X \sim H * F^n\right].$$

Putting these two conditions together, we obtain the determination of the equilibrium.

Proposition 3.2. *Under disagreement, the price and quantity of firms in the unique equilibrium are given by the unique solution to*

$$\begin{aligned} W'(n) &= \mathbb{E}\left[\max\left(p, X\right)|X \sim H * F^n\right], \\ 1 &= H * F^n(p)(1 + np). \end{aligned}$$

3.2.3 Comparing equilibria

We can now compare the outcomes of the situation with and without disagreement. First, note that in this case disagreement gives rise to an extreme form of portfolio concentration. In the presence of detail disagreement investors buy shares in at most one firm. By comparison, with agreement, all investors are indifferent to invest in various firm and in a symmetric allocation would hold diversified portfolios of asset. Importantly this specialization of portfolios is the result of detail disagreement. With a degenerated distribution F, agents would either invest in all stocks or none of the stocks, but never have a strong preference for one stock agains all others.

Second, let us focus the number of firms in each sector. For this consider first equation in the two propositions determining the equilibria. Two main differences appear: the expectation taken for the marginal benefit of an additional firm is over $\max(p, X)$ rather than X with disagreement. This first effect is the standard Miller (1977) effect. In the presence of disagreement and short-sell constraints, as only optimistic investors buy firms, beliefs larger than the price are overrepresented, which pushes the price over the average belief, and subsequently the number of firms. This effect is present with aggregate and detail disagreement. The second main difference is specific to detail disagreement. The $1/n$ factor in the equilibrium with agreement disappears, again creating more entry. We call this effect *competition neglect* as it results from the fact that investors have a preferred firm and ignore the presence of other firms in their evaluation of that firm. With agreement, each new firm lowers the valuation investors put on other firms. This displacement effect is not present with detail disagreement, leading to competition neglect. We come back to this effect in the next section.

Finally we prove in appendix that the asset price is always higher under disagreement that under agreement. Again, this reflects the two selection effects of matching optimistic investors and investors who prefer them to each firm. These three types of predictions reflect the differences we documented in Section 2 when comparing high-tech sectors to the rest of the economy.

3.3 Understanding Competition neglect

To further investigate the seeming disappearance of competition in the model with detail disagreement, we study an alternative structure for the profits of firms. We keep the same data generating process for the types θ_i, but now do not include the displacement effect. All firms produce output $y_i = \theta_i$. In other words, contrary to the previous model, the presence of other firms does not reduce the potential profit of a firm. The actual productive capacity of this economy is much larger than the previous one and all agents in the economy are aware of it. We consider in turns a comparison of this economy to the displacement economy with agreement and without disagreement.

3.3.1 Agreement

Under agreement, each firm is valued at its expected productivity, being the expected value of its type θ_i. The labor supply and firm creation decisions are as in the previous setting. Therefore, the equilibrium is given by

$$\begin{aligned} \mathbb{E}\left[X|X \sim H * F\right] &= W'(n), \\ p &= W'(n). \end{aligned}$$

Creating firms is indeed much more valuable in the eyes of the market. Investors recognize they will always collect the type of the firm, independently of whether it is the most productive or not. Therefore, more firms are created and the price of a given firm is larger.

3.3.2 Disagreement

Under disagreement, even though all investors recognize that all firms will produce, they still decide to specialize their portfolios. Indeed, in a symmetric equilibrium, all firms have the same price. Investors only buy firms for which they expect an output larger than the price. Overall pessimistic investors for which no such firm exists stay out of the market. And, when multiple firms represent such a good deal, the investor only buys the firm with the largest expected gain. Just as in the previous model, investors only buy their favorite firm, even if it is not the only productive firm in their view. Similar derivations as the model with displacement provide the equilibrium conditions:

$$\begin{aligned} W'(n) &= \mathbb{E}\left[\max(p, X) | X \sim H * F^n\right], \\ 1 &= H * F^n(p)(1 + np). \end{aligned}$$

The equilibrium is exactly identical to the model with displacement. This confirms the explanation that once investors specialize in their favorite firm, they do not consider the effect of other firms on the market. This effect is strong as all investors, whatever their belief is, agree that the total productive capacity of this economy is larger than the economy with displacement. Yet, they include as many firms whether this is the case or not. The reason this knowledge is not reflected in market outcomes, is that investors are "blinded" by the good opportunity they face in their favorite firm and extend all their resources to this firm.

Further, note that competition neglect also impacts the economy without displacement. Indeed, beyond the selection effect of firms being evaluated at the expectation of $\max(p, X)$ rather than X, the specialization of each investor in her favorite firm, transforms the distribution at which the type is evaluated from $H * F$ to $H * F^n$. This transformation, once again, is the result of detail disagreement rather than the aggregate disagreement as H is not transformed, only F is.

Summary — To summarize, we showed detail disagreement leads to a form of competition neglect. As each firm is financed by investors who believe it is more productive than competitors, it receives funding as if it was alone in the market. This effect leads to more firm creation and higher asset prices. Naturally, for competition neglect to have a large impact on outcomes, the market needs to be competitive to start with. We showed the increase in firm creation due to competition neglect is much more pronounced in a winner-take-all market than in one where firms all produce independently from each other.

4 A Complete Model

In Section (3) we exposed the role played by a new form of disagreement: "firm level" disagreement. We find firm disagreement generates higher level of innovation: more firms enter the economy as investors are ready to finance them. We now introduce along with firm level disagreement what we call "field level" disagreement. We show that not only firm level disagreement affects the level of innovation but it also affects the diversity of innovation. We show how both types of disagreement interact and affect both the level and type of innovation.

If in Section (3) investors were considering whether to invest in twitter or facebook based on their beliefs, they generally agreed that the industry looked promising, leading to their unit of profit. Now we add a layer of disagreement, where investors are separated into different groups: some might find the idea of social networks attractive, while other think the sector has little chance of success. That second group might for example prefer the newspaper business and within that group we will find individual investors disagreeing over the success chances of the New York Times against the Wall Street Journal.

This additional layer of disagreement not only captures the incentives for diversity in innovation, but also the interaction with firm level disagreement. We show our initial and novel concept of disagreement at the firm level interacts with incentives for increasing diversity. We think our two-tier model captures a richer picture where outcomes are not only the quantity but also the diversity of innovation.

We find conditions under which investors chooses to specialize into one industry: both groups decide they'd better invest in social network industry. We also find conditions under which disagreement leads to more diversity and investors split and invest in each field in proportion.

4.1 Setup

We follow closely the model presented in Section 3, we consider an economy populated by a unit mass of investors indexed by $j \in [0, 1]$, and an infinite number of firm creators that create firms. Now firms can be produced in two fields indexed by $k \in \{1, 2\}$. Within fields k, firms are indexed by $i_k \in \{1_1, ..., n_1, 1_2, ..., n_2\}$.

We assume a winner-takes-all setting. In particular, each field has type λ_k, potentially unknown to investors. The larger the number of firms n_k in a field, the faster the progress of the field. The total speed of progress in field k is $n_k\lambda_k$. The field with the larger value of $n_k\lambda_k$ collects the unit of profit. Within the field, one of the firms collects the profit.

Firm creators — As in our previous model, firm creators decide whether or not to create a firm. Their decision is static and they behave as price takers in competitive markets. Therefore, each firm creator decides to create a firm if and only if:

$$\pi_i = p_i - w \geq 0$$

Investors — All investors are ex-ante identical and endowed with possession of all firm creators and one unit of date 0 consumption good. They have linear utility across periods. In addition, they can provide l_j units of labor with disutility $W(l_j)$ in exchange for wage w. We assume $W', W'' > 0$. Investor j buys share $s^j_{i_k}$ of firm i_k to obtain future consumption. We assume a short sale constraint $s^j_{i_k} \geq 0$. Investors have potentially different priors on firm types. Investors therefore solve

$$\begin{aligned} &\max_{c_0, \{s_{i_k}\}, l} c_0 + \mathbb{E}_j \left[\sum_{i_k} s_{i_k} y_{i_k} \right] - W(l) \\ \text{s.t.} \quad & c_0 + \sum_{i_k} s_{i_k} p_{i_k} \leq \sum_{i_k} \pi_{i_k} + wl \end{aligned}$$

Beliefs — We introduce two types of disagreement. Firstly, there is disagreement over field types. We assume that investors can be divided into two groups. Group 1 believes that $\lambda_1 \sim$ uniform $[1-\alpha, 1]$ and $\lambda_2 \sim$ uniform $[0, \alpha]$, while group 2 believes the converse. $\alpha \in [0, 1]$ parameterizes the level of agreement: $\alpha = 1$ corresponds to agreement, while $\alpha = 0$ corresponds to dogmatic disagreement. For example when $\alpha = 0$, investors in group 1 believe that $(\lambda_1, \lambda_2) = (1, 0)$. Hence they believe the probability of getting the unit of profit from investing in field 2 is impossible no matter the investment as $n_2\lambda_2 = 0$.

The other form of disagreement follows Section 3: investors disagree over firms probability of winning the final profit unit. Each investor has a favorite firm within each field. The investor believes that conditional on the field winning, this firm wins with probability $n_k^{-\beta}$. $\beta \in [0, 1]$ parameterizes the level of agreement - $\beta = 1$ corresponds to agreement about firms within the same field, while $\beta = 0$ corresponds to investors believing that conditional on the field winning, their firm will win with probability one. To relate to Section 3, β parameterized the level of disagreement within a field directly through competition neglect within a field: how far are investors from internalizing the externalities of other firms in the field.

Market clearing We have market clearing conditions, now extended to both fields:

$$\begin{aligned} n_1 + n_2 &= l \\ \int_j s_i^j &= 1 \\ \int_j c_0^j &= 1 \end{aligned}$$

for labor, firm ownership, and time 0 consumption.

4.2 Equilibrium

In equilibrium, we require that the analogous aggregate equilibrium conditions to those of the first model are satisfied. In addition, we require that no investor would rather invest in a different field. When solving the investor's problem for which field to invest in, it is important to consider the discreteness of the number of firms. However, when solving for the equilibrium level of firm entry, taking the number of firms as continuous is without much loss of generality.

Firstly, free entry implies $\pi_i = 0$, for every firm i. Therefore $w = p_i$. The investor's first-order condition for labor implies that $W'(l) = w$. Hence, denoting the total number of firms by $N = n_1 + n_2$, we have

$$W'(N) = p_i$$

The price of the firm will be determined by the willingness of the most optimistic investor to pay for shares in the firm, i.e.

$$p_i = \max_j \mathbb{E}_j [y_{i_k}]$$

In Appendix (A), we derive expressions for $\mathbb{E}_j$ and we are able to determine the willingness to pay by group k for a firm in field k': $X_k(k'; n_1, n_2, \alpha, \beta)$

Before we state our first results, we define two types of equilibrium that are likely to arise in our framework, the *specialized* equilibrium and the *diverse* equilibrium.

Definition 4.1. *Equilibrium.*

- A *specialized* equilibrium of the economy is an equilibrium where there is only investment in one field k, such that $n_k = N$ and $n_{-k} = 0$.
- A *diverse* equilibrium of the economy is an equilibrium where investors invest in both fields equally: $n_1 = N/2$ and $n_2 = N/2$.

In appendix, we show these two equilibria are the only possible equilibria of our economy. Now we state our main proposition. Depending on the two types of disagreement, field disagreement and firm disagreement, we are able to characterize which equilibrium arises. Then we interpret the results.

Proposition 4.1. On the existence of an equilibrium of each type.

- *The cut-off for the existence of a specialized equilibrium is increasing in α and decreasing in β.*
- *The cut-off for diverse equilibrium is decreasing in α and increasing in β.*

All proofs are in appendix (see Section A.2). The proposition highlights two of our key results: First, an increase in field level disagreement makes the specialized disagreement less likely to exist. Second an increase in firm level disagreement makes the specialized equilibrium more likely to exit.

The first mechanism is very intuitive. A tighter prior on a specific field being successful implies there is a greater incentives for each group to invest in their favorite field. Regarding the second mechanism and an increase in firm disagreement: investors believe that their firm is more likely to win conditionally on their field winning. The conditional probability is always one if there is only one firm in the field. However in a field with a large number of firms, increasing this firm disagreement creates a competition neglect effect, thus a greater incentives for investors to "follow their crowd".

Regarding the second cut-off the intuition follows the first. Specialized and diverse equilibrium are naturally opposed: an increase in field disagreement increases incentives for diversity while firm level disagreement decreases them.

4.3 Firm Entry

Now we examine the extensive margin, figuring how many firms do enter depending on both disagreements and the equilibrium they sustain. First we define $\beta_S(\alpha; N)$, the cut-off such that the specialized equilibrium if and only if $\beta \leq \beta_S(\alpha; N)$, for a given N. According to our previous proposition, $\beta_S(\alpha; N)$ is increasing in α.

We also define $\beta_D(\alpha; N)$ the increasing function such that for a given N the diverse equilibrium exists if and only if: $\beta \geq \beta_D(\alpha; N)$.

Proposition 4.2. About the cut-off increasing with innovation N.

- $N \mapsto \beta_S(\alpha; N)$ *is an increasing function.*
- $N \mapsto \beta_D(\alpha; N)$ *is an increasing function.*

Both propositions state that as N increases (a) the specialized equilibrium is more likely to exist and (b) the diverse equilibrium is less likely to exist.

The implication is that if we have a W such that the marginal cost of firm creation is more costly, the equilibrium number of firms will on average be smaller and therefore the diverse equilibrium is more likely to exist.

The result is partly dependent on the distributional assumptions we have made. More importantly, however, it relies on the fact that field progress varies linearly with the number of firms. If there were diminishing returns, e.g. the field has speed of progress $N^\gamma \lambda_k$, then as γ falls, the effect of increasing N reverses.

In appendix we show that when N is exactly equal to 2, the diverse equilibrium always exists. In that case the specialized equilibrium only exists when $\alpha = 1$. When $N = 2$, we have minimum willingness to pay of $N^{-\beta}$ when $\alpha = 1$, which is the investor's willingness to pay for the other field.

Proposition 4.3. Innovation in the specialized and diverse equilibrium.

- *The number of firms N in the specialized equilibrium is decreasing in β and independent of α.*
- *The number of firms N in the diverse equilibrium is decreasing in both α and β.*

The above proposition says that more disagreement on either the firm or field level increases the equilibrium number of firms. The intuition is similar to the first model –increased disagreement implies that investors are more confident that their own firm will win, increasing prices and thus increasing firm entry.

4.4 Special Case: No Firm Level Disagreement

In our simple model in Section (3), we examine a world where there is a no field level disagreement: investors agree on the single field offered for investment, they only disagree on which firm will eventually succeed.

In this Section we examine the case of a world where investors do agree on firms' probability of success but disagree on the field overall probability such that $\beta = 1$ but $\alpha \in]0, 1[$.

The analysis of field disagreement for itself contrasts our results where both types not only coexist but interact with one another. Comparing both propositions for the existence of a diverse and specialized equilibrium in the context serves as a baseline for the role of firm level disagreement.

Let $N_S(\beta)$ denote the equilibrium number of firms in the specialized equilibrium, i.e. N_S satisfies

$$W'(N_S) = N_S^{-\beta}$$

Proposition 4.4. *With $\beta = 1$, and $N_S \geq 2$, the specialized equilibrium exists if and only if*

$$\alpha \geq \left(\frac{3}{2} - \frac{1}{N_S}\right)^{-1} \tag{4.1}$$

Let $N_D(\alpha, \beta)$ denote the equilibrium number of firms in the diverse equilibrium, i.e. N_D satisfies

$$W'(N_D) = \left(\frac{N_D}{2}\right)^{-\beta} \left[\frac{1}{\alpha} - \frac{1}{2}\left(1 + \left(\frac{1-\alpha}{\alpha}\right)^2\right)\right]$$

For convenience, suppose N_D is even.

Proposition 4.5. *With $\beta = 1$, and $N_D \geq 4$, the diverse equilibrium exists if and only if*

$$\alpha \leq \alpha^\star(N_D) \tag{4.2}$$

When $N_D = 2$, the diverse equilibrium always exists.

5 Welfare Analysis

5.1 Welfare

We introduce a central planner to explore the role of government interventions in the context of both firm level and field level disagreement. The planner maximizes the total expected utility of investors while respecting their individual beliefs:

$$\max\left\{c_0+\int_0^1\sum_{i_k}\mathbb{E}_j\left[s^j_{i_k}y_{i_k}\right]dj-W(N)\right\} \tag{5.1}$$

We consider a sequence of constrained optimizations to understand the welfare tradeoffs in our framework.

Proposition 5.1. *For a fixed amount of firm entry N, the corresponding diverse equilibrium is prefered by the social planner to the specialized equilibrium if and only if*

$$\alpha \quad \leq \quad \frac{1}{2-\sqrt{2-2^{1-\beta}}} \tag{5.2}$$

The relative preference for the diverse equilibrium increases as α decreases and β increases.

Note that the cutoff (5.2) is independent of N.

Proposition 5.2. *Given the type of equilibrium, firm entry is greater than is optimal if $\beta<1$, and optimal if $\beta=1$. The level of over-entry is characterized by a wedge of β.*

5.2 Financial contracts

Suppose we force investors to sign financial contracts and invest in a field rather than in individual firms. All investors then split the profit equally if the field wins. The resulting equilibrium will be as if $\beta=1$.

Proposition 5.3. *Suppose (4.1) is satisfied and we have a resulting specialized equilibrium. Then the equilibrium without financial contracts is prefered to the one with financial contracts if the equilibrium number of firms without financial contracts is greater than 2. We have indifference when $\beta=1$.*

Intuitively, a smaller β implies greater disagreement. Investors are therefore more confident that their firm will win, which increases the expected returns. Introducing financial contracts kills off this force, which leads to lower utility.

6 Conclusion

Innovation, a jump in the unknown, favors the emergence of *detail disagreement*: disagreement about the details of which technologies are likely to be successful. In this paper, we showed the presence of detail disagreement can generate innovation booms. Such episodes are characterized by high asset prices that subsequently drop, large number of firms being created, and a particular spurt in the diversity of technologies explored. We showed this pattern of events occured during the Internet boom of 1998-2001 and seems to emerge again, albeit less violently since 2009.

When considering the real side of the economy, detail disagreement has distinct implications from aggregate disagreement. Detail disagreement yields a form of competition neglect. Investors match with the firm they prefer, and ignore the presence of other firms when evaluating profit prospects. We showed this effect has a profound impact on the competitive structure of the economy, particularly in sectors with a winner-take-all aspect. Further, this effect also affects the nature of innovation undertaken, yielding more diversity in technologies. Finally, competition neglect also changes the normative evaluation of economies. In the presence of detail disagreement, economies with real frictions can have efficient equilibria.

Still, many questions remain. We briefly discussed which financing structures are favorable to the expression of detail disagreement. The emergence of new financing structures in the last few years — such as online financing platforms, or more and more early financing of projects — is likely to interact strongly with detail disagreement. Should we encourage the development of such structures? Further, in our models firms are passive. In practice, firms can react, decide and adapt over time the type of products they sell. Rajan (2012) studies the lifecycle of a firm's originality in the perspective of securing financing; clearly, the presence of speculation can also affect firms' decisions over their lifecycle. Finally, a more in-depth empirical study of detail disagreement is in order. We see two main challenges in this direction. First, measuring beliefs of investor is challenging, and relative beliefs across firms seems even more complex. An indirect approach we have taken in this paper is study the portfolios held by investors as a signal of those beliefs, but a tighter connection would be desirable. Second, an important outcome of our models is the diversity of innovation. Very little measures of such diversity have been undertaken in the literature. Given the potentially large growth consequences of the level of exploration in the economy, it would be useful to get a better grasp at this quantity.

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Appendix

A A Complete Model

A.1 Solving the Complete Model

First we derive the expression for $\mathbb{E}_j$, that will eventually determine the willingness to pay of group k for a firm in field k'.

Consider the perspective of group 1. We have

$$
\begin{aligned}
P_1\,[\text{field 1 wins}] &= \begin{cases} \left(\frac{1}{\alpha}-\frac{1}{2}\right)\frac{n_1}{n_2} & \frac{1}{\alpha}\le\frac{n_2}{n_1} \\ \int_{1-\alpha}^{\frac{n_2}{n_1}\alpha}\frac{1}{\alpha}\Pr\left[\lambda_1<\frac{n_2}{n_1}\lambda_2\right]d\lambda_1+\frac{1-\alpha}{\alpha}\frac{n_1}{n_2} & \frac{1-\alpha}{\alpha}\le\frac{n_2}{n_1},\ \frac{1}{\alpha}\ge\frac{n_2}{n_1} \\ 1 & \frac{1-\alpha}{\alpha}\ge\frac{n_2}{n_1} \end{cases} \\
&= \begin{cases} \left(\frac{1}{\alpha}-\frac{1}{2}\right)\frac{n_1}{n_2} & \alpha\ge\frac{n_1}{n_2} \\ \frac{1}{\alpha}-\frac{1}{2}\left[\frac{n_2}{n_1}+\left(\frac{1-\alpha}{\alpha}\right)^2\frac{n_1}{n_2}\right] & \frac{n_1}{N}\le\alpha\le\frac{n_1}{n_2} \\ 1 & \alpha\le\frac{n_1}{N} \end{cases}
\end{aligned}
$$

and

$$
P_1\,[\text{field 2 wins}] = \begin{cases} 1-\left(\frac{1}{\alpha}-\frac{1}{2}\right)\frac{n_1}{n_2} & \alpha\ge\frac{n_1}{n_2} \\ -\frac{1-\alpha}{\alpha}+\frac{1}{2}\left[\frac{n_2}{n_1}+\left(\frac{1-\alpha}{\alpha}\right)^2\frac{n_1}{n_2}\right] & \frac{n_1}{N}\le\alpha\le\frac{n_1}{n_2} \\ 0 & \alpha\le\frac{n_1}{N} \end{cases}
$$

Denote the willingness to pay by group k for a firm in field k' by $X_k\,(k';n_1,n_2,\alpha,\beta)$. We have

$$
\begin{aligned}
X_1\,(1;n_1,n_2,\alpha,\beta) &= \begin{cases} \left(\frac{1}{\alpha}-\frac{1}{2}\right)n_1^{1-\beta}n_2^{-1} & \alpha\ge\frac{n_1}{n_2} \\ \frac{1}{\alpha}n_1^{-\beta}-\frac{1}{2}\left[n_1^{-(1+\beta)}n_2+\left(\frac{1-\alpha}{\alpha}\right)^2 n_1^{1-\beta}n_2^{-1}\right] & \frac{n_1}{N}\le\alpha\le\frac{n_1}{n_2} \\ n_1^{-\beta} & \alpha\le\frac{n_1}{N} \end{cases} \\
X_1\,(2;n_1,n_2,\alpha,\beta) &= \begin{cases} n_2^{-\beta}-\left(\frac{1}{\alpha}-\frac{1}{2}\right)n_1n_2^{-(1+\beta)} & \alpha\ge\frac{n_1}{n_2} \\ -\frac{1-\alpha}{\alpha}n_2^{-\beta}+\frac{1}{2}\left[n_1^{-1}n_2^{1-\beta}+\left(\frac{1-\alpha}{\alpha}\right)^2 n_1n_2^{-(1+\beta)}\right] & \frac{n_1}{N}\le\alpha\le\frac{n_1}{n_2} \\ 0 & \alpha\le\frac{n_1}{N} \end{cases}
\end{aligned}
$$

A.2 Proofs

Specialized Equilibrium Cut-off — There exists a specialized equilibrium where $(n_1,n_2)=(0,N)$ if and only if

$$
X_1\,(1;1,N-1,\alpha,\beta) \quad\le\quad X_1\,(2;0,N,\alpha,\beta) \tag{A.1}
$$

We have

$$
\begin{aligned}
X_1\,(1;1,N-1,\alpha,\beta) &= \begin{cases} \left(\frac{1}{\alpha}-\frac{1}{2}\right)\frac{1}{N-1} & \alpha\ge\frac{1}{N-1} \\ \frac{1}{\alpha}-\frac{1}{2}\left[N-1+\left(\frac{1-\alpha}{\alpha}\right)^2\frac{1}{N-1}\right] & \frac{1}{N}\le\alpha\le\frac{1}{N-1} \\ 1 & \alpha\le\frac{1}{N} \end{cases} \\
X_1\,(2;0,N,\alpha,\beta) &= N^{-\beta}
\end{aligned}
$$

The left-hand side of of (A.1) is decreasing in α and independent of β if $\alpha > \frac{1}{N}$. The right-hand side of (A.1) is decreasing in β and independent of α.

Therefore, we can define a decreasing function $\beta_S(\alpha; N)$ for which (A.1) holds with equality. The specialized equilibrium exists if and only if $\beta \leq \beta_S(\alpha; N)$.

We have the exact same proof for the specialized equilibrium with $(n_1, n_2) = (N, 0)$. ∎

Diverse Equilibrium Cut-off — There exists a diverse equilibrium where $(n_1, n_2) = \left(\frac{N}{2}, \frac{N}{2}\right)$ if and only if

$$X_1\left(1; \frac{N}{2}, \frac{N}{2}, \alpha, \beta\right) \geq X_1\left(2; \frac{N}{2} - 1, \frac{N}{2} + 1, \alpha, \beta\right) \tag{A.2}$$

Suppose $\alpha > \frac{1}{2} - \frac{1}{N}$. We have

$$\begin{aligned}
X_1\left(1; \frac{N}{2}, \frac{N}{2}, \alpha, \beta\right) &= \frac{1}{\alpha}\left(\frac{N}{2}\right)^{-\beta} - \frac{1}{2}\left[\left(\frac{N}{2}\right)^{-\beta} + \left(\frac{1-\alpha}{\alpha}\right)^2\left(\frac{N}{2}\right)^{-\beta}\right] \\
&= \left(\frac{N}{2}\right)^{-\beta}\left[\frac{1}{\alpha} - \frac{1}{2}\left(1 + \left(\frac{1-\alpha}{\alpha}\right)^2\right)\right] \\
X_1\left(2; \frac{N}{2} - 1, \frac{N}{2} + 1, \alpha, \beta\right) &= \begin{cases} \left(\frac{N+2}{2}\right)^{-\beta}\left(1 - \left(\frac{1}{\alpha} - \frac{1}{2}\right)\frac{N-2}{N+2}\right) & \alpha \geq \frac{N-2}{N+2} \\ \left(\frac{N+2}{2}\right)^{-\beta}\left[-\frac{1-\alpha}{\alpha} + \frac{1}{2}\frac{N+2}{N-2} + \frac{1}{2}\left(\frac{1-\alpha}{\alpha}\right)^2\frac{N-2}{N+2}\right] & \alpha \leq \frac{N-2}{N+2} \end{cases}
\end{aligned}$$

Thus we can rewrite (A.2) as

$$\left(\frac{N+2}{N}\right)^{\beta} \geq \begin{cases} \frac{1 - \left(\frac{1}{\alpha} - \frac{1}{2}\right)\frac{N-2}{N+2}}{\frac{1}{\alpha} - \frac{1}{2}\left(1 + \left(\frac{1-\alpha}{\alpha}\right)^2\right)} & \alpha \geq \frac{N-2}{N+2} \\ \frac{-\frac{1-\alpha}{\alpha} + \frac{1}{2}\frac{N+2}{N-2} + \frac{1}{2}\left(\frac{1-\alpha}{\alpha}\right)^2\frac{N-2}{N+2}}{\frac{1}{\alpha} - \frac{1}{2}\left(1 + \left(\frac{1-\alpha}{\alpha}\right)^2\right)} & \alpha \leq \frac{N-2}{N+2} \end{cases} \tag{A.3}$$

The left-hand side is clearly increasing in β and independent of α. The right-hand side is increasing in α and independent of β.

If $\alpha \leq \frac{1}{2} - \frac{1}{N}$, then the equilibrium always exists, since $X_1\left(2; \frac{N-2}{2}, \frac{N+2}{2}, \alpha, \beta\right) = 0$.

Therefore, we can define a decreasing function $\beta^D(\alpha)$ for which (A.2) holds with equality. The diverse equilibrium exists if and only if $\beta > \beta^D(\alpha)$. ∎

Proposition ?? — We look to prove: $N \mapsto \beta_S(\alpha; N)$ is increasing. Suppose $\alpha \geq \frac{1}{N-1}$. The specialized equilibrium exists if and only if

$$N^{-\beta} \geq \left(\frac{1}{\alpha} - \frac{1}{2}\right)\frac{1}{N-1}$$

We can write this as

$$(N-1)\, N^{-\beta} \geq \frac{1}{\alpha} - \frac{1}{2}$$

Differentiating the log of the left-hand side, we find that the left-hand side is increasing in N. Since the left-hand side is decreasing in β, $\beta_S(\alpha; N)$ is increasing in N.

Suppose $\alpha \leq \frac{1}{N-1}$. The specialized equilibrium exists if and only if

$$N^{-\beta} \geq \frac{1}{\alpha} - \frac{1}{2}\left[N - 1 + \left(\frac{1-\alpha}{\alpha}\right)^2 \frac{1}{N-1}\right]$$

or

$$N^{-\beta} - \frac{1}{\alpha} + \frac{1}{2}\left[N - 1 + \left(\frac{1-\alpha}{\alpha}\right)^2 \frac{1}{N-1}\right] \geq 0$$

Differentiating with respect to N, we have

$$-\beta N^{-(1+\beta)} + \frac{1}{2} - \left(\frac{1-\alpha}{\alpha}\right)^2 (N-1)^{-2} > 0$$

Therefore $\beta_S(\alpha; N)$ is increasing in N. ∎

Proposition ?? — We look to prove: $N \mapsto \beta_D(\alpha; N)$ is increasing. Recall the condition (A.3) for the existence of the diverse equilibrium. The left-hand side is decreasing in N and increasing in β. The right-hand side is increasing in N. Therefore $\beta_D(\alpha; N)$ is increasing in N.

Proposition 4.3 — In the specialized equilibrium, we have the equilibrium condition

$$W'(N) = N^{-\beta}$$

The left-hand side is an increasing function of N. The right-hand side is a decreasing function of N that shifts downwards as β increases (assuming $N > 1$). Therefore the intersection shifts to the left as we increase β, i.e. a decrease in the equilibrium N.

In the diverse equilibrium, we have the equilibrium condition

$$W'(N) = \left(\frac{N}{2}\right)^{-\beta}\left[\frac{1}{\alpha} - \frac{1}{2}\left(1 + \left(\frac{1-\alpha}{\alpha}\right)^2\right)\right]$$

The left-hand side is an increasing function of N. The right-hand side is a decreasing function of N that shifts downwards as β increases (assuming $N > 1$) or as α increases. Therefore the intersection shifts to the left as we increase α or β, i.e. a decrease in the equilibrium N. ∎

A.3 Special Cases

Proposition 4.4 — With $\beta = 1$, we have

$$X_1(1; 1, N_S - 1, \alpha, \beta = 1) = \begin{cases} \left(\frac{1}{\alpha} - \frac{1}{2}\right)\frac{1}{N_S - 1} & \alpha \geq \frac{1}{N_S-1} \\ \frac{1}{\alpha} - \frac{1}{2}\left[N_S - 1 + \left(\frac{1-\alpha}{\alpha}\right)^2 \frac{1}{N_S-1}\right] & \frac{1}{N_S} \leq \alpha \leq \frac{1}{N_S-1} \\ 1 & \alpha \leq \frac{1}{N_S} \end{cases}$$

$$X_1(2; 0, N_S, \alpha, \beta = 1) = \frac{1}{N_S}$$

Suppose $\alpha \geq \frac{1}{N_S - 1}$. Then we have the following condition for the specialized equilibrium

$$\frac{1}{N_S} \quad \geq \quad \left(\frac{1}{\alpha} - \frac{1}{2}\right) \frac{1}{N_S - 1}$$

This reduces to

$$\frac{1}{\alpha} \quad \leq \quad \frac{3}{2} - \frac{1}{N_S}$$

which is equivalent to (4.1). We claim that $N - 1 \geq \frac{3}{2} - \frac{1}{N}$ if and only if $N \geq 2$. In particular,

$$\begin{aligned} \frac{d}{dN_S}\left(N - 1 - \left(\frac{3}{2} - \frac{1}{N}\right)\right) &= 1 - \frac{1}{N^2} \\ &\geq 0 \text{ for } N \geq 1 \end{aligned}$$

and

$$N - 1 - \left(\frac{3}{2} - \frac{1}{N}\right) \quad = \quad 0$$

when $N = 2$. Therefore condition (4.1) is binding in the case of interest with $N_S \geq 2$. ∎

Proposition 4.5 — First we define the cutoff for α:

$$\alpha^\star(N_D) = \frac{(N_D + 2)^2}{3N_D^2 + 6N_D + 8 - \sqrt{\left(3N_D^2 + 6N_D + 8\right)^2 - (N_D + 2)^2 \left(5N_D^2 + 8N_D + 8\right)}}$$

Suppose $\beta = 1$ and $N_D \geq 4$.
Consider the region $\alpha \leq \frac{N-2}{N+2}$. We have the condition

$$\frac{N_D + 2}{N_D} \quad \geq \quad \frac{-\frac{1-\alpha}{\alpha} + \frac{1}{2}\frac{N_D+2}{N_D-2} + \frac{1}{2}\left(\frac{1-\alpha}{\alpha}\right)^2 \frac{N_D-2}{N_D+2}}{\frac{1}{\alpha} - \frac{1}{2}\left(1 + \left(\frac{1-\alpha}{\alpha}\right)^2\right)}$$

for the existence of the diverse equilibrium. The right-hand side is increasing in α. Hence it is sufficient that the inequality is satisfied when $\alpha = \frac{N_D-2}{N_D+2}$, or $\frac{1-\alpha}{\alpha} = \frac{4}{N_D-2}$, which yields a right-hand side of

$$\begin{aligned} \frac{-\frac{4}{N_D-2} + \frac{1}{2}\frac{N_D+2}{N_D-2} + \frac{1}{2}\left(\frac{4}{N_D-2}\right)^2 \frac{N_D-2}{N_D+2}}{\frac{N_D+2}{N_D-2} - \frac{1}{2} + \frac{1}{2}\left(\frac{4}{N_D-2}\right)^2} &= \frac{N_D - 6 + \frac{16}{N_D+2}}{N_D + 6 + \frac{16}{N_D-2}} \\ &= \left(\frac{N_D - 2}{N_D + 2}\right)^3 \\ &< 1 \\ &< \frac{N_D + 2}{N_D} \end{aligned}$$

Therefore the equilibrium always exists for $\alpha \leq \frac{N_D-2}{N_D+2}$.

Consider the region $\alpha \geq \frac{N-2}{N+2}$. We have the condition

$$\frac{N_D + 2}{N_D} \quad \geq \quad \frac{1 - \left(\frac{1}{\alpha} - \frac{1}{2}\right)\frac{N_D - 2}{N_D + 2}}{\frac{1}{\alpha} - \frac{1}{2}\left(1 + \left(\frac{1-\alpha}{\alpha}\right)^2\right)}$$

for the existence of the diverse equilibrium. Define $\phi = \frac{1}{\alpha}$. We seek a lower bound for ϕ. Rewrite the inequality as

$$\begin{aligned}
0 \quad &\geq \quad 1 - \frac{N_D - 2}{N_D + 2}\phi + \frac{1}{2}\frac{N_D - 2}{N_D + 2} + \frac{N_D + 2}{N_D}\left(\frac{1}{2}\phi^2 - 2\phi + 1\right) \\
&= \quad \frac{N_D + 2}{2N_D}\phi^2 - \left(\frac{N_D - 2}{N_D + 2} + \frac{2\left(N_D + 2\right)}{N_D}\right)\phi + 1 + \frac{1}{2}\frac{N_D - 2}{N_D + 2} + \frac{N_D + 2}{N_D} \\
&= \quad \frac{1}{2N_D\left(N_D + 2\right)}\left[\left(N_D + 2\right)^2\phi^2 - 2\left(3N_D^2 + 6N_D + 8\right)\phi + 5N_D^2 + 8N_D + 8\right]
\end{aligned}$$

This holds with equality when

$$\phi \quad = \quad \frac{3N_D^2 + 6N_D + 8 \pm \sqrt{\left(3N_D^2 + 6N_D + 8\right)^2 - \left(N_D + 2\right)^2\left(5N_D^2 + 8N_D + 8\right)}}{\left(N_D + 2\right)^2}$$

We can verify that

$$\begin{aligned}
\frac{3N_D^2 + 6N_D + 8 + \sqrt{\left(3N_D^2 + 6N_D + 8\right)^2 - \left(N_D + 2\right)^2\left(5N_D^2 + 8N_D + 8\right)}}{\left(N_D + 2\right)^2} \quad &> \quad \frac{N_D + 2}{N_D - 2} \\
\frac{3N_D^2 + 6N_D + 8 - \sqrt{\left(3N_D^2 + 6N_D + 8\right)^2 - \left(N_D + 2\right)^2\left(5N_D^2 + 8N_D + 8\right)}}{\left(N_D + 2\right)^2} \quad &< \quad \frac{N_D + 2}{N_D - 2}
\end{aligned}$$

for $N \geq 4$. Therefore, the diverse equilibrium exists if and only if (**??**) is satisfied.

When $N_D = 2$, then the probability of winning when there is one firm in each field is just the probability of the field winning, which is greater than or equal to $\frac{1}{2}$. On the other hand, moving a firm to the other field will leave two firms in one field and zero firms in the other field. The implied probability of each firm winning is exactly $\frac{1}{2}$. ∎

B Welfare

Proposition 5.1 — Fix the total number of firms N. Suppose $\alpha > \frac{1}{2}$. The diverse equilibrium is socially optimal if

$$2^{\beta}N^{1-\beta}\left[\frac{1}{\alpha} - \frac{1}{2}\left(1 + \left(\frac{1-\alpha}{\alpha}\right)^2\right)\right] \quad \geq \quad N^{1-\beta} \tag{B.1}$$

or

$$\frac{1}{\alpha} - \frac{1}{2}\left(1 + \left(\frac{1-\alpha}{\alpha}\right)^2\right) \quad \geq \quad 2^{-\beta}$$

$2^{-\beta}$ is decreasing in β. Therefore, for a fixed total number of firms, increased firm-level disagreement (β decreasing) makes the diverse equilibrium less favored relative to the specialized equilibrium. On the other hand, $\frac{1}{\alpha} - \frac{1}{2}\left(1 + \left(\frac{1-\alpha}{\alpha}\right)^2\right)$ is decreasing in α if $\alpha > \frac{1}{2}$. Thus increased field-level disagreement (α decreasing) makes the diverse equilibrium more favored relative to the specialized equilibrium. This proves the second part of the proposition.

(B.1) is satisfied if and only if

$$\frac{1}{\alpha^2} - \frac{4}{\alpha} + 2 + 2^{1-\beta} \leq 0$$

which is satisfied when $2 - \sqrt{2 - 2^{1-\beta}} \leq \frac{1}{\alpha} \leq 2 + \sqrt{2 - 2^{1-\beta}}$. However, $1 \leq \frac{1}{\alpha} < 2$ by assumption. Hence (B.1) holds if and only if (5.2) holds.

For $\alpha \leq \frac{1}{2}$, the left-hand side of (B.1) is replaced with $2^{\beta} N^{1-\beta}$, in which case the corresponding inequality is clearly satisfied. Therefore the first part of the proposition holds. ∎

Proposition 5.2 — Suppose we solved (5.1), restricting firms to only open in one field. In the specialized equilibrium, we have social utility of $c_0 + N_S^{1-\beta} - W(N_S)$ where N_S satisfies

$$W'(N_S) \quad = \quad N_S^{-\beta}$$

On the other hand, the optimum number of firms satisfies

$$\begin{aligned} W'\left(\widetilde{N}_S\right) &= (1-\beta)\, \widetilde{N}_S^{-\beta} \\ &\leq \widetilde{N}_S^{-\beta} \end{aligned}$$

Therefore the competitive equilibrium results in more firms than is optimal as long as $\beta > 0$.

If we restrict the social planner to having investors invest only in their favored field, we have social utility of $c_0 + 2^{\beta} N_D^{1-\beta}\left[\frac{1}{\alpha} - \frac{1}{2}\left(1 + \left(\frac{1-\alpha}{\alpha}\right)^2\right)\right] - W(N_D)$ where N_D satisfies

$$W'(N_S) \quad = \quad 2^{\beta} N_D^{-\beta}\left[\frac{1}{\alpha} - \frac{1}{2}\left(1 + \left(\frac{1-\alpha}{\alpha}\right)^2\right)\right]$$

On the other hand, the optimum number of firms satisfies

$$W'\left(\widetilde{N}_D\right) \quad = \quad (1-\beta)\, 2^{\beta} N_D^{-\beta}\left[\frac{1}{\alpha} - \frac{1}{2}\left(1 + \left(\frac{1-\alpha}{\alpha}\right)^2\right)\right]$$

which gives us the same wedge of β. ∎

B.1 Taxation

Consider a planner solving

$$\begin{aligned} &\max_{\tau} \left\{ c_0 + \int_0^1 \sum_{i_k} (1-\tau)\, \mathbb{E}_j\left[s_{i_k}^j y_{i_k}\right] dj - W(N) + \tau \right\} \\ \text{s.t.} \quad 0 &= W'(N) - \max_j \left\{(1-\tau)\, \mathbb{E}_j\left[y_{i_k}\right]\right\} \end{aligned}$$

In particular, the planner imposes a tax τ on the winner which is returned to investors lump sum.

Consider the specialized equilibrium. We can rewrite the problem as

$$\max_{\tau}\left\{c_0+(1-\tau)\,N(\tau)^{1-\beta}-W(N(\tau))+\tau\right\}$$

which yields first order condition

$$\begin{aligned}0 &= -N(\tau)^{1-\beta}+(1-\tau)(1-\beta)\,N(\tau)^{-\beta}N'(\tau)-W'(N(\tau))\,N'(\tau)+1\\ &= -\left[N(\tau)^{1-\beta}-1\right]-(1-\tau)\,N(\tau)^{-\beta}N'(\tau)\,\beta\end{aligned}$$

The first term is the loss from taxation. Even though ex-post the tax merely redistributes income, ex-ante there is a loss, as investors believe their firms have a probability of winning that is more than $1/N$. The second term is the welfare effects arising because taxation results in a change in the equilibrium number of firms. Taxation produces a tradeoff between these two forces.

As we increase β, there are several effects:

(a) the wedge between the investors' and social planner's first-order conditions increases;

(b) the ex-ante loss from taxation decreases since the equilibrium number of firms decreases and investors are more pessimistic.

Both these forces generally imply that the optimal tax increases as β increases. However, this does not seem to be true in general. In particular, the effect of β on the second term is ambiguous.

B.2 Financial Contracts

Proposition 5.3 — Denote the equilibrium number of firms with financial contracts by $N_{S,F}$, i.e.

$$W'(N_{S,F}) = N_{S,F}^{-1}$$

We note that we will have a specialized equilibrium without financial contracts. This follows from Propositions **??** and **??** as well as the fact that equilibrium firm entry is decreasing in β.

Financial contracts are prefered if and only if

$$1-W(N_{S,F}) \geq N_S^{1-\beta}-W(N_S)$$

This holds with equality when $\beta=1$. The left-hand side is fixed.

To show that the inequality does not hold, it is sufficient to show that $N_S(\beta)^{1-\beta}-W(N_S(\beta))$ is decreasing in β. We want to show that

$$\left[N_S(\beta)^{1-\beta}-W(N_S(\beta))\right]-\left[N_S(\beta+\Delta_\beta)^{1-\beta-\Delta_\beta}-W(N_S(\beta+\Delta_\beta))\right] > 0$$

Recall that $N_S(\beta)\geq N_S(\beta+\Delta_\beta)$. Since W' is increasing, we have that $W'(N_S(\beta))\geq W'(N_S(\beta+\Delta_\beta))$ Now

$$\begin{aligned}W(N_S(\beta))-W(N_S(\beta+\Delta_\beta)) &\leq W'(N_S(\beta+\Delta_\beta))\,\Delta_\beta\\ &= N_S(\beta+\Delta_\beta)^{-\beta-\Delta_\beta}\,\Delta_\beta\end{aligned}$$

In addition,

$$N_S(\beta)^{1-\beta} - N_S(\beta+\Delta_\beta)^{1-\beta-\Delta_\beta} \quad \geq \quad N_S(\beta+\Delta_\beta)^{-\beta-\Delta_\beta} N_S(\beta+\Delta_\beta)\left(N_S(\beta)^{\Delta_\beta} - 1\right)$$

Now

$$\frac{d}{d\Delta_\beta} \log N_S(\beta)^{\Delta_\beta} \quad = \quad \log N_S(\beta)$$

Thus

$$\frac{d}{d\Delta\beta} N_S^{\Delta_\beta} \quad = \quad N_S^{\Delta_\beta} \log N_S(\beta)$$

Therefore

$$N_S(\beta)^{\Delta_\beta} - 1 \quad = \quad \Delta_\beta\left(N_S^{\Delta_\beta} \log N_S(\beta)\right) + O\left(\Delta_\beta^2\right)$$

The first term is larger than $\Delta_\beta/2$ for $N_S(\beta) > 2$, and $\lim_{\Delta_\beta \to 0} N_S(\beta+\Delta_\beta) \geq 2$. Therefore it is optimal not to have financial contracts. ∎