Risk Sharing in International Economies and Market Incompleteness

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Abstract

We develop an incomplete markets framework to show that international risk sharing can be low, particularly among countries with large interest-rate differentials. Furthermore, risk sharing computed from asset returns can be consistent with that computed from consumption. The fundamental departure in our paper is that exchange rate growth need not equal the ratio of stochastic discount factors (SDFs), and we develop a restriction that precludes "good deals" in international economies with incomplete markets. Our innovation is to compute the lowest risk sharing consistent with SDFs that (i) are nonnegative, (ii) correctly price returns, and (iii) disallow "good deals."

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1 Introduction

How high is risk sharing in international economies? The bottom line view is spearheaded by Brandt, Cochrane, and Santa-Clara (2006), who feature economies in which $m(e_{t+1}/e_t) - m^\star = 0$ (m (respectively, $m^\star$) is the domestic (foreign) stochastic discount factor, and e_{t+1}/e_t is the growth rate of the exchange rate) and show that the degree of international risk sharing is high. In a potential paradigm shift, we propose an incomplete markets setting in which there is an infinite number of m and $m^\star$ pairs that may or may not satisfy $m(e_{t+1}/e_t) - m^\star = 0$, and show that international risk sharing can be low, especially among country pairs with large interest-rate differentials.

What is the statement of the international risk sharing puzzle? While imposing $m(e_{t+1}/e_t) - m^\star = 0$, Brandt, Cochrane, and Santa-Clara (2006, equation (2)) advocate using a risk sharing index, defined by

$$\text{Risk Sharing Index} \equiv 1 - \frac{\text{Var}\left[\ln(e_{t+1}/e_t)\right]}{\text{Var}\left[\ln(m)\right] + \text{Var}\left[\ln(m^\star)\right]} = \frac{2\,\text{Cov}\left[\ln(m), \ln(m^\star)\right]}{\text{Var}\left[\ln(m)\right] + \text{Var}\left[\ln(m^\star)\right]}, \qquad (1)$$

where $\text{Var}[\cdot]$ and $\text{Cov}[\cdot, \cdot]$ denote variance and covariance, respectively, and m and $m^\star$ are computed as the minimum variance stochastic discount factors (SDFs) that correctly price the traded assets. With a 10% exchange rate volatility and a 50% annualized volatility of marginal utility growth in each country, the asset return data implied risk sharing index is $1 - \left(0.1^2 / \left(0.5^2 + 0.5^2\right)\right) = 98\%$. With volatilities of consumption growth assumed (approximately) equal in each country and correlations between them of around 0.3, they infer risk sharing to be around 30%. The puzzle is that the risk sharing index, based on asset return data, is high, indicating a high degree of risk sharing, whereas the corresponding one based on consumption growth data is low (e.g., Cole and Obstfeld (1991), Lewis (1996), and Lewis (2000)).

What do we do differently in this paper? First, we consider a discrete-time economy and a risk sharing index $\text{RSI} \equiv \frac{2\,\text{Cov}[m,m^\star]}{\text{Var}[m]+\text{Var}[m^\star]}$, with the understanding that there is an infinite number of SDFs in incomplete markets. Second, unlike most others, we do *not* assume that $m(e_{t+1}/e_t) - m^\star = 0$, and we develop a restriction that precludes "good deals" in international economies with incomplete markets. Specifically, a good deal is the possibility to form a portfolio, which has an implausibly high reward-for-risk. We show that ruling out an implausibly high reward-for-risk in an international

economy places an upper bound on the dispersion of SDFs. Third, we develop a new programming problem (with quadratic objective and both quadratic and linear constraints) that searches for the lowest risk sharing index in the space of the two SDFs subject to the constraint that the two SDFs preclude good deals, the SDFs price the returns on traded assets, and the SDFs be nonnegative. The key question is: what is the minimum amount of risk sharing that is consistent with the data?

With our new framework, we see values of the risk sharing index, based on asset return data, which are broadly in the same range as those estimated from consumption growth data, and our analysis is backed by evidence from ten industrialized countries. The level of risk sharing varies by country, and for pairs of countries that comprise typical legs of the carry trade, risk sharing can be low. We do observe a slight trade-off between the covariation and the volatility of the SDFs in international economies. Nevertheless, low risk sharing can be achieved without requiring the SDFs to be highly volatile.

We additionally argue that our findings are supported by economic intuition. For instance, there is evidence that high Sharpe ratios are possible for a carry strategy (e.g., Burnside, Eichenbaum, Kleshchelski, and Rebelo (2011, Table 1) and Lustig, Roussanov, and Verdelhan (2011, Table 1)). But, if risk sharing were to be high, why would that be the case? If, as some argue, risk sharing is almost perfect, the net position has little risk. Thus, little reward should be expected in equilibrium.

Related Literature: Our work connects to a literature that is at the intersection of consumption risk sharing and asset pricing. For example, Backus, Kehoe, and Kydland (1992) demonstrate that international consumption correlations are too low to be explained by models with complete markets. Lewis (1996) supports the view that international consumption correlations cannot be explained by complete markets models. She proposes, and finds some supporting empirical evidence for, the idea that incomplete markets may be able to explain low consumption correlations and, thus, low international risk sharing. Baxter (2011) contrasts international risk sharing at short versus long horizons and finds that it is lower at short horizons and higher at long horizons. Additionally, Lewis (1999) makes the observation that, in theory, an implication of complete markets is that a country's consumption growth should be more highly correlated with world output growth than with domestic output growth. However, in practice, the data indicates the opposite.

Related to our approach, the study of Becker and Hoffmann (2006) argues that the lack of international consumption insurance is, in fact, a failure to insure against permanent fluctuations, whereas insurance against transitory shocks seems almost complete. They favor endogenous market incompleteness, which makes insuring against permanent shocks expensive and hampers international risk sharing. Overall, the work of Backus, Kehoe, and Kydland (1992), Lewis (1996), Lewis (1999), Becker and Hoffmann (2006), and Corsetti, Dedola, and Leduc (2008) motivate us to formalize the implications of incomplete markets for international risk sharing.

Finally, the studies of Colacito and Croce (2011, 2013) consider a two-country asset pricing model with long-run risk and recursive preferences to reconcile the correlation between asset returns and the correlation between consumption growth. Their model, through the mechanism of a high correlation of the long-run components of consumption growth, produces correlated SDFs, while maintaining a low unconditional correlation between the consumption growth of the two countries. In contrast, we showcase an incomplete markets setting, and provide evidence that SDFs could share a low correlation between country pairs with high interest-rate differentials.

While many scholars are beginning to expose the consequences of incomplete markets for explaining puzzles in international finance, the adopted approaches are different from ours and interesting in their own right. For example, Lustig and Verdelhan (2015, Proposition 1) leave the home and foreign SDFs unchanged and reconfigure the determination of exchange rates. They conclude that their setup of incomplete markets faces a hurdle addressing exchange rate puzzles. Favilukis, Garlappi, and Neamati (2015) employ restrictions in financial trade to induce market incompleteness, and show that their framework could explain certain facts, including a positive correlation between currency appreciation and consumption growth. The study of Gabaix and Maggiori (2015) allows for incomplete markets in their substantive work on international trade and exchange rates. The novelty of our approach is that we emphasize that there is an infinite number of domestic and foreign SDFs in incomplete markets and, hence, an infinite number of risk sharing indexes. We develop a framework in which SDFs are not unique, and an incomplete markets setting is at the center of a restriction that precludes extremely lucrative trading opportunities.

Our investigation reveals that incomplete markets are relevant to the mechanism by which risks are shared – or not shared – across international borders.

2 A framework for analyzing risk sharing in incomplete markets

We consider a discrete time economy with two dates, namely, t, the current time, and $t+1$, the time one period ahead. There are $\mathbb{J}$ (finite or infinite) possible states of the world at time $t+1$. We consider two countries, denoted domestic and foreign, and use a superscript $\star$ to denote quantities in the foreign country.

The exchange rate, defined as the number of units of domestic currency per unit of foreign currency, at time t, is denoted by e_t (the foreign currency is the reference). We assume that asset markets are frictionless. For example, there are no bid-ask spreads and no short sale constraints.

We assume that there are N assets that can be traded by both domestic and foreign investors and that none of the assets is redundant. Denote by $\mathbf{R}$ and $\mathbf{R}^\star$ the N-dimensional vector of domestic and foreign gross returns. Included within the return vectors $\mathbf{R}$ and $\mathbf{R}^\star$ are (one period) risk-free bonds in the domestic and foreign country with gross return R_f and $R_f^\star$, respectively.

More generally, we include all available asset returns, in either currency, in both $\mathbf{R}$ and $\mathbf{R}^\star$. This means that $\mathbf{R}$ and $\mathbf{R}^\star$ are related by

$$\mathbf{R} = (e_{t+1}/e_t)\,\mathbf{R}^\star. \tag{2}$$

Hence, when we refer to domestic and foreign returns, "domestic" and "foreign" refer to the currency in which the return is made – not, for example, to the country in which the equity index is based.

Let m and $m^\star$ denote the domestic and foreign stochastic discount factors (SDFs). Importantly, we do not assume complete markets, so m and $m^\star$ are not unique.

To rule out extreme counterexamples, assume that the first and second moments of all relevant quantities, for example, m, $m^\star$, $\mathbf{R}$ and $\mathbf{R}^\star$, exist and are finite. In particular, $\mathbb{E}_t[m^2] < +\infty$ and $\mathbb{E}_t[(m^\star)^2] < +\infty$ and, hence, $\left|\mathbb{E}_t[m\,m^\star]\right| < +\infty$ by the Cauchy-Schwarz inequality, where $\mathbb{E}_t[\,.\,]$ indicates time t conditional expectation.

2.1 Implications of incomplete markets

Since m and $m^\star$ price domestic returns $\mathbf{R}$ and foreign returns $\mathbf{R}^\star$, we have

$$\mathbb{E}_t[m\,\mathbf{R}] = \mathbf{1} \qquad \text{and} \qquad \mathbb{E}_t[m^\star\,\mathbf{R}^\star] = \mathbf{1}, \tag{3}$$

where $\mathbf{1}$ denotes an N-dimensional vector of ones. However, since $\mathbf{R} = (e_{t+1}/e_t)\,\mathbf{R}^\star$, we also have $\mathbb{E}_t[m\,(e_{t+1}/e_t)\,\mathbf{R}^\star] = \mathbf{1}$.

Subtracting $\mathbb{E}_t[m^\star\mathbf{R}^\star] = \mathbf{1}$ from $\mathbb{E}_t[m\,(e_{t+1}/e_t)\,\mathbf{R}^\star] = \mathbf{1}$ implies:

$$\mathbb{E}_t[(m\,(e_{t+1}/e_t) - m^\star)\,\mathbf{R}^\star] = 0. \tag{4}$$

Since equation (4) is true for all foreign returns, it is also true for the foreign risk-free return $R_f^\star$:

$$\mathbb{E}_t[(m\,(e_{t+1}/e_t) - m^\star)\,R_f^\star] = 0, \quad \text{which implies} \quad \mathbb{E}_t[m\,(e_{t+1}/e_t) - m^\star] = 0. \tag{5}$$

In the case of complete markets, there is an Arrow-Debreu security tradeable for every time $t+1$ state of the world, which implies, in the absence of arbitrage, that $e_t m^\star = e_{t+1} m$, or equivalently, we obtain the relation (e.g., Backus, Foresi, and Telmer (2001, Proposition 1, equation (7))),

$$m\,(e_{t+1}/e_t) - m^\star = 0 \quad \text{in a complete markets setting.} \tag{6}$$

In incomplete markets, some m and $m^\star$ satisfy $m\,(e_{t+1}/e_t) - m^\star = 0$, and some do not.

The statement that "$m\,(e_{t+1}/e_t) - m^\star$ need not equal zero" is intuitive, because, in incomplete markets, there are some outcomes for which no Arrow-Debreu security trades, and different investors will place different marginal utility on those outcomes. For example, if a representative agent exists in each country and if, say, the agent in the domestic country is more risk averse than the foreign counterpart, then the former will assign greater marginal utility to unpalatable states of the world.

We do stress that equations (4) and (5) always hold, regardless of whether the market is complete or incomplete. Brandt, Cochrane, and Santa-Clara (2006, Section 1.2) highlight the potential significance of incomplete markets, but in the end their analysis features $m\,(e_{t+1}/e_t) - m^\star = 0$.

It simplifies the exposition and analytical characterizations if we define (note $e_{t+1}/e_t > 0$) the N-dimensional vector $\mathbf{Z}$ by

$$\mathbf{Z} \equiv \sqrt{e_{t+1}/e_t}\,\mathbf{R}^{\star} = \mathbf{R}/\sqrt{e_{t+1}/e_t}. \tag{7}$$

Further, we define

$$y \equiv m\sqrt{e_{t+1}/e_t}\,, \qquad \text{and} \qquad y^{\star} \equiv m^{\star}/\sqrt{e_{t+1}/e_t}\,, \tag{8}$$

which implies $y\,y^{\star} = m\,m^{\star}$. For later use, observe that $y\,y^{\star}$ has a bounded expectation, since $\left|\mathbb{E}_t[y\,y^{\star}]\right| = \left|\mathbb{E}_t[m\,m^{\star}]\right| < +\infty$. Furthermore, from equation (3), it must hold that

$$\mathbb{E}_t[y\mathbf{Z}] = \mathbf{1} \quad \text{and} \quad \mathbb{E}_t[y^{\star}\mathbf{Z}] = \mathbf{1}. \tag{9}$$

Equations (7) and (9) imply that the vector $\mathbf{Z}$ can be interpreted as the gross returns in a hypothetical economy in which the gross returns are the geometric average of $\mathbf{R}$ and $\mathbf{R}^{\star}$ (since equation (7) implies $(\mathbf{R}\mathbf{R}^{\star})^{\frac{1}{2}} = \mathbf{Z}$). Further, y and $y^{\star}$ can be interpreted as SDFs in this hypothetical economy. The transformations in (7) and (8) are merely devices that allow cash flow pricing in a symmetric fashion, circumventing the need to duplicate calculations in different currency units.

If the market were to be complete, equation (6) would imply $y = y^{\star}$. Further, if the market were to be complete, without loss of generality, we can assume that Arrow-Debreu securities, for every time $t + 1$ state of the world, trade and then $y = y^{\star}$ is equivalent to saying that domestic investors and foreign investors agree on their prices.

Whether markets are complete or incomplete, domestic investors and foreign investors agree on the prices of securities in the linear span of $\mathbf{R}$ (or $\mathbf{R}^{\star}$). The takeaway is that in incomplete markets, $m\,(e_{t+1}/e_t) - m^{\star}$ need not equal zero, y need not equal $y^{\star}$, and the valuations of domestic and foreign investors need not coincide for securities that are not in the linear span of $\mathbf{R}$ (or $\mathbf{R}^{\star}$).

2.2 Motivating a constraint on reward-for-risk in the international economy

How different can those valuations be in incomplete markets? The purpose of this subsection is to derive a restriction of the form $\mathbb{E}_t[(y - y^\star)^2] \leq \Theta^2$ for some constant Θ, by showing that it is equivalent to placing an economically motivated bound on the differences in valuation of securities that are not in the linear span of $\mathbf{R}$ (or $\mathbf{R}^\star$). The substantive content of an upper bound $\mathbb{E}_t[(y - y^\star)^2] \leq \Theta^2$ arises in incomplete markets when y need not equal $y^\star$, and is related to the amount of risk sharing in international economies.

Following Cochrane (2005, pages 94–95), we project y and $y^\star$ onto the space of gross returns $\mathbf{Z}$:

$$y = y_{\mathbf{z}} \; + \; \frac{1}{2} \, q_0 \, \eta, \quad \text{and} \quad y^\star = y_{\mathbf{z}} \; + \; \frac{1}{2} \, q_0^\star \, \eta, \tag{10}$$

where q_0 and $q_0^\star$ are constant scalars and $y_{\mathbf{z}}$ and η are random variables that satisfy

$$\mathbb{E}_t[y_{\mathbf{z}} \mathbf{Z}] = 1, \quad \mathbb{E}_t[y_{\mathbf{z}} \, \eta] = 0, \quad \mathbb{E}_t[\eta \mathbf{Z}] = 0 \;\; \text{for each element of } \mathbf{Z}, \quad \text{and} \quad \mathbb{E}_t[\eta^2] = 1. \tag{11}$$

The decomposition in equation (10) breaks y and $y^\star$ into two components. The first component $y_{\mathbf{z}} = \mathbf{1}' \left(\mathbb{E}_t[\mathbf{ZZ}'] \right)^{-1} \mathbf{Z}$ can be interpreted as the minimum second moment stochastic discount factor in the hypothetical economy in which gross returns are $\mathbf{Z}$. The second components $\frac{1}{2} q_0 \, \eta$ and $\frac{1}{2} q_0^\star \, \eta$ are orthogonal to $\mathbf{Z}$, where η is normalized to have second moment equal to unity.

To illustrate that domestic and foreign investors disagree, in general, when $q_0 \neq q_0^\star$, on their valuations of securities outside the linear span of $\mathbf{R}$ (or $\mathbf{R}^\star$), consider, for example, a synthetic security that pays $\eta \sqrt{e_{t+1}/e_t}$ units of domestic currency, at time $t + 1$. This would be privately valued, in domestic currency, at time t, at $\mathbb{E}_t[m \, \eta \, \sqrt{e_{t+1}/e_t}] = \mathbb{E}_t[y \, \eta] = \frac{1}{2} q_0$ by domestic investors and at $e_t \, \mathbb{E}_t[m^\star \, \eta \, (\sqrt{e_{t+1}/e_t})/e_{t+1}] = e_t \, \mathbb{E}_t[y^\star \, \eta/e_t] = \frac{1}{2} q_0^\star$ by foreign investors. If q_0 were to equal $q_0^\star$, these valuations would be the same, but $q_0 = q_0^\star$ also implies that (i) $y = y^*$, and (ii) $m \, (e_{t+1}/e_t) - m^\star = 0$.

The discrepancy between valuations of the synthetic security is greater when $|q_0 - q_0^\star|$ is larger, which implies that $|y - y^\star|$ is larger and $|m \, (e_{t+1}/e_t) - m^\star|$ is larger. This is a situation that financial intermediaries may, potentially, wish to exploit, and they can do this by creating a synthetic security

that offers payoffs outside the linear span of $\mathbf{R}$ (or $\mathbf{R}^\star$). Our approach, broadly speaking, is to ask: By how much can domestic investors and foreign investors diverge on the valuation of securities outside the linear span of $\mathbf{R}$ (or $\mathbf{R}^\star$) before financial intermediaries would be presented with a "good deal"? The larger $|q_0 - q_0^\star|$ is, the greater is the potential profit for financial intermediaries. Hence, we study the consequences of a class of "good deals" characterized by

$$q_0 = -q_0^\star \equiv q \neq 0. \tag{12}$$

Specifically, equation (12), in conjunction with (10), translates into two restrictions on y and y^*:

$$\mathbb{E}_t[y - y^*] = \frac{1}{2}(q_0 - q_0^*)\,\mathbb{E}_t[\eta] = q\,\mathbb{E}_t[\eta], \tag{13}$$

$$\mathbb{E}_t[(y - y^*)^2] = \frac{1}{4}(q_0 - q_0^*)^2\,\mathbb{E}_t[\eta^2] = q^2. \tag{14}$$

We use equations (13) and (14) to rule out implausibly high reward-for-risk strategies in the international economy with incomplete markets.

Consider now a financial intermediary that considers the *possibility* of privately negotiating a contract between itself and a domestic investor. We assume that the possible opportunity to enter into this private contract does not materially alter m, $m^\star$, $\mathbf{R}$, or $\mathbf{R}^\star$.

If entered into, the private contract with the domestic investor would require the investor to buy a synthetic security with payoff $x[\mathbf{Z}, e_{t+1}]$ in units of domestic currency, at time $t+1$, from the financial intermediary, where

$$x[\mathbf{Z}, e_{t+1}] = \sqrt{e_{t+1}/e_t}\,\boldsymbol{w}'\mathbf{Z} + \eta\,\sqrt{e_{t+1}}. \tag{15}$$

Here, $\boldsymbol{w}$ is an N-dimensional vector of portfolio weights in the traded assets assumed to be of the form $\boldsymbol{w} = -\frac{1}{2}\left(q\sqrt{e_t}\right)\boldsymbol{v}$, where $\boldsymbol{v}'\mathbf{1} = 1$. The domestic investor computes the value, at time t, in units of domestic currency, of this synthetic security as:

$$
\begin{aligned}
\mathbb{E}_t\left[m\left(\sqrt{e_{t+1}/e_t}\,\boldsymbol{w}'\mathbf{Z} + \eta\,\sqrt{e_{t+1}}\right)\right] &= \mathbb{E}_t[m\,(\boldsymbol{w}'\mathbf{R})] + \mathbb{E}_t[(y_{\mathbf{z}} + \frac{1}{2}q\,\eta)\eta\sqrt{e_t}], \\
&= \boldsymbol{w}'\mathbf{1} + \frac{1}{2}q\sqrt{e_t} = 0. \tag{16}
\end{aligned}
$$

Thus, the cash flow postulated in (15) can be synthesized at zero cost.

If the financial intermediary were to enter into this private contract, it would have a short exposure to the cash flow $x[\mathbf{Z}, e_{t+1}]$ at time $t+1$. Substituting $\boldsymbol{w} = -\frac{1}{2}(q\sqrt{e_t})\boldsymbol{v}$ into equation (15)):

$$
\begin{aligned}
X &\equiv -x[\mathbf{Z}, e_{t+1}] = \frac{1}{2}(q\sqrt{e_t})\,\boldsymbol{v}'\mathbf{R} \;-\; \eta\sqrt{e_{t+1}}, && \text{(in units of domestic currency)} & (17)\\
&= \frac{1}{2}q\,\boldsymbol{v}'\mathbf{Z} \;-\; \eta, && \text{(in currency units of the hypothetical economy, i.e., } \sqrt{e_{t+1}}) & (18)
\end{aligned}
$$

where in moving from equation (17) to (18), we have divided by $\sqrt{e_{t+1}}$ because, since there are, at time $t+1$, e_{t+1} units of domestic currency per unit of foreign currency, there are $\sqrt{e_{t+1}}$ units of domestic currency per unit of currency of the hypothetical economy (using equation (7)).

The financial intermediary faces a trade-off between the risks and rewards inherent in the cash flows X. We evaluate this trade-off in the currency units of the hypothetical economy to emphasize symmetry (without repeating the cash flow calculations in different currency units). Then,

$$
\begin{aligned}
\mathbb{E}_t[X] &= \frac{1}{2}q\,\mathbb{E}_t[\boldsymbol{v}'\mathbf{Z}] \;-\; \mathbb{E}_t[\eta], & (19)\\
\mathbb{E}_t[(X - \mathbb{E}_t[X])^2] &= \mathrm{Var}_t[\frac{1}{2}q\,\boldsymbol{v}'\mathbf{Z}] + \mathbb{E}_t[\eta^2] - (\mathbb{E}_t[\eta])^2 \;-\; \underbrace{q\,\mathbb{E}_t[\boldsymbol{v}'\mathbf{Z}\,\eta]}_{=0 \text{ from eq. } (11)} + q\,\mathbb{E}_t[\boldsymbol{v}'\mathbf{Z}]\,\mathbb{E}_t[\eta], & (20)\\
&= \mathrm{Var}_t[\frac{1}{2}q\,\boldsymbol{v}'\mathbf{Z}] + \mathbb{E}_t[\eta^2] - (\mathbb{E}_t[\eta])^2 + q\,\mathbb{E}_t[\boldsymbol{v}'\mathbf{Z}]\,\mathbb{E}_t[\eta]. & (21)
\end{aligned}
$$

In Appendix A, we show two results. First, $|\mathbb{E}_t[y - y^\star]|$ is always bounded above by an easily computable quantity that we argue will, in practice, be small. Second, in the special case that m, $m^\star$, and e_{t+1}/e_t are log-normally distributed, $\mathbb{E}_t[y - y^\star]$ is identically equal to zero. Log-normality will only be an approximation to reality, but both results suggest that $|\mathbb{E}_t[y - y^\star]|$ will not be far from zero. From equation (13), this implies $\mathbb{E}_t[\eta] \approx 0$ given that $q \neq 0$.

Using the approximation $\mathbb{E}_t[\eta] \approx 0$ in equation (19), the expected payoff, denoted by EP, to the financial intermediary can be approximated as:

$$
\begin{aligned}
\text{EP} &\equiv \frac{1}{2}q\,\mathbb{E}_t[\boldsymbol{v}'\mathbf{Z}],\\
&= \frac{1}{2}\mathbb{E}_t[\boldsymbol{v}'\mathbf{Z}]\sqrt{\mathbb{E}_t[(y - y^\star)^2]}, & (22)
\end{aligned}
$$

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where we have substituted for q, using equation (14). The expression for EP in equation (22) is a measure of the potential reward to the financial intermediary. Analogously, by equation (21)

$$\text{Var}_t[X] - \text{Var}_t\left[\frac{1}{2}q\, \boldsymbol{v}'\mathbf{Z}\right] \approx \mathbb{E}_t[\eta^2] = 1. \tag{23}$$

In words, the incremental variance of X over and above that of the payoff $\frac{1}{2}q\, \boldsymbol{v}'\mathbf{Z}$ is unity.

The quantity $\text{EP}/\sqrt{\text{Var}_t[X] - \text{Var}_t[\frac{1}{2}q\, \boldsymbol{v}'\mathbf{Z}]} = \text{EP}$ is a measure of the reward-for-risk potentially available to the financial intermediary. It is equal to (or analogous to - definitions in the literature vary) what is variously (Sharpe (1981), Roll (1992), and Grinold and Kahn (2007)) termed the Information Ratio or the *Appraisal Ratio* (the latter being the term we will use), in that the reward is an excess return and the risk is measured as the square root of the incremental variance over and above that of a risky benchmark (in our setting, this risky benchmark is the portfolio with return $\frac{1}{2}q\, \boldsymbol{v}'\mathbf{Z}$). This incremental variance is unity (by equation (23)).

If the reward-for-risk EP were high enough, a financial intermediary would have an incentive to privately negotiate contracts with investors that exploit the fact that domestic and foreign investors disagree on the valuations of securities outside the linear span of $\mathbf{R}$ (or $\mathbf{R}^\star$).

We therefore place an upper bound on $\text{EP}/\sqrt{\text{Var}_t[X] - \text{Var}_t[\frac{1}{2}q\, \boldsymbol{v}'\mathbf{Z}]}$, which has the effect of ruling out a potential contract that is too good to be true, i.e., a "good deal" (e.g., Cochrane and Saá-Requejo (2000)).

Specifically, for some $\widehat{\Theta}$, satisfying $0 \leq \widehat{\Theta} < +\infty$, the reward-for-risk is bounded:

$$|\text{EP}| = \left|\frac{1}{2}\,\mathbb{E}[\boldsymbol{v}'\mathbf{Z}]\,\sqrt{\mathbb{E}_t[(y - y^*)^2]}\right| \leq \widehat{\Theta}. \tag{24}$$

Alternatively, defining Θ by $\mathbb{E}_t[v'\mathbf{Z}]\,\Theta \equiv 2\,\widehat{\Theta}$, and substituting into equation (24) and henceforth, for brevity, dropping the subscript t from the expectation operator, we exclude good deals in the international economy by placing an upper bound Θ as follows:

$$\mathbb{E}[(y - y^\star)^2] \leq \Theta^2, \tag{25}$$

for some economically motivated choice of the Appraisal Ratio Θ (for $0 \leq \Theta < +\infty$). The value of Θ can be changed if required to analyze the sensitivity of our analysis, and, thus, of our risk sharing index, to the choice of Θ.

Importantly, equation (25) places a restriction on y and $y^\star$ and, thus, on the set of admissible m and $m^\star$. Equation (25) is central to our analysis, and can be distinguished from the setting where $m\left(e_{t+1}/e_t\right) - m^\star = 0$, which entails $y - y^\star = 0$, which in turn, imposes $\mathbb{E}[(y - y^\star)^2] = 0$.

2.3 Operationalizing a risk sharing index when $m\left(e_{t+1}/e_t\right) - m^\star$ need not equal zero

In complete markets, $m\left(e_{t+1}/e_t\right) - m^\star = 0$ tightly link exchange rate growth and m and $m^\star$. Moreover, Brandt, Cochrane, and Santa-Clara (2006) show that the *minimum variance* m and $m^\star$ recovered from asset return data, also satisfy $m\left(e_{t+1}/e_t\right) - m^\star = 0$ in an incomplete market. In contrast, we study different choices of m and $m^\star$ (restricted by a feasible set outlined shortly) that may or may not be consistent with $m\left(e_{t+1}/e_t\right) - m^\star = 0$. Our particular interest is in assessing the prevailing view that international risk sharing is high and that contrasting estimates of risk sharing, from asset return data and from consumption data, constitute a puzzle.

Now turn to the question of specifying a risk sharing index. We work in discrete-time, so it is tractable for us to consider covariances between proportional changes, rather than changes in logs of the pricing kernel. This leads us to possibly consider specifying a risk sharing index of the form:

$$\text{RSI} \equiv \frac{2\,\text{Cov}\,[m, m^\star]}{\text{Var}\,[m] + \text{Var}\,[m^\star]} = \frac{2\,\mathbb{E}[m\,m^\star] - 2/(R_f R_f^\star)}{\text{Var}\,[m] + \text{Var}\,[m^\star]}. \tag{26}$$

In the setting of incomplete markets, there is an infinite number of m and $m^\star$ and, thus, an infinite number of possible values of such a proposed risk sharing index, which leads us to take infimums, over m and $m^\star$, of a risk sharing index in equation (26). Thus, we ask what is a plausible, but economically justified, lower bound for the risk sharing index based on asset return data?

To operationalize our choice of the risk sharing index using asset return data, we compute the risk sharing index in equation (26). Recalling from equation (8) that $y \equiv m\sqrt{e_{t+1}/e_t}$ and

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$y^\star \equiv m^\star / \sqrt{e_{t+1}/e_t}$, we consider the following problem (which, since $y\,y^\star = m\,m^\star$, is equivalent to the objective $\inf_{m,m^\star} \mathbb{E}[2\,m\,m^\star]$):

Problem 1 *Choose y and $y^\star$ to*

$$\inf_{y,y^\star} \mathbb{E}[2\,y\,y^\star] \tag{27}$$

subject to

$$\mathbb{E}[(y - y^\star)^2] \;\leq\; \Theta^2, \qquad \text{(relaxes the restriction that } m\,(e_{t+1}/e_t) - m^\star = 0) \tag{28}$$

$$\mathbb{E}[y\mathbf{Z}] = \mathbb{E}[y^\star \mathbf{Z}] = \mathbf{1}, \qquad \text{(correct pricing)} \tag{29}$$

$$y \geq 0 \;\; \text{and} \;\; y^\star \geq 0. \qquad \text{(nonnegativity constraints)} \tag{30}$$

In Problem 1, the inequality constraint (28) arises as a consequence of incorporating the incomplete markets assumption in the international economy, whereby $m\,(e_{t+1}/e_t) - m^\star$ need not equal zero. Specifically, Θ^2 is the upper bound on $\mathbb{E}[(y - y^\star)^2]$. The equality constraint $\mathbb{E}[y\mathbf{Z}] = \mathbb{E}[y^\star \mathbf{Z}] = \mathbf{1}$ in equation (29) is equivalent to $\mathbb{E}[m\mathbf{R}] = \mathbf{1}$ and $\mathbb{E}[m^\star \mathbf{R}^\star] = \mathbf{1}$ and enforces that m and $m^\star$ must price the returns $\mathbf{R}$ and $\mathbf{R}^\star$.

We are interested in analyzing what exchange rates have to say about international risk sharing in incomplete markets. At the same time, we are interested in inferring marginal utility growth rates that are consistent with the data. The marginal utilities are nonnegative, so, following Hansen and Jagannathan (1991), we focus on nonnegative SDFs in the admissible set. The constraints $y \geq 0$ and $y^\star \geq 0$ in equation (30) are equivalent to $m \geq 0$ and $m^\star \geq 0$.

In the objective function (27), we essentially compute a lower bound on the value of the numerator of equation (26) consistent with repricing the returns $\mathbf{R}$ and $\mathbf{R}^\star$, consistent with the absence of arbitrage, and consistent with the upper bound $\mathbb{E}[(y - y^\star)^2] \leq \Theta^2$.

Still, the optimization problem could become ill-posed if one could find m and $m^\star$, where the objective is unbounded. Such an outcome is disallowed with our constraints and via $\big|\mathbb{E}[yy^*]\big| < +\infty$. The solution depends critically on $\mathbb{E}[(y - y^\star)^2] \;\leq\; \Theta^2$.

We use the minimizing y and $y^\star$ in Problem 1. Then we use equation (8) to compute m and $m^\star$, which we insert into equation (26) to compute the risk sharing index.

Observe further that if, instead, we were to minimize the ratio $2\operatorname{Cov}\left[m, m^\star\right] / \left(\operatorname{Var}\left[m\right] + \operatorname{Var}\left[m^\star\right]\right)$ subject to the constraints in equations (28) through (30), we would obtain a value of the risk sharing index that is (weakly) lower than that obtained from Problem 1. Hence, the solution to Problem 1 could be viewed as a conservative lower bound on international risk sharing.

Our goal in computing the risk sharing index RSI in equation (26) is not to establish that values obtained from (26) are identical to, or even close to, those obtained from consumption data. Instead, our key insight is to show that there may be $(m, m^\star)$ pairs that have low correlations (or covariances), that generate an RSI that is not necessarily inconsistent with those computed from consumption growth data, implying that international risk sharing can, in some cases, be low.

2.4 Solving the problem

Based on the problem in equations (27)–(30), we look for solutions for y and $y^\star$ of the form:

$$ y = y_{\mathbf{z}} \ + \ \frac{1}{2} d\,\Theta\,\eta, \quad \text{and} \quad y^\star = y_{\mathbf{z}} \ + \ \frac{1}{2} d^\star\,\Theta\,\eta, \tag{31} $$

where $y_{\mathbf{z}} \geq 0$, η, d, and d^* are yet to be determined (the conjectured solution inherits the form in equation (10), but with $q_0 = d\,\Theta$ and $q_0^* = d^*\,\Theta$). For now, d and $d^\star$ are constant scalars satisfying $|d - d^\star| \leq 2$. Further the random variables $y_{\mathbf{z}}$ and η satisfy,

$$ \mathbb{E}[y_{\mathbf{z}}\,\eta] = 0, \qquad \mathbb{E}[\eta\mathbf{Z}] = 0 \ \text{ for each element of } \mathbf{Z}, \qquad \text{and} \qquad \mathbb{E}[\eta^2] = 1. \tag{32} $$

Because $y - y^\star = \frac{1}{2}\left(d - d^\star\right)\Theta\eta$, the constraint $\mathbb{E}[(y - y^\star)^2] \leq \Theta^2$ is automatically satisfied. The constraint $\mathbb{E}[y\mathbf{Z}] = \mathbb{E}[y^\star\mathbf{Z}] = \mathbf{1}$ in equation (29) is satisfied provided $\mathbb{E}[y_{\mathbf{z}}\mathbf{Z}] = \mathbf{1}$.

With the conjectured forms of y and $y^\star$ in equation (31), $\mathbb{E}[m\,m^\star] = \mathbb{E}[y\,y^\star] = \mathbb{E}[y_{\mathbf{z}}^2] + \frac{1}{4} d\,d^\star\Theta^2$. Hence, the infimum $\inf_{y,y^\star} \mathbb{E}[y\,y^\star]$ in Problem 1 separates into two distinct problems:

$$ \inf_{y_{\mathbf{z}}} \{\mathbb{E}[y_{\mathbf{z}}^2]\} \ + \ \inf_{d,d^\star} \{\frac{1}{4} d\,d^\star\Theta^2\} \tag{33} $$

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subject to $\mathbb{E}[y_{\mathbf{z}}\mathbf{Z}] = \mathbf{1}$, $y_{\mathbf{z}} \geq 0$, $y \geq 0$, and $y^{\star} \geq 0$.

Exploiting this feature of the solution, we sequentially solve for $y_{\mathbf{z}}$, then for η and, finally, for d and $d^{\star}$. We first, determine $y_{\mathbf{z}}$ by solving:

$$\inf_{y_{\mathbf{z}}} \mathbb{E}[y_{\mathbf{z}}^2] \quad \text{such that} \quad \mathbb{E}[y_{\mathbf{z}}\mathbf{Z}] = \mathbf{1}, \quad y_{\mathbf{z}} \geq 0. \tag{34}$$

Here, $y_{\mathbf{z}}$ can be interpreted as the minimum second moment stochastic discount factor with non-negativity in the hypothetical economy in which gross returns are $\mathbf{Z}$.

Introduce an N-dimensional vector of Lagrange multipliers $\boldsymbol{\lambda}$. Then the solution to the problem in equation (34) is the solution to:

$$\max_{\boldsymbol{\lambda}}\{ \inf_{y_{\mathbf{z}} \geq 0} \{\mathbb{E}[y_{\mathbf{z}}^2] \; - \; 2\boldsymbol{\lambda}^{'}\left(\mathbb{E}[y_{\mathbf{z}}\mathbf{Z}] - \mathbf{1})\right)\}\}. \tag{35}$$

The first-order condition implies $0 = 2y_{\mathbf{z}} - 2\boldsymbol{\lambda}^{'}\mathbf{Z}$. Both the first-order condition and the constraint $y_{\mathbf{z}} \geq 0$ will be satisfied if $y_{\mathbf{z}} = \max(\boldsymbol{\lambda}^{'}\mathbf{Z}, 0)$. Then, substituting and simplifying, the solution to (34) is the solution to

$$\max_{\boldsymbol{\lambda}}\{2\boldsymbol{\lambda}^{'}\mathbf{1} - \mathbb{E}[(\max(\boldsymbol{\lambda}^{'}\mathbf{Z}, 0))^2]\}, \quad \text{or, equivalently,} \quad -\min_{\boldsymbol{\lambda}}\{\mathbb{E}[(\max(\boldsymbol{\lambda}^{'}\mathbf{Z}, 0))^2] - 2\boldsymbol{\lambda}^{'}\mathbf{1}\}. \tag{36}$$

Next, to solve for η, we note that η is proportional to $e_{\mathbf{z}}$, where $e_{\mathbf{z}}$ is the residual from the projection of one onto the space of returns $\mathbf{Z}$. Hence, $e_{\mathbf{z}}$ can be computed from the Ordinary Least Squares regression formula (e.g., Cochrane (2005, page 95)). Then η is obtained by multiplicatively scaling $e_{\mathbf{z}}$ in such a way that $\mathbb{E}[\eta^2] = 1$. More formally,

$$e_{\mathbf{z}} = 1 - \mathbb{E}[\mathbf{Z}]^{'}\left(\mathbb{E}[\mathbf{Z}\mathbf{Z}^{'}]\right)^{-1}\mathbf{Z}, \quad \text{and then} \quad \eta = e_{\mathbf{z}}/\sqrt{\mathbb{E}[e_{\mathbf{z}}^2]}. \tag{37}$$

In the degenerate case that $e_{\mathbf{z}} = 0$, in every state, we set $\eta = 0$.

Finally, we solve for d and $d^{\star}$. The second part of equation (33) minimizes $\frac{1}{4}d\,d^{\star}\Theta^2$. It is clear that the minimum requires that d and $d^{\star}$ must be of opposite signs. Hence, without loss of generality, we assume $d \geq 0$, $d^{\star} \leq 0$. The solution of Problem 1 must also accommodate $y = y_{\mathbf{z}} + \frac{1}{2}d\Theta\eta \geq 0$ and $y^{\star} = y_{\mathbf{z}} + \frac{1}{2}d^{\star}\Theta\eta \geq 0$.

Let d_p (respectively, d_n) be the smallest positive value (respectively, largest negative, i.e., least negative value) of $-y_{\mathbf{z}}/\left(\frac{1}{2}\Theta\eta\right)$ across the $\mathbb{J}$ possible states of the world. Then $y \geq 0$, and $y^\star \geq 0$ requires that $d \leq d_p$ and $d_n \leq d^\star$. Taken together with the constraint $d - d^\star \leq 2$, the solution for d and $d^\star$ can be summarized as:

$$
\begin{cases}
\text{If } d_p \geq 1 \text{ and } d_n \leq -1, \text{ then } d = 1 \text{ and } d^\star = -1; \\
\text{else } d = \min(d_p, -d_n) \text{ and } d^\star = -d.
\end{cases}
\tag{38}
$$

These values for d and $d^\star$, together with equations (36) and (37), provide the values of y and $y^\star$ and, thus, of m and $m^\star$, which solve Problem 1. Utilizing these values of m and $m^\star$ with nonnegativity enforced, we compute the risk sharing index postulated in equation (26).[1]

Note that the original equations (27)–(30) involved minimizations over y and $y^\star$. With the calculations of the risk sharing index spanning the period of January 1975 to June 2014 at monthly frequency (which corresponds to 474 months), the minimizations over y and $y^\star$ (or m and $m^\star$) involve minimizations over $2 \times 474 = 948$ variables. By contrast, the maximization in the left-hand side of equation (36) is a maximization over a total of four variables ($\boldsymbol{\lambda}$) when $N = 4$.

Additionally, we solve a particularly parameterized economy with five states of the world in the Internet Appendix (Section I and Table Internet-I), which helps to synthesize, in a simplified setting, the various elements of our approach to study international risk sharing under incomplete markets.

3 What does our approach tell us about risk sharing?

Our empirical investigation employs a risk-free bond and a broad-based equity index denominated in each currency for ten countries – namely, Australia (AUD), New Zealand (NZD), United Kingdom (STG), France (FRA), Canada (CAD), United States (USD), Netherlands (NLG), Germany (GER), Japan (JPY), and Switzerland (SWI).

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[1]The numerical computation of equation (36) is also worthy of some clarifications. It is solved numerically by choice of Lagrange multipliers $\boldsymbol{\lambda}$, using standard multidimensional nonlinear maximizing software (a "solver" in C++) as in Press, Teukolsky, Vetterling, and Flannery (2007). Since the maximand in the left-hand side of problem (36) is globally concave, if the algorithm finds a maximum, it is the global maximum.

When computing, for example, the risk sharing index for the US and Australia, the gross return vector $\mathbf{R}$ includes *real* returns on four assets, namely, on the US risk-free bond, on the US equity index, on the Australian risk-free bond and on the Australian equity index, all denominated in US dollars, while $\mathbf{R}^\star$ includes returns on the same four assets, but now all the *real* returns are denominated in Australian dollars.

The country-specific data on the LIBOR interest-rate, equity index, currency, and inflation are described in Appendix B.

3.1 Risk sharing is high across all country pairs when $m\left(e_{t+1}/e_t\right) - m^\star = 0$

To benchmark our analysis, we first compute the risk sharing index of Brandt, Cochrane, and Santa-Clara (2006, equation (17)) and report the values in Panel A of Table 1. Note that our results are displayed in the order of decreasing average interest-rates (as shown in Panel A of Table Internet-II), so Australia has the highest average interest-rate, while Switzerland has the lowest.

The risk sharing index of Brandt, Cochrane, and Santa-Clara (2006) depends on the variance of the exchange rate growth and the variance of the domestic and foreign minimum variance stochastic discount factors. Even when their analysis of countries and currencies is expanded from three to ten and the time period extended by an additional 16 years, the *average* risk sharing index across the 45 pairs of countries is 93.2%, with a standard deviation of 6.95%.

Our evidence further shows that 44 out of 45 RSI values are above 80%, most exceed 94% and the maximum value reached is 99.8%. Thus, the result that international risk sharing is high is at the heart of the $m\left(e_{t+1}/e_t\right) - m^\star = 0$ setting of Brandt, Cochrane, and Santa-Clara (2006). The high values of the computed risk sharing index reflect the low variance of exchange rate growth in relation to the estimated variances of the minimum variance stochastic discount factors.

3.2 Risk sharing is low across country pairs with high interest-rate differentials when $m\left(e_{t+1}/e_t\right) - m^\star \neq 0$

The constraint $\mathbb{E}[(y - y^\star)^2] \leq \Theta^2$, in equation (25), is pivotal to our characterizations of the risk sharing index but leaves open the question of how to choose Θ. In our empirical work, we initially

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feature choices of the Appraisal Ratio Θ of $\Theta = 0.175$, $\Theta = 0.35$, $\Theta = 0.525$, and $\Theta = 0.70$. Accordingly, we report in Panel B of Table 1, the estimates of the risk sharing index by varying Θ, as defined in equation (26).

To understand the range of our choices of Θ, we note that a portfolio earning a return 0.065 (i.e., 6.5% annualized) over and above a benchmark whose return is 0.08, when the standard deviation of the portfolio is 0.4 (i.e., 40% annualized) and the standard deviation of the benchmark is 0.16, has an Appraisal Ratio of $0.065/\sqrt{(0.4^2 - 0.16^2)} \approx 0.175$. Alternatively, a portfolio earning a return 0.084 over and above a benchmark whose return is 0.08, when the standard deviation of the portfolio is 0.2 and the standard deviation of the benchmark is 0.16, has an Appraisal Ratio of $0.084/\sqrt{(0.2^2 - 0.16^2)} = 0.7$.

Furthermore, Table Internet-III reports the Appraisal Ratios available to domestic investors for investing in the foreign equity index for each pair of currencies. The largest Appraisal Ratio is 0.26. Cochrane and Saá-Requejo (2000, page 82) suggest eliminating good deals by ruling out Sharpe ratios greater than twice the Sharpe ratio available on a major broad-based equity index. Written in these terms, our exploration of Appraisal Ratios around 0.175 to 0.70 is economically motivated and not overly stringent.

Panel B of Table 1 shows that for $\Theta = 0.175$ (respectively, $\Theta = 0.35$), the risk sharing index varies from 66.2% to 94.1% (respectively, from 28.9% to 79.9%). Among the four of them, Germany, France, Netherlands, and Switzerland do have high values of the risk sharing index (over 99% from the approach of Brandt, Cochrane, and Santa-Clara (2006)) and, for our methodology, well over 91% for $\Theta = 0.175$ and well over 70% for $\Theta = 0.35$).

These results possibly suggest that international risk sharing can be high, at least, for some pairs of countries. On the other hand, for pairs of currencies involving New Zealand, the values of the risk sharing index are much lower.

Using low values of Appraisal Ratios Θ (e.g., $\Theta=0.175$) brings the risk sharing index closer to the $m\left(e_{t+1}/e_t\right) - m^\star = 0$ case. In contrast, higher values of Θ magnify the departures between the results highlighted in Brandt, Cochrane, and Santa-Clara (2006) and ours. In particular, for $\Theta=0.525$, the *average* risk sharing index is 42.7%, and it varies from -6.1% to 60.3%. With $\Theta = 0.70$, the *average* risk sharing index is 18.4%, varying from -31.9% to 39.1%.

While our evidence supports the view that the estimates of risk sharing obtained using equation (26) (for the featured choices of Θ in Panel B of Table 1) are lower than that when $m\left(e_{t+1}/e_t\right) - m^\star = 0$ is imposed, is the difference statistically significant? The two-sided p-values associated with the non-parametric Wilcoxon-Mann-Whitney test (implemented in SAS using "proc npar1way wilcoxon") are all 0.000, indicating that there is a significant difference between the two risk sharing index distributions. The takeaway is that relaxing the constraint $m(e_{t+1}/e_t) - m^\star = 0$ translates into less correlated m and $m^\star$ pairs, resulting in lower estimates of the risk sharing index.

3.3 Squaring the evidence on risk sharing from consumption and asset returns

How do the estimates of international risk sharing recovered from asset return data compare to the ones from real consumption growth?

To establish our own estimates of the risk sharing based on consumption growth, we follow Brandt, Cochrane, and Santa-Clara (2006, equation (23)), and replace the variances and covariances of the SDFs with variances and covariances of changes in log consumption:

$$\text{RSI}^C \equiv \frac{2\,\text{Cov}\left[d\ln(c), d\ln(c^\star)\right]}{\text{Var}\left[d\ln(c)\right] + \text{Var}\left[d\ln(c^\star)\right]}. \tag{39}$$

The values of risk sharing, RSI^C, reported in Panel A of Table 2, use *annual* consumption growth data from Barro and Ursua (2008), updated using World Development Indicators.

Brandt, Cochrane, and Santa-Clara (2006) describe as a puzzle the fact that their values of the risk sharing index, based on asset return data, are much higher than those based on consumption growth data. In order to reconcile estimates of risk sharing from asset returns with those from consumption growths, a key step is to recognize that $m\left(e_{t+1}/e_t\right) - m^\star$ need not equal zero.

With our methodology (equations (26) and (27)), we see values of the risk sharing index, based on asset return data, which are broadly in the same range as those based on consumption growth data. We emphasize only "broadly in the same range" because the values do show considerable variation, both across different currency pairs and across different values of Θ. If we compare the results when the Appraisal Ratio $\Theta = 0.525$ (Panel B of Table 1) with those in Panel A of Table 2,

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11 out of a possible 45 entries produce a higher risk sharing from consumption growth data than from asset return data. For the case when the Appraisal Ratio $\Theta = 0.70$, that number rises to 29.

Complementing the above evidence, we ask: Are there values of Θ at which the risk sharing index from consumption data (Panel A of Table 2) most closely corresponds, in a statistical sense using the non-parametric Wilcoxon-Mann-Whitney test statistic, to the counterparts from asset return data? To answer this question, we conduct an exercise in which we compute the risk sharing index from asset return data (for each of the 45 country pairs) using values of $\Theta = \{0.10, 0.15, 0.20, \ldots, 0.70, 0.75, 0.80\}$. This part of our investigation reveals that the associated two-sided p-value with $\Theta = 0.60$ (respectively, $\Theta = 0.65$) is 0.32 (respectively, 0.58), indicating that we cannot reject that risk sharing extracted from consumption and asset returns have the same underlying distribution. The two-sided p-values for other values of Θ are all below 0.05.

Our confidence is reinforced by a finding that the two-sided p-values from a t-test, allowing for unequal variance, favors the same conclusion.

Our investigation, thus, supports the view that values of the risk sharing index, based on asset return data, can be aligned with those based on consumption data, for certain values of Θ. The values of risk sharing index (26) corresponding to $\Theta = 0.60$ and $\Theta = 0.65$ are reported in Panels B and C of Table 2, respectively.

How sensible are the values of Θ that align asset market data with data from consumption growth? To offer a perspective, we set $q = \Theta$ in equations (22) and (23), then $\text{EP} = \frac{1}{2}\Theta\mathbb{E}[\boldsymbol{v}'\mathbf{Z}]$ and $\text{Var}[X] - \frac{1}{4}\Theta^2\text{Var}[\boldsymbol{v}'\mathbf{Z}] = 1$. If we use benchmark values of an excess return of $\mathbb{E}[\boldsymbol{v}'\mathbf{Z}] = 0.08$ and standard deviation of $\text{Var}[\boldsymbol{v}'\mathbf{Z}] = 0.16$, then $\mathbb{E}[\boldsymbol{v}'\mathbf{Z}] = 1.08$ and $\text{Var}[\boldsymbol{v}'\mathbf{Z}] = 0.16^2$. Then choosing $\Theta = 0.65$ results in a, reward-for-risk, $\text{EP}/\sqrt{1} \approx 0.35$. Ruling out "good deals," associated with this reward-for-risk, is instrumental in lowering the risk sharing index, implied from asset return data, sufficiently to be able to align it with that computed from consumption data.

The big picture is that it is possible to reconcile the estimates of risk sharing from consumption data with those from asset returns. Moreover, the most pessimistic estimates of risk sharing obtained from asset returns can be lower than that from consumption data (for example, in 24 out of 45 instances, when $\Theta = 0.65$), potentially reversing the order of the risk sharing puzzle.

3.4 Low risk sharing index is accompanied by reasonable volatility of SDFs

One may surmise that a low value of the risk sharing index could be due to the high volatility of the domestic and foreign country SDFs, which enter into the denominator of equation (26). Adopting a higher value of the Appraisal Ratio Θ, in equation (25), increases the volatility of y and $y^\star$ (see equation (31)), and, thus, likely increases the volatility of m and $m^\star$. Thus, a potentially important question is: Are the SDFs *unrealistically* volatile in our incomplete markets setting?

To address the aforementioned question, we compute and display in Table 3 the values of the SDF volatilities, denoted by $\sigma[m]$ and $\sigma[m^*]$, together with the pairwise correlation between the SDFs, denoted by CORR. Our quantitative evaluation sheds light on two aspects of the economic environment. First, as is to be expected, raising Θ increases the volatility of m and $m^\star$ (see Panel A of Table 3), but the new insight is: Not by much. In contrast, the decline in correlation is stronger.

Second, with a value of $\Theta = 0.65$, the average SDF volatility, across the 45 distinct country pairs, is not out of line with the 50% minimum volatility of the SDF that is suggestive from the bounding exercise in Cochrane (2005, page 456). The 95th percentile value is 60% and 62% for $\sigma[m]$ and $\sigma[m^*]$, respectively. The SDF volatilities for each country pair reported in Panel C of Table 2 convey the same message. We recall that for the value of $\Theta = 0.65$ (as in Table 2), we cannot reject the null that the distribution of risk sharing obtained using asset returns is the same as the distribution of risk sharing obtained using consumption growth.

Further evidence of our SDFs having realistic properties is that, for the values of Θ in Table 1, the nonnegativity constraints $y \geq 0$ and $y^\star \geq 0$ happen not to be binding, suggesting that, in practice, m and $m^\star$ are *strictly* positive, a feature also illustrated in the context of Table Internet-I and Section I of the Internet Appendix.

3.5 Synthesizing and understanding the patterns of international risk sharing

In Panels A of Table 2, eight out of a possible 45 distinct off-diagonal entries of risk sharing, obtained from real consumption growth data, are negative. The currency pairs that produce negative values of the risk sharing, RSI^C, are AUD/GER, AUD/JPY, NZD/FRA, NZD/NLG, NZD/GER, NZD/JPY, NZD/SWI and CAD/JPY. Our estimates imply that many of the currency pairs that

produce low values of the risk sharing involve either a funding currency for the carry trade or an investment currency for the carry trade (and usually both).

Conversely, interest-rates for USD, STG, and CAD are typically close. From Panel A of Table 2, we see that STG/USD, STG/CAD, and CAD/USD produce larger (positive) values of risk sharing using consumption growth data. Similarly, FRA/GER, FRA/SWI, and GER/SWI also have relatively larger values of risk sharing using consumption growth data. Hence, there is a fairly consistent pattern in the relationship between the risk sharing index using consumption growth data and the average interest-rate differential.

The low risk sharing obtained for country pairs with high interest rate differentials cannot be attributed to unreasonably high volatility of the SDFs, as can be inferred from Panel B of Table 3. Here, we isolate the evidence on risk sharing for the country pairs with the three highest and the three lowest interest-rates, together with the respective SDF volatilities. For example, consider the case of NZD/JPY. For $\Theta = 0.65$, the risk sharing index is -25.4%. The SDF volatilities of 40% (New Zealand) and 46% (Japan) are reasonable.

How could one justify the observation that for country pairs with high interest differentials, risk sharing is relatively low? To examine this when $m\left(e_{t+1}/e_t\right) - m^\star \neq 0$, assume, for illustration, that m and e_{t+1}/e_t are jointly log-normally distributed. We show in Online Appendix (Section II) that

$$
\begin{aligned}
\ln(R_f^\star) - \ln(R_f) + \mathbb{E}_t\left[\ln\left(\frac{e_{t+1}}{e_t}\right)\right] = {} & -\frac{1}{2}\mathrm{Cov}_t[\ln(m), \ln\left(\frac{e_{t+1}}{e_t}\right)] - \frac{1}{2}\mathrm{Cov}_t[\ln(m^\star), \ln\left(\frac{e_{t+1}}{e_t}\right)] \\
& -\frac{1}{4}\mathrm{Var}_t[\ln(m) + \ln\left(\frac{e_{t+1}}{e_t}\right) - \ln(m^\star)] \\
& +\frac{1}{4}\left(\mathrm{Var}_t[\ln(m)] + \mathrm{Var}_t[\ln(m^\star)] - 2\mathrm{Cov}_t[\ln(m), \ln(m^\star)]\right) \\
& -\frac{1}{4}\mathrm{Var}_t[\ln\left(\frac{e_{t+1}}{e_t}\right)].
\end{aligned}
\tag{40}
$$

Replacing conditional by unconditional expectations and using an approximation to go from logs to levels and vice versa, $\mathrm{Cov}_t[\ln(m), \ln(m^\star)]$, appearing in the third line of equation (40), is approximately proportional to the risk sharing index. If the domestic currency is Japanese yen and the foreign currency is Australian dollar, then *ceteris paribus* the expected excess return to borrowing in Japanese yen and investing in Australian dollar can be higher when the risk sharing index is lower. This provides some rationale for what we observe in the data.

Both approaches, that is, those from asset returns and real consumption growth data, reinforce the notion of low risk sharing, especially among country pairs with high interest-rate differentials.[2]

4 Summary and conclusions

In contrast to the literature, we find that estimates of international risk sharing can be low, especially for pairs of countries that have large interest-rate differentials. Further, risk sharing computed from asset return data need not be inconsistent with that computed from consumption growth data. Our conclusions are based on consumption and asset return data, spanning nearly 40 years, for ten industrialized countries.

The key issue that drives the difference from the extant literature is that we do not require that exchange rate growth be the ratio of stochastic discount factors. Instead, we develop a restriction that precludes "good deals" in international economies with incomplete markets. We present the lowest risk sharing index consistent with stochastic discount factors that are nonnegative, that correctly price asset returns in both countries, and that rule out "good deals." Our work highlights the role of incomplete financial markets in understanding extant puzzles in international finance.

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[2]Numerous authors have contributed to the debate on international risk sharing and the cost of shutting down financial markets. Among others, we refer the reader to the treatments in Tesar (1993, 1995), van Wincoop (1999), Baxter and Jermann (1997), Obstfeld and Rogoff (2000), Martin (2010), and Stathopoulos (2014). Gourinchas (2005), Wickens (2012, Section 11.9), and Devereux and Kollmann (2010) provide a synthesis of the outstanding issues.

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A   Appendix A: $|\mathbb{E}[y - y^\star]|$ should, in practice, be very small and proof that $\mathbb{E}[y - y^\star] = 0$ when m, $m^\star$ and e_{t+1}/e_t are log-normally distributed

Our objective is to show that (i) $|\mathbb{E}[y - y^\star]|$ should, in practice, be very small, and (ii) $\mathbb{E}[y - y^\star]$ is identically zero when m, $m^\star$ and e_{t+1}/e_t are log-normally distributed.

We focus first on the case where $\ln(m)$, $\ln(m^\star)$, and $\ln(e_{t+1}/e_t)$ are jointly normally distributed, and prove an exact result. We have

$$\mathbb{E}[m] = 1/R_f, \qquad \mathbb{E}[m^\star] = 1/R_f^\star, \qquad \text{and} \qquad \mathbb{E}[e_{t+1}/e_t] \equiv \exp\left(\mu_e\right), \tag{A1}$$

and the variances are $\text{Var}\left[\ln(m)\right]$, $\text{Var}\left[\ln(m^\star)\right]$ and $\text{Var}\left[\ln(e_{t+1}/e_t)\right] \equiv \sigma_e^2$, respectively.

Using standard results on moment generating functions, $\mathbb{E}[(e_{t+1}/e_t)^{1/2}] = \exp\left(\frac{1}{2}\mu_e - \frac{1}{8}\sigma_e^2\right)$ and $\mathbb{E}[(e_{t+1}/e_t)^{-1/2}] = \exp\left(-\frac{1}{2}\mu_e + \frac{3}{8}\sigma_e^2\right)$. Equation (5), namely, $\mathbb{E}_t[m\left(e_{t+1}/e_t\right) - m^\star] = 0$, implies

$$\frac{1}{R_f^\star} = \frac{1}{R_f}\exp\left(\mu_e + \text{Cov}\left[\ln(m), \ln(e_{t+1}/e_t)\right]\right), \tag{A2}$$

while $\mathbb{E}[m^\star/\left(e_{t+1}/e_t\right) - m] = 0$ implies (i.e., Balakrishnan and Lai (2009, equation (11.69), page 526)):

$$\frac{1}{R_f} = \frac{1}{R_f^\star}\exp\left(-\mu_e + \sigma_e^2 - \text{Cov}\left[\ln(m^\star), \ln(e_{t+1}/e_t)\right]\right). \tag{A3}$$

Further,

$$\mathbb{E}[y] = \mathbb{E}[m(e_{t+1}/e_t)^{1/2}] = (1/R_f)\exp\left(\frac{1}{2}\mu_e + \frac{1}{2}\text{Cov}\left[\ln(m), \ln(e_{t+1}/e_t)\right] - \frac{1}{8}\sigma_e^2\right), \tag{A4}$$

$$= 1/\sqrt{R_f R_f^\star}\exp\left(-\frac{1}{8}\sigma_e^2\right), \qquad \text{(using (A2))} \tag{A5}$$

and also,

$$\mathbb{E}[y^*] = \mathbb{E}[m^\star(e_{t+1}/e_t)^{-1/2}] = (1/R_f^\star)\exp\left(-\frac{1}{2}\mu_e - \frac{1}{2}\text{Cov}\left[\ln(m^\star), \ln(e_{t+1}/e_t)\right] + \frac{3}{8}\sigma_e^2\right), \tag{A6}$$

$$= 1/\sqrt{R_f R_f^\star}\exp\left(-\frac{1}{8}\sigma_e^2\right), \qquad \text{(using (A3))}, \tag{A7}$$

or $\mathbb{E}[y - y^\star] = 0$. Our assertion is proved. $\blacksquare$

Next, we show $|\mathbb{E}[y - y^\star]| \approx 0$, in general. With $y = m\sqrt{e_{t+1}/e_t}$ and $y^\star = m^\star/\sqrt{e_{t+1}/e_t}$, the analog to equation (5) is:

$$\mathbb{E}[(y - y^\star)\,(e_{t+1}/e_t)^{1/2}] = 0 \qquad \text{and likewise} \qquad \mathbb{E}[(y - y^\star)\,\frac{1}{(e_{t+1}/e_t)^{1/2}}] = 0. \qquad (A8)$$

More generally, defining $g[\phi] \equiv (\phi\,e_{t+1}/e_t)^{1/2} + 1/(\phi\,e_{t+1}/e_t)^{1/2}$, for any ϕ, satisfying $0 < \phi < +\infty$, we have

$$\mathbb{E}[(y - y^\star)(\phi\,e_{t+1}/e_t)^{1/2} + (y - y^\star)/(\phi\,e_{t+1}/e_t)^{1/2}] \equiv \mathbb{E}[(y - y^\star)\,g[\phi]] = 0. \qquad (A9)$$

Hence, $\text{Cov}\,[y - y^\star, g[\phi]] = -\mathbb{E}[y - y^\star]\mathbb{E}[g[\phi]]$. Thus, $|\mathbb{E}[y - y^\star]|^2(\mathbb{E}[g[\phi]])^2 \leq \text{Var}\,[y - y^\star]\,\text{Var}\,[g[\phi]]$. Or,

$$|\mathbb{E}[y - y^\star]|^2\,\{(\mathbb{E}[g[\phi]])^2 - \text{Var}\,[g[\phi]]\} \leq \mathbb{E}[(y - y^\star)^2]\text{Var}\,[g[\phi]]. \qquad (A10)$$

We further note that $\text{Var}\,[g[\phi]] = \mathbb{E}[((\phi\,e_{t+1}/e_t) + 1/(\phi\,e_{t+1}/e_t)) - 2] - \mathbb{E}[g[\phi] - 2]\,\mathbb{E}[g[\phi]] + 2]$.

In practice, for $\phi \approx 1$, $\text{Var}\,[g[\phi]]$ may be very small. For example, empirical data suggests e_{t+1}/e_t is close to a martingale, so $\mathbb{E}[e_{t+1}/e_t] \approx 1$. Moreover, e_{t+1}/e_t has relatively low volatility, suggesting $\mathbb{E}[1/(e_{t+1}/e_t)]$ may also be close to unity. Hence, $\mathbb{E}[((e_{t+1}/e_t) + 1/(e_{t+1}/e_t)) - 2] \approx 0$, $\mathbb{E}[g[1]] \approx 2$, and $\mathbb{E}[g[1] - 2] = \mathbb{E}[(e_{t+1}/e_t)^{1/2} + 1/(e_{t+1}/e_t)^{1/2} - 2] \approx 0$, and, thus, $\text{Var}\,[g\,(1)]$ should typically be close to zero. Hence, in practice, equation (A10), evaluated in the special case of $\phi = 1$, should imply a tight bound on $|\mathbb{E}[y - y^\star]|$. Our conclusion follows. ∎

B Appendix B: Data description and sources

We obtain monthly data from January 1975 to June 2014 for the following ten countries: Australia (AUD), New Zealand (NZD), United Kingdom (STG), France (FRA), Canada (CAD), United States (USD), Netherlands (NLG), Germany (GER), Japan (JPY), and Switzerland (SWI).

The nominal returns of bonds, equity indexes, and currencies are converted into real returns by adjusting by ex-post realized inflation. Table Internet-II presents the summary statistics (in

annualized percentage units) for the data used in our study. The sources of the data are described below:

Interest rates: Risk-free bonds are synthesized from LIBOR quotes as $1/(1 + \tau \, \text{LIBOR})$, where τ is the day count fraction, that is, $\tau = 1/12$ for monthly. When LIBOR is not available, we use the nearest substitute, such as 30-day Bank Bill rates (which are money market rates, not to be confused with Treasury bill yields). At the start of the historical time period considered, interest-rate data for JPY, AUD, and NZD was not available from Datastream so we use data from the Bank of Japan, the Reserve Bank of Australia, and the Reserve Bank of New Zealand.

Equity returns: The equity index return data is MSCI data from Datastream, and we employ total returns (including dividends). MSCI data is not available for New Zealand prior to 1988, so for the period 1975-1987, we use returns data supplied by Martin Lally and Alastair Marsden (documented and used in Lally and Marsden (2004)). The data is at monthly frequency.

Exchange rates: The spot exchange rate data for all country pairs is the midpoint of the bid and ask quotes (from Datastream). The exchange rates for France, Germany, and Netherlands from January 1999 (the introduction of the Euro) onward are taken to be the relevant fixed conversion rate to the Euro (e.g., DM 1.95583 = 1 Euro).

Inflation: Country-specific inflation data is from Datastream and is CPI data except for United Kingdom and France, which only have data for CPI from 1988 and 1989, respectively. For United Kingdom and France, we splice the CPI data with retail price index data for the periods 1975 to 1988 and 1975 to 1989, respectively. ∎

Table 1: **Risk sharing index imputed from returns data**

The reported risk sharing index RSI (in %) in Panel A is based on the methodology of Brandt, Cochrane, and Santa-Clara (2006, equation (17), Table 2). The reported risk sharing index RSI (in %) in Panel B is based on equation (26) for various Appraisal Ratios Θ. When computing, for example, the risk sharing index for JPY/AUD, **R** includes returns on four assets, namely, on the Japanese risk-free bond, on the Japanese equity index, on the Australian risk-free bond, and on the Australian equity index, all denominated in Japanese yen, while $\mathbf{R}^\star$ includes returns on the same four assets but now all the returns are denominated in Australian dollars. There are ten countries and the sample period is January 1975 to June 2014. CI_l and CI_u, respectively, represent the 90% lower and upper bootstrap confidence intervals.

	Panel A: $m(e_{t+1}/e_t) - m^\star = 0$			Panel B: Incomplete markets (varying Θ) $\Theta = 0.175$			$\Theta = 0.35$			$\Theta = 0.525$			$\Theta = 0.70$		
	RSI	CI_l	CI_u	RSI	CI_l	CI_u	RSI	CI_l	CL_u	RSI	CI_l	CI_u	RSI	CI_l	CI_u
AUD/NZD	91.2	80	99	83.5	56	96	54.2	7	88	21.8	-32	76	-5.8	-55	61
AUD/STG	92.8	66	98	87.7	62	96	67.7	20	89	42.4	-16	77	17.7	-41	64
AUD/FRA	89.2	63	98	85.6	59	96	62.3	13	88	34.3	-23	76	8.2	-47	61
AUD/CAD	93.9	84	99	86.4	66	96	61.2	19	87	31.7	-19	75	4.8	-43	59
AUD/USD	95.2	81	99	89.3	65	97	70.5	22	90	46.3	-15	79	22.1	-40	66
AUD/NLG	95.1	85	99	90.5	70	97	74.4	31	91	52.8	-5	81	30.2	-32	70
AUD/GER	89.8	68	98	85.8	57	96	63.4	8	89	36.1	-29	79	10.4	-52	66
AUD/JPY	81.0	50	98	80.9	56	95	55.7	15	88	26.4	-20	77	0.1	-45	63
AUD/SWI	94.6	81	99	89.9	72	97	74.3	37	91	53.2	2	82	31.0	-25	71
NZD/STG	93.2	78	98	87.1	64	96	65.9	22	89	39.5	-15	77	14.2	-41	64
NZD/FRA	86.4	57	98	82.0	47	96	54.0	-3	87	22.6	-38	74	-4.5	-60	58
NZD/CAD	89.7	57	99	82.7	38	96	54.6	-16	88	23.2	-48	75	-3.9	-67	60
NZD/USD	94.4	85	99	87.9	58	96	68.0	13	89	42.8	-24	78	18.0	-49	65
NZD/NLG	95.3	85	99	90.0	66	97	73.1	24	90	50.8	-12	81	27.8	-37	69
NZD/GER	86.8	57	98	81.2	52	96	53.2	4	89	21.8	-32	77	-5.3	-54	64
NZD/JPY	57.3	-7	98	66.2	15	95	28.9	-34	86	-6.1	-61	72	-31.9	-74	57
NZD/SWI	95.1	82	99	89.7	63	97	73.0	21	91	51.0	-14	83	28.2	-39	72
STG/FRA	96.6	88	99	89.9	74	97	69.4	33	89	43.6	-5	77	18.4	-32	63
STG/CAD	95.4	82	99	89.4	67	97	69.8	22	90	44.9	-15	79	20.2	-41	65
STG/USD	96.4	86	99	91.1	73	97	74.2	35	90	51.8	-1	79	28.7	-28	67
STG/NLG	97.5	92	99	92.5	76	97	76.9	35	91	55.8	-2	82	33.5	-29	70
STG/GER	96.7	89	99	90.6	70	97	71.6	26	90	47.2	-12	80	22.8	-38	68
STG/JPY	93.0	76	99	87.6	66	96	67.5	23	89	42.1	-14	79	17.3	-38	66
STG/SWI	97.1	90	99	92.3	79	97	77.3	44	92	56.9	9	83	35.1	-19	72
FRA/CAD	92.0	69	98	86.2	57	96	62.1	8	87	33.3	-29	74	6.8	-52	59
FRA/USD	95.6	84	99	89.7	70	96	70.8	28	89	46.5	-9	78	22.2	-35	65
FRA/NLG	99.7	99	100	94.1	84	98	78.9	50	92	58.3	14	82	36.3	-15	71
FRA/GER	99.5	98	100	91.7	64	97	71.4	9	90	45.7	-30	80	20.5	-54	67
FRA/JPY	87.6	70	98	82.7	59	96	55.5	13	87	24.7	-24	75	-2.4	-48	60
FRA/SWI	99.0	96	100	93.2	79	98	77.4	41	92	56.1	5	82	33.6	-24	71
CAD/USD	98.4	93	100	91.2	70	97	71.5	24	90	46.3	-14	78	21.5	-41	65
CAD/NLG	96.3	87	99	91.4	74	97	75.5	37	91	54.1	1	82	31.6	-26	70
CAD/GER	91.3	61	99	85.8	54	96	61.6	2	89	32.8	-34	78	6.3	-56	64
CAD/JPY	84.7	40	98	81.8	12	95	55.2	-38	86	24.8	-63	73	-2.0	-76	57
CAD/SWI	95.6	80	99	90.7	66	97	74.9	23	91	53.7	-14	82	31.5	-39	71
USD/NLG	96.5	88	99	91.9	78	97	76.7	44	91	56.1	10	82	34.2	-18	70
USD/GER	95.3	83	99	89.5	74	97	70.7	36	90	46.5	0	80	22.3	-28	67
USD/JPY	94.8	75	99	89.5	68	97	70.9	26	89	47.0	-12	79	22.9	-37	66
USD/SWI	96.0	86	99	91.5	75	97	76.8	37	91	56.8	1	83	35.4	-26	72
NLG/GER	99.8	100	100	94.1	85	98	78.6	52	91	57.6	18	82	35.3	-11	70
NLG/JPY	95.5	85	99	90.9	74	97	75.0	38	91	53.7	3	82	31.3	-24	71
NLG/SWI	99.1	96	100	94.1	81	98	79.9	44	92	60.3	8	83	39.1	-20	72
GER/JPY	86.8	62	98	83.1	55	96	56.5	8	89	26.0	-29	77	-1.0	-53	64
GER/SWI	99.1	97	100	93.3	77	98	77.4	36	92	55.9	-2	83	33.4	-30	71
JPY/SWI	95.9	83	99	91.1	72	97	75.4	34	91	54.3	-3	82	32.1	-29	71

Table 2: **Risk sharing imputed from consumption data and consistency with risk sharing index imputed from returns data**

We report the estimates of risk sharing, RSI^C, based on equation (39), which uses real consumption growth data. Reported also is the risk sharing index, RSI, that are based on equation (26) for Appraisal Ratios $\Theta = 0.60$ and $\Theta = 0.65$, and the associated annualized (in %) volatilities of the SDFs, denoted by $\sigma[m]$ and $\sigma[m^*]$, respectively. We focus on $\Theta = 0.60$ and $\Theta = 0.65$, as for these values of Θ, we cannot reject the null that the distribution of risk sharing obtained from asset returns is the same as that from consumption growth (based on the Wilcoxon-Mann-Whitney test).

| | Risk sharing from consumption growth | Risk sharing index (RSI) from asset returns, and volatilities of m and m^* | | | | | |
| | Panel A | Panel B: $\Theta = 0.60$ | | | Panel C: $\Theta = 0.65$ | | |
	RSI^C	RSI	$\sigma[m]$	$\sigma[m^*]$	RSI	$\sigma[m]$	$\sigma[m^*]$
AUD/NZD	26.2	9.2	46	46	1.4	47	47
AUD/STG	19.7	31.5	50	55	24.5	52	57
AUD/FRA	24.1	22.7	48	52	15.3	49	53
AUD/CAD	41.6	19.6	47	50	12.0	48	52
AUD/USD	29.4	35.7	52	57	28.8	53	59
AUD/NLG	4.4	43.0	55	60	36.6	57	62
AUD/GER	-11.1	24.7	48	53	17.3	49	55
AUD/JPY	-18.6	14.6	45	51	7.1	47	53
AUD/SWI	13.8	43.6	56	61	37.3	57	63
NZD/STG	54.8	28.3	50	53	21.1	51	55
NZD/FRA	-8.3	10.3	44	48	2.6	46	50
NZD/CAD	39.9	10.9	44	49	3.2	46	51
NZD/USD	31.7	31.9	51	56	24.8	52	57
NZD/NLG	-7.0	40.8	54	59	34.2	56	60
NZD/GER	-28.1	9.5	43	49	1.8	45	51
NZD/JPY	-17.0	-18.3	38	44	-25.4	40	46
NZD/SWI	-17.4	41.1	55	60	34.6	56	61
STG/FRA	34.9	32.5	51	53	25.3	53	54
STG/CAD	53.6	34.1	53	54	27.0	54	55
STG/USD	60.6	41.8	56	58	35.2	57	59
STG/NLG	32.1	46.2	58	60	39.8	59	61
STG/GER	20.1	36.6	53	55	29.6	54	57
STG/JPY	35.5	31.2	51	54	24.2	52	56
STG/SWI	10.0	47.5	58	61	41.3	59	63
FRA/CAD	25.9	21.5	48	50	14.0	50	51
FRA/USD	44.7	35.9	53	55	28.9	55	57
FRA/NLG	39.0	48.8	59	61	42.5	60	62
FRA/GER	45.9	34.6	52	54	27.4	53	56
FRA/JPY	32.9	12.4	45	48	4.8	47	50
FRA/SWI	53.4	46.4	57	60	39.9	59	61
CAD/USD	66.0	35.4	53	54	28.3	54	56
CAD/NLG	34.1	44.4	57	58	38.0	59	60
CAD/GER	21.8	20.9	48	49	13.4	50	51
CAD/JPY	-5.9	12.6	45	48	5.1	47	50
CAD/SWI	35.6	44.1	57	59	37.7	58	61
USD/NLG	49.1	46.7	59	59	40.4	60	60
USD/GER	41.6	35.9	54	54	29.0	55	55
USD/JPY	24.0	36.5	54	55	29.6	55	56
USD/SWI	24.5	47.6	59	60	41.5	61	61
NLG/GER	58.5	48.0	59	59	41.6	60	60
NLG/JPY	15.5	44.0	58	59	37.6	59	60
NLG/SWI	36.5	51.2	61	62	45.1	62	63
GER/JPY	46.9	13.8	46	48	6.2	48	49
GER/SWI	50.9	46.2	58	59	39.7	59	60
JPY/SWI	2.7	44.7	58	58	38.4	60	60

Table 3: **Risk sharing index and the volatility of the stochastic discount factors**

This table first reports the mean, the standard deviation, and the percentiles for (i) the volatility of the SDFs, and (ii) the pairwise correlations of SDFs, across the 45 country pairs (Panel A). Reported also are the RSI and volatilities of the SDFs across the countries with high interest rate differentials (Panel B). $\sigma[m]$ and $\sigma[m^*]$ are the annualized volatilities (i.e., standard deviation, reported as %) of the domestic and foreign SDFs and CORR is the correlation. The three countries with the highest interest-rates are Australia (AUD), New Zealand (NZD), and United Kingdom (STG), and the three countries with the lowest interest-rates are Germany (GER), Japan (JPY), and Switzerland (SWI).

Panel A: Volatility and correlations of the SDFs across the 45 distinct country pairs

	$\Theta = 0.01$			$\Theta = 0.175$			$\Theta = 0.35$			$\Theta = 0.525$			$\Theta = 0.65$		
	$\sigma[m]$	$\sigma[m^*]$	CORR	$\sigma[m]$	$\sigma[m^*]$	CORR	$\sigma[m]$	$\sigma[m^*]$	CORR	$\sigma[m]$	$\sigma[m^*]$	CORR	$\sigma[m]$	$\sigma[m^*]$	CORR
Mean	43	45	0.96	44	46	0.88	46	49	0.68	50	53	0.43	54	56	0.25
Stdev.	7	6	0.02	7	6	0.05	6	6	0.10	6	5	0.14	5	5	0.15
Percentiles															
5th	33	36	0.93	34	37	0.82	37	40	0.54	42	45	0.22	46	49	0.02
25th	37	39	0.95	38	41	0.86	41	44	0.62	46	49	0.33	49	53	0.14
50th	44	45	0.97	44	46	0.90	47	49	0.71	51	53	0.47	54	57	0.29
75th	49	50	0.98	50	51	0.91	52	54	0.75	56	57	0.54	59	60	0.38
95th	51	52	1.00	52	53	0.94	54	56	0.78	57	59	0.57	60	62	0.42

Panel B: RSI and volatility of SDFs for countries with high interest rate differentials

	$\Theta = 0.01$			$\Theta = 0.175$			$\Theta = 0.35$			$\Theta = 0.525$			$\Theta = 0.65$		
	$\sigma[m]$	$\sigma[m^*]$	RSI	$\sigma[m]$	$\sigma[m^*]$	RSI	$\sigma[m]$	$\sigma[m^*]$	RSI	$\sigma[m]$	$\sigma[m^*]$	RSI	$\sigma[m]$	$\sigma[m^*]$	RSI
AUD/GER	37	43	**94.7**	38	44	**85.8**	41	47	**63.4**	46	51	**36.1**	49	55	**17.3**
AUD/JPY	34	39	**91.1**	35	41	**80.9**	38	44	**55.7**	43	49	**26.4**	47	53	**7.1**
AUD/SWI	47	52	**95.8**	48	53	**89.9**	50	56	**74.3**	54	59	**53.2**	57	63	**37.3**
NZD/GER	32	37	**93.0**	33	38	**81.2**	36	41	**53.2**	41	46	**21.8**	45	51	**1.8**
NZD/JPY	24	30	**83.9**	25	31	**66.2**	30	35	**28.9**	35	41	**-6.1**	40	46	**-25.4**
NZD/SWI	46	50	**95.9**	47	51	**89.7**	49	54	**73.0**	53	58	**51.0**	56	61	**34.6**
STG/GER	44	46	**97.8**	45	47	**90.6**	47	49	**71.6**	51	53	**47.2**	54	57	**29.6**
STG/JPY	42	44	**95.4**	43	45	**87.6**	45	48	**67.5**	49	52	**42.1**	52	56	**24.2**
STG/SWI	50	53	**97.9**	51	53	**92.3**	53	56	**77.3**	56	59	**56.9**	59	63	**41.3**

Risk Sharing in International Economies and Market Incompleteness

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Internet Appendix: Not for Publication

Abstract

This internet appendix provides additional theoretical and empirical results. The thrust of Section I is an example economy in which $m\left(e_{t+1}/e_t\right) - m^\star$ need not be zero in each state of the world, and the domestic country has low interest-rates, while the foreign country has high interest-rates. We describe the parameterized economy with five states and the results on risk sharing index in Table Internet-I. Next, Section II shows a relation between the excess return to borrowing in domestic currency and investing in foreign currency, to risk sharing.

Table Internet-II displays the summary statistics on interest-rates, equity returns, and inflation across our sample of ten countries, whereas Table Internet-III provides the Appraisal Ratios for a strategy in which the domestic investor invests in the foreign equity index. The highest Appraisal Ratio is 0.26, which helps to benchmark our choice of Appraisal Ratios used in the study. Table Internet-IV summarizes the data on consumption growth for each of the ten countries.

I Risk sharing parameterized by five states in an incomplete markets setting where $m\left(e_{t+1}/e_t\right) - m^\star \neq 0$

In Table Internet-I, we present a particularly parameterized two-country economy with five states of the world, featuring that $m\left(e_{t+1}/e_t\right) - m^\star$ need not be zero in each state.

The economy is constructed to capture some relevant features. First, the domestic country supports a low risk-free interest-rate (say, Japan), whereas the foreign country (say, Australia) a high risk-free interest rate. Second, the returns of the risky asset (i.e., equity) display positive correlation.

Our objective is to illustrate the solution technique and highlight the volatilities and covariance between m and $m^\star$. We also compute the extent of risk sharing. We further show that the problem is well-posed with a finite objective and well-defined Lagrange multipliers, and the solution supports $m > 0$ and $m^\star > 0$ over a wide range of values of Θ.

We compute $e_{\mathbf{z}}$ by solving equation (37) and then d and $d^\star$ as described in equation (38). The solution method for computing $e_{\mathbf{z}}$ is to regress one on $\mathbf{Z}$. Armed with $y_{\mathbf{z}}$, η, d, and $d^\star$, we compute y and $y^\star$ using equation (31). We verify our solution and check if $\mathbb{E}[\eta\mathbf{Z}] = 0$, for each element of $\mathbf{Z}$. As in Subsection 2.2, we verify that $\mathbb{E}[y - y^\star]$ is not far from zero.

With the computed values of y and $y^\star$, we obtain m and $m^\star$ across the five states using equation (8). Prompted by the specifics of our solution, we compute (i) the risk sharing index in equation (26), (ii) $\mathbb{E}[m]$ and $\mathbb{E}[m^\star]$, and (iii) $\sqrt{\mathrm{Var}[m]}$ and $\sqrt{\mathrm{Var}[m^\star]}$.

Comparison of the risk sharing index in equation (26) with the risk sharing index if the Appraisal Ratio Θ were to be assumed close to zero shows how much the risk sharing index is lowered by ruling out "good deals," that is, $\mathbb{E}[(y - y^\star)^2] \leq \Theta^2$ in comparison with $\mathbb{E}[(y - y^\star)^2] \approx 0$. As seen, the risk sharing index is lowered but with the added effect of slightly raising the volatility of m and $m^\star$. We check our solution by directly minimizing the objective in (27), subject to the constraints in (28)–(30).

In summary, we construct a discrete-time, five-state economy where $m\left(e_{t+1}/e_t\right) - m^\star$ need not be zero and the domestic and foreign country pairs have high and low risk-free interest-rates, respectively. We show that the nonnegativity constraints do not bind, and m and $m^\star$ are strictly

positive in each state. Moreover, increasing Θ lowers risk sharing by reducing the covariance between m and $m^\star$, while slightly increasing the volatility of m and $m^\star$, possibly exposing a source of tension between the attributes of m and $m^\star$ in the international economy. ♣

II Excess return to borrowing in domestic currency and investing in foreign currency and its relation to the risk sharing index

In what follows, we do not constrain $\text{Var}_t\left[\ln(m) + \ln\left(\frac{e_{t+1}}{e_t}\right) - \ln(m^\star)\right]$ to equal zero.

However, we do assume, but *only within this appendix*, first, that m and e_{t+1}/e_t are jointly log-normally distributed. Using the joint moments formula of the bivariate log-normal distribution (e.g., Balakrishnan and Lai (2009, equation (11.69), page 526)), we can express $\mathbb{E}_t[m\,(e_{t+1}/e_t)\,R_f^\star] = 1$ as:

$$\ln(R_f^\star) - \ln(R_f) + \mathbb{E}_t\left[\ln\left(\frac{e_{t+1}}{e_t}\right)\right] = -\text{Cov}_t\left[\ln(m), \ln\left(\frac{e_{t+1}}{e_t}\right)\right] - \frac{1}{2}\text{Var}_t\left[\ln\left(\frac{e_{t+1}}{e_t}\right)\right], \quad \text{(B1)}$$

where we use a subscript t to denote time t conditional expectation.

We recognize $\ln(R_f^\star) - \ln(R_f) + \mathbb{E}_t\left[\ln\left(\frac{e_{t+1}}{e_t}\right)\right]$ is the expected (log) excess return to a trade, which borrows one unit of domestic currency at time t, sells it for foreign currency, holds the foreign currency until time $t+1$, and then liquidates the position. The next step is to use the identity:

$$\begin{aligned}
\text{Var}_t[\ln(m) - \ln(m^\star) + \ln\left(\frac{e_{t+1}}{e_t}\right)] &= 2\text{Cov}_t\left[\ln(m), \ln\left(\frac{e_{t+1}}{e_t}\right)\right] - 2\text{Cov}_t\left[\ln(m^\star), \ln\left(\frac{e_{t+1}}{e_t}\right)\right] \\
&\quad + \text{Var}_t[\ln(m) - \ln(m^\star)] + \text{Var}_t\left[\ln\left(\frac{e_{t+1}}{e_t}\right)\right]. \quad \text{(B2)}
\end{aligned}$$

Rearranging equation (B2), we then have

$$\begin{aligned}
-\frac{1}{2}\text{Cov}_t\left[\ln(m), \ln\left(\frac{e_{t+1}}{e_t}\right)\right] &= -\frac{1}{2}\text{Cov}_t\left[\ln(m^\star), \ln\left(\frac{e_{t+1}}{e_t}\right)\right] - \frac{1}{4}\text{Var}_t\left[\ln(m) - \ln(m^\star) + \ln\left(\frac{e_{t+1}}{e_t}\right)\right] \\
&\quad + \frac{1}{4}\left\{\text{Var}_t[\ln(m)] + \frac{1}{4}\text{Var}_t[\ln(m^\star)] - 2\text{Cov}_t[\ln(m), \ln(m^\star)]\right\} \\
&\quad + \frac{1}{4}\text{Var}_t\left[\ln\left(\frac{e_{t+1}}{e_t}\right)\right]. \quad \text{(B3)}
\end{aligned}$$

2

Writing equation (B1) as:

$$\ln(R_f^\star) - \ln(R_f) + \mathbb{E}_t\left[\ln\left(\frac{e_{t+1}}{e_t}\right)\right] = -\frac{1}{2}\text{Cov}_t\left[\ln(m), \ln\left(\frac{e_{t+1}}{e_t}\right)\right] - \frac{1}{2}\text{Cov}_t\left[\ln(m), \ln\left(\frac{e_{t+1}}{e_t}\right)\right]$$

$$-\frac{1}{2}\text{Var}_t\left[\ln\left(\frac{e_{t+1}}{e_t}\right)\right]. \tag{B4}$$

Or, substituting the expression for (B3) into equation (B4), the analog for the excess returns in an incomplete markets economy is as presented in equation (40). ∎

Next, to go from $\text{Cov}_t[\ln(m), \ln(m^\star)]$ to $\text{Cov}_t[m, m^\star]$, we assume that m and m^* are jointly log-normally distributed. Exploiting the formula for joint moments (e.g., Balakrishnan and Lai (2009, equation (11.69), page 526)), we obtain

$$\text{Cov}_t[m, m^\star] = \mathbb{E}_t[m\, m^\star] - \frac{1}{R_f\, R_f^\star}, \tag{B5}$$

$$= \exp\left(-\ln(R_f) - \ln(R_f^\star) + \text{Cov}_t[\ln(m), \ln(m^\star)]\right) - \frac{1}{R_f\, R_f^\star}, \tag{B6}$$

$$= \frac{1}{R_f\, R_f^\star}\left\{\exp\left(\text{Cov}_t[\ln(m), \ln(m^\star)]\right) - 1\right\}, \tag{B7}$$

$$\approx \frac{1}{R_f\, R_f^\star}\text{Cov}_t[\ln(m), \ln(m^\star)], \qquad (\text{using } e^a \approx 1 + a) \tag{B8}$$

Going from conditional to unconditional expectations, $\text{Cov}[\ln(m), \ln(m^\star)]$ is, loosely speaking, proportional to RSI in equation (26). This possibly suggests the following: If the domestic currency is Japanese yen and the foreign currency is Australian dollar, then *ceteris paribus* the expected excess return to borrowing in Japanese yen and investing in Australian dollar is higher when the risk sharing index is lower. ♣

Table Internet-I: **Properties of m and m^* in an example economy with five states and where $m\left(e_{t+1}/e_t\right) - m^\star \neq 0$**

In our illustrative calculations, the domestic country is Japan (with a low risk-free interest-rate), and the foreign country is Australia (with a high risk-free interest-rate). The exchange rate growth e_{t+1}/e_t is denominated in ¥|AD. We follow the steps in Subsection 2.4 and compute $e_{\mathbf{z}}$ by solving equation (37) and then d and $d^\star$ via equation (38). The computer code for obtaining the solution by minimizing the objective in (27), subject to the constraints in equations (28)–(30), is available from the authors (in C++ and in an Excel spreadsheet setting). CORR$[m, m^*]$ is the correlation and RSI is the risk sharing index.

Panel A: Parametrization of the economy

	States of the world				
	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$
Probability	0.07	0.20	0.45	0.21	0.07
Risk-free (domestic)	1.01	1.01	1.01	1.01	1.01
Risk-free (foreign)	1.04	1.04	1.04	1.04	1.04
Risky (domestic)	0.87	1.01	1.04	1.05	1.08
Risky (foreign)	0.87	1.01	1.08	1.04	1.42
Exchange rate growth	0.90	0.89	1.00	1.11	1.10

Panel B: Properties of m and m^* obtained by varying Θ

	States of the world					Mean	Std.	CORR$[m, m^*]$	RSI
	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$				
$\Theta = 0.05$									
m	1.9887	1.3949	0.9410	0.5724	0.4036	0.9901	0.4054		
$m^\star$	1.7763	1.2975	0.8889	0.6841	0.4858	0.9615	0.3216		
								0.9941	0.9680
$\Theta = 0.40$									
m	2.0415	1.1745	1.1232	0.4188	0.2706	0.9901	0.4438		
$m^\star$	1.7288	1.4937	0.7068	0.8547	0.6321	0.9615	0.3702		
								0.5493	0.5404
$\Theta = 0.60$									
m	2.0716	1.0485	1.2273	0.3309	0.1946	0.9901	0.4934		
$m^\star$	1.7017	1.6058	0.6027	0.9522	0.7157	0.9615	0.4285		
								0.1904	0.1885
$\Theta = 0.80$									
m	2.1017	0.9225	1.3314	0.2431	0.1186	0.9901	0.5569		
$m^\star$	1.6746	1.7179	0.4986	1.0496	0.7993	0.9615	0.4999		
								-0.1068	-0.1062

Table Internet-II: **Summary statistics of interest-rates, excess equity returns, and inflation**

We report the summary statistics for interest-rates, excess equity returns, and inflation over the sample period of January 1975 to June 2014. The reported numbers are in annual percentage units, except the Sharpe ratios which are reported as annualized decimals. There are ten countries in our sample: Australia (AUD), New Zealand (NZD), United Kingdom (STG), France (FRA), Canada (CAD), United States (USD), Netherlands (NLG), Germany (GER), Japan (JPY), and Switzerland (SWI). Panel A reports the mean nominal annualized interest-rates, and the results are displayed in the order of decreasing average interest-rates (so Australia (AUD) has the highest average interest-rates, while Switzerland (SWI) has the lowest average interest-rates). Panel B reports the properties of average excess equity returns, computed as the real equity return over and above the real interest-rate and in the local currency. Finally, Panel C reports the properties of the CPI inflation. Because some of the data is unavailable for the United Kingdom and France at the beginning of the sample, we use Retail Price Index inflation data.

Currency	AUD	NZD	STG	FRA	CAD	USD	NLG	GER	JPY	SWI
Panel A: Average one-month money market interest-rates										
Mean interest-rate	8.83	8.63	7.86	7.10	6.71	5.86	4.99	4.58	3.14	2.99
Panel B: Excess equity returns										
Mean	5.85	3.65	7.94	6.72	5.47	6.93	8.99	6.70	4.20	8.04
Standard deviation	17.90	19.12	19.16	20.28	16.96	15.77	18.26	19.98	18.66	16.14
Sharpe ratio	0.33	0.19	0.41	0.33	0.32	0.44	0.49	0.34	0.23	0.50
Panel C: CPI inflation										
Mean	5.03	5.93	5.08	4.00	3.92	3.92	2.84	2.36	1.67	1.89
Standard deviation	1.23	1.74	2.18	1.27	1.45	1.11	1.51	1.59	1.79	1.25

Table Internet-III: **Appraisal Ratios for a strategy in which the domestic investor invests in the foreign equity index**

The table reports Appraisal Ratios for a strategy whereby the domestic investor invests in the foreign equity index. The return from the strategy is

$$r_E \;\equiv\; \left(\frac{e_{t+1}}{e_t}\right) R^\star_{\text{equity}} \;-\; R_{\text{equity}},$$

where R_{equity} and $R^\star_{\text{equity}}$, respectively, are the domestic and foreign equity index returns, measured in their own currencies. The Appraisal Ratio of this strategy is computed using the domestic equity index as the benchmark:

$$\frac{\mathbb{E}[r_E]}{\sqrt{\text{Var}\,[r_E]}}.$$

To clarify the reported Appraisal Ratios below, the currency of the domestic investor is specified by the rows, while the currency of the foreign investor is specified by the first column. As an example of how to read the entries in the table, 0.26 is the Appraisal Ratio for a Japanese (JPY) investor investing in the Netherlands (NLG) equity index. The sample period is January 1975 to June 2014, and there are ten countries in our sample: Australia (AUD), New Zealand (NZD), United Kingdom (STG), France (FRA), Canada (CAD), United States (USD), Netherlands (NLG), Germany (GER), Japan (JPY), and Switzerland (SWI). The Appraisal ratios are reported as annualized decimals.

	Domestic investor									
	AUD	NZD	STG	FRA	CAD	USD	NLG	GER	JPY	SWI
Foreign										
AUD	0.00	0.14	-0.01	0.06	0.13	0.10	-0.01	0.10	0.20	0.06
NZD	-0.10	0.00	-0.12	-0.06	-0.01	-0.03	-0.13	-0.02	0.09	-0.07
STG	0.08	0.17	0.00	0.09	0.17	0.15	0.01	0.13	0.24	0.10
FRA	0.01	0.11	-0.06	0.00	0.09	0.07	-0.10	0.06	0.18	0.01
CAD	-0.08	0.06	-0.12	-0.04	0.00	-0.01	-0.14	0.01	0.13	-0.04
USD	-0.04	0.09	-0.10	-0.02	0.05	0.00	-0.14	0.03	0.16	-0.02
NLG	0.08	0.19	0.03	0.10	0.19	0.19	0.00	0.18	0.26	0.12
GER	-0.03	0.07	-0.10	-0.05	0.04	0.02	-0.18	0.00	0.14	-0.05
JPY	-0.12	-0.02	-0.18	-0.12	-0.06	-0.09	-0.21	-0.08	0.00	-0.15
SWI	0.02	0.13	-0.05	0.01	0.11	0.09	-0.10	0.07	0.20	0.00

Table Internet-IV: **Summary statistics for real consumption growth**

We report the summary statistics for real consumption growth (annualized and in percentage units). AR_1 represents the first-order autocorrelation. We also report the correlation between country consumption growth and country real equity returns, and between country consumption growth and OECD consumption growth. The consumption growth data is from Barro and Ursua (2008), updated using World Development Indicators. The data on OECD consumption growth is from Datastream. There are ten countries in our sample: Australia (AUD), New Zealand (NZD), United Kingdom (STG), France (FRA), Canada (CAD), United States (USD), Netherlands (NLG), Germany (GER), Japan (JPY), and Switzerland (SWI).

	AUD	NZD	STG	FRA	CAD	USD	NLG	GER	JPY	SWI
Mean consumption growth	1.87	0.96	2.27	1.45	1.62	2.06	1.12	1.74	1.91	0.89
Standard deviation	1.48	2.45	2.61	1.25	1.64	1.70	1.96	1.44	1.59	1.12
AR_1	0.06	0.42	0.57	0.39	0.44	0.53	0.69	0.69	0.50	0.38
Correlation with real equity returns	0.16	0.36	0.30	0.26	0.46	0.51	0.38	0.29	0.47	0.21
Correlation with OECD consumption growth	0.23	0.24	0.77	0.64	0.62	0.88	0.65	0.45	0.49	0.47