Identifying Collusion in English Auctions

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Introduction

- Collusion: Practical concern.
- Econometrics: Can we identify/detect collusive behavior from data?
- Literature:
  - Timber: Baldwin, Marschall and Richard (JPE, 1997). **English auctions.**
  - Highway procurement: Porter and Zona (JPE, 1993), Bajari and Ye (Restat, 1993), Pesendorfer (Restud, 2000). First-price sealed-bid auctions.
  - School milk: Porter and Zona (Rand, 1999). First-price sealed-bid auctions.

Contribution: In English Auctions with Independent Private Values

- Random censoring methods can be used to identify latent distribution of values under competition or collusion.
- Framework with asymmetric bidders.
- Fully nonparametric identification and estimation procedure.
- Bidder-by-bidder test of collusion (non-competitive behavior).
- Valid bootstrap procedure (with validity established via the Extended Functional Delta Method).
- Application to Internet GIC auctions.

Collusive behavior is identified when:

- Data on many independent auctions with the same participants.
- There is *at least one known* competitive bidder.
- *Efficient Cartel*: A cartel member with the highest value (Leader) faces *no competition from other cartel members.*
- Every cartel member gets to be the leader with positive probability.
- Cartel can lose with positive probability.

Framework

- N bidders participate in multiple independent auctions.
- Independent private values.
- Heterogeneous distributions of values: V_i is the value of bidder i drawn from the CDF $F_i(\cdot)$, pdf $f_i(\cdot)$, $i = 1, \ldots, N$.
- The support of f_i is $[0, \bar{v}]$, common.
- $w_i = 1$ if bidder i wins the auction; $w_i = 0$ otherwise.

Competition and censoring

- Consider bidder i (everything is done bidder-by-bidder).
- The maximum value for the rivals of bidder i:
  $V_{-i} = \max_{j \neq i} V_j$.
- **Under competition,** the bid of bidder i: $B_i = \min\{V_i, V_{-i}\}$.
- $B_i = V_i$ only if $w_i = 0$ (losing bid) $\Longrightarrow$ Random Censoring from above by V_{-i}.
- Let $F_{-i}(v)$ denote the CDF of V_{-i}.

- "Observable" distribution: The CDF of V_i (or B_i) conditional on losing: $G_i(v|w_i = 0)$, with the PDF:

$$g_i(v|w_i = 0) = \frac{f_i(v)\,(1 - F_{-i}(v))}{P\,(w_i = 0)}.$$

- Re-arrange, and let $G_i(\cdot)$ denote the CDF of B_i (unconditional):

$$\frac{f_i(v)}{1 - F_i(v)} = \frac{g_i(v|w_i = 0)P(w_i = 0)}{(1 - F_i(v))(1 - F_{-i}(v))} = \frac{g_i(v|w_i = 0)P(w_i = 0)}{1 - G_i(v)}.$$

- Solution: De-censored CDF of values:

$$F_i(v) = 1 - \exp\left(-\int_0^v \frac{dG_i(u|w_i = 0)}{1 - G_i(u)} P(w_i = 0)\right).$$

Efficient cartel (a simple example)

- Three bidders: V_1, V_2, V_3.
- Suppose $V_3 > V_2 > V_1$.
  - Under full competition, 3 wins with $B_3 = V_2 + \epsilon$.
  - If 1 is competitive, 2 & 3 are an efficient cartel, 3 wins with $B_3 = V_1 + \epsilon < V_2$.
  - Bidder 2 (cartel member) loses, but bids only $B_2 = V_1 < V_2$ (does not reveal his value).
- Suppose $V_1 > V_3 > V_2$, and 2 & 3 are a cartel:
  - Bidder 1 wins.
  - Cartel leader loses with $B_3 = V_3$ and reveals his true value.
- Cartel leader always bids competitively!

De-censoring under collusion

- Bidders $i \in \mathcal{N}_{com}$ are competitive. Assume there is at least one! Let
$$V^*_{com} = \max_{j \in \mathcal{N}_{com}} V_j.$$
- $i \in \mathcal{N}_{col}$ are (suspect) cartel members. Cartel's leader value:
$$V^*_{col} = \max_{j \in \mathcal{N}_{col}} V_j.$$
- Efficient cartel: Cartel members do not compete with each other. For $i \in \mathcal{N}_{col}$,
$$B_i = \min\{V_i, V^*_{com}\}.$$
- The leaders V^*_{col} and V^*_{com} are two competitive bidders $\Longrightarrow$ Their CDFs $F^*_{col}(\cdot)$ and $F^*_{com}(\cdot)$ are identified!

- New variable: Let $\ell_i = 1$ if bidder i is cartel's leader.
- Pick a suspect $i \in \mathcal{N}_{col}$.
- The PDF of V_i conditional on $\ell_i = 1$ & $w_i = 0$ (leader & lost):

$$g_i(v|w_i = 0, \ell_i = 1) = \frac{f_i(v)F_{-i}^{col}(v)(1 - F_{com}^*(v))}{P(w_i = 0, \ell_i = 1)},$$

where $F_{-i}^{col}(\cdot)$ is the CDF of $\max_{j \in \mathcal{N}_{col}, j \neq i} V_j$.

De-censoring for cartel members

$$g_i(v|w_i = 0, \ell_i = 1) = \frac{f_i(v) F_{-i}^{col}(v)(1 - F_{com}^*(v))}{P(w_i = 0, \ell_i = 1)}$$
$$= \frac{f_i(v)}{F_i(v)} \frac{F_{col}^*(v)(1 - F_{com}^*(v))}{P(w_i = 0, \ell_i = 1)}.$$

- or

$$\frac{f_i(v)}{F_i(v)} = \frac{d \log F_i(v)}{dv} = \frac{dG_i(v|w_i = 0, \ell_i = 1)}{F_{col}^*(v)(1 - F_{com}^*(v))} P(w_i = 0, \ell_i = 1).$$

- Solution:

$$F_i(v) = \exp\left(- \int_v^{\bar{v}} \frac{dG_i(u|w_i = 0, \ell_i = 1)}{F_{col}^*(u)(1 - F_{com}^*(u))} P(w_i = 0, \ell_i = 1) \right).$$

Identifying collusive (non-competitive) behavior

- The counterfactual CDF of bids for bidder i if he is competitive:

$$C_i(v) = P(\min\{V_i, V_{-i}\} \leq v) = 1 - (1 - F_i(v))(1 - F_{-i}(v)).$$

- The observed CDF of bids for bidder i: $G_i(\cdot)$ (from data).
- **Under competition:** $C_i(v) = G_i(v)$ for all v.
- **Under collusion:** Cartel's member bids $\min\{V_i, V^*_{com}\} \leq \min\{V_i, V_{-i}\} \Longrightarrow$

$$\begin{aligned} C_i(v) &\leq G_i(v) \text{ for all } v, \\ C_i(v) &< G_i(v) \text{ for some } v. \end{aligned}$$

Test if bidder i's behavior is non-competitive

- $\hat{G}_{i,L}(\cdot)$ and $\hat{C}_{i,L}(\cdot)$: the observed (empirical) and estimated counterfactual CDFs of bids for bidder i from data on L auctions.
- Take differences: $\hat{\Delta}_{i,L}(\cdot) = \hat{G}_{i,L}(\cdot) - \hat{C}_{i,L}(\cdot)$.
- **Statistic:** Let $[x]_+ = \max\{0, x\}$. Kolmogorov-Smirnov-type:

$$KS_{i,L} = \sup_v \sqrt{L}\left[\hat{\Delta}_{i,L}(v)\right]_+.$$

Nonparametric bootstrap

- Draw a bootstrap sample (by sampling entire auctions).
- $\hat{\Delta}^{\dagger}_{i,L,m}(\cdot)$ is the analogue of $\hat{\Delta}_{i,L}(\cdot)$ for bootstrap sample $m = 1, \ldots, M$.
- $KS^{\dagger}_{i,L,m} = \sup_v \sqrt{L}\left[\hat{\Delta}^{\dagger}_{i,L,m}(v) - \hat{\Delta}_{i,L}(v)\right]_+$.
- Crit. val.'s $(KS^{\dagger}_{i,L,1-\alpha})$: the $(1-\alpha)$-th sample quantile of $\{KS^{\dagger}_{i,L,m} : m = 1, \ldots, M\}$.
- Reject H_0 that bidder i is competitive if

$$KS_{i,L} > KS^{\dagger}_{i,L,1-\alpha}.$$

Validity of the Bootstrap: Quantile transformation

- Quantile transformation:

$$
\begin{aligned}
t &= G_i(v), \quad t \in [0,1] \\
S_i(t) &= F_i(G_i^{-1}(t)), \\
\mu_i(t) &= G_i(G_i^{-1}(t), w_i = 0).
\end{aligned}
$$

- $S_i(t)$ is the "same" as $F_i(v)$, except we now consider $t \in [0,1]$.
- De-censoring formula for competition:

$$
S_i(t) = 1 - \exp\left(-\int_0^t \frac{d\mu_i(\tau)}{1-\tau}\right).
$$

- Hadamard differentiability fails because of $1/(1-\tau)$: the linearization error in the Functional Delta Method explodes as $t \to 1$.

Validity of the Bootstrap: Trimming

- Choose a trimming parameter $t_L \to 1$ (we need the entire support): $1 - t_L = L^{-\beta} \quad$ for $1/2 < \beta < 3/4$.
- Trimmed estimator:

$$\tilde{S}_{i,L}(t) = \hat{S}_i(t \wedge t_L) = 1 - \exp\left(\int_0^{t \wedge t_L} \frac{d\hat{\mu}_i(\tau)}{1-\tau}\right).$$

- The error due to trimming: For $t > t_L$ and since $S_i(t)$ is differentiable,

$$\sqrt{L}(S_i(t) - S_i(t \wedge t_L)) \sim \sqrt{L}(1 - t_L).$$

$\beta > 1/2 \implies \sqrt{L}(1 - t_L) \to 0$ (no asymptotic bias).

- We show that $\beta < 3/4$ is sufficient to control the uniform rate of the linearization error.

Validity of the Bootstrap: Extended Functional Delta Method

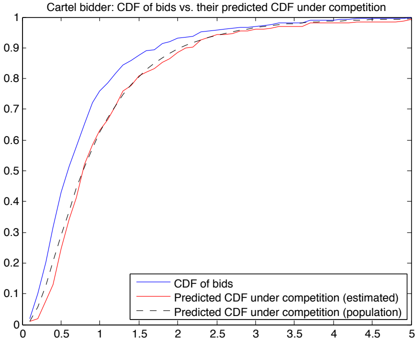
Let $\phi_L(\mu)$ be the trimmed functional: $\phi_L(\mu_i) = S_i(t \wedge t_L)$. Let $\|\psi\| \equiv \sup_t |\psi(t)|$. Suppose that as $\delta_L \to 0$:

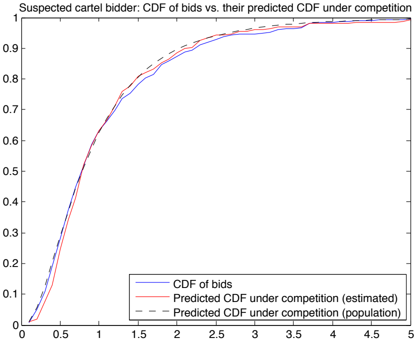
- Linearization error is controlled uniformly: $\sup_{h_L} \left\| \frac{\phi_L(\mu+\delta_L h_L)-\phi_L(\mu)}{\delta_L} - \phi'_{\mu,L}(h_L) \right\| \to 0$.
- No asy bias due to trimming: $\delta_L^{-1} \|\phi_L(\mu) - \phi(\mu)\| \to 0$.
- Sample-size-dependent derivative converges: $\|\phi'_{\mu,L}(h_L) - \phi'_\mu(h)\| \to 0$ for all $\|h_L - h\| \to 0$.
- Weak convergence of the empirical and bootstrap processes: $\delta_L^{-1}(\hat{\mu}_L - \mu) \rightsquigarrow \mathbb{M}$ and $\delta_L^{-1}(\hat{\mu}_L^\dagger - \hat{\mu}_L) \rightsquigarrow \mathbb{M}$.

$\Longrightarrow$

$$\delta_L^{-1}(\phi_L(\hat{\mu}_L) - \phi(\mu)) \rightsquigarrow \phi'_\mu(\mathbb{M}) \text{ (Estimator)}.$$
$$\delta_L^{-1}(\phi_L(\hat{\mu}_L^\dagger) - \phi(\hat{\mu}_L)) \rightsquigarrow \phi'_\mu(\mathbb{M}) \text{ (Bootstrap)}.$$

Monte Carlo: Cartel (left) and Competition (right), $N = 3$, $L = 400$.





Monte Carlo: Average rejection rates of the bootstrap test

significance level	$L = 100$	$L = 400$	$L = 100$	$L = 400$
	Competition (H_0)		Collusion (H_1)	
0.01	0.009	0.008	0.403	0.934
0.05	0.030	0.043	0.626	0.981
0.10	0.067	0.080	0.732	0.994

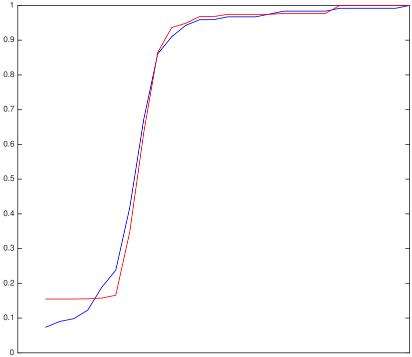
Empirical Application: Internet Guarantee Investment Certificates (GIC) Auctions

- Municipalities raise funds through bonds and invest them by soliciting bids from banks.
- Traditionally, they employed brokers. There were numerous legal cases of collusion involving brokers and banks.
- Internet GIC auctions - no brokers involved.
- Ascending-bid, closed-exit: Participants observe the current status of their bid (winning or losing), and can increase it at any time during the auction. Other bidders do not know the identity of the current winner.

Internet GIC Auctions

- Among the bidders:
  - AEGON has never been implicated in bid rigging.
  - Rabobank has been implicated (along with many other banks in the sample) in bid rigging in regular GIC auctions
- We test if Rabobank is competitive in internet GIC auctions.

Blue = the CDF of bids
Red = the predicted CDF of bids under competition



H_0 of competitive behavior cannot be rejected (the p-value is 0.26)