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Stabilising house prices: the role of housing futures trading

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Abstract

This study investigates the effects of housing futures trading on housing demand, house price volatility and housing bubbles in a theoretical framework. The baseline model is an application of the De Long, Shleifer, Summers and Waldmann (1990) model of noise traders to the housing market, when the risky asset is housing. This adds new features to the model as households receive utility from housing services and cannot short-sell houses. The existence of noise traders in the housing market creates uncertainty about house prices, causes prices to deviate away from their fundamental values, and leads to a distortion in housing consumption. To investigate the impact of housing derivatives trading on the housing market, a new financial instrument, housing futures, is introduced into the baseline model. Housing futures trading affects house price stability through three channels: by (i) enabling households to disentangle their housing consumption decisions from investment decisions; (ii) allowing short-selling; and (iii) attracting an additional set of traders (pure speculators) looking for portfolio diversification opportunities. The results show that, for a large set of admissible parameter values, housing futures trading decreases the volatility of house prices and increases the welfare of households and investors when noise trader (sophisticated) households are always relatively optimistic (pessimistic), and the share of pure speculators that are sophisticated is higher than the share of households that are sophisticated.

Key words: Housing derivatives market, speculation, house price volatility, short-selling, noise traders.

JEL classification: G13, R21.

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1 Introduction

Housing is one of the largest asset classes in the world. However, the housing market is imperfect due to its illiquid and lumpy nature, high transaction costs, short-sale constraints and the absence of financial instruments that would permit investors to hedge their exposure to house price risk. Moreover, housing has the dual role of providing a flow of consumption services and being an investment asset. When households' optimal consumption and investment decisions do not coincide, either their housing consumption or investment choices become distorted.

This paper studies how the imperfections and distortions in the housing market can be addressed by introducing financial instruments, in particular house price index derivatives. These financial instruments address the imperfections by allowing investors to gain exposure to house price returns through investing incrementally with low transaction costs, and giving them scope to short-sell in a more liquid market, and by permitting property and real estate developers, banks, mortgage lenders, home suppliers and homeowners to hedge their exposure to house price risk. These instruments also address the distortions by enabling households to separate their housing consumption decisions from housing investment decisions (Englund, 2010).

In contrast to other major asset classes, including stocks and bonds, housing does not have well-developed derivatives markets. Although in the last two decades there have been several initiatives to launch house price index derivatives markets, and various derivative products - futures/forwards, options, swaps and structured notes - have been developed and traded most notably in the United Kingdom and the United States, housing derivatives trading is still at a nascent stage. The main barriers to growth in these markets are a lack of sufficient pricing models, a lack of liquidity, a lack of a secondary market, a lack of education and acceptance, and legislative uncertainties and impediments over the treatment of housing derivatives. However, this paper argues that well-developed housing derivatives markets can be effective in solving the imperfections and distortions in the housing market. Furthermore, they can play a role in stabilising house prices and dealing with housing bubbles.

By exploring the role of housing derivatives in separating housing consumption decisions from investment decisions, and solving the imperfections in the housing market by facilitating short-selling, hedging and speculation, in a theoretical framework this study investigates the effects of housing derivatives trading on housing demand, house price volatility and housing bubbles.

The baseline model is an application of the De Long, Shleifer, Summers, and Waldmann (1990) model to the housing market. This study employs a two-period overlapping generations model. There is no consumption in the first period, no labour supply decision and there is no bequest. There are two types of assets in the economy: a risk-free asset and housing. The supply of housing is fixed. Housing cannot be sold short[1], and its price fluctuates over time. Households receive utility from housing services, which are assumed to be available in either of two mutually exclusive ways, renting or owning. To capture the imperfect substitution between owner-occupying and renting, as a modeling device a difference in the maintenance cost of rental-occupied and owner-occupied housing is assumed.[2]

Two types of agents are present in the baseline model: sophisticated households and noise trader households. While young sophisticated households in the first period accurately perceive the next period expected price of housing, young noise trader households misperceive the expected house price. Noise traders' misperception enters into the model as an i.i.d. uniform (common) random variable. The only source of uncertainty in the model is the size of noise traders' misperceptions, which changes stochastically between generations. The existence of noise traders in the housing market creates uncertainty in house prices, causes prices to deviate away from their fundamental values, and leads to a distortion in housing consumption.

To investigate the impact of housing derivatives trading on the housing market, a new financial instrument, housing futures, is introduced into the baseline model. In the first period, young households invest in the risk-free asset and housing, and also trade housing futures contracts among themselves. Housing futures are in zero net supply and settled in cash. Relatively optimistic households take long positions in housing futures, and relatively pessimistic households take short positions. At maturity, if the realised house price is less (more) than the futures price set in the contract, households with long (short) positions in housing futures pay the price difference to households who have short (long) positions in housing futures.

[1] In financial markets, short-selling is defined as the sale of a security or financial instrument which is not currently owned. However, this practice is not possible in the physical housing market.

[2] Although the rental market enables households to separate their housing investment decisions from their housing consumption decisions, services from owner-occupied housing and rental housing are imperfect substitutes. Most households express a strong preference for owning rather than renting, which makes it difficult to disentangle investment and consumption decisions.

Housing futures trading eliminates distortions in households' housing consumption decisions by separating the price dynamics of owner-occupied homes from the housing services they contain and allowing speculation about house prices in the futures market. Moreover, by allowing short-selling housing derivatives trading enables relatively pessimistic households to participate in the housing market.

The introduction of the housing futures market can attract an additional set of traders, institutional investors such as hedge funds, pension funds and insurers, that use the markets to diversify their portfolios. They may also strengthen the presence of speculative trading in the futures market. To capture this, I incorporate into the model pure speculators, who do not invest in the housing market but can trade housing futures.

Housing derivatives trading affects the level and volatility of house prices through three mechanisms. The first mechanism is related to enabling households to disentangle their housing consumption decisions from housing investment decisions. The second mechanism is the short-selling opportunity provided by housing futures trading. The third mechanism is related to attracting pure speculators looking for portfolio diversification opportunities. The analysis shows that, for a large set of admissible parameter values, housing futures trading stabilises house prices and increases the welfare of households and investors. The key assumptions behind these results are that noise trader (sophisticated) households are always relatively optimistic (pessimistic), and the share of pure speculators that are more sophisticated is higher than the share of households that are sophisticated.

The contributions of this paper are fourfold. First, this paper adapts the De Long et al. (1990) model in order to study the housing market, in doing so incorporating a short-selling constraint and utility derived from housing services. The modified model shows that the existence of noise traders creates fluctuations in house prices directly and also indirectly through the rental market by changing participation in the housing market. Second, whereas in the literature the impact of the derivatives market on the underlying spot market is mainly investigated through the effects of speculators, this study presents two other mechanisms - the rental market and the short-selling constraint - through which housing futures trading can affect house price volatility in the absence of pure speculators. Third, the study provides a theoretical framework with which the potential benefits of house price index derivatives can be analysed. It demonstrates how the housing derivatives market can solve the imperfections in the housing market, stabilise house prices and increase the welfare of society. Fourth, this paper contributes to the debate on how to deal with house price bubbles by analysing the conditions under which housing derivatives trading decreases housing bubbles.

Related Literature

There are three strands of literature related to this study.

The first is the literature on housing and house price index derivatives. In recent years, there is a growing body of literature focusing on the impact of house price risk on housing choices. The literature on hedging house price risk and house price index derivatives is, however, limited.[3] In the last two decades, several papers have supported the introduction of housing derivatives markets by demonstrating their potential benefits (Case and Shiller, 1989; Case, Shiller, and Weiss, 1993; Shiller, 2008). Empirical evidence from Sweden (Englund, Hwang, and Quigley, 2002), the United Kingdom (Iacoviello and Ortalo-Magne, 2003), and the United States (Bertus, Hollans, and Swidler, 2008) suggests that households could benefit from a well-functioning housing derivatives markets.

A second strand of literature is concerned with the impact of derivatives market on the underlying markets. As a result of the strong growth of futures and options markets since the 1970s, the effect of derivatives trading on the volatility of underlying spot markets has received considerable attention from practitioners, regulators and academics. Nevertheless, it is still an open question as to what impact derivatives trading has. Existing theoretical models make ambiguous predictions about the effects of derivatives markets: while some predict that derivatives trading should have a stabilising effect, others reach the opposite conclusion.[4]

The third relevant strand of the literature is related to asset price bubbles. House price booms have been experienced in a number of countries historically and the latest boom, which started in the late 1990s, has affected many countries[5] in the world (Shiller, 2007). Since the aim of this study is

[3] To my knowledge, there are only two theoretical papers (Voicu and Seiler, 2013; De Jong, Driessen, and Van Hemert, 2008) that study the role of housing futures in hedging house price risk. However, these two papers do not discuss the effects of the introduction of the housing derivatives market on house prices and its role in solving the imperfections and distortions in the housing market.

[4] Derivatives markets may reduce spot volatility by supporting price discovery and transferring risk (Kawai, 1983; Turnovsky, 1983; Sarris, 1984; Demers and Demers, 1989). On the other hand, it has been argued that derivatives markets may destabilise spot markets by attracting uninformed speculative investors through the higher degree of leverage, low transaction costs and low margins (Danthine, 1978; Stein, 1987; Newbery, 1987; Chari et al., 1990).

[5] Australia, Canada, China, France, Ireland, Italy, New Zealand, Norway, Russia, South Africa, Spain, the United Kingdom and the United States, among other countries.

to investigate the effect of derivatives trading on house prices, considering bubbles in house prices is inevitable. In the asset price bubbles literature, there are mainly two classes of models, *rational versus irrational*, that derive conditions under which bubbles can exist.[6] In this analysis, the second class of models is followed.

The rest of the paper is organised as follows. Section 2 builds the baseline model without housing futures trading. Section 3 introduces the housing futures market, and Section 4 incorporates pure speculators into the model. The effects of housing futures trading on housing demand and house price volatility are analysed in Section 3 and Section 4. Section 5 presents a numerical example and Section 6 concludes. Some of the proofs and extensions are presented in the Appendix.

2 Noise Trader Risk in the Housing Market

This section presents the baseline model, which is an application of the De Long et al. (1990) model to the housing market. The risky asset is housing in this application. This adds new features to the model as households receive utility from housing services and cannot short-sell houses. The existence of noise traders in the housing market creates uncertainty about house prices and causes prices to deviate away from their fundamental values. Moreover, heterogeneity in beliefs leads to a distortion in households' housing consumption decisions.

2.1 The Model Set-up

This study adopts a two-period overlapping generation model. There is no consumption in the first period, no labour supply decision and there is no bequest motive. There are two types of assets in the economy: housing, h and a risk free asset, s. Housing is homogeneous in terms of quality, its supply is fixed and normalised to one. Its price is denoted by p, which fluctuates over time.[7] On the other hand, the risk-free asset is in perfectly elastic supply, pays a fixed real dividend, r, and its price is normalised to one.

[6] The first class of models assumes that all investors have rational expectations: they can either have identical information (Samuelson (1958); Blanchard and Watson (1982); Tirole (1985); Santos and Woodford (1997); Martin and Ventura (2012)) or be asymmetrically informed (Allen and Gorton (1993); Allen, Morris, and Postlewaite (1993)). In the second class of models, bubbles can occur due to heterogeneous beliefs among investors (Miller (1977); Harrison and Kreps (1978); Scheinkman and Xiong (2003)) or due to the interaction between rational and behavioral traders (De Long et al. (1990); Shleifer and Vishny (1997); Abreu and Brunnermeier (2003)).

[7] In the model there is no fundamental uncertainty. House prices vary as a result of stochastic changes in noise traders' opinions between generations.

Two types of households are present in the model: noise traders (n) of measure μ and sophisticated agents (i) of measure $1 - \mu$. Both types of households choose the quantity of housing to consume and invest in, and savings in the risk-free asset to maximize expected utility given their own beliefs when young. While young sophisticated households in period t accurately perceive the expected price of housing at $t + 1$, young noise trader households misperceive the expected house price. Their misperception enters into the model as an i.i.d. uniform (common) random variable ρ_t, with mean $\overline{\rho}$ and variance σ_ρ^2:

$$E_t^n(p_{t+1}) = E_t^i(p_{t+1}) + \rho_t = \overline{p_{t+1}} + \rho_t, \tag{1}$$

where the mean misperception $\overline{\rho}$ is a measure of the average bullishness of noise traders, and σ_ρ^2 is the variance of the noise traders' misperception.

Households receive utility from housing services, which are assumed to be available in either of two mutually exclusive ways, renting or owning. The consumption of housing services depreciates the housing investment. Therefore to keep the size or the quality of the investment position constant maintenance is required. The maintenance cost per unit of rental-occupied housing, δ_R, is higher than the maintenance cost per unit of owner-occupied housing, δ_O, as a result of a moral hazard problem.[8] This difference in the maintenance costs induces a reduction in the implicit cost of owner-occupied housing and a premium reflecting the additional maintenance cost in the rental price of housing (Chambers, Garriga, and Schlagenhauf, 2009).

2.2 The Households' Optimisation Problem

Young households choose their housing consumption h^c, housing investment h^l, and savings s, to maximise their expected utility, which is received from housing consumption $u(h^c)$, and wealth w, when old:

$$u(h_t^c) + E(w_{t+1}) - \gamma\sigma_{w_{t+1}}^2. \tag{2}$$

The utility derived from housing consumption is characterised by the quadratic utility function, $u(h^c) = ah^c - \frac{b}{2}(h^c)^2$, where $a > 0$, $b > 0$ and $u_{h^c} = a - bh^c > 0$. Households are assumed to have mean-

[8] Moral hazard problem occurs in the rental markets as it is hard to ensure a high standard of maintenance by tenants. Since the maintenance efforts of tenants cannot be observed by landlords, they assume tenants will choose low maintenance efforts. Hence, tenants pay a premium reflecting the additional maintenance cost.

variance preferences over their terminal wealth.[9] Additionally, it is assumed that households consume housing ($h_t^c > 0$) when young and cannot short-sell houses ($h_t^l \geq 0$).

Following Henderson and Ioannides (1983), households are classified into three groups according to their housing consumption and housing investment choices.[10]

Definition 1. *Owner-occupiers are homeowners that choose to consume all services generated from their housing investment position,* $(h^l = h^c)$. *Landlords are homeowners that owner-occupy their housing investment up to their housing consumption and rent out the rest,* $(h^l > h^c)$. *Tenants are households that rent their consumed housing and, if they have any housing investment rent it out,* $(h^l < h^c)$.

In the first period, when young, households invest their exogenous income, y_t, in housing and the risk-free asset, and consume housing. In the second period, when old, they receive interest on their holdings of the risk-free asset, sell their houses at a price $\widetilde{p_{t+1}}$ to the new young, and pay maintenance expenses for their housing investment mc_t. Landlords receive rent from their investment in housing, which is rented out to others at a price of R_t, tenants pay rent for their housing consumption, and old households consume all of their wealth. Budget, wealth and non-negativity constraints, respectively, are as follows:

$$p_t h_t^l + s_t \leq y_t, \tag{3}$$

$$w_{t+1} = (1+r)s_t + \widetilde{p_{t+1}} h_t^l + R_t(h_t^l - h_t^c) - mc_t, \tag{4}$$

$$h_t^c > 0, h_t^l \geq 0.$$

The maintenance costs differ for landlords, owner-occupiers and tenants; the optimisation problems for each group and their respective housing consumption and investment demand functions reflecting these differences in maintenance costs are expressed below.

The Optimisation Problem for Landlords, for whom the maintenance expenses, $\delta_O h_t^c + \delta_R(h_t^l - h_t^c)$,

9 With normally distributed returns, maximizing the expected value of the CARA utility function is equivalent to maximizing the mean-variance utility function. However, for tractability, this analysis assumes a uniform distribution, and uses explicitly a mean-variance preference as it gives closed-form solutions.

10 Henderson and Ioannides (1983) introduce an investment constraint, $h^l \geq h^c$, which requires owner-occupiers' housing investment to be at least as large as their housing consumption. Therefore, as consumption tenure can not be split, when $h^l < h^c$, households rent for their consumption and rent out their housing investment.

depend on the fraction of services consumed and the fraction rented-out to other households:

$$\max_{h_t^l \geq 0, h_t^c > 0, h_t^l > h_t^c} \quad ah_t^c - \frac{b}{2}(h_t^c)^2 + (1+r)y_t + [R_t - \delta_R + p_{t+1}^e - (1+r)p_t]h_t^l - [R_t - (\delta_R - \delta_O)]h_t^c - \gamma[\sigma_P^2 (h_t^l)^2] \quad (5)$$

Housing consumption and investment demand functions are given by

$$h_t^c = \frac{a - R_t + (\delta_R - \delta_O)}{b}, \quad (6)$$

$$h_t^l = \frac{R_t - \delta_R + p_{t+1}^e - (1+r)p_t}{2\gamma\sigma_P^2}. \quad (7)$$

The Optimisation Problem for Tenants, who only pay the maintenance costs for their housing investment positions, $\delta_R h_t^l$:

$$\max_{h_t^l \geq 0, h_t^c > 0, h_t^c > h_t^l} \quad ah_t^c - \frac{b}{2}(h_t^c)^2 + (1+r)y_t + [R_t - \delta_R + p_{t+1}^e - (1+r)p_t]h_t^l - R_t h_t^c - \gamma[\sigma_P^2 (h_t^l)^2] + \lambda h_t^l \quad (8)$$

where λ is the Lagrange multiplier for the short-selling constraint on housing investment. The tenants' optimisation problem yields the following demand functions for housing consumption and housing investment:

$$h_t^c = \frac{a - R_t}{b}, \quad (9)$$

$$h_t^l = max\{0, \frac{R_t - \delta_R + p_{t+1}^e - (1+r)p_t}{2\gamma\sigma_P^2}\}. \quad (10)$$

Housing consumption demand functions for landlords and tenants differ as the spread in the maintenance cost reduces the implicit cost of owner-occupied housing, which increases the housing demand of landlords. While landlords hold positive housing investment positions, tenants' short-selling constraint can be binding (when $p_t \geq \frac{R_t - \delta_R + p_{t+1}^e}{1+r}$).

The Optimisation Problem for Owner-occupiers, who incur a maintenance expense equal to $\delta_O h_t$:

$$\max_{h_t>0} \quad ah_t - \frac{b}{2}(h_t)^2 + (1+r)y_t + [-\delta_O + p^e_{t+1} - (1+r)p_t]h_t - \gamma[\sigma^2_P(h_t)^2] \tag{11}$$

yields the following housing demand:

$$h_t = \frac{a - \delta_O + p^e_{t+1} - (1+r)p_t}{2\gamma\sigma^2_P + b}. \tag{12}$$

2.3 Market Clearing Conditions

Old households sell their holdings of houses, so the housing investment demand of young households must sum to one in equilibrium. The market clearing condition in the housing market is as follows:

$$\mu h^l_{n,t} + (1-\mu)h^l_{i,t} = 1. \tag{13}$$

The rental price is determined by the tenants' demand for rental housing services and the supply of rental housing.[11] In equilibrium, the housing consumption demand is equal to the housing investment demand, which is equal to housing supply. The rental market clearing condition is

$$\mu h^c_{n,t} + (1-\mu)h^c_{i,t} = 1. \tag{14}$$

2.4 Equilibrium Analysis

In this section, a rational expectations equilibrium is solved for the defined economy.

Definition 2. *Given preferences, endowments and beliefs, a stationary noisy rational expectations equilibrium (SNREE) consists of*

- ▷ *a house price function $p(\rho)$ and a rental price function $R(\rho)$,*
- ▷ *allocations of housing services $h^c_n(\rho), h^c_i(\rho)$ and housing investments $h^l_n(\rho), h^l_i(\rho)$,*

[11] Heterogeneity in noise trader and sophisticated households' house price expectations can give rise to an active rental market (without requiring additional heterogeneity in tastes or incomes). Relatively optimistic households invest more in housing for speculative purposes. When their housing investment demand is higher than their housing consumption demand, to avoid the higher maintenance cost, they can owner-occupy their housing investment up to their housing consumption and rent out the rest. Relatively pessimistic households, on the other hand, invest less in housing. When their housing investment demand is less than their housing consumption demand, they can rent their consumed housing since they cannot own only part of their consumption. Therefore, while relatively optimistic households can choose to be landlords, relatively pessimistic households can choose to be tenants depending on the dispersion in their beliefs.

such that

1. *$h^c_j(\rho), h^l_j(\rho)$ are the solutions to household j's consumption-portfolio problem given his/her perceived price process for $j \in \{i, n\}$,*
2. *housing and rental markets clear in every state:*

$$\mu h^l_n(\rho) + (1-\mu) h^l_i(\rho) = 1,$$

$$\mu h^c_n(\rho) + (1-\mu) h^c_i(\rho) = 1.$$

To characterise the equilibrium, first the ranges of values of noise traders' misperceptions for which the short-sale constraint binds and the rental market is active are analysed. In the following analysis, it is assumed that the noise traders' misperception is uniformly distributed over $[0, \rho^u]$, where ρ^u is the maximum value that ρ can take.[12] The optimality conditions from the households' problem and the market clearing conditions yield the following two lemmas.

Lemma 1. *The short-selling constraint is binding for sophisticated households when $\rho_t \geq \frac{\Psi}{\mu}$.*

Where $\Psi = 2\gamma\sigma^2_P$. When the disagreement between sophisticated and noise trader households about house price expectations is large enough, the relatively pessimistic sophisticated households desire to short housing. Hence, the short-sale constraint binds for them.[13]

Lemma 2. *In equilibrium, the rental market is active when $\rho_t > \frac{\Psi(\delta_R - \delta_O)}{b}$.*

Proof. See Appendix 7.1. □

The rental market is always active when the short-selling constraint is binding for sophisticated households, as renting is the only way for them to receive housing services. On the other hand, when $\rho_t < \frac{\Psi}{\mu}$, both noise traders and sophisticated households participate in the housing market, and choose between renting and owner-occupying for their housing consumption.

[12] To simplify the analysis noise traders are assumed to be optimistic. However, when this assumption is relaxed to allow noise traders to be pessimistic, the main results still hold. See Appendix 7.4 for the extended analysis.

[13] See Appendix 7.1 for the housing investment demand of noise traders and sophisticated households (equation (56)).

Due to their heterogeneous beliefs, the housing investment demands of noise traders and sophisticated households differ. The relatively optimistic noise trader households invest more in housing, while the relatively pessimistic sophisticated households invest less in housing. Since housing has a dual role of providing a flow of consumption services and being an investment asset, households‘ housing consumption is tied to their housing investment. Therefore, heterogeneity in beliefs leads to a distortion in the households' housing consumption, which would otherwise be equal for both noise trader and sophisticated households as they have the same preferences and incomes. While noise trader households consume more housing, sophisticated households consume less. The rental market, by separating housing consumption from housing investment, can eliminate the distortion caused by heterogeneous beliefs. However, at the same time, the difference in maintenance costs between rental-occupied and owner-occupied housing creates another distortion.

When the distortion in housing investment due to heterogeneous beliefs is greater than the rental market distortion in housing consumption, ($\frac{(\delta_R - \delta_O)}{b} < \frac{\rho_t}{\Psi}$), while noise traders prefer to owner-occupy their housing consumption and rent out the rest of their housing investment, sophisticated households prefer to rent. Hence, the rental market becomes active. On the other hand, when the distortion in housing investment as a result of heterogeneous beliefs is less than the rental market distortion in housing consumption, households owner-occupy their housing investment, and the rental market becomes inactive.

Lemma 1 and Lemma 2 indicate that equilibrium consists of three regions, depending on whether the short-sale constraint is binding, and whether the rental market is active. Figure 1 shows these regions, with the noise traders' misperception shown on the x-axis.[14]

Theorem 1. *The stationary noisy rational expectations equilibrium house price function is expressed as*

$$p_t = \frac{(a-b)-\delta_R}{r} + \frac{\theta_t(\delta_R-\delta_O)}{(1+r)} + \frac{\overline{\theta}(\delta_R-\delta_O)}{(1+r)r} + \frac{\kappa_t\rho_t}{(1+r)} + \frac{\overline{\kappa\rho}}{(1+r)r} - \frac{\eta_t\Psi}{(1+r)} - \frac{\overline{\eta}\Psi}{(1+r)r} \quad (15)$$

[14] The assumption of positive housing consumption implies $\frac{(\delta_R-\delta_O)}{b} < \frac{1}{\mu}$. See equation (55) in Appendix 7.1.

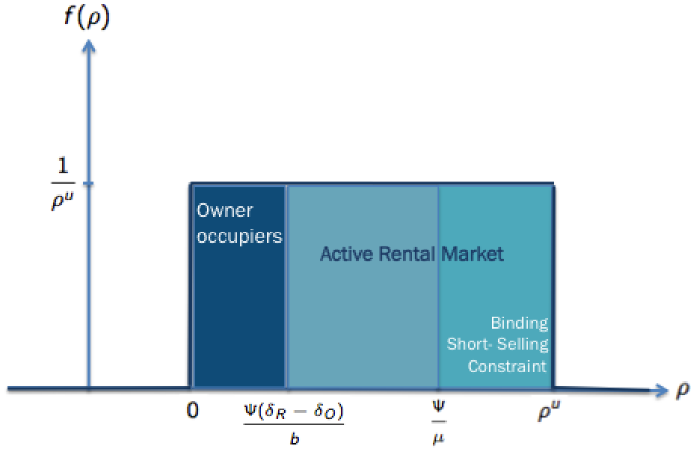


Figure 1

$$(\kappa_t, \eta_t, \theta_t) = \begin{cases} (1, \frac{1}{\mu}, \mu) & \textit{if} \quad \rho_t \geq \frac{\Psi}{\mu} \\ (\mu, 1, \mu) & \textit{if} \quad \frac{\Psi(\delta_R - \delta_O)}{b} < \rho_t < \frac{\Psi}{\mu} \\ (\mu, 1, 1) & \textit{if} \quad 0 \leq \rho_t \leq \frac{\Psi(\delta_R - \delta_O)}{b}, \end{cases} \tag{16}$$

where κ represents the share of noise traders in the population of <u>*housing investors*</u> *(households that participate in housing market), η is the housing stock per* <u>*housing investor*</u>*, and θ is the share of owner-occupiers. $\overline{\theta}, \overline{\kappa\rho}, \overline{\eta}$ denote the expected values of respective variables.*

Proof. Together with the rental market clearing condition, equations (6), (9) and (12) imply the following rental price function:

$$R_t = \begin{cases} (a-b) + \mu(\delta_R - \delta_O) & \text{if} \quad \rho_t > \frac{\Psi(\delta_R - \delta_O)}{b} \\ (a-b) + (\delta_R - \delta_O) & otherwise. \end{cases} \tag{17}$$

When the noise traders' misperception is larger than $\frac{\Psi(\delta_R - \delta_O)}{b}$, noise trader households owner-occupy their housing investments and rent out the rest to sophisticated households. Otherwise, the rental market becomes inactive as both types of households choose to owner-occupy their housing investments. In that case, the rental price represents the imputed rent for owner-occupiers.

Together with the housing market clearing condition, equations (7), (10) and (12) imply the house price function:

$$p_t = \begin{cases} \frac{1}{1+r}[R_t - \delta_R + \overline{p_{t+1}} + \rho_t - \frac{\Psi}{\mu}] & \text{if} \quad \rho_t \geq \frac{\Psi}{\mu} \\ \frac{1}{1+r}[R_t - \delta_R + \overline{p_{t+1}} + \mu\rho_t - \Psi] & otherwise. \end{cases} \quad (18)$$

If noise traders are very optimistic, $\rho_t \geq \frac{\Psi}{\mu}$, and the short-selling constraint becomes binding for sophisticated households; they do not invest in housing, and only rent for their housing consumption. Otherwise, both types of households enter into the housing market.

By combining equations (17) and (18), the house pricing rule can be summarized as follows:

$$p_t = \frac{1}{1+r}[(a-b) - \delta_R + \theta_t(\delta_R - \delta_O) + \overline{p_{t+1}} + \kappa_t\rho_t - \eta_t\Psi], \quad (19)$$

where κ_t, η_t and θ_t are as expressed in equation (16), and $\Psi = 2\gamma\sigma_P^2$. Considering only stationary equilibria, in which the unconditional distribution of p_{t+1} is the same as that of the distribution of p_t, $\overline{p_{t+1}}$ can be eliminated from equation (19) by solving recursively, and the final form of the stationary noisy rational expectations equilibrium (SNREE) price function is obtained (equation (15)). □

Due to the short-selling constraint and utility received from housing services, the house price equation is more complicated than the pricing rule of the risky asset in the De Long et al. (1990) model. If there are no noise traders in the economy, $\mu = 0$, the house price is equal to its fundamental value. In that case, the fundamental value of housing is equal to $\frac{(a-b)-\delta_O}{r}$ as all households owner-occupy their housing investments ($\theta = 1$). The existence of noise traders drives prices away from their fundamental values and creates uncertainty. The last two terms in the pricing rule indicate that households must be compensated for bearing noise trader risk. The uncertainty over what next period's noise traders will believe makes the otherwise riskless asset risky, and drives its price down and its return up. Variations in noise traders' misperceptions lead to fluctuations in house prices directly through changes in ρ_t, and indirectly through changes in κ_t (the share of noise traders in the population of housing investors), η_t (the housing stock per housing investor), and θ_t (the share of owner-occupiers).

To understand the house price function in equation (15) better, consider the following special cases:

Case 1. *Suppose that the noise traders' misperception is uniformly distributed over* $[0, \rho^u]$, *where* $\rho^u \leq$

$\frac{\Psi(\delta_R - \delta_O)}{b}$.

For this interval of misperception, the equilibrium corresponds to the left most region in Figure 1. The rental market is inactive and households consume their own housing investment as the rental market distortion in housing consumption is greater than the distortion in housing investment due to noise traders' misperception. Since both noise traders and sophisticated households participate in the housing market, the share of noise traders in housing investors is equal to μ and the housing stock per housing investor is equal to one. As everyone owner occupies their housing investment, the share of owner-occupiers in the economy is equal to one. Therefore, the house price function presented in Theorem 1 takes the following form:[15]

$$p_t = \frac{(a-b) - \delta_O}{r} + \frac{\mu(\rho_t - \overline{\rho})}{(1+r)} + \frac{\mu\overline{\rho}}{r} - \frac{\Psi}{r}. \tag{20}$$

The house price function presented in equation (20) is quite similar to the pricing rule of the risky asset in the De Long et al. (1990) model. The first term is the fundamental value of housing. The second term reflects the fluctuations in the house price resulting from stochastic changes in noise traders' misperceptions. The third term represents the deviation of the house price from its fundamental value as a result of the average bullishness of noise traders. The last term indicates that households must be compensated for bearing noise trader risk.

Case 2. *Suppose that the noise traders' misperception is uniformly distributed over* $[0, \rho^u]$*, where* $\rho^u < \frac{\Psi}{\mu}$.

In the defined interval, the equilibrium corresponds to two left most regions in Figure 1. Since the short-selling constraint for sophisticated households is not binding for this range of values for ρ_t, all households participate in the housing market ($\kappa = \mu, \eta = 1$). However, in each period, the share of owner-occupiers may change depending on the interval in which the noise traders' misperceptions lie. If $0 \leq \rho_t \leq \frac{\Psi(\delta_R - \delta_O)}{b}$, both noise traders and sophisticated households owner-occupy their housing investments, and thus the share of owner-occupiers is equal to one. On the other hand, if $\rho_t > \frac{\Psi(\delta_R - \delta_O)}{b}$, the rental market becomes active, which means that sophisticated households rent and noise traders owner-occupy. In that case, the share of owner-occupiers is equal to μ. The house price function presented in

[15] House price variance is given as $\sigma_P^2 = \frac{\mu^2 \sigma_\rho^2}{(1+r)^2}$, where $\sigma_\rho^2 = \frac{(\rho^U)^2}{12}$ for a uniformly distributed ρ. For Case 1 to be valid, the following condition must be satisfied: $\rho^U \geq \frac{6}{\gamma} \frac{b}{(\delta_R - \delta_O)} (\frac{1+r}{\mu})^2$.

Theorem 1 takes the following form:

$$p_t = \frac{(a-b)-\delta_R}{r} + \frac{\theta_t(\delta_R-\delta_O)}{(1+r)} + \frac{\overline{\theta}(\delta_R-\delta_O)}{(1+r)r} + \frac{\mu(\rho_t-\overline{\rho})}{(1+r)} + \frac{\mu\overline{\rho}}{r} - \frac{\Psi}{r}, \tag{21}$$

where $\theta_t = \begin{cases} \mu & \text{if} \quad \rho_t > \frac{\Psi(\delta_R-\delta_O)}{b} \\ 1 & \text{if} \quad 0 \le \rho_t \le \frac{\Psi(\delta_R-\delta_O)}{b}. \end{cases}$

The second and third terms in equation (21) result from the variation in the share of owner-occupiers due to the rental market friction.

The equilibrium corresponds to all three regions in Figure 1 when the disagreement between sophisticated and noise trader households about house price expectations becomes large enough that the short-selling constraint for sophisticated households binds. If $\rho_t \ge \frac{\Psi}{\mu}$, only noise traders invest in housing, and thus the share of noise traders in housing investors is equal to one and the housing stock per housing investor is equal to $\frac{1}{\mu}$. Therefore, the difference between the house price function presented in equation (21) and in Theorem 1 is due to the variations in the share of noise traders in the population of housing investors and the housing stock per housing investor.

The housing consumption and investment demand of sophisticated and noise trader households are presented below:

$$(h^c_{i,t}, h^I_{i,t}) = \begin{cases} (1-\frac{\mu(\delta_R-\delta_O)}{b}; 0) & \text{if} \quad \rho_t \ge \frac{\Psi}{\mu} \\ (1-\frac{\mu(\delta_R-\delta_O)}{b}; 1-\frac{\mu\rho_t}{\Psi}) & \text{if} \quad \frac{\Psi(\delta_R-\delta_O)}{b} < \rho_t < \frac{\Psi}{\mu} \\ (1-\frac{\mu\rho_t}{\Psi+b}; 1-\frac{\mu\rho_t}{\Psi+b}) & \text{if} \quad 0 \le \rho_t \le \frac{\Psi(\delta_R-\delta_O)}{b}, \end{cases} \tag{22}$$

$$(h^c_{n,t}, h^I_{n,t}) = \begin{cases} (1+\frac{(1-\mu)(\delta_R-\delta_O)}{b}; 1+\frac{(1-\mu)\rho_t}{\Psi}) & \text{if} \quad \rho_t > \frac{\Psi(\delta_R-\delta_O)}{b} \\ (1+\frac{(1-\mu)\rho_t}{\Psi+b}; 1+\frac{(1-\mu)\rho_t}{\Psi+b}) & \text{if} \quad otherwise. \end{cases} \tag{23}$$

When noise traders are extremely optimistic, $\rho_t \ge \frac{\Psi}{\mu}$, they buy all the housing stock and sophisticated households rent from them. When ρ_t is in the interval $(\frac{\Psi(\delta_R-\delta_O)}{b}, \frac{\Psi}{\mu})$, both sophisticated households and noise traders invest in housing. While noise traders owner-occupy their housing investment and rent out the rest, sophisticated households rent for their housing consumption. On the other hand, when ρ_t is in the interval $[0, \frac{\Psi(\delta_R - \delta_O)}{b}]$, both types of households owner-occupy their housing investments.

3 The Introduction of the Housing Futures Market

In this section, a housing futures market is introduced into the model. Housing futures trading enables households to handle imperfections in the housing market and eliminate distortions in their housing consumption.

The basics of the model are the same as before. However, now there is a new financial instrument in the economy, housing futures, x, which is in zero net supply. Young households can trade futures contracts among themselves by taking long positions $(x < 0)$ or short positions $(x > 0)$. The return on a futures contract is defined as the futures price, k_t, minus the spot price at maturity, $\widetilde{p_{t+1}}$.[16] At maturity, if the realised house price is less (more) than the futures price set at the contract, agents with long (short) positions in housing futures pay the price difference to agents who have short (long) positions in housing futures.

3.1 The Households' Optimisation Problem

The optimisation problem of households when housing futures are available in the economy is as follows. When young they maximise their expected utility, which is received from housing services and terminal wealth as described in the previous section. In the first period, households invest in the risk-free asset and housing with their exogenous income. Additionally, they trade housing futures contracts.[17] In the second period, when old, they receive interest on their holdings of the risk-free asset, sell their houses at a price $\widetilde{p_{t+1}}$ to the new young, pay maintenance costs for their housing investment, and receive returns from housing futures contracts. Landlords receive rent from their investment in housing that is rented out to others at a price of R_t and tenants pay rent for their housing consumption, and households consume

[16] Housing futures contracts are based on a house price index and settled in cash. Therefore, the buyer and the seller exchange the difference between the realised index on maturity and the contract price agreed upon.

[17] In the first period, households trade housing futures by writing a contract without making any financial transaction. For simplicity the margin account requirement for futures trading is not taken into consideration. In the second period, households settle by paying (receiving) the loss (gain) related to the contract in cash.

all of their wealth. Budget, wealth and non-negativity constraints are

$$p_t h_t^l + s_t \leq y_t, \tag{24}$$

$$w_{t+1} = (1+r)s_t + \widetilde{p_{t+1}} h_t^l + R_t(h_t^l - h_t^c) - mc_t + [k_t - \widetilde{p_{t+1}}]x_t, \tag{25}$$

$$h_t^c > 0, h_t^l \geq 0.$$

Households choose their housing consumption, and their investment in housing and housing futures in order to maximise their expected utility. The introduction of the housing futures market allows households to hedge their exposure to house price risk, and moves the speculative investment from the housing market to the futures market. As a result, housing investment demand becomes a function of the housing futures price rather than the expected housing price, which in turn eliminates the difference in the housing investment demand of noise trader and sophisticated households. Since there will be no difference in households' investment, their consumption demand would be the same as well. In that case, there is no rental market equilibrium, as all households owner-occupy their own housing investment due to higher maintenance costs in the rental market. Therefore, only the optimisation problem of owner-occupiers is presented below.

The Optimisation Problem of Owner-occupiers, who incur a maintenance expense equal to $\delta_O h_t$:

$$\begin{aligned} \max_{x_t, h_t > 0} \quad & a h_t - \frac{b}{2}(h_t)^2 + (1+r)y_t + [-\delta_O + p_{t+1}^e - (1+r)p_t]h_t \\ & + (k_t - p_{t+1}^e)x_t - \gamma[\sigma_P^2(h_t - x_t)^2] \end{aligned} \tag{26}$$

This yields the following housing investment and futures demands:

$$h_t = \frac{a - \delta_O + k_t - (1+r)p_t}{b}, \tag{27}$$

$$x_t = h_t + \frac{k_t - p_{t+1}^e}{2\gamma\sigma_P^2}. \tag{28}$$

As shown by equation (27), the housing decision is made separately from the futures trading decision, and it does not depend on risk attitudes and the probability distribution of house prices. Hence, the introduction of housing derivatives eliminates the difference in noise trader and sophisticated households' housing demands, as discussed above. This separation result is consistent with Kawai (1983)[18], and as stated by him, futures contract demand is composed of two parts. The first term is the hedging component, which says h_t should be sold in a futures market if households were to hedge perfectly against house price risk. The second term is the speculation component that reflects the difference between the futures price and household's subjective expectation about the period $t+1$ house price. Note that absolute volume of speculation declines as households become more risk averse or the house price becomes more uncertain.

3.2 Equilibrium Analysis

In this section, a rational expectations equilibrium is solved for the economy where housing futures trading is available.

Definition 3. *Given preferences, endowments and beliefs, a stationary noisy rational expectations equilibrium (SNREE) with the housing futures market consists of*

- $\triangleright$ *a house price function $p(\rho)$ and a housing futures price function $k(\rho)$,*
- $\triangleright$ *allocations of housing $h_n(\rho), h_i(\rho)$ and housing futures $x_n(\rho), x_i(\rho)$,*

such that

1. *$h_j(\rho), x_j(\rho)$ are the solutions to household j's consumption-portfolio problem given his/her perceived price process for $j \in \{i, n\}$,*
2. *housing and housing derivatives markets clear in every state:*

$$\mu h_n(\rho) + (1-\mu)h_i(\rho) = 1,$$

$$\mu x_n(\rho) + (1-\mu)x_i(\rho) = 0.$$

[18] Danthine (1978), Holthausen (1979) and Feder, Just, and Schmitz (1980) show that separation result can also be derived in a general expected utility maximization framework.

Theorem 2. *When housing futures trading is available in the economy, stationary noisy rational expectations equilibrium house price function is expressed as*

$$p_t = \frac{(a-b)-\delta_O}{r} + \frac{\mu(\rho_t - \overline{\rho})}{(1+r)} + \frac{\mu\overline{\rho}}{r} - \frac{\Psi}{r}. \tag{29}$$

Proof. The market clearing condition in the housing market and equation (27) imply

$$p_t = \frac{1}{1+r}[(a-b) - \delta_O + k_t]. \tag{30}$$

The housing futures market clearing condition and equation (28) yield the futures price function:

$$k_t = \overline{p_{t+1}} + \mu\rho_t - \Psi. \tag{31}$$

Combining the housing and housing futures price equations gives

$$p_t = \frac{1}{1+r}[(a-b) - \delta_O + \overline{p_{t+1}} + \mu\rho_t - \Psi]. \tag{32}$$

Considering only stationary equilibria, and eliminating $\overline{p_{t+1}}$ from above equation by solving recursively yields the SNREE house price function in equation (29). □

Since in equation (29) only the second term is variable, the variance of p_t is a function of the variance of the noise traders' misperception ρ_t. House price variance is represented as $\sigma_P^2 = \frac{\mu^2\sigma_\rho^2}{(1+r)^2}$.

Proposition 1. *Noise trader and sophisticated households' optimal housing and housing futures holdings are*

$$h_{i,t} = h_{n,t} = 1, \tag{33}$$

$$x_{n,t} = -\frac{(1-\mu)\rho_t}{\Psi}, \tag{34}$$

$$x_{i,t} = \frac{\mu\rho_t}{\Psi}. \tag{35}$$

The difference in beliefs generates an incentive for housing futures trading. While relatively optimistic noise traders take long positions in housing futures, relatively pessimistic sophisticated households take short positions. With housing futures trading, the housing demands of the noise trader and sophisticated households are equalised, and households owner-occupy their housing investment. Therefore, the introduction of the futures market overcomes the distortion in the households' housing consumption, which otherwise arises due to heterogeneous beliefs about the expected house price, and separates the price dynamics of houses from the associated housing services by allowing speculation in the housing futures market.

3.3 The Effects of Housing Futures Trading on House Price Volatility

The effects of the introduction of the futures market on the housing market are analysed in detail below. To understand the effects of housing futures trading on the housing market through the rental market friction and the short-selling constraint separately, the same special cases are considered as in Section 2.4.

Proposition 2. *If the noise traders' misperception is uniformly distributed over $[0, \rho^u]$, where $\rho^u \leq \frac{\Psi(\delta_R - \delta_O)}{b}$, the introduction of the derivatives market does not have any effect on house prices and volatility.*

Proof. For the defined interval of ρ_t, the house price function without a futures market presented in equation (20) is equal to the house price function with the futures market (Theorem 2). □

For this interval of misperception, without the futures market both noise trader and sophisticated households participate in the housing market, and consume their own housing investment. Noise traders invest more in housing than sophisticated households. With the introduction of the futures market, both noise traders and sophisticated households invest and consume the same amount of housing, and trade housing futures according to their house price expectations. In the absence of rental market, binding short-selling constraint and additional set of traders, the futures market allows only the reallocation of resources between housing and housing futures, where households make their speculative investments in the futures market rather than the housing market, therefore it does not have any influence on house prices and volatility. This result is consistent with Oh (1996)'s finding that financial innovation does not affect risk pricing in a standard mean-variance setting unless it changes the participation set by attracting new entrants.

Proposition 3. *If the noise traders' misperception is uniformly distributed over* $[0, \rho^u]$*, where* $\rho^u < \frac{\Psi}{\mu}$ *and* $(\delta_R - \delta_O)\sigma_\theta^2 + 2\mu\sigma_{\theta,\rho} > 0$*, housing futures trading decreases the house price volatility by crowding out the rental market.*

Proof. When misperception takes values in the interval $(\frac{\Psi(\delta_R - \delta_O)}{b}, \frac{\Psi}{\mu})$, in the absence of a futures market, the rental market becomes active. The house price function for the defined interval of ρ_t is presented in equation (21). House price variance is given by

$$\sigma_P^2 = \frac{1}{(1+r)^2}[(\delta_R - \delta_O)^2\sigma_\theta^2 + 2\mu(\delta_R - \delta_O)\sigma_{\theta,\rho} + \mu^2\sigma_\rho^2]. \tag{36}$$

On the other hand, with the futures market, house price variance takes the following form:

$$\sigma_{PD}^2 = \frac{\mu^2\sigma_\rho^2}{(1+r)^2}. \tag{37}$$

Equation (36) consists of the expression given in equation (37) and two additional terms. While the first term in the square bracket is positive, the sign of the second term depends on the sign of the covariance between θ and ρ, which is negative for the defined interval.[19] If $(\delta_R - \delta_O)\sigma_\theta^2 + 2\mu\sigma_{\theta,\rho} > 0$, the introduction of the futures market decreases house price volatility. □

For the defined interval of misperception, the introduction of the futures market affects the level and volatility of house prices through a change in rental prices. Without the futures market, in each period, the share of owner-occupiers may change depending on the noise traders' misperception. Therefore, variation in the share of owner-occupiers leads to fluctuations in the rental prices. However, housing futures trading enables all households to owner-occupy their own housing investment, and as the share of owner-occupiers does not change over time, it stabilises (imputed) rental prices.

Proposition 4. *If the noise traders' misperception is uniformly distributed over* $[0, \rho^u]$*, where* $\rho^u > \frac{\Psi}{\mu}$ *and* $\delta_R = \delta_O$*, trading housing futures decreases house price volatility.*

For the defined intervals of misperception, housing futures trading affects house prices through two channels: crowding out the rental market; and allowing short-selling. To isolate the effect of short-selling,

[19] Define the critical value as $c = \frac{\Psi(\delta_R - \delta_O)}{b}$ to simplify the notation for the following covariance expression: $\sigma_{\theta,\rho} = E(\theta, \rho) - E(\theta)E(\rho) = (1-\mu)\frac{c}{2}(\frac{c}{\rho^u} - 1) < 0$.

suppose that there is no difference in the maintenance cost between rental-occupied and owner-occupied housing. This is the case when renting and owner-occupying are perfect substitutes.[20] The house price equation takes the following form by letting $\delta_R = \delta_O$ in Theorem 1:

$$p_t = \frac{(a-b)-\delta_O}{r} + \frac{\kappa_t \rho_t}{(1+r)} + \frac{\overline{\kappa\rho}}{(1+r)r} - \frac{\eta_t \Psi}{(1+r)} - \frac{\overline{\eta}\Psi}{(1+r)r}, \tag{38}$$

where $(\kappa_t, \eta_t) = \begin{cases} (1, \frac{1}{\mu}) & \text{if} \quad \rho_t \geq \frac{\Psi}{\mu} \\ (\mu, 1) & \text{if} \quad 0 \leq \rho_t < \frac{\Psi}{\mu}. \end{cases}$

For the defined interval of misperception, the short-selling constraint is binding for sophisticated households when $\rho_t \geq \frac{\Psi}{\mu}$. Trading housing futures enables sophisticated households to participate in the housing market. In other words, with the introduction of the futures market, sophisticated households can short-sell housing futures and at the same time invest in the housing market. This, in turn, decreases the effect of the noise traders' misperception on the house prices and volatility as relatively pessimistic households‘ beliefs are reflected in house prices. The proof is presented in Appendix 7.2.[21]

4 New Investors in the Housing Derivatives Market

In practice, the introduction of a futures market could attract an additional set of traders (institutional investors such as hedge funds, pension funds and insurers) who invest in order to diversify their portfolios and may strengthen the presence of speculative trading. To capture this effect, the analysis is extended by incorporating investors into the model who do not invest in the housing market but trade housing futures.

In the economy, there are now two types of agents: households and investors. Each type of agent consists of noise traders and sophisticated agents: noise trader households of measure α, sophisticated households of measure ν, sophisticated investors of measure φ and noise trader investors of measure $1-\alpha-\nu-\varphi$.

Without the futures market, while households derive utility from housing services and invest in

[20] This case can be considered as analysing the effect of housing futures trading when shared equity schemes are available in the economy. Shared equity schemes allow households to receive utility from the full range of housing services in a property while only owning a fraction of it. They also give the resident household all the management controls and right to decide when to sell. Therefore, shared equity schemes help to eliminate the differences in services received from renting and owner-occupying (Caplin, Chan, Freeman, and Tracy, 1997). In practice, shared ownership/equity schemes are not common. Either they are not available in many countries or only available for first time buyers and people with limited funds.

[21] It is also shown in Appendix 7.2 that if $\rho^u = \frac{\Psi}{\mu}$, house price volatility does not change with housing derivatives trading.

housing and the risk-free asset, investors do not hold housing and only invest in the risk-free asset. The housing market analysis without the futures market is the same as in Section 2. The analysis of the implications of a futures market is modified by the introduction of investors into the housing futures market in the ways shown in the rest of this section. (The optimisation problem of households remains the same as in Section 3.)

4.1 The Optimization Problem of Investors

Since investors do not hold and consume housing, they only have mean-variance preferences over their terminal wealth, $E(w)-\gamma\sigma_w^2$. In the first period, they invest all of their exogenous income in the risk-free asset and trade housing futures contracts. In the second period, when old, they receive interest on their holdings of the risk-free asset, receive a return from their housing futures contracts and consume all of their wealth. The budget and wealth constraints are as follows:

$$s_t \leq y_t, \tag{39}$$

$$w_{t+1} = (1+r)s_t + [k_t - \widetilde{p_{t+1}}]x_t. \tag{40}$$

Investors choose the quantity of housing futures to maximise their expected utility

$$\max_{x_t} \quad (1+r)y_t + [k_t - p^e_{t+1}]x_t - \gamma\sigma_P^2(x_t)^2 \tag{41}$$

Optimisation yields the housing futures demand of sophisticated investors ($x_{is,t}$) and noise trader investors ($x_{ns,t}$):

$$x_{is,t} = \frac{k_t - \overline{p_{t+1}}}{2\gamma\sigma_P^2}, \tag{42}$$

$$x_{ns,t} = \frac{k_t - \overline{p_{t+1}}}{2\gamma\sigma_P^2} - \frac{\rho_t}{2\gamma\sigma_P^2}. \tag{43}$$

4.2 Market Clearing Conditions

In equilibrium, the demand for housing should be equal to its supply. Since only households hold houses, the supply of housing is normalised to $\alpha+\nu$ (one per household). Hence, the market clearing condition in the housing market is defined as

$$\alpha h_{n,t} + \nu h_{i,t} = \alpha + \nu. \tag{44}$$

As housing futures are in zero net supply, and both households and investors trade housing futures contracts, the housing futures market clearing condition is defined as

$$\alpha x_{n,t} + \nu x_{i,t} + \varphi x_{is,t} + (1 - \alpha - \nu - \varphi) x_{ns,t} = 0. \tag{45}$$

4.3 Equilibrium Analysis

In this section, a rational expectations equilibrium is solved for the case in which the introduction of housing futures market attracts an additional set of traders.

Definition 4. *Given preferences, endowments and beliefs, a stationary noisy rational expectations equilibrium (SNREE) with a housing futures market and additional investors consists of*

- ▷ *a house price function $p(\rho)$ and a housing futures price function $k(\rho)$,*
- ▷ *allocations of housing $h_n(\rho), h_i(\rho)$ and housing futures $x_n(\rho), x_i(\rho)$,$x_{ns}(\rho), x_{is}(\rho)$,*

such that

1. *$h_j(\rho), x_j(\rho)$ are the solutions to household j's consumption-portfolio problem given his/her perceived price process for $j \in \{i, n\}$,*
2. *$x_{js}(\rho)$ is the optimal solution to investor j's portfolio problem given his/her perceived price processes for $j \in \{i, n\}$,*
3. *the housing and housing derivatives markets clear in every state:*

$$\alpha h_n(\rho) + \nu h_i(\rho) = \alpha + \nu,$$

$$\alpha x_n(\rho) + \nu x_i(\rho) + \varphi x_{is}(\rho) + (1 - \alpha - \nu - \varphi) x_{ns}(\rho) = 0.$$

Theorem 3. *With the introduction of the housing derivatives market and additional investors, the sta-*

tionary noisy rational expectations equilibrium house price function is expressed as

$$p_t = \frac{(a-b)-\delta_O}{r} + \frac{\varkappa(\rho_t - \overline{\rho})}{(1+r)} + \frac{\varkappa\overline{\rho}}{r} - \phi\frac{\Psi}{r}, \tag{46}$$

where $\varkappa = 1 - \nu - \varphi$*, and* $\phi = \alpha + \nu$*.*

Proof. Using the market clearing condition for the housing futures market and equations (28), (42) and (43), and defining $2\gamma\sigma_P^2 = \Psi$ yields the futures price function:

$$k_t = \overline{p_{t+1}} + (1-\nu-\varphi)\rho_t - \Psi(\alpha+\nu). \tag{47}$$

Defining the noise traders' share as $\varkappa = 1 - \nu - \varphi$, and the housing per capita as $\phi = \alpha + \nu$, and combining the above futures price function with the housing price equation (30) gives

$$p_t = \frac{1}{1+r}[(a-b) - \delta_O + \overline{p_{t+1}} + \varkappa\rho_t - \Psi\phi]. \tag{48}$$

Considering only stationary equilibria, in which the unconditional distribution of p_{t+1} is the same as that of the distribution of p_t, $\overline{p_{t+1}}$ can be eliminated from equation (48) by solving recursively, and the final pricing rule for housing is obtained (equation (46)). House price variance is denoted as $\sigma_P^2 = \frac{\varkappa^2\sigma_\rho^2}{(1+r)^2}$. □

Introducing investors into the analysis changes the house price function by modifying the share of noise traders and per capita housing stock.

Proposition 5. *The noise trader and sophisticated households' optimal housing and housing futures holdings are*

$$h_{i,t} = h_{n,t} = 1, \tag{49}$$

$$x_{n,t} = (1-\phi) - \frac{(1-\varkappa)\rho_t}{\Psi}, \tag{50}$$

$$x_{i,t} = (1-\phi) + \frac{\varkappa\rho_t}{\Psi}. \tag{51}$$

The noise trader and sophisticated investors' optimal housing futures holdings are

$$x_{ns,t} = -\phi - \frac{(1-\varkappa)\rho_t}{\Psi}, \tag{52}$$

$$x_{is,t} = -\phi + \frac{\varkappa \rho_t}{\Psi}. \tag{53}$$

4.4 The Effects of Housing Futures Trading with the Introduction of Additional Investors on House Price Volatility and Housing Bubbles

To isolate the effect of the introduction the futures market on the housing market by attracting an additional set of investors, analysis focuses on the interval of misperception $[0, \rho^u]$, where $\rho^u \leq \frac{\Psi(\delta_R - \delta_O)}{b}$. As shown in Proposition 2, for this interval, in the absence of investors trading housing futures among households does not have any effect on house prices and volatility. Therefore, the results in the following analysis indicate only the effects of the presence of investors.

Proposition 6. *If the introduction of the futures market attracts an additional set of investors:*

i. when the shares of noise traders among households and investors are the same, the introduction of the futures market increases house prices but does not change the volatility of house prices;

ii. when the share of investors that are sophisticated is higher than the share households that are sophisticated, the introduction of the futures market has an ambiguous effect on house prices but decreases the volatility of house prices;

iii. when all investors are sophisticated and risk neutral, the introduction of the futures market eliminates noise trader risk and drives prices to their fundamental levels.

Proof. ***i.*** Recall that house price variance without derivatives is $\sigma_P^2 = \frac{\mu^2 \sigma_\rho^2}{(1+r)^2}$, and house price variance with derivatives is $\sigma_{PDS}^2 = \frac{\varkappa^2 \sigma_\rho^2}{(1+r)^2}$. If the share of noise traders in households and investors are the same ($\mu = \varkappa$), then the volatility of house prices is unchanged by the introduction of the futures market. In this model, the only source of uncertainty is noise traders. If the introduction of new traders does not change the share of noise traders in the market, then volatility does not change.

On the other hand, house prices, and in particular the deviation from their fundamental values (the bubble component), change as a result of the introduction of new investors. Denoting the house price rule equation (20) as p_t and equation (46) as p_t^{DS}, the difference between these two prices is

$$p_t^{DS} - p_t = \frac{\Psi}{r}[1 - \phi] > 0.$$

The presence of new investors in the futures market increases house prices due to a fall in the risk premium. Risk premium is proportional to the variance of per unit of housing and the total number of housing per capita. Although the volatility does not change, the risk premium decreases as the housing stock per capita decreases with the participation of pure speculators. ***ii.*** If the investors are considered to be institutional investors, it is reasonable to assume that the share of sophisticated agents in the population of investors is greater than in the population of households. In this case, the introduction of the futures market means to the introduction of more sophisticated investors, which decreases the share of noise traders $\mu > \varkappa$ in the market, and hence, reduces the house price variance ($\sigma_P^2 > \sigma_{PDS}^2$). Therefore, the introduction of the derivatives market stabilises house prices.

The difference in the house price levels is given as

$$p_t - p_t^{DS} = [\frac{(\rho_t - \overline{\rho})}{1+r} + \frac{\overline{\rho}}{r}][\mu - \varkappa] - \frac{2\gamma\sigma_\rho^2}{r(1+r)^2}[\mu^2 - \varkappa^2\phi] \gtrless 0. \tag{54}$$

The derivatives market affects the house price level through two channels. First, it reduces the impact of noise traders' misperception, thus decreases prices. Secondly, as a result of the decreased risk premium, due to both decreased variance and increased risk sharing, it increases prices. The net effect depends on the magnitude and volatility of the misperception of noise traders.

iii. Suppose, as a limiting case, investors are sophisticated and risk neutral. It can be argued that institutional investors are likely to be risk neutral as they are well diversified. In this case, the futures price is equal to the true expected house price, $k_t = \overline{p_{t+1}}$. In this case, while sophisticated households hedge their housing exposure perfectly ($x_{i,t} = h_{i,t}$), noise trader households hedge less than their entire housing position, ($x_{n,t} = h_{n,t} - \frac{\rho_t}{\Psi^{DS}}$), depending on their misperception. The house prices becomes equal to their fundamental values, ($p_t = \frac{a-b-\delta_O}{r}$). As a result, if housing futures trading attracts sophisticated risk neutral investors, then the introduction of the futures market eliminates noise trader risk, drives prices to fundamentals, and eliminates the imperfections and distortions in the housing market. □

5 Numerical Exercise

In the previous sections, special cases were considered in order to understand the effects of housing futures trading on the housing market by considering the impact on the rental market friction, the short-selling constraint and increased speculation separately. Calculating the distribution of house prices analytically would be complicated as both prices and participation in the housing market are determined in equilibrium. Whether the rental market is active or inactive and whether sophisticated households participate in the housing market or not depend on critical values which are functions of the house price volatility. Therefore, a numerical exercise is conducted to analyse the overall impact of the introduction of housing futures market on the housing market, and to study the welfare implications of housing futures trading.[22] Table 1 presents the parameter values used in this exercise.

Table 1: **Parameters**

α	0.3	the share of noise trader households
ν	0.2	the share of sophisticated households
φ	0.4	the share of sophisticated investors
μ	0.6	the share of noise traders in households
ψ	0.4	the share of noise traders in all agents
ϕ_1	1	the stock of housing per capita in households
ϕ_2	0.5	the stock of housing per capita in all agents
γ	2	the coefficient of risk aversion
r	0.7	the interest rate
a	14	utility function parameters
b	2	utility function parameters
δ_O	3.5	the maintenance cost for owner-occupied housing
δ_R	4	the maintenance cost for rental housing
$[\rho^L, \rho^U]$	$[0, 3.2]$	the range of noise traders' misperceptions

The analysis is conducted for a period of 20 years. The parameters regarding maintenance costs are derived from the annual depreciation rates 1.43% and 1.64% for owner-occupied and rental-occupied housing obtained by Halket and Vasudev (2012).[23] These depreciation rates are multiplied by the fundamental house price value to find the annual maintenance cost and then multiplied by 20 to obtain the maintenance expense for the whole period.[24] The interest rate for the whole period is calculated as the

[22] See Appendix 7.3 for details.

[23] Halket and Vasudev (2012) use the Current-cost Net Stock of Residential Fixed Assets and Current-cost Depreciation of Residential Fixed Assets tables in the National Income and Product Accounts (NIPA) to compute the rate of depreciation of non-farm owner-occupied housing and tenant-occupied housing.

[24] The fundamental value of house prices, $\frac{(a-b)-\delta_O}{r}$, is used to calculate the annual maintenance cost. Otherwise, fluctuations in house

compounded yield rate of 20-Year US Treasury Bond.[25] The range of the noise traders' misperception is chosen so that the house price volatility matches the estimated US house price index volatility. However, the results are robust to a wide range of noise trader misperception.[26]

To investigate the effects of housing futures trading via the presence of speculating investors, the numerical exercise is performed with and without investors. Table 2 presents results relating to the house price volatility without derivatives trading (σ_p^2) and with derivatives trading ($\sigma_{p^D}^2$); and welfare of sophisticated ($\triangle EU_i$) and noise trader ($\triangle EU_n$) households, and sophisticated ($\triangle EU_{is}$) and noise trader ($\triangle EU_{ns}$) investors.[27]

Table 2: **The Effects of Housing Futures Trading on House Price Volatility and Welfare**

	I	II	III
σ_p^2	0.2099	0.2040	0.2040
$\sigma_{p^D}^2$	0.1063	0.1063	0.0472
$\triangle EU_i$	0.64	0.90	1.35
$\triangle EU_n$	0.50	0.33	2.59
$\triangle EU_{is}$	–	–	1.14
$\triangle EU_{ns}$	–	–	3.75

The first column shows the effect of housing futures trading only through the short-selling mechanism (hence it is assumed that there is no difference in maintenance cost, $\delta_O = \delta_R = 3.5$). As indicated by the theoretical analysis, the introduction of the housing derivatives market decreases volatility and increases the welfare of households. The second column presents the effects of housing futures trading through allowing both short-selling and crowding out the rental market. In fact, rental market friction,

prices as a result of noise traders' misperceptions create variations in maintenance costs as well.

[25] The yield rate of 20-Year US Treasury Bond is taken as 2.7% in the calculations.

[26] The volatility estimate is computed as the standard deviation of the annualised percentage change in a house price index over 20 years. In the analysis, the range of the noise traders' misperception is chosen so that the baseline model (without the futures market) matches the house price volatility with the estimated volatility of Federal Housing Finance Agency's House Price Index (4.9%) between 1994:Q1 and 2013:Q4. In fact, volatility measures may differ significantly with different house price indices. For example, the volatility of S&P/Case-Shiller House Price Index is 8% over the same period. However, the results are robust to a wide range of noise trader misperception between [0, 0.1] and [0,13] with respective volatility measures 0.2% and 25%.

[27] A change in welfare is calculated as the difference between the expected utility received from housing consumption and terminal wealth with and without housing futures trading. The introduction of the derivatives market affects the welfare of households by causing changes in housing consumption, speculative investment demand, and the return on housing investment through variations in house price volatility and risk premium.

the difference in the maintenance cost, decreases the volatility of house prices (0.2040) compared to the economy where rental-occupied housing is a perfect substitute for owner-occupied housing (0.2099). Also in this case, the introduction of housing futures trading decreases volatility and increases the welfare of households. The third column displays the effects of housing futures trading through all three mechanisms: allowing short-selling, crowding out of the rental market, and attracting pure speculators. The results show that the introduction of the housing futures market further decreases house price volatility and increases the welfare of both households and investors.

6 Conclusion

The aim of this study is to investigate the effects of housing derivatives trading on housing demand, house price volatility and housing bubbles in a theoretical framework. The analysis is an extension of De Long et al. (1990) model. The existence of noise traders in the housing market creates uncertainty in house prices and causes prices to deviate away from their fundamental values. Moreover, heterogeneity in beliefs leads to a distortion in households' housing consumption decisions. The introduction of a futures market eliminates this distortion by separating the price dynamics of owner-occupied homes from the housing services they contain and allowing speculation in the housing futures market.

Housing futures trading affects house prices through three channels. The first channel is related to the crowding-out of rental housing by the introduction of the housing futures market. Stochastic changes in the noise traders' misperceptions lead to fluctuations in the house prices directly and also indirectly through the rental market. The introduction of the futures market closes this indirect channel, as all households owner-occupy their housing investment by trading housing futures. Depending on the parameter values, the volatility in house prices may increase or decrease with housing futures trading.

The second channel is the short-selling opportunity provided by the housing futures market. When the noise traders' misperception becomes extreme (very optimistic), only they enter into the housing market, while sophisticated households consume housing by renting and do not invest in housing. However, by allowing short-selling, housing futures trading also allows sophisticated households to enter into the housing market. As a result, housing futures trading decreases the effect of the noise traders' misperception on house prices and volatility.

The last channel is related to attracting pure speculators looking for portfolio diversification opportunities. When futures trading attracts sophisticated pure speculators (such as institutional investors), the volatility of house prices decreases. If investors are assumed to be sophisticated and risk-neutral, then the housing bubble is eliminated by housing futures trading.

In summary, the introduction of the derivatives market enables households to separate their housing consumption decisions from their housing investment choices, and solves the imperfections in the housing market by permitting households to hedge their housing investment positions, allowing both households and investors to gain exposure to house price returns and take short positions. Consistent with the conventional wisdom about futures contracts, the introduction of housing futures trading could stabilise house prices by increasing risk sharing. Moreover, the introduction of the housing futures market can increase the participation of sophisticated households in the housing market by allowing short-selling, hence stabilising house prices further. The results of a numerical exercise show that, for a large set of admissible parameter values for noise trader misperception, housing futures trading decreases the volatility of house prices and increases the welfare of households and investors.

7 Appendix

7.1 Required Conditions for the Rental Market

The rental market becomes active when optimistic households (noise traders) prefer to owner-occupy their housing consumption and rent out the rest of their housing investment, and relatively pessimistic households (sophisticated households) prefer to rent. The optimization problems of the noise trader landlords and sophisticated household tenants yield the following housing investment and consumption demands:

$$h^c_{i,t} = 1 - \frac{\mu(\delta_R - \delta_O)}{b}; \quad h^c_{n,t} = 1 + \frac{(1-\mu)(\delta_R - \delta_O)}{b}. \tag{55}$$

$$h^l_{i,t} = max\{1 - \frac{\mu\rho_t}{\Psi}, 0\}; \quad h^l_{n,t} = max\{1 + \frac{(1-\mu)\rho_t}{\Psi}, \frac{1}{\mu}\}. \tag{56}$$

where $\Psi = 2\gamma\sigma^2_P$. For an active rental market, two inequalities must be satisfied: $h^l_{n,t} > h^c_{n,t}$ and $h^l_{i,t} < h^c_{i,t}$, which yields the following condition: $\frac{(\delta_R - \delta_O)}{b} < \begin{cases} \frac{\rho_t}{\Psi} & \text{if} \quad \rho_t < \frac{\Psi}{\mu} \\ \frac{1}{\mu} & \text{if} \quad \rho_t \geq \frac{\Psi}{\mu}. \end{cases}$

Additionally, the assumption of positive housing consumption requires that $\frac{(\delta_R - \delta_O)}{b} < \frac{1}{\mu}$. These necessary conditions indicate the rental market becomes active if $\rho_t > \frac{\Psi(\delta_R - \delta_O)}{b}$.

7.2 Proposition 4

If the noise traders' misperception is uniformly distributed over $[0, \rho^u]$, where $\rho^u > \frac{\Psi}{\mu}$ and $\delta_R = \delta_O$, the equilibrium house price function without derivatives market is

$$p_t = \frac{(a-b) - \delta_O}{r} + \frac{\kappa_t\rho_t}{(1+r)} + \frac{\overline{\kappa\rho}}{(1+r)r} - \frac{\eta_t\Psi}{(1+r)} - \frac{\overline{\eta}\Psi}{(1+r)r}, \tag{57}$$

where $\Psi = 2\gamma\sigma^2_P$ and $(\kappa_t, \eta_t) = \begin{cases} (1, \frac{1}{\mu}) & \text{if} \quad \rho_t \geq \frac{\Psi}{\mu} \\ (\mu, 1) & \text{if} \quad 0 \leq \rho_t < \frac{\Psi}{\mu}. \end{cases}$ House price variance is determined by

$$\sigma^2_P = \frac{1}{(1+r)^2}[Var(\kappa\rho) - 2Cov(\kappa\rho, \eta)(2\gamma\sigma^2_P) + Var(\eta)(2\gamma\sigma^2_P)^2]. \tag{58}$$

The aim of this analysis is not to solve for the house price variance but to compare house price variance with and without the futures market. Therefore, I try to simplify the analysis as much as possible in order to have an expression which allows this comparison. Once the house price variance is known, it is possible to denote the upper bound value as $\rho^u = \frac{\chi\gamma\sigma^2_P}{\mu}$, where $\chi > 2$ to guarantee that the short-selling constraint is binding for noise traders. After substituting in the respective expressions of the moments of variables, equation (58) can be expressed as follows:

$$(1+r)^2\sigma^2_P = [-(1-\mu)^2\frac{4}{\chi^2} + (2-\mu)(1-\mu)\frac{8}{3\chi} - 2(1-\mu) + \frac{\chi^2}{12}]\frac{\gamma^2}{\mu^2}(\sigma^2_P)^2, \tag{59}$$

$$\sigma^2_P = \frac{(1+r)^2}{[-(1-\mu)^2\frac{4}{\chi^2} + (2-\mu)(1-\mu)\frac{8}{3\chi} - 2(1-\mu) + \frac{\chi^2}{12}]\frac{\gamma^2}{\mu^2}}. \tag{60}$$

With the introduction of the futures market both type of households buy houses. Hence, the futures market increases participation in the housing market. The house price function is given by

$$p_t^D = \frac{(a-b)-\delta_O}{r} + \frac{\mu(\rho_t - \overline{\rho})}{(1+r)} + \frac{\mu\overline{\rho}}{r} - \frac{2\gamma\sigma^2_{P^D}}{r}, \tag{61}$$

where $\sigma^2_{p^D} = \frac{\mu^2\sigma^2_\rho}{(1+r)^2}$. The house price variance expression, when futures trading is available, can be rewritten by substituting in $\sigma^2_\rho = \frac{U^2}{12} = \frac{[\frac{\chi\gamma\sigma^2_P}{\mu}]^2}{12}$ and σ^2_P from equation (60) as follows:

$$\sigma^2_{P^D} = \sigma^2_P \frac{\mu^2\chi^2}{12[-(1-\mu)^2\frac{4}{\chi^2} + (2-\mu)(1-\mu)\frac{8}{3\chi} - 2(1-\mu) + \frac{\chi^2}{12}]}. \tag{62}$$

Since $\mathcal{M} = \frac{\mu^2\chi^2}{12[-(1-\mu)^2\frac{4}{\chi^2}+(2-\mu)(1-\mu)\frac{8}{3\chi}-2(1-\mu)+\frac{\chi^2}{12}]} < 1$, for $\chi > 2$ and $\forall\mu$, the introduction of the futures market decreases the volatility of house prices. [28]

7.3 Numerical Exercise

Calculating the variance of the house price function analytically would be complicated as both prices and participation in the housing market are determined in equilibrium, and moreover, participation depends on a critical value which is a function of the house price volatility. For this reason, a numerical exercise is conducted.

The price function in Theorem 1 is

$$p_t = \frac{(a-b)-\delta_R}{r} + \frac{\theta_t(\delta_R-\delta_O)}{(1+r)} + \frac{E(\theta)(\delta_R-\delta_O)}{(1+r)r} + \frac{\kappa_t\rho_t}{(1+r)} + \frac{E(\kappa\rho)}{(1+r)r} - \frac{\eta_t 2\gamma\sigma^2_P}{(1+r)} - \frac{E(\eta)2\gamma\sigma^2_P}{(1+r)r}, \tag{63}$$

where

$$(\kappa_t, \eta_t, \theta_t) = \begin{cases} (1, \frac{1}{\mu}, \mu) & \text{if} \quad \rho_t \geq \frac{2\gamma\sigma^2_P}{\mu} \\ (\mu, 1, \mu) & \text{if} \quad 2\gamma\sigma^2_P\frac{(\delta_R-\delta_O)}{b} < \rho_t < \frac{2\gamma\sigma^2_P}{\mu} \\ (\mu, 1, 1) & \text{if} \quad 0 \leq \rho_t \leq 2\gamma\sigma^2_P\frac{(\delta_R-\delta_O)}{b}. \end{cases} \tag{64}$$

House price variance is given as

$$\begin{aligned} \sigma^2_P = \frac{1}{(1+r)^2}[&(\delta_R-\delta_O)^2 Var(\theta) + Var(\kappa\rho) + Var(\eta)(2\gamma\sigma^2_P)^2 \\ &+ 2(\delta_R-\delta_O)Cov(\theta,\kappa\rho) - 2(\delta_R-\delta_O)(2\gamma\sigma^2_P)Cov(\theta,\eta) \\ &- 2(2\gamma\sigma^2_P)Cov(\eta,\kappa\rho)]. \end{aligned} \tag{65}$$

[28] Checking whether $\mathcal{M} = \frac{\mu^2\chi^2}{12[-(1-\mu)^2\frac{4}{\chi^2}+(2-\mu)(1-\mu)\frac{8}{3\chi}-2(1-\mu)+\frac{\chi^2}{12}]} \gtreqless 1$, is equivalent to checking if $(1-\mu^2)\chi^4 - 24(1-\mu)\chi^2 + (2-\mu)(1-\mu)32\chi - 48(1-\mu)^2 \lesseqgtr 0$. When $\mu \to 1$, the expression approaches zero, and when $\mu \to 0$, as $\chi > 2$ by assumption, the expression is positive, indicating that $\mathcal{M} < 1$. However, if $\chi = 2$ housing futures trading does not change the volatility of house prices, as $\mathcal{M} = 1$ in that case.

Moments of the variables are calculated as follows:

$$E(\theta) = \int_0^{2\gamma\sigma_P^2 \frac{(\delta_R - \delta_O)}{b}} 1 f(\rho) d\rho + \int_{2\gamma\sigma_P^2 \frac{(\delta_R - \delta_O)}{b}}^{\rho^u} \mu f(\rho) d\rho, \tag{66}$$

$$Var(\theta) = \int_0^{2\gamma\sigma_P^2 \frac{(\delta_R - \delta_O)}{b}} [1 - E(\theta)]^2 f(\rho) d\rho + \int_{2\gamma\sigma_P^2 \frac{(\delta_R - \delta_O)}{b}}^{\rho^u} [\mu - E(\theta)]^2 f(\rho) d\rho, \tag{67}$$

$$E(\eta) = \int_0^{\frac{2\gamma\sigma_P^2}{\mu}} 1 f(\rho) d\rho + \int_{\frac{2\gamma\sigma_P^2}{\mu}}^{\rho^u} \frac{1}{\mu} f(\rho) d\rho, \tag{68}$$

$$Var(\eta) = \int_0^{\frac{2\gamma\sigma_P^2}{\mu}} [1 - E(\eta)]^2 f(\rho) d\rho + \int_{\frac{2\gamma\sigma_P^2}{\mu}}^{\rho^u} [\frac{1}{\mu} - E(\eta)]^2 f(\rho) d\rho, \tag{69}$$

$$E(\kappa\rho) = \int_0^{\frac{2\gamma\sigma_P^2}{\mu}} (\mu\rho) f(\rho) d\rho + \int_{\frac{2\gamma\sigma_P^2}{\mu}}^{\rho^u} (1\rho) f(\rho) d\rho, \tag{70}$$

$$Var(\kappa\rho) = \int_0^{\frac{2\gamma\sigma_P^2}{\mu}} [(\mu\rho) - E(\kappa\rho)]^2 f(\rho) d\rho + \int_{\frac{2\gamma\sigma_P^2}{\mu}}^{\rho^u} [(1\rho) - E(\kappa\rho)]^2 f(\rho) d\rho, \tag{71}$$

$$E(\eta\kappa\rho) = \int_0^{\frac{2\gamma\sigma_P^2}{\mu}} (\mu\rho) f(\rho) d\rho + \int_{\frac{2\gamma\sigma_P^2}{\mu}}^{\rho^u} (\frac{\rho}{\mu}) f(\rho) d\rho, \tag{72}$$

$$E(\theta\eta) = \int_0^{2\gamma\sigma_P^2 \frac{(\delta_R - \delta_O)}{b}} 1 f(\rho) d\rho + \int_{2\gamma\sigma_P^2 \frac{(\delta_R - \delta_O)}{b}}^{\frac{2\gamma\sigma_P^2}{\mu}} \mu f(\rho) d\rho + \int_{\frac{2\gamma\sigma_P^2}{\mu}}^{\rho^u} 1 f(\rho) d\rho, \tag{73}$$

$$E(\theta\kappa\rho) = \int_0^{2\gamma\sigma_P^2 \frac{(\delta_R - \delta_O)}{b}} (\mu\rho) f(\rho) d\rho + \int_{2\gamma\sigma_P^2 \frac{(\delta_R - \delta_O)}{b}}^{\frac{2\gamma\sigma_P^2}{\mu}} (\mu^2\rho) f(\rho) d\rho + \int_{\frac{2\gamma\sigma_P^2}{\mu}}^{\rho^u} (\mu\rho) f(\rho) d\rho. \tag{74}$$

Since all of the moments can be written as a function of the house price variance, equation (65), a fourth-order polynomial with one unknown, σ_P^2, is solved numerically in Matlab. After solving for σ_P^2, whether the thresholds, $2\gamma\sigma_P^2 \frac{(\delta_R - \delta_O)}{b}$ and $\frac{2\gamma\sigma_P^2}{\mu}$, are within the range of noise traders' misperceptions is checked.[29] Then, the effects of housing futures trading on the housing market via three channels are analysed.

Finally, a welfare analysis is conducted. A change in welfare is calculated as the difference between the expected utility received from housing consumption and terminal wealth with and without housing futures trading. The threshold for active rental market is defined as $\zeta = 2\gamma\sigma_P^2 \frac{(\delta_R - \delta_O)}{b}$. The changes in expected utility of households and investors with (EV^D) / without (EV) derivatives trading are expressed as follows:

[29] Additionally, whether marginal utility of consumption and housing consumption are positive, and whether the condition for the active rental market is satisfied are checked for the defined parameter values.

Sophisticated Households

$$EV_i^D - EV_i = \int_0^{\zeta} (\gamma[\sigma_{PD}^2\theta^2 - \sigma_P^2] - \rho_t(\psi\theta - \mu) + \frac{1}{2}[\frac{(\psi\rho_t)^2}{2\gamma\sigma_{PD}^2} - \frac{(\mu\rho_t)^2}{2\gamma\sigma_P^2 + b}])f(\rho)d\rho$$
$$+ \int_{\zeta}^{\frac{2\gamma\sigma_P^2}{\mu}} (\mu(\delta_R - \delta_O)[1 - \frac{\mu(\delta_R - \delta_O)}{2b}] + \gamma[\sigma_{PD}^2\theta^2 - \sigma_P^2] - \rho_t(\psi\theta - \mu) + \frac{1}{2}[\frac{(\psi\rho_t)^2}{2\gamma\sigma_{PD}^2} - \frac{(\mu\rho_t)^2}{2\gamma\sigma_P^2 + b}])f(\rho)d\rho$$
$$+ \int_{\frac{2\gamma\sigma_P^2}{\mu}}^{\rho^u} (\mu(\delta_R - \delta_O)[1 - \frac{\mu(\delta_R - \delta_O)}{2b}] + \frac{[2\gamma\sigma_{PD}^2\theta - \psi\rho_t]^2}{4\gamma\sigma_{PD}^2})f(\rho)d\rho \tag{75}$$

Noise Trader Households

$$EV_n^D - EV_n = \int_0^{\zeta} (\gamma[\sigma_{PD}^2\theta^2 - \sigma_P^2] + \rho_t[(1-\psi)\theta - (1-\mu)] + \frac{1}{2}[\frac{[(1-\psi)\rho_t]^2}{2\gamma\sigma_{PD}^2} - \frac{[(1-\mu)\rho_t]^2}{2\gamma\sigma_P^2 + b}])f(\rho)d\rho$$
$$+ \int_{\zeta}^{\frac{2\gamma\sigma_P^2}{\mu}} (-(1-\mu)(\delta_R - \delta_O)[1 + \frac{(1-\mu)(\delta_R - \delta_O)}{2b}])f(\rho)d\rho$$
$$+ \int_{\zeta}^{\frac{2\gamma\sigma_P^2}{\mu}} (\gamma[\sigma_{PD}^2\theta^2 - \sigma_P^2] + \rho_t[(1-\psi)\theta - (1-\mu)] + \frac{1}{2}[\frac{[(1-\psi)\rho_t]^2}{2\gamma\sigma_{PD}^2} - \frac{[(1-\mu)\rho_t]^2}{2\gamma\sigma_P^2 + b}])f(\rho)d\rho$$
$$+ \int_{\frac{2\gamma\sigma_P^2}{\mu}}^{\rho^u} (-(1-\mu)(\delta_R - \delta_O)[1 + \frac{(1-\mu)(\delta_R - \delta_O)}{2b}] + \frac{[2\gamma\sigma_{PD}^2\theta + (1-\psi)\rho_t]^2}{4\gamma\sigma_{PD}^2} - \frac{\gamma\sigma_P^2}{\mu^2})f(\rho)d\rho \tag{76}$$

Sophisticated Investors

$$EV_{is}^D - EV_{is} = \int_0^{\rho^u} \frac{[2\gamma\sigma_{PD}^2\theta - \mu\rho_t]^2}{4\gamma\sigma_{PD}^2} f(\rho)d\rho \tag{77}$$

Noise Trader Investors

$$EV_{ns}^D - EV_{ns} = \int_0^{\rho^u} \frac{[2\gamma\sigma_{PD}^2\theta + (1-\mu)\rho_t]^2}{4\gamma\sigma_{PD}^2} f(\rho)d\rho \tag{78}$$

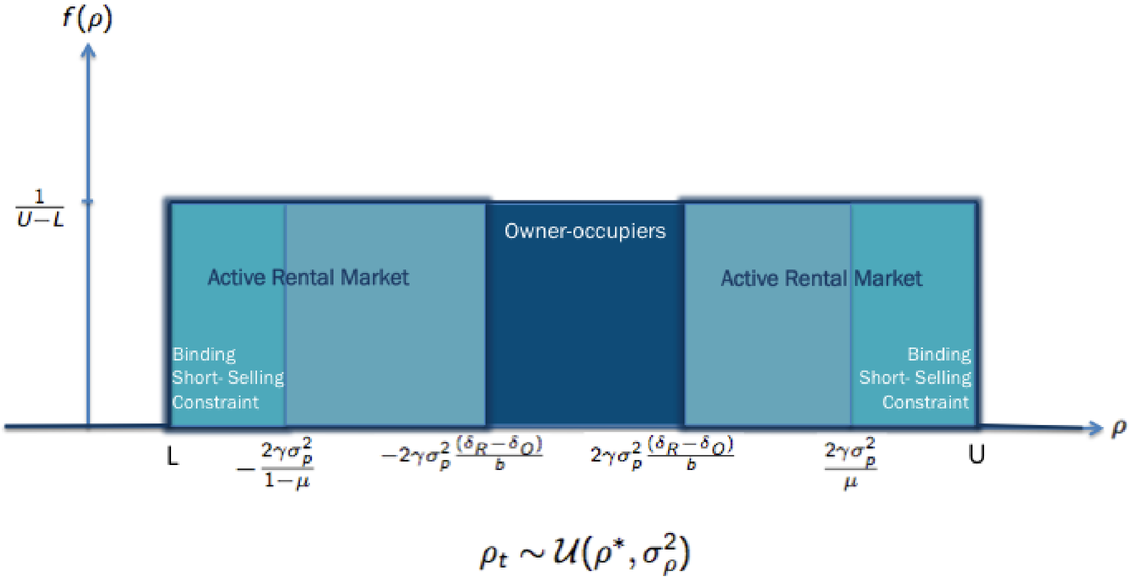
The introduction of the futures market impacts the welfare of households by causing changes in housing consumption[30], speculative investment demand, and return on housing investment through variations in house price volatility and risk premium.

7.4 Noise Traders' Misperceptions: Optimism & Pessimism

When the baseline model is extended by allowing noise traders also to be pessimistic in their house price expectation, the solutions to the optimisation problems yield an equilibrium consisting of five regions:

Propositions 2 and 3 are still valid after allowing pessimistic misperception of noise traders, while Proposition 4 has to be revised as follows:

[30] Housing futures trading leads a change in homeownership structure. It has a positive effect on welfare for sophisticated households, who are renters without the futures market, as they become homeowners and consume more when they are able to trade housing futures. On the other hand, it has a negative effect for noise trader households, who are owner-occupiers without the futures markets. Although, with the introduction of the futures markets they still owner-occupy housing, their housing consumption decreases as the reduction in the implicit cost of owner-occupied housing (due to the spread in maintenance costs) is eliminated.



Proposition 7. *If the noise traders' misperception is uniformly distributed over* $[\rho^L, \rho^U]$*,*

i. where $\rho^L > -\frac{\Psi}{1-\mu}$ *and* $\rho^U > \frac{\Psi}{\mu}$*, trading housing futures decreases the house price volatility;*

ii. where $\rho^L < -\frac{\Psi}{1-\mu}$ *and* $\rho^U > \frac{\Psi}{\mu}$*, the volatility of house prices can increase or decrease with housing futures trading.*

For the defined interval of the misperception in (i), the short-selling constraint is binding for sophisticated households when $\rho_t > \frac{\Psi}{\mu}$. Trading housing futures enables sophisticated households to participate to the housing market, and hence decreases the effect of the noise traders' misperception on house prices and volatility. On the other hand, for the interval defined in (ii), the short-selling constraint can be binding also for noise traders (when $\rho_t < -\frac{\Psi}{1-\mu}$), and hence volatility might increase for some parameter values by allowing them to short housing futures and invest in housing. Indeed, the introduction of futures market can increase the volatility if a majority of the households are noise traders.

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