Customer Friendly Finance*

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Abstract

In the United States, customer owned firms are responsible for 35% of consumer insurance and 10% of consumer banking, yet receive little theoretical or empirical attention. In this paper, I propose a theory of internal finance for the customer owned firm. I show that its growth, pricing, and capital structure are tied together: *higher* sales tomorrow are achieved through *higher* prices today and *lower* leverage today. This result does not hold for a shareholder owned firm. I document stylized facts from the credit union industry and find that they are consistent with the theory's predictions. I discuss empirical implications for other customer owned firms, such as mutual insurance companies and agricultural credit associations.

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"Cooperative undertakings account for a substantial share of developed market economies and that share is likely to grow with the advent of the new economy" (Rey and Tirole, 2007).

1 Introduction

Two firms operate in identical, but independent markets. They face identical demands for their goods and identical costs of production. One firm is owned by shareholders—the "corporation." It maximizes profit. The other firm is owned by its customers—the "cooperative." It maximizes total surplus, as its owners are both purchasers of its good and suppliers of its capital. How do growth, pricing, and capital structure compare between these two firms? I show that in the cooperative—unlike the corporation—they are fundamentally linked and should covary in predictable ways.

Suppose, for concreteness, that a corporate bank observes an opportunity to grow its loan book next quarter. Perhaps a new source of information will reduce the riskiness of making loans—like credit reports—or perhaps a new technology will lower the cost of day-to-day transactions—like smartphone apps. When next quarter rolls around, the bank will offer lower loan rates to bring more applicants through the door. But it won't offer this quarter's loan applicants lower (or higher) rates. This quarter's rates are set to maximize this quarter's profit. They don't tell us anything about credit reports or smartphone apps. What if the bank is running low on equity? Can it use this quarter's rates to shore up its balance sheet? It can't. It already makes as much profit as it can. If it needs more equity, it will have to get it somewhere else.

Suppose, on the other hand, that a cooperative bank observes an opportunity to grow its loan book next quarter. Like the corporate bank, its costs will fall. Now the cooperative bank would happily set loan rates so that it makes no profit.[1] If it has ample equity, it will do just this. But what if it doesn't have ample equity? Unlike the corporate bank, the cooperative bank *can* do better than offer its "first best" rate. It can offer positive-profit rates, retain the earnings and use them to finance next quarter's loan growth. The cooperative bank balances the gains from loan growth next quarter against the deadweight loss from profit this quarter.

In this paper, I show that the cooperative's growth, prices, and leverage will covary in predictable ways. I show that higher sales next quarter are achieved through higher prices this quarter and lower leverage this quarter. In insurance-speak, higher policy growth is achieved through a higher insurance margin and a higher solvency ratio. In bank-speak, higher loan growth is achieved through a higher net interest margin and a higher capital

[1] Profit creates a deadweight loss for its customer-owners.

ratio.

While this paper's contribution is primarily theoretical, I document a number of stylized facts about US credit unions, which are cooperative banks. I find that industry loan growth is positively correlated with the industry net interest margin and positively correlated with the industry capital ratio. These correlations don't hold for traditional, shareholder owned banks and are consistent with the theory's predictions.

Cooperatives appear most commonly in insurance—as mutual insurance companies—and banking—as credit unions, mutual savings banks, and agricultural credit associations. Mutual insurance companies are responsible for 35% of consumer insurance in the US and 27% of consumer insurance worldwide. They manage \$7.7 trillion in assets.[2] Credit unions are responsible for 10% of consumer banking in the US.[3] Approximately 8% of adults worldwide are members of a credit union and collectively they manage \$1.8 trillion in assets in 105 countries.[4] A non-negligible amount of banking services are provided by mutual savings banks, and agricultural credit associations. Cooperatives appear outside of the financial sector as well, most notably in electricity provision. Rural electricity cooperatives supply 13% of US electricity and hold \$150 billion in assets.[5] Despite their economic importance, customer owned firms receive little theoretical or empirical attention.[6] In this paper, I show that they should defy some basic theories of corporate finance and industrial organization.

The model works as follows. There are two firms: the shareholder owned corporation and the customer owned cooperative. The corporation maximizes profit while the cooperative maximizes total surplus. Firms operate for two periods and then liquidate. They face identical demands for their goods and identical costs of production. In the first period, they observe an opportunity to grow—a *technology shock*—which lowers their costs of doing business. Firms use the prices of their goods to manage their sales and internal funds.

The model has three working assumptions. The first assumption is that firms enjoy market power. They can increase sales by lowering prices and decrease sales by raising prices. In the model, I assume that firms are monopolists. This isn't true in reality, but it's a good benchmark.

The second assumption is that firms face capital requirements. There is only so much leverage that regulators, creditors or investors can stomach. In the model, firms must hold a minimum ratio of equity-to-sales. If "sales" is off-putting, recall that in an insurance

[2] 2012 Global Mutual Market Share. International Cooperative and Mutual Insurance Federation.
[3] Credit Union Report Year-End 2014. Credit Union National Association.
[4] 2014 Statistical Report. World Council of Credit Unions.
[5] Co-op Facts & Figures. National Rural Electric Cooperative Association.
[6] The basic contracting issues of customer ownership have been studied by several authors (see section 2). What remains undeveloped is an operational theory of the customer owned firm.

company, "sales" means "policies" (a liability) and in a bank, "sales" means "loans" (an asset) or "deposits" (a liability). Equity-to-sales is a measure of leverage. I don't impose a capital structure *per se*. Firms can hold equity in excess of the minimum.

The third assumption is that firms rely on internal funds for equity. The cooperative can't raise external equity. If it did, it would cede control to non-customers. It would no longer be "customer owned." There's nothing stopping the corporation from raising external equity. For the sake of a clean comparison, I assume that it can't. I show that differences in behavior are driven by differences in ownership and not by differences in financial constraints.

The first set of results concerns the behavior of prices. I show that the corporation's prices never depend on the technology sock. For sufficiently large technology shocks, the cooperative's prices are *increasing* in the technology shock.

The second set of results concerns the covariation of growth, pricing and leverage. For sufficiently large technology shocks, the cooperative's growth will covary positively with its prices and negatively with its leverage. The corporation's growth will vary with the technology shock, but it won't *covary* with its prices or leverage.

The theory proposed in this paper offers a novel explanation for the sensitivity of cooperatives' investment to cash flow.[7] The standard explanation says that financially constrained firms turn positive cash flow directly back into investment. The explanation in this paper is quite different. It says that cooperatives observe investment opportunities and adjust their (expected) cash-flow by adjusting their prices.

The theory also offers a novel example of a dynamic pricing problem. In the model, the corporation's problem is time-separable. The corporation can choose today's prices without regard for the future. The cooperative's problem is not. It adjusts today's prices according to future growth opportunities. That pricing should be a dynamic problem in insurance or banking deserves pause. As demonstrated by Bulow (1982), a durable goods monopolist may have an incentive to intertemporally price discriminate, in which case her problem problem is "dynamic" (it's not time-separable). But if she can lease the good instead of selling it, her problem is static again (it's time-separable). Lending is leasing cash. Insurance is leasing a state contingent claim to cash. Borrowers can refinance. Policyholders can surrender their policies and obtain new ones. We should not expect Bulow's intertemporal price discrimination in the financial sector.

[7] I don't develop a Q-theory for the cooperative, so I only conjecture that this channel is at-play.

2 Literature

It has long been postulated that cooperatives should, because of their peculiar organization, offer competitive prices (see, for example, Enke, 1945). The implication is powerful: a monopolist may enjoy economies of scale, but society suffers a deadweight loss; Bertrand competitors may offer competitive prices, but miss out on economies of scale. The cooperative skirts both costs. It can offer competitive prices while enjoying economies of scale. As noted by Hansmann (2009), many insurance mutuals in nineteen century America were formed precisely to avoid paying the rates offered by corporate carriers.

Also noted by Hansmann (2009) is the reality that contracting is costly when ownership lies with customers. Whether customer owned firms offer prices that are consistent with their ownership is an empirical question, and a challenging one at that. Consider a firm that operates for one period (and one period only). If it shows a profit, then we can conclude that its prices are inconsistent with customer ownership.[8] Now suppose that it operates for *more* than one period. If it still shows a profit, then we can't be so sure that it's prices are inconsistent with customer ownership. Maybe it needed to grow, creating a need for profits. A dynamic perspective is essential. This paper takes a step toward answering the "pricing and ownership" question by providing a dynamic theory of the customer owned firm, with particular emphasis on those found in insurance and banking.

A substantial literature beginning with Fama and Jensen (1983b,a), Hansmann (1985, 2009), Mayers and Smith Jr (1981, 1986) and Smith and Stutzer (1990a,b, 1995) explores the basic contracting problems of customer ownership. Broadly, this literature argues that customer ownership reduces informational costs inherent to some businesses. By making its policyholders owners, for example, a mutual insurance company attenuates moral hazard by tying its policyholders' welfare to its own.

A parallel, empirical literature studies the effects of customer ownership on firm policies. O'Hara (1981) examines the impact of customer ownership on costs, profitability, risk and growth in the savings and loan industry. She finds that savings and loan associations are less profitable and grow slower than their stock association counterparts. Mayers and Smith Jr (1988) conduct a similar study in the property/casualty market. They examine the impact of ownership on lines-of-business specialization, line-of-business concentration and geographic concentration. More recently, Ostergaard et al. (2015) examine the survivability of savings banks in the presence of competition.

What seems absent from this literature is an applied theory of corporate finance and industrial organization for customer owned firms. Because of their ownership, they rely

[8] I'm assuming that profit is deterministic, as it will be in the model.

heavily—if not exclusively—on internal funds for growth. Internal finance is an inherently dynamic problem, in the sense that today's decisions affect tomorrow's availability of funds. The static models that kickstarted the literature are insufficient to explain real-world growth, pricing and capital structure decisions.

The predictions of the current paper are consistent with a number of recent findings. Zanjani (2007) finds that when regulation of the life insurance industry changed from federal to state hands, mutual life insurance companies survived in states with less stringent capital requirements. The theory presented in this paper predicts that capital requirements are a limiting factor for the growth of mutual insurance companies. Ramcharan et al. (2014), who study the transmission of financial shocks through bank networks, find that loan growth in US credit unions is highly sensitive to their capital ratios, a relationship that will emerge from the theory in this paper. This paper strongly complements the recent empirical work of Adelino, Lewellen, and Sundaram (2015), who examine investment policies of non-profit hospitals. The authors point out that non-profits dominate the healthcare sector, which accounts for 15% of the US economy. Non-profits do not share objectives with the profit-maximizing, shareholder-owned firms of standard theory. The standard theory of corporate finance simply doesn't apply. While this paper considers customer owned firms—such as mutual insurance companies and credit unions—many of the insights can be translated to the problem of the non-profit, which also has incentives for growth and also relies on internal funds for equity. Complimenting their findings, I explore a novel mechanism behind the investment cash-flow sensitivity.

That customer ownership should affect pricing dates back to Enke (1945), who argued that a cooperative should offer competitive prices, even if it enjoyed market power. The importance of a dynamic theory for customer owned firms cannot be understated. Cooperatives regularly make profits, which are justified on the grounds of "risk management" and "growth." Asking an empirical question, like "do credit unions offer rates consistent with customer ownership?" requires an understanding of their internal finance. To the best of my knowledge, this is the first paper to theoretically investigate the effect of a customer friendly objective on pricing by benchmarking to the behavior of a profit-maximizer.

Several attempts have been made at a dynamic theory of customer owned financial institutions. Deshmukh, Greenbaum, and Thakor (1982) model a mutual financial institution and take the pessimistic view that its objective is to maximize the manager's welfare. Smith (1988) presents a dynamic model of a credit union, but obtains qualitative results only. Brown and Davis (2009) present an ad-hoc model of credit union capital management for estimation purposes. Rubin et al. (2013) present a dynamic model of a credit union, where members' concern for the viability of the institution drive rate decisions. None of these models explicitly benchmark to a shareholder owned, profit-maximizing firm.

3 A Deterministic, Two-Period Model

I consider two types of firms: the cooperative, whose ownership lies with its customers, and the corporation, whose ownership lies with shareholders (who are not customers). For ease of exposition, I model firm behavior using the standard model of a single-product, two-period monopolist. I make two adjustments to the standard model. First, I adjust the cooperative's objective to reflect its customers' ownership. Second, I assume that firms face capital requirements, as they would if they operated in the insurance or banking industries. To examine the effect of customer ownership on growth, pricing and capital structure, I benchmark my results for the cooperative against those of the corporation, which faces capital requirements but has the usual objective of profit maximization.

There are four periods, $t = 0, 1, 2, 3$. Firms are capitalized in period zero. They operate in periods one and two. They liquidate in period three. There is no discounting.

Firms face constant marginal costs $c_t > 0$ for $t \in \{1, 2\}$. Growth emerges as a feature of the model because of an exogenous shock to marginal costs. Put $\delta := (c_1 - c_2)/c_1$, so that δ measures the percentage by which marginal costs *fall* between the first and second periods. For example, if $\delta = 1/2$, then the marginal cost falls by 50%. I will refer to δ as the *technology shock*. The technology shock can be interpreted as an innovation in the transaction technology (e.g. the Internet, smartphone apps), risk assessment (e.g. actuarial tables, credit reports) or knowledge (e.g. learning-by-insuring, learning-by-lending).

Firms face a twice continuously differentiable demand D, which is defined for strictly positive prices. D satisfies $D > 0$, $D' < 0$, and $D'' > 0$. Put $P := D^{-1}$. Define the elasticity and curvature of D to be $\epsilon(p) := -pD'(p)/D(p)$ and $\sigma(D) := -pD''(p)/D'(p)$ respectively. D is such that for all $p < 0$,

(**A1**) $\sigma(p) < 2\epsilon(p)$,

(**A2**) $\sigma(p) \leq 1 + \epsilon(p)$,

(**A3**) $\int_p^\infty D(s)ds < \infty$, and

(**A4**) $\lim_{\rho \to \infty} \rho D(\rho) = 0$.

(**A1**) guarantees the strict quasiconcavity of one-period profit (and hence the uniqueness of the profit-maximizing price). (**A2**)—without too much loss of generality—simplifies the proof of Proposition 2. (**A3**) guarantees the existence of the consumer surplus. (**A4**) guarantees the existence of a profit-maximizing price. Exponential demand and constant elasticity demand (with elasticity strictly greater than one) satisfy **A1** – **A4**. Linear demand doesn't, but it results in particularly well-posed problems for both the cooperative and corporation, as the objectives and constraints are all concave.

The first assumption deals with firms' market power.

Assumption 1. *Firms are monopolists.*

Reality is more complicated. Mutual insurance companies compete with a plethora of carriers. Credit unions compete with just about anyone who makes home and auto loans or issues credit cards. To model this rich ecosystem of financial institutions would cloud the basic intuition of the theory. For simplicity and clarity, I assume that firms are monopolists and leave the task of developing a richer theory to future work.

Define the profit $\Pi(p;c) := (p-c)D(p)$ and the consumer surplus $\mathcal{S}(p) := \int_p^\infty D(s)ds$. As it will appear frequently, define the monopoly price to be $p^m(c) := \operatorname{argmax}_p \Pi(p;c)$. $p^m(c)$ exists and is unique (see Lemma 4 in the appendix).

The next assumption deals with firms' sources of equity.

Assumption 2. *A firm's internal funds are its only source of equity.*

Assumption 2 might seem strong, but it turns out to be natural for the cooperative. Suppose a firm needs equity. By equity, I mean the residual claim. The residual claim, because of its riskiness, gives its holder two rights: the right to control the firm and the right to appropriate the firm's profits. These rights constitute ownership (Hansmann, 2009). If the firm is a cooperative, then it can't issue residual claims to non-customers. If it did, it would no longer be customer owned. Now it can issues residual claims to its customers, but the practice is uncommon.[9]

There's no reason why the corporation can't issue equity. I make Assumption 2 for the corporation so that I can make a clean, clear and fair comparison with the cooperative. All of the paper's main results go through without it.

To fix ideas, I assume that firms pay dividends upon liquidation. Again, this assumption doesn't change the paper's main results. It just relieves us from the burden of deriving a dividend policy.

Assumption 2 implies that a firm's stock of equity, m_t, evolves according to

$$m_{t+1} = \Pi(p_t; c_t) + m_t. \tag{1}$$

The firm makes profits and increases its stock of equity by retaining earnings. The stock is neither depleted through dividend payments nor replenished through equity issuance.

[9]See Lund (2013) and Section 4.

The last assumption deals with firms' capital structures.

Assumption 3. *Firms face capital requirements.*

Whether to appease regulators, creditors, or investors, firms must holds some minimum fraction of equity in their capital structure. They are free to hold a larger fraction. The assumption can be interpreted in a variety of ways, depending on the nature of the business. In insurance and banking, firms face state imposed capital requirements. In non-financial industries, the "capital requirement" amounts to "risk management." I will defer discussion of real-world capital requirements to Section 4.

Formally, firms face a capital requirement of the form

$$kD(p_t) \leq m_t \tag{CRt}$$

for $k \in (0, 1)$. Taken literally, CRt says that firms must hold a fraction k of sales in equity. Recall that in an insurance company, "sales" means "policies" (a liability); in a bank, "sales" means "loans" (an asset) or "deposits" (a liability), so the capital requirement is a constraint on leverage.

I assume that the corporation maximizes profit. For the cooperative, I adopt the objective postulated by Enke (1945): the cooperative maximizes total surplus, which is the sum of consumer surplus and profit.[10] The total surplus has a number of desirable properties. First, it clearly reflects the fact that the cooperative's members are both customers (consumer surplus) and owners (profit). Second, the optimal price is the marginal cost, so the cooperative is a natural non-profit. What Enke's objective lacks in specificity, it makes up for in clarity, elegance and generality.

The timeline is as follows:

Period 0 : Owners learn c_1 and capitalize the firm with equity $m_1 > 0$.

Period 1 : Owners learn c_2 and set price p_1.

Period 2 : Owners earn profit $\Pi(p_1; c_1)$ and set price p_2.

Period 3 : Owners earn profit $\Pi(p_2; c_2)$ and pay liquidating dividend m_3.

To widen the scope of the model's applications, I don't attempt to model the capitalization process in period 0. For the remainder of the paper, I treat m_1 as if it were exogenous.

[10] Because firms are assumed to be monopolists (Assumption 1), the industry profit and the firm profit are the same.

3.1 What's in a Price?

In this section, I define the cooperative's and corporation's problems. I show that they always have solutions and I present the paper's core result: the cooperative's first period price is increasing in the technology shock, while the corporation's first period price is constant.

Formally, the *cooperative's problem* is to

$$\max_{p>0} \int_{p_1}^{\infty} D(s)ds + \int_{p_2}^{\infty} D(s)ds + m_3 \tag{2}$$

$$\text{subject to} \tag{3}$$

$$kD(p_1) \leq m_1, \tag{CR1}$$

$$kD(p_2) \leq m_2 = m_1 + \Pi(p_1; c_1) \text{ and} \tag{CR2}$$

$$m_3 = \Pi(p_1; c_1) + \Pi(p_2; c_2) + m_1. \tag{4}$$

The cooperative operates in the first and second periods, offering favorable prices to its customer-owners. In the third period, it pays a liquidating dividend. By substituting m_3, we see that it maximizes total surplus each period.

The solution to the cooperative's *unconstrained* problem is to offer the marginal cost price each period. Any other price creates a deadweight loss for its customer-owners. The solution to its *constrained* problem is far more nuanced and is the subject of this section.

It's worth pointing out how growth, pricing and capital structure enter the cooperative's problem. The role of pricing should be evident (prices are the choice variables). Growth is the percentage by which sales increase between the first and second periods. Capital structure comes from the capital requirements. If the capital requirement binds, then the firm is very levered; if it doesn't, then the firm is less levered. At the heart of the paper's results is the profit term in the second period capital requirement. It represents the firm's internal funds. By adjusting this term in the first period, the firm adjusts the amount of growth that it can support in the second period.

Fortunately, the cooperative's problem admits solutions under the assumptions made thus far.

Lemma 1. *There is a solution to the cooperative's problem. Moreover, it satisfies the Kuhn-Tucker conditions.*

Throughout the paper, I benchmark the cooperative's behavior to that of the corporation.

The *corporation's problem* is to

$$\max_{p>0} m_3 = \Pi(p_1; c_1) + \Pi(p_2; c_2) + m_1 \tag{5}$$

$$\text{subject to} \tag{6}$$

$$kD(p_1) \leq m_1 \text{ and} \tag{CR1}$$

$$kD(p_2) \leq m_2 = m_1 + \Pi(p_1; c_1) \tag{CR2}$$

and it also admits solutions.

Lemma 2. *There is a solution to the corporation's problem. Moreover, it satisfies the Kuhn-Tucker conditions.*

Our first result deals with the corporation's prices.

Proposition 1. *The corporation's first period price is constant with respect to the technology shock.*

Proposition 1 says that the corporation sets today's price to maximize today's profit. It may need additional equity to support tomorrow's growth, but today's prices won't be of any help.

It's instructive to look at the first-order conditions for p_1. If the first period capital requirement binds, then $p_1^* = P(m_1/k)$, which is constant with respect to δ. Now suppose that the first period capital requirement doesn't bind. Let λ denote the Langrange multiplier on the second period capital requirement. The corporation's first-order condition reads

$$0 = \underbrace{\Pi'(p_1^*; c_1)}_{\mathbf{A}} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{\mathbf{B}}. \tag{Corporation}$$

Since $\lambda \geq 0$, we have that $\Pi'(p_1^*; c_1) = 0$. p_1^* doesn't depend on λ and λ is the only variable tying p_1^* to δ, so p_1^* doesn't depend on δ either. In fact, $p_1^* = p^m(c_1)$. The corporation's first period price is constant with respect to the technology shock. Increasing its internal funds (**B**) is consistent with maximizing profit (**A**).

Now that we have a benchmark in place, let's turn our attention back to the cooperative. Before proceeding, I segregate cooperatives into those that are *adequately capitalized* and those that are *poorly capitalized*. This segregation has no economic importance and is meant only to ease the exposition.

Definition 1. *The cooperative is adequately capitalized if $m_1 > kD(c_1 - k)$ and poorly capitalized otherwise.*[11]

[11] I have to assume that the minimum equity-to-sales ratio is sufficiently small ($k < c_1$).

The corporation's pricing problem is time-separable. It sets today's price to maximize today's profit, without regard for tomorrow's growth opportunity. This is always true, regardless of the state of its internal funds. With this result in mind, the following result about the cooperative is surprising.

Proposition 2. *If the cooperative is adequately capitalized, then for sufficiently large technology shocks ($\delta > 1 - P(m_1/k)/c_1$), its first period price is strictly increasing in the technology shock. Its first period price is constant for all other technology shocks. If the cooperative is poorly capitalized, then its first period price is non-decreasing in the technology shock.*

Proposition 2 says that the cooperative adjusts today's price according to tomorrow's growth opportunity. Its pricing problem is *not* time-separable. Today's prices depend on information about the future.

The first-order conditions are again informative. Let λ be as before. The cooperative's first-order condition reads:

$$0 = \underbrace{\mathcal{S}'(p_1^*) + \Pi'(p_1^*; c_1)}_{\mathbf{A}} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{\mathbf{B}} \tag{Cooperative}$$

(recall that $\mathcal{S}$ is the consumer surplus). Now, p_1^* depends on λ and so p_1^* depends on δ. Increasing its internal funds (**B**) is *inconsistent* with maximizing first period total surplus (**A**), although it is consistent with maximizing *second* period total surplus. Unlike the corporation, the cooperative faces a trade-off. Increasing its internal funds creates slack in the second period capital requirement—which makes growth possible—but it also creates a deadweight loss.

The proof is slightly more involved than the argument that I've just made, but the result is deep. I said that $\mathcal{S}$ is the consumer surplus, but any function that depends on p_1 will tie the first period price to the technology shock. Put differently, the cooperative's first period price will depend on the technology shock, *even if I've misspecified its objective.*

3.2 Growth, Pricing and Leverage

Propositions 1 and 2 suggest that while the corporation's problem is time-separable, the cooperative's is not. The cooperative adjusts today's prices according to tomorrow's growth opportunity. The result is theoretically interesting, but it doesn't lend itself to predictions about the real world.

In what follows, I show that the cooperative's growth is positively related to prices and negatively related to leverage.

In this model, the technology shock is the natural source of variation, so I index all variables by δ. Define growth to be

$$Growth(\delta) = \gamma(\delta) := \frac{D(p_2^*(\delta)) - D(p_1^*(\delta))}{D(p_1^*(\delta))}. \tag{7}$$

It is the percentage by which sales increase between the first and second periods. If, for example, D is the demand for insurance policies, then *Growth* is policy growth. If D is the demand for loans, then *Growth* is loan growth.

Define the profit margin to be

$$ProfitMargin(\delta) = \mu(\delta) := p_1^*(\delta) - c_1. \tag{8}$$

In insurance and banking, $ProfitMargin$ is the insurance margin and the net interest margin respectively. Note that c_1 is fixed, so $ProfitMargin$ will vary one-for-one with p_1^*.

In the model, the natural measure of leverage is the ratio of equity-to-sales. If D is the demand for insurance policies, then the equity-to-sales ratio is the *solvency ratio*. If D is the demand for loans—as it is in my data work—the equity-to-sales ratio is the equity-to-loans ratio or the *capital ratio*. I will use the banking nomenclature. Define the capital ratio to be

$$CapitalRatio(\delta) = \kappa(\delta) := \frac{m_1}{D(p_1^*(\delta))}. \tag{9}$$

Leverage and the capital ratio are inverses: the higher the leverage, the lower the capital ratio and the higher the capital ratio, the lower the leverage.

The first implication of the model is that the cooperative's growth is increasing with its profit margin.

Proposition 3. *If the cooperative is adequately capitalized, then for sufficiently large technology shocks ($\delta > 1 - P(m_1/k)/c_1$), its growth is strictly increasing with its profit margin. Its profit margin is constant for all other technology shocks. If the cooperative is poorly capitalized, then its growth is non-decreasing with its profit margin.*

Proof. Fix $\delta > 0$. Consider first the adequately capitalized cooperative. If $\delta > 1 - P(m_1/k)/c_1$, then there is an open interval of technology shocks around δ in which the second period capital requirement binds, but the first period capital requirement does not (see Lemma 14 in the appendix). The second period capital requirement reads

$$kD(p_2^*(\delta)) = m_1 + \Pi(p_1^*(\delta); c_1) = m_1 + (p_1^*(\delta) - c_1)D(p_1^*(\delta)). \tag{10}$$

Rewriting it in terms of growth and the profit margin, we obtain

$$\gamma(\delta) = \frac{m_1}{kD(\mu(\delta) + c_1)} + \frac{\mu(\delta)}{k} - 1. \tag{11}$$

Differentiating γ with respect to δ, we obtain

$$\frac{d\gamma}{d\delta} = -\frac{m_1 D'(\mu + c_1)}{kD(\mu + c_1)^2} \cdot \frac{d\mu}{d\delta} + \frac{1}{k} \cdot \frac{d\mu}{d\delta}. \tag{12}$$

Proposition 2 yields

$$\frac{d\mu}{d\delta} = \frac{dp_1^*}{d\delta} > 0 \tag{13}$$

so we can safely write

$$\frac{d\gamma}{d\mu} = \frac{d\gamma}{d\delta} \Big/ \frac{d\mu}{d\delta} = -\frac{m_1 D'(\mu + c_1)}{kD(\mu + c_1)^2} + \frac{1}{k} > 0. \tag{14}$$

as desired. If $\delta \leq 1 - P(m_1/k)/c_1$, then neither the first nor the second period capital requirements bind (see, again, Lemma 14 in the appendix). In fact, $p_1^* = c_1$ so the profit margin is constant.

Suppose that the cooperative is poorly capitalized. We can no longer appeal to Lemma 14, but the arguments made for the adequately capitalized cooperative can be carried over to the poorly capitalized cooperative. If the first period capital requirement binds, then $p_1^* = P(m_1/k)$. If neither the first nor the second period capital requirements bind, then $p_1^* = c_1$. In either case, the first period price is constant with respect to the technology shock—and hence the profit margin is constant with respect to the technology shock. If the second period capital requirement binds, but the first period capital requirement does not, then we can appeal to the argument for the adequately capitalized cooperative to conclude that growth is strictly increasing with the profit margin. □

In section 4, I will take Proposition 3 to the data. Suppose we ran the regression

$$Growth_t = \alpha + \beta ProfitMargin_t + \epsilon_t. \tag{15}$$

Proposition 3 says that either $ProfitMargin_t$ is constant—in which case the regression suffers from multicollinearity—or $Growth_t$ is positively correlated with $ProfitMargin_t$, in which case β will load with a positive sign.

As a benchmark, note the following result.

Proposition 4. *The corporation's profit margin is constant.*

Proof. This is a direct consequence of Proposition 1. □

The second implication of the model is that the cooperative's growth is decreasing with its leverage (equivalently, *increasing* with its capital ratio). The intuition is a bit subtle. We know from Proposition 2 that the cooperative's first period price is increasing in the technology shock. If the cooperative observes a growth opportunity but doesn't have ample equity, it will raise its first period price, make a profit, retain its earnings and use them to support growth. By raising its first period price, it lowers its first period sales, which lowers its first period leverage (equivalently, *raises* its first period capital ratio). If the cooperative is a bank—for example—then expanding its first period net interest margin means contracting its first period loan book. It will appear *less* levered in the first period.

Proposition 5. *If the cooperative is adequately capitalized, then for sufficiently large technology shocks ($\delta > 1 - P(m_1/k)/c_1$), its growth is strictly increasing with its capital ratio. Its capital ratio is constant for all other technology shocks. If the cooperative is poorly capitalized, then its capital ratio is non-decreasing with its capital ratio.*

Proof. The proof follows the proof of Proposition 3. Consider first the adequately capitalized cooperative. Rewriting the second period capital requirement in terms of growth and the capital ratio, we obtain

$$\gamma(\delta) = \frac{\kappa(\delta)}{k} + \frac{P(m_1/\kappa(\delta))}{k} - 1. \tag{16}$$

Differentiating γ with respect to δ, we obtain

$$\frac{d\gamma}{d\delta} = \frac{1}{k} \cdot \frac{d\kappa}{d\delta} - \frac{P'(m_1/\kappa)}{k\kappa^2} \cdot \frac{d\kappa}{d\delta} \tag{17}$$

Proposition 2 yields

$$\frac{d\kappa}{d\delta} = -\frac{m_1 D'(p_1^*)}{D(p_1^*)^2} \cdot \frac{dp_1^*}{d\delta} > 0 \tag{18}$$

so we can safely write

$$\frac{d\gamma}{d\kappa} = \frac{d\gamma}{d\delta} \Bigg/ \frac{d\kappa}{d\delta} = \frac{1}{k} - \frac{P'(m_1/\kappa)}{k\kappa^2} > 0 \tag{19}$$

as desired. If $\delta \leq 1 - P(m_1/k)/c_1$, then neither the first nor the second period capital requirements bind (see, again, Lemma 14 in the appendix). $p_1^* = c_1$ so the capital ratio is constant. The argument for the poorly capitalized cooperative follows the one contained in the proof of Proposition 3. □

Again, we can think about Proposition 5 in terms of a regression:

$$Growth_t = \alpha + \beta CapitalRatio_t + \epsilon_t. \tag{20}$$

Proposition 3 says that either $CapitalRatio_t$ is constant—in which case the regression suffers from multicollinearity—or $Growth_t$ is positively correlated with $CapitalRatio_t$, in which case β will load with a negative sign.

As a benchmark, note the following result.

Proposition 6. *The corporation's capital ratio is constant.*

The profit margin and capital ratio emerge as the key determinants of the cooperative's growth. It's natural to ask how growth covaries with both the profit margin and the capital ratio. Doing so, we obtain the following elegant result:

Proposition 7. *If the cooperative is adequately capitalized, then for sufficiently large technology shocks ($\delta > 1 - P(m_1/k)/c_1$), the cooperative's growth can be written as a linear function of its profit margin and capital ratio.*

Proof. The proof follows the proof of Proposition 3. Rewriting the second period capital requirement in terms of growth, the profit margin and the capital ratio, we obtain

$$\gamma(\delta) = k^{-1}\mu(\delta) + k^{-1}\kappa(\delta) - 1. \tag{21}$$

as desired. □

At first blush, it might seem that Propositions 3, 5, and 7 are just restatements of the second period capital requirement. This is not the case. The corporation's second period capital requirement may bind, but variation in its profit margin and its capital ratio will not translate into variation in its growth.

4 Applications

In this section, I discuss customer owned firms to which the theory can be applied. For each type of firm, I assess the extent to which Assumptions 1 (market power), 2 (internal finance) and 3 (capital requirements) are satisfied. In the case of credit unions, I document two stylized facts that are consistent with Propositions 3 and 5.

4.1 Credit Unions

Credit unions are savers' cooperatives. They are owned by the people from whom they accept deposits, known as *members*. Like any other bank, credit unions sell deposits to savers and buy loans from borrowers. Borrowers are typically nominal savers and enjoy control rights comparable to those of savers.[12] Members who borrow are offered low rates on loans while members who save are offered high rates on deposits. Members' control rights are similar to those of the shareholders of large, publicly held corporations. The Federal Credit Union Act requires boards "...to be elected annually by and from the members...." In the event of a voluntary liquidation, credit unions distribute their equity to their members through a liquidating dividend.

US credit unions hold \$1.1 trillion in assets.[13] They hold 9.5% of consumer savings (\$971.2 billion) and originate 10.3% of installment credit (\$345.6 billion).[14] They originate 11% of home loans and 17% of auto loans.[15] Over 100 million US adults are members of a credit union.[16]

The theory proposed in this paper assumes that internal funds—amassed through earnings retention—are the cooperative's only source of equity. Credit unions satisfy this assumption well. The Federal Credit Union Act states that credit unions "...do not issue capital stock...[and]...must rely on retained earnings to build net worth."

US credit unions face capital requirements under federal law. They must maintain a capital asset ratio of 6%. They are also evaluated using more contemporary Risk Based Capital measures.

Because of data availability, I use the US credit union industry as a laboratory to test the theory's predictions. Data on credit unions comes from the NCUA call reports; data on traditional banks comes from the FDIC call reports. Observations are industry aggregates for the first quarter of 2003 through the last quarter of 2012. Figure 1 plots loan growth against the net interest margin. For both samples I plot a regression line. The point estimates and the standard errors can be found in Table 1. For credit unions, loan growth is positively and significantly related to the net interest margin, a finding consistent with Proposition 3. I'm unable to document a relationship for banks. This finding is either consistent with Proposition 4 or a symptom of high variance.

A similar story emerges for leverage. Figure 1 plots loan growth against the industry

[12]To illustrate the point, consider the following allegory. Bonnie and Clyde choose to become members of a credit union. They each make a minimum deposit of—say—\$5. Credit unions operates under a *one-member-one-vote rule*. So Bonnie receives the right to exactly one vote and Clyde receives the right to exactly one vote. After opening their accounts, Bonnie deposits \$9,995 and Clyde borrows \$10,005. Bonnie is a net \$10,000 depositor and Clyde is a net \$10,000 borrower. Bonnie could care less about the rate on loans and Clyde could care less about the rate on deposits, but each has one vote just the same.

[13]2017 Statistical Report. World Council of Credit Unions.

[14]Credit Union Report Year-End 2014. Credit Union National Association.

[15]First statistic: David Morrison. Credit Unions Score 11% Mortgage Market Share. Credit Union Times, August 2015. Second statistic: Michael Muckian. Credit Unions Gain Auto Loan Market Share: Experian. Credit Union Times, September 2014

[16]2017 Statistical Report. World Council of Credit Unions.

capital ratio. Again, point estimates and the standard errors can be found in Table 1. For credit unions, loan growth is positively and significantly related to the capital ratio (negatively related to leverage), a finding consistent with Proposition 5. I'm unable to document a relationship for banks. It is unclear whether this result is consistent with Proposition 6.

Finally, I attempt to test Proposition 7. Neither the net interest margin nor the capital ratio load. The two covariates have a correlation of .84 (compare with .46 for banks) so it is likely that the regression suffers from multicollinearity.

4.2 Mutual Insurance Companies

Mutual insurance companies are policyholders' cooperatives. They are owned by the people to whom they underwrite insurance policies, also known as members. Members can vote for directors and have a claim to the company's equity upon dissolution. Mutual insurance companies can—and do—pay non-obligatory dividends.[17]

In 2012, US mutual insurance companies enjoyed a 34.5% market share (28.7% for life and 39.2% for non-life); globally, mutual insurance companies enjoyed a 26.7% market share (25.0% for life and 28.9% for non-life). The two largest mutual life insurance companies are New York Life (4.8% market share) and MassMutual (2.8% market share) while the two largest mutual home/auto insurance companies are State Farm (20.3% for home, 18.7% for auto) and Liberty Mutual (6.6% for home, 5.0% for auto).[18]

Like their credit union cousins, mutual insurance companies rely heavily on retained earnings for equity.[19] They are state regulated. Most of the statutory capital requirements specify the amount of *initial* equity needed to be chartered.[20] That being said, state regulators use Risk Based Capital measures as part of their ongoing oversight role. The National Association of Insurance Commissioners (NAIC) has developed "model laws" to unify insurance regulation through the US.

4.3 The Farm Credit System

The Farm Credit System is a network of cooperative banks known as Agricultural Credit Associations (ACAs). ACAs are borrowers' cooperatives. They are owned by the people to whom they make loans. Unlike credit unions, ACAs don't have saver-members. Members

[17] New York Life, the largest mutual life insurance company in the US, announced its largest dividend in company history on November 23, 2015. http://www.newyorklife.com/about/nyl-dividend-payout.

[18] Global Mutual Market Share 2012. International Cooperative and Mutual Federation.

[19] Robert Detlefsen. Focus On The Future: Options For The Mutual Insurance Company. National Association of Mutual Insurance Companies. March, 2010.

[20] http://www.naic.org/documents/industry_ucaa_chart_min_capital_surplus.pdf.

finance their loans by issuing bonds through Farmer Mac. ACA members—like credit union and mutual insurance company members—can vote for directors.

In 2014, the Farm Credit System was responsible for 42.5% ($135 billion) of total farm debt. The Farm Credit System is regulated by the Farm Credit Administration, which has adopted capital requirements "comparable to the Basel III framework."[21] ACAs are unique among cooperative banks in that they require borrowers to purchase "at-risk stock." This practice violates Assumption 2 and should attenuate the effects predicted by Propositions 3 and 5. That being said, it does not seem that borrower stock completely covers ACA equity needs. In 2014, ACAs financed sizable 17% of their assets with retained earnings.

4.4 Rural Electricity Cooperatives

This paper has focused on cooperatives in the financial sector. Cooperatives appear in other industries as well. One of the most interesting and economically important examples comes from rural electricity provision. Rural electricity cooperatives supply 13% of US electricity (killowatt-hours) and serve 42 million Americans. Rural electricity cooperatives don't face capital requirements like banks do, but equity still seems to be important. In 2010, the industry maintained a capital-asset ratio of 30%[22].

5 Conclusion

Despite their economic significance, customer owned firms have received little theoretical or empirical attention. This paper presents a simple theory of internal finance for the customer owned firm. It shows that a customer friendly objective ties together the firm's growth, pricing and capital structure. It shows that high sales growth is achieved through high prices and low leverage. Underlying this result is a dynamic pricing problem. The price of the firm's good depends on future growth opportunities. These results stand in stark contrast to traditional theories of corporate finance and industrial organization.

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6 Appendix B: Proofs

This section contains proofs omitted from the text.

6.1 A Few Definitions

As they will appear frequently, put

$$\overline{p}_1 := \max\{P(m_1/k), p^m(c_1)\}, \tag{22}$$

$$\underline{p}_1 := \max\{P(m_1/k), c_1\}, \tag{23}$$

$$\overline{p}_2 := \max\{P(m_1/k + \Pi(\underline{p}_1; c_1)/k), p^m(c_2)\}, \tag{24}$$

$$\underline{p}_2 := \max\{P(m_1/k + \Pi(\overline{p}_1; c_1)/k), c_2\}. \tag{25}$$

It will transpire that solutions to both the cooperative's problem and the corporation's problem live in the box $B := [\underline{p}_1, \overline{p}_1] \times [\underline{p}_2, \overline{p}_2]$. That $\underline{p}_1 \leq \overline{p}_1$ follows immediately from the fact that $p^m(c_1) > c_1$. That $\underline{p}_2 \leq \overline{p}_2$ will follow from Lemma 6. Put

$$g^1(p) := m_1 - kD(p_1), \tag{26}$$

$$g^2(p) := m_1 + \Pi(p_1; c_1) - kD(p_2). \tag{27}$$

In terms of g^1 and g^2, the constraint set for both the cooperative's problem and the corporation's problem is given by

$$C := \{p \in \mathbb{R}^2_{++} \mid g^1(p) \geq 0, g^2(p) \geq 0\}. \tag{28}$$

The cooperative's problem is equivalent to

$$\max_{p \in C} f^{\mathcal{S}+\Pi}(p) := \mathcal{S}(p_1) + \Pi(p_1; c_1) + \mathcal{S}(p_2) + \Pi(p_2; c_2) \tag{29}$$

and the corporation's problem is equivalent to

$$\max_{p \in C} f^{\Pi}(p) := \Pi(p_1; c_1) + \Pi(p_2; c_2). \tag{30}$$

I will frequently use the definitions $\delta := (c_1 - c_2)/c_1$ and $p^m(c) := \operatorname{argmax}_p \Pi(p; c)$ and the assumptions **A1** – **A4**. Definitions and assumptions not contained in this appendix can be found in the body of the text.

6.2 General Results

Lemma 3. $\Pi(\bullet; c)$ *is strictly quasiconcave.*

Proof. If $p > 0$ is such that $0 = \Pi'(p; c) = D(p) + (p - c)D'(p)$, then

$$\begin{aligned} \Pi''(p; c) &= 2D'(p) + (p - c)D''(p) && (31) \\ &= 2D'(p) - \frac{D(p)D''(p)}{D'(p)} && (32) \\ &= D'(p)\left(2 - \frac{D(p)D''(p)}{D'(p)^2}\right) && (33) \\ &= D'(p)\left(2 - \frac{\sigma(p)}{\epsilon(p)}\right) > 0 && (34) \end{aligned}$$

where the last line follows by (**A1**). We conclude that $\Pi(\bullet; c)$ is strictly quasiconcave. $\square$

Lemma 4. *$p^m(c)$ exists and is unique.*

Proof. $\Pi'(c;c) = D(c) > 0$, so there is some $p_0 > c$ such that $\Pi(p_0;c) > 0 = \Pi(c;c)$. Now $\lim_{p\to\infty} pD(p) = 0$ (**A4**) and $\lim_{p\to\infty} D(p) = 0$ (a direct consequence of **A3**), so $\lim_{p\to\infty} \Pi(p;c) = 0$: there is some $p_1 > c$ such that $\Pi(p;c) < \Pi(p_0;c)$ for all $p > p_1$. $p_m(c)$, if it exists, is an element of $[c, p_1]$. But $\Pi(p;c)$ is continuous, so it achieves $p^m(c)$ on $[c, p_1]$ (Extreme Value Theorem). The uniqueness of $p^m(c)$ follows by the quasiconcavity of $\Pi(\bullet;c)$. □

Lemma 5. *Let $c > 0$. c is the unique maximizer of $\mathcal{S} + \Pi(\bullet\,;c)$.*

Proof. Let $p > 0$, $p \neq c$. Then

$$\mathcal{S}(p) + \Pi(p;c) = \int_p^\infty D(s)ds + (p-c)D(p) \tag{35}$$

$$= \int_c^\infty D(s)ds + \int_p^c D(s)ds + (p-c)D(p) \tag{36}$$

$$= \int_c^\infty D(s)ds - \int_p^c (s-c)D'(s)ds \tag{37}$$

$$< \int_c^\infty D(s)ds = \mathcal{S}(c) + \Pi(c;c). \tag{38}$$

Use integration by parts to obtain the third line; use the fact that $D' < 0$ to obtain the fourth. □

Lemma 6. *If $\underline{p}_1 \leq p_1 \leq \overline{p}_1$, then $\Pi(p_1;c_1) \geq \Pi(\underline{p}_1;c_1)$.*

Proof. If $\underline{p}_1 = c_1$, then $p_1 \geq \underline{p}_1 = c_1$ and hence $\Pi(p_1;c_1) \geq 0 = \Pi(c_1;c_1) = \Pi(\underline{p}_1;c_1)$. If $\underline{p}_1 = \overline{p}_1 = P(m_1/k)$, then the result follows immediately. If $\underline{p}_1 = P(m_1/k)$ and $\overline{p}_1 = p^m(c_1)$, then $\underline{p}_1 \leq p_1 \leq p^m(c_1)$ and hence

$$\Pi(p_1;c_1) \geq \min\{\Pi(\underline{p}_1;c_1), \Pi(p^m(c_1);c_1)\} = \Pi(\underline{p}_1;c_1) \tag{39}$$

by the strict quasiconcavity of $\Pi(\bullet\,;c_1)$ and the definition of $p^m(c_1)$. □

Lemma 7. *Let $p \in C$, $t \in \{1,2\}$. If $p_t > \overline{p}_t$, then $\Pi(\overline{p}_t;c_t) > \Pi(p_t;c_t)$.*

Proof. If $\overline{p}_t = p^m(c_t)$, then $p_t > \overline{p}_t = p^m(c_t)$ and hence $\Pi(\overline{p}_t;c_t) = \Pi(p^m(c_t);c_t) > \Pi(p_t;c_t)$. If $\overline{p}_t \neq p^m(c_t)$, then $p_t > \overline{p}_t > p^m(c_t)$ and hence

$$\Pi(\overline{p}_t;c_t) > \min\{\Pi(p_t;c_t), \Pi(p^m(c_t);c_t)\} = \Pi(p_t;c_t) \tag{40}$$

by the strict quasiconcavity of $\Pi(\bullet\,; c_t)$ and the definition of $p^m(c_t)$. □

Lemma 8. *Let $p \in C$, $t \in \{1,2\}$. If $p_t > \overline{p}_t$, then $\mathcal{S}(\overline{p}_t) + \Pi(\overline{p}_t; c_t) > \mathcal{S}(p_t) + \Pi(p_t; c_t)$.*

Proof. Use Lemma 7 and the fact that $\mathcal{S}$ is strictly decreasing. □

Lemma 9. *Let $p \in C$, $t \in \{1,2\}$. If $p_t < \underline{p}_t$, then $\underline{p}_t = c_t$.*

Proof. Suppose $\underline{p}_t \neq c_t$. If $t = 1$, then $p_1 < \underline{p}_1 = P(m_1/k) \leq p_1$ (recall that $p \in C$). We have a contradiction. If $t = 2$, then $p_2 < \underline{p}_2 = P(m_1/k + \Pi(\overline{p}_1; c_1)/k)$. Now $p \in C$, so $p_1 \geq P(m_1/k)$ and $p_2 \geq P(m_1/k + \Pi(p_1; c_1)/k)$. If $\overline{p}_1 = p^m(c_1)$, then $\Pi(\overline{p}_1; c_1) = \Pi(p^m(c_1); c_1) \geq \Pi(p_1; c_1)$. If $\overline{p}_1 \neq p^m(c_1)$, then $p_1 \geq P(m_1/k) = \overline{p}_1 > p^m(c_1)$ and hence

$$\Pi(\overline{p}_1; c_1) \geq \min\{\Pi(p_1; c_1), \Pi(p^m(c_1); c_1)\} = \Pi(p_1; c_1) \tag{41}$$

by the strict quasiconcavity of $\Pi(\bullet\,; c_1)$ and the definition of $p^m(c_1)$. Therefore,

$$p_2 < \underline{p}_2 = P(m_1/k + \Pi(\overline{p}_1; c_1)/k) \leq P(m_1/k + \Pi(p_1; c_1)/k) \leq p_2. \tag{42}$$

We again have a contradiction. □

Lemma 10. *Let $p \in C$, $t \in \{1,2\}$. If $p_t < \underline{p}_t$, then $\Pi(\underline{p}_t; c_t) > \Pi(p_t; c_t)$.*

Proof. $\underline{p}_t = c_t$ (Lemma 9) so $p_t < \underline{p}_t = c_t < p^m(c_t)$ and hence

$$\Pi(\underline{p}_t; c_t) > \min\{\Pi(p_t; c_t), \Pi(p^m(c_t); c_t)\} = \Pi(p_t; c_t) \tag{43}$$

by the strict quasiconcavity of $\Pi(\bullet\,; c_t)$ and the definition of $p^m(c_t)$. □

Lemma 11. *Let $p \in C$, $t \in \{1,2\}$. If $p_t < \underline{p}_t$, then $\mathcal{S}(\underline{p}_t) + \Pi(\underline{p}_t; c_t) > \Pi(p_t; c_t) + \mathcal{S}(p_t)$.*

Proof. $\underline{p}_t = c_t$ (Lemma 9) so $p_t < \underline{p}_t = c_t$. The result follows by Lemma 5. □

Lemma 12. *Let $p^* \in C$ be a solution to either the cooperative's problem or the corporation's problem. The constraint qualification holds at p^*.*

Proof. The derivative of (g^1, g^2) has full rank at p^*:

$$\begin{vmatrix} g^1_1(p^*) & g^1_2(p^*) \\ g^2_1(p^*) & g^2_2(p^*) \end{vmatrix} = \begin{vmatrix} -kD'(p^*_1) & 0 \\ \Pi'(p^*_1; c_1) & -kD'(p^*_2) \end{vmatrix} = k^2 D'(p^*_1) D'(p^*_2) > 0 \tag{44}$$

so the constraint qualification holds. □

Lemma 13. *Let $p^* \in C$ be a solution to either the cooperative's problem or the corporation's problem. If $g^1(p^*) = 0$, then $m_1 < D(c_1 - k)$.*

Proof. If $g^1(p^*) = 0$, then $m_1 = kD(p_1^*)$. $g^2(p^*) \geq 0$ implies that

$$0 < kD(p_2^*) \leq m_1 + \Pi(p_1^*; c_1) = kD(p_1^*) + (p_1^* - c_1)D(p_1^*) = (k + p_1^* - c_1)D(p_1^*), \tag{45}$$

which implies that $p_1^* > c_1 - k$. We conclude that $m_1 = kD(p_1^*) < kD(c_1 - k)$. □

6.3 The Cooperative

Proof of Lemma 1. I will start by showing that solutions, if they exist, live in the box B: for each $p \in C \cap B^c$, there is a $\tilde{p} \in C \cap B$ such that $f^{\Pi+\mathcal{S}}(\tilde{p}) > f^{\Pi+\mathcal{S}}(p)$. Fix $p \in C \cap B^c$. Each element of B is strictly positive in each of its coordinates. Moreover, $g^1(p_1, \bullet) \geq 0$, $g^1(\overline{p}_1, \bullet) \geq 0$ and $g^1(\underline{p}_1, \bullet) \geq 0$ by construction. Consider each of the following cases:

1. Suppose $p_1 > \overline{p}_1$ and $p_2 > \overline{p}_2$. Observe that $(\overline{p}_1, \overline{p}_2) \in C \cap B$:

$$\begin{aligned} g^2(\overline{p}_1, \overline{p}_2) &= m_1 + \Pi(\overline{p}_1; c_1) - kD(\overline{p}_2) && (46)\\ &\geq m_1 + \Pi(\overline{p}_1; c_1) - kD(P(m_1/k + \Pi(\underline{p}_1)/k)) && (47)\\ &= \Pi(\overline{p}_1; c_1) - \Pi(\underline{p}_1; c_1) \geq 0 && (48) \end{aligned}$$

   (Lemma 6). Apply Lemma 8 twice to obtain $f^{\Pi+\mathcal{S}}(\overline{p}_1, \overline{p}_2) > f^{\Pi+\mathcal{S}}(p_1, p_2)$.

2. Suppose $p_1 > \overline{p}_1$ and $p_2 < \underline{p}_2$. Observe that $(\overline{p}_1, \underline{p}_2) \in C \cap B$:

$$\begin{aligned} g^2(\overline{p}_1, \underline{p}_2) &= m_1 + \Pi(\overline{p}_1; c_1) - kD(\underline{p}_2) && (49)\\ &\geq m_1 + \Pi(\overline{p}_1; c_1) - kD(P(m_1/k + \Pi(\overline{p}_1; c_1)/k)) = 0. && (50) \end{aligned}$$

   Apply Lemmas 8 and 11 to obtain $f^{\Pi+\mathcal{S}}(\overline{p}_1, \underline{p}_2) > f^{\Pi+\mathcal{S}}(p_1, p_2)$.

3. Suppose $p_1 < \underline{p}_1$ and $p_2 > \overline{p}_2$. Observe that $(\underline{p}_1, \overline{p}_2) \in C \cap B$:

$$\begin{aligned} g^2(\underline{p}_1, \overline{p}_2) &= m_1 + \Pi(\underline{p}_1; c_1) - kD(\overline{p}_2) && (51)\\ &\geq m_1 + \Pi(\underline{p}_1; c_1) - kD(P(m_1/k + \Pi(\underline{p}_1; c_1)/k)) = 0. && (52) \end{aligned}$$

   Apply Lemmas 11 and 8 to obtain $f^{\Pi+\mathcal{S}}(\underline{p}_1, \overline{p}_2) > f^{\Pi+\mathcal{S}}(p_1, p_2)$.

4. Suppose $p_1 < \underline{p}_1$ and $p_2 < \underline{p}_2$. Observe that $(\underline{p}_1, \underline{p}_2) \in C \cap B$:

$$g^2(\underline{p}_1, \underline{p}_2) = m_1 + \Pi(\underline{p}_1; c_1) - kD(\underline{p}_2) \tag{53}$$
$$> m_1 + \Pi(p_1; c_1) - kD(p_2) = g^2(p_1, p_2) \geq 0 \tag{54}$$

   (Lemma 10). Apply Lemma 11 twice to obtain $f^{\Pi+\mathcal{S}}(\underline{p}_1, \underline{p}_2) > f^{\Pi+\mathcal{S}}(p_1, p_2)$.

5. Suppose $p_1 > \overline{p}_1$ and $\underline{p}_2 \leq p_2 \leq \overline{p}_2$. Observe that $(\overline{p}_1, p_2) \in C \cap B$:

$$g^2(\overline{p}_1, p_2) = m_1 + \Pi(\overline{p}_1; c_1) - kD(p_2) \tag{55}$$
$$> m_1 + \Pi(p_1; c_1) - kD(p_2) = g^2(p_1, p_2) \geq 0 \tag{56}$$

   (Lemma 7). Apply Lemma 8 to obtain $f^{\Pi+\mathcal{S}}(\overline{p}_1, p_2) > f^{\Pi+\mathcal{S}}(p_1, p_2)$.

6. Suppose $p_1 < \underline{p}_1$ and $\underline{p}_2 \leq p_2 \leq \overline{p}_2$. Observe that $(\underline{p}_1, p_2) \in C \cap B$:

$$g^2(\underline{p}_1, p_2) = m_1 + \Pi(\underline{p}_1; c_1) - kD(p_2) \tag{57}$$
$$> m_1 + \Pi(p_1; c_1) - kD(p_2) = g^2(p_1, p_2) \geq 0 \tag{58}$$

   (Lemma 10). Apply Lemma 11 to obtain $f^{\Pi+\mathcal{S}}(\underline{p}_1, p_2) > f^{\Pi+\mathcal{S}}(p_1, p_2)$.

7. Suppose $\underline{p}_1 \leq p_1 \leq \overline{p}_1$ and $p_2 > \overline{p}_2$. Observe that $(p_1, \overline{p}_2) \in C \cap B$:

$$g^2(p_1, \overline{p}_2) = m_1 + \Pi(p_1; c_1) - kD(\overline{p}_2) \tag{59}$$
$$\geq m_1 + \Pi(p_1; c_1) - kD(P(m_1/k + \Pi(\underline{p}_1; c_1)/k)) \tag{60}$$
$$= \Pi(p_1; c_1) - \Pi(\underline{p}_1; c_1) \geq 0 \tag{61}$$

   (Lemma 6). Apply Lemma 8 to obtain $f^{\Pi+\mathcal{S}}(p_1, \overline{p}_2) > f^{\Pi+\mathcal{S}}(p_1, p_2)$.

8. Suppose $\underline{p}_1 \leq p_1 \leq \overline{p}_1$ and $p_2 < \underline{p}_2$. Observe that $(p_1, \underline{p}_2) \in C \cap B$:

$$g^2(p_1, \underline{p}_2) = m_1 + \Pi(p_1; c_1) - kD(\underline{p}_2) \tag{62}$$
$$> m_1 + \Pi(p_1; c_1) - kD(p_2) = g^2(p_1, p_2) \geq 0. \tag{63}$$

   Apply Lemma 11 to obtain $f^{\Pi+\mathcal{S}}(p_1, \underline{p}_2) > f^{\Pi+\mathcal{S}}(p_1, p_2)$.

The solution, if it exists, must be an element of B. Put $\mathcal{C} := C \cap B$. $\mathcal{C}$ is non-empty because $(\overline{p}_1, \overline{p}_2) \in \mathcal{C}$. $\mathcal{C}$ is closed because it is defined by weak inequalities. $\mathcal{C}$ is bounded because it lives in the box B. So $\mathcal{C}$ is compact. Now $f^{\mathcal{S}+\Pi}$ is continuous. The Extreme Value Theorem tells us that $f^{\mathcal{S}+\Pi}$ attains its maximum on $\mathcal{C}$, which we've shown is its maximum on C. We have a solution, the objective and constraints are twice continuously differentiable and we know that the constraint qualification holds (Lemma 12). We appeal to Kuhn-Tuckers' Theorem: there are multipliers satisfying the Kuhn-Tucker conditions at the solution. □

Lemma 14. *Suppose that the cooperative is adequately capitalized. Then its first period capital requirement does not bind. Its second period capital requirement binds if and only if* $\delta > 1 - P(m_1/k)/c_1$.

Proof. Let $p^* \in C$ be a solution to the cooperative's problem (Lemma 1 guarantees that p^* exists). Because the cooperative is adequately capitalized, we have that $m_1 > kD(c_1 - k)$ and so Lemma 13 guarantees that $g^1(p_1^*) > 0$. Put differently, the first period capital requirement doesn't bind. If $\delta > 1 - P(m_1/k)/c_1$, then we have that $m_1 < kD((1-\delta)c_1) = kD(c_2)$ and so Lemma 5 guarantees that $g^2(p^*) = 0$. The second period capital requirement binds. Conversely, if $\delta \leq 1 - P(m_1/k)/c_1$, then we have that $m_1 \geq kD(c_2)$ and Lemma 5 guarantees that $g^2(p^*) = 0$. The second period capital requirement doesn't bind. □

Proof of Proposition 2. Suppose that the cooperative is adequately capitalized. If $\delta > 1 - P(m_1/k)/c_1$, then Lemma 14 guarantees that there's an open interval of technology shocks around δ in which the second period capital requirement binds, but its first period capital requirement doesn't. In this neighborhood, the first-order conditions are:

$$0 = -D(p_1^*) + \Pi'(p_1^*; c_1) + \lambda \Pi'(p_1^*; c_1) \tag{64}$$

$$0 = -D(p_2^*) + \Pi'(p_2^*; c_2) - \lambda k D'(p_2^*) \tag{65}$$

$$kD(p_2^*) = m_1 + \Pi(p_1^*; c_1). \tag{66}$$

Eliminating λ yields

$$kD(p_1^*) = (k + p_2^* - c_2)\Pi'(p_1^*; c_1) \tag{67}$$

$$kD(p_2^*) = m_1 + \Pi(p_1^*; c_1). \tag{68}$$

Now $kD(p_1^*) > 0$, so $\Pi'(p_1^*; c_1) \neq 0$ and we can safely write

$$p_2^* = c_2 - k + \frac{kD(p_1^*)}{\Pi'(p_1^*; c_1)}. \tag{69}$$

Using the second period capital requirement, we can write

$$P(m_1/k + \Pi(p_1^*; c_1)/k) = c_2 - k + \frac{kD(p_1^*)}{\Pi'(p_1^*; c_1)}. \tag{70}$$

Rearranging, we obtain

$$c_2 = \phi(p_1^*) := k + P(m_1/k + \Pi(p_1^*; c_1)/k) - \frac{kD(p_1^*)}{\Pi'(p_1^*; c_1)}. \tag{71}$$

All that's left to show is the invertability of the right-hand-side. Put $\psi(p_1^*) := D(p_1^*)/\Pi'(p_1^*; c_1)$. Differentiating ψ, we obtain

$$\begin{aligned}
\psi'(p_1^*) &= (\Pi'(p_1^*; c_1))^{-2}(D'(p_1^*)\Pi'(p_1^*; c_1) - D(p_1^*)\Pi''(p_1^*; c_1)) && (72)\\
&= (\Pi'(p_1^*; c_1))^{-2}(D'(p_1^*)(D(p_1^*) + (p_1^* - c_1)D'(p_1^*)) - D(p_1^*)(2D'(p_1^*) + (p_1^* - c_1)D''(p_1^*))) && (73)\\
&= (\Pi'(p_1^*; c_1))^{-2}((p_1^* - c_1)(D'(p_1^*))^2 - D(p_1^*)D'(p_1^*) - (p_1^* - c_1)D(p)D''(p_1^*)) && (74)\\
&= (\Pi'(p_1^*; c_1))^{-2}((D'(p_1^*))^2((p_1^* - c_1)(1 - D(p)D''(p_1^*)/(D'(p_1^*))^2) - D(p_1^*)/D'(p_1^*))) && (75)\\
&= (\Pi'(p_1^*; c_1))^{-2}(D'(p_1^*))^2((p_1^* - c_1)(1 - \sigma(p_1^*)/\epsilon(p_1^*)) + p_1^*/\epsilon(p_1^*)) && (76)\\
&= (\Pi'(p_1^*; c_1))^{-2}(D'(p_1^*))^2(p_1^* - c_1)(\epsilon(p_1^*) - \sigma(p_1^*) + p_1^*)/\epsilon(p_1^*) && (77)\\
&\geq (\Pi'(p_1^*; c_1))^{-2}((D'(p_1^*))^2(-(p_1^* - c_1) + p_1^*)/\epsilon(p_1^*)) && (78)\\
&= (\Pi'(p_1^*; c_1))^{-2}c_1(D'(p_1^*))^2/\epsilon(p_1^*) && (79)\\
&> 0, && (80)
\end{aligned}$$

having used the facts that $p_1^* \geq c_1$ (Lemma 1) and $\sigma(p) \leq 1 + \epsilon(p)$ (**A2**). Together with the fact that $P' < 0$, we conclude that $\phi' < 0$ by the inverse function theorem. Finally,

$$\frac{dp_1^*}{d\delta} = \frac{dp_1^*}{dc_2} \cdot \frac{dc_2}{d\delta} = -c_1\phi'(p_1^*) > 0 \tag{81}$$

so that the first period price is strictly increasing in the technology shock.

If $\delta \leq 1 - P(m_1/k)/c_1$, then Lemma 14 says that neither the first nor the second period capital requirements bind. In fact, $p_1 = c_1$ so the first period price is constant with respect to the technology shock.

Suppose that the cooperative is poorly capitalized. We can no longer appeal to Lemma 14, but the arguments made for the adequately capitalized cooperative can be carried over to the poorly capitalized cooperative. If the first period capital requirement binds, then $p_1^* = P(m_1/k)$. If neither the first nor the second period capital requirements bind, then $p_1^* = c_1$. In either case, the first period price is constant with respect to the technology shock. If the second period capital requirement binds, but the first period capital requirement doesn't, then we can appeal to the argument for the adequately capitalized cooperative to conclude that the first period price is strictly increasing in the technology shock.

$\square$

6.4 The Corporation

Proof of Lemma 2. The proof is identical to that of Lemma 1, except that Lemma 7 should be used in place of Lemma 8, Lemma 10 should be used in place of Lemma 11 and f^{Π} should be used in place of $f^{\mathcal{S}+\Pi}$. $\square$

Proof of Proposition 1. Let $p^* \in C$ be a solution to the corporation's problem and let $\lambda \geq 0$ denote the Lagrange multiplier on the constraint $g^2(p) \geq 0$. If $g^1(p^*) = 0$, then $p_1^* = P(m_1/k)$. If $g^1(p^*) > 0$, then the condition for p_1 is $0 = (1+\lambda)\Pi'(p_1^*; c_1)$, which implies that $p_1^* = p^m(c_1)$. $\square$

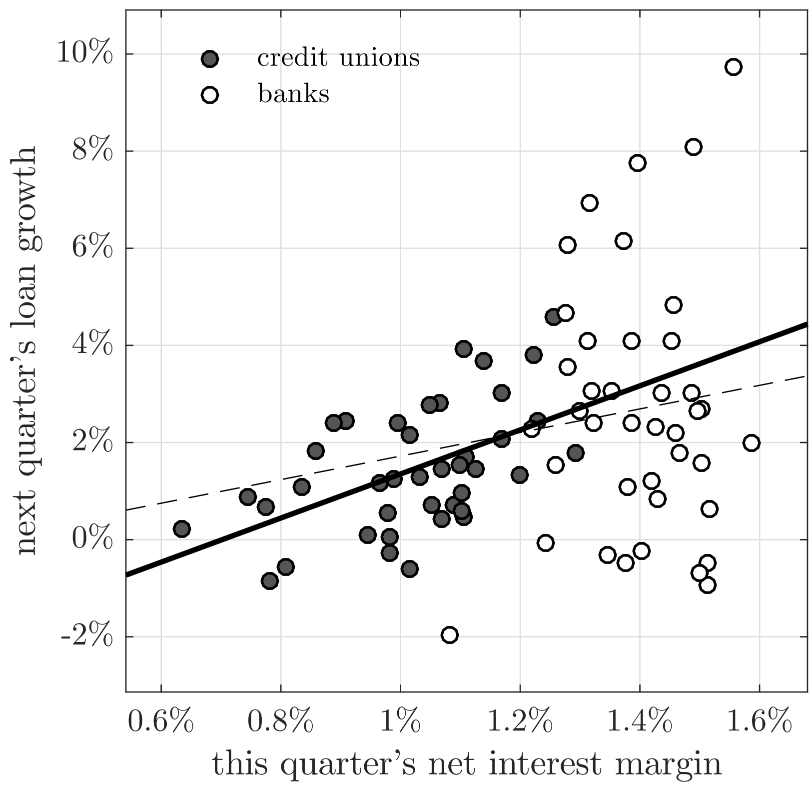

Figure 1: **Loan Growth and Net Interest Margins.** This figure plots loan growth against the net interest margin for credit unions and banks. An observation is a quarter between the first quarter of 2003 and the fourth quarter of 2012. Loan growth in quarter t is the aggregate change in loans from quarter t to quarter $t+1$ divided by the aggregate loans in quarter t. The net interest margin in quarter t is the aggregate net interest income in quarter t divided by the aggregate loans in quarter t. The coefficient on the net interest margin is significant and positive for credit unions and insignificant for banks (see Table 1). The R^2 is 27.3% for the credit union regression and 1.0% for the bank regression. The solid line is the regression line for credit unions; the dashed line is the regression line for banks. Data on credit unions comes from the NCUA call reports; data on banks comes from the FDIC call reports.

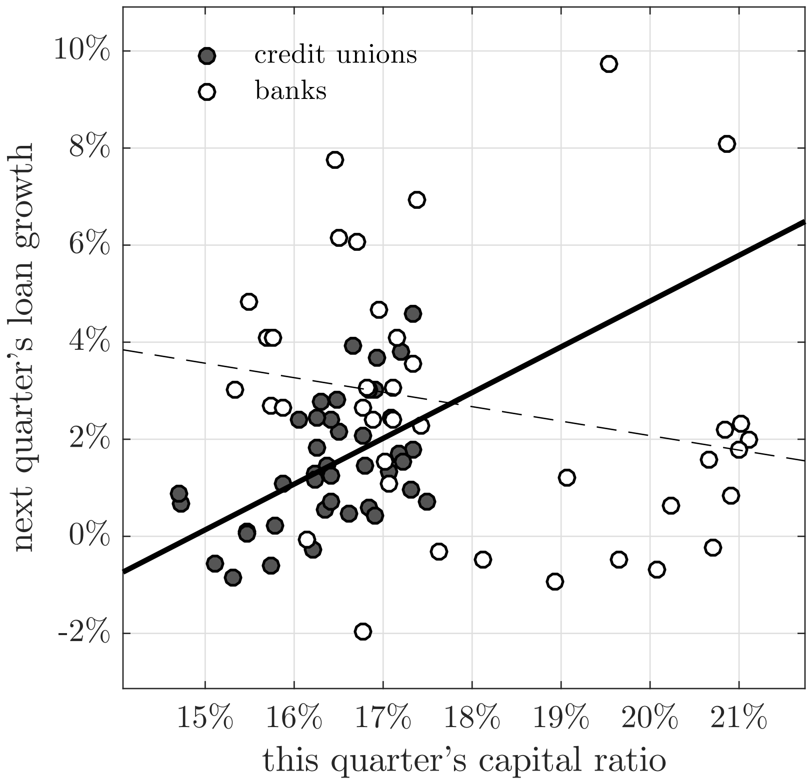

Figure 2: **Loan Growth and Capital Ratios.** This figure plots loan growth against the capital ratio for credit unions and banks. An observation is a quarter between the first quarter of 2003 and the fourth quarter of 2012. Loan growth in quarter t is the aggregate change in loans from quarter t to quarter $t+1$ divided by the aggregate loans in quarter t. The capital ratio in quarter t is the aggregate equity capital in quarter t divided by the aggregate loans in quarter t. The coefficient on the capital ratio is significant and positive for credit unions and insignificant for banks (see Table 1). The R^2 is 26.8% for the credit union regression and 4.6% for the bank regression. The solid line is the regression line for credit unions; the dashed line is the regression line for banks. Data on credit unions comes from the NCUA call reports; data on banks comes from the FDIC call reports.

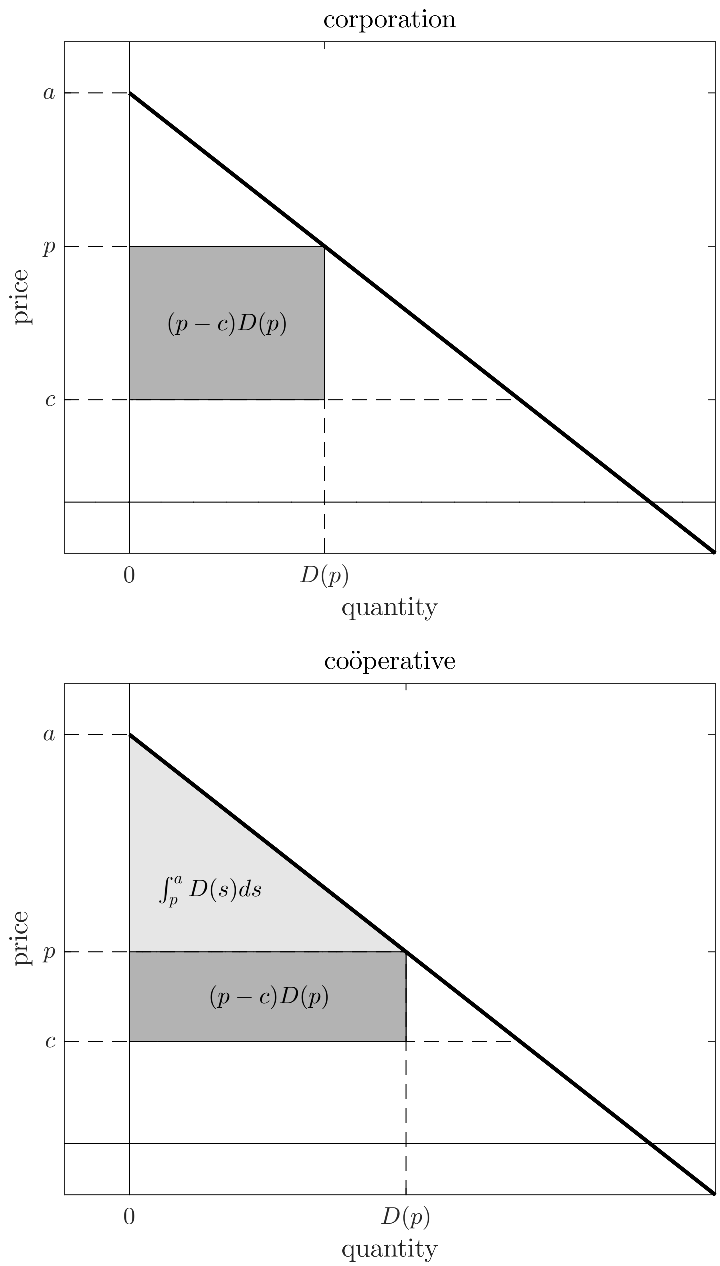

Figure 3: **The Corporation's (Top) and Cooperative's (Bottom) Objectives.** This figure illustrates the corporation's and cooperative's objectives under linear demand. The corporation maximizes the profit $\Pi(p;c) = (p-c)D(p)$ (the dark rectangle). The co-operative's members are customers, and so it maximizes the consumer surplus $\int_0^1 D(s)ds$ (the light triangle), but its members are also owners, and so it maximizes the profit $\Pi(p;c) = (p-c)D(p)$ (the dark rectangle). It should be clear that $p > c$ is inefficient in a static setting (although it will be in a dynamic setting).

	Loan Growth					
	Credit Unions			Banks		
	(1)	(2)	(3)	(4)	(5)	(6)
Net Interest Margin	4.537***		2.585	2.427		6.143
	(1.200)		(2.179)	(4.004)		(4.361)
Capital Ratio		0.943***	0.490		−0.298	−0.456*
		(0.252)	(0.457)		(0.220)	(0.244)
Constant	−0.032**	−0.140***	−0.092	−0.007	0.080*	0.023
	(0.012)	(0.041)	(0.058)	(0.056)	(0.040)	(0.057)
Observations	40	40	40	40	40	40
R^2	27.3%	26.8%	29.5%	1.0%	4.6%	9.5%
F Statistic	14.290***	13.939***	7.748***	0.367	1.840	1.936

Note: *p<0.1; **p<0.05; ***p<0.01

Table 1: **Loan Growth, Net Interest Margins, and Capital Ratios.** This table displays loan growth regressions. An observation is a quarter between the first quarter of 2003 and the fourth quarter of 2012. Loan growth in quarter t is the aggregate change in loans from quarter t to quarter $t+1$ divided by the aggregate loans in quarter t. The net interest margin in quarter t is the aggregate net interest income in quarter t divided by the aggregate loans in quarter t. The capital ratio in quarter t is the aggregate equity capital in quarter t divided by the aggregate loans in quarter t. Data on credit unions comes from the NCUA call reports; data on banks comes from the FDIC call reports.