Lending of Last Resort in an Open Economy*

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Abstract

The paper analyzes a small open economy with flexible exchange rates in which the financial sector is subject to confidence crises. A feedback between the health of the financial sector and the exchange rate amplifies the effects of the crisis. The presence of dollarized liabilities is crucial for this mechanism. Dollarized liabilities arise endogenously when domestic investors expect a currency crisis with sufficiently high probability and switch their deposits from domestic to foreign currency denomination. We use this framework to analyze operations of lending of last resort. The presence of the exchange rate channel raises the amount of resources necessary to backstop a confidence crisis. Precautionary reserve accumulation by the fiscal authority facilitates effective lending of last resort, and can lead to a less dollarized financial sector and to a more stable exchange rate.

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1 Introduction

Banking panics are a common feature of financial crises, both in emerging and in developed economies. The main feature of a banking panic is that investors lose confidence in the short term liabilities of financial institutions, leading to a loss of funding for these institutions, to depressed asset values, and, eventually, to a contraction in credit and investment. In an open economy, with an open capital account, the problem is often compounded by a generalized loss in confidence in *all* domestic assets, leading to a capital flight. The central bank can respond to a banking panic by acting as a lender of last resort, providing emergency lending facilities to the banks in trouble. However, the combination of banking panic and capital flight makes lending of last resort particularly challenging in open economies.

Fixed exchange rate regimes offer stark examples of situations in which the domestic central bank lacks sufficient reserves to satisfy the demand of investors who are trying to convert domestic bank liabilities into foreign currency assets at a fixed rate. These situations eventually lead to a joint banking and currency crisis, i.e., a twin crisis.[1] An open question is whether a regime of flexible exchange rates makes it easier or harder for the domestic central bank to act as a lender of last resort and mitigate a financial crisis. In this paper we explore this question by building a macroeconomic model with an intermediation sector that borrows both from foreign investors and from domestic consumers to finance purchases of risky assets.

The model features multiple equilibria which have the typical features of a banking panic with capital flight. In the bad equilibrium, all investors, domestic and foreign, are less willing to extend credit to banks and, at the same time, the economy overall experiences a current account reversal. We consider interventions by a central bank with limited resources, namely, foreign exchange reserves and a fixed amount of domestic fiscal revenue. We show that the presence of a fully flexible exchange rate and an inflation targeting regime does not eliminate the possibility of multiple equilibria.

Our analysis proceeds in two stages. First, we focus on the period in which the panic occurs, taking as given the economy's initial conditions, including the assets and liabilities of the banks. We show that a flexible exchange rate regime is especially exposed to panics when three weaknesses are present: high leverage of domestic banks; high levels of foreign denominated liabilities; the fact that the fiscal resources that back up central bank interventions are in domestic currency. The ingredients behind these results are not new, leverage plays a similar role as in Gertler and Kiyotaki (2015) in a closed econ-

[1] See for instance Kaminsky and Reinhart (1999).

omy context, and the role of foreign liabilities is in line with the literature following the Asian crisis of 1997-1998, for example Aghion, Bacchetta, and Banerjee (2001), Corsetti, Pesenti, and Roubini (1999) and Chang and Velasco (2001). However, the connection between banks' weak balance sheets and the real exchange rate is derived in a novel (and, we think, realistic) way and the role of the fiscal backing of the central bank is new.

In the second stage of the analysis, we take a step back and analyze the determinants of the banks' balance sheets. Here, we focus on the endogenous choice of domestic consumers between domestic-denominated and dollar-denominated deposits. Our crucial result here is that this endogenous choice does not eliminate multiplicity, but actually adds a new layer to it. Consumers who anticipate banking panics, associated to large fluctuations of the exchange rate, tend to prefer dollar deposits, since they give them some protection against a devaluation. However, their preference for dollar deposits is exactly what pushes banks towards greater degrees of mismatch and thus to a higher probability of a panic. On the other hand, if consumers do not expect banking panics to occur, they have a natural preference for domestic deposits, because, due to the central bank's inflation targeting, domestic deposits provide more stability in terms of domestic purchasing power. So the presence or absence of panics feeds back into the asset choices of consumers, because it changes the nature of exchange rate fluctuations. Without panics, exchange rate risk is just an unwanted additional source of risk for domestic agents. With panics, foreign assets become a good hedge, because the currency tends to depreciate exactly when the country is in a crisis. The endogenous nature of exchange rate risk in presence of financial crises is the main innovative contribution of our paper.

Literature. (To be added. Here is a partial list of closely related papers) Twin crises and the exchange rate regime: Jeanne and Wyplosz (2003), Céspedes, Chang, and Velasco (2000). Financial intermediation and open economy macro: Gertler and Karadi (2011), Maggiori and Gabaix (2015), Bruno and Shin (2014). Foreign reserves: Obstfeld, Shambaugh, and Taylor (2010), Aizenman and Lee (2007), Ranciere and Jeanne (2006).

2 Model

We consider a small open economy that lasts three periods, $t = 0, 1, 2$. There are three domestic agents in the economy: households, banks and capital good producers. The economy produces two types of goods. The tradable goods can be traded with the rest of the world, while the non-tradables are produced and consumed domestically. Households work in every period for the sector producing tradable goods, earning a wage, and they get the profits from capital good producers. They then face a standard consumption-saving problem. The households can save in two types of assets: bonds denominated in domestic currency and bonds denominated in foreign currency. We assume that domestic currency bonds are issued only by the domestic banks, while the bonds that pay in foreign currency are issued by both foreigners and domestic banks. The banks borrow from domestic households and from foreign investors in order to buy new capital goods from capital good producers. These capital goods are then used by the banks to produce the final goods.[2]

We now turn to a description of the environment, the agents' decision problem, and a definition of the equilibrium for this economy.

2.1 Agents and their decision problems

2.1.1 Households

The households enter period t with claims from domestic banks and foreigners. We denote by a_t the claims denominated in domestic currency (the nummeraire), while a_t^* are claims that pay in foreign currency. We denote by s_t the exchange rate, expressed in units of domestic currency per unit of foreign currency. Beside this capital income, the households earn a wage w_t from working for the producers of tradable goods, and they receive every period an endowment of non-tradable goods, e^N. They also earn profits from the capital good producers, Π_t. The households use this income to buy tradable and non-tradable goods, and to invest in assets. Accordingly, the period t budget constraint for the households is given by

$$a_{t+1} + s_t q_t^* a_{t+1}^* + p_t^T C_t^T + p_t^N C_t^N \leq w_t + p_t^N e^N + \Pi_t + a_t + s_t a_t^*, \tag{1}$$

where q_t and q_t^* are the prices of bonds that return, respectively, one unit of domestic and foreign currency next period, p_t^N is the price of non-tradable goods, p_t^T the price of tradable goods, and C_t^N and C_t^T their respective consumption.

The household flow utility function is $U(C_t)$, where C_t is a consumption aggregator,

$$C_t = (C_t^T)^\omega (C_t^N)^{1-\omega}.$$

[2]Effectively, our banks consolidate the banking and the productive sector of the economy.

The households choose $\{a_{t+1}, a^*_{t+1}, C^T_t, C^N_t\}$ in order to maximize lifetime utility

$$\mathbb{E}_0 \left[\sum_t U(C_t) \right]$$

subject to the sequence of budget constraints in (1).

We can further simplify the households' problem by noting that the optimal allocation between tradable and non-tradable goods satisfies

$$\frac{\omega}{1-\omega} \left(\frac{C^N_t}{C^T_t} \right) = \frac{p^T_t}{p^N_t}. \tag{2}$$

Substituting this expression into the definition of consumption expenditure we have

$$p^T_t C^T_t + p^N_t C^N_t = \underbrace{\left(\frac{\omega}{1-\omega} \right)^{1-\omega} (p^N_t)^{1-\omega} (p^T_t)^{\omega}}_{P_t} C_t, \tag{3}$$

where P_t is domestic CPI. The problem can then be collapsed into choosing the sequence of consumption baskets $\{C_t\}$, with the households allocating a fraction ω of consumption expenditures to tradable goods and a fraction $(1-\omega)$ to non-tradable goods.

2.1.2 Banks

The banks enter period t with assets and liabilities. On the asset side, the banks hold two types of capital goods, k^T_t and k^N_t. The former are used to produce the tradable good y^T_t while the latter are used to produce the non-tradable good y^N_t. These capital goods trade at market prices Ψ^T_t and Ψ^N_t. They also earn rents. Specifically, the banks use the capital stocks to produce tradable and non-tradable goods which are sold in competitive markets. The technology for producing tradable goods is given by,

$$y^T_t = (k^T_t)^{\alpha} (l_t)^{1-\alpha},$$

while the technology for producing non-tradables is linear in k^N_t. We denote by R^T_t and R^N_t the rents accrued to capital from the production of the final goods. The rental rates, expressed in domestic currency, are then given by

$$R^T_t = p^T_t \alpha (k^T_t)^{\alpha-1} l^{\alpha},$$

$$R_t^N = p_t^N. \tag{4}$$

On the liability side, the banks repay the bonds issued in the previous period in domestic and foreign currency, respectively b_t and b_t^*. Accordingly, the time t net worth of the banks (expressed in domestic currency) is given by

$$n_t = (\Psi_t^T + R_t^T)k_t^T + (\Psi_t^N + R_t^N)k_t^N - b_t - s_t b_t^*. \tag{5}$$

Given their accumulated net worth, the banks borrow from households and foreigners to buy new capital goods,

$$\Psi_t^T k_{t+1}^T + \Psi_t^N k_{t+1}^N = n_t + q_t b_{t+1} + s_t q_t^* b_{t+1}^*. \tag{6}$$

Importantly, they face a collateral constraint,

$$q_t b_{t+1} + s_t q_t^* b_{t+1}^* \leq \theta \Psi_t^T k_{t+1}^T. \tag{7}$$

We assume that banks consume all their accumulated net worth in the final period $(t = 2)$, and they do not discount the future. Therefore, the decision problem of the banks consists in choosing $\{k_{t+1}^T, k_{t+1}^N, b_{t+1}, b_{t+1}^*\}$ in order to maximize the final period net worth n_2 subject to the law of motion of net worth implied by the banks' choices and by their balance sheet, and subject to the sequence of collateral constraints in (7).

2.1.3 Capital good producers

The capital good producers can transform tradable goods into capital for the tradable sector and vice-versa. In order to produce i_t units of capital, the producers need $G(i_t)$ units of tradable goods. This function is convex, with the property that $G(i_t) > 0$ if $i > 0$, $G(i) < 0$ if $i < 0$, and $G(0) = 0$. Negative investment is interpreted as the activity of transforming capital into tradable goods. The profit function for the capital good producers is then,

$$\Pi_t = \Psi_t^T i_t - p_t^T G(i_t).$$

The decision problem of capital good producers is static: they choose i_t period by period in order to maximize their profits.

We assume that it is not feasible to produce new capital for the production of non-tradable goods. This effectively amounts to assume that every period the economy has a fixed endowment of non-tradable goods equal to $y^N = k_0^N + e^N$.

2.1.4 Foreign Investors

Foreign investors act as if they are risk neutral toward the small open economy, and they demand bonds issued by the domestic banks. We assume that foreign investors cannot lend to the banks in domestic currency. In equilibrium, we must therefore have $b_t = a_t$. The Euler equation of foreign investors determines the price of bonds issued in foreign currency,

$$q_t^* = \beta_f, \tag{8}$$

where β_f is their discount factor.

We denote by p_t^{T*} the price of tradable goods abroad. Because the law of one price holds for tradable goods, one must have that

$$p_t^T = s_t p_t^{T*}. \tag{9}$$

We assume that the price of tradable goods abroad is subject to random shocks at $t = 1$. Specifically, at $t = 1$ a shock realizes, and the price of non-tradables is $p_1^{T*} = \varepsilon$. The shock is assumed to be permanent, $p_1^{T*} = p_2^{T*}$. Further, we assume that $\mathbb{E}[\varepsilon] = 1$, and we denote by σ_ε the variance of ε. These purely nominal disturbances generate fluctuations in the exchange rate, and they are introduced in the model in order to capture the notion that exchange rates movements are often orthogonal to the economic fundamentals of an economy.

2.1.5 Monetary authority

We assume that the monetary authority successfully targets a constant CPI. Domestic CPI is given by the price of the consumption basket optimally chosen by the households,

$$P_t = \xi (p_t^T)^\omega (p_t^N)^{1-\omega}.$$

Keeping $P_t = \xi$, the behavior of the monetary authority implies a relation between the price of tradables and the price of non-tradable goods,

$$p_t^T = \left(p_t^N\right)^{-\frac{(1-\omega)}{\omega}}.$$

Coupled with the law of one price in equation (9), this assumption implies that the economy operates under a flexible exchange rate regime, with the exchange rate being given by

$$s_t = \frac{\left(p_t^N\right)^{-\frac{(1-\omega)}{\omega}}}{p_t^{T*}}. \tag{10}$$

2.2 Equilibrium

A competitive equilibrium for the small open economy is a collection of prices in the capital markets $\{\Psi_t^T, \Psi_t^N\}$, prices in domestic and foreign bond markets $\{q_t, q_t^*\}$, prices in the factor markets $\{R_t^T, R_t^N, w_t\}$, prices in the good markets $\{p_t^T, p_t^N, p_t^{T*}\}$, the exchange rate $\{s_t\}$, asset and consumption choices for households $\{a_{t+1}, a_{t+1}^*, C_t^N, C_t^T\}$, asset choices for banks $\{k_t^T, k_t^N, b_{t+1}, b_{t+1}^*\}$, and investment choices for the capital good producers $\{i_t\}$ such that **(i)** The choices of households, banks, and capital good producers solve their respective decision problem; **(ii)** all domestic markets clear; **(iii)** q_t^* and $p_t^{T^*}$ are determined abroad; **(iv)** and the law of one price holds; **(v)** the monetary authority is committed to price stability.

3 Twin Crises with flexible exchange rates

We now show that the model can generate multiple equilibria. Specifically, we will focus on two types of outcomes, that we will label the "good" and the "bad" equilibrium. The former is supposed to mimic the behavior of the small open economy in normal times, while the latter is characterized by a twin crisis- a financial crisis that occurs in conjunction with a depreciation of the domestic currency. The main ingredient that allows the model to generate these bad equilibria is the households' ability to change the currency denomination of their savings.

In the good equilibrium, households are willing to hold assets denominated in domestic currency. Thus, the domestic banking sector can finance its operations mostly by issuing domestic currency liabilities. Because of that, fluctuations in the exchange rate have limited impact on the net worth of the banking sector, making the latter less prone to twin crises.

The bad equilibrium, instead, originates from households' desire to hold assets denominated in foreign currency. When this motive is sufficiently strong, banks are forced to finance themselves mostly by issuing liabilities in foreign currency. As the currency mismatch in the banks' balance sheet opens up, the domestic banking sector becomes more exposed to confidence crises, which take the form of depressed asset values, depressed economic activity and a depreciation of the exchange rate. This situation is self-fulfilling because households have a precautionary motive to denominate their savings in foreign currency when the prospects of a confidence crisis loom. Foreign currency assets are, in fact, a good hedge during a crisis because they appreciate precisely when households' wealth plummets.

In order to illustrate how these two types of equilibria can arise in the model, we proceed in three steps. In the first step, we consider the allocations that are consistent with a competitive equilibrium from $t = 1$ onward, taking as given the asset structure inherited from period 0. In lack of a better term, we will refer to these allocations as *continuation equilibria*. In the second step, and taking as given the optimal behavior from $t = 1$ onward, we will study the determination of the asset structure at $= 0$. Finally, we construct examples of self-fulfilling twin crises.

3.1 Continuation equilibria

We start characterizing the continuation equilibria of the model by describing households' optimal behavior. There is no uncertainty left from $t = 1$ onward, and the households decide how to allocate their life-time wealth between period 1 and period 2, and in which currency to denominate their assets. The first order conditions of their optimization problem are then

$$\begin{aligned} q_1 U'(C_1) &= U'(C_2), && (11) \\ q_1^* s_1 U'(C_1) &= s_2 U'(C_2). && (12) \end{aligned}$$

To simplify the algebra going forward, we assume that $q_1^* = 1$. Given this restriction, one can easily guess and verify that $s_1 = s_2 = s$ and that households' consumption is constant over time. Indeed, if $s_1 = s_2$ and $q_1^* = 1$, we have that $C_1 = C_2 = C$ by equation (12). This also implies that the demand of non-tradable goods, $(1 - \omega)P_t C_t = (1 - \omega)C$ is constant. Because the endowment of non-tradable goods is also constant, one has that $p_1^N = p_2^N = p^N$ by the market clearing condition for non-tradables,

$$(1 - \omega)C = p_t^N y^N. \tag{13}$$

The guess is then verified because the price of non-tradable goods abroad is constant between period 1 and period 2, and equal to ε.

Therefore, the optimal behavior of the households consists in consuming half of their lifetime wealth in period 1 and the remaining half in period 2,

$$C = \frac{1}{2}[a_1 + sa_1^* + w_1 + w_2 + 2p^N e^N + \Pi_1]. \tag{14}$$

Given households' consumption, one can then find an expression for the equilibrium exchange rate. Using equations (10), (13) and (14), we have that s must satisfy

$$\frac{(1-\omega)}{2}[a_1 + sa_1^* + w_1 + w_2 + 2(s\varepsilon)^{-\frac{\omega}{1-\omega}} e^N + \Pi_1] = (s\varepsilon)^{-\frac{\omega}{1-\omega}} y^N, \tag{15}$$

Note that wages in period 2 and the profits of capital good producers are not predetermined, but they depend on the capital stock that banks use to produce tradable goods in period 2, k_2^T, and potentially on the exchange rate.

Equation (15) implicitly defines a relation between s and k_2^T. When banks invest more in productive capital, households' wages in period 2 increase. Therefore, households' consumption increase. The associated increase in the demand for non-tradable goods leads to an increase in their price by equation (13). Because the monetary authority is committed to price stability, the price of domestic tradable goods declines, and the exchange rate appreciates. The opposite pattern emerges when capital in the tradable sector declines. Therefore, in the continuation equilibrium, the exchange rate depreciates when banks' investment declines. We summarize this equilibrium relationship by writing the exchange rate as a function of the capital stock in the tradable sector, $s(k_2^T)$.

Importantly, given (Ψ_1^T, k_2^T), one can use the above relations and obtain all other quantities and prices consistent with households' optimality at $t = 1$ and $t = 2$. Because of that, the determination of the continuation equilibria can be simplified into finding the (Ψ_1^T, k_2^T) pair that clears the capital market at $t = 1$.

The demand for capital goods in period 1 is determined by banks' optimal behavior. The banks enter period 1 with assets (k_1^T, k_0^N) and with liabilities (b_1, b_1^*), and they solve their portfolio problem subject to the collateral constraint. In describing their optimality, we consider two cases, depending on whether the collateral constraint binds or not in period 1.

When the collateral constraint does not bind, banks' optimality is governed by the Euler equations

$$\Psi_1^j = q_1 R_2^j \qquad \text{for } j = \{T, N\}. \tag{16}$$

When the banks are unconstrained, the price of the two capital goods must equal the present discounted value of the dividends these assets offer.[3] Moreover, because the collateral constraint is slack, we must have by equations (4)-(7) that

$$k_2^T \leq \frac{1}{(1-\theta)\Psi_1^T}[(\Psi_1^T + R_1^T)k_1^T + p^N k_0^N - b_1 - sb_1^*]. \tag{17}$$

These derivations clarify that the demand for capital used in the production of tradable goods is a decreasing function of its price when the banks are unconstrained. This can be seen from equation (16), and from the fact that R_2^T is a decreasing function of k_2^T because of the concavity of the production function.

When the collateral constraint binds, instead, there will be a wedge between the price of capital and the fundamental value of the asset,

$$\Psi_1^j < q_1 R_2^j \qquad \text{for } j = \{T, N\}. \tag{18}$$

The demand for k_2^T will then satisfy

$$k_2^T = \frac{1}{(1-\theta)\Psi_1^T}[(\Psi_1^T + R_1^T)k_1^T + p^N k_0^N - b_1 - sb_1^*]. \tag{19}$$

Importantly, the relation between the demand for capital goods and their price changes when the collateral constraint binds. When Ψ_1^T declines, the banks experience balance sheet losses, and the associated decline in their net worth reduces the resources that banks have to buy capital goods. This balance sheet channel is amplified by the equilibrium behavior of the exchange rate. As k_2^T declines, the domestic currency depreciates and banks' net worth declines further if the banks entered the period with liabilities in foreign currency, $b_1^* > 0$. Thus, a decline in the price of capital could be associated to a decline in the demand for capital goods when the collateral constraint binds in period 1. As we will discuss momentarily, this non-monotonicity in the demand schedule for capital goods can lead to multiple continuation equilibria.

The supply of capital goods at $t = 1$ is determined by the optimal behavior of capital good producers. As described in Section 2, these agents solve a static optimization problem, and their optimal behavior is governed by the first order condition

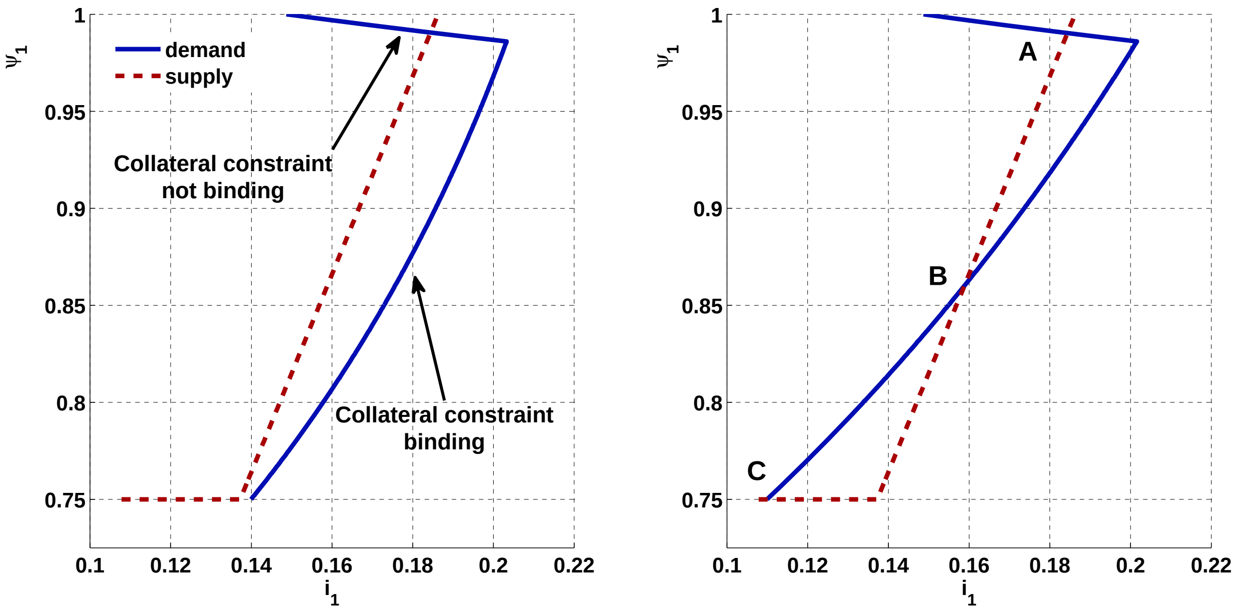
$$\Psi_1^T = p_1^T G'(i_1), \tag{20}$$

[3] Note that the capital goods earn a dividend only in period 2, R_2^j, because $t = 2$ is the last period. Also, the rental rates are known with certainty at $t = 1$ because there are no shocks hitting the economy in the last period.

where $i_1 = k_2^T - k_1^T$. With $G(.)$ convex, the supply of capital goods is upward sloping: the higher the price of capital, the higher i_1.

Having described the demand and supply of capital goods, we can now study the determination of the equilibrium in the capital market. In Figure 1 we report some numerical examples that are meant to illustrate different situations that can arise in the capital market.[4] The solid lines plot the demand for capital goods by the bankers, while the dotted line its supply. As explained earlier, the demand for capital goods is non-monotone. In the unconstrained region, the lower the price of capital goods the higher the demand of capital by the bankers. In the constrained region, instead, demand increases with the price. The supply of capital is, instead, upward sloping, with the flat region arising because capital good producers have no incentives to supply capital goods when the price falls below $\underline{A}$.

Figure 1: **The equilibrium in the capital market**



In the left panel of Figure 1 we describe a situation where there is only one pair of prices and quantities consistent with market clearing. Given this pair, we can derive uniquely all other quantities and prices at $t = 1$ and $t = 2$. Therefore, in this case, the model features only one continuation equilibrium.

In the right panel of the figure, instead, we describe a situation in which more than one equilibrium can arise in the capital market. We can see that there are three points in which

[4]For these examples, we assume that $G(i) = \min\{\phi_0 + \phi_1 i + \frac{\phi_2}{2} i^2, \underline{A} i\}$.

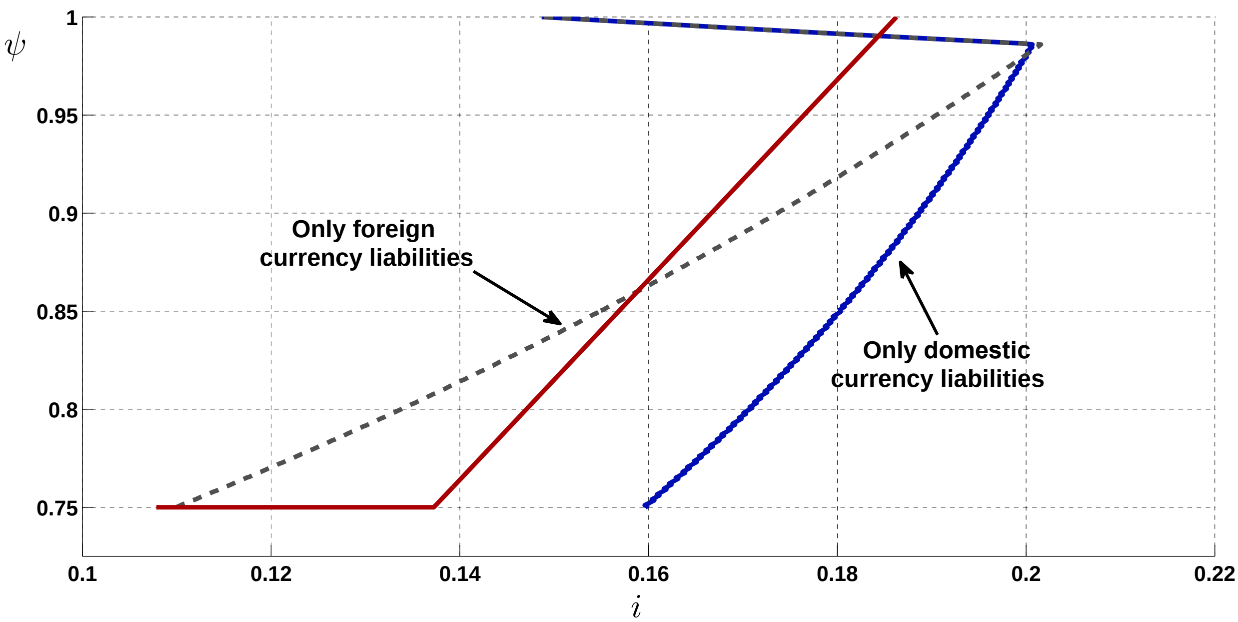
the demand of capital crosses the supply schedule. Point A describes an equilibrium in which banks are unconstrained, while in point B and C the price of capital is so low that the collateral constraint of the banks binds. Associated to these equilibria in the capital market, there are different levels of aggregate consumption and of the exchange rate. As discussed earlier, households' consumption increases in k_2^T, while domestic currency appreciates the higher the capital stock in the tradable sector. Therefore, the equilibria are Pareto-ranked, in the sense that households' and banks' utility are the highest in the unconstrained equilibrium, and they decline monotonically with the prices and quantities that clear the capital market. Note also that some of the equilibria may be "unstable", for example the one in point B in the figure.

The model can thus feature multiple continuation equilibria, and which one will be played depends on agents' expectations. When agents expect asset prices to be high, banks are unconstrained, and their demand of capital is high. This translates into high consumers' wealth and high aggregate consumption. When agents expect low asset prices, instead, the economy will experience a crisis: banks' financial wealth plummets, their demand for productive assets declines, leading to a decline in the wealth of households and in their consumption. We select among these possible outcomes using a sunspot. In going forward, we will assume that agents never coordinate on an unstable continatuion equilibrium, and that the economy features at most two stable continuation equilibria. When confidence crisis are possible, we will assume that with probability π the agents will play the "bad" (constrained) equilibrium, and with probability $(1-\pi)$ they will play the "good" (unconstrained) one.

The existence of these multiple continuation equilibria depends on models' parameters and on the initial asset positions that bank' and households inherit in period 1. Before closing this section it is important to briefly comment on the role played by the banks' initial balance sheet. As we have discussed earlier, exchange rates depreciates when the capital stock declines, and this amplifies the negative effects that a drop in asset prices has on the demand for capital when the banks are financially constrained. From equation (19), we can see that this amplification is stronger the higher b_1^*, the more foreign currency debt banks owe. We can illustrate this point graphically by considering the equilibrium in the capital market for two sets of initial conditions. In the first case, banks enter period 1 with only domestic currency liabilities $(b_1 > 0, b_1^* = 0)$, while in the second case they inherit only foreign currency liabilities $(b_1 = 0, b_1^* > 0)$. We adjust the level of b_1 and b_1^* so that banks have the same leverage in these two cases, and we keep all remaining parameters and initial conditions fixed.

In Figure 2 we plot the demand schedules in these two scenarios. The presence of

Figure 2: **The role of the banks' balance sheet**



foreign currency liabilities for the banking sector flattens the upward sloping portion of the demand schedule. When banks do not inherit foreign currency liabilities, movements in the exchange rate have little impact on the banks' net worth. When banks have debt denominated in foreign currency, instead, the depreciation of the exchange rate that is set in motion by the decline in the price of capital have more dramatic effects on banks' net worth and on the demand of productive capital in the economy. This channel, that distinguish our environment from that of a closed economy, can be sufficiently strong to expose the economy to the confidence crises studied in this section. Indeed, in the figure presented above, the twin crises equilibria arise only when banks enter the period with foreign currency liabilities, and not otherwise.

This latter discussion makes clear that the portfolio choices of households and banks in period 0 determine whether the economy is exposed to confidence crises in the continuation equilibria. We now turn to the analysis of such choices.

3.2 Optimal portfolio choices at $t = 0$

We start analyzing the portfolio choices of households at $t = 0$. The Euler equations characterizing their behavior are given by

$$q_0 U'(C_0) = \mathbb{E}_0[U'(C_1)], \tag{21}$$

$$s_0 q_0^* U'(C_0) = \mathbb{E}_0[s_1 U'(C_1)]. \tag{22}$$

Differently from the case analyzed in the previous section, households at $t = 0$ face risk. This risk takes two forms. First, the price of foreign tradable goods is stochastic, and these shocks to p_1^{T*} generate fluctuations in the exchange rate. Second, there might be the risk of a twin crisis in the future. Given our selection rule, the economy experiences such a crisis with probability π if the balance sheet inherited by the banks at $t = 1$ exposes the economy to multiple continuation equilibria.

These two sources of risk have different implications for the households' choices regarding the currency composition of their assets. To illustrate this point, we can combine equations (21) and (22) as follows,

$$\frac{\mathbb{E}_0[R_1^{\text{fc}}] - R_0^{\text{dc}}}{R_0^{\text{dc}}} = -\text{Cov}_0\left[R_1^{\text{fc}}, \frac{U'(C_1)}{U'(C_0)}\right], \tag{23}$$

where $R_1^{\text{fc}} = s_1/(s_0 q_0^*)$ are the $t = 1$ realized returns for bonds denominated in foreign currency, and $R_0^{\text{dc}} = 1/q_0$ are the returns for bonds denominated in domestic currency. Equation (23) is a standard asset pricing condition that determines the yields on foreign currency assets relative to those on assets denominated in domestic currency.

Consider first a scenario in which only the unconstrained continuation equilibrium is played from $t = 1$ onward.[5] In such a case, shocks to foreign tradable goods affect the exchange rate, but do not impact the equilibrium level of the capital stock, and they are thus orthogonal to households' labor income and to the profits of capital good producers. Because of that, households have an incentive to save in domestic currency because foreign currency bonds increase the volatility of their consumption at $t = 1$. This can be formally seen by inspecting $t = 1$ consumption in the unconstrained continuation equilibrium, see equation (14). When $a_1^* > 0$, C_1 will be positively associated to the equilibrium exchange rate s_1. Therefore, $\text{Cov}_0\left[R_1^{\text{fc}}, \frac{U'(C_1)}{U'(C_0)}\right] < 0$ when $a_1^* > 0$, and by equation (23) we must have that foreign currency bonds pay a risk premium to make households willing to hold them.

The prospect of a crisis at $t = 1$, however, generates an incentive fo households to

[5] This happens if the asset structure at $t = 1$ does not permit multiple continuation equilibria, or if $\pi = 0$.

denominate their savings in foreign currency. To illustrate this point, consider a second scenario in which the twin crisis equilibrium is played at $t = 1$ with probability $\pi > 0$, and assume for simplicity that $\sigma_\varepsilon = 0$. In this case, there are only two states of the world at $t = 1$. With probability $(1 - \pi)$, the economy is in the good continuation equilibrium, with high consumption and a relatively appreciated exchange rate. With probability π, the economy experiences a twin crisis: households' consumption is low and the exchange rate relatively appreciated. Foreign currency bonds are a good crisis hedge from the households' perspective because they pay a high return in the bad state of the world, which implies $\text{Cov}_0\left[R_1^{\text{fc}}, \frac{U'(C_1)}{U'(C_0)}\right] > 0$. In equilibrium, households are willing to accept lower yields on foreign currency bonds because of their hedging property.

While households wish to save in foreign currency when they expect a twin crisis in the future, banks' have the opposite incentive: they would like to issue debt denominated in domestic currency. To understand this point in the simplest possible way, we assume that initial conditions at $t = 0$ are such that the banks' collateral constraint does not bind, and we assume that the banks cannot invest/disinvest in capital, $k_1^T = k_0^T$. Hence, their problem consists in choosing the currency composition of their liabilities, b_1 and b_1^*. The first order conditions for the banks at $t = 0$ are similar to those of the households,

$$q_0 \lambda_0 = \mathbb{E}_0[\lambda_1], \tag{24}$$

$$s_0 q_0^* \lambda_0 = \mathbb{E}_0[s_1 \lambda_1], \tag{25}$$

where λ_t is the banks' marginal value of wealth at time t. The marginal utility of wealth for the banks may be stochastic at $t = 1$ even though the banks are risk neutral and consume only in the final period. This is due to the possibility that the collateral constraint may bind in period 1.[6] When the collateral constraint does not bind at $t = 1$, the value of one unit of domestic currency at $t = 1$ equals

$$\lambda_1^{\text{not bind}} = \frac{1}{q_1 p_2^T}. \tag{26}$$

That is, the banks can use that unit of net worth to invest, obtain the return $1/q_1$ at $t = 2$, and convert the proceeds into tradable goods. When the collateral constraint binds at $t = 1$, instead, the value of one unit of domestic currency at $t = 1$ equals

$$\lambda_1^{\text{bind}} = \frac{1}{(1-\theta)\Psi_1^T}\left[\frac{R_2^T - \theta\Psi_1^T/q_1}{p_2^T}\right]. \tag{27}$$

[6]See, for example, Aiyagari and Gertler (1999) and Bocola (2016).

The banks can in fact lever that unit of net worth up to $1/(1-\theta)\Psi_1^T$, invest in tradable capital, and obtain a return of $\left[\frac{R_2^T-\theta\Psi_1^T/q_1}{p_2^T}\right]$ tradable goods in period $t=2$.

Importantly, one can verify that $\lambda_1^{\text{bind}} \geq \lambda_1^{\text{not bind}}$: a unit of net worth allows the banks to relax their collateral constraint, and it is more valuable when the latter binds. Rearranging equations (24) and (25), we then have

$$\frac{\mathbb{E}_0[R_1^{\text{fc}}]-R_0^{\text{dc}}}{R_0^{\text{dc}}} = -\text{Cov}_0\left[R_1^{\text{fc}}, \frac{\lambda_1}{\lambda_0}\right]. \tag{28}$$

This expression clarifies that banks have little incentives to issue foreign currency debt when they anticipate a twin crisis at $t=1$. A twin crisis is, in fact, associated to binding collateral constraints for the banks, implying an high λ_1. Moreover, it is a state of the world in which the domestic currency depreciates, and this raises the payments that banks have to make on their foreign currency debt. Therefore, banks have a precautionary motive to issue domestic currency debt when a twin crisis is anticipated.

3.3 Self-fulfilling twin crises

Having discussed optimal portfolio choices at $t=0$, we can now turn to the analysis of the competitive equilibria in the model. Specifically, we ask whether the portfolio choices of banks and households at $t=0$ can expose the economy to the bad continuation equilibria at $t=1$, or whether they prevent these crises from happening. As we argue below, the answer to this question depends on the risk aversion of the households relative to the (implicit) risk aversion of the banks.

In order to understand why, suppose that at $t=0$ agents attach a high likelihood to a future twin crisis. As explained earlier, households have an incentive to save in foreign currency while banks have a precautionary motive to issue debt in domestic currency. If the former effect dominates, the banks will have to issue foreign currency liabilities at $t=0$. The presence of foreign currency liabilities in the banks' balance sheet could then expose the financial sector to confidence crises at $t=1$. This would validate agents' expectation of a twin crisis at $t=1$. On the contrary, when banks' precautionary motives are sufficiently strong, the expectation of future twin crises may lead to a reduction of their foreign currency liabilities. This would make the economy less exposed to the twin crises continuation equilibria studied in Section 3.1.

In what follows, we construct a numerical example and show that the model is indeed capable of generating self-fulfilling twin crises. Specifically, we adopt the following utility function for households,

$$U(C_t) = \frac{(C_t - \underline{C})^{1-\sigma}}{1-\sigma}.$$

Households' risk aversion could therefore be extremely high when consumption gravitates around the consumption commitment $\underline{C}$.

We can now turn to the problem of deriving a competitive equilibrium. We have already determined all quantities and prices from $t = 1$ onward. Therefore, we just need to determine the $t = 0$ portfolio choices $\{a_1, a_1^*, b_1, b_1^*\}$, the prices $\{p_0^N, p_0^T, q_0\}$, the exchange rate s_0, and households' consumption C_0. The equilibrium conditions these variables need to satisfy are the Euler equations of households and banks (21)-(25), their budget constraints, market clearing in the non-tradable good sector, the law of one price, and market clearing in domestic currency bond market,

$$a_1 = b_1. \tag{29}$$

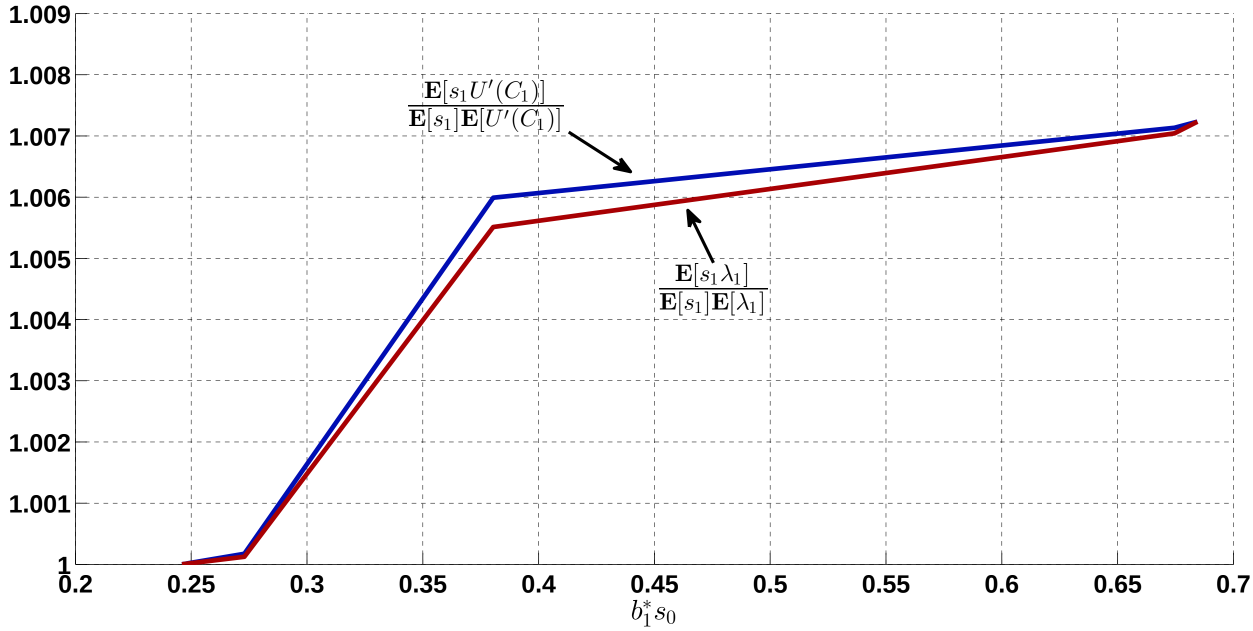
We can use all these equilibrium conditions and collapse the problem into that of choosing a b_1^* such that the Euler equations of banks and households are both satisfied,

$$\frac{\mathbb{E}_0[s_1 U'(C_1)]}{\mathbb{E}_0[s_1]\mathbb{E}_0[U'(C_1)]} = \frac{\mathbb{E}_0[s_1\lambda_1]}{\mathbb{E}_0[s_1]\mathbb{E}_0[\lambda_1]}. \tag{30}$$

In Figure 3 we plot the left and right hand side of equation (30) as a function of the foreign currency liabilities that the banking sector takes in period 0. As we move b_1^*, the asset structure in the economy adjusts in order to fulfill the remaining equilibrium conditions. Specifically, by the bankers' budget constraint, high b_1^* is associated to a low debt in domestic currency and, by equation (29), to a low level of households' savings in domestic currency. We can verify that the two curves cross twice, demonstrating the existence of multiple continuation equilibria.

Lets' consider first the equilibrium at the low level of b_1^*. In our example, the banking sector is not exposed to confidence crisis at $t = 1$ when b_1^* is low. Hence, from period $t = 1$ onward, the economy operates only in the good continuation equilibrium, and the only source of risk for this economy arises because of the fluctuations in the price of foreign tradable goods. Because of that, we know from our previous discussion that households are not willing to save in foreign currency unless foreign currency debt provides a risk premium. However, neither foreigners nor banks wishes to pay such premium, as they both act as risk neutral in this equilibrium. Therefore, households save exclusively in

Figure 3: **Portfolio choices and multiple equilibria**



domestic currency, $a_1 > 0$ and $a_1^* = 0$.[7] These choices by the households allows the banks to issue in the first place mostly domestic currency assets, sustaining this outcome as an equilibrium of the model.

In the second equilibrium, instead, the banks issue large amounts of foreign currency liabilities. In our example, the large amount of foreign currency debt exposes the banking sector to a confidence crisis at $t = 1$. Therefore, households now face the risk of a twin crisis at $t = 1$. We know from the previous discussion that foreign currency assets have good hedging properties during a crisis, and this pushes households to save mostly in foreign currency. The scarcity of savings in domestic currency forces the banking sector to finance its operation by issuing foreign currency liabilities. As the “dollarization" of banks’ liabilities occurs, the economy becomes exposed to the twin crisis equilibria in period 1.

This second equilibrium captures, in our view, salient features of the financial fragility for economies with an open capital account and flexible exchange rates. The economy may operate most of the time in the good equilibrium, where the domestic financial sector does not have hard time financing itself in domestic currency. However, the expectation of a confidence crisis may suddenly lead households to “run" on the domestic currency

[7] We can verify this in Figure 1, as $\mathbb{E}_0[s_1 U'(C_1)] = \mathbb{E}_0[s_1]\mathbb{E}_0[U'(C_1)]$. This means that s_1 and $U'(C_1)$ are orthogonal in the continuation equilibria, a situation that occurs only when $a_1^* = 0$.

and redenominate their assets in foreign currency. This capital flights increase the fragility of the banking sector and they contribute to further expose the economy to a confidence crisis. We now turn to analyze how the interactions of capital flights and financial crisis affects the operations of lending of last resort for an open economy.

4 Lending of Last Resort

We consider a government that at $t = 0$ levies a lump sum tax τ_0 on the households. The government can use these resources to accumulate reserves in foreign and in domestic currency,

$$q_0 h_1 + s_0 q_0^* h_1^* = \tau_0.$$

To make the problem interesting, we assume that the government faces some upper bound in the taxes that it can collect from households, $\tau_0 \leq \overline{\tau}$.

The reserves can be used in period 1 to conduct operations of lending of last resort. Specifically, we assume that the government can commit to a demand schedule for capital goods at $t = 1$.[8] We denote by $z_1(\Psi^T)$ the demand of capital goods by the government given the price Ψ^T,

$$\Psi_1^T z_1(\Psi_1^T) \leq h_1 + s_1 h_1^*.$$

The government holds the capital stock for one period and operates an alternative linear technology that returns $\underline{A} z_1$ units of tradable goods in period 2. These resources are then rebated back to the consumers as a transfer in period 2.

These operations can potentially eliminate the bad continuation equilibria at $t = 1$. By adding to private capital demand, the government can curb the fall in asset prices and the depreciation of the exchange rate that characterizes the bad equilibrium, limiting in this fashion the decline in the banks' net worth that sustains the bad equilibria. However, to be effective, they must also be credible. That is, the government needs to have enough fiscal resources to shift out the demand for capital and eliminate the bad equilibria.

In order to formalize this last point, we can define $Z(\Psi^T)$ to be the difference between the supply of capital and its private demand when the price is Ψ^T. The government can credibly eliminate the bad continuation equilibria if

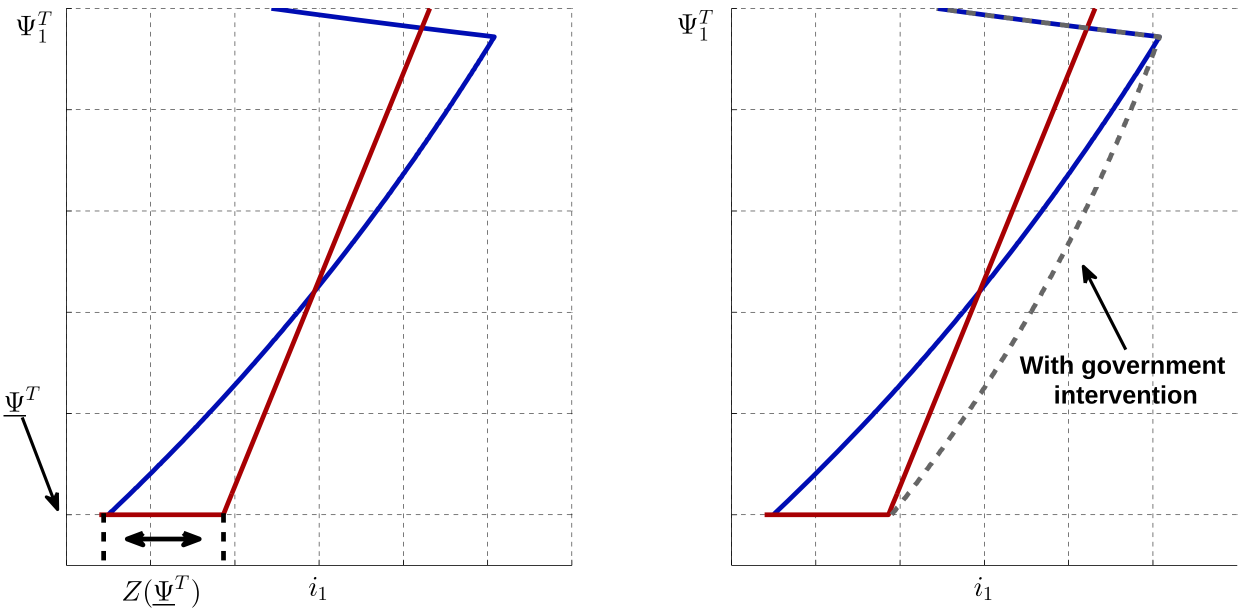
$$\Psi^T Z(\Psi^T) \leq h_1 + s_1 h_1^*,$$

[8] Equivalently, we could have modeled operation of lending of last resort as a direct loan that the government extends to the bank at some penalty rate.

for all Ψ^T that are below the good equilibrium price.

Figure 4 clarifies this discussion. In the left panel we can see that $Z(\underline{\Psi}_1^T)$ is the excess supply of capital in the bad equilibrium of the model. If the government has enough reserves to purchase $Z(\underline{\Psi}_1^T)$ at the market prices $\underline{\Psi}_1^T$, it could prevent the confidence crisis in period 1: by committing to purchase capital goods, the government can shift the demand schedule to the right and implement uniquely the good equilibrium, see the right panel in Figure 4 for an example of a successful operation. If the government does not have enough fiscal resources, however, the bad equilibrium is unavoidable and lending of last resort is less effective.

Figure 4: **Effective lending of last resort**



While this section of the paper is still largely a work in progress, we wish to point out two results that naturally emerge in our environment. First, the possibility of the capital flights makes it harder for the government to credibly avert a confidence crisis in period 1 through lending of last resort. As we have seen earlier, the presence of foreign currency liabilities for banks flattens the capital demand schedule in its upward sloping portion, see Figure 2. *Ceteris paribus*, this implies an increase in the amount of fiscal resources necessary to avert the bad equilibrium, $\underline{\Psi}_1^T Z(\underline{\Psi}_1^T)$. This is the sense in which an open economy with flexible exchange rates makes effective lending of last resort more difficult.

Second, the model provides a rationale for the ex-ante accumulation of foreign reserves. Foreign reserves have the property of appreciating in the bad equilibrium. As such, they increase the fiscal resources that the government has at its disposal to avert the bad equilibrium, and to effectively conduct lending of last resort. This last result can rationalize the findings of Obstfeld, Shambaugh, and Taylor (2010) and Aizenman and Lee (2007) that the large accumulation of foreign reserves among emerging markets over the last 20 years is strongly associated to the extent of financial openness and financial depth of these economies, and the informal arguments brought in the literature that such accumulation was done in order to facilitate effective lending of last resort.

5 Conclusion

[to be completed]

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