

# Innovation Waves, Investor Sentiment, and Mergers\*

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## Abstract

We develop a theory of innovation waves, investor sentiment, and merger activity based on uncertainty aversion. Investors must typically decide whether or not to fund an innovative project with very limited knowledge of the odds of success, a situation that is best described as “Knightian uncertainty.” We show that uncertainty-averse investors are more optimistic on an innovation if they can also make contemporaneous investments in other innovative ventures. This means that uncertainty aversion makes investment in innovative projects strategic complements, which results in innovation waves. Innovation waves occur in our economy when there is a critical mass of innovative companies and are characterized by strong investor sentiment, high equity valuation in the technology sector, and “hot” IPO and M&A markets. We also argue that M&A promotes innovative activity and leads to greater innovation rates and firm valuations.

Keywords: Innovation, Ambiguity Aversion, Hot IPO Markets

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Innovation is the most important value driver of modern corporations and a key source of economic growth (Solow, 1957). There are times when innovation is stagnant, but other times when technology leaps forward. Further, investors must typically decide whether or not to fund an innovative project with very limited knowledge of the odds of success, a situation that is best described as “Knightian uncertainty” (Knight, 1921). This paper studies the impact of uncertainty aversion on the incentives to innovate. We show that uncertainty aversion can generate innovation waves that are associated with strong investor sentiment and high stock market valuations.<sup>1</sup>

There are many reasons why innovation develops in waves. These include fundamental reasons such as random scientific breakthroughs in the presence of externalities and technological spillovers. In this paper, we focus on the impact of financial markets on the incentives to create innovation. We argue that innovation waves can be the product of investor uncertainty aversion. We show that investor uncertainty aversion creates externalities in innovative activities which results in innovation waves characterized by strong investor sentiment and high stock market valuations. We also show that innovation waves lead to an active M&A market which further promotes innovation activities. Finally, our model suggests that innovation waves may lead to “hot” IPO markets associated with strong investor sentiment, high equity valuations, and lower long term equity returns.

We study an economy with multiple entrepreneurs endowed with project-ideas. Project-ideas are risky and, if successful, may lead to innovations. The innovation process consists of two stages. In the first stage, entrepreneurs must decide whether or not to invest personal resources, such as effort, to innovate. If the first stage of the process is successful, further development of the innovation requires additional investment in the second stage. Entrepreneurs raise funds for the additional investment by selling shares of their firms to uncertainty-averse investors. The second stage of the innovation process is uncertain in that outside investors are uncertain of the exact distribution of the residual success probability of the innovation process. Following Epstein and Schneider (2010), we model uncertainty aversion by assuming that outside investors are Minimum Expected Utility (MEU) maximizers and that they hold a set of priors, or “beliefs,” rather than a single prior as is the case for Subjective Expected Utility (SEU) agents.

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<sup>1</sup>A positive effect of investor sentiment on innovation has been documented in Aramonte (2015).

In our model, probabilistic assessments (or “beliefs” in the sense of de Finetti, 1974) on the future returns of investments held by uncertainty-averse investors are endogenous, and depend on the composition of their portfolios. In particular, uncertainty-averse investors prefer to hold an uncertain asset if they can also hold other uncertain assets, a feature that is denoted as “uncertainty hedging.” This happens because, by holding uncertain assets in a portfolio, investors can lower their overall exposure to the sources of uncertainty in the economy. In addition, because of uncertainty hedging, an investor will hold more favorable probabilistic beliefs toward an innovation – and thus be more optimistic – if he/she is able to invest in other innovations as well. We will refer to the probabilistic assessments held by investors on the success of innovations as characterizing their “sentiment.” Thus, uncertainty-averse investors have stronger sentiment and are willing to pay more for equity in a given entrepreneur’s firm when other entrepreneurs innovate as well. This means that investors are more willing to fund an entrepreneur’s innovation if they can also fund other entrepreneurs at the same time. It also implies that the market value of equity of a new firm will be greater when multiple new firms are on the market as well. Thus, investments in different innovative companies are effectively complements and have positively correlated market valuations.

We also show that investor uncertainty aversion can generate inefficient equilibria where potentially valuable innovation is not pursued. When the initial personal cost to the entrepreneur is sufficiently low, entrepreneurs’ dominant strategy is to innovate, irrespective of other entrepreneurs’ decisions. Similarly, when the initial personal cost is very large, the dominant strategy is not to innovate. For intermediate levels of the initial personal cost, an entrepreneur is willing to initiate the innovation process only if she expects other entrepreneurs to innovate as well. Thus, multiple equilibria, with and without innovation, may exist. Existence of the inefficient equilibrium without innovation depends on the correlation between the success rates of the innovation processes, that is, on the degree of “relatedness” of the innovation.

Strategic complementarity between innovative activities due to uncertainty aversion may result in innovation waves. We show that an innovation wave occurs if the number of innovative companies in a technological sector reaches critical mass. Arrival of innovation opportunities in the economy may be random and due to exogenous technological progress. However, we argue that

such technological advances, while seeding the ground for an innovation wave, may not be sufficient to ignite one. Rather, an innovation wave will start when in the economy (or in a specific technological sector) a critical mass of innovators is attained, which will spur a “hot” market for innovative companies. Thus, innovation waves are characterized by strong investor sentiment and a wave of “rational exuberance” with high equity market valuations. In our model, equity market “booms” in technology markets can materialize, and these booms are beneficial since they can spur valuable innovation.

Note that the channel we propose in our model, based on uncertainty aversion, differs substantially from an alternative explanation based on pure risk aversion and the benefits from diversification (see, for example, Acemoglu and Zilibotti, 1997). The traditional portfolio diversification argument can only generate innovation waves and high stock market valuations as the outcome of a reduction of the economy-wide market price of risk. In this case, innovation waves will necessarily be associated with economy-wide equity market booms. Our approach, in contrast, can explain the apparent “boom and bust” behavior of technology sectors, such as the Life Sciences and the Information Technology, where hot periods alternate with cold periods in innovation rates, mergers and asset valuations. In addition, we provide a decision-theoretic foundation of and build on the importance of the notion of “investor sentiment.”

Our paper also has implications for the impact of M&A activity and, more generally, of the ownership structure on innovation rates. Specifically, in the new channel we propose, based on uncertainty aversion, mergers of innovative firms create synergies and spur innovation. In our paper, positive synergies in an acquisition are created endogenously, and are the direct outcome of the beneficial spillover (i.e., externality) on the probabilistic assessments of future returns on innovation due to uncertainty aversion.<sup>2</sup> In addition, our model predicts that merger activities involving innovative firms will be associated with strong investor sentiment and, thus, greater firm valuations.

Finally, we argue that uncertainty aversion has implications for the composition of venture capital portfolios, and the structure of the venture capital industry. This happens because of the

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<sup>2</sup>Hart and Holmstrom (2010) develop a model where mergers create value by internalizing externalities, such as coordinating on a technological standard.

possible beneficial role that venture capitalists can play to remedy a coordination failure that causes the inefficient no-innovation equilibrium.

Our paper rests at the intersection of three strands of literature. First, and foremost, our paper belongs to the rapidly expanding literature on the determinants of innovation and innovation waves (see Fagerberg, Mowery and Nelson, 2005, for an extensive literature review).<sup>3</sup> The critical role of innovation and innovation waves in modern economies has been extensively studied at least since Schumpeter (1939) and (1942), Kuznets (1940), Kleinknecht (1987) and, more recently, Aghion and Howitt (1992). Early research focused mostly on the “fundamentals” behind innovation waves, such as the positive spillover effects across different technologies. More recent research has focused on the link between innovation waves, the availability of financing, and stock market booms. Scharfstein and Stein (1990) suggest that reputation considerations by investment managers may induce them to herd their behavior in the stock market, and thus facilitate the financing of technology firms. Gompers and Lerner (2000) find that higher venture capital valuations are not necessarily linked to better success rates of portfolio companies. Perez (2002) shows that technological revolutions are associated with “overheated” financial markets. Gompers et al. (2008) suggest that increased venture capital funding is the rational response to positive signals on technology firms’ investment opportunities. Nanda and Rhodes-Kropf (2013) find that in “hot markets” VCs invest in riskier and more innovative firms. Nanda and Rhodes-Kropf (2016) argue that favorable financial market conditions reduce refinancing risk for VCs, promoting investment in more innovative projects.

To our knowledge, ours is the first paper that models explicitly the role of uncertainty aversion on the innovation process and its impact on innovation waves and stock market valuations.<sup>4</sup> We show that investor uncertainty aversion can generate innovation waves that are driven by investors’ optimism, that is, their positive sentiment. In our model, due to uncertainty aversion, investors’ probabilistic assessments are endogenous, and they respond to the availability of investments in

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<sup>3</sup>See also Chemmanur and Fulghieri (2014) for a discussion of current issues related to entrepreneurial finance and innovation.

<sup>4</sup>This paper is part of the growing literature studying ambiguity aversion in finance, including Mukerji and Tallon (2001), Maenhout (2004), Epstein and Schneider (2008) and (2011), Easley and O’Hara (2009) and (2013), Caskey (2009), Illeditsch (2011), Jahan – Parker and Liu (2014), Mele and Sangiorgi (2015), Dimmock, Kouwenberg, Mitchell, and Peijnenburg (2016), Garlappi, Giammarino, and Lazrak (2016), Byun (2014), Dicks and Fulghieri (2016a), and Dicks and Fulghieri (2016b).

innovative projects. Innovation waves and stock market “exuberance” are jointly determined in equilibrium in a model where investors are sophisticated. In our model, greater investment in innovation activities occurs simultaneously with investor optimism and stock market booms.

The second stream of literature is the recent debate on the links between technological innovation and stock market prices. Nicholas (2008) shows that an important driver of the stock market run-up experienced in the American economy in the late 1920’s was the strong innovative activity by industrial companies which affected the market valuation of corporate “knowledge assets.” Pastor and Veronesi (2009) argue that technological revolutions can generate dynamics in asset prices in innovative firms that are observationally similar to assets bubbles followed by a valuation crash. Their paper argues that this “bubble-like” behavior of stock prices is the rational outcome of learning about the productivity of new technologies, where the risk is essentially idiosyncratic, followed by the adoption of the new technologies on large scale, where the risk becomes systematic. Our paper proposes a new explanation for the link between innovative activity and stock market booms. In Pastor and Veronesi (2009) stock market booms (and subsequent crashes) are the outcome of the changing nature of risk that characterizes technological revolutions, from idiosyncratic to systematic, and its impact on discount rates. In our model, periods of strong innovative activity are accompanied by high valuations because innovation waves are, in equilibrium, associated with more optimistic expectations on future expected cash flows from innovations. Thus, our model, which focuses on expected cash flows, complements theirs, that focus on discount rates. Furthermore, similar to Pastor and Veronesi (2009), in our model high valuations imply lower long-term returns.

The third stream of literature focuses on the drivers of merger waves and the impact of M&A activity – and, more generally, of the ownership structure – on the incentives to innovate. High stock market valuations are also associated with strong M&A activity in merger waves (see, for example, Maksimovic and Phillips, 2001, and Jovanovic and Rousseau, 2001). Rhodes-Kropf and Viswanathan (2004) argue that such correlation is the outcome of misvaluation of the true synergies created in a merger in periods when the overall market is overvalued. The impact of M&A activity on corporate innovative activity has been documented by several empirical studies. For example,

Phillips and Zhdanov (2013) show that a firm's R&D expenditures increase in periods of strong M&A activity in the same industry. Bena and Li (2014) argue that the presence of technological overlap between two firms innovative activities is a predictor of the probability of a merger between firms. Sevilir and Tian (2012) show that acquiring innovative target firms is positively related to acquirer abnormal announcement returns and long-term stock return performance. The importance of the presence of technological overlaps between acquiring firms and targets is confirmed by Seru (2014), which finds that innovation rates are lower in diversifying mergers, where the technological benefits of a merger are likely to be absent.

In our model we are able to jointly generate the observed positive correlations between stock market valuations, the level of M&A activity, and innovation rates. Specifically, our paper creates a novel direct link between stock price valuations, M&A activity, and greater innovation rates that is based on investors' uncertainty aversion. Endogeneity of probabilistic assessments creates an externality between innovations that is at the heart of the synergy creation in mergers of innovative companies. This externality results in greater innovation rates and innovation waves that are characterized by strong investor sentiment and greater stock market valuations.

The paper is organized as follows. In Section 1, we introduce the basic model of our paper. In Section 2, we derive the paper's main results. Section 3 examines the impact of mergers on the incentives to innovate. In Section 4, we develop a simple dynamic extension of our basic model which generates innovation waves associated with strong investor sentiment and stock market booms. Section 5 shows that our results hold also in the case of process innovation. Section 6 presents the main empirical implications of our model. Section 7 concludes. All proofs are in the appendix.

## 1 The Basic Model

We study a two-period model, with three dates,  $t \in \{0, 1, 2\}$ . The economy has two classes of agents: investors and entrepreneurs. Entrepreneurs are endowed with unique project-ideas that may lead to an innovation. Project-ideas are risky and require an investment both at the beginning of the period,  $t = 0$ , and at the interim date,  $t = 1$ , as discussed below. If successful, project-ideas generate a valuable innovation at the end of the second period,  $t = 2$ . If the project-idea is

unsuccessful, it has zero payoff. For simplicity, we assume initially that there are only two types of project-ideas, denominated by  $\tau$ , with  $\tau \in \{A, B\}$ .

Entrepreneurs are penniless and require financing from investors. There is a unit mass of investors. Investors are endowed at the beginning of the first period,  $t = 0$ , with  $w_0$  units of the riskless asset. The riskless asset can either be invested in one (or both) of the two types of project-ideas, or it can be invested in the riskless technology. A unit investment in the riskless technology can be made either at  $t = 0$  or  $t = 1$ , and yields a unit return in the second period,  $t = 2$ , so that the (net) riskless rate of return is zero.

We assume that project-ideas are specific to each entrepreneur, that is, an entrepreneur can invest in only one type of project-idea, which will determine entrepreneur's type  $\tau$ ,  $\tau \in \{A, B\}$ . This assumption captures the notion that project-ideas are creative innovations that can be successfully pursued only by the entrepreneur who generated them.

The innovation process is structured in two stages. To implement a project-idea, and thus “innovate,” an entrepreneur must first make at  $t = 0$  a fixed, non-pecuniary investment  $k_\tau$ . We interpret the initial investment  $k_\tau$  as representing all the preliminary personal effort that the entrepreneur must exert in order to generate the idea and make it potentially viable. We will denote the initial personal investment made by the entrepreneur,  $k_\tau$ , as a “discovery cost” that is necessary for the innovation. The innovation process is inherently risky, and we denote with  $q_\tau$  the success probability of the first stage of the process. We allow the first-stage success probabilities of the two project-ideas to be correlated. Specifically, we assume that the probability that both entrepreneurs are successful in the first stage is  $q_A q_B + r$ , while the probability that only entrepreneur  $\tau$  is successful is  $q_\tau(1 - q_{\tau'}) - r$ , with  $\tau', \tau \in \{A, B\}$ ,  $\tau' \neq \tau$  and  $r \in \left(-\min\{q_A q_B, (1 - q_A)(1 - q_B)\}, \min_{\tau} q_\tau(1 - q_\tau)\right)$ .<sup>5</sup> The parameter  $r$  captures the possibility of the presence of similarities between entrepreneurial project-ideas. Thus,  $r$  characterizes the degree of “relatedness” of the innovations.

If the first stage is successful, at  $t = 1$  entrepreneurs enter the second stage of the process. In this second stage, the entrepreneur must decide the level of intensity of the innovation process, for example, the level of R&D expenditures. Innovation intensity will affect the ultimate value of the

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<sup>5</sup>It can be quickly verified that the correlation of the first-stage projects is  $r [q_A (1 - q_A) q_B (1 - q_B)]^{-\frac{1}{2}}$ .



innovation that can be realized at  $t = 2$ , and that is denoted by  $y_\tau$ . Innovation intensity is costly, and we assume that an entrepreneur of type  $\tau$  choosing an innovation intensity  $y_\tau$  will sustain a cost  $c_\tau(y_\tau) = \frac{1}{Z_\tau(1+\gamma)}y_\tau^{1+\gamma}$ , where  $Z_\tau$  represents the productivity of entrepreneur  $\tau$ 's project-idea. To obtain interior solutions, we will assume that the productivity parameters,  $Z_\tau$ , for the two entrepreneurs are not too dissimilar.<sup>6</sup> Entrepreneurs will pay for the cost  $c_\tau(y_\tau)$  by selling equity to a large number of well-diversified investors.<sup>7</sup> The second stage of the innovation process is also uncertain and, if successful, the innovation will generate at the end of the second period,  $t = 2$ , the payoff  $y_\tau$  with probability  $p_\tau$ , and zero otherwise (if the project fails in the first stage, it is similarly worthless). We will assume, for simplicity, that the success probabilities of the second stage are independent, and will show that innovation waves can occur even if success probabilities are uncorrelated.<sup>8</sup>

We assume that entrepreneurs are impatient and that they will sell at the interim period,  $t = 1$ , their firms to outside investors at total price  $V_\tau$ . An important feature of our model is that we assume investors are uncertain about the success probability of the second stage of project-ideas,  $p_\tau$ . We model uncertainty (or ‘‘ambiguity’’) aversion by adopting the minimum expected utility (MEU) approach promoted in Epstein and Schneider (2010).<sup>9</sup> In this framework, economic agents do not have a single prior on future events but, rather, they believe that the probability distribution of future events belongs to a given set  $\mathcal{M}$ , denoted as the investor’s ‘‘core beliefs set.’’ Thus, uncertainty-averse agents maximize their MEU utility

$$\mathcal{U} = \min_{\mu \in \mathcal{M}} E_\mu [u(\cdot)], \tag{1}$$

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<sup>6</sup>Formally, we assume that  $\frac{Z_A}{Z_B} \in \left(\frac{1}{\psi}, \psi\right)$  where  $\psi \equiv \frac{1}{4}e^{2\alpha(\gamma+1)}\left(1 + \frac{1}{2\gamma}\right)^{2\gamma}$ . This assumption guarantees that if both first-stage projects are successful, entrepreneurs execute innovation intensity levels so that investors have interior beliefs in equilibrium.

<sup>7</sup>The sale of equity may, for example, take place in the form of an Initial Public Offering, IPO.

<sup>8</sup>Our model can easily be extended to the case where second-stage success probabilities are correlated.

<sup>9</sup>MEU was originally derived by Gilboa and Schmeidler (1989). An alternative approach is ‘‘smooth ambiguity’’ developed by Klibanoff, Marinacci, and Mukerji (2005). In their model, agents maximize expected felicity of expected utility. Agents are uncertainty averse if the felicity function is concave. The main results of our paper will hold also in this latter approach, but at the cost of requiring a substantially greater analytical complexity. Similarly, our results also hold in the context of variational preferences of Maccheroni, Marinacci, and Rustichini (2006) if the ambiguity index  $c(p)$  has a positive cross-partial.

where  $\mu$  is a probability distribution over future events, and  $u(\cdot)$  is a von-Neumann Morgenstern (vNM) utility function.<sup>10</sup> In addition, following Epstein and Schneider (2010), we assume that uncertainty-averse agents are sophisticated with consistent planning. In this setting, agents are sophisticated in that they correctly anticipate their future uncertainty aversion and, thus, correctly take into account how they will behave at future dates in different states of the world.<sup>11</sup>

We model investor uncertainty aversion by assuming that investors are uncertain on the success probability of the second stage of the innovation process,  $p_\tau$ , and we characterize the core beliefs set by using the notion of relative entropy. For given pair of (discrete) probability distributions  $(p, \hat{p})$ , the *relative entropy* of  $p$  with respect to  $\hat{p}$  is defined as the Kullback-Leibler divergence of  $p$  from  $\hat{p}$ , and is given by

$$R(p|\hat{p}) \equiv \sum_i p^i \log \frac{p^i}{\hat{p}^i}. \quad (2)$$

The core beliefs set for the uncertainty-averse investors in our economy is then given by

$$\mathcal{M} \equiv \{p : R(p|\hat{p}) \leq \eta\}, \quad (3)$$

where  $p$  is the joint distribution of the success probability of the second stage of the two project-ideas, and  $\hat{p}$  is a certain, exogenously given “reference” probability distribution of such success probabilities. From (2), it is easy to see that the relative entropy of  $p$  with respect to  $\hat{p}$  represents the (expected) likelihood ratio of the distribution  $\hat{p}$  when the “true” probability distribution is  $p$ . In other words, it is a measure of the degree of confidence that an agent has on the probability distribution  $\hat{p}$ .<sup>12</sup> The core beliefs set  $\mathcal{M}$  can therefore be interpreted as the set of probability distributions,  $p$ , with the property that, if true, the investor would expect not to reject the (“null”) hypothesis  $\hat{p}$  in a likelihood-ratio test.

Intuitively, the core belief set  $\mathcal{M}$  is the set of probability distributions that are not “too unlikely” to be the true (joint) probability distribution that characterizes the two technologies, given the reference distribution  $\hat{p}$ . Note that a small value of  $\eta$  represents situations where agents have more

<sup>10</sup>In the traditional framework, players have a single prior  $\mu$  and maximize expected utility  $E_\mu [u(\cdot)]$ .

<sup>11</sup>Siniscalchi (2011) describes this framework as preferences over trees.

<sup>12</sup>As in Epstein and Schneider (2010), Hansen and Sargent (2001), (2007), and (2008), relative entropy can also be interpreted as characterizing the extent of “misspecification error” that affects investors.

confidence that the probability distribution  $\hat{p}$  is a good representation of the success probability of the two technologies, while a large value of  $\eta$  corresponds to situations where there is great uncertainty on the true probabilities underlying the two technological processes.

It is immediate to verify the following property of the core beliefs set  $\mathcal{M}$ .

**Lemma 1** *Let  $\eta < \underline{\eta}(\hat{p})$ , defined in the appendix. The core beliefs set  $\mathcal{M}$  is a strictly convex set with smooth boundary. If investors have nonnegative investments in both innovations, the solution to (1) is on the lower left-hand boundary of  $\mathcal{M}$ .*

Lemma 1 is an implication of the fact that relative entropy  $R(p|\hat{p})$  is a strictly convex function.<sup>13</sup> Lemma 1 also shows that uncertainty-averse investors with positive investment in both project ideas will select their probability assessments that lie in the “lower-left” boundary of the core beliefs set  $\mathcal{M}$ . Thus, the relevant part of the core beliefs set  $\mathcal{M}$  is a smooth, decreasing, and convex function. See Figure 1 on page 44.

In addition, it is easy to see that restricting investors’ beliefs to belong to the core beliefs set (3) has the effect of ruling out probability distributions that are very unlikely, given the reference probability  $\hat{p}$ . In other words, the maximum entropy criterion implied by (3) excludes from the core belief set probability distributions too far from  $\hat{p}$ . Because uncertainty-averse investors (from Lemma 1) are essentially concerned about the probability of “left-tail” events, we denote this property as “trimming pessimism” (see again Figure 1).

Because there is no closed-form solution for the level set of relative entropy for binomial distributions in (3), for ease of exposition, we model the relevant portion of the core beliefs set (namely, the decreasing and convex “lower-left” boundary) by using a lower-dimensional parametrization, as follows. We assume that the success probability of a project idea of type- $\tau$  depends on the value of an underlying parameter  $\theta_\tau$ , and is denoted by  $p_\tau(\theta_\tau)$ , with  $\theta_\tau \in [\theta_L, \theta_H] \subseteq [\theta_m, \theta_M]$ . For analytical tractability, we assume that  $p_\tau(\theta) = e^{\theta_\tau - \theta_M}$ , with  $\tau \in \{A, B\}$ . Uncertainty-averse agents treat the vector  $\vec{\theta} \equiv (\theta_A, \theta_B)$  as ambiguous and assess that  $\vec{\theta} \in C \subset \{(\theta_A, \theta_B) : (\theta_A, \theta_B) \in [\theta_L, \theta_H]^2\}$ . We interpret the parameter combination  $\vec{\theta}$  as describing the state of the economy at  $t = 2$  and we denote  $C$  as the set of “core beliefs” of our uncertainty-averse investors. In light of Lemma

<sup>13</sup>For a general discussion, see Theorem 2.5.3 and 2.7.2 of Cover and Thomas (2006).

1 and subsequent discussion, we assume that for  $\vec{\theta} \in C$  we have that  $(\theta_A + \theta_B)/2 = \theta_T$ , where  $\theta_T \equiv (\theta_H + \theta_L)/2$ . Importantly, note that, given  $\theta$ , the success probabilities of the second-stage of project-ideas are independent. We will characterize the extent of technological uncertainty as  $\alpha = \theta_T - \theta_L$ .

Payoffs are determined as follows. If entrepreneur  $\tau$  innovates, and the first stage of the innovation process is successful, he develops an innovation with a (potential) value  $y_\tau$ . At the interim date,  $t = 1$ , each entrepreneur sells her entire firm to outside investors for a value  $V_\tau$ , which thus represents her payoff from innovation. In turn, an uncertainty-averse investor can purchase a fraction  $\omega_\tau$  of firm  $\tau$ , with  $\tau \in \{A, B\}$ , and thus holding the residual value  $w_0 - \omega_A V_A - \omega_B V_B$  in the risk-free asset. To avoid (uninteresting) corner solutions, we assume that the endowment of the risk-free asset is sufficiently large that the budget constraint will not be nonbinding in equilibrium:  $w_0 > \omega_A V_A + \omega_B V_B$ . Investors' final payoff will then depend on their holdings of the risk-free asset and on the success/failure of each innovation at the second stage and on their holdings in the innovation,  $\omega_\tau$ . Finally, we assume that, while outside investors are uncertainty averse with respect to the parameter  $\theta$ , there are no other sources of uncertainty (as opposed to “risk”) in the economy,<sup>14</sup> and that all agents (investors and entrepreneurs) are otherwise risk-neutral.

We will at times benchmark the behavior of uncertainty-averse agents with the behavior of an uncertainty-neutral SEU agent, and we will assume that an uncertainty-neutral investor has  $\theta_L = \theta_H$ , so that she assesses  $\theta_\tau = \theta_T$ . This assumption guarantees that the uncertainty-neutral investor has the same probability assessment on the success probability of each project-idea as a well-diversified uncertainty-averse investor (and thus there is no “hard-wired” difference between the two types of investors).

## 1.1 Endogenous Investor Sentiment

An important implication of uncertainty aversion is that the investor’s probabilistic assessment at the interim date on the parameter  $\theta$  depends on their overall exposure to the source of risk and, thus, on the structure of their portfolios. This means that the probability assessment (i.e.,

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<sup>14</sup>If there is uncertainty on  $q$  or  $r$ , entrepreneurs will assume the worst, selecting  $q_{\min}$  and  $r_{\min}$ , because entrepreneurs’ payoffs are increasing in  $q$  and  $r$ .

the “beliefs”) held by an uncertainty-averse investor (that is, the parameter combination  $\vec{\theta}$ ) are endogenous, and depend on the agent’s overall exposure to the risk factors.

Endogeneity of beliefs is the outcome of the fact that the minimization operator in (1), which determines the probability assessment held by an investor on the success probability of the second stage of the project-ideas, in general depends on the composition of the investor’s overall portfolio. It is useful to note that this property, which plays a critical role in our paper, implies that uncertainty-averse agents are more willing to hold uncertain assets if they can hold such assets in a portfolio rather than in isolation. This happens because, by holding uncertain assets in a portfolio, investors can lower their overall exposure to the sources of uncertainty in the economy. Namely, by investing in both project-ideas, the investor will limit her exposure to the “tail event” that both project-ideas have a very low success probability in the second stage, a property that we will refer to as uncertainty hedging.<sup>15</sup>

The effect of uncertainty hedging in our model is that investors hold more favorable probability assessments on the success probability of project-ideas if they invest in both projects, rather than in just one project. Specifically, if an investor decides to purchase a proportion  $\omega_\tau$  of entrepreneur  $\tau$ ’s firm, with innovation intensity  $y_\tau$ , the investor will hold a risky portfolio  $\Pi = \{\omega_A y_A, \omega_B y_B, w_0 - \omega_A V_A - \omega_B V_B\}$ . Because investors are uncertainty averse (since, they believe  $\theta \in C$ ) but otherwise risk neutral, a portfolio  $\Pi$  provides the investor with utility

$$U(\Pi) = \min_{\theta \in C} \left\{ e^{\theta_A - \theta_M} \omega_A y_A + e^{\theta_B - \theta_M} \omega_B y_B + w_0 - \omega_A V_A - \omega_B V_B \right\}.$$

Because of uncertainty aversion, the investor’s assessment at  $t = 1$  on the state of the economy,  $\vec{\theta}^a$ , is the solution to the minimization problem

$$\vec{\theta}^a(\Pi) = \arg \min_{\theta \in C} U(\Pi),$$

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<sup>15</sup>This property can be loosely interpreted as the analogue for MEU investors of the more traditional “benefits of diversification” displayed by SEU preferences. The property may be seen immediately by noting that, given two random variables,  $y_k$ , with distributions  $\mu_k \in \mathcal{M}$ ,  $k \in \{1, 2\}$ , which are ambiguous to agents, by the property of the minimum operator we have, for  $q \in [0, 1]$ , that

$$q \min_{\mu \in \mathcal{M}} E_\mu [u(y_1)] + (1 - q) \min_{\mu \in \mathcal{M}} E_\mu [u(y_2)] \leq \min_{\mu \in \mathcal{M}} \{q E_\mu [u(y_1)] + (1 - q) E_\mu [u(y_2)]\}.$$

and is characterized in the following lemma.

**Lemma 2** *Increasing an investor’s exposure to one innovation risk induces a more favorable assessment of the other innovation risk. Formally, given a portfolio  $\Pi$ , and letting*

$$\check{\theta}_\tau(\Pi) = \theta_T + \frac{1}{2} \ln \frac{\omega_{\tau'} y_{\tau'}}{\omega_\tau y_\tau}, \quad (4)$$

*an uncertainty-averse investor holds an assessment  $\theta_\tau^a$  on the uncertain parameter  $\theta_\tau$  equal to*

$$\theta_\tau^a(\Pi) = \begin{cases} \theta_L & \check{\theta}_\tau(\Pi) \leq \theta_L \\ \check{\theta}_\tau(\Pi) & \check{\theta}_\tau(\Pi) \in (\theta_L, \theta_H) \\ \theta_H & \check{\theta}_\tau(\Pi) \geq \theta_H \end{cases} . \quad (5)$$

Lemma 2 shows that an investor’s assessment on  $\vec{\theta}$  is endogenous, and it depends crucially on the composition of her portfolio,  $\Pi$ . Thus, we will at times refer to  $\vec{\theta}^a(\Pi)$  as the “portfolio-distorted” assessments. We will say that the agent has “*interior assessments*” when  $\theta_\tau^a \in (\theta_L, \theta_H)$ , in which case, the agent’s assessments are equal to  $\check{\theta}_\tau(\Pi)$  as in (4). Otherwise, we will say that the investor holds “*corner assessments*.” Further, an uncertainty-averse investor’s assessment of  $\vec{\theta}$  determines the views held by the investor on the future state of the economy. Thus, we will also refer to the assessment  $\theta_\tau^a$  as “*investor sentiment*.”

Lemma 2 shows that when an investor has a relatively smaller proportion of her portfolio invested in innovation  $\tau$ ,  $\omega_\tau y_\tau < \omega_{\tau'} y_{\tau'}$ , she will be relatively more optimistic about the return on that innovation. This happens because a smaller exposure to the risk generated by a given innovation, relative to another innovation, will make an uncertainty-averse investor relatively more concerned about priors that are less favorable to the other innovation. Correspondingly, the investor will give more weight to the states of nature that are more favorable to the first innovation. In other words, the investor will be more “optimistic” on the success probability of that innovation (i.e., will have a stronger sentiment), and more “pessimistic” with respect to the other innovation.

Suppose entrepreneur of type  $A$  decides to innovate, but entrepreneur  $B$  decides not to innovate. Because  $y_B = 0$ , by Lemma 2, we have that  $\theta_A^a(\Pi) = \theta_L$  for any  $\omega_A y_A > 0$ . Correspondingly, if

entrepreneur  $B$  decides to innovate, but entrepreneur  $A$  does not, we have that  $\theta_B^a(\Pi) = \theta_L$ . Similar situations emerge if only one entrepreneur has a successful first-stage project-idea, while the other entrepreneur fails. In this case, at the interim date,  $t = 1$ , investors hold more pessimistic assessments about the successful innovation than if both entrepreneurs have a successful first-stage project-idea. This means that investors, when facing only one innovation, will be more pessimistic on that innovation than when facing both innovations. This happens because, by investing in only one project-idea, investors forego the benefits of uncertainty-hedging and hold a portfolio with greater exposure to the possibility that the second-stage success probability is very low. In contrast, by investing in both technologies, the investor protects herself from the situation that both technologies have very low success probability, a hypothesis rejected by the relative entropy criterion (3).

In our model, portfolio-distorted assessments determine investors' expectations on the ultimate success probability of the innovation processes in the economy, and thus characterize investors' "sentiment" toward innovations. An important implication of Lemma 2 that will play a key role in our analysis is that investor sentiment about one innovation will crucially depend on the availability of other innovations in the economy, and their innovation intensity. In particular, an investor will be more optimistic about an innovation success probability, and she values it more, if she will be able to also invest in the other innovation. Thus, investors' probabilistic assessments create an externality for entrepreneurs, in that an entrepreneur's successful innovation will be more valuable if other entrepreneurs have successful innovations as well. In other words, if both entrepreneurs innovate and are successful at the first stage, investor sentiment toward both innovations improves making both innovations more valuable. Note that this spillover effect from one innovation to another is driven by investors' assessments of the success probability of the second stage of the innovation processes and, thus, by their sentiment.

## 2 The Innovation Decision

We will solve the model recursively. First, we find the choice by entrepreneurs that are successful at the first stage of the innovation process of the optimal innovation intensity,  $y_\tau$ , and the value  $V_\tau$

that investors are willing to pay at the interim date for innovations. Next, we solve for the initial choice by entrepreneurs on whether or not to initiate the innovation process by incurring the initial discovery cost  $k_\tau$ . As a benchmark, we start the analysis by characterizing the two entrepreneurs' innovation decisions when investors are uncertainty-neutral SEU agents, then we consider the case where investors are uncertainty-averse MEU agents.

The implementation of the second stage of the innovation process requires entrepreneurs to raise capital from investors by selling equity in the capital markets at  $t = 1$ . For simplicity, we assume that an entrepreneur of type  $\tau$  sells her entire firm to investors, uses the proceeds to pay for the intensity costs  $c_\tau(y_\tau)$ , and pockets the difference. We assume that  $y_\tau$  is observable and contractible with outside investors, thus ruling out moral hazard. In this case, the choice of innovation intensity  $y_\tau$  by a type- $\tau$  entrepreneur depends on the price that outside investors are willing to pay for her firm, that is, on the market value of the equity of the firm. This, in turn, depends on the assessments held by investors on the success probability of the innovation,  $p_\tau(\theta)$ .

**Lemma 3** *Given investors' assessments and risk-neutrality, entrepreneurs' firms are priced at their expected value, that is,  $V_\tau = p_\tau(\theta_\tau^a)y_\tau$  for uncertainty-averse investors, and  $V_\tau = p_\tau(\theta_T)y_\tau$  for uncertainty-neutral investors, with  $\tau \in \{A, B\}$ . In equilibrium, it is (weakly) optimal for investors to hold a balanced portfolio:  $\omega_A^* = \omega_B^*$  for both type of investors (SEU and MEU).*

Lemma 3 shows that, given our assumption of universal risk-neutrality, investors price equity at its expected value, given their assessments. Investors' assessments, however, depend on their attitude toward uncertainty, that is whether they are uncertainty-neutral investors or uncertainty-averse investors. Endogeneity of assessments is critical because it will lead to different market valuation of equity, and thus, different behavior by entrepreneurs. Also, it is weakly optimal for investors to hold balanced portfolios. SEU investors are indifferent on their portfolio composition, because of risk neutrality. In contrast, uncertainty-averse investors strictly prefer a balanced portfolio, due to uncertainty-hedging. For notational simplicity, we normalize investors' portfolio holding and set  $\omega_A^* = \omega_B^* = 1$ .<sup>16</sup>

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<sup>16</sup>This is WLOG optimal because there is one unit mass of investors.



## 2.1 The Uncertainty-Neutral Benchmark

As a benchmark, we start with the simpler case in which investors are uncertainty-neutral. When investors are uncertainty-neutral, equity prices depend only on their prior  $\theta_T$  and on the level of innovation intensity,  $y_\tau$ , chosen by the firm, giving

$$V_\tau^S = p_\tau(\theta_T) y_\tau, \text{ for } \tau \in \{A, B\}. \quad (6)$$

Equation (6) shows that equity value for an innovation of type  $\tau$  depends only the investors' assessments of the success probability of the second stage of the innovation process,  $p_\tau(\theta_T) = e^{\theta_T - \theta_M}$ , and its level of innovation intensity,  $y_\tau$ : it does not depend on the innovation intensity decision of the other firm,  $y_{\tau'}$ , for  $\tau' \neq \tau$ . This means that, without uncertainty aversion, there are no interactions between the choice of the innovation intensities by the two entrepreneurs. In this case, if the first stage of the project-idea was successful, entrepreneur  $\tau$ 's chooses the level of innovation intensity for the second stage,  $y_\tau$ , by solving

$$\max_{y_\tau} \mathcal{U}_\tau^S \equiv V_\tau^S - c_\tau(y_\tau) = p_\tau(\theta_T) y_\tau - \frac{1}{Z_\tau(1+\gamma)} y_\tau^{1+\gamma}. \quad (7)$$

From (7) it immediately follows that the optimal innovation intensity,  $y_\tau$ , chosen by entrepreneur  $\tau$ , is equal to

$$y_\tau^* \equiv [p_\tau(\theta_T) Z_\tau]^\frac{1}{\gamma}, \quad (8)$$

By direct substitution of  $y_\tau^*$  into (7),<sup>17</sup> we obtain that the ex-ante expected payoff for entrepreneur  $\tau$  from initiating the innovation process, and thus incurring discovery cost  $k_\tau$ , is equal to

$$EU_\tau^S = q_\tau \frac{\gamma}{1+\gamma} [p_\tau(\theta_T)]^\frac{1+\gamma}{\gamma} Z_\tau^\frac{1}{\gamma} - k_\tau.$$

Thus, entrepreneur  $\tau$  innovates at  $t = 0$  if  $EU_\tau^S \geq 0$ , leading to the following theorem.

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<sup>17</sup>Because  $\frac{\partial^2 U_\tau^S}{\partial y_\tau^2} = -\frac{\gamma}{Z_\tau} y_\tau^{\gamma-1} < 0$ , first-order conditions are sufficient for a maximum.

**Theorem 1** *When investors are uncertainty-neutral, entrepreneurs of type  $\tau$  innovate iff*

$$k_\tau \leq k_\tau^S \equiv q_\tau \frac{\gamma}{1+\gamma} [p_\tau(\theta_T)]^{\frac{1+\gamma}{\gamma}} Z_\tau^{\frac{1}{\gamma}}, \quad \tau \in \{A, B\},$$

*and the innovation processes of the two entrepreneurs are independent.*

Theorem 1 shows that when investors are uncertainty neutral, the investment decisions by the two entrepreneurs are effectively independent from each other, with no spillover effects. When investors are uncertainty averse, however, the innovation processes of the two firms are interconnected.

## 2.2 Uncertainty Aversion and Innovation

We now derive optimal innovation decisions when investors are uncertainty averse. In this case, from Lemma 2, we know that investor sentiment toward the success probability of the second stage of each innovation process,  $p_\tau(\theta_\tau^a)$ , depends on the overall risk exposure of their portfolios. Specifically, sentiment is endogenous, and depends on the innovation intensities chosen by both firms,  $y_\tau$ .

**Lemma 4** *If investors are uncertainty averse, the market value of entrepreneur  $\tau$ 's firm is*

$$V_\tau^U(\Pi) = \begin{cases} p_\tau(\theta_H) y_\tau & y_\tau \leq e^{-2\alpha} y_{\tau'} \\ p_\tau(\theta_T) y_\tau^{\frac{1}{2}} y_{\tau'}^{\frac{1}{2}} & y_\tau \in (e^{-2\alpha} y_{\tau'}, e^{2\alpha} y_{\tau'}) \\ p_\tau(\theta_L) y_\tau & y_\tau \geq e^{2\alpha} y_{\tau'} \end{cases}, \quad (9)$$

*where  $y_\tau$  is the innovation intensity selected by entrepreneur of type- $\tau$ , with  $\tau, \tau' \in \{A, B\}$ ,  $\tau \neq \tau'$ .*

Lemma 4 shows that, when investors are uncertainty averse, the market value of equity of one firm depends on the level of innovation intensity chosen by its entrepreneur as well as on the level chosen by the other firm. The interaction between the market values of the equity of the two firms creates a strategic externality between the two entrepreneurs, which will be critical in the analysis below.

Note the linkage between the market value of the two firms occurs through endogenous investor sentiment. From Lemma 2 an increase of the innovation intensity of one firm will increase the relative exposure of investors to that firm's risk relative to the other firm's risk, making (all else equal)

investors relatively more pessimistic about that firm's success probability and, correspondingly, relatively more optimistic about the other firm's success probability.

Lemma 4 also implies that an increase of the level of innovation intensity in one firm,  $y_\tau$ , has two opposing effects on its value  $V_\tau^U$ . The first is the positive direct effect that greater innovation intensity has on the ultimate value of the innovation. This positive effect can however be mitigated (in the case of "interior assessments") by a second negative effect that an increase in innovation intensity has, all else equal, on investor sentiment. This implies that firm value is a (weakly) increasing function of the innovation intensities of both firms.

Finally, note that if one of the two firms does not innovate or the innovation is not successful in the first stage, the level of innovation intensity for that firm is necessarily equal to zero. From Lemma 4, this implies that the market value of the one firm will be determined at the worst-case scenario for that firm, that is  $V_\tau(\Pi) = p_\tau(\theta_L)y_\tau$ .

We can now determine the optimal level of innovation intensity for each entrepreneur. If the first stage of the project-idea was successful, entrepreneur  $\tau$  chooses the level of innovation intensity for the second stage,  $y_\tau$ , by solving

$$\max_{y_\tau} \mathcal{U}_\tau^U \equiv V_\tau(\Pi) - \frac{1}{Z_\tau(1+\gamma)} y_\tau^{1+\gamma}, \quad (10)$$

where  $\Pi = \{y_A, y_B, w_0 - V_A - V_B\}$  and  $V_\tau(\Pi)$  is given in (9). To simplify the exposition, in what follows we assume that the two types of firms are not too dissimilar. Specifically, we assume that the values  $Z_A$  and  $Z_B$  are not too far away from each other:  $\frac{Z_A}{Z_B} \in \left(\frac{1}{\psi}, \psi\right)$  where  $\psi \equiv \frac{1}{4}e^{2\alpha(\gamma+1)}\left(1 + \frac{1}{2\gamma}\right)^{2\gamma}$ . This assumption ensures that if both firms have successful first-stage projects, they find it optimal to chose levels of innovation intensity  $\{y_A, y_B\}$  that in equilibrium result in interior assessments for investors.

The solution to problem (10) depends on whether one or both firms decide to initiate the innovation process and pay the discovery costs  $k_\tau$  and, if they do so, whether they are successful at the first stage of the innovation process. Thus, there are four possible states of the world that we need to analyze: (i) when both entrepreneurs had a successful first stage, state  $SS$ ; (ii) when only one entrepreneur has a successful first-stage, state  $SF$  with the symmetric  $FS$  state, (iii) when

both entrepreneur fail in the first stage and no innovation can take place, state  $FF$ . Since the last state  $FF$  is trivial, we now focus on the first two.

### 2.2.1 Only One Firm Has Successful First-Stage Project, State SF

Consider first the case in which only one entrepreneur had a successful first-stage project-idea, state SF. For future reference, note that this state may emerge either because the other entrepreneur has not initiated the innovation process (that is, she did not sustain the discovery cost, or because the first stage was unsuccessful.

**Lemma 5** *If only one entrepreneur has a successful first stage project-idea (state SF), she selects innovation intensity equal to*

$$y_{\tau}^{U,SF} = [p_{\tau}(\theta_L) Z_{\tau}]^{\frac{1}{\gamma}}; \quad (11)$$

*the market value of the entrepreneur's firm is equal to*

$$V_{\tau}^{U,SF} = [p_{\tau}(\theta_L)]^{\frac{1+\gamma}{\gamma}} Z_{\tau}^{\frac{1}{\gamma}}, \quad (12)$$

*giving a continuation utility for the entrepreneur equal to*

$$\mathcal{U}_{\tau}^{U,SF} \equiv [p_{\tau}(\theta_L)]^{\frac{1+\gamma}{\gamma}} Z_{\tau}^{\frac{1}{\gamma}} \frac{\gamma}{1+\gamma}. \quad (13)$$

If only one entrepreneur successfully develops a first-stage project, there will only be one type of uncertain innovation available to investors, so investors will believe the worst-case scenario about that innovation resulting in negative investor sentiment and low equity valuations. Therefore, the lone entrepreneur will chose a low level of innovation intensity, consistent with the negative sentiment.

### 2.2.2 Both Firms Have Successful First-Stage Projects, State SS

If both entrepreneurs have successful first-stage projects, market valuation is given in Lemma 4, which leads to the following lemma.

**Lemma 6** Let  $\frac{Z_A}{Z_B} \in \left(\frac{1}{\psi}, \psi\right)$ . If both entrepreneurs innovate and have a successful first stage (state SS), they select innovation intensities equal to

$$y_\tau^{U,SS}(y_{\tau'}) = \left[ \frac{Z_\tau}{2} p_\tau(\theta_T) (y_{\tau'})^{1/2} \right]^{\frac{1}{\gamma + \frac{1}{2}}}, \quad \text{with } \tau \neq \tau', \text{ and } \tau, \tau' \in \{A, B\}. \quad (14)$$

Lemma 6 establishes that there is strategic complementarity in entrepreneurs' production decisions. In particular, an entrepreneur's choice of innovation intensity,  $y_\tau^{U,SS}(y_{\tau'})$ , is an increasing function of the other entrepreneur's innovation intensity,  $y_{\tau'}$ . The strategic complementarity originates in investor uncertainty aversion and endogenous investor sentiment. From Lemma 2 and Lemma 4, we know that the sentiment of uncertainty-averse investors on the success probability of the second stage of an innovation process and, thus, their market valuations at the interim date, depend on the innovation intensities chosen by both entrepreneurs. Thus, because of the effect on sentiment, investors perceive innovations effectively as complements. This complementarity is then transferred from investors' sentiment to entrepreneurs' innovation decisions.

We can now determine the equilibrium levels of innovation intensities chosen by the two entrepreneurs in the SS state.

**Theorem 2** If both entrepreneurs innovate and have successful first-stage projects (state SS), the equilibrium level of innovation intensities for an entrepreneur is

$$y_\tau^{U,SS} = \left[ \frac{1}{2} p_\tau(\theta_T) Z_\tau^{\frac{2\gamma+1}{2\gamma+2}} Z_{\tau'}^{\frac{1}{2\gamma+2}} \right]^{\frac{1}{\gamma}}. \quad (15)$$

In equilibrium, firm value for each firm is

$$V_\tau^{U,SS} = 2^{-\frac{1}{\gamma}} [p_\tau(\theta_T)]^{\frac{1+\gamma}{\gamma}} (Z_\tau Z_{\tau'})^{\frac{1}{2\gamma}}, \quad (16)$$

and continuation utility is equal to

$$\mathcal{U}_\tau^{U,SS} = 2^{-\frac{1}{\gamma}} [p_\tau(\theta_T)]^{\frac{1+\gamma}{\gamma}} (Z_\tau Z_{\tau'})^{\frac{1}{2\gamma}} \frac{2\gamma + 1}{2\gamma + 2}. \quad (17)$$

The following corollary compares the equilibrium values when one or both entrepreneurs have successful first-stage projects.

**Corollary 1** *An entrepreneur is better off when the other entrepreneur also has a successful first-stage projects:  $U_\tau^{U,SS} > U_\tau^{U,SF}$ . If entrepreneurs' productivities are not too dissimilar,  $\frac{Z_{\tau'}}{Z_\tau} \in \left(\frac{1}{\psi_1}, \psi_1\right)$ , equity values are higher when both entrepreneurs have successful first-stage projects:  $V_\tau^{U,SS} > V_\tau^{U,SF}$ . In addition, if entrepreneurs' productivities are sufficiently close together,  $\frac{Z_{\tau'}}{Z_\tau} \in \left(\frac{1}{\psi_2}, \psi_2\right)$ , entrepreneurs innovate with greater intensity when both have successful first-stage projects:  $y_\tau^{U,SS} > y_\tau^{U,SF}$ . Finally,  $\psi_2 < \psi_1 < \psi$ .*

An important implication of Corollary 1 is that, if entrepreneurs' productivities are not too dissimilar, because of the complementarity of innovations generated by uncertainty aversion, investors value one type of innovation more when they can also invest in the other type of innovation, yielding  $V_\tau^{U,SS} > V_\tau^{U,SF}$ .

### 2.3 The Innovation Decision

In the previous sections we have shown that investor uncertainty aversion affects equity valuations and generates strategic complementarity in the interim choice of innovation intensity,  $y_\tau$ . The interim strategic complementarity of the choice of innovation intensity generates a strategic complementarity also in the entrepreneurs' decisions to innovate at the beginning of the innovation process,  $t = 0$ , that is, to incur the discovery cost  $k_\tau$ .

If entrepreneur  $\tau'$  chooses to innovate, the expected utility for entrepreneur  $\tau$  from sustaining at  $t = 0$  the initial discover cost  $k_\tau$  and, thus, initiating the innovation process is

$$EU_\tau^{U,I} = (q_\tau q_{\tau'} + r)\mathcal{U}_\tau^{U,SS} + (q_\tau(1 - q_{\tau'}) - r)\mathcal{U}_\tau^{U,SF} - k_\tau$$

for  $\tau, \tau' \in \{A, B\}$  and  $\tau \neq \tau'$ . Conversely, if entrepreneur  $\tau'$  does not innovate at  $t = 0$ , the expected utility for entrepreneur  $\tau$  from choosing to innovate at  $t = 0$  is

$$EU_\tau^{U,N} = q_\tau \mathcal{U}_\tau^{U,SF} - k_\tau.$$

We can now characterize the equilibrium of the innovation decision at the beginning of the period,  $t = 0$ .

**Theorem 3** *For low levels of discover cost,  $k_\tau \leq \underline{k}_\tau$ , the entrepreneur always innovates. For high levels of discovery cost,  $k_\tau \geq \bar{k}_\tau$ , the entrepreneur never innovates. For intermediate levels of the discovery cost,  $k_\tau \in (\underline{k}_\tau, \bar{k}_\tau)$ , the entrepreneur is willing to innovate only if the other entrepreneur innovates. If both entrepreneurs have intermediate levels of discovery cost, there are multiple equilibria, one where both entrepreneurs innovate and one where neither innovate. The innovation equilibrium dominates the no-innovation equilibrium.*

For very small levels of discovery costs,  $k_\tau \leq \underline{k}_\tau$ , it is a dominant strategy for the entrepreneur to innovate. For very large levels of discovery costs,  $k_\tau \geq \bar{k}_\tau$ , it is a dominant strategy for the entrepreneur to not innovate. For intermediate levels of discovery costs,  $k_\tau \in (\underline{k}_\tau, \bar{k}_\tau)$ , entrepreneur  $\tau$  wishes to innovate only if the other entrepreneur innovates as well. Theorem 3 shows this strategic complementarity in entrepreneurs' innovation decisions.

When both entrepreneurs have intermediate levels of the discovery cost, there are multiple equilibria, with and without innovation. In this case, entrepreneurs face a classic “assurance game,” in which there is a Pareto-dominant equilibrium, where both entrepreneurs innovate, yet there is also an inefficient, Pareto-inferior equilibrium, where neither entrepreneur innovates. Multiplicity of equilibria depends on the fact that it is profitable for one entrepreneur to innovate only if he expects the other entrepreneur to innovate as well. Such multiplicity of equilibria in the innovation game is the direct outcome of investors' uncertainty aversion.

We conclude this section by characterizing the impact of the model's parameters on the threshold levels  $\{\underline{k}_\tau, \bar{k}_\tau\}_{\tau \in \{A, B\}}$ .

**Corollary 2** *The threshold levels  $\{\bar{k}_\tau\}_{\tau \in \{A, B\}}$  are increasing functions of  $q_\tau, q_{\tau'}, Z_\tau, Z_{\tau'}$  and  $r$ , and the threshold levels  $\{\underline{k}_\tau\}_{\tau \in \{A, B\}}$  are increasing functions of  $q_\tau$  and  $Z_\tau$ .*

Corollary 2 has the interesting implication that an increase in one firm's probability of success,  $q_\tau$ , makes not only that firm, but also other firms, more willing to attempt first-stage discovery of a product-idea. This follows because the strategic complementarity induced by uncertainty aversion.

In the absence of uncertainty aversion, an increase in the probability of discovery affects only that entrepreneur, with no effect on other entrepreneurs. Corollary 2 also shows that entrepreneurs are more willing to innovate if her innovation is more related to other entrepreneurs' innovations, that is  $r$  is greater. This happens because greater degree of relatedness increases the probability that both project-ideas are simultaneously successful in the first-stage, increasing the market value of the innovations. Finally, Corollary 2 also shows that an increase in productivity of an entrepreneur increases not only that entrepreneur's willingness to innovate, but also makes other entrepreneurs willing to innovate as well.

### 3 Acquiring Innovation

In the previous sections, we have shown that investors' uncertainty aversion creates externalities across innovations. These externalities are due to endogeneity of investor sentiment, and create the possibility of value dissipation due to coordination failures. This means that there may be gains from internalizing such externalities via acquisitions.

There are two externalities at work in our model. The first externality is due to the valuation spillover discussed in Lemma 2. This happens because, for any given set of choices of innovation intensities,  $\{y_\tau, y_{\tau'}\}$ , the two firms are more valuable to uncertainty-averse investors when they are held in the same portfolio than when they are owned separately.

The second externality is due to the strategic complementarity between the choices of innovation intensity  $y_\tau$ , discussed in Lemma 4: the market value of an individual firm,  $V_\tau^U$ , is an increasing function of the innovation intensity chosen by both firms,  $\{y_\tau, y_{\tau'}\}$ , through its effect on investor sentiment. When a firm chooses their own optimal level of innovation intensity, they ignore the positive externality that choice has on the other firm's valuation.

We extend our analysis by examining the effect of the strategic complementarity between innovation intensities. We modify the basic model as follows. If both entrepreneurs are successful in the first stage, we now allow for the possibility that at the interim date,  $t = 1$ , both entrepreneurs merge their firms in a new firm.<sup>18</sup> After the merger, the entrepreneurs jointly determine the inno-

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<sup>18</sup>Alternatively, the merger between the two firms may be initiated by a third firm which may acquire the innovation



vation intensity,  $y_\tau$ , for both innovation processes  $\tau \in \{A, B\}$ . After the selection of the innovation intensities  $y_\tau$ , the merged firm will again sell all its equity in the public equity market. The two innovations processes may be sold to the public equity market either as a single multi-divisional firm, or as two independent firms.<sup>19</sup>

After the merger of the first-stage innovations, the problem of the merged firm is to maximize the combined value of the two innovation projects. By identical reasoning to the proof of Lemma 3, the merged firm will value the projects at  $V_\tau = p_\tau(\theta_\tau^I) y_\tau$ , for  $\tau \in \{A, B\}$ , where  $\vec{\theta}^I$  is the investors' assessment when the merged firm is sold on the public equity market. Thus, the merged firm's objective at this stage is now to solve

$$\max_{\{y_A, y_B\}} \mathcal{U}^M = p_A(\theta_A^I) y_A + p_B(\theta_B^I) y_B - c_A(y_A) - c_B(y_B).$$

If investors are uncertainty neutral,  $\theta_\tau^I = \theta_T$ , so the choice of  $y_A$  and  $y_B$  are independent of each other. In this case, the merged firm solves the same problem as the original entrepreneurs (7):  $U^M = U_A^S + U_B^S$ . This implies that the optimal levels of innovation intensity chosen by the merged firm are again given by (8), that is, the values the entrepreneurs would choose if the two firms were independent. Thus, if investors are uncertainty neutral, the merger does not add value.

In contrast, if investors are uncertainty averse,  $\vec{\theta}^I = \vec{\theta}^a$  which, from (5), depends on the choice of both  $y_A$  and  $y_B$ . As shown in Lemma 4, for interior assessments (which we will show is the case in equilibrium), we now have that

$$V_A = V_B = e^{\theta_T - \theta_M} y_A^{\frac{1}{2}} y_B^{\frac{1}{2}}.$$

This implies that the maximization problem of the merged firm becomes

$$\max_{y_A y_B} \mathcal{U}^M = 2e^{\theta_T - \theta_M} y_A^{\frac{1}{2}} y_B^{\frac{1}{2}} - \frac{1}{Z_A(1+\gamma)} y_A^{1+\gamma} - \frac{1}{Z_B(1+\gamma)} y_B^{1+\gamma},$$

leading to the following theorem.

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from both entrepreneurs.

<sup>19</sup>Remember that, if the two innovations are sold in two separate firms, from Lemma 3, investors will optimally invest in both firms.

**Theorem 4** *If investors are uncertainty averse, the merged firm will select a greater innovation intensity in both projects*

$$y_\tau^M \equiv \left[ p_\tau (\theta_T) Z_{\tau'}^{\frac{1}{2\gamma+2}} Z_\tau^{\frac{2\gamma+1}{2\gamma+2}} \right]^{\frac{1}{\gamma}} > y_\tau^{U,SS} ,$$

*and will have a greater value than these firms would have as a stand-alone:*

$$V^M = 2 [p_\tau (\theta_T)]^{\frac{1+\gamma}{\gamma}} [Z_A Z_B]^{\frac{1}{2\gamma}} > V_A^{U,SS} + V_B^{U,SS} .$$

Theorem 4 shows that a merger can add value to the innovative process by merging both firms from the original entrepreneurs and then choosing an innovation intensity at both firms that is greater than the one that the entrepreneurs would chose individually. Because of the positive externality between investment levels  $y_\tau$ , inefficiently low levels of investment occur when each entrepreneur maximizes his own payoff. By merging, the post-acquisition firm internalizes the spillover effects of investment, leading to greater firm valuation.

We now examine the impact of the possibility of a merger at the interim date  $t = 1$  on the entrepreneurs' ex-ante incentives to innovate, that is, to sustain at  $t = 0$  the discovery cost  $k_\tau$ . The initial decision to innovate by an entrepreneur will depend on the terms at which the entrepreneur anticipates the merger with take place. The acquisition price, in turn, will depend on the allocation of the surplus generated by the acquisition, that is, on how the synergies are divided between the two entrepreneurs.

The allocation of the synergies created in the merger occurs through bargaining, and we will assume that the two entrepreneurs will split the surplus equally. Thus, if both innovations are successful in the first stage, entrepreneur  $\tau$  earns

$$\Upsilon_\tau = u_\tau^{U,SS} + \frac{1}{2} \left( u^M - u_A^{U,SS} - u_B^{U,SS} \right) .$$

The incentives to pay the initial discover cost are discussed in the following.

**Theorem 5** *For low levels of discover cost,  $k_\tau \leq \underline{K}_\tau$ , the entrepreneur always innovates. For high levels of discovery cost,  $k_\tau \geq \bar{K}_\tau$ , the entrepreneur never innovates. For intermediate levels*

of the discovery cost,  $k_\tau \in (\underline{K}_\tau, \bar{K}_\tau)$ , the entrepreneur is willing to innovate only if the other entrepreneur innovates. If both entrepreneurs have intermediate levels of discovery cost, there are multiple equilibria, one where both entrepreneurs innovate and one where neither innovate. The innovation equilibrium dominates the no-innovation equilibrium. Finally  $\underline{K}_\tau = \underline{k}_\tau < \bar{k}_\tau < \bar{K}_\tau$ : the possibility of a merger induces entrepreneurs to innovate more ex-ante.

Theorems 4 and 5 have the interesting implication that an active M&A market promotes innovative activity and leads to greater innovation rates, stronger investor sentiment, and higher firm valuations. Synergies created in the merger are a direct consequence of endogenous investor sentiment due to uncertainty aversion. A merger allows entrepreneurs to internalize the positive impact that the choice of the innovation intensity in one innovation has on other innovations, and leads to greater innovation rates. Thus, the merger of innovations endogenously promotes stronger investor sentiment and leads to greater valuations.

## 4 Innovation Waves

In this section we extend our basic model to the case of multiple innovators in the context of a simple dynamic model. We show that entrepreneurs initiate innovation only if in the economy there is a sufficiently large number of potentially active innovators. In particular, when the number of potential innovators is low, entrepreneurs do not engage in innovation because they expect weak investor sentiment and, thus, the market for innovation to be “cold,” and potential innovations remain latent in the economy. In contrast, when the number of innovators reaches critical mass, innovation is triggered and a wave of innovations takes place in the economy amid strong investor sentiment.

We modify the basic model of our paper as follows. We consider a simple discrete-time dynamic model, where  $t$  denotes time. At each date  $t$ , a new project-idea arrives with probability  $\pi$ , where each project idea is owned by a unique entrepreneur. The economy is “bounded” in that at any point of time only up to  $N$  project-ideas can exist in economy. Let  $\mathcal{N}_t$  be the set of entrepreneurs endowed with a project-idea at any given time  $t$ , and let  $\nu_t \equiv |\mathcal{N}_t|$ . Different from the basic model,

we now assume that an entrepreneur endowed with a project-idea can delay its implementation to a future date. Waiting to implement the project-idea is costly, however: entrepreneurs and investors are impatient and have discount factor  $\delta$ .

An entrepreneur endowed with a project-idea at time  $t$  must decide whether or not to implement the innovation. We now assume that the decision to initiate an innovation is made by the entrepreneur after observation of a public signal  $\sigma_t$  which is informative on  $\nu_t$ . For analytical simplicity, we assume that the public signal is perfectly informative on  $\nu_t$  and we set  $\sigma_t = \nu_t$ .<sup>20</sup> If an entrepreneur decides to innovate at time  $t$ , she must pay at that time the non-pecuniary discovery cost  $k$  to implement the first stage of the innovation process. For analytical tractability, we now assume that the first stage of the innovation process is always successful, if it is implemented, setting  $q_n = 1$ .<sup>21</sup> At time  $t + 1$ , entrepreneurs proceed with the second stage of the innovation process by implementing innovation intensity  $y_n$  at cost  $c(y_n)$ , which is paid for by selling equity to investors. Finally, at time  $t + 2$  project-ideas have a payoff  $y_n$  with probability  $p_n$ . For simplicity, we assume that entrepreneurs' project ideas have the same productivity,  $Z_n = Z$  and that the innovation intensity is fixed,  $y_n = y$ .

The success probability of the second stage of a project implement at date  $t$ ,  $p_n$ , is uncertain, and depends again on the value of  $\theta_{nt}$ , where  $p_n(\theta_{nt}) = e^{\theta_{nt} - \theta_M}$ , with  $\theta_{nt} \in [\theta_L, \theta_H] \subseteq [\theta_m, \theta_M]$ . For simplicity, we assume that uncertainty on  $p_n$  is stationary and independent across time periods.<sup>22</sup> Thus, at any time  $t$ , investors are uncertain over the vector  $\vec{\theta} \equiv \{\theta_n\}_{n=1}^N$ , and believe that  $\vec{\theta} \in \mathcal{C} \subset [\theta_m, \theta_M]^N$ , and that  $\frac{1}{N} \sum_{n=1}^N \theta_n = \theta_T$  for  $\theta_T \in (\theta_L, \theta_H)$ .

Uncertainty-averse investors form at any time  $t + 1$  their portfolios of uncertain assets by buying equity from successful entrepreneurs (if any) available at that time. We denote by  $S_t$  the set of successful entrepreneurs at  $t + 1$ , and we let  $s_t \equiv |S_t|$ . Note that, given our assumption that all innovations that are undertaken by an entrepreneur have a successful first-stage,  $S_t$  is also equal to the set of entrepreneurs that have initiated their project ideas at time  $t$ . Similar to the basic model, each investor chooses a portfolio of the uncertain assets,  $\{\omega_n\}_{n \in S_t}$ , given their fair

<sup>20</sup>It is possible, although messy, to extend the model to the case in which the public signal is noisy.

<sup>21</sup>Our results will go through for the case in which  $q_n < 1$ .

<sup>22</sup>This assumption rules out, for example, interesting issues such as learning, which can be included in the analysis and we leave for future research.

market valuations  $\{V_n\}_{n \in S_t}$ . By identical reasoning behind that of Lemma 3, it is easy to show that investors optimally invest equally in all innovations, giving  $\omega_n = \omega_{n'}$  for all  $(n, n') \in S_t$ . Furthermore, given investor sentiment, denoted now by the vector  $\vec{\theta}^a(S_t) = \{\theta_n^a\}_{n=1}^N$ , equity is priced at its expected value, giving  $V_n = p_n(\theta_n^a)y$ , for  $n \in S_t$ .

Investor sentiment depends on the number of entrepreneurs that innovate, as follows.

**Lemma 7** *There is a threshold  $\bar{s} \equiv N \frac{\theta_H - \theta_T}{\theta_H - \theta_L}$  such that if a small number of entrepreneurs innovate,  $s_t \leq \bar{s}$ , the market will assess all entrepreneurs in the market with very pessimistic sentiment, setting  $\theta_{nt} = \theta_L$  for all  $n \in S_t$ ; the value of equity is*

$$V_n = \delta p_n(\theta_L)y. \quad (18)$$

*If a large number of entrepreneurs innovate,  $s_t > \bar{s}$ , investors' sentiment satisfies:*

$$\theta_{nt}^a = \theta_H - \frac{N}{s_t}(\theta_H - \theta_T) > \theta_L, \quad \text{for } n \in S_t, \quad (19)$$

*and the market value of equity is*

$$V_n(s_s) = \delta p_n(\theta_{nt}^a)y. \quad (20)$$

*Finally,  $V_n(s_t)$  is increasing in  $s_t$ .*

Lemma 7 shows that investor sentiment at any date  $t$  depends on the number of successful first-stage innovations,  $s_t$ . When the number of entrepreneurs that innovate is small,  $s_t \leq \bar{s}$ , investor sentiment is low, and the capital market values innovations very conservatively. In contrast, when a large number of entrepreneurs decide to innovate,  $s_t > \bar{s}$ , investors have a strong sentiment about ongoing innovation projects, leading to greater equity valuations.

Note that, for  $s_t > \bar{s}$ , from (19) investor sentiment is increasing in the number of projects available at that time  $s_t$ . This happens because, for greater value of  $s_t$ , uncertainty-averse investors will be relatively less concerned on each individual project that is actually available at that time, relative to the set of possible projects, including those potentially yet to come.<sup>23</sup> Note also that

<sup>23</sup>Intuitively, this property can be seen immediately in light of the discussion in Section 1. For given total entropy (i.e. uncertainty), investors must limit the extent of the pessimism that they can have on each individual project.

investor sentiment is negatively affected by the extent of uncertainty in the economy, which we measure by the difference  $\alpha \equiv \theta_H - \theta_T$ . This happens because, from (19), a greater value of  $\alpha$  will make uncertainty-averse investors more pessimistic.

At the beginning of each period  $t$ , entrepreneurs endowed with a project-idea must decide whether or not to pay the discovery cost  $k$  and innovate, or to postpone the initiation of the innovation to a later period. This decision is made after the observation of the public signal  $\sigma_t$  which perfectly reveals the number of entrepreneurs endowed with a project idea at that time,  $\nu_t$ , and is characterized as follows.

**Theorem 6** *Let  $k \in (\underline{k}_d, \bar{k}_d)$  (where  $\underline{k}_d$  and  $\bar{k}_d$  are defined in the Appendix). There is a threshold  $\bar{\nu} > \bar{s}$  (defined in the Appendix) such that if  $\nu_t \geq \bar{\nu}$ , it is optimal for an entrepreneur with a project-idea to pay the discovery cost  $k$  and innovate if all other entrepreneurs with project-ideas innovate.*

Lemma 7 and Theorem 6 imply that investor sentiment, market valuations of firm equity, and innovation decisions are endogenous, and depend on the number of innovative firms available on the market. If few entrepreneurs are endowed with a project idea, they rationally anticipate that in the following period investor sentiment will be cold, and correspondingly, market valuations will be low. The expectation of “cold equity markets” will induce entrepreneurs not to innovate, and to delay the decision to another date. In contrast, when the number of entrepreneurs with an innovation is greater than a certain critical mass,  $\bar{\nu}$ , entrepreneurs anticipate that, if they innovate, investors will have strong sentiment in the following period, and correspondingly, market valuations will be high. The expectation of “hot equity markets” will thus induce entrepreneurs to innovate.

From Theorem 6, we know that innovations may occur any time the number of entrepreneurs with a project-ideas exceeds the critical mass  $\bar{\nu}$ . The following theorem shows that, if the probability that a new project-idea arrives,  $\pi$ , is sufficiently low (or, equivalently, the discount factor  $\delta$  is sufficiently small) it is best for entrepreneurs with project-ideas to pay the discovery cost  $k$  and innovate as soon as their number  $\nu_t$  exceeds the critical mass  $\bar{\nu}$ .

**Theorem 7** *There a threshold  $\bar{\pi}$  (or, equivalently,  $\bar{\delta}$ ) such that if  $\pi \leq \bar{\pi}$  (or, equivalently,  $\delta \leq \bar{\delta}$ )*

*the efficient equilibrium is for all entrepreneurs to innovate as soon as the number of entrepreneurs with project-ideas exceeds critical mass,  $\nu_t \geq \bar{\nu}$ .*

The factors affecting the value of the critical mass  $\bar{\nu}$  are characterized in the following corollary.

**Corollary 3** *The critical mass  $\bar{\nu}$  is increasing in  $\{\alpha, N, k\}$ , and decreasing in  $\delta$ .*

The critical mass  $\bar{\nu}$  depends positively on the level of uncertainty  $\alpha$ , and the number of potential project-ideas,  $N$ . This happens because, from (19), a greater value of  $\alpha$  and  $N$  will make uncertainty-averse investors more pessimistic (all else equal). Thus, a greater number of project ideas is needed to generate a level of (expected) investor sentiment that is sufficiently strong to ignite innovation. Similarly, a greater discovery cost  $k$  will require a stronger expected investor sentiment, and thus greater equity valuations, to induce entrepreneurs to pay the initial cost and initiate the innovation process. Finally, a smaller discount factor  $\delta$  will make entrepreneurs more impatient, so they will require more positive sentiment to be willing to invest, requiring a larger critical mass.

Our model has the following implications for the innovation process in an economy. Theorem 7 implies that innovation activity remains latent in the economy when the number of entrepreneurs with project-ideas is below critical mass. During this time, entrepreneurs with project ideas delay their innovation, the market for entrepreneurial equity is “cold,” and dominated by low investor sentiment with a negative outlook. When the number of entrepreneurs with project-ideas reaches critical mass, entrepreneurs expect a substantial improvement in investor sentiment and a “hot” equity market for innovations. The improved expectations on the future market conditions spark an innovation wave that ripples through the economy. In addition, Corollary 3 implies that greater uncertainty, or a greater discovery cost, will lead to less frequent innovation waves, but when the wave is taking place it will involve a larger number of innovations and will be characterized by stronger investor sentiment and equity valuations. In addition, if we interpret  $N$  as characterizing the complexity on an industry, Corollary 3 implies that less complex industries are characterized by more frequent innovation waves, of smaller intensity, and with less ebullient equity markets. In contrast, more complex industries are characterized by relatively less frequent innovation waves but

that, when they occur, are of greater intensity, and with more ebullient equity markets.

## 5 Process Innovation

An important distinction that has been identified in the literature on innovation is the difference between “product innovation” and “process innovation.”<sup>24</sup> Product innovation refers to the generation of a new product that did not exist before, while process innovation involves the improvement of an already existing product. Process innovation is interpreted broadly as involving the improvement of any part of the production process of an existing product, which typically results in efficiency gains due to productivity increases and/or cost reductions.

The innovation process that we have considered so far in our analysis is well suited to describe the case of “product innovation,” whereby a firm invests resources, such as R&D, to develop an innovative product. If the R&D is successful, the firm obtains a new product, while if the R&D is not successful, the innovation process has no value.

In this section we show that our analysis extends very easily to the case of process innovation. We model process innovation by assuming that, by paying at  $t = 0$  a fixed cost of  $\kappa_\tau$ , a firm can increase the productivity of its second-stage innovation process from  $Z_\tau$  to  $I Z_\tau$  ( $1 < I < \psi$ ). In addition, we assume that the first stage of the innovation process is not risky,  $q_\tau = 1$ , for  $\tau \in \{A, B\}$ . The rest of the model unfolds as before.

The following theorem characterizes the equilibrium innovation decision by the two firms.

**Theorem 8** *For low levels of cost,  $\kappa_\tau \leq \underline{\kappa}_\tau$ , the firm always innovates. For high levels of discovery cost,  $\kappa_\tau \geq \bar{\kappa}_\tau$ , the firm never innovates. For intermediate levels of the discovery cost,  $\kappa_\tau \in (\underline{\kappa}_\tau, \bar{\kappa}_\tau)$ , the firm is willing to innovate only if the other firm innovates. If both firms have intermediate levels of discovery cost, there are multiple equilibria, one where both firms innovate and one where neither innovates. The innovation equilibrium dominates the no-innovation equilibrium. There are strategic complementarities in process innovation iff investors are uncertainty averse.*

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<sup>24</sup>The distinction between process innovation and product innovation goes back at least to Utterback and Abernathy (1975). More recent work includes Klepper (1996), among many others.



Similar to the case of product innovation, a firm with low cost of innovation,  $\kappa_\tau \leq \underline{\kappa}_\tau$ , is willing to implement the process innovation, independent of what the other firm does. A firm with high costs,  $\kappa_\tau \geq \bar{\kappa}_\tau$ , is never willing to implement process innovation. For intermediate values of the initial fixed cost,  $\kappa_\tau \in [\underline{\kappa}_\tau, \bar{\kappa}_\tau]$ , a firm is willing to innovate only if the other firm innovates. If both firms have intermediate levels of discovery cost, there are multiple equilibria, generating again an assurance game. The presence of multiple equilibria is again a direct consequence of the strategic complementarities created by investors' aversion to uncertainty. If, on the contrary, investors are uncertainty neutral,  $\underline{\kappa}_\tau = \bar{\kappa}_\tau$ , and the innovation processes in the two firms are independent from each other.

## 6 Empirical Implications

Our paper has several novel empirical implications on the relationship between innovation waves, equity valuations in the technology sectors, “hot” IPO markets and M&A activity.

1. *Innovation waves.* The strategic complementarity between entrepreneurs' innovation decisions in our model creates the possibility of innovation waves. An innovation wave occurs if the number of entrepreneurs endowed with project-ideas reaches critical mass. Arrival of innovation opportunities (i.e. project-ideas) in the economy may be random, and it may depend on classic “fundamentals” such as technological advances in certain sectors, say in Information Technologies or Life Sciences. Our paper suggests that such technological advances, while necessary, may not be sufficient to start a wave. Rather, an innovation wave will occur when a critical mass of (potential) innovators is attained which will spur a “hot” market for innovative companies.

Note that an innovation wave may start in one “sector” and then spill over to other “sectors,” even if they are unrelated. This can happen, for example, when a positive shock in the project idea of entrepreneurs in one sector lowers their discovery cost from a high level,  $k_\tau > \bar{k}_\tau$ , to a low level,  $k_\tau < \underline{k}_\tau$ , while the other entrepreneur faces a moderate discovery cost,  $k_{\tau'} \in (\underline{k}_{\tau'}, \bar{k}_{\tau'})$ ,  $\tau \neq \tau'$ . If the discovery costs of the first set of entrepreneurs are subject to a shock and decrease to a low level,  $k_\tau < \underline{k}_\tau$ , it now becomes optimal for them to initiate the innovation process. This decision makes it profitable for other entrepreneurs to innovate as well, in anticipation of the possibility of

higher equity prices. Thus, a positive idiosyncratic shock to the technology in one sector spills over to other entrepreneurs, triggering an innovation wave in another sector.<sup>25</sup> The “contagion” across sectors may be due to an “equity valuation” channel which is driven by strong investor sentiment, rather than a pure technological channel. Similar results hold for the productivity of innovation,  $Z_\tau$ , and the probability of success,  $q_\tau$ . Note that the beneficial spillover effect is more likely to occur the greater the degree of relatedness of the two technologies (the greater the value of  $r$ ).

*2. Innovation waves, investor sentiment, and hot IPO markets.* In our model, the market value of an entrepreneur’s firm is (weakly) increasing in the number of successful firms in the market. This is because uncertainty-averse investors are more optimistic when they can invest in the equity of a larger set of new firms, leading to higher equity valuations. Given our discussion above, this means that innovation waves will be associated with strong investor sentiment toward innovations and, thus, to booms in the equity of technology firms. This also means that innovation waves can be associated with hot IPO markets, which are then followed by lower stock returns. Thus, our model can explain the relationship between IPO volume and stock market valuations and the subsequent lower returns documented in the literature (see, for example, Ritter and Welch, 2002, for an extensive survey of the IPO literature).

*3. Innovation waves and venture capitalists.* An additional implication of our model is a new role for venture capitalists. If discovery costs fall in the intermediate range,  $k_\tau \in (\underline{k}_\tau, \bar{k}_\tau)$ , entrepreneurs face an “assurance game” in that each entrepreneur will be willing to incur the discovery cost and innovate only if she is assured that also other entrepreneurs will do the same. Lacking such assurance, entrepreneurs may be confined to the inefficient equilibrium with no innovation. In this setting, a venture capitalist may indeed play a positive role by addressing the coordination failure among entrepreneurs. By investing in several technology firms, the venture capitalist can help coordination among entrepreneurs and lead to greater innovation. Note that companies in the VC portfolio do not need to have directly related technologies for the VC to have a beneficial role. In addition, as discussed above, coordination among entrepreneurs’ innovative activities will be associated with greater equity market valuations. These observations imply that venture capital

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<sup>25</sup>For example, a positive technological shock to, say, LinkedIn may be a boost to Uber, even if no direct technological link is present.

activity will be associated with innovation waves and greater equity valuations.

4. *Innovation, investor sentiment and merger activity.* Our paper presents a new channel in which merger activity can generate synergies and spur innovative activity. In our paper, synergistic gains are the direct outcome of the beneficial spillover effect of the merger on the expected value of the innovation. In the post-merger firm, innovators will improve their assessment of the success probability of the innovation and, thus, will choose greater levels of innovation intensity, leading to greater innovation rates for the merged firms. In addition, our model predicts that merger activities involving innovative firms will be associated with strong investor sentiment and greater firm valuations.

## 7 Conclusion

In this paper, we show that uncertainty aversion generates innovation waves. Uncertainty aversion causes investors to treat different uncertain lotteries as complements, a property that we refer to as uncertainty hedging. Uncertainty hedging by investors produces strategic complementarity in entrepreneurial behavior, producing innovation waves. Specifically, when one entrepreneur has a successful first-stage project, equity valuation, entrepreneur utility, and the intensity of innovation increase for other entrepreneurs. Thus, entrepreneurs are more willing to innovate if they expect other entrepreneurs are going to innovate as well, resulting in multiple equilibria. Our model can thus explain why there are some periods when investment in innovation is “hot,” and investors are more willing to invest in risky investment projects tainted by significant uncertainty. Finally, we argue that mergers can add value because the positive spillover effects of innovation due to uncertainty hedging. Thus, our model predicts simultaneous innovation waves, merger waves, and positive investor sentiment in “hot” equity markets.

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## A Appendix: Proofs

**Proof of Lemma 1.** Let  $x = \{x_A, x_B\}$  be a vector of indicator variables for success of type  $A$  and  $B$  assets:  $x \in \{0, 1\}^2$ . If the probability of success is  $p = \{p_A, p_B\}$  the probability of  $x$  is  $p_A^{x_A} p_B^{x_B} (1 - p_A)^{1-x_A} (1 - p_B)^{1-x_B}$ . Thus, the relative entropy of  $p$  w.r.t.  $\hat{p}$  is

$$R(p|\hat{p}) = \sum_{x \in \{0,1\}^2} p_A^{x_A} p_B^{x_B} (1 - p_A)^{1-x_A} (1 - p_B)^{1-x_B} \ln \frac{p_A^{x_A} p_B^{x_B} (1 - p_A)^{1-x_A} (1 - p_B)^{1-x_B}}{\hat{p}_A^{x_A} \hat{p}_B^{x_B} (1 - \hat{p}_A)^{1-x_A} (1 - \hat{p}_B)^{1-x_B}}.$$

Because the log of a product is the sum of the logs, and probabilities sum to one, we can express this as

$$R(p|\hat{p}) = R_A(p_A|\hat{p}_A) + R_B(p_B|\hat{p}_B)$$

where  $R_\tau(p_\tau|\hat{p}_\tau) = p_\tau \ln \frac{p_\tau}{\hat{p}_\tau} + (1 - p_\tau) \ln \frac{1-p_\tau}{1-\hat{p}_\tau}$ . Because  $\frac{\partial^2 R_\tau}{\partial p_\tau^2} = \frac{\hat{p}_\tau}{p_\tau} + \frac{1-\hat{p}_\tau}{1-p_\tau}$ ,  $R_\tau(p_\tau|\hat{p}_\tau)$  is strictly convex in  $p_\tau$ . Thus,  $R(p|\hat{p})$  is strictly convex in  $p = \{p_A, p_B\}$ . Also,  $\lim_{p_\tau \rightarrow 0^+} R_\tau(p_\tau|\hat{p}_\tau) = \ln \frac{1}{1-\hat{p}_\tau}$  and  $\lim_{p_\tau \rightarrow 1^-} R_\tau(p_\tau|\hat{p}_\tau) = \ln \frac{1}{\hat{p}_\tau}$ . Define  $\underline{\eta}(\hat{p}) = \min_{x \in Q} \ln \frac{1}{x}$ , where  $Q = \{\hat{p}_A, 1 - \hat{p}_A, \hat{p}_B, 1 - \hat{p}_B\}$ . Therefore, if  $\eta < \underline{\eta}(\hat{p})$ ,  $\mathcal{M}$ , as the lower level set of a strictly convex function, is strictly convex. Note that this result generalizes: Theorem 2.5.3 of Cover and Thomas (2006) shows that relative entropy is additively separable in independent variables, and their Theorem 2.7.2 shows that it is strictly convex.

Suppose that the agent receives  $w_A$  if lottery  $A$  is successful and  $w_B$  if lottery  $B$  is successful, both of which are strictly positive. It can be quickly proved that  $R$  achieves a minimum of zero at  $p = \hat{p}$  and that  $R_\tau$  is strictly convex in both arguments (most importantly  $p_\tau$  here). This implies that  $\frac{\partial R_\tau}{\partial p_\tau} < 0$  for  $p_\tau < \hat{p}_\tau$  and  $\frac{\partial R_\tau}{\partial p_\tau} > 0$  for  $p_\tau > \hat{p}_\tau$ . The worst-case scenario solves

$$\begin{aligned} \min \quad & \{p_A w_A + p_B w_B\} \\ & R(p|\hat{p}) \leq \eta \end{aligned}$$

Let  $\lambda$  be the multiplier for the constraint, and  $L$  be the Lagrangian function. Thus,  $L = -(p_A w_A + p_B w_B) - \lambda(R(p|\hat{p}) - \eta)$ . Because  $\frac{\partial R}{\partial p_\tau} = \frac{\partial R_\tau}{\partial p_\tau}$ , this implies  $\frac{dL}{dp_\tau} = -w_\tau - \lambda \frac{\partial R_\tau}{\partial p_\tau}$ . At the worst-case scenario,  $\frac{dL}{dp_\tau} = 0$ . Because  $w_\tau > 0$ , it must be that  $\lambda \frac{\partial R_\tau}{\partial p_\tau} < 0$ . This requires not only that the constraint binds,  $\lambda > 0$ , but also that  $p_\tau$  is on the decreasing portion of  $R_\tau$ , or equivalently, that  $p_\tau < \hat{p}_\tau$ . If the agent has strictly positive exposure to only one uncertain lottery, but not the other, say  $w_\tau > 0$  but  $w_{\tau'} = 0$ , the worst-case scenario involves choosing the worst possible value of  $p_\tau$ ,  $R(p_\tau|\hat{p}_\tau) = \eta$  for  $p_\tau < \hat{p}_\tau$ , and setting  $p_{\tau'} = \hat{p}_{\tau'}$ . Finally, if the agent has no exposure to either lottery,  $w_A = w_B = 0$ , the claim holds WLOG. ■

**Proof of Lemma 2.** Define  $u(\theta; \Pi) = e^{\theta_A - \theta_M} \omega_A y_A + e^{\theta_B - \theta_M} \omega_B y_B + w_0 - \omega_A V_A - \omega_B V_B$ , so that  $U(\Pi) = \min \{u(\theta; \Pi)\}$  s.t.  $\frac{1}{2}(\theta_A + \theta_B) = \theta_T$ . Let  $L$  be the Lagrangian for the minimization problem, and  $\psi$  be the multiplier on the constraint. Thus,  $\frac{\partial L}{\partial \theta_\tau} = -e^{\theta_\tau - \theta_M} \omega_\tau y_\tau + \frac{\psi}{2}$ . Because  $U$  is strictly convex in  $\theta$ , FOCs are sufficient for a minimum. Setting  $\frac{\partial L}{\partial \theta_\tau} |_{\theta_\tau = \hat{\theta}_\tau} = 0$ , and substituting into  $\frac{1}{2}(\theta_A + \theta_B) = \theta_T$ , this implies

$$\hat{\theta}_\tau = \theta_T + \frac{1}{2} \ln \frac{\omega_{\tau'} y_{\tau'}}{\omega_\tau y_\tau}$$

Thus, if  $\hat{\theta}_\tau^\alpha(\Pi) \in [\theta_L, \theta_H]$ ,  $\theta_\tau^\alpha = \hat{\theta}_\tau^\alpha$ . If  $\hat{\theta}_\tau^\alpha < \theta_L$ ,  $\frac{\partial L}{\partial \theta_\tau} < 0$  for all  $\theta_\tau \in [\theta_L, \theta_H]$ , so  $\theta_\tau^\alpha = \theta_L$ . If  $\hat{\theta}_\tau^\alpha > \theta_H$ ,  $\frac{\partial L}{\partial \theta_\tau} > 0$  for all  $\theta_\tau \in [\theta_L, \theta_H]$ , so  $\theta_\tau^\alpha = \theta_H$ . Therefore, (5) corresponds to the worst-case scenario for an investor with portfolio  $\Pi$ . ■

**Proof of Lemma 3.** Each investor's objective function is  $U(\Pi) = \min_{\theta \in C} u(\theta; \Pi)$  where  $u(\theta; \Pi) = e^{\theta_A - \theta_M} \omega_A y_A + e^{\theta_B - \theta_M} \omega_B y_B + w_0 - \omega_A V_A - \omega_B V_B$ . Thus, for  $\tau \in \{A, B\}$ ,

$$\frac{dU}{d\omega_\tau} = \frac{\partial u}{\partial \omega_\tau} + \frac{\partial u}{\partial \theta_A} \frac{d\theta_A}{d\omega_\tau} + \frac{\partial u}{\partial \theta_B} \frac{d\theta_B}{d\omega_\tau}.$$

If investors are uncertainty-neutral, they believe  $\theta_\tau = \theta_T$ , so the second term disappears ( $\theta_\tau$  is constant). If investors are uncertainty averse,  $\vec{\theta}^a$  solves the minimization problem. For interior solutions, by Lemma 2,  $\frac{\partial u}{\partial \theta_A} = \frac{\partial u}{\partial \theta_B} = \frac{\psi}{2}$ , so the last two terms sum to  $\frac{\psi}{2} \frac{\partial(\theta_A + \theta_B)}{\partial \omega_\tau}$ , which is zero because  $\theta_A + \theta_B$  is constant. For corner solutions,  $\frac{\partial \theta_A}{\partial \omega_\tau} = \frac{\partial \theta_B}{\partial \omega_\tau} = 0$ . Therefore,  $\frac{dU}{d\omega_\tau} = \frac{\partial u}{\partial \omega_\tau}$  for  $\tau \in \{A, B\}$ . Note  $\frac{\partial u}{\partial \omega_\tau} = p_\tau(\theta_\tau^a) y_\tau - V_\tau$ . Thus, market clearing requires that  $V_\tau = p_\tau(\theta_\tau^a) y_\tau$ . Note that it is WLOG optimal for all investors to set  $\omega_A = \omega_B = 1$ , because innovations are priced at expected value given market assessments. Further, if investors are uncertainty-averse, they will hold identical positions in the risky portfolio (formally,  $\frac{\omega_A}{\omega_B}$  is constant across all investors), because there would be gains from trade if they did not. ■

**Proof of Lemma 4.** From Lemma 2,  $\theta_\tau^a(\Pi) = \theta_L$  iff  $\check{\theta}_\tau^a(\Pi) \leq \theta_L$  iff  $y_\tau \geq e^{2\alpha} y_{\tau'}$ . Thus, if  $y_\tau \geq e^{2\alpha} y_{\tau'}$ ,  $V_\tau = p_\tau(\theta_L) y_\tau$  and  $V_{\tau'} = p_{\tau'}(\theta_H) y_{\tau'}$ . The  $\theta_\tau^a(\Pi) = \theta_H$  case is symmetric. Finally, from Lemma 2,  $\theta_\tau^a(\Pi) \in (\theta_L, \theta_H)$  iff  $\check{\theta}_\tau^a(\Pi) \in (\theta_L, \theta_H)$  iff  $y_\tau \in (e^{-2\alpha} y_{\tau'}, e^{2\alpha} y_{\tau'})$ . Because  $\theta_\tau^a(\Pi) = \check{\theta}_\tau^a(\Pi)$ ,  $p_\tau(\theta^a(\Pi)) = e^{(\theta_T - \theta_M)} y_{\tau'}^{\frac{1}{2}} y_\tau^{-\frac{1}{2}}$ , which implies the market values entrepreneur  $\tau$ 's firm at  $V_\tau = e^{\theta_T - \theta_M} y_{\tau'}^{\frac{1}{2}} y_\tau^{\frac{1}{2}}$ . There is strategic complementarity in production because  $\frac{\partial V_\tau}{\partial y_{\tau'}} \geq 0$  for  $\tau' \neq \tau$ , with strict inequality for  $y_\tau \in (e^{-2\alpha} y_{\tau'}, e^{2\alpha} y_{\tau'})$ . ■

**Proof of Lemma 5.** Because only entrepreneur  $\tau$  has a successful first-stage project-idea, so  $y_{\tau'} = 0$ . By Lemma 2,  $\theta_\tau^a = \theta_L$ . By Lemma 3,  $V_A = p_\tau(\theta_L) y_A$ , so the entrepreneur's payoff is

$$\mathcal{U}_\tau = p_\tau(\theta_L) y_\tau - \frac{1}{Z_\tau(1+\gamma)} y_\tau^{1+\gamma}.$$

Note that  $\frac{\partial \mathcal{U}_\tau}{\partial y_\tau} = p_\tau(\theta_L) - \frac{1}{Z_\tau} y_\tau^\gamma$ , and  $\frac{\partial^2 \mathcal{U}_\tau}{\partial y_\tau^2} = -\frac{\gamma}{Z_\tau} y_\tau^{\gamma-1} < 0$ , so FOCs are sufficient for a maximum. Thus, entrepreneur  $\tau$  selects  $y_\tau^{U,SF} = [p_\tau(\theta_L) Z_\tau]^{-\frac{1}{\gamma}}$ , sells for  $V_\tau^{U,SF} = [p_\tau(\theta_L)]^{\frac{1+\gamma}{\gamma}} Z_\tau^{-\frac{1}{\gamma}}$ , and earns continuation payoff  $\mathcal{U}_\tau^{U,SF} = [p_\tau(\theta_L)]^{\frac{1+\gamma}{\gamma}} Z_\tau^{-\frac{1}{\gamma}} \frac{\gamma}{1+\gamma}$ . ■

**Proof of Lemma 6.** Suppose it is optimal for entrepreneurs to produce output resulting in interior assessments:  $y_A \in (e^{-2\alpha} y_B, e^{2\alpha} y_B)$ , which will be optimal because the assumptions on  $Z_A$  and  $Z_B$ . For  $\tau \in \{A, B\}$  and  $\tau' \neq \tau$ , when entrepreneur  $\tau'$  produces  $y_{\tau'}$ , entrepreneur  $\tau$  produces  $y_\tau$  and earns continuation utility

$$\mathcal{U}_\tau = p_\tau(\theta_T) y_\tau^{\frac{1}{2}} y_{\tau'}^{\frac{1}{2}} - \frac{1}{Z_\tau(1+\gamma)} y_\tau^{1+\gamma}.$$

Thus,  $\frac{\partial \mathcal{U}_\tau}{\partial y_\tau} = \frac{1}{2} p_\tau(\theta_T) y_\tau^{-\frac{1}{2}} y_{\tau'}^{\frac{1}{2}} - \frac{1}{Z_\tau} y_\tau^\gamma$  and  $\frac{\partial^2 \mathcal{U}}{\partial y_\tau^2} = -\frac{1}{4} p_\tau(\theta_T) y_\tau^{-\frac{3}{2}} y_{\tau'}^{\frac{1}{2}} - \frac{\gamma}{Z_\tau} y_\tau^{\gamma-1}$ . Because  $\frac{\partial^2 \mathcal{U}}{\partial y_\tau^2} < 0$ , FOCs are sufficient for a local maximum. Thus,  $y_\tau = \left[ \frac{Z_\tau}{2} p_\tau(\theta_T) y_{\tau'}^{\frac{1}{2}} \right]^{\frac{1}{\gamma+\frac{1}{2}}}$ .

On this region, optimal output by one firm is strictly increasing in the output of the other firm. Inspection of the revenue function from Lemma 4 immediately shows that each entrepreneur's problem is locally concave almost everywhere, the exception being at  $y_\tau = e^{2\alpha} y_{\tau'}$ . Because there is a kink, there may be multiple critical points, resulting in a discontinuous best response function. Thus, there are some parameter values for which there is no pure strategy equilibrium. However, it can be verified (after messy calculations) that so long as  $\frac{Z_A}{Z_B} \in \left( \frac{1}{\psi}, \psi \right)$  where  $\psi = \frac{1}{4} e^{2\alpha(\gamma+1)} \left( 1 + \frac{1}{2\gamma} \right)^{2\gamma}$ , if both firms succeed in the first stage, there is a unique equilibrium – it is optimal for the firms to produce output levels such that investors have interior assessments. Thus, entrepreneurs' best response functions are  $y_\tau^{U,SS}(y_{\tau'}) = \left[ \frac{Z_\tau}{2} p_\tau(\theta_T) y_{\tau'}^{\frac{1}{2}} \right]^{\frac{1}{\gamma+\frac{1}{2}}}$ . ■

**Proof of Theorem 2.** In equilibrium, both entrepreneurs select innovation intensity optimally, given the intensity the other entrepreneur is innovating. From Lemma 6, the best response functions are  $y_\tau^{U,SS}(y_{\tau'}) = \left[ \frac{Z_\tau}{2} p_\tau(\theta_T) y_{\tau'}^{\frac{1}{2}} \right]^{\frac{1}{\gamma+\frac{1}{2}}}$ . Because the other entrepreneur also selects intensity optimally, selecting  $y_{\tau'}^{U,SS}(y_\tau)$ ,

$$y_\tau = \left[ \frac{Z_\tau}{2} p_\tau(\theta_T) \left[ \frac{Z_{\tau'}}{2} p_{\tau'}(\theta_T) y_{\tau'}^{\frac{1}{2}} \right]^{\frac{1}{2\gamma+1}} \right]^{\frac{1}{\gamma+\frac{1}{2}}}.$$



After some messy calculations, this holds iff

$$y_\tau^{U,SS} = \left[ \frac{1}{2} p_\tau (\theta_T) Z_\tau^{\frac{2\gamma+1}{2\gamma+2}} Z_{\tau'}^{\frac{1}{2\gamma+2}} \right]^{\frac{1}{\gamma}}$$

for  $\tau \in \{A, B\}$  and  $\tau' \neq \tau$ .

Because the market price is  $V_\tau^{U,SS} = p_\tau (\theta_T) y_\tau^{\frac{1}{2}} y_{\tau'}^{\frac{1}{2}}$ , it follows that

$$V_\tau^{U,SS} = 2^{-\frac{1}{\gamma}} [p_\tau (\theta_T)]^{\frac{\gamma+1}{\gamma}} [Z_\tau Z_{\tau'}]^{\frac{1}{2\gamma}}.$$

Similarly, entrepreneur  $\tau$  earns continuation utility  $\mathcal{U}_\tau^{U,SS} = V_\tau^{U,SS} - \frac{1}{Z_\tau(1+\gamma)} y_\tau^{1+\gamma}$ , which can be expressed as

$$\mathcal{U}_\tau^{U,SS} = 2^{-\frac{1}{\gamma}} [p_\tau (\theta_T)]^{\frac{\gamma+1}{\gamma}} Z_\tau^{\frac{1}{2\gamma}} Z_{\tau'}^{\frac{1}{2\gamma}} \frac{2\gamma+1}{2\gamma+2},$$

for  $\tau \in \{A, B\}$  and  $\tau' \neq \tau$ . Thus, there are strategic complementarities in production and profit. ■

**Proof of Corollary 1.** Recall  $\mathcal{U}_\tau^{U,SS} = \frac{1}{2^{\frac{1}{\gamma}}} e^{(\theta_T - \theta_M) \frac{\gamma+1}{\gamma}} Z_\tau^{\frac{1}{2\gamma}} Z_{\tau'}^{\frac{1}{2\gamma}} \frac{2\gamma+1}{2\gamma+2}$  and  $\mathcal{U}_\tau^{U,SF} = e^{(\theta_L - \theta_M) \frac{\gamma+1}{\gamma}} Z_\tau^{\frac{1}{1+\gamma}}$ . Thus,  $\mathcal{U}_\tau^{U,SS} > \mathcal{U}_\tau^{U,SF}$  iff  $\frac{Z_{\tau'}}{Z_\tau} > \frac{1}{\psi}$  where  $\psi = \frac{1}{4} e^{2\alpha(\gamma+1)} \left(1 + \frac{1}{2\gamma}\right)^{2\gamma}$ . Recall that we assumed  $\frac{Z_{\tau'}}{Z_\tau} \in \left(\frac{1}{\psi}, \psi\right)$  where  $\psi = \frac{1}{4} e^{2\alpha(\gamma+1)} \left(1 + \frac{1}{2\gamma}\right)^{2\gamma}$ , so this is always satisfied – entrepreneurs are better off when other entrepreneurs have a successful first-stage project.

Recall that  $V_\tau^{U,SS} = 2^{-\frac{1}{\gamma}} e^{(\theta_T - \theta_M) \frac{\gamma+1}{\gamma}} [Z_\tau Z_{\tau'}]^{\frac{1}{2\gamma}}$  and  $V_\tau^{U,SF} = e^{(\theta_L - \theta_M) \frac{1+\gamma}{\gamma}} Z_\tau^{\frac{1}{\gamma}}$ . After some messy algebra, it can be shown that  $V_\tau^{U,SS} > V_\tau^{U,SF}$  iff  $\frac{Z_{\tau'}}{Z_\tau} > 4e^{-2\alpha(1+\gamma)}$ . Define  $\psi_1 = \frac{1}{4} e^{2\alpha(1+\gamma)}$ .

Finally,  $y_\tau^{U,SS} = \left[ \frac{1}{2} e^{\theta_T - \theta_M} Z_\tau^{\frac{2\gamma+1}{2\gamma+2}} Z_{\tau'}^{\frac{1}{2\gamma+2}} \right]^{\frac{1}{\gamma}}$  and  $y_\tau^{U,SF} = [e^{\theta_L - \theta_M} Z_\tau]^{\frac{1}{\gamma}}$ . After some messy algebra, it can be shown that  $y_\tau^{U,SS} > y_\tau^{U,SF}$  iff  $\frac{Z_{\tau'}}{Z_\tau} > 4^{\gamma+1} e^{-2\alpha(1+\gamma)}$ . Thus, define  $\psi_2 = \left[\frac{1}{4}\right]^{\gamma+1} e^{2\alpha(1+\gamma)}$ . Further, because  $\gamma > 0$ , it follows immediately that  $\psi_2 < \psi_1 < \psi$ . ■

**Proof of Theorem 3.** If only one entrepreneur innovates, he earns payoff  $EU_\tau^{U,N} = q_\tau \mathcal{U}_\tau^{U,SF} - k_\tau$  (Lemma 5). Thus, if an entrepreneur does not expect the other entrepreneur to innovate, he will innovate iff  $k_\tau \leq \underline{k}_\tau \equiv q_\tau \mathcal{U}_\tau^{U,SF}$ . Conversely, if the other entrepreneur innovates, the entrepreneur earns payoff  $EU_\tau^{U,I} = (q_\tau q_{\tau'} + r) \mathcal{U}_\tau^{U,SS} + [q_\tau (1 - q_{\tau'}) - r] \mathcal{U}_\tau^{U,SF} - k_\tau$  if he innovates as well. Thus, if the other entrepreneur innovates, he innovates iff  $k_\tau \leq \bar{k}_\tau \equiv (q_\tau q_{\tau'} + r) \mathcal{U}_\tau^{U,SS} + [q_\tau (1 - q_{\tau'}) - r] \mathcal{U}_\tau^{U,SF}$ . By Corollary 1,  $\mathcal{U}_\tau^{U,SF} < \mathcal{U}_\tau^{U,SS}$ , so  $\underline{k}_\tau < \bar{k}_\tau$  (because coefficients on the terms in  $\bar{k}_\tau$  sum to  $q_\tau$ ). ■

**Proof of Corollary 2.** Comparative Statics follow immediately from inspection of the expressions for  $\underline{k}_\tau$  and  $\bar{k}_\tau$ , and because  $\mathcal{U}_\tau^{U,SS}$  is increasing in  $Z_\tau$  and  $Z_{\tau'}$ , and  $\mathcal{U}_\tau^{U,SF}$  is increasing in  $Z_\tau$ . ■

**Proof of Theorem 4.** The merged firm seeks to maximize the combined value of the two projects. By identical reasoning to Lemma 3,  $V_A = p_A (\theta_A^I) y_A$  and  $V_B = p_B (\theta_B^I) y_B$ , where  $\vec{\theta}^I$  is the market assessment at  $t = 1$  on  $\theta$ . Thus, the merged firm's objective is

$$\mathcal{U}^M = p_A (\theta_A^I) y_A + p_B (\theta_B^I) y_B - c_A (y_A) - c_B (y_B).$$

Because investors are uncertainty averse,  $\vec{\theta}^I = \vec{\theta}^a$ , which depend on the choice of  $y_A$  and  $y_B$ . As shown in Lemma 4,  $V_A = V_B = e^{\theta_T - \theta_M} y_A^{\frac{1}{2}} y_B^{\frac{1}{2}}$ , so the objective function of the merged firm becomes

$$\mathcal{U}^M = 2e^{\theta_T - \theta_M} y_A^{\frac{1}{2}} y_B^{\frac{1}{2}} - \frac{1}{Z_A (1 + \gamma)} y_A^{1+\gamma} - \frac{1}{Z_B (1 + \gamma)} y_B^{1+\gamma}.$$

$$\frac{\partial \mathcal{U}^M}{\partial y_\tau} = e^{\theta_T - \theta_M} y_\tau^{-\frac{1}{2}} y_{\tau'}^{\frac{1}{2}} - \frac{1}{Z_\tau} y_\tau^\gamma$$

for  $\tau \in \{A, B\}$  and  $\tau' \neq \tau$ . This implies<sup>26</sup> that

$$y_\tau = \left[ p_\tau(\theta_T) y_{\tau'}^{\frac{1}{2}} Z_\tau \right]^{\frac{1}{\gamma + \frac{1}{2}}},$$

so

$$y_\tau^M = \left[ p_\tau(\theta_T) Z_{\tau'}^{\frac{1}{2\gamma+2}} Z_\tau^{\frac{2\gamma+1}{2\gamma+2}} \right]^{\frac{1}{\gamma}}.$$

Thus, each project within the merged firm has value  $V_\tau^M = [p_\tau(\theta_T)]^{\frac{1+\gamma}{\gamma}} [Z_{\tau'} Z_\tau]^{\frac{1}{2\gamma}}$ . ■

**Proof of Theorem 5.** If entrepreneur  $\tau$  does not expect entrepreneur  $\tau'$  to innovate, he innovates iff  $k_\tau \leq \underline{K}_\tau \equiv q_\tau \mathcal{U}_\tau^{U,SF}$ , the same cutoff as without the possibility of a merger. However, if entrepreneur  $\tau$  expects entrepreneur  $\tau'$  to innovate, he innovates iff  $k_\tau \leq \bar{K}_\tau \equiv (q_\tau q_{\tau'} + r) \Upsilon_\tau + [q_\tau(1 - q_{\tau'}) - r] \mathcal{U}_\tau^{U,SF}$ . Because  $\Upsilon_\tau = \mathcal{U}_\tau^{U,SS} + \frac{1}{2} (\mathcal{U}^M - \mathcal{U}_A^{U,SS} - \mathcal{U}_B^{U,SS})$ , if  $\Upsilon_\tau > \mathcal{U}_\tau^{U,SS}$ ,  $\bar{K}_\tau > \underline{k}_\tau$ , so the cutoff will be larger when mergers are possible, resulting in more innovation. Thus, it is sufficient to show that  $\mathcal{U}^M > \mathcal{U}_A^{U,SS} + \mathcal{U}_B^{U,SS}$ .

Because  $V^M = V_A^M + V_B^M$ , the merged firm earns utility

$$\mathcal{U}^M = 2 \frac{\gamma}{1+\gamma} e^{(\theta_T - \theta_M) \frac{1+\gamma}{\gamma}} [Z_A Z_B]^{\frac{1}{2\gamma}}.$$

Each entrepreneur could earn utility  $\mathcal{U}_\tau^{U,SS} = \frac{1}{2\gamma} e^{(\theta_T - \theta_M) \frac{1+\gamma}{\gamma}} Z_\tau^{\frac{1}{2\gamma}} Z_{\tau'}^{\frac{1}{2\gamma}} \frac{2\gamma+1}{2\gamma+2}$  if they did not merge, so

$$\begin{aligned} \mathcal{U}_A^{U,SS} + \mathcal{U}_B^{U,SS} &= 2 \frac{1}{2\gamma} e^{\frac{1}{2}(\theta_T - \theta_M) \frac{1+\gamma}{\gamma}} Z_A^{\frac{1}{2\gamma}} Z_B^{\frac{1}{2\gamma}} \frac{2\gamma+1}{2\gamma+2} \\ &= \mathcal{U}^M \frac{1}{2\gamma} \frac{2\gamma+1}{2\gamma} \end{aligned}$$

Because<sup>27</sup>  $\frac{1}{2\gamma} \frac{2\gamma+1}{2\gamma} \in (0, 1)$  for all  $\gamma \in (0, \infty)$ , the merger adds value:  $\mathcal{U}^M > \mathcal{U}_A^{U,SS} + \mathcal{U}_B^{U,SS}$ . Because surplus is divided evenly, entrepreneur  $\tau$  receives utility  $\Upsilon_\tau = \mathcal{U}_\tau^{U,SS} + \frac{1}{2} (\mathcal{U}^M - \mathcal{U}_A^{U,SS} - \mathcal{U}_B^{U,SS})$ . ■

**Proof of Lemma 7.** At  $t+1$ , entrepreneurs in  $S_t$  chose to implement their project-ideas and thus have a successful first-stage innovation. Only implemented projects can be traded, so investors choose portfolio weights  $\{\omega_n\}_{n \in S_t}$  to maximize their minimum expected payoff,  $\min_{\vec{\theta} \in C} u(\vec{\theta})$ , where  $u(\vec{\theta}) = \sum_{n \in S_t} \omega_n [\delta p_n(\theta_{nt}) y_n - V_n] + w_0$ . By identical proof to Lemma 3, in equilibrium,  $\omega_n = 1$  for all  $n \in S_t$  and  $V_n = \delta p_n(\theta_{nt}^a) y_n$ . Recall that  $\vec{\theta}$  is in  $C$  iff  $\sum_{n=1}^N \theta_{nt} = N\theta_T$  and  $\theta_{nt} \in [\theta_L, \theta_H]$  for all  $n$ . Let  $L$  be the Lagrangian function for the minimization problem, let  $\lambda$  be the multiplier for the sum, and let  $\gamma_{nL}$  and  $\gamma_{nH}$  be the constraints that  $\theta_{nt} \geq \theta_L$  and  $\theta_{nt} \leq \theta_H$  respectively. Thus,  $\frac{\partial L}{\partial \theta_{nt}} = -e^{\theta_{nt} - \theta_M} y_n + \lambda + \gamma_{nL} - \gamma_{nH}$ . For  $n \in S_t$ ,  $y_n = y > 0$ , while for  $n \notin S_t$ ,  $y_n = 0$ . Thus, symmetry of the FOCs implies that WLOG that there will be symmetry in the worst-case scenarios:  $\theta_{nt}$  is constant for all  $n \in S_t$  and  $\theta_{nt}$  is constant for all  $n \notin S_t$ .

If  $\lambda = 0$ , then  $\gamma_{nL} > 0$  for  $n \in S$ , so  $\theta_{nt} = \theta_L$  (the market has negative sentiment toward all implemented projects). WLOG, for  $n \notin S_t$ ,  $\theta_n = \frac{N\theta_T - s\theta_L}{N-s}$ . This is feasible if  $\frac{N\theta_T - s\theta_L}{N-s} < \theta_H$ , or equivalently, if  $s \leq \bar{s} \equiv N \frac{\theta_H - \theta_T}{\theta_H - \theta_L}$ . If  $\lambda > 0$ , then  $\gamma_{nH} > 0$  for all  $n \notin S_t$ , so  $\theta_{nt} = \theta_H$ . Substituting into  $\sum_{n=1}^N \theta_{nt} = N\theta_T$ , this implies that  $s\theta_{nt}^a + (N-s)\theta_H = N\theta_T$ , or equivalently, that  $\theta_{nt}^a = \theta_H - \frac{N}{s}(\theta_H - \theta_T)$ . Note that this is feasible,  $\theta_{nt}^a \geq \theta_L$ , iff  $s \geq \bar{s}$ . Market valuation follows by substitution, and is increasing in  $s$  because market sentiment,  $\theta_{nt}^a$ , is increasing in  $s$ . ■

<sup>26</sup> Because the cost functions are convex, the problem is globally concave, so first-order conditions are sufficient.

<sup>27</sup> Define  $x = \frac{1}{\gamma}$ , and  $f(x) = 2^{-x-1}(2+x)$ . Note  $\lim_{x \rightarrow 0^+} f(x) = 1$ , and  $\lim_{x \rightarrow +\infty} f(x) = 0$ , and  $f'(x) = 2^{-x-1}[1 - (2+x)\ln 2]$ , which is strictly negative because  $2\ln 2 \approx 1.3863 > 1$ . Therefore,  $\frac{1}{2\gamma} \frac{2\gamma+1}{2\gamma} < 1$  for all  $\gamma \in (0, \infty)$ .

**Proof of Theorem 6.** Suppose that  $k \in (\underline{k}_d, \bar{k}_d)$ , where  $\underline{k}_d = \delta^2 e^{\theta_L - \theta_M} y - c(y)$  and  $\bar{k}_d = \delta^2 e^{\theta_T - \theta_M} y - c(y)$ . Suppose an entrepreneur believes that all other entrepreneurs are going to innovate, and there are  $s - 1$  other entrepreneurs with project ideas. Entrepreneurs with a project-idea earn 0 if they do not innovate, while they earn  $U(s)$  if they do innovate. If  $s \leq \bar{s}$ ,  $U(s) = \delta^2 e^{\theta_L - \theta_M} y - c(y)$ . For  $s > \bar{s}$ ,  $U(s) = \delta^2 e^{\theta_H - \frac{N}{s}(\theta_H - \theta_T) - \theta_M} y - c(y) - k$ , which is increasing in  $s$ , which implies that  $U(N) = \delta^2 e^{\theta_T - \theta_M} y - c(y) - k$ . Because  $k \in (\underline{k}_d, \bar{k}_d)$ ,  $U(\lfloor \bar{s} \rfloor) < 0$  and  $U(N) > 0$ .  $U$  is strictly increasing in  $s$  for  $s \in (\bar{s}, N)$ , so define  $\bar{\nu} \equiv \min \{s | U(s) > 0, s \in \mathbb{N}\}$ . Because  $U(\lfloor \bar{s} \rfloor) < 0$ ,  $\bar{\nu} > \bar{s}$ . Thus, if an entrepreneur believes all other entrepreneurs are going to innovate, and  $\nu_t \geq \bar{\nu}$ , it is optimal for her to innovate as well. However, if she does not believe other entrepreneurs to innovate, it is suboptimal for her to innovate as well. Thus, there are multiple threshold equilibria – it is also an equilibrium for entrepreneurs to wait until  $\bar{\nu} + k$  project-ideas exist, for  $\bar{\nu} \leq \bar{\nu} + k \leq N$  for  $k \in \mathbb{N}$ . ■

**Proof of Theorem 7.** Suppose that there are currently  $\tilde{\nu} \geq \bar{\nu}$  projects for entrepreneurs to initiate. If entrepreneurs innovate immediately, they receive a payoff of  $U(\tilde{\nu})$ . If they wait for another project to arrive, they will receive  $U(\tilde{\nu} + 1)$  when the next project arrives. Because arrival is stochastic,  $\pi < 1$ , and entrepreneurs are impatient,  $\delta < 1$ , waiting is costly. Using standard techniques, the expected value of waiting is  $\frac{\pi \delta}{1 - (1 - \pi) \delta} U(\tilde{\nu} + 1)$ . Thus, it is inefficient to wait when at  $\tilde{\nu}$  if  $\delta \leq \frac{U(\tilde{\nu})}{\pi U(\tilde{\nu} + 1) + (1 - \pi) U(\tilde{\nu})}$ , or equivalently, if  $\pi \leq \frac{U(\tilde{\nu})}{U(\tilde{\nu} + 1) - U(\tilde{\nu})} (\delta^{-1} - 1)$  (this cutoff is well-defined because  $U$  is strictly increasing for  $s \geq \bar{s}$  and  $\tilde{\nu} \geq \bar{\nu} > \bar{s}$ ) Finally, if this is satisfied for all  $\tilde{\nu} \geq \bar{\nu}$ , if  $\delta \leq \min_{\tilde{\nu} \geq \bar{\nu}} \frac{U(\tilde{\nu})}{\pi U(\tilde{\nu} + 1) + (1 - \pi) U(\tilde{\nu})}$ , or equivalently,  $\pi \leq \min_{\tilde{\nu} \geq \bar{\nu}} \frac{U(\tilde{\nu})}{U(\tilde{\nu} + 1) - U(\tilde{\nu})} (\delta^{-1} - 1)$ , it is inefficient to wait for another project idea for all  $\tilde{\nu} \geq \bar{\nu}$ . Therefore, the efficient equilibrium is to invest immediately whenever  $\bar{\nu}$  projects are available. ■

**Proof of Corollary 3.** For  $s > \bar{s}$ , we can express  $U(s) = \delta^2 e^{\theta_H - \frac{N}{s} \alpha - \theta_M} y - c(y) - k$ , so  $U' > 0$ , and  $\bar{\nu} \equiv \min \{s | U(s) > 0, s \in \mathbb{N}\}$ . Thus, anything that increases  $U$  decreases  $\bar{\nu}$ , and vice versa.  $\frac{\partial U}{\partial N} = -\frac{\delta^2}{s} e^{\theta_H - \frac{N}{s} \alpha - \theta_M} y \alpha < 0$ , so  $\bar{\nu}$  is increasing in  $N$ .  $\frac{\partial U}{\partial \alpha} = -\frac{N}{s} \delta^2 e^{\theta_H - \frac{N}{s} \alpha - \theta_M} y < 0$ , so  $\bar{\nu}$  is increasing in  $N$ .  $\frac{\partial U}{\partial k} = -1$ , so  $\bar{\nu}$  is increasing in  $k$ .  $\frac{\partial U}{\partial \delta} = 2\delta e^{\theta_H - \frac{N}{s}(\theta_H - \theta_T) - \theta_M} y > 0$ , so  $\bar{\nu}$  is decreasing in  $\delta$ . ■

**Proof of Theorem 8.** Similar to Theorem 2, if neither firm innovates, firm  $\tau$  receives utility  $\mathcal{U}_\tau^{U,N} = \frac{1}{2^{\frac{1}{\gamma}}} e^{(\theta_T - \theta_M) \frac{1+\gamma}{\gamma}} Z_\tau^{\frac{1}{2\gamma}} Z_{\tau'}^{\frac{1}{2\gamma}} \frac{2\gamma+1}{2\gamma+2}$ , while, if either firm  $\tau$  or firm  $\tau'$  innovates,  $\mathcal{U}_\tau^{U,S} = I^{\frac{1}{2\gamma}} \mathcal{U}_\tau^{U,N}$ , and if both firms innovate,  $\mathcal{U}_\tau^{U,B} = I^{\frac{1}{\gamma}} \mathcal{U}_\tau^{U,N}$ . Thus, if firm  $\tau'$  does not innovate, firm  $\tau$  executes process innovation iff  $\mathcal{U}_\tau^{U,S} - \kappa_\tau > \mathcal{U}_\tau^{U,N}$ , or equivalently, iff  $\kappa_\tau < \underline{\kappa}_\tau \equiv \left( I^{\frac{1}{2\gamma}} - 1 \right) \mathcal{U}_\tau^{U,N}$ . Similarly, if firm  $\tau'$  innovates, firm  $\tau$  executes process innovation iff  $\mathcal{U}_\tau^{U,B} - \kappa_\tau > \mathcal{U}_\tau^{U,S}$ , or equivalently, iff  $\kappa_\tau < \bar{\kappa}_\tau \equiv I^{\frac{1}{2\gamma}} \left( I^{\frac{1}{2\gamma}} - 1 \right) \mathcal{U}_\tau^{U,N}$ . Because  $I > 1$ ,  $\underline{\kappa}_\tau < \bar{\kappa}_\tau$ . ■

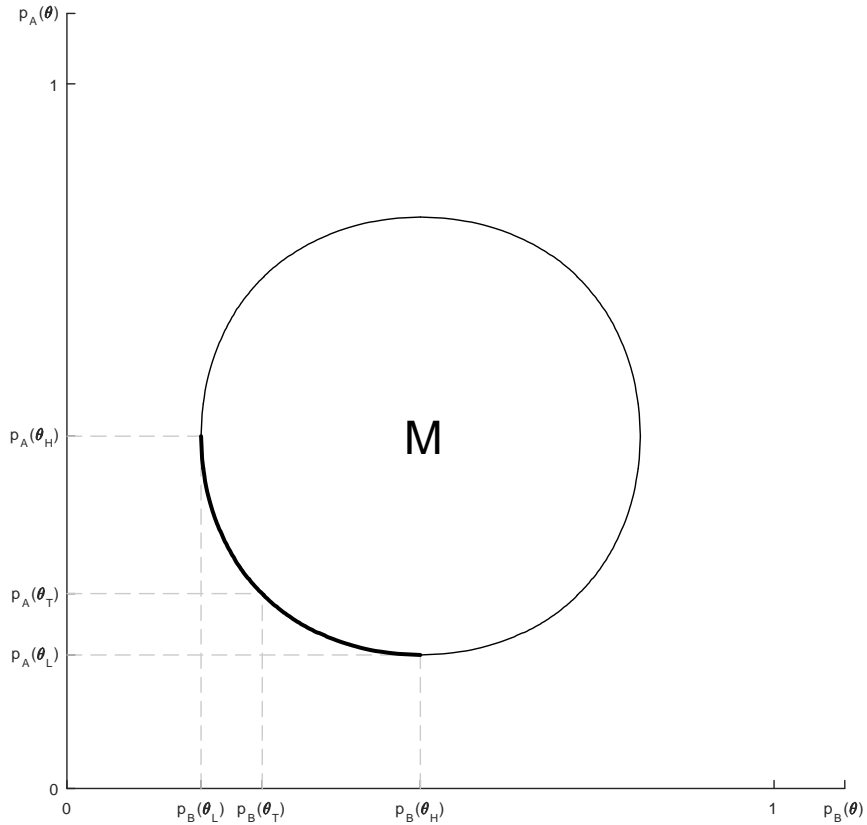


Figure 1: Core Belief Set. This figure represents the core belief set implied by the relative entropy criterion.

This figure shows the core of beliefs when the maximum relative entropy criteria is applied. That is, it shows the set of probability distributions  $p = (p_A, p_B)$  that satisfy  $\{p|R(p|\hat{p}) \leq \eta\}$  when  $\hat{p}_A = \hat{p}_B = \frac{1}{2}$  and  $\eta = \frac{3}{10} \ln 2$ . If  $p_B = \hat{p}_B = \frac{1}{2}$ , the maximum relative entropy criteria implies that  $p_A \in [0.1893, 0.8107]$ . In contrast, if  $p_A = p_B$ , then the maximum relative entropy criterion implies that  $p_A \in [0.2760, 0.7240]$ .

This illustrates that an uncertainty-averse investor treats different uncertain lotteries as complements. The lower left boundary, which is darkened, represents the relevant portion of the core beliefs for investors with long positions in both risky assets. Specifically, if an uncertainty-averse investor invested in only one innovative project, he would value it as though it would succeed about 18.93% of the time. In contrast, if he could invest equally in both, he would believe that both would succeed about 27.60% of the time. Therefore, there will be strategic complementarities in innovation, because entrepreneurs rationally anticipate that they will face more positive sentiment if they both arrive at the same time.