Earnings Inequality and Other Determinants of Wealth Inequality

Jess Benhabib, Alberto Bisin, Mi Luo*

New York University

First draft: December 2016

Abstract: We study the relation between the distribution of stochastic earnings and the distribution of wealth in microfounded dynamic models. Our findings suggest, both theoretically as well as empirically, that while stochastic earnings and precautionary savings are important factors for explaining the distribution of wealth, they cannot by themselves account for the top shares in the distribution of wealth. Therefore other factors, like stochastic and idiosyncratic returns, as well as savings rates and rates of returns increasing in wealth may help to better explain the top wealth shares as well as wealth mobility.

Key words: wealth distribution; thick tails; inequality

JEL codes: E13, E21, E24

*Corresponding author: alberto.bisin@nyu.edu. We thank the Washington Center for Equitable Growth for their financial support.

1 Some Theory

Increasing income and wealth inequality has led to renewed interest in understanding and explaining wealth and income distributions, and in particular the recent growth in their top shares. The earlier literature had emphasized the role of earnings inequality in explaining wealth inequality. The early work of Aiyagari (1994), which emphasizes the role of precautionary savings, has been very influential in this respect. However, models of earnings inequality and precautionary savings find it generally difficult to reproduce the top tail of the wealth distribution observed in the data. A simple linear model is useful to highlight several theoretical aspects underlying this difficulty.

Consider a linear individual wealth accumulation equation,

$$w_{t+1} = r_t w_t + y_t - c_t; \tag{1}$$

where w_t, y_t, c_t and r_t are wealth, earnings, consumption and rate of return at time t. We may assume $\{y_t, r_t\}$ are stationary stochastic processes.[1] For simplicity we may also assume a linear consumption function,

$$c_t = \psi w_t + \chi_t.\text{[2]} \tag{2}$$

[1]We may think of r_t as an exogenous partial equilibrium representation of the rate of return at t, or as coming from an endogenous growth model with AK technology; after all the capital labor ratio has grown substantially over the last 150 years without any discernible Marxian "falling rate of profit" or fall in mean returns.

[2]Of course it is well understood that infinitely-lived agent models with stochastic earnings and precautionary savings are concave, but with CRRA (DARA) preferences the consumption function becomes asymptotically linear at high wealth levels. Therefore the results that can be derived for the tails of wealth with linear consumption functions carry over to models with non-linear consumption functions that become asymptotically linear. See Benhabib, Bisin and Zhu (2016) for a rigorous exposition and proofs, allowing for idiosyncratic stochastic r_t. Linear consumption policies can also be obtained with quadratic preferences, giving rise to a wealth accumulation process that is stationary (rather than a random walk) if $\beta r > 1$. This is because under quadratic utility we have certainty equivalence and precautionary savings are avoided. A disadvantage of quadratic utility however is that for large wealth and therefore consumption above the "bliss point," marginal utility can become negative, creating complications.

We can then write the accumulation equation as

$$w_{t+1} = (r_t - \psi)\, w_t + (y_t - \chi_t)\,. \tag{3}$$

We can now apply a theorem due to Grey (1994), extending results of Kesten (1973), to (3). Suppose that $(r_t - \psi)$ and $(y_t - \chi_t)$ are both random variables, independent of w_t and $\chi_t \geq 0$, $r_t - \psi > 0$.[3] Suppose finally that the accumulation equation 3 satisfies the following: i) (r_t, y_t) are independent and $i.i.d$ over time; and ii)

$$y > 0, \;\; 0 < E(r_t) - \psi < 1, \;\text{and}\; prob\,(r_t - \psi > 1) > 0,$$

for any $t \geq 0$.[4] Now let $(y_t - \chi_t)$ have a thick right-tail, with tail-index $\beta > 0$. The stationary distribution for w_t can then be characterized as follows.

Proposition 1 *If $E\left((r_t - \psi)^\beta\right) < 1$, and $E\left((r_t - \psi)^\gamma\right) < \infty$ for some $\gamma > \beta > 0$, then under some regularity assumptions, the right-tail of the stationary distribution of wealth will be β. If instead $E\left((r_t - \psi)^\gamma\right) = 1$ for $\gamma < \beta$, then the right-tail index of the stationary distribution of wealth will be γ.*

The Proposition makes clear that the right-tail index of the wealth distribution induced by Equation 3 is either γ, which depends on the stochastic properties of returns, or β, the right-tail of $\{y_t - \chi_t\}$). With $\chi_t \geq 0$ the right tail of $\{y_t - \chi_t\}$ will be no thicker than that of than that of $\{y_t\}$. Under our assumptions, for stochastic process describing the accumulation of wealth, the tail index of earnings could not amplify the tail index of the wealth distribution; it's either the accumulation process or the skewed earnings which determine the thickness of

[3]Note that χ_t will depend on the stochastic properties (i.e. the persistence and variance of its innovations) of the earnings process. In a deterministic model, with constant r and y for example, if $\beta r = 1$, we have $\chi = y$. In a certainty equivalence quadratic utility $(-(bc_t - b(c_t)))$ model of an infinitely lived representative agent (ignoring the bliss point) with i.i.d. earnings shocks, assuming that consumption takes place at the beginning of each period prior to observing the earnings innovation ε_t, we have $\chi_t = aE\,(y) - k$, where a and k are positive constants and k goes to zero as b_1 goes to zero.

[4]Some additional regularity conditions are required; see Benhabib, Bisin, Zhu (2011) for details.

the right- tail of the wealth distribution.

2 Empirical Findings

This result is of course obtained under very simplified assumptions that in particular minimize the role of precautionary savings, especially at low wealth levels, and assume a linear consumption function. It applies abstractly to the right tail of wealth, which empirically may be far out to the right. However, it does point to the potential difficulty of matching the upper tail of wealth distribution by relying solely on earnings. Since until recently data was top coded and information on the top end of the income distribution was sparse, several studies that match the wealth distribution via earnings had postulated some extraordinarily high earnings states, and introduced them into Bewley-Aiyagari type models to match the wealth distribution. The introduction of high earnings states, appropriately called *awesome states* in the literature, invariably induces thickness in the distribution of earnings in excess of that which can be documented in earnings data. These results suggest that, if earnings were the main determinant of the thickness in the tail of the distribution of wealth, a much thicker distribution of the tail of earnings relative to the tail of actual earnings data would be required to fit the wealth data. For example, Castaneda et al. (2003) estimate the properties of an *awesome state* in a rich overlapping-generation model with various demographic and life-cycle features. It requires the top 0.039% earners have about $1,000$ times the average labor endowment of the bottom 61%, while this ratio, even for the top .01%, is at most of the order of 200 in the World Wealth and Income Database (WWID) by Facundo Alvaredo, Anthony B. Atkinson, Thomas Piketty, Emmanuel Saez, and Gabriel Zucman (since 2011).[5] Similarly, Krueger and Kindermann's (2014) *awesome state* in their Aiyagari-Bewley model requires the top 0.25% earnings to be 400 to 600 larger than the median. Instead, according to the WWID, even the top 0.1% are just about 34 times larger than the median. Finally,

[5] We use WWID earnings data, which is not top-coded, for 2014. The argument is not much changed even when considering average income, excluding capital gains.

Diaz et al. (2003) estimate a top 6% of the population to earn 46 times the labor earnings of the median; while the top 5% in WWID earns about 5 times the median.[6]

Many of the above cited studies, as well as others, introduce, in addition to stochastic earnings, additional features to help match the wealth distribution.We briefly discuss some of them.

1. Age Heterogeneity or Perpetual Youth Models: Allowing random death with constant probability introduces extreme age heterogeneity. The constancy of the death rate, independent of age, allows wealth levels to depend on age. This is certainly true in models where all new born agents start life with the same initial wealth and collect annuities, but with bequests the heterogeneity is reflected in the average lifespan of dynasties. For example a standard calibration used in many models makes expected working life 45 years, but this implies that 11% of the population works more than 100 years, with some working and accumulation for a thousand years. While this mechanism does induce additional wealth heterogeneity, it does not seem to survive the realistic assumption of uncertain but finite lives. Recently De Nardi et al (2016), adapted earnings data from Guvenen, Karahan, Ozkan, and Song (2015), which they introduce into a finite-life OLG model, to show that earnings processes derived from data, including the one that they use, generate a much better fit of the wealth holdings of the bottom 60% of people, but generates too little wealth concentration at the top of the wealth distribution (See De Nardi et al, 2016, p. 44).

2. Stochastic Idiosyncratic Returns: Stochastic returns had been introduced into wealth distribution models by Krusell and Smith (1998) (who equivalently used stochastic discount rates), as well as Quadrini (2000) and Cagetti and De Nardi (2006, 2007) who introduced idiosyncratic entrepreneurial returns. Early work of Moskowitz and Vissing-Jorgensen (2000) also found high variance to individual portfolio returns. The variability occurs particularly due to returns to private business equity which on average tends to be

[6]Furthermore, while the precautionary savings motive is the driving force of the Aiyagari-Bewley model, Guvenen and Smith (2014) note that "... the amount of uninsurable lifetime income risk that individuals perceive is substantially smaller than what is typically assumed in calibrated macroeconomic models with incomplete markets."

held as large fractions of portfolios, as well as variable returns to housing ownership. Recently Benhabib, Bisin and Zhu (2011, 2016) also studied a micro-founded model and showed how stochastic returns can generate fat wealth distributions. Fagereng, Guiso, Malacrino and Pistaferri (2015, 2016) using Norwegian data, as well as Bach, Calvet and Sodini (2015) using Swedish data, documented idiosyncratic returns to individual agent portfolios.Benhabib, Bisin, Luo (2016) explicitly estimate the stochastic properties of the Markov process for r_t to match the distribution of wealth. Interestingly, the mean and standard deviation of estimated returns, 2.76% and 2.54%, respectively, closely match those estimated by Fagereng, Guiso, Malacrino and Pistaferri (2015) for the idiosyncratic component of the lifetime rate of return on wealth Benhabib, Bisin, Luo (2016) explicitly estimate the stochastic properties of the Markov process for r_t to match the distribution of wealth. Interestingly, the mean and standard deviation of estimated returns, 2.76% and 2.54%, respectively, closely match those estimated by Fagereng, Guiso, Malacrino and Pistaferri (2015) for the idiosyncratic component of the lifetime rate of return on wealth.[7]

3. Savings Rates Increasing in Wealth: Early theoretical work introduced by Atkinson (1971) studies OLG models of savings and accumulation with savings rates that could increase in wealth. De Nardi (2004), similarly introduced bequest motives non-homogenous in wealth which generate savings rates increasing in wealth. Benhabib, Bisin, Luo (2016) also find support for savings rates increasing in wealth in an estimated model.

4. Returns Increasing in Wealth: Returns increasing in wealth has also been recently introduced by Kaplan, Moll and Violante (2015) due to fixed costs of holding high return assets, and documented by Fagereng, Guiso, Malacrino and Pistaferri (2015) as well as Bach, Calvet and Sodini (2015). Benhabib, Bisin and Luo (2016) also find some empirical evidence for returns increasing in wealth. However the degree to which savings rates and returns increase in wealth have to be tempered by social mobility data in estimations be-

[7] Interestingly, as noted by Gabaix et al, (2016) exogenous stochastic returns on wealth can fill the tail of wealth distribution quickly and generate realistic convergece speeds in reponse to exogenous change in parameters.

cause if they are too strong, the rich will tend to stay rich across generations, contradicting the wealth mobility we observe in data, maybe even result in non-ergodicity. Since micro-founded wealth distribution models also generate social mobility, it seems desirable to model and calibrate models to be consistent with wealth transitions, especially across generations.

Empirical work by Benhabib, Bisin, Luo (2016) suggest that stochastic earnings, idiosyncratic returns, and savings rates increasing in wealth are all important for explaining wealth distribution and mobility, though for different reasons. Stochastic earnings with enough variance are important to avoid poverty traps, not apparent in mobility transitions, since the poor do not hold much in terms of wealth. Idiosyncratic returns are important both for generating mobility as well as filling the right tail of wealth distribution. The estimation of Benhabib, Bisin, Luo (2016) also find that savings rates increasing in wealth helps to match the right tail of wealth and shutting down this channel weakens the match. The estimation also suggests that average returns also increase in wealth, mostly at the high end of wealth, consistent with Kaplan, Moll and Violante (2015).

3 Earnings and wealth inequality across countries

We started by suggesting that stochastic earnings alone cannot by themselves match the upper right tail of wealth. We end by at best a suggestive exercise, looking at the correlation of Gini coefficients for wealth and earnings across countries for which data is available for both in Figure 1. The earnings Gini's are mostly available from country specific papers of the special issue of the Review of Economic Dynamics (2010), titled "Cross-sectional facts for macroeconomists," edited by Krueger, Perri, Pistaferri, and Violante, while wealth Ginis are from James B. Davies, Susanna Sandström, Anthony Shorrocks, and Edward N. Wolff (2011). We have pairs of Gini's for US, Canada, UK, Germany, Italy, Spain, Sweden, Russia and Mexico.

Figure 1: Earnings and Wealth Gini

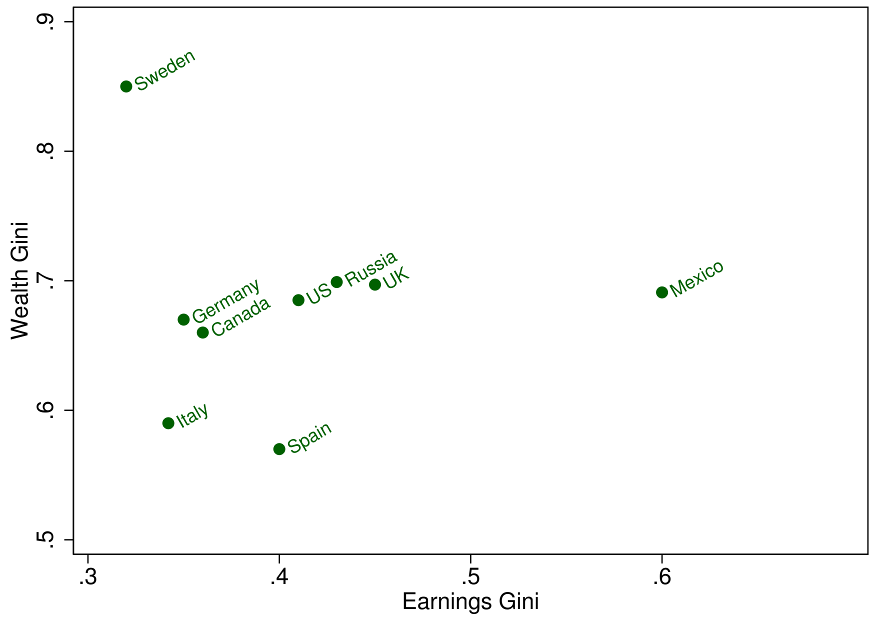

Table 1: Tail Indices for Wealth and Earnings

Country	Wealth	Earnings
US	1.48 - 1.55	2
Sweden	1.63 - 1.85	3
Canada	1.33 - 1.54	2

The correlation coefficient is -0.063. This is some evidence that earnings inequality does not explain wealth inequality, as conjectured at the beginning.

Unfortunately there is little country data that we could find on pairs of earnings and wealth tails. We were able to obtain three pairs in Table 1. For wealth tails we have US (Vermullen (2014), Table 8)=1.48-1.55, Sweden (Cowell (2011))=1.634-1.852, Canada (Cowell (2011))=1.333-1.536, and for earnings tails we have from Badel, Dayl, Huggett and Nybom (2016) tail indices for the US of about 2, Canada about 2, and Sweden about 3. This establishes that wealth tails are indeed thicker than earnings tails.

References

Atkinson, A.B. (1971): "Capital taxes, the redistribution of wealth and individual savings," *Review of Economic Studies,* 38(2), 209-27.

Alvaredo, F., Atkinson, A. B., Piketty, T., Saez, E.,(2013). "The Top 1 Percent in International and Historical Perspective," *Journal of Economic Perspectives*, American Economic Association, vol. 27,3-20.

Alvaredo, F., Atkinson, A. B., Piketty, T., Saez, E., Zuchmn, G., (since 2011). World Wealth and Income Database, http://52.209.180.1/

Aiyagari, S.R. (1994): "Uninsured Idiosyncratic Risk and Aggregate Savings," *Quarterly Journal of Economics,*109 (3), 659-684.

Bach, L., Calvet, L., and P. Sodini (2015): "Rich Pickings? Risk, Return, and Skill in the Portfolios of the Wealthy," mimeo, Stockholm School of Economics.

Badel, A., Dayl, M., Huggett, M. and M. Nybom (2016). "Top Earners: Cross-Country Facts", mimeo.

Benhabib, J., Bisin, A. and Zhu, S. (2011). "The Distribution of Wealth and Fiscal Policy in Economies with Finitely-Lived Agents" *Econometrica* (79) 2011, 123-157.

Benhabib, J., Bisin, A. and Zhu, S. (2016). "The Wealth Distribution in Bewley Models with Capital Income", *Journal of Economic Theory,* Part A, September 2015, 489-515.

Benhabib, J., Bisin, A. and Luo, M., (2015) "Wealth distribution and social mobility in the US: A quantitative approach," NBER working paper w21721.

Cagetti, C. and M. De Nardi (2005): "Wealth Inequality: Data and Models," Federal Reserve Bank of Chicago, W. P. 2005-10.

Cagetti, M. and M. De Nardi (2006), "Entrepreneurship, Frictions, and Wealth", *Journal of Political Economy*, 114, 835-870.

Cagetti, M. and C. De Nardi (2007): "Estate Taxation, Entrepreneurship, and Wealth," NBER Working Paper 13160.

Cagetti, C. and M. De Nardi (2008): "Wealth inequality: Data and models," *Macroeco-*

nomic Dynamics, 12(Supplement 2), 285-313.

Castaneda, A., J. Diaz-Gimenez, and J. V. Rios-Rull (2003): "Accounting for the U.S. Earnings and Wealth Inequality," *Journal of Political Economy*, 111, 4, 818-57.

Cowell F.A., (2011): "Inequality Among the Wealthy," mimeo, Centre for Analysis of Social Exclusion, LSE.

Davies, J.B., S. Sandström, A. Shorrocks, E. Wolff (2011): "The Level and Distribution of Global Household Wealth," *Economic Journal,* 121(551), 223-254.
http://onlinelibrary.wiley.com/doi/10.1111/j.1468-0297.2010.02391.x/epdf

De Nardi, M. (2004): "Wealth Inequality and Intergenerational Links," *Review of Economic Studies*, 71, 743-768.

De Nardi, M., Fella, G. and G. P. Pardo (2016), "The Implications of Richer Earnings Dynamics for Consumption, Wealth, and Welfare", NBER Working paper No, 21917.

Diaz, A, Josep Pijoan-Masb, J., and Rios-Rull, J-F, (2003). "Precautionary savings and wealth distribution under habit formation preferences,"*Journal of Monetary Economics* 50, 1257–1291.

Fagereng, A., L. Guiso, D. Malacrino, and L. Pistaferri (2015): "Wealth Return Dynamics and Heterogeneity," mimeo, Stanford University.

Facundo A., A. B. Atkinson, T. Piketty, E. Saez, and G. Zucman (2016), *The World Wealth and Income Database*, http://www.wid.world

Fagereng, A., Guiso, L., Malacrino, D., and Pistaferri, L. (2016): "Heterogeneity in Returns to Wealth, and the Measurement of Wealth Inequality" *American Economic Review Papers and Proceedings*, 106, 651-655(5).

Gabaix, X., Lasry, J-M., Lions, P-L, and B. Moll, (2016): "The Dynamics of Inequality," *Econometrica* 84, 2071-2111.

Grey, D.R. (1994): "Regular variation in the tail behavior of solutions of random difference equations," *The Annals of Applied Probability*, 4)(1), 169-83.

Guvenen, F. and A. A. Smith (2014): "Inferring Labor Income Risk and Partial Insurance

From Economic Choices," *Econometrica*, 82, 2085–2129.

Guvenen, F., Karahan, F., Ozkan, S. and J. Song (2015): "What Do Data on Millions of U.S. Workers Reveal about Life-Cycle Earnings Risk?," National Bureau of Economic Research, Inc. NBER Working Papers 20913.

Kaplan, G, Moll, B., and Gianluca Violante (2015), "Asset Illiquidity, MPC Heterogeneity and Fiscal Stimulus," in preparation.

Kesten, H. (1973): "Random Difference Equations and Renewal Theory for Products of Random Matrices," *Acta Mathematica,* 131, 207–248.

Krueger, D., F. Perri, L. Pistaferri, G.L. Violante (eds.) (2010). *Review of Economic Dynamics* 13.

Krueger, D. and Kindermann, F., (2014). "High marginal tax rates on the top 1%? Lessons from a life-Cycle model with idiosyncratic income risk," NBER
WP 20601 http://www.nber.org/papers/w20601

Krusell, P. and A. A. Smith (1998): "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, 106, 867-896.

Moskowitz, T. and A. Vissing-Jorgensen (2002): "The Returns to Entrepreneurial Investment: A Private Equity Premium Puzzle?", *American Economic Review*, 92, 745-778.

Quadrini, V. (2000): "Entrepreneurship, Savings and Social Mobility," *Review of Economic Dynamics*, 3, 1-40.

Vermeulen, P. (2014), "How fat is the top tail of the wealth distribution?" *European Central Bank*, Working Paper Series no.1692.