Assignment of Stock Market Coverage*

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Abstract

Price efficiency plays an important role in financial markets. Firms influence it, particularly when they issue public equity. They can hire a reputable underwriter with a star analyst to generate public signals about profits to reduce uncertainty and increase valuations. We develop an assignment model of this labor market. The value of a match between firms, that differ in multiple dimensions, and agents, that differ in precision, is endogenously generated from a stock-market equilibrium. We characterize the multidimensional-to-one assignment and obtain testable predictions. Extensions allow firms to value efficiency for other reasons and apply to other labor markets like media-or-investor relations professionals.

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1. Introduction

The information efficiency of stock prices (Grossman and Stiglitz (1980)) plays an important role in many aspects of financial economics. It determines the cost of capital when a firm goes public (Diamond and Verrecchia (1991)), shapes agency and incentive problems in the firm (Holmström and Tirole (1993)) and influences the level of capital investments (Chen et al. (2007), Bai et al. (2013))). In practice, firms can affect the level of information efficiency, particularly when they issue public equity. During this time, a firm can hire a reputable underwriter (typically an investment bank) with a talented analyst to generate public signals about firm profits, thereby reducing uncertainty and increasing valuations.[1]

Empirically, a firm's hiring decision is heavily influenced by the presence of such analysts (often called an "All-American"), who typically work for reputable underwriters (Krigman et al. (2001), Hong and Kubik (2003)). Firms pay for this accurate coverage through both underwriting fees and underpricing (Cliff and Denis (2004)). Consistent with firms competing and spending significant resources for quality coverage, there is significant analyst wage dispersion. The most accurate and influential analysts earn millions of dollars a year in renumeration covering initial public offerings (see, e.g., Stickel (1992)).

Despite the importance of coverage in determining the informational efficiency of firms in need of public capital, there has been little theoretical research on this labor market. As such, we provide an assignment model, where coverage is endogenously determined by the labor market matching of firms with agents who can generate coverage, defined as a public signal about the firm's fundamental pay-off.[2] The agents have heterogeneous precision (indexed

[1] A large body of evidence points to the ability of this coverage to improve informational efficiency and mean stock prices. Analyst coverage in the cross-section is correlated with more informative prices and deeper markets (see, e.g., Brennan and Subrahmanyam (1995)). Exogenous shocks to analyst coverage generated by brokerage house mergers (Hong and Kacperczyk (2010)) or closures (Kelly and Ljungqvist (2012)) show that the effect of coverage on price efficiency is causal. Media coverage variation due to differential investor access to local newspapers or newspaper strikes lead to similar causal conclusions for the benefits of coverage for stock market pricing (Engelberg and Parsons (2011) and Peress (2014)).

[2] A typical starting point for thinking about assignment problems with heterogeneous agents is the model of Becker (1973) and Rosen (1974). Building on this, there are extensive works that study labor market sorting.

by h), which is known to the market.[3] Given that the best analysts typically work at the best underwriters and their tasks of coverage and obtaining high equity valuations for their client firm are one and the same, our model can be thought of as applying to an underwriter where the heterogeneity is the informational precision of the coverage.

Consistent with practice, firms hire *at most* one agent (an underwriter/analyst pair) to generate coverage when they issue equity. We only allow the agent to work for one firm for simplicity.[4] The outcome of the labor market matching between firms and agents determines the accuracy or precision of coverage across firms, i.e. the endogenous informational efficiency in the stock market. We assume firms can differ potentially over a number of different dimensions, including investment scale, amount of share issuance, risk-absorption capacity of the firm's investors, cashflow volatility, and its informational environment before the purchase of coverage.

Empirical work points to the importance of assortative matching in the initial or secondary public offering stages between firms and underwriters (Fernando et al. (2005),Akkus et al. (2013)). Our paper provides a theory for this empirical work. In particular, our analysis emphasizes the importance of sorting on multiple firm dimensions and we derive the complementarity endogenously from the firm's stock pricing and trading environment. That is, the noisy rational expectations equilibrium of Grossman and Stiglitz (1980) tells us the value of the match to an agent of a given precision depending on multiple firm dimensions. Our model extends earlier matching models used in the firm-CEO matching literature (see, e.g., Terviö (2008); Gabaix and Landier (2008)), where complementarities are exogenously specified in the firm's production function.

There are three dates. At $t = 0$, a firm issuing equity and seeking to maximize its share price decides which agent it will hire, i.e. the assignment function $\mu(y)$ a firm of characteristic y, where y can denote potentially different firm characteristics, to an agent of precision h.

[3]Heterogeneity of ability for coverage agents and the observability of ability are realistic assumptions given the observability of forecasting track records.

[4]In practice, agents can work for more than one firm. We can extend our set-up to account for this setting.

Firms pay a competitive wage to hire an agent. Each agent works for and covers one firm, producing the information for that firm at a fixed cost.

At $t = 1$, asset markets are open for each firm. The asset market follows a traditional noisy rational expectations set-up. Investors submit price-contingent demand based on their own private signals and whatever public signal the firm purchases in the labor market for coverage.[5] There are also noise traders in the market. At $t = 2$, the pay-off of the firm is realized. Following the literature (see, e.g., Merton (1987)), asset markets are segmented.[6]

We solve for an equilibrium consisting of: (1) optimality of firm coverage decisions; (2) optimality of investor's decisions; (3) market clearing in the labor market, i.e. an assignment function $\mu(y)$ and wage function $\omega(h)$ based on the optimal choices of the firms for coverage and the talent distribution of agents; and (4) market-clearing in the asset markets.

In equilibrium, coverage improves the estimation of fundamentals by investors, i.e. the market is more efficient. Since idiosyncratic risk is priced, this also means a higher stock price for the company at $t = 1$. [7] In other words, coverage improves both price efficiency and the mean price, consistent with empirical evidence on the importance of coverage for the pricing of the stock market.

Our model generates several sets of results. First, we derive the multi-dimensional-to-one matching of firm characteristics of talent. We show that the many firm characteristics that determine the value of a match can be collapsed into two dimensions. The first is scale variables, including the size of the share issuance and the risk absorption capacity of the firm, that lead to a higher risk premium. The second is transparency variables, including

[5] We consider an asset market along the line of Grossman and Stiglitz (1980); Diamond and Verrecchia (1981). Specifically, our setup is closet to the one in Hellwig et al. (2006).

[6] We use segmentation for convenience. We show that our results obtain as along as asset markets are incomplete and the idiosyncratic risk of a firm is priced as a result.

[7] However, we have not modeled the potential bias in coverage as a result of conflicts of interest or incentive issues (see, e.g., Michaely and Womack (1999), Hong and Kubik (2003), Dyck and Zingales (2003)). One could introduce bias into our setting by assuming the noise traders are influenced by the bias. But assuming that there is talent in spinning the news to get stock prices higher, we would end up with the same outcome in terms of valuations but through an alternative channel to information efficiency. But information efficiency is, nonetheless, the most natural route to model this effect and the data also points to information efficiency effects as well as mean price effects of coverage.

the firm's cash-flow precision, the precision of private signals and the degree of noise trading, which lead to a lower risk premium.

Holding fixed scale, we show that there is positive assortative matching using an index of the firm's information environment that adds up the transparency variables. This result follows from a closed-form characterization of the matching surplus function between the firms and the agents. It holds for any distribution of precision among agents and distribution of firm characteristics. Firms with a higher opacity have more risk that investors have to bear and hence pay more for accurate coverage. Holding fixed opacity, firms with a higher scale have more risk that the investors have to bear. Hence, higher scale firms benefit the most from paying for the most precise agents.

Under regularity conditions from Chiappori et al. (2016), we can characterize the assignment equilibrium that maps (assigns) these two dimensions of firms into the talent of agents. Our results on the assignment function here naturally provide a theory for the aforementioned empirical work on sorting in this labor market that will generate sharper tests of the importance of sorting.

We also derive a new asset pricing test of the influence of labor market sorting on the informational efficiency of the firm. Under a null where the firm's level informational efficiency is not endogenous, we expect that the expected return of a stock to increase with firm scale and firm opacity. But if informational efficiency is endogenous, then the relationships between expected return between firm scale and firm opacity are no longer monotonically increasing. There are two forces then that shape the cross-section of expected returns. The first is the usual (exogenous) risk-sharing force whereby firms with higher scale and opacity required higher expected return. The other is an (endogenous) information efficiency force whereby firms with higher scale and opacity can purchase coverage to improve their efficiency. In general, the shape between the cross-section of expected returns and these underlying parameters can be non-monotonic, depending on the strength of the relative strength of these two forces.

In other words, the assignment function is an omitted variable in determining the cross-section of expected returns. When we control for this omitted variable, we can then decompose these two forces. The model offers a very simple functional form to take to the data. Using a first-stage assignment function along the lines of Akkus et al. (2013), we can test this asset pricing prediction. This is a test of the endogeneity of assignment and of the value matching function o being driven by a noisy rational-expectations equilibrium.

Importantly, we also prove that if there is a scarcity of talent to evaluate firms, the endogenous information efficiency dominates, leading to a flatter relationship between expected return with firm size and opacity. Since we have a static model, our scarcity of talent prediction can be interpreted as the exogenous arrival of new initial public offering (IPO) or technologies that only some of the existing analysts in the labor market can accurately decipher. As such, it is easy to look to time series variation in IPO waves and test if our predictions are true.

Second, we show this competitive-sorting effect is captured by the steepness of the wage distribution. The wage of an agent is rising in his talent. The stronger is our competitive-sorting effect, the steeper the wage profile is with talent.[8] As a result, the strength of the complementarity depends on external factors such as the amount of market noise and the extent of the precision that the market has about firm profits. We expect compensation to be more skewed when there is more market noise or less market precision.

Consistent with this prediction, the compensation of the top analysts are highly skewed. During the Internet Bubble Period of 1997-2000, security analysts' pay were especially skewed as the prices of dot-com stocks were noisy and underlying dot-com pay-offs were also highly uncertain, consistent with our model's prediction. More recently, during the 2005-2007 period, with the advent of the second Internet Bubble in social media stocks, analyst wages became again skewed. A number of commentators expresses surprise given the regulations on

[8]Wages rise with firm scale, firm pay-off volatility, and the ratio of noise trading to the precision of private signals in the market.

conflicts of interest that were established after the Internet Period.[9] Our model shows that such a superstar effect can occur even independent of regulatory concerns about conflicts of interest leading to skewed wages for security analysts. This second set of predictions on wages and IPO waves complements our earlier asset pricing test of endogenous market efficiency.

Third, we show that sorting can offer a satisfying explanation of the neglected firm effect.[10] The traditional explanation of why some firms have no coverage and suffer from low prices as a result requires that there be high fixed cost (see, e.g., Merton (1987)). But we show that the neglect effect is magnified by the labor market matching effect. That is, we can get a neglect effect even at low fixed cost. When there are fixed costs to covering a firm, firms below a certain cut-off (depending on scale and opacity) do not pay for coverage since they do not benefit as much as other firms. Less informationally opaque firms are only able to compete for and hire low precision agents since the high precision ones work for higher opacity firms and get paid more. This is a different take on the traditional interpretation of the neglect effect in the literature. We show that, absent this labor market matching effect, the cut-off firm opacity below which there would be no coverage is larger.

We have emphasized the labor market for underwriter/analysts when firms go public. But firms can have other motives for wanting their stock prices to be informationally efficient as we alluded above, including using informative stock prices to alleviate moral hazard (Holmström and Tirole (1993)) and making wise capital investments (Chen et al. (2007), Bai et al. (2013))) even in the absence of a need for external capital. As a result, it is not surprising that more mature firms hire media or investor relations professionals to improve their disclosures.

Our model is similar to voluntary disclosure models (see, e.g., Diamond and Verrecchia

[9]Susan Craig, "Star Analysts Are Back (No Autographs, Please), August 20, 2011, NYTIMES DealBook

[10]The earliest study on the neglected firm effect by Arbel et al. (1983) finds that stocks with zero or low analyst coverage out-perform stocks with high analyst coverage. Subsequent studies such as Foerster and Karolyi (1999) used quasi-experiments to establish the importance of coverage or investor recognition in explaining the neglect effect. Recently, Tetlock (2007) and Fang and Peress (2009) find that stocks with more media coverage have lower expected returns.

(1991), Fishman and Hagerty (1989), Admati and Pfleiderer (2000)). Firms might pre-commit to disclosing a public signal of their earnings to improve the price efficiency and risk discount for their shares. Results depend on the nature of the cost of disclosure, which is exogenously given and requires convexity in the cost structure. The costs in our model are determined by the entire stock market via labor market sorting and wages.

We show in the extension section that we can model the value of matching in this labor market using Holmström and Tirole (1993), whereby the firm derives benefits from better contracting with employees. Our model can be applied to savvy media or investor relations professionals who help the firm improve disclosure (Bushee and Miller (2012), Karolyi and Liao (2015)). Indeed, Karolyi and Liao (2015) find also substantial wage dispersion for investor relations professionals, consistent with such a matching model.[11]

2. Model

The model lasts for three dates. There is a continuum of heterogeneous firms who issue equity through stock markets with measure one. Specifically, a firm originally owns $(1+\psi)$ measure of shares and wants to raise capital by issuing one measure of their equity to investors. There is a distribution of agents with measure one, denoted by $G^A(h)$ with support $[h^L, h^U] \equiv H$, who differ in terms of skill (i.e., the precision of information they can produce). The quality of information for each firm's profits thus depends on which agent a firm hires. At date 0, the allocation of agents across firms and agents' fees are pinned down in a competitive assignment equilibrium. At date 1, agents produce information, and trade takes place in the stock markets. Finally, at date 2, the cash flow is realized, and all agents consume their realized gains.

Firm: Firms seek to maximize mean share price at $t = 1$. A firm's capital stock is denoted by k and each firm owns a risky project with volatility σ_θ. The payoff of the project

[11] Similarly, Dougal et al. (2012) find that certain journalists are more influential in stock markets, pointing to the importance of heterogeneity in talent when it comes to media coverage.

for the firm with capital k, is given by $k\theta$, where θ is a firm-specific payoff drawn from a Normal distribution with mean $\bar{\theta}$ and variance σ_θ^2. We assume that the fundamental payoffs are uncorrelated across firms.

We assume that there is a segmented capital market for each firm in the spirit of Merton (1987). In each market, there is a unit measure of a continuum of risk-averse investors. They have CARA expected utility with coefficient of risk aversion γ_I. Investors are imperfectly and heterogeneously informed. Specifically, each investor receives a private signal

$$x_i = \theta + \sigma_x \epsilon_i,$$

where $\epsilon_i \sim N(0,1)$. The precision of investors' private signal is then given by $\tau_x \equiv \frac{1}{\sigma_x^2}$.

For simplicity, we assume that each investor only has access to one market, and each investor can submit their demand based on their information set. There are also noise traders in each market. To solve the model in closed form, we assume that noise traders purchase a random quantity $\Phi(\tilde{u})$ of stock, where $\tilde{u} \sim N(0, \sigma_u)$ and Φ is the standard normal CDF. This specific functional form assumed here is close to that in Hellwig et al. (2006).

We allow firms to differ in multiple characteristics, where $y = (\sigma_\theta, \sigma_x, \sigma_u, k, \psi, \gamma_I) \in Y \subseteq \mathbb{R}_+^6$ represent firm types. That is, firms can differ in their project risks (σ_θ), clientele (captured by precision σ_x and risk aversion γ_I of their investors), market condition (captured by market noise σ_u), capital size (k) and the proportion of issued shares $\frac{1}{1+\psi}$. The types of firms are distributed according to a probability measure ν^F on Y, which is assumed to be absolutely continuous with respect to the Lebesgue measure.

Agent: When the agent $h \in H$ works for the firm y, he can produce a report z at cost C at date 1. Coverage, or the report, is a noisy, unbiased signal regarding the payoff:

$$z = \theta + \sigma_h \eta,$$

where $\eta \sim N(0,1)$, and the variance of the report is parameter as

$$\sigma_h^2 = \frac{1}{h}.$$

Higher values of h denote more precise agents.

Labor Market for Agents ($t = 0$): At date 0, each firm can at most hire one agent to cover the firm assuming the firm hires any at all. The fee paid to the agent is denoted by $\omega(h)$. That is, the fee is independent of the signal and realized payoff.[12] The fee $\omega(h)$ will be determined in equilibrium. The payoff of agent h is then given by $\omega(h) - C$. It is assumed the firm is be able to pre-commit to the hiring decision. The end-of-period cash flows for firm y is then the profit from its project minus the fee that the firm commits to pay:

$$\pi_y = k\theta - \omega(h).$$

At date 0, given the fee required to hire agent $\omega(h)$, a firm of type y, rationally anticipating how different agents affect the stock price at date 1, chooses the optimal agent to maximize the firm's expected payoff,

$$U^*(y) \quad = \quad \max_{h \in H \cup \{\emptyset\}} E\left[\left(\frac{\psi}{1+\psi}\right)(k\theta - \omega(h)) + \tilde{p}_{hy}\right], \tag{1}$$

where $\tilde{p}_{hy}$ denotes the realized share price at date 1 for firm y if it hires agent h. That is, a firm effectively maximizes its expected share price. In a noisy rational expectations stock market equilibrium, the realized price will be a function of the fundamental θ, public signal z, and the demand of noise traders $\tilde{u}$. That is, $\tilde{p}_{hy} = P(\theta, z, u|h, y)$.

[12] Since the fee is not contingent on the report z, we thus assume away the agent's incentive for biased report.

Financial Market for Stocks ($t = 1$): All investors observe the public signal z produced by the agent and know its precision. Each can purchase at most one share for each stock or none at all based on their information set. That is, they submit a price-contingent demand schedule for stock y, which specifies their demand $d_i(p) \in \{0, 1\}$ conditional on price p to solve:

$$\max_{d\in\{0,1\}} \left\{ d\mathbb{E}[(\frac{\pi_y}{1+\psi} - p)|x_i, z, p] - \frac{\gamma_I}{2} d^2 Var(\frac{\pi_y}{1+\psi} - p|x_i, z, p) \right\}. \tag{2}$$

These bid functions determine the aggregate demand by informed investors. We maintain the restriction on demand for tractability. Alternatively, one can allow investors to submit a bidding schedule $d_i(p) \in \mathbb{R}$, which will not change the key economic results.[13] Together with the demand from the noisy traders, the auctioneer selects a price P to clear the market.

Equilibrium Definition: An equilibrium consists of an assignment $\mu(y)$: $Y \to H \cup \{\emptyset\}$, competitive fee for agents $\omega(h) : H \to \mathbb{R}^+$, demand function for each investor in the market (h, y), $D(x_i, z, p|h, y)$, and a price function $P(\theta, z, u|h, y)$ such that the following three conditions are met.

First, in the labor market for agents, the optimality conditions for both firms and agents are satisfied, which means that, given the wage $\omega(h)$, $\mu(y)$ is the type of agent that firm y optimally chooses to hire. That is, $\mu(y)$ maximizes (1). Second, in each market (h, y), investors choose their demand schedules to maximize (2). Third and lastly, the market-clearing condition holds for both the labor and asset markets.

3. Equilibrium

We first analyze the surplus generated by agent h for firm y, taking into account how the produced information affects the price movement in the stock market at $t = 0$. With this

[13] In the alternative setup, one would need to use a different assumption on the noise traders' demand. Specifically, the noise traders' demand is given by $\tilde{u}$ instead of $\Phi(\tilde{u})$. As standard, there exists a price which is linear in $(\theta, z, \tilde{u})$.

surplus function, we then analyze the matching of agents and firms in the labor market.

3.1. Financial Markets

When firm y hires agent h, investors thus obtain a public signal with precision $h = \frac{1}{\sigma_h^2}$. Aggregating the demand decisions of all investors in market (h, y), market clearing then implies

$$\int D(x_i, z, p|h, y)dF(x_i|\theta) + \Phi(\tilde{u}) = 1. \tag{3}$$

From the investor's optimization problem (2),

$$\begin{aligned} D(x_i, z, p|h, y) &\in \arg\max_{d\in\{0,1\}} \left\{ d\mathbb{E}[\left(\frac{k\theta - \omega(h)}{1+\psi}\right) - p|x_i, z, p] - \frac{\gamma_I}{2} d^2 Var(\frac{(k\theta - \omega(h))}{1+\psi} - p|x_i, z, p) \right\} \\ &= \arg\max_{d\in\{0,1\}} \left\{ d\mathbb{E}[\left(\frac{k\theta - \omega(h)}{1+\psi}\right) - p|x_i, z, p] - \frac{\gamma_I}{2} \frac{k^2 d^2}{(1+\psi)^2} Var(\theta|x_i, z, p) \right\}, \end{aligned}$$

Since $Var(\theta|x_i, z, p)$ is constant over the realization of (x_i, z, p), the demand $D(x_i, z, p|h, y) \in \{0, 1\}$ can be characterized by a cutoff, such that $D(x_i, z, p|h, y) = 1$ if and only if $x_i > \hat{x}(z, p)$.

Recall that each investor receives a private signal $x_i = \theta + \sigma_x \epsilon_i$, where $\epsilon_i \sim N(0, 1)$. This cut-off equilibrium then implies that only investors with good signals will buy, i.e. those investors with

$$\epsilon_i > \frac{\hat{x}(z, p) - \theta}{\sigma_x}.$$

With our specifications, the market-clearing condition can then be conveniently rewritten as

$$1 - \Phi(\frac{\hat{x}(z, p) - \theta}{\sigma_x}) + \Phi(\tilde{u}) = 1. \tag{4}$$

For the market to clear,

$$\hat{x} = \theta + \sigma_x \tilde{u}.$$

Hence, observing price in our model is informationally equivalent to a public signal (i.e. this cut-off value $\hat{x}$) with the precision $\frac{1}{\sigma_x^2\sigma_u^2}$.[14] An investor's information set can be summarized by $I_i = (x_i, z, \hat{x})$. Thus, the conditional expectation of the fundamental is then given by

$$\mathbb{E}[\theta|I_i] = \frac{\sigma_\theta^{-2}\bar{\theta} + \sigma_x^{-2}x_i + (\sigma_x\sigma_u)^{-2}\hat{x} + hz}{\sigma_\theta^{-2} + \sigma_x^{-2} + (\sigma_x\sigma_u)^{-2} + h}. \tag{5}$$

For the cut-off investor $\hat{x}$, the price must be equalized to the payoff of holding one share. Hence,

$$\begin{aligned} P(\theta, z, u|h, y) &= \mathbb{E}\left[\left(\frac{k\theta - \omega(h)}{1+\psi}\right)|x_i = \hat{x}, z, \hat{x}\right] - \frac{\gamma_I}{2}\frac{k^2}{(1+\psi)^2}Var(\theta|x, z, p) \\ &= -\frac{\omega(h)}{1+\psi} + \frac{k}{1+\psi}\mathbb{E}[\theta|x_i = \hat{x}, z, \hat{x}] - \frac{\gamma_I}{2}\frac{k^2}{(1+\psi)^2}Var(\theta|x, z, p). \end{aligned} \tag{6}$$

Notice that since investors rationally take into account the fee expense, a higher fee thus decreases the price, which shows up in the first term of Equation (6). The second term represents the risk premium for firm y that hires analyst h, which has the following expression

$$\frac{\gamma_I}{2}\frac{k^2}{(1+\psi)^2}Var(\theta|x, z, p) = \frac{\kappa(y)}{\tau(y) + h}, \tag{7}$$

where $\tau(y)$ is an one-dimensional transparency index that summarizes the information characteristic of firm:

$$\tau(y) \equiv \frac{1}{\sigma_x^2} + \frac{1}{\sigma_x^2\sigma_u^2} + \frac{1}{\sigma_\theta^2}$$

And, $\kappa(y)$ is the scale index of a firm y :

$$\kappa(y) \equiv \frac{\gamma_I k^2}{2(1+\psi)^2}.$$

Lemma 1 summarizes the properties of the risk premium.

[14] In general, as shown in Albagli et al. (2011), there exists a random variable that is only a function of θ and $\tilde{u}$, and contains the same information as the price.

Lemma 1. *The risk premium for firm y that hires analyst h is given by Equation (7), which strictly decreases with the precision of the agent h and strictly increases with firm volatility σ_θ, market noise σ_u, investor's noise σ_x, risk aversion γ_I, firm size k, and the proportion of issued shares $\frac{1}{1+\psi}$.*

That is, a higher level of precision h decreases the risk premium charged by investors, since it improves investor's estimation of the fundamental pay-off. Firms with higher volatility and with a noisier capital market have lower transparency index. All things being equal, the risk premium is then higher for less transparent firms, since the investors have to bear more risk for those firms. Similarly, the risk premium is higher for firms with a larger scale. But not all things will be equal in equilibrium, as firms with different characteristics have varying incentives or purchase varying degrees of accurate coverage, or even none at all.

3.2. Matching in The Labor Market

Taking into account how the agent affects the price in the asset market, the allocation of agents across firms can be solved as a matching problem. As is well-known in a matching model, the allocation depends on the property of the matching surplus of firm and agent. In our model, this surplus is driven by the coverage effect in financial markets. Specifically, the firm's expected utility when hiring agent h can be conveniently rewritten as the expected payoff of the project minus the risk premium and the agent fee:

$$
\begin{aligned}
U(y,h) &= \mathbb{E}\left[\left(\frac{\psi}{1+\psi}\right)(k\theta - \omega(h)) + P(\theta, z, u|h, y)\right] \\
&= k\bar{\theta} - \frac{\kappa(y)}{\tau(y)+h} - \omega(h).
\end{aligned} \tag{8}
$$

Despite that the firm only pays some portion of the fees at the end of period (i.e., $\frac{\psi}{1+\psi}\omega(h)$), the reduction in asset price due to the hiring is $\frac{\omega(h)}{1+\psi}$. Hence, from a firm's view point, the total cost is simply the agent fee. As a result, the surplus between firm with y and agent h, which is the sum of their payoff minus their outside option, yields

$$\begin{aligned}\Omega(y,h) \equiv& U(y,h) - U(y,\emptyset) + \{\omega(h) - C\} \\ =&\kappa(y)\left\{\frac{1}{\tau(y)} - \frac{1}{\tau(y)+h}\right\} - C,\end{aligned} \tag{9}$$

where $\emptyset$ denotes the case in which a firm hires no agent (i.e., the firm's autarky value) and the worker's unemployed value is normalized to zero. The first term thus represents the gain of firm y when he hires agent h relatively to no hiring. The second term represents the payoff of a worker, which is the fee minus the production cost. Hence, the surplus is simply the value of coverage, which is the reduction in risk premium (relative to no hiring), minus the cost of producing information.

Technically, given multiple characteristics of a firm, our environment is a multidimensional-to-one matching problem. Chiappori et al. (2016) has established regularity conditions under which a stable matching exists and the assignment function $\mu(y)$ is pure and unique. Given our surplus function in (9) and the measure of firms ν^F is absolutely continuous with respect to the Lebesgue measure, we show that these conditions are indeed satisfied and extend their results to the environment where some agents remain unmatched.

Proposition 1. *There exists a unique equilibrium* $\{\mu(y), \omega(h)\}$*. The market price for firm* y *is characterized by Equation (6), setting* $(h, y) = (\mu(y), y)$*, and the demand function for firm* y *is given by* $D(x_i, z, p|\mu(y), y)$*.*

Despite firms differing in multiple characteristics, from the surplus function, one can see that the characteristics of firms can be simply reduced to two aggregated indices: a transparency index $\tau(y)$ and a scale index $\kappa(y)$, thereby simplifying our characterization. With a slight abuse of notation, we now denote the assignment function as a function of these two indices directly. That is, the agent hired by firm y is denoted by $\mu(\kappa(y), \tau(y))$. Proposition 2 first establishes the property of the assignment function as a function of these two indices directly.

Proposition 2. *(1) Conditional on firms with the same scale index $\kappa(y') = \kappa(y)$, firm y' hires a more precise agent than firm y if and only if firm y' has a lower transparency index: $\mu(\kappa, \tau) \leq \mu(\kappa, \tau')$ iff $\tau' \leq \tau$. (2) Conditional on firms with the same transparncy index $\tau(y') = \tau(y)$, firm y' hires a more precise agent than firm y if and only if firm y' has a higher scale index: $\mu(\kappa, \tau) \leq \mu(\kappa', \tau)$ iff $\kappa' \geq \kappa$.*

These results can be seen easily from the firms' optimization problem. Specifically, given that all firms facing the same cost function $\omega(h)$, a firm that has a higher marginal benefit of increasing precision must hire a more precise agent in equilibrium. From Equation (8), one can see that the marginal value of more precise information is given by

$$\frac{\partial U(y,h)}{\partial h} = \frac{\kappa(y)}{(\tau(y)+h)^{\,2}} - \omega_h(h). \tag{10}$$

In other words, there is a *complementarity* between the precision of an agent and firms' scale and the transparency of the firm's endowed informational environment before hiring from the labor market. The intuition is very simple: fixing the scale of a firm, when investors' estimation of firm cashflow is less precise (i.e. a lower $\tau(y)$), any increase in the precision of public information improves their estimation more substantially. A lower transparency index can be driven by a more risky project (a higher σ_θ) or a less informative capital market (a higher level of investors' noise σ_x or market noise σ_u). On the other hand, if any of the variances converges to zero, $\tau(y) \to \infty$, and the marginal value of public information thus goes to zero. Hence, fixing the scale of a firm, there is a negative sorting between agent precision and the firm transparency. That is, a more opaque firm must hire a more precise agent.

Similarly, since firms with a higher scale $\kappa(y)$ have more risk that the investors have to bear, a better investor estimation reduces the risk premium more substantially. Thus, fixing the transparency, there is a positive sorting between agent precision and firm scale.

Proposition 3. *(1) The assignment function $\mu(\kappa,\tau)$ must satisfy the following partial differential equation:*

$$\left\{\frac{\partial\mu}{\partial\tau}+\frac{2\kappa}{\tau+\mu(\kappa,\tau)}\frac{\partial\mu}{\partial\kappa}\right\}=0. \tag{11}$$

(2) There exists a cutoff of agent $h^*\geq h_L$ such that are agents actively matched if and only if $h\geq h^*$ and $\omega_h(h)>0$ for $h\geq h^*$. The cutoff type h^* must satisfy the following conditions: (a) $\nu^F(Y_0)=G^A(h^*)$, where Y_0 denote the set of firms who are not matched (i.e. $Y_0\equiv\{y\in Y|\mu(y)=\emptyset\}$); (b) If $h^*>h_L$, $\Omega(\mu^{-1}(h^*),h^*)=0$ and $\Omega(y,h^*)<0$ for all firm $y\in Y_0$.

To see this, for all firms with index (κ,τ) :

$$U^*(\kappa,\tau)\equiv U(\kappa,\tau,\emptyset)+\max_{\tilde{h}\in H\cup\{\emptyset\}}\left\{\Omega(\kappa,\tau,\tilde{h})-\left(\omega(\tilde{h})-C\right)\right\}.$$

Assuming double differentiability of U^*, we thus have $\frac{\partial^2\Omega}{\partial\kappa\partial h}\frac{\partial\mu}{\partial\tau}=\frac{\partial^2\Omega}{\partial\tau\partial h}\frac{\partial\mu}{\partial\kappa}$, which gives the expression in Equation (11).

Without any fixed cost (i.e., $C=0$), the surplus is positive for all agents and thus all should be matched. On other hand, given any positive fixed cost $C>0$, agents with lower precision may not be hired since he cannot generate enough value to compensate the fixed cost.

Given that $\Omega(y,h)$ increases with precision, the surplus function is then positive if and only if the precision of an agent above a cutoff value $\hat{h}(\kappa,\tau)$, which is the precision that solves $\Omega(y,h)=0$. Moreover, all firms essentially compete for the more precise agents. This thus explains (1) the cutoff rule: if a less precise agent is hired in equilibrium, then a more precise agent must be hired as well; (2) the agent fee increases with precision $\omega_h(h)>0$.

Generally, the assignment function must satisfy Equation (11) and together with the restriction on the market-clearing condition. As discussed in Chiappori et al. (2016), when type spaces are multidimensional, it is generally not possible to derive a closed form solution for the assignment function. We now consider the following *nested* environment that allows for a full characterization. Most of our results below do not require a nested environment but some of the wage distribution implications do. This nested environment is empirically plausible, however, as we explain below. It also offers the reader an illustration of the matching patterns and deeper insight into the assignment solution.

3.3. Full Characterization under Nested Matching

Facing the equilibrium fee $\omega(h)$, $\omega_h(h)$ represents the marginal cost of a particular precision from the view point of firms. From the first order condition, if a firm y chooses to match with an agent $\mu(y)$ in equilibrium, then his marginal benefit of precision must equal the marginal cost. That is, $\Omega_h(y, \mu(y)) = \omega_h(\mu(y))$.

In other words, once we have figured out the value of $\omega_h(h)$, one can then find out the set of firms that are matched to agent h. Note that when firms differ in both indices, two different types of firms may have the same marginal value of h. To facilitate the analysis, define the set of firms y whose marginal benefit of precision h is given by a value of m :

$$\Upsilon(h, m) \equiv \{y \in Y \mid \Omega_h(y, h) = m\}.$$

That is, if the marginal cost of hiring agent h is given by $\omega_h(h)$, then $\Upsilon(h, \omega_h(h))$ is the set of firms that are matched to agent h.

Clearly, $\omega_h(h)$ is an equilibrium object, which depends on the underlying distribution. We now consider the following algorithm that allows us to construct an explicit solution for this multi-dimensional environment. For simplicity, we assume $C = 0$ throughout this section; nevertheless, similar characterization can be obtained for $C > 0$. The basic idea of the equilibrium construction is the following.

First, for each h, we will need to choose some level $m \in R$ that satisfies the following condition:

$$\nu^F\left(\{y \in Y \mid \Omega_h(y,h) \le m\}\right) = G^A(h). \tag{12}$$

That is, by choosing m properly for each agent h, the measure of firms whose marginal benefit of h is lower than m, denoted by $\underline{Y}(h,m) \equiv \{y \in Y \,|\, \Omega_h(y,h) \le m\}$, exactly coincides with the measure of agents below h.

Second, set $\omega_h(h) = m$ and assign agent h to the set of firms in $\Upsilon(h, \omega_h(h))$. That is, $\mu^{-1}(h) = \Upsilon(h, \omega_h(h))$. Figure 1 illustrates the construction above. For each value of h, the set of firms whose marginal value of h is m (i.e. $\underline{Y}(h,m)$) can be represented by a quadratic relationship between κ and τ: $\kappa = m(\tau+h)^2$, as the marginal value is given by $\Omega_h(y,h) = \frac{\kappa}{(\tau+h)^2}$. Firms below this curve thus constitute the set of firms whose marginal benefit of precision is lower than m. Intuitively, if m were the price for precision h, all firms below (above) the line find this type of agent too expensive (cheap).

Choosing m for each h is thus as if we are choosing the price for any given precision. Equation (12) requires that, in equilibrium, the price for any particular precision must be chosen in a way so that the measure of firms that find this type of agent too expensive exactly coincides the measure of agents that are below this agent.

By setting $m = \omega_h(h)$, it is only optimal for firms in the set $\Upsilon(h, \omega_h(h))$ to match with agent h. The set of firms who are matched to h is illustrated in Figure 1, where each line is given by $\kappa = \omega_h(h)(\tau+h)^2$ and $\omega_h(h)$ is chosen so that Equation (12) is satisfied.

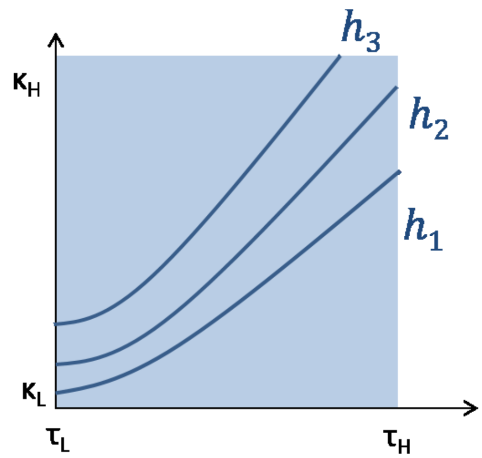

Figure 1: Figure 1 illustrates the set of firms who hires agent with precision h, where $h_3 > h_2 > h_1$. Each line is given by $\kappa = \omega_h(h)(\tau + h)^2$ and $\omega_h(h)$ is chosen so that the measure of firms below the line coincides with $G^A(h)$.

As established in Chiappori et al. (2015), this algorithm works only in the environment where the constructed $\omega_h(h)$ in the above procedure satisfies the following *nested* condition:

$$\underline{Y}(h, \omega_h(h)) \subset \underline{Y}(h', \omega_h(h')) \; \forall h' > h. \tag{13}$$

That is, the construction of fee schedule is such that if a firm finds hiring an agent h is too expensive, then he must find a more precise agent $h' > h$ too expensive as well. In other words, the constructed indifference set in Figure 1 never intersect.

Observe that Condition (13) together with Condition (12) guarantee that (1) the set of firms who found h are too expensive are always matched to firms below agent h and (2) market clears in the sense that the measure of these firms who hire agents below h coincides with the measure of agents below h. As a result, the optimality condition of firms and market-clearing condition are satisfied. Proposition 4 summarizes the characterization.

Proposition 4. *Let $\omega_h(h)$ be the value that solves $\underline{Y}(h, \omega_h(h)) = G^A(h)$. Under nested matching (i.e., if condition (13) holds), the optimal assignment is characterized by $\mu^{-1}(h) = \Upsilon(h, \omega_h(h))$.*

As discussed in Chiappori et al. (2015), whether condition (13) actually holds generally depends on the underlying measure of agents and firms. Chiappori et al. (2015) further provides criteria for this condition to hold and establishes that it is always possible to find such a underlying distribution that satisfies this condition. Note that, in our environment, if the constructed $\omega(h)$ is convex, then condition (13) must hold. That is, if a firm $y' = (\kappa', \tau')$ is such that $\kappa' < \omega_h(h)(\tau' + h)^2$, then $\kappa' < \omega'(h')(\tau' + h')^2$ for any $h' > h$ when $\omega''(h) \geq 0$. Hence, whenever the underlying distribution leads to a convex $\omega(h)$, we know immediately that the above procedure indeed characterizes the stable matching.

Firms with Homogeneous Scales We now look at the case when firms have the same $\kappa(y) = \kappa \ \forall y$ but with different cash flow volatility (σ_θ), different clientele (captured by investor's precision σ_x), and different market noise (σ_u). As discussed below, this is in fact a special case where condition (13) holds for any distribution.

Since the types for firm can be summarized by an one-dimensional transparency index $\tau(y)$, the model then simply collapses to the standard model with one-dimensional heterogeneity. Let $G^F(\tau)$ denote the measure of firms that has a transparency index lower than τ : $G^F(\tau) \equiv \nu^F(\{y \in Y : \tau(y) \leq \tau\})$. Specifically, since a more opaque firm (i.e., with a lower transparency index) have higher marginal value of precision, such a firm must hire a more precise agent.

According to Equation (12), $\omega_h(h)$ is then simply the marginal value of precision h for firms $\tilde{\tau}$, where the measure of firms above firm $\tilde{\tau}$ (i.e., those who found h are too expensive) coincides with the measure of agent below h. That is, $\mu_h^{-1}(h)$ is firm with index $\tilde{\tau}$ such that $1 - G^F(\tilde{\tau}) = G^A(h)$.

Since a more precise agent is matched to a more opaque firm (i.e. $\mu^{-1}(h) > \mu^{-1}(h')$ for

$h' > h$), one can show that condition (13) is always satisfied. Formally,

$$\begin{aligned}\underline{Y}(h, \omega_h(h)) &= \{\tau \in Y \mid \frac{\kappa}{(\tau + h)^2} \leq \frac{\kappa}{(\mu^{-1}(h) + h)^2}\} \\ &= \{\tau \in Y \mid \tau \geq \mu^{-1}(h)\} \\ &\subset \{\tau \in Y \mid \tau \geq \mu^{-1}(h')\} = \underline{Y}(h', \omega_h(h')).\end{aligned}$$

Hence, the wage profile and assignment function can be simply characterized by the following equations:

$$\omega_h(h) = \frac{\kappa}{(\mu^{-1}(h) + h)^2}, \tag{14}$$

$$\mu_h^{-1}(h) = -\frac{dG^A(h)}{dG^F(\mu^{-1}(h))}. \tag{15}$$

As standard in the one-dimensional matching model, Equation (14) shows that the marginal increase in the fee of agent h is his contribution to the surplus $\Omega_h(\mu^{-1}(h), h)$ within the match, given his optimal assignment $\mu^{-1}(h)$. For all agents that are actively matched, Equation (15) can be derived directly from the familiar market-clearing condition:

$$1 - G^F(\mu^{-1}(h)) = G^A(h).$$

As discussed above, condition (12) in two-dimensional case is in fact equivalent to this market-clearing condition in the one-dimensional case.

4. Implications

We now establish empirical implications for cross-sectional variation in coverage and asset returns, as well as agent wages.

4.1. Asset Pricing Test of Endogenous Information Efficiency and Labor Market Sorting

We examine how the cross-sectional variation in coverage affects the expected return across firms. Given any price p, the asset return is given by $\mathbb{E}[\pi_y - p|p]$. The (unconditional) expected asset return for firm y is then given by

$$R(y) \equiv \mathbb{E}\left[\left(\frac{k\theta - \omega(\mu(q(y)))}{1+\psi}\right) - P(\theta, z, u|\mu(y), y)\right] = \frac{\kappa(y)}{\tau(y) + \mu(y)}.$$

If coverage were homogeneous across firms, one would expect that the risk premium must increase with the scale and decrease with transparency of the firm. This is, however, no longer true when the quality of coverage is endogenous. To see this formally, the change in the risk premium respect to transparency and scale of a firm is given by:

$$\begin{aligned}
\frac{\partial R(y)}{\partial \tau} &\propto \left\{1 + \mu_\tau(\kappa(y), \tau(y))\right\}, \\
\frac{\partial R(y)}{\partial \kappa} &\propto \left\{\tau(y) + \mu(y) - \kappa\mu_\kappa(\kappa(y), \tau(y))\right\}.
\end{aligned}$$

Comparison to No-Sorting benchmark As expected, if all firms hire the same agent ($\mu_\tau = 0$ and $\mu_\kappa = 0$) , the risk premium must increase with the scale and decrease with transparency: $\frac{\partial R(y)}{\partial \tau} < 0$ and $\frac{\partial R(y)}{\partial \kappa} > 0$. However, according to Proposition 2, the sorting generates an opposite force: the more opaque and/or the larger firm hires a more precise agent in equilibrium (i.e. $\mu_\tau < 0$ and $\mu_\kappa > 0$), which in turns decreases the risk premium.

To highlight the sorting effect, we further consider a counterfactual environment where agents and firms are matched randomly. That is, the type of agent hired by each firm is randomly drawn from the distribution $G^A(h)$. In this case, a firm y will hire an analyst h as long as the surplus $\Omega(y, h)$ is positive (i.e., when $h \geq \hat{h}(\kappa, \tau)$). Hence, under random matching, the average expected return for firm y conditional on having coverage is then given by

$$R^{RM}(y) \equiv \int_{\hat{h}(\kappa,\tau)}^{h_H} \left(\frac{\kappa(y)}{\tau(y) + h} \right) dG^A(h).$$

Since the less transparent and/or the larger firm are less selective, the average coverage quality in fact decreases with the scale and increase with transparency: $\mu_\kappa^{RM}(\kappa, \tau) < 0$ and $\mu_\tau^{RM}(\kappa, \tau) > 0$. Hence, under random matching, the risk premium always increases with scale and decreases with transparency, which is in sharp contrast to the sorting model. As established in Proposition 5, only the coverage pattern under sorting can overturn the standard opacity and size prediction on expected return.

Proposition 5. *(Coverage Effect and Non-Monotonic Returns)*

The expected returns can be non-monotonic in transparency and scale as a result of the coverage effect under competitive sorting. In contrast, under no-sorting benchmark, the expected returns must increase with scale and decrease with transparency.

Figure 2 below shows the expected return under sorting v.s. random matching, which gives a concrete example where the coverage effect dominates. The coverage for each firm $\mu(\kappa, \tau)$ is shown in Figure 3. As shown in the left panel, the expected returns can be non-monotonic in transparency and scale, which is in sharp contrast to the random matching environment in the right panel. Figure

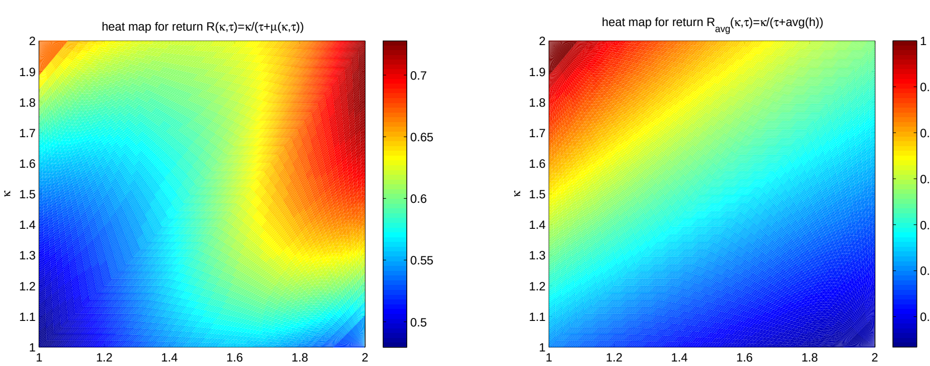

Figure 2: The heat map shows the expected return under sorting $R(\kappa, \tau)$ (left panel) and under random matching $R^{RM}(\kappa, \tau)$ (right panel), respectively. The distribution for agents and firms are assumed to be uniform, where $h \sim U[0, 2]$, $\kappa \sim [1, 2]$, and $\tau \sim U[1, 2]$.

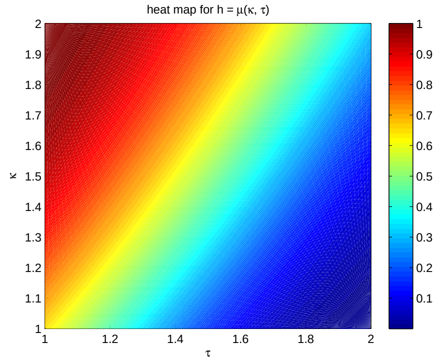

Figure 3: The heat map shows the assignment $\mu(\kappa, \tau)$ when the distribution for agents and firms are assumed to be uniform, where $h \sim U[0, 2]$, $\kappa \sim [1, 2]$, and $\tau \sim U[1, 2]$, and $C = 0$.

The magnitude of the coverage effect The strength of this coverage effect–captured by μ_τ and μ_κ– moreover, endogenously depends on the underlying distribution of firms and agents (i.e., the competition). To see this clearly, first consider the simple case where firms have homogeneous scales. In this case, the change in the precision as a function of firm's transparency is characterized by Equation (15), which shows explicitly how the underlying distribution affects the the magnitude of the coverage effect. Without loss of generality, assume that $G^A(h)$ are uniformly distributed between $[h^L, h^U]$. One can then clearly see that the strength of this coverage effect increases with dispersion of agents' talents. That is, Equation (15) can be rewritten as:

$$\mu_\tau(\tau) = -dG^F(\tau)(h^U - h^L).$$

When firms differ in both indices, the assignment function does not have a simple analytical expression. Nevertheless, same intuition holds. The more dispersed talents simply suggest that it generates a large cross-sectional difference in the coverage, which thus lead to higher slope of μ_τ and μ_κ and to Proposition 6:

Proposition 6. *Assume $G^A(h)$ are uniformly distributed between $\Delta = [h_L, h_H]$. Under nested matching, a higher dispersion of talents leads to a stronger endogenous coverage effect (i.e., a steeper μ_τ and μ_κ)*

Discussion on Empirical Tests From these propositions, we can derive the following simple set of equations to take to the data:

$$\ln R(\kappa,\tau) = \ln\kappa - \ln(\tau + \mu(\kappa,\tau)) \tag{16}$$

$$\frac{\partial \ln R}{\partial \ln \kappa} = 1 - \frac{\kappa\mu_\kappa(\kappa,\tau)}{\tau + \mu(\kappa,\tau)} \tag{17}$$

$$\frac{\partial \ln R}{\partial \ln \tau} = -\tau\left(\frac{1+\mu_\tau(\kappa,\tau)}{\tau + \mu(\kappa,\tau)}\right). \tag{18}$$

Note that if there is no sorting, $\mu_k = \mu_\tau = 0$, we get that the log of expected return of a firm rises with the log scale k and decreases with log transparency τ (this is the risk-sharing force). Empirically, since there is measurement error, we might not get exactly coefficients of one but expect something close. But if there is sorting (μ_k and μ_τ are non-zero), it is easy to see from these equations that these relationships need not be monotonic anymore and can actually be highly non-monotonic as we described in the 3-D plots above, depending on the strength of the sorting or the endogenous information efficiency effect.

One way to test these equations structurally is to using a first-stage assignment function along the lines of Akkus et al. (2013). We can test these asset pricing predictions. This is a test of the endogeneity of assignment and of the value matching function o being driven by a noisy rational-expectations equilibrium. We expect the coefficients in front of log size and transparency to be steeper once we control for the assignment function.

Another way to test our model is to use Proposition 6. Our scarcity of talent prediction can be interpreted as the exogenous arrival of new initial public offering (IPO) or technologies that only some of the existing analysts in the labor market can accurately decipher. As such, it is easy to look to time series variation in IPO waves and test if our predictions are true. All else equal, we expect the coefficients in front of log size and transparency to be flatter when there is an IPO wave.

4.2. Wage Dispersion and Superstar Effect

We now establish empirical implications for agent wages. Clearly, the wage of an agent is rising in his talent (i.e., precision). The slope of the wage profile thus represents the return of talent. As in the CEO literature (see, e.g., Terviö (2008); Gabaix and Landier (2008)), a higher return of talent can thus be interpreted as a stronger superstar effect. We now establish how such an effect depends the underlying parameters, linking our predictions to the rise of superstar effect among securities analysts.

Recall that in equilibrium, firms choose agent h if and only if their marginal value precision of h is equal to marginal cost $\omega_h(h)$. In other words, the marginal increase in the fee of agent h is then his contribution to the surplus within the match: $\omega_h(h) = \Omega_h(\mu^{-1}(h), h)$ where $\mu^{-1}(h)$ is the set of firms that are matched to agent h. Since the sorting predicts that a more precise agent is matched to a firm who has a higher value of precision (i.e., either a larger or more opaque firm), it thus further amplifies the superstar effect.

Comparison to No-Sorting Benchmark To see how sorting amplifies the superstar effect formally, we again compare to the environment under random matching. Notice that, in our model, the wage of an analyst is pinned down competitively by his next best competitor so that it is optimal for each firm to hire their optimal agent. Such a mechanism is absent in the random matching setting. Thus, following the standard random matching setup, we assume that the wage is determined by Nash Bargaining, which yields

$$\omega^{RM}(y,h) = (1-\beta)C + \beta \left\{ \frac{\kappa(y)}{\tau(y)} - \frac{\kappa(y)}{\tau(y)+h} \right\}, \tag{19}$$

where β is the (exogenous) bargain power of workers. For simplicity, assume that the surplus is positive for all pairs, the average wage for agent h under random matching is then given by $\omega^{RM}(h) \equiv \int_Y \omega^{RM}(y,h) d\nu^F$, and its slope is given by:

$$\omega_h^{RM}(h) = \beta \int_Y \left(\frac{\kappa(y)}{(\tau(y)+h)^2} \right) d\nu^F. \tag{20}$$

That is, the slope is given by the average marginal value of precision across all firms. Since the marginal value of precision is decreasing in h, Equation (20) thus implies that that the wage must be concave in talent under random matching: $\omega_{hh}^{RM}(h) < 0$.

Clearly, in both cases, a more precise analyst receives a higher wage due to a higher ability. However, Equation (20) shows how the prediction on the slope differs across two settings: the marginal value for a more precise agent is given by his added value at a larger or more opaque firm in the sorting environment, instead of an average value across all firms under random matching.

This thus suggests that the slope of wage profile for agents at the top is much higher in the sorting framework than the one under random matching. In contrast, the slope of wage profile for agents at the bottom is much lower in the sorting framework, as such agents can only match to firms with lower marginal value. Thus, sorting amplifies the superstar effect in the sense that it increases the return of talent for agents with higher precision but decreases the return of agents with lower precision.

The Steepness of Wage Profile We now establish that, under nested matching, how the return of talent depends the underlying parameters. Recall that the value of $\omega_h(h)$ is given by m that solves Equation (12). That is, among agents that are actively matched,

$$\nu^F \left(\{ y \in Y \mid \frac{\kappa(y)}{(\tau(y)+h)^2} \leq m \} \right) = G^A(h). \tag{21}$$

Equation (21) clearly shows that, given any agent h, when there are more agents below him (i.e., less talents below h), the value of m that clears the market must increase, which thus implies the wage becomes steeper for agent h. In contrast, this force is again absent under random matching. This can be seen clearly from Equation (19), $\omega_h^{RM}(h)$ is independent of the talent distribution $G^A(h)$.

Proposition 7. *Consider two talent distributions $G_1^A(h)$ and $G_2^A(h)$ with the same range and assuming nested matching, if $G_2^A(h)$ is first-order stochastically dominated by $G_1^A(h)$ (i.e., $G_1^A(h) \leq G_B^2(h)$), $\omega_h(h)$ is higher under distribution $G_2^A(h)$. That is, the slope of wage profile $\omega_h(h)$ is higher when talents are scarce.*

We now turn to analyze how the wage depends on the distribution of firms. Consider the case when all firms become larger or more opaque. That is,$\kappa' = \lambda\kappa$ and/or $\tau' = \frac{\tau}{\lambda}$ for all firms for some constant $\lambda \geq 1$.

Proposition 8. *Under nested matching, the slope of wage profile $\omega_h(h)$ increases when all firms have larger scale or become more opaque.*

Intuitively, the marginal value of precision is now higher. One would then expect a higher return of talent. Moreover, since sorting leads to a stronger superstar effect compared to random matching, one can easily see that such an effect is further amplified. That is, for example, when firms are multiplied by a constant λ, the difference in the slope $\omega_h(h) - \omega_h^{RM}(h)$ is then simply scaled up by λ.

Our result thus connects the superstar effect to underlying distribution of firms. During the Internet Bubble Period of 1997-2000, security analysts' pay were highly skewed as the prices of dot-com stocks were noisy and underlying dot-com payoffs were also highly uncertain. Top analysts such as Henry Blodgett and Mary Meeker had compensation in the millions of dollars per year similar to top investment bankers. After the dot-com bubble, it is generally thought that compensation was significantly scaled back because of regulatory reforms such as Reg-FD which curtailed conflicts of interests in the industry.

Our model predicts that, beyond regulatory pressures, this compensation is also fundamentally tied to the uncertainty in the stock market and the scarcity of talent. Indeed, during the Second Internet Boom of the mid-2000s, compensation for analysts have returned with again very skewed pay-offs for the top analysts.

Through the lens of our model, one can thus interpret both the first and the second internet boom are the periods where firms become more opaque. As discussed above, the competitive sorting environment can generate much higher wage dispersion and stronger superstar effect.

4.3. Coverage and Neglected Stocks

Due to the fix cost of information production, one would expect that some firms are being neglected. As shown in Proposition 4, certain firms do not hire any agent in equilibrium $\mu(y) = \emptyset$. In this section, we show that the competition (i.e., sorting) further amplifies this neglected effect.

To measure the neglected effect, we look at the total measure of firms that are being neglected, which is given by $\nu^F(Y_0)$. In other words, $1-\nu^F(Y_0)$ then represents the extensive margin of aggregate market coverage. In order to show how sorting amplifies the neglected effect, we compare our results to a counterfactual environment in which all firms can choose to hire an average quality of agents $\bar{h} = \int h dG^A(h)$. Let $\mu^{ns}(y) \in \{\emptyset, \bar{h}\}$ denote whether a firm has an agent covering it in the no-sorting benchmark. The set of firms who receive zero coverage without sorting is denoted by

$$Y_0^{ns} \equiv \{y \in Y| : \mu^{ns}(y) = \emptyset\}.$$

Proposition 9. *(Results on the Measure of Neglected Firms)*

There exists $\bar{c} > 0$ and $\underline{c} > 0$, sorting leads to a higher measure of neglected stocks: $\nu^F(Y_0) > \nu^F(Y_0^{ns}) = 0$.

The first result establishes how the competition leads to more firms being neglected. The intuition is very simple. Firms that are smaller or more transparent are most selective in terms of their hiring. That is, recall that firm (κ, τ) hires agent if and only if $h \geq \hat{h}(\kappa, \tau)$. Given the surplus function increases with scale and precision but decreases with transparency, smaller and more transparent firms thus have a higher threshold. However, the sorting suggests that these firms can only matched to agent with lower precision, since they could not compete with other firms. As a result, these firms rather being unmatched (i.e., neglected).

The condition on the production cost simply guarantees that the cost is not too high so that it is valuable to hire an average analyst (i.e., $\Omega(\kappa^L, \tau^H, \bar{h}) \geq 0$) but it is also not too low so that the worst agent does not generate enough surplus to compensate the production cost (i.e., $\Omega(\kappa^L, \tau^H, h^L) < 0$). Hence, within this parameter region, the neglected firm effect is purely driven by the sorting.

5. Extension

5.1. One Agent for Multiple Firms

We have so far considered a one-to-one matching environment, that is one agent can only be hired by one firm. The model, however, can be easily extended to the environment where an agent is allowed to work for $N \geq 1$ firms instead. The parameter N thus captures the capacity constraint of the underwriter or analyst.

Intuitively, given that an agent can work for N firms, there are effectively more talent available from the viewpoint of the firms. That is, N effectively changes the talent distribution. Since all firms would prefer to hire a better agent, the market clearing condition under nested matching can then be rewritten as:

$$\nu^F\left(\{y \in Y \mid \Omega_h(y, h) \geq m\}\right) = N(1 - G^A(h)). \tag{22}$$

In words, left hand side represents the measure of firms whose marginal value of h is above m. Those firms must hire an agent above h. Since each agent above h can work for N firms, the measure of these firm must equal $N(1 - G^A(h))$. If all firms hire an agent in equilibrium, then the cutoff type h_N^* must be given by $1 = N(1 - G^A(h_N^*))$. In other words, the equilibrium is thus equivalent to the environment where $\tilde{G}^A(h) = \frac{G^A(h)-G(h_N^*)}{1-G^A(h_N^*)}$ with the support $[h_N^*, h^U]$. One can thus solve for $\omega(h)$ under the distribution $\tilde{G}^A(h)$ and the payoff for all agents above h^* is then given by $N\omega(h)$.

Moreover, observe from Equation 22, it shows that the solution of m (and thus $\omega_h(h)$) must decrease in N. The intuition is simple: as a result of competition, the return for talent becomes lower when there is more talent out there.

5.2. Contracting and Firm Demand for Information Efficiency

Firms in our main model hire agents in order to reduce the cost of capital when going public. Another possible reason for why firms care about market efficiency is to use informative stock prices to alleviate moral hazard, as in Holmström and Tirole (1993). We now consider this monitoring channel as an alternative way that an agent provides value to the firm, and analyze the sorting outcome in this environment.

For simplicity, we assume all investors are risk neutral instead (i.e., shutting down the risk premium channel in the main model) and all firms face a moral hazard problem. The firm is run by a risk neutral manager, who exerts effort at $t = 0$ that affects the final output of the firm. The payoff of firm is given by

$$\theta = v + e,$$

where v is a firm-specific payoff drawn from a Normal distribution with mean $\bar{v}$ and variance σ^2, and e is the component of earning determined by managerial effort. Hence, $\bar{\theta} = \bar{v} + \bar{e}$, where $\bar{e}$ denote the manager's equilibrium effort.

Managers are paid at $t = 1$. In the spirit of Holmström and Tirole (1993), we assume that the compensation of the manager (denoted by I) is based on a fixed salary w_0 and can be contingent on the stock price at $t = 1$. That is,

$$I = w_0 + \alpha P,$$

where (w_0, α) are chosen optimally by the insider, and P is the realized market price. For simplicity, we assume that insider pays the manager and agent out of his own pocket at $t = 1$, and thus the price of stock only depends on the final payoff. We further assume all investors are risk neutral. The price is then simply the expected payoff. That is, the price expression can be obtained by setting $\bar{\theta} = \bar{v} + \bar{e}$, where $\bar{e}$ denote the manager's equilibrium effort:

$$\begin{aligned} P(\theta, z, \tilde{u}|h, y) &= \mathbb{E}\left[\left(\frac{k\theta}{1+\psi}\right)|x_i = \hat{x}, z, \hat{x}\right] \\ &= \frac{k}{1+\psi}\left(\frac{\tau_\theta \bar{\theta} + \tau_m(\theta + \sigma_x \tilde{u}) + hz}{\tau_\theta + \tau_m + h}\right), \end{aligned} \tag{23}$$

where $\tau_m \equiv \sigma_x^{-2} + (\sigma_x \sigma_u)^{-2}$ and $\tau_\theta = \sigma_\theta^{-2}$. Given the compensation and effort level e, the utility of risk-neutral manager is then given by

$$W(w_0, \alpha, e) = w_0 + \alpha P(v + e, z, \tilde{u}|h, \sigma) - kc(e),$$

where $c(e) = \frac{1}{2}e^2$ is the manager's effort cost.

Facing the agency problem of the manager, the insider is now choosing both the analyst h as well as the managerial contract (w_0, α) optimally at $t = 0$. The payoff of firm when hiring an agent h is given by

$$J(y, h) = \max_{\{w_0, \alpha, e\}} \mathbb{E}[\frac{k(v+e)\psi}{1+\psi} - (w_0 + \alpha P(v + e, z, \tilde{u}|h, y))] \tag{24}$$

subject to manager's (1) the individual compatibility (IC) constraint: $e \in \arg\max_e \mathbb{E}[W(w_0, \alpha, e)]$ and (2) individual rationality (IR) constraint: $\mathbb{E}[W(w_0, \alpha, e)] \geq 0$.

Similar our baseline model, the firm chooses an agent h optimally so as to solve the following problem: $U^*(y) = \max_h\{J(y,h) - \omega(h)\}$. Hence, the sorting pattern is determined by the complementarity of the payoff function $J(y,h)$. Since the manager is risk-neutral, the firm does not have incentive to pay the fixed fee w_0 as it does not provide incentives for a manager to exert effort. That is, $w_0 = 0$ for the optimal contract.

One can show that the solution to problem (24) can be simply reduced to the following:

$$J(y,h) = k \max_e \left\{ \left(\frac{\psi}{1+\psi} - \phi(y,h)e \right) (\bar{v} + e) \right\},$$

where

$$\phi(y,h) \equiv \left(\frac{\tau_\theta}{\tau_m + h} + 1 \right).$$

The function $\phi(y,h)$ can be interpreted as the cost of incentivizing effort. Such cost is lower if the market information is more precise (i.e., higher h and τ_m) and when the firm is more volatile (i.e., lower τ_θ). Intuitively, when firms are less volatile, investors put a higher weight on the prior $\bar{\theta} = (\bar{v} + \bar{e})$ in equilibrium (as shown in Equation (23)) and thus the price responds less to the true state and thus harder to incentivize the manager.

Clearly, an more precise agent leads to a higher payoff for firms, as it is easier to incentivize the manager. Formally,

$$J_h(y,h) = -ke^*(y,h)(\bar{v} + e^*(y,h)) \frac{\partial \phi(y,h)}{\partial h} > 0.$$

Similar as before, the firm with a larger scale k or issue a higher share $\frac{1}{1+\psi}$ have a higher marginal value of precision. Furthermore, under certain parameters, one can show that $J_{\tau_m h} < 0$ and $J_{\tau_\theta h} > 0$. Intuitively, since the market condition τ_m and the precision h are substitutes, the marginal value for h is thus lower for higher τ_m. On the other hand, when firms are less volatile (i.e., high τ_θ), any additional precision is more valuable, as it reduces the cost more significantly (i.e., $\frac{\partial^2 \phi}{\partial h \partial \tau_\theta} < 0$). The detailed derivation is left in Appendix A.2

6. Conclusion

We provide a theory of endogenous stock market efficiency emphasizing the sorting or assignment of firms of heterogeneous characteristics in a variety of labor markets where agents generate a public signal about the firm's fundamental value, i.e. coverage, with heterogeneous precision. For instance, in a noisy rational-expectations stock market equilibrium when a firm is going public, we show that there is positive assortative matching. In general, firms with higher noise or scale index benefit more and pay more for accurate coverage as it leads to greater price efficiency and less risk discount.

It turns out that this assortative matching effect has wide-reaching implications for thinking about a host of issues in stock markets, including coverage patterns, wage distributions and even well known asset pricing anomalies including the neglected firm effect. The model generates a number of new testable implications for future empirical work. The distinguishing feature of our model is that coverage, compensation and stock pricing are all endogenously determined across firms. As such, there are a large number of sharp predictions that have not yet been tested.

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A. Appendix

A.1. Omitted Proofs

A.1.1. Proof for Lemma 1

Proof.

$$Var(\theta|x,z,p) = \sigma^2 - \begin{bmatrix} \sigma^2 \\ \sigma^2 \\ \frac{k}{1+\psi}\frac{(h+\tau_m)\sigma^2}{(h+\tau_m+\tau_\theta)} \end{bmatrix}^T \Sigma_{dd}^{-1} \begin{bmatrix} \sigma^2 \\ \sigma^2 \\ \frac{k}{1+\psi}\frac{(h+\tau_m)\sigma^2}{(h+\tau_m+\tau_\theta)} \end{bmatrix} = \frac{1}{(\tau_m + h + \frac{1}{\sigma^2})},$$

where $\Sigma_{dd} \equiv \begin{bmatrix} \sigma^2+\sigma_x^2 & \sigma^2 & \frac{k}{1+\psi}\frac{(h+\tau_m)\sigma^2}{(h+\tau_m+\tau_\theta)} \\ \sigma^2 & \sigma^2+\sigma_h^2 & \frac{k}{1+\psi}\frac{(h+\tau_m)\sigma^2+h\sigma_h^2}{(h+\tau_m+\tau_\theta)} \\ \frac{k}{1+\psi}\frac{(h+\tau_m)\sigma^2}{(h+\tau_m+\tau_\theta)} & \frac{k}{1+\psi}\frac{(h+\tau_m)\sigma^2+h\sigma_h^2}{(h+\tau_m+\tau_\theta)} & \left(\frac{k}{1+\psi}\right)^2 \frac{(\tau_\theta+\tau_m)\tau_\theta^{-1}\theta+h+\tau_m^2(\sigma_x^2\sigma_u^2)}{(\tau_\theta+h+\tau_m)^2} \end{bmatrix}.$

Hence, the expression for the risk premium is given by

$$\gamma_I k^2 \frac{Var(\theta|x,z,p)}{2(1+\psi)^2} = \frac{\gamma_I k^2}{2(1+\psi)^2(\frac{1}{\sigma_x^2} + \frac{1}{\sigma_x^2\sigma_u^2} + \frac{1}{\sigma_\theta^2} + h)} = \frac{\kappa(y)}{\tau(y)+h}$$

The risk premium strictly decreases with precision h and increases with all noise variables σ_j as $\frac{\partial\tau(y)}{\partial\sigma_j} < 0$, where $j \in \{\theta, x, u\}$, and factors that increases the scale $\kappa(y)$: k,γ_I, and $\frac{1}{1+\psi}$.

□

A.1.2. Proof for Proposition 1

Proof. Given that the set of agents and firms' types are bounded and the surplus is continuous, according to Theorem 4 inChiappori et al. (2015), there exists a stable match in the labor market and the stable match must maximize the aggregate surplus.

Moreover, when the information cost is small enough (for example, when $C = 0$) so that the matching surplus generated by all agents across all firms are positive (i.e.,$\Omega(y,h) > 0$

$\forall\, y,\ h$), the surplus function satisfies the following condition: for any given y, $D_y\Omega(y,h) \neq D_y\Omega(y,h')$ for $h \neq h'$, where D_y denote the derivatives respect to the vector y. That is, for a fixed firm y, different h implies different marginal surplus for y. Hence, Theorem 6 in Chiappori et al. (2015) guarantees that there is an unique stable matching outcome, and the assignment function $\mu(y)$ is pure. That is, a firm y chooses an unique agent h and never randomizes between different agents h.

We now consider the case for large information costs so that the information value does not worth the production cost for *some* matching pairs. Clearly, among the subset of agents that are actively matched, the surplus must be positive (i.e.,$\Omega(y,\mu(y)) > 0$), hence, for the same logic, the matching solution for these firms and agents must be unique and pure.

We now turn to the set of firms that are being neglected. Note that, for any given $\omega(h)$, firms' optimization problem can be rewritten as:

$$\begin{aligned} U^*(\kappa,\tau) &= \max\left\{\max_{h\in[h^*,h^U]}\{-\frac{\kappa}{\tau+h} - \omega(h)\},\ -\frac{\kappa}{\tau}\right\}. \\ &= \max\left\{-\frac{\kappa}{\tau+h^*(\kappa,\tau)} - \omega(h^*(\kappa,\tau)),\ -\frac{\kappa}{\tau}\right\}, \end{aligned}$$

where $h^*(\kappa,\tau) = \arg\max_{h\in[h^*,h^U]}\{-\frac{\kappa}{\tau+h} - \omega(h)\}$. Hence, this shows that a firm either hires an agent $h^*(\kappa,\tau)$ or remain in autarky $\mu(y) = \{\emptyset\}$. Thus, $\mu(y)$ is pure and unique for all firms, including those who choose to be unmatched. Furthermore, given $\omega(h)$, the set of firms who are being neglected can be characterized as

$$Y_0 = \{y \in Y |\ \frac{\kappa}{\tau} - \frac{\kappa}{\tau+h^*(\kappa,\tau)} - \omega(h^*(\kappa,\tau)) < 0\}. \tag{25}$$

Furthermore, since $\Omega(y,h^*) < 0\,\forall y \in Y_0$ and the surplus increases with precision, the surplus between neglected firms and agents below the cutoff types must be negative as well (i.e., $\Omega(y,h) < 0 \forall h < h^*$). This thus shows that it is indeed optimal for those firms to remain unmatched $\mu(y) = \{\emptyset\}$.

$\square$

A.1.3. Proof for Proposition 2

Proof. Given that $U_h(\kappa, \tau, h)$ increases with scale κ and decreases with transparency τ, by Milgrom and Segal (2002), $\mu(\kappa, \tau)$, the solution to Equation (1), must increase with κ and decrease with τ. Since $U_h(\kappa, \tau, h) = \Omega_h(\kappa, \tau, h)$, this is equivalent to looking at the complementarity of the surplus function, as standard in the matching model. $\square$

A.1.4. Proof for Proposition 3

Proof. (1) From Equation (8), the payoff for firms with index (κ, τ) yields:

$$U^*(\kappa, \tau) \equiv U(\kappa, \tau, \emptyset) + \max_{\tilde{h} \in H \cup \{\emptyset\}} \left\{ \Omega(\kappa, \tau, \tilde{h}) - (\omega(\tilde{h}) - C) \right\}$$

By the Envelope Theorem $U^*_\kappa(\kappa, \tau) = U_\kappa(\kappa, \tau, \emptyset) + \Omega_\kappa(\kappa, \tau, \mu(\kappa, \tau))$ and $U^*_\tau(\kappa, \tau) = U_\tau(\kappa, \tau, \emptyset) + \Omega_\tau(\kappa, \tau, \mu(\kappa, \tau))$. The cross partial derivatives yields:

$$U^*_{\kappa\tau} = U_{\kappa\tau}(\kappa, \tau, \emptyset) + \Omega_{\kappa\tau}(\kappa, \tau, \mu) + \Omega_{\kappa h}(\kappa, \tau, \mu) \frac{\partial \mu}{\partial \tau}$$
$$U^*_{\tau\kappa} = U_{\kappa\tau}(\kappa, \tau, \emptyset) + \Omega_{\kappa\tau}(\kappa, \tau, \mu) + \Omega_{\tau h}(\kappa, \tau, \mu) \frac{\partial \mu}{\partial \kappa}$$

Assuming double differentiability of U^*, we thus have $\Omega_{\kappa h}(\kappa, \tau, \mu) \frac{\partial \mu}{\partial \tau} = \Omega_{\tau h}(\kappa, \tau, \mu) \frac{\partial \mu}{\partial \kappa}$, which gives the expression in Equation (11).

(2) We now show that an agent is actively matched iff $h \geq h^*$. The cutoff role can be proved by contradiction. Suppose there exists $h_2 > h_1$ such that agent h_2 is not matched but agent h_1 is actively matched with firm y'. That is, $\mu^{-1}(h_2) = \emptyset$ and $\mu^{-1}(h_1) = y' \in Y$. Given that agent $h_1$is actively matched with firm y', the surplus must be postive: $\Omega(y', h_1) > 0$. Since the surplus increases with precision, the surplus between firm y' and agent h_2 must

be positive as well. Hence, there is a profitable deviation for firm y' by offering offer wage $\omega(h_1)$ to agent h_2. Both firm y' and worker h_2 are better off in this case: firm y' is strictly better off since he receives better coverage by paying the same wage; while the agent h_2 now receive $\omega(h_1) - C$, which must be (weakly) larger than zero (i.e., the aurtarky value). That is, the pair-wise stability condition is violated.

Furthermore, the utility of agent be expressed as

$$\omega(h) - C = \max_{y \in Y \cup \{\emptyset\}} \{\Omega(y, h) - (U^*(y) - U(y, \emptyset))\}$$

Hence, by the Envelope Theorem, it shows $\omega_h(h) = \Omega_h(\mu^{-1}(h), h)) > 0$ for $\mu^{-1}(h) \neq \{\emptyset\}$.

Condition (a) $\nu^F(Y_0) = G^A(h^*)$ is the market cleaning condition. Since both firms and agents have the same measure, the measure of neglected firms and unemployed agents must be the same. We now establish condition (b): if $h > h_L$ (i.e., some agents are unmatched), $\Omega(\mu^{-1}(h^*), h^*) = 0$. We prove this by contradiction. Suppose that in equilibrium, $\Omega(\mu^{-1}(h^*), h^*) > 0$ and let y^*denote a firm that is matched to agent h^*. Then there exist a firm y_ϵ such that $\kappa(y_\epsilon) = \kappa(y^*) - \epsilon$ and $\tau(y_\epsilon) = \tau(y^*) + \epsilon$ such that $\Omega(y_\epsilon, h^* - \epsilon') > 0$ for some $\epsilon, \epsilon' > 0$. By Proposition 2, since this firm has a smaller scale and higher transparency, he must hire a lower agent than firm y^*. However, since h^* is the cutoff type by definition, this firm must be inactive $\mu(y_\epsilon) = \{\emptyset\}$. However, there then exists a profitable deviation: firm y_ϵ can hire agent $h^* - \epsilon'$ by paying a wage C. In this case, an agent is indifferent, while the firm is strictly better off:

$$U(y_\epsilon, h^* - \epsilon') = \frac{\kappa(y_\epsilon)}{\tau(y_\epsilon) + h^* - \epsilon'} - C > U(y_\epsilon, \emptyset) = \frac{\kappa(y_\epsilon)}{\tau(y_\epsilon)}$$

The inequality follows from the fact that $\Omega(y_\epsilon, h^* - \epsilon') > 0$.

Lastly, since $\Omega(\mu^{-1}(h^*), h^*) = 0$, then the cutoff type must earn zero (i.e., $\omega(h^*) = C$). Hence, for firms being neglected, it must be the case that they rather choose not to hire agent h^*. That is,

$$U(y,h^*) = \frac{\kappa(y)}{\tau(y)+h^*} - C < U(y,\emptyset) = \frac{\kappa(y)}{\tau(y)}$$

This thus establishes that $\Omega(y,h^*) < 0$ for for $y \in Y_0$.

$\square$

A.1.5. Proof for Proposition 4

Proof. By construction, Equation (12) and(13) guarantees the market clearing is satisfied: the measure of agents below h is the same as the measure of firms that hire agents whose precision is lower than h. We now examine firms' optimality condition.

Recall that$\Upsilon(h,\omega_h(h))$ is the set of firms that are matched to agent h,

$$\Upsilon(h,\omega_h(h)) \equiv \{y \in Y \mid \Omega_h(y,h) = \omega_h(h)\}.$$

Since $\Omega_h(y,h) = \frac{\kappa}{(\tau+\mu(y))^2} = U_h(y,\mu(y))$, it thus shows that FOC of firms is satisfied as

$$U_h(y,\mu(y)) = \omega_h(\mu(y)).$$

We now show that $\mu(y)$ is indeed the *maximum* of $U(y,h)$. Condition (13) suggests that, for any $h' > \mu(y)$, $y \in Y(\mu(y),\omega_h(\mu(y)) \subset Y(h',\omega_h(h'))$. That is, the marginal cost of a hiring a better agent h' is too high:

$$U_h(y,h') < \omega_h(h)\,\forall h' > \mu(y).$$

That is, $U(y,h)$ decreases with h for $h > \mu(y)$. Similarly, hiring an agent with lower precision is too cheap:

$$U_h(y,h') > \omega_h(h)\,\forall h' < \mu(y).$$

And thus, $U(y,h)$ increases with h for $h < \mu(y)$. Hence, the constructed $\mu(y)$ solves the firm's optimization problem.

$\square$

A.1.6. Proof for Proposition 5 and 6

Proof. Let $\hat{h}(\kappa,\tau)$ denote the solution such that $\Omega(\kappa,\tau,h) = 0$. Since the surplus increases with scale κ and decreases with transparency τ, it must be the case that $\hat{h}_\tau(\kappa,\tau) \geq 0$ and $\hat{h}_\kappa(\kappa,\tau) \leq 0$. By Leibniz's rule, below shows that the expected return under random matching must increase in size and decrease in transparency.

$$R_\tau^{RM}(\kappa,\tau) = -\int_{\hat{h}(\kappa,\tau)}^{h_H} \frac{\kappa}{(\tau+h)^2} dG^A(h) - \frac{\kappa}{\tau+\hat{h}(\kappa,\tau)}\hat{h}_\tau(\kappa,\tau) < 0$$
$$R_\kappa^{RM}(\kappa,\tau) = \int_{\hat{h}(\kappa,\tau)}^{h_H} \frac{1}{(\tau+h)} dG^A(h) - \frac{\kappa}{\tau+\hat{h}(\kappa,\tau)}\hat{h}_\kappa(\kappa,\tau) > 0.$$

The coverage effect dominates whenever

$$\frac{\partial R(y)}{\partial \tau} \propto \{1 + \mu_\tau(\kappa(y),\tau(y))\} > 0$$
$$\frac{\partial R(y)}{\partial \kappa} \propto \{\tau(y) + \mu(y) - \kappa\mu_\kappa(\kappa(y),\tau(y))\} < 0$$

In one-dimension case, we have $\mu_\tau(\tau) = -dG^F(\tau)(h^U - h^L)$ and similarly, $\mu_\kappa(\kappa) = dG^F(\kappa)(h^U - h^L)$. Hence, the larger the dispersion, the more likely that the coverage effect dominates. For two-dimensional case, under nested matching, define

$$M(\mu,m) \equiv \int \left(\int_{\kappa_L}^{m(\tilde{\tau}+\mu)^2} g^F(\tilde{\kappa},\tilde{\tau}) d\tilde{\kappa} \right) d\tilde{\tau},$$

where $g^F(\tilde{\kappa},\tilde{\tau})$ denotes the density function. Clearly, $M_\mu > 0$ and $M_m > 0$. Moreover, Condition (12) can then be rewritten as

$$M(\mu, \frac{\kappa}{(\tau+\mu)^2}) - \left(\frac{\mu - h_L}{\Delta}\right) = 0.$$

We thus have,

$$\frac{\partial \mu}{\partial \tau} = -\left(\frac{-2M_m \frac{\kappa}{(\tau+\mu)^3}}{\frac{dM}{d\mu} - \frac{1}{\Delta}} \right), \ \frac{\partial \mu}{\partial \kappa} = -\left(\frac{M_m \frac{1}{(\tau+\mu)^2}}{\frac{dM}{d\mu} - \frac{1}{\Delta}} \right)$$

Hence, the larger the dispersion Δ, the steeper $\frac{\partial \mu}{\partial \tau}$ and $\frac{\partial \mu}{\partial \kappa}$.

□

A.1.7. Proof for Proposition 7

Proof. For any given h, recall that $\omega_h(h)$ is the value of m that solves

$$\nu^F \left(\{ y \in Y \mid \Omega_h(y,h) \leq m \} \right) = G^A(h).$$

Since RHS increases with m (recall that $M_m > 0$), thus, a higher $G^A(h)$ implies that a higher m. Hence, if $G_1^A(h) \leq G_2^A(h)$, then $\omega_h(h)$ is higher under $G_2^A(h)$.

□

A.1.8. Proof for Proposition 8

Proof. One can easily see that, if all firms scales are multiplied by some constant λ. Then, the value of m that solves the equation below is simply also multiplied λ :

$$\nu^F \left(\{ y \in Y \mid \frac{\kappa(y)}{\tau(y)+h} \leq m \} \right) = \nu^F \left(\{ y \in Y \mid \frac{\lambda \kappa(y)}{\tau(y)+h} \leq \lambda m \} \right) = G^A(h).$$

When all firms become more opaque, the marginal value increase for all h, hence, the value of m that solves the above equation must goes up as well. Hence, a higher $\omega_h(h)\ \forall h$.

□

A.1.9. Proof for Proposition 9

Proof. Assume that there exists y^L such that $\kappa(y^L) \leq \kappa(y)$ and $\tau(y^L) \geq \tau(y)$ $\forall y$. That is, intuitively, y^L is the firm that has the lowest marginal value of precision since he has smallest scale and highest transparency. Let $\underline{c}$ be the value of the information cost such that $\Omega(\kappa(y^L), \tau(y^L), h^L) = 0$ and let $\bar{c}$ be the value such that $\Omega(\kappa(y^L), \tau(y^L), \bar{h}) = 0$. Hence, for any $C \in (\underline{c}, \bar{c})$, the surplus between y^L and the lowest agent h^L is negative: $\Omega(\kappa(y^L), \tau(y^L), h^L) < 0$. By continuity, there exists a set of firm $y = (\kappa, \tau)$ that are in the neighbourhoods of $(\kappa(y^L), \tau(y^L))$ such that $\Omega(\kappa, \tau, h^*(\kappa, \tau)) < 0$. Hence, $\nu^F(Y_0) > 0$. On the other hand, those firms will hire agent $\bar{h}$ since $\Omega(\kappa, \tau, \bar{h}) > \Omega(\kappa(y^L), \tau(y^L), \bar{h}) = 0$. Hence, for any $C \in (\underline{c}, \bar{c})$, $\nu^F\left(Y_0^{ns}\right) = 0 < \nu^F(Y_0)$.

□

A.2. Derivation for Contracting Channel

Firms design the contract $\{\alpha, e\}$ to solve the following constrained optimization problem:

$$J(y, h) = \max_{\{\alpha, e\}} \left\{ \frac{k}{1+\psi}(\psi - \alpha)(\bar{v} + e) \right\}$$

subject to

$$c'(e) = e = \frac{\alpha}{1+\psi} \frac{\tau_m + h}{\tau_\theta + \tau_m + h} \tag{26}$$

and

$$\frac{\alpha}{1+\psi}(\bar{v} + e) - c(e) \geq 0. \tag{27}$$

Plugging in Equation (26) into the participation constrain, one can see that PC is always satisfied, assuming $\bar{v} > 0$:

$$\begin{aligned}
\frac{\alpha}{1+\psi}(\bar{v} + e) - c(e) &= e\left(\left(\frac{\tau_m + h}{\tau_\theta + \tau_m + h}\right)(\bar{v} + e) - \frac{c(e)}{e}\right) \\
&\propto \left\{ \bar{v} + e - \left(\frac{\tau_\theta + \tau_m + h}{\tau_m + h}\right)\frac{1}{2}e \right\} > 0
\end{aligned}$$

Thus, the problem can be rewritten as

$$J(y,h) = k \max_{e} \left\{ \left(\frac{\psi}{1+\psi} - \phi(y,h)e \right) (\bar{v}+e) \right\},$$

where $\phi(y,h) \equiv \frac{(\tau_\theta+\tau_m+h)}{\tau_m+h} = \left(\frac{\tau_\theta}{\tau_m+h}+1\right)$. The parameter ϕ thus captures of cost of incentivizing effort. FOC of e yields:

$$\frac{\psi}{1+\psi} - 2\phi(y,h)e - \phi(y,h)\bar{v} = 0.$$

One can show that SOC is also satisfied, and thus the solution is given by $e^*(\phi,\psi) = \frac{\frac{\psi}{1+\psi}-\phi\bar{v}}{2\phi}$. Furthermore, given that $J_h = J_\phi \frac{\partial \phi}{\partial h}$, and $J_\phi = -e^*(\bar{v}+e^*) = -\left(\left(\frac{\frac{\psi}{1+\psi}}{2\phi}\right)^2 - \left(\frac{\bar{v}}{2}\right)^2\right) < 0$ and $J_{\phi\phi} = -(\bar{v}+2e^*)\frac{\partial e}{\partial \phi} = (\bar{v}+2e^*)\left(\frac{\psi}{1+\psi}\right)\frac{1}{\phi^2} = \left(\frac{\psi}{1+\psi}\right)^2 \frac{1}{\phi^3}$, for $x \in \{\tau_\theta, \tau_m\}$,

$$\frac{\partial^2 J}{\partial h \partial x} = \left(\frac{\psi}{1+\psi}\right)^2 \frac{1}{\phi^3} \left(\frac{\partial \phi}{\partial h}\frac{\partial \phi}{\partial x} - \frac{\phi}{4}\left(1 - \frac{\bar{v}^2}{\left(\frac{\psi}{1+\psi}\right)^2}\right)\frac{\partial^2 \phi}{\partial x \partial h}\right).$$

Hence, $\frac{\partial^2 J}{\partial h \partial \tau_m} \propto \left(\frac{\tau_\theta^2}{(\tau_m+h)^4} - \frac{\left(\frac{\tau_\theta}{\tau_m+h}+1\right)}{4}\left(1-\frac{\bar{v}^2}{\left(\frac{\psi}{1+\psi}\right)^2}\right)\frac{2\tau_\theta}{(\tau_m+h)^3}\right) \propto \left(1 - \frac{\left(1+\frac{\tau_m+h}{\tau_\theta}\right)}{2}\left(1-\frac{\bar{v}^2}{\left(\frac{\psi}{1+\psi}\right)^2}\right)\right)$ and $\frac{\partial^2 J}{\partial h \partial \tau_\theta} \propto \left(\frac{-\tau_\theta}{(\tau_m+h)^3} + \frac{\left(\frac{\tau_\theta}{\tau_m+h}+1\right)}{4}\left(1-\frac{\bar{v}^2}{\left(\frac{\psi}{1+\psi}\right)^2}\right)\frac{2}{(\tau_m+h)^2}\right) \propto \left(-1 + \frac{\left(1+\frac{\tau_m+h}{\tau_\theta}\right)}{2}\left(1-\frac{\bar{v}^2}{\left(\frac{\psi}{1+\psi}\right)^2}\right)\right)$. For both cases, when $\tau_m + h$ is large enough compared to $\tau_{\theta,}$ the second effect dominates: $\frac{\partial^2 J}{\partial h \partial \tau_m} < 0$ and $\frac{\partial^2 J}{\partial h \partial \tau_\theta} > 0$. Lastly, $J_\psi = \frac{(\bar{v}+e^*)}{(1+\psi)^2} > 0$ and

$$J_{\psi h} = -2\frac{\left(\frac{\bar{v}}{2}+\frac{\frac{\psi}{1+\psi}}{2\phi}\right)}{(1+\psi)^3} + \frac{1}{(1+\psi)^4}\frac{1}{2\phi} \propto \left(-2\left((1+\psi)\phi\bar{v}+\psi\right)+1\right) < 0$$

for large enough $\bar{v}$. Hence, firms who issue a higher proportion of shares $\frac{1}{1+\psi}$ has a higher marginal value of precision.

A.3. Setting with Multiple assets

Our results can be easily extended to the environment where the same (deep-pocket) investor i can invest multiple assets in different markets and submit the demand function $d_i^j(p^j) \in \{0,1\}$ for firm j. For simplicity, we assume all investors have the same precision and risk aversion. Specifically, assume that each investor has access to N market and his private signal for each market (i.e., firm) is given by $x_i^j = \theta^j + \sigma_x \epsilon_i$, And $I_i = (x_i^j, z^j, p^j)_{j=1,...N}$ that contain the information for each market j. The investor's wealth is the given by $W_i = \sum_{j\in N} d_i^j(\frac{k\theta^j}{1+\psi} - p^j)$. Hence, an investor then solves the following maximization problem:

$$\begin{aligned}
U^i &= \max_{d^j} E_{I_i}\left[\sum_{j\in N} d^j(\theta^j - p^j)\right] - \frac{\gamma_I}{2} Var(\sum_{j\in N}(d^j)^2\left(\frac{k\theta^j}{1+\psi} - p^j\right)|I_i). \\
&= \Sigma_{j\in N}\left\{\max_{d^j} E_{I_i}\left[d^j(\theta^j - p^j)\right] - \frac{\gamma_I k^2 (d^j)^2}{2(1+\psi)^2} Var(\theta^j|I_i).\right\}
\end{aligned}$$

Given that all assets and signals are uncorrelated, one can see that our result remains unchanged since the optimization for each asset can be solved separately.