Intervention Policy in a Dynamic Environment: Coordination and Learning*

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Abstract

We model a dynamic economy with strategic complementarity among investors and endogenous government interventions that mitigate coordination failures. We establish equilibrium existence and uniqueness, and show that one intervention can affect subsequent interventions through altering public information structures. Our results suggest that optimal policy often emphasize initial interventions because coordination outcomes tend to correlate. Neglecting informational externalities of initial interventions results in over- or under-interventions depending on intervention costs. Moreover, saving smaller funds before saving the big ones under certain circumstances costs less and generates greater informational benefits. Our paper is applicable to intervention programs such as those during the 2008 financial crisis.

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1 Introduction

Coordination failures are prevalent and socially costly. Thus, effective interventions may ameliorate such damaging outcomes. For example, financial systems, especially short-term credit markets, are vulnerable to liquidity shocks and runs by investors. The 2008 financial crisis witnessed a series of runs on both financial and non-financial institutions. In response, governments and central banks around the globe employed an array of policy actions over time. Given the novelty, the scale, and the intertwined nature of such interventions, it is natural to study how interventions dynamically relate to each other.

More broadly, how should a government formulate intervention policy in a dynamic economy with strategic complementarity? How does intervention in one institution or market affect subsequent interventions in other institutions or markets? This paper tackles these questions by modeling the government as a large player in sequential global games. We find that many interventions not only improve welfare within the current episode of events, but also dynamically affect future coordination among agents. Consequently, optimal dynamic policy features an emphasis on initial intervention as opposed to a subsequent intervention. A myopic government may over- or under-intervene, depending on the intervention cost structure. Moreover, the optimal policy entails saving smaller funds first. The results hold both when the government faces a hard resource constraint, as well as when interventions incur welfare costs. The insights apply to many situations with strategic complementarity and multiple interventions. Examples include interventions in currency attacks, stock market crises, bank runs, cross-sector industrialization, and technology subsidy programs.

Specifically, we introduce the model in the context of runs on money market mutual funds (MMMFs) and the commercial paper market.[1] In a two-period economy, a group of atomistic investors in each period choose whether to run or stay with a fund. Running

[1]Runs on MMMFs in September 2008 and subsequently on commercial paper (primarily in financial commercial paper as opposed to ABCPs, according to Kacperczyk and Schnabl (2010)) were both triggered by investors' interpretation of Lehman's failure as a revelation of the credit risk and systemic illiquidity of commercial paper. The initial strong intervention with unlimited insurance to all MMMF depositors and the Asset-Backed Commercial Paper Money Market Mutual Fund Liquidity Facilities (AMLF) quickly stopped the runs on MMMFs and arguably made investors react more positively to later interventions in the commercial paper market such as the Commercial Paper Funding Facility (CPFF).

guarantees a higher payoff if the fund fails, whereas staying pays more if the fund survives. The fund survives if and only if the total measure of investors who choose to stay is above a fundamental threshold θ_t–interpreted as an unhedgeable system-wide illiquidity shock or as a measure of the persistent quality of the underlying investment, and is ex-ante positively correlated across the two periods. Following the global games framework, θ of the period is unobservable and each investor receives a noisy signal. Prior literature has established that in static settings, there exists a unique equilibrium in which the fund survives as long as the true θ is below a threshold θ^*, and each investor stays if and only if his private signal is below a certain threshold x^*.

Policy responses in a crisis are often about managing expectations, yet the formation of expectations in this context is understudied. Our model tackles this issue by incorporating both the contemporaneous coordination with intervention, and dynamic learning from intervention outcomes. Specifically, we model intervention as direct liquidity injections to the funds.[2] The equilibrium θ^* increases strictly with the size of government's intervention: a greater liquidity injection makes the fund more likely to survive. This is the contemporaneous effect of intervention on coordinating investors. Therefore, in a static economy, a benevolent government should always increase intervention up to the point at which the contemporaneous marginal benefit equals the marginal cost.

In a dynamic setting, government intervention in the first period alters investors' beliefs in the second period. Indeed, agents' prior beliefs on θ_2 are truncated, since whether or not a run occurs during the first period is public information. When the fund has survived in the first period, agents learn that $\theta_1 < \theta_1^*$, and with positive probability that their belief on θ_2 shifts downwards, making coordination easier. The opposite holds if the fund has failed in the first period. Therefore, the optimal policy has to consider the initial intervention's informational effect and trade off two competing forces: intervening more to increase the

[2]Many crisis interventions are indeed direct liquidity injections. For example, Duygan-Bump, Parkinson, Rosengren, Suarez, and Willen (2013) discuss how AMLF and CPFF were essentially liquidity injections that alleviated funds' pressure to meet redemptions without suffering fire sales. Other examples include the Economic Stimulus Act of 2008 that reduced firms' tax obligations directly, or TARP which intended to improve the liquidity of hard-to-value assets through secondary market mechanisms. We discuss how the intuition and insights carry over to other forms of intervention in Section 5.5.

likelihood of good news (truncating from above), and intervening less for more favorable conditional updates (lower θ^*).

We establish results on the existence and uniqueness of equilibrium and study the optimal policy of a benevolent government. Under fairly general conditions, optimal policy emphasizes initial intervention: the scale of intervention in the first period always exceeds that in the second period. The intuition for this result relies on the dynamic coordination effect of initial intervention. If the initial intervention is successful, the government needs less intervention to induce investors to stay in the second period, as investors now are more optimistic about the fundamental. If, however, the initial intervention fails, it becomes more costly to intervene in the second period because investors believe a fund experiencing a run is more likely to fail. Therefore, optimal intervention endogenously induces an equilibrium in which the intervention outcomes are more correlated across two periods (*endogenous correlation effect*). Initial intervention is then more important as it increases the probability of survival in both periods. As such, consideration of the dynamic informational effect leads to greater initial intervention.

However, strong initial intervention also has an informational cost, and thus its magnitude must be tempered. When a large intervention is combined with a fund's survival, investors may infer that the outcome is due to the intervention itself and not on strong fundamentals. Conversely, if the fund fails despite a large initial intervention, investors become even more pessimistic about the market's fundamentals. This *conditional inference effect* on beliefs harms investors' welfare. It dominates when the costs to intervene across the two periods differ drastically, so much so that survival in the first period does not guarantee survival in the second period (when second period cost is too high relative to the first and private signals are relevant for the marginal investor), nor does failure lead to failure (when second period cost is so low that one can intervene more despite the negative update from first period's failure). Then the more the government considers the dynamic informational effect, the more it shades intervention. This intuition also applies to countries and regions sharing common fundamentals where one country's investors learn from another country's intervention outcome. In that sense, the dynamic effect of coordination becomes an informational externality and a global social planner such as the European Union may have a role in mitigating inefficient interventions in member countries or states.

Finally, when the government endogenously decides the order of interventions in funds of different sizes, intervention outcomes tend to be correlated and the larger fund is "too big to save first", for two reasons. First, it generally costs less to intervene in the smaller fund first to induce the same updating on the fundamentals. Second, the larger fund benefits more in the subsequent coordination game with reduced uncertainty. This complements studies on institutions deemed "too big to fail" in that though bigger funds could be systemically important, they are generally not the best targets in an initial intervention.

This paper contributes to our understanding of how interventions shape the information environment during a crisis, hence is useful for studying and assessing policies that aim to avoid inefficient equilibria. In particular, we highlight the role of government intervention on the information structure: not only does it affect the probability of good news versus bad news, but it also affects the informativeness of news.[3] It thus complements existing work on government interventions during financial crises. Strategic complementary in financial markets is well-recognized in prior literature such as Diamond and Dybvig (1983), and more recently by empirical studies including Chen, Goldstein, and Jiang (2010) and Hertzberg, Liberti, and Paravisini (2011). Closely related are Acharya and Thakor (2014) which considers how liquidation decisions by informed creditors of one bank signal systematic shocks to other creditors and create contagions, and [cite Angeletos et al on signaling]. Three other related papers are Bebchuk and Goldstein (2011), which examines the effectiveness of various forms (rather than the extent) of exogenous government policies in avoiding self-fulfilling credit market freezes, Sakovics and Steiner (2012) which analyzes who matters in coordination failures and how to set intervention targets, and Choi (2014) which shows

[3]Bernanke and Geithner spoke of the financial crisis as a bank run and emphasized the need to combat a financial crisis with the "use of overwhelming force to quell panics" (p. 397 in Geithner's Memoir), a tactic of "shock and awe" that often connotes signaling by the government. However, governments may not have superior information and political constraints are real, at least at the onset of the crisis (Swagel (2015)). Therefore, information signaling alone cannot fully justify the conventional wisdom that emphasizes strong initial intervention. Similarly, not only do the financial networks matter for intervention policies, information structure also plays a crucial role in coordination. Although it is challenging to isolate the informational aspect from systemic connectedness, it was an important element of both the Lehman Brothers episode and the Eurozone bank bailouts in 2010 and 2011.

the importance of bolstering the strong in order to prevent contagion. Like these studies that focus on one particular aspect of intervention, we demonstrate how information structure design should play an important role in formulating intervention policies, and should be considered together with previously discussed factors. Different from them, this paper concerns the dynamic interaction of endogenous interventions under general cost functions.

This paper is also related to global games and equilibrium selection (Carlsson and Van Damme, 1993; Morris and Shin, 1998) in dynamic settings (Frankel and Pauzner, 2000; Angeletos, Hellwig, and Pavan, 2007), with government as a large player (Corsetti, Dasgupta, Morris, and Shin, 2004). Our paper builds on the insights of prior studies and explicitly model the government as a large player that endogenously selects coordination equilibrium through both static and dynamic channels. Two closely related papers are Angeletos, Hellwig, and Pavan (2007) and Goldstein and Huang (2016). The first entails endogenous learning from outcomes of previous coordination games while the second has policy-makers endogenously design information to affect the likelihood of failure. This paper complements them by incorporating both endogenous intervention and learning from previous coordination outcomes.

Specifically, Angeletos, Hellwig, and Pavan (2007) extend global games to a dynamic setup where agents take actions over multiple periods and can learn about the fundamental over time. The authors point out that multiplicity originates from the interaction between endogenous learning based on regime survivals and exogenous learning induced by private news arrivals. We demonstrate that unique equilibrium can be obtained either when private signal is not precise and does not get infinitely precise over time, or the government intervenes in a way that private information and public information do not interact. Goldstein and Huang (2016) use a Bayesian persuasion framework to endogenize the truncation of beliefs introduced in Angeletos, Hellwig, and Pavan (2007). They focus on a one-shot intervention where the government pre-commits to a costless regime change policy in order to increase the probability of the survival of the status-quo. Our paper adds by underscoring the role of intervention costs in determining whether an initial coordination success always leads to a subsequent success, as well as characterizing the tradeoffs present in designing the information structure in closed-forms. Most distinctly, we derive novel implications on how multiple endogenous interventions relate to one another in coordinating agents' behaviors in a broad class of situations.

The rest of the paper is organized as follows: Section 2 lays out the basic framework and establishes a static benchmark. Section 3 characterizes the equilibrium in dynamic settings. Section 4 solves for the optimal policy and presents its implications. Section 5 extends the model and discusses its robustness. Section 6 concludes.

2 Model

This section introduces the baseline model and specializes to a representative intervention form: government directly infusing liquidity to funds subject to runs in each period.[4] We start by analyzing a static model as our benchmark in Section 2.1 and move to the dynamic setup in Section 2.2. For analytical solutions, we follow Goldstein and Pauzner (2005) and Bouvard, Chaigneau, and Motta (2015) by assuming uniform distribution for signals.[5]

2.1 Static Benchmark

Model setup

A fund has a continuum of investors indexed by i and normalized to unit measure. Each has one unit capital invested in the fund, and simultaneously choose between two actions: stay ($a_i = 1$) or withdraw ($a_i = 0$). For the remaining analysis, we interpret withdrawals as "runs" on the fund, and staying can be interpreted as rolling over short-term debts. The net payoff from running on the fund and investing the proceeds in an alternative vehicle (such as treasury bill) is always equal to r, whereas the payoff to each investor from staying is R if the fund survives the run ($s = S$), and is 0 if the fund fails ($s = F$). Let $R > r$. Therefore, an investor finds it optimal to stay if and only if she expects the probability of survival exceeds the cost of illiquidity defined as $c \equiv \frac{r}{R}$. Table 1 (left panel) shows the net payoff of each action under different states and actions. In the right panel of Table 1, we

[4]Section 5.5 discusses how this setup captures other intervention forms.

[5]Section 5 discusses Normally distributed signals and the role of bounded support on equilibrium multiplicity.

normalize the payoff matrix by subtracting r and scaling by $\frac{1}{R}$. For notational convenience, we use the normalized net payoffs for the remainder of the paper.

Table 1: Net Payoffs and Normalized Net Payoffs

	Stay	Run
Survive	R	r
Fail	0	r

	Stay	Run
Survive	$1-c$	0
Fail	$-c$	0

Agents' decisions are complements: the fund is more likely to survive as more agents choose to stay. Specifically, the fund survives if and only if

$$A + m \geq \theta \tag{1}$$

where A represents total measure of agents who choose to stay, $m \in [0, \bar{m}]$ is the size of the government's liquidity injection to the fund and is bounded above by a constant $\bar{m} > 0$.[6] $\theta \in \mathbb{R}$ summarizes the underlying fundamental. θ can be interpreted as the system-wide illiquidity shock. For the fund to survive, the remaining liquidity $A + m$ must dominate the liquidity shock θ. The government cares about social welfare comprised of investors' total payoff less the intervention cost $k(m)$, which is weakly increasing and quasiconvex. $k(m)$ captures the legal political capital expended, tax distortion, or moral hazard associated with the intervention policy.[7]

Apparently, coordination is needed when both θ and m are commonly known by all agents. Indeed, if $\theta - m \in (0, 1)$, two equilibria coexist. In one equilibrium, all investors stay and in the other one, all investors run. Global games resolve this issue of multiple equilibria through introducing incomplete information. We apply the same technique to assume that agents each observe a noisy private signal of θ. In particular, agent i observes,

$$x_i = \theta + \varepsilon_i \tag{2}$$

where the noise $\varepsilon_i \sim Unif[-\delta, \delta]$ is i.i.d. across investors. For simplicity, we assume that the

[6]Similar to Goldstein and Huang (2016), we assume the government publicly commits to the intervention.
[7]In Section 5.6, we microfound the cost using moral hazard and cash diversion.

prior distribution of θ is uniform on $[-B, B]$ where $B \gg \max\{\delta, m\}$.[8] We also assume the government does not know the realization of the fundamental θ and does not have private signal about it. Essentially we are assuming that institutional investors are typically more informed about the fundamental state of the market, which is consistent with Diamond and Kashyap (2015) (financial institutions know more about the fundamental illiquidity), Bond and Goldstein (2015) (government relies on market prices to learn fundamentals), and Goldstein and Pauzner (2005) and Sakovics and Steiner (2012) (governments' inferior knowledge on utilizing or allocating resources leads to tax and subsidy distortions).

Partial Equilibrium Given Intervention

We restrict the equilibrium set to symmetric Perfect Bayesian equilibria (PBE) in monotone strategies: all agents' strategies are symmetric and monotonic *w.r.t.* x and m. Specifically, agent i's strategy $a_i(x_i, m)$ is non-increasing in x_i and non-decreasing in m.

Since $B \gg \max\{\delta, m\}$, it is *w.l.o.g.* to further restrict the equilibrium set to threshold equilibria denoted by (θ^*, x^*). The fund survives if and only if $\theta \leq \theta^*$ and each investor stays if and only if his signal $x \leq x^*$. Lemma 1 below summarizes the equilibrium outcome in the static game.

Lemma 1

In the static game, there exists a unique symmetric PBE in monotone strategies (θ^*, x^*), where

$$\begin{cases} \theta^* & = 1 + m - c \\ x^* & = 1 + m - c + \delta(1 - 2c). \end{cases} \tag{3}$$

Each investor's strategy follows $a_i = \mathbb{1}\{x_i \leq x^*\}$. The fund's outcome $s = S$ if $\theta \leq \theta^*$ and $s = F$ otherwise.

According to Lemma 1, the fund survives if and only if $\theta \leq \theta^*$. Each agent stays if and only if his private signal $x_i \leq x^*$. Note that θ^* increases in m and so is x^*. In other

[8] Uninformative prior corresponds to $B \to \infty$.

words, the fund is more likely to survive and investors are more inclined to stay if the size of government intervention increases. This is the static effect of government intervention on coordination. In the next section, we show that government intervention has have dynamic coordination effects.

Welfare and Optimal Intervention

Let V_i be investor i's net payoff and $W = E\left[\int_0^1 V_i di\right]$. Then investors' welfare is

$$W = \frac{1}{2B}\left[\underbrace{\int_{-B}^{\theta^*}(1-c)\,d\theta}_{\text{fundamental}} - \underbrace{\int_{x^*-\delta}^{\theta^*}(1-c)\left(1-\frac{x^*-(\theta-\delta)}{2\delta}\right)d\theta}_{\text{overrun}} - \underbrace{\int_{\theta^*}^{x^*+\delta} c\frac{x^*-(\theta-\delta)}{2\delta}d\theta}_{\text{underrun}}\right]$$

Let us interpret the above payoff function. $\frac{1}{2B}$ is the probability density of the uniform distribution. The terms inside the square bracket split into three terms. The first term, *fundamental*, equals to the net payoff if all agents stay when the fund survives. The second term, *overrun*, represents the net payoff loss due to the fact that some agents choose to run when the fund survives. The last term, *underrun*, is the net loss from agents who choose to stay when the fund fails.

Simple calculation suggests that total welfare is

$$W - k(m) = \frac{(1-c)\left[1+B-c\left(1+\delta\right)+m\right]}{2B} - k(m)$$

The marginal benefit of m on W is a constant, $\frac{(1-c)}{2B}$. This result comes from the fact that an increase in m also raises θ^* linearly, making the fund more likely to survive. $\frac{(1-c)}{2B}$ is the net payoff from stay $1-c$, scaled by the probability density $\frac{1}{2B}$. Therefore, intervention improves coordination. Because the intervention cost lies in a compact set, there always exists an optimal intervention:

$$m^* = \sup\left\{m \in [0,\bar{m}] : \lim_{\epsilon\to 0}\frac{k(m+\epsilon)-k(m)}{\epsilon} \leq \frac{1-c}{2B}\right\}$$

For example, if $k(m) = \frac{1}{2}zm^2$, then $m^* = \min\left\{\frac{1-c}{2zB}, 1\right\}$.

2.2 Dynamic Economy

We now extend the static model to a two-period dynamic economy. In each period, there is a continuum of agents of measure 1, where they choose whether or not to stay or run. The government chooses intervention policies for each period, m_1 and m_2. Agents in period two observe whether or not there was a run in period one. To focus on Bayesian learning from public intervention outcomes, we assume that the mass of agents in each period are non-overlapping, in that they do not observe the private signals in other periods. The government's cost of intervention now is $K(m_1, m_2)$, which is weakly increasing and quasiconvex in both arguments, and satisfies $K(0,0) = 0$, where $\{m_1, m_2\} \in I$, and $I \subset R^2$ indicates a convex set of feasible interventions.[9] For ease of exposition, we assume for the remaining of the paper that the cost is defined on $\mathcal{C}[0, \bar{m}_1] \times \mathcal{C}[0, \bar{m}_2]$, where $\bar{m}_1$ and $\bar{m}_1$ are finite constants.

Importantly, the two periods are linked: (a) the fundamentals $\{\theta_t\}_{t=1,2}$ are identical across two periods;[10] (b) agents in period 2 also observe the public outcome of whether investment has succeeded in the first period, indicated by $s_1 = S$ or $s_1 = F$; (c) there could potentially be interaction between the costs of intervention across the two periods. For the rest of the analysis, we will omit the subscript of θ.

The government chooses interventions to maximize investors' welfare subtracting the intervention cost $K(m_1, m_2)$. In each period, agents simultaneously choose between stay with the fund ($a_t = 1$) or run ($a_t = 0$). The period-by-period normalized payoff structure is identical to the static game: running ($a_t = 0$) always guarantees 0 payoff whereas staying ($a_t = 1$) pays off $1 - c$ in survival and $-c$ in failure. Agents' decisions within the same period are complements: investment in period t succeeds if and only if

$$A_t + m_t \geq \theta, \tag{4}$$

[9] Notice the cost function nests the static benchmark in that we can set $K(m, 0) = k(m)$.

[10] We made this assumption for simplicity. More generally, we need the fundamentals to be highly correlated to have non-trivial learning, which is natural as the periods are relatively short in the setup.

where A_t is the total measure of investors who choose to invest, m_t denotes the size of liquidity injected by the government. Again, θ represents the fundamental. Similar to the interpretation of the static game, θ represents the market-wide illiquidity that affects both periods.

The timing within each period goes as follows. First, government announces m_t. Second, each investor i in period t receives a private signal $x_{it} = \theta + \varepsilon_{it}$ about the fundamental where $\varepsilon_{it} \sim Unif\,[-\delta, \delta]$. Lastly, investors choose whether to stay and their payoffs realize. The setup is dynamic in the sense that period 1's outcome is revealed before investors take actions in period 2.

In the baseline, we study a problem in which the government maximizes welfare by solving

$$\max_{m_1, m_2} E\left[\int_0^1 V_{1i} di + \int_0^1 V_{2i} di\right] - K(m_1, m_2). \tag{5}$$

Given $\mathcal{I}$ is compact, an optimal policy exists in general, which exhibits interesting features. We solve this problem in two steps. The next section takes government interventions as given, and derive the coordination equilibrium. Section 4 then examines a benevolent government's optimal policy design.

3 Coordination Equilibrium

Our equilibrium concept is symmetric Perfect Bayesian equilibria (PBE) in monotone strategies, taking the intervention as given. Specifically, all agents' strategies are symmetric and monotonic $w.r.t.$ x_t and m_t: agent i's strategy in period t, $a_{it}(x_{it})$, is non-increasing in x_{it} and non-decreasing in m_t, $t = 1, 2$.

3.1 Equilibrium and Social Welfare in Period 1

The analysis in period 1 is identical to the static game. We relabel the unique threshold equilibrium with time subscripts $(\theta_1^*, x_1^*) = (1 + m_1 - c, 1 + m_1 - c + \delta(1 - 2c))$. The fate of the fund is $s_1 = S$ if $\theta \leq \theta_1^*$ and $s_1 = F$ otherwise. Agents adopt a threshold strategy

$a_{i1} = \mathbb{1}\{x_{i1} \le x_1^*\}$.

The social welfare in period 1 is also identical to the static economy,

$$W_1 - K(m_1, 0) = \frac{(1-c)\left[1 + B - c(1+\delta) + m_1\right]}{2B} - K(m_1, 0).$$

3.2 Equilibrium in Period 2

In period 2, the outcome of period 1 intervention (henceforth referred to as public news) is publicly known. As a result, beliefs on θ are truncated either from above or from below.

Unless specified otherwise, we assume for the remainder of the paper $2\delta > 1$ and $\frac{1}{2\delta+1} < c < \frac{2\delta}{1+2\delta}$. These assumptions correspond to the fact that during crisis uncertainty is high and cost of illiquidity is in an intermediate range where agents do not overwhelmingly prefer staying or running. These assumptions ensure a unique threshold equilibrium in period 2 for both $s_1 = S$ and $s_1 = F$, and for all values that m_1 and m_2 take on.[11]

3.2.1 Survival News

If the fund in period 1 has survived ($s_1 = S$), the prior belief on θ is bounded above at θ_1^*: $\theta \sim Unif\,[-B, \theta_1^*]$. In this case, it is possible that investors stay regardless of their signals. In fact, this is the equilibrium if and only if $m_2 > m_1 - c$. In this equilibrium, the (hypothetical) threshold x_2^* satisfies $x_2^* \ge \theta_1^* + \delta$, which is always above all agents' realized signals. We call such equilibrium *Equilibrium with Dynamic Coordination* because the government's intervention in the first period has a dominant effect on improving coordination among investors in the second period:

Lemma 2 (Subgame Equilibrium with Dynamic Coordination)
If $s_1 = S$, $(\theta_2^*, x_2^*) = (\infty, \infty)$ consists an equilibrium if and only if $m_2 > m_1 - c$.[12]

Next, we turn to threshold equilibria with $\theta_2^* < \theta_1^*$ so that the fate of the fund in period 2 still has uncertainty. Likewise, any threshold equilibrium (θ_2^*, x_2^*) necessarily satisfies

[11] Similarly, some global games literature takes the limit of $\delta \to 0$ to ensure uniqueness. However, it would not be realistic in our setting and leads to multiplicity, as discussed in Section 5.1.

[12] Now that $\theta \le \theta_1^*$ is common knowledge, any equilibrium with $(\theta_2^* > \theta_1^*, x_2^* > \theta_1^* + \delta)$ is equivalent to one with $(\theta_2^*, x_2^*) = (\infty, \infty)$.

two conditions. First, when $\theta = \theta_2^*$, $A_2 + m_2 = \Pr\left(x_2 < x_2^* \middle| \theta = \theta_2^*\right) + m_2 = \theta_2^*$. Second, the marginal agent who receives the signal x_2^* is just indifferent between stay and run, $\Pr\left(\theta \leq \theta_2^* \middle| x_2 = x_2^*, \theta \in [-B, \theta_1^*]\right) = c$.

We analyze the equilibrium in two cases, depending on whether the marginal investor finds the public news "useful". Ignoring the public news, the marginal investor's posterior belief on θ is simply $\Pr\left(\theta \middle| x_2 = x_2^*\right) \sim Unif\left[x_2^* - \delta, x_2^* + \delta\right]$. If $x_2^* + \delta < \theta_1^*$, then $\Pr\left(\theta \leq \theta_2^* \middle| x_2 = x_2^*, \theta \in [-B, \theta_1^*]\right) = \Pr\left(\theta \leq \theta_2^* \middle| x_2 = x_2^*\right)$ and he finds the public news useless. We call such equilibrium *Equilibrium without Dynamic Coordination* because intervention in the first period has no effect on coordination in the second period.

Lemma 3 (Subgame Equilibrium without Dynamic Coordination)
If $s_1 = S$ and $m_2 < m_1 - 2\delta(1-c)$, there exists an equilibrium with thresholds (θ_2^*, x_2^*) where

$$\begin{cases} \theta_2^* = 1 + m_2 - c \\ x_2^* = 1 + m_2 - c + \delta(1 - 2c) \end{cases} \tag{6}$$

Notice that when public news is useless, the dynamic game is simply a repeated version of the static game. However, if $x_2^* + \delta > \theta_1^*$, $\Pr\left(\theta \leq \theta_2^* \middle| x_2 = x_2^*, \theta \in [-B, \theta_1^*]\right) \neq \Pr\left(\theta \leq \theta_2^* \middle| x_2 = x_2^*\right)$, and the marginal investor finds the public news useful. We call this equilibrium *Equilibrium with Partial Dynamic Coordination* as government intervention in the first period has partially improved the coordination among investors in the second period. Equilibrium without dynamic coordination is an artifact of bounded noise in the private signals. For unbounded noise, there is always partial dynamic coordination.

Lemma 4 (Subgame Equilibrium with Partial Dynamic Coordination)
If $s_1 = S$ and $m_1 - 2\delta(1-c) < m_2 < m_1 - c$, there exists an equilibrium with thresholds

$$\begin{cases} \theta_2^* = 1 + m_2 - c + \frac{c[m_2 - m_1 + 2\delta(1-c)]}{2\delta - c(1+2\delta)} \\ x_2^* = 1 + m_2 - c + \delta(1 - 2c) + \frac{c(1+2\delta)[m_2 - m_1 + 2\delta(1-c)]}{2\delta - c(1+2\delta)}. \end{cases} \tag{7}$$

Combining Lemma 2, 3 and 4, Proposition 1 describes the equilibrium outcome given any (m_1, m_2) and $s_1 = S$.

Proposition 1 (Equilibrium in period 2 when $s_1 = S$)

1. *If $m_2 < m_1 - 2\delta(1-c)$, the unique equilibrium is the Subgame Equilibrium without Dynamic Coordination.*

2. *If $m_1 - 2\delta(1-c) < m_2 < m_1 - c$, the unique equilibrium is the Subgame Equilibrium with Partial Dynamic Coordination.*

3. *If $m_1 - c < m_2$, the unique equilibrium is the Subgame Equilibrium with Dynamic Coordination.*

3.2.2 Failure News

If the fund in period 1 has failed ($s_1 = F$), the prior belief on θ is bounded below at θ_1^*: $\theta \sim Unif[\theta_1^*, B]$. Proposition 2 below summarizes the equilibrium outcome in this case. The detailed derivation can be found in Appendix B. The only difference is that, in the Subgame Equilibrium with Dynamic Coordination, investors choose to run regardless of their signals.

Proposition 2 (Equilibrium in period 2 when $s_1 = F$)

1. *If $m_2 < m_1 + 1 - c$, the unique equilibrium is the Subgame Equilibrium with Dynamic Coordination.*

2. *If $m_1 + 1 - c < m_2 < m_1 + 2c\delta$, the unique equilibrium is the Subgame Equilibrium with Partial Dynamic Coordination.*

3. *If $m_1 + 2c\delta < m_2$, the unique equilibrium is the Subgame Equilibrium without Dynamic Coordination.*

3.2.3 Investors' Welfare and Dynamic Coordination

Let $W_{2S} = E\left[\int_0^1 V_{2i}di \middle| s_1 = S\right]$ be the total expected payoff in period 2 conditional on $s_1 = S$. Also, let $W_{2F} = E\left[\int_0^1 V_{2i}di \middle| s_1 = F\right]$ be the total expected payoff in period 2 when $s_1 = F$. Applying results from Proposition 1 and 2, we are able to obtain W_{2S} and W_{2F} for different values of m_1 and m_2. Corollary 1 below shows the results.

Corollary 1 (Investors' Welfare in Period 2)

1. *Conditional on $s_1 = S$*

   (a) *If* $m_2 < m_1 - 2\delta(1-c)$, $W_{2S}^{nc} = \frac{(1-c)[1+B-c(1+\delta)+m_2]}{B+\theta_1^*}$.

   (b) *If* $m_1 - 2\delta(1-c) < m_2 < m_1 - c$,
   $W_{2S}^{pc} = \frac{1-c}{\theta_1^*+B}\left[\theta_1^* + B + \frac{\delta c(c-m_1+m_2)^2+2\delta(c-m_1+m_2)[2\delta-c(1+2\delta)]}{[2\delta-c(1+2\delta)]^2}\right]$.

   (c) $m_2 > m_1 - c$, $W_{2S}^{c} = (1-c)$.

2. *Conditional on $s_1 = F$*

   (a) *If* $m_2 < m_1 + 1 - c$, $W_{2F}^{c} = 0$.

   (b) *If* $m_1 + 1 - c < m_2 < m_1 + 2c\delta$, $W_{2F}^{pc} = \frac{1-c}{B-\theta_1^*}\frac{c\delta(-1+c-m_1+m_2)^2}{(-1+c+2c\delta)^2}$.

   (c) *If* $m_2 > m_1 + 2c\delta$, $W_{2F}^{nc} = \frac{1-c}{B-\theta_1^*}(m_2 - m_1 - c\delta)$.

The superscripts of W_{2S} and W_{2F} refer to equilibrium types. nc, pc and c respectively stand for equilibrium without dynamic coordination, with partial coordination, and with coordination.

The left panel of Figure 1 plots W_{2S} against m_2, including the welfare function in all three different types of equilibria. Given m_1, W_{2S} is continuous, increasing in m_2, and convex in the region that involves partial dynamic coordination. Unlike in the first period, the marginal effect of m_2 on W_{2S} is no longer a constant. Initially, W_{2S} increases linearly in m_2 , in which case the intervention in the first period has no dynamic coordination effect. When $m_1 - 2\delta + 2c\delta < m_2 < m_1 - c$, the marginal effect of m_2 is increasing, due to the dynamic coordination effect of period 1 intervention. When $m_2 > m_1 - c$, the dynamic coordination effect is maximized and all agents' decisions are well coordinated towards an equilibrium without any run. In that case, further increasing m_2 has no effect.

Similarly, the right panel of Figure 1 plots W_{2F} against m_2, including the welfare function in all three different types of equilibria. Given m_1, W_{2F} is continuous, increasing in m_2, and convex when the equilibrium involves partial dynamic coordination. The effect of m_2 on W_{2F} is not a constant either. When $m_2 < m_1 + 1 - c$, the failed intervention in period 1 makes all agents very pessimistic. A slight increase in m_2 does not change people's belief and therefore, the marginal effect of m_2 on W_{2F} is zero. When $m_1 + 1 - c < m_2 < m_1 + 2c\delta$, the marginal effect of m_2 on W_{2F} is positive and increasing. Finally, when $m_2 > m_1 + 2c\delta$, the dynamic effect is zero and W_{2F} increases linearly in m_2.

Clearly, m_1 affects both W_{2S} and W_{2F} by altering θ_1^* and thus the resulting informational structure. Since W_{2S} and W_{2F} are piecewise in m_1 and thus not everywhere differentiable, we define left-hand derivative of W_{2S} and W_{2F} *w.r.t.* m_1 as the conditional inference effect, as Figure 2 illustrates.

Proposition 3 (Conditional Inference Effect)
Investors' welfare W_{2S} and W_{2F} decrease in m_1 conditional on s_1 and m_2.

However, the overall effect of m_1 on W_2 is non-monotone. Indeed, the probability of $s_1 = S$ increases linearly with m_1. Figure 3 shows this non-monotonic property by plotting $E[W_2] = \Pr(s_1 = S) W_{2S} + (1 - \Pr(s_1 = S)) W_{2F}$ against m_1, taking m_2 as given. Obviously, the overall effect attains its highest level at $m_1 = m_2 + c$, and starts to decline afterwards. Intuitively, conditional on s_1, a larger m_1 leads to a more negative update on θ_1^* because investors attribute fund's survival more to the large intervention. However, they become really pessimistic about the fundamental when the fund fails.

3.2.4 Equilibrium Comparison

It is interesting to compare thresholds across different types of equilibria. When $s_1 = S$ and $m_2 \in (m_1 - 2\delta(1-c), m_1 - c)$, both x_2^* and θ_2^* in the Equilibrium with Partial Dynamic Coordination exceed their counterparts in the Equilibrium without Dynamic Coordination.

Indeed this shows the dynamic coordination effect of government intervention. When the marginal agent finds the public news useful and realizes that the expected threshold level suggested by his signal alone is too high, he behaves more aggressively by choosing a higher threshold. As a result, θ_2^* is also higher and the fund is more likely to survive. Similarly, when $s_1 = F$, both x_2^* and θ_2^* are lower in the Equilibrium with Partial Dynamic Coordination.

4 Dynamic Intervention and Optimal Policy

Given the costs and constraints of intervention, how should the government allocate resources across two periods, and how does the information structure channel affect the scale and sequence of interventions? This section discusses three key implications on the optimal policy: emphasis on initial intervention, under- and over-intervention by myopic governments, and the "too big to save first" phenomenon. To illustrate the main tradeoffs in explicit closed-forms, we consider first the case in which the government faces a budget constraint, $m_1 + m_2 = M$, before showing the results and intuition hold more generally. This section also focuses on the case of **committed intervention**, which corresponds to choosing m_2 before s_1 is realized. Section 5.4 examines the case of **contingent intervention**, which corresponds to choosing m_2 after s_1 is realized. The former describes situations in which the government has to roll out policy programs before knowing the outcome of previous interventions.

4.1 Emphasis on Initial Intervention

First suppose the government has a total budget M that can be costless used across the two periods. In other words, $K(m_1, m_2) = \frac{\mathbb{I}_{\{m_1+m_2>M\}}}{1-\mathbb{I}_{\{m_1+m_2>M\}}}$. A benevolent government solves the following problem,

$$\max_{m_1,m_2} E\left[\int_0^1 V_{1i}di + \int_0^1 V_{2i}di\right] \tag{8}$$

$$s.t. m_1 + m_2 = M. \tag{9}$$

We have shown earlier the information channel that arises from dynamic learning: while W_1 increases linearly with m_1, W_2 is non-monotonic in m_1 and increases with m_2 in a non-linear manner. Since the government also faces a hard budget constraint $m_1 + m_2 = M$, an increase in m_1 necessarily crowds out $m_2$through the budget channel. When the government optimally allocates resources in two periods, it needs to consider both.

Figure 4 plots a typical social welfare W as m_1 varies. The pattern delivered by the figure holds for all parameters. (a) W is always flat for either small or large m_1. (b) W always attains its maximum at $m_1 = \frac{M+c}{2}$. Therefore, whenever M is large, the government should invest $m_1^* = \frac{M+c}{2}$. Lemma 6 in the Appendix summarizes the aggregate social welfare and the net benefit of initial intervention.

Therefore, the optimal intervention plan also depends on M, the total resources available to the government. When M is small ($M < \frac{M+c}{2}$), it is optimal to set $m_1 = M$. In contrast, when M gets larger, increasing m_1 may actually decrease the total payoff and the optimal $m_1 = \frac{M+c}{2}$.

Proposition 4 below characterizes the optimal intervention under different Ms.

Proposition 4 (Optimal Intervention)
The optimal intervention under budget constraint M is $\min\left(\frac{c+M}{2}, M\right)$*. Optimal intervention always emphasizes initial intervention: $m_1^* > m_2^*$.*

At the optimal intervention level, the fund in period 2 survives if and only if the fund in period 1 survives. The endogenous correlation effect completely dominates. The intuition for $m_1^* > m_2^*$ is then apparant. To see this, suppose the government equally splits the budget and invests $\frac{M}{2}$ in each period. Two periods' intervention outcomes are completely correlated. Knowing this, government always has incentives to kill two birds with one stone – increasing

m_1 to increases the survival probability in period-1 (and period-2) fund.[13]

One may question whether the results are driven by the fact that imposing the budget constraint takes away the flexibility of m_2 after m_1 is chosen. By specifying a very general $K(m_1, m_2)$, we show that emphasizing initial intervention is a very robust phenomenon. For committed interventions, the government separately chooses m_1 and m_2 all before s_1 is realized. This corresponds to situations where governments have to setup funding facilities or provide subsidies even before the outcomes of earlier interventions are known yet.

Proposition 5 (Emphasis on Early Intervention)
If intervention cost satisfies $K(m_1, m_2) > K\left(\frac{1}{2}[m_1 + m_2], \frac{1}{2}[m_1 + m_2 - 2c]\right)$*, optimal policy strictly emphasizes initial intervention, i.e.,* $m_1^* > m_2^*$*.*

The condition in the proposition is satisfied by many plausible cost functions, such as one that is separable and symmetric in m_1 and m_2, or one that emphasizes consistency in the sense that $K(m_1, m_2)$ only depends on $m_1 + m_2$ and $|m_1 - m_2|$ and is increasing in $|m_1 - m_2|$. It is worth pointing out that this proposition is not about comparing the absolute sizes of the interventions. Given that we have normalized the total capital in the economy to one in both periods, we are really talking about a notion of intervention relative to the market size. Therefore, the conclusion could apply more broadly, especially when the coordination games are scale-invariant, i.e., the normalized intervention, cost, and participation scale proportionally with the market size.[14]

4.2 Information Externality and Myopic Intervention

This section examines the situations where the decision-maker for the initial intervention does not fully take into consideration the informational impact on subsequent interventions.

[13] The ratio $\frac{m_2^*}{m_1^*}$ is weakly increasing in M and weakly decreasing in c, thus the tilt towards initial intervention is most significant when the government has a small budget or the illiquidity cost is high.

[14] Indeed, the eligible ABCPs for AMLF constitutes less than half of the commercial paper markets, thus the scale of AMLF (\$150 billion in the first 10 days relative to the magnitude of the run-\$172 billion plummet from the \$3.45-trillion MMF sector) is higher than CPFF (\$144 billion usage in the first week, relative to a reduction of commercial paper outstanding, larger both in percentage (15%) and in level (330 billion)) that targets almost the entire commercial paper markets. AMLF and its success also seem to have helped later interventions. For example, CPFF was also effective and even generated \$5 billion in net income for the government.

This happens in the real world when the incumbent government is not expecting to be re-elected and does not fully consider the impact of current intervention on coordination games and interventions under the future government. This could also happen when one EU countrys intervention does not fully consider the informational externality on neighboring countries with correlated fundamentals but not necessarily similar intervention costs.

We characterize how the information externality on the second period in our model affects m_1^*. In general, a myopic government–one who ignores this negative impact–may fail to formulate a welfare-maximizing policy. Understanding such myopic interventions can facilitate formulating forward-looking policies and coordinated efforts among multiple governments.

To highlight the information externality from the initial intervention, we shut down the budget channel in our general intervention cost function by setting $K_{12}(m_1, m_2) = 0$.[15]

In general, the government chooses $\{m_1, m_2\}$ to maximize welfare. For a given m_1, define the objective as

$$\begin{aligned} Y(m_1;\chi) \quad = \quad & W_1 - K(m_1, 0) + \chi \max_{m_2} \left[\frac{B + m_1 + 1 - c}{2B} [W_{2S} - (K(m_1, m_2) - K(m_1, 0))] \right. \\ & \left. + \frac{B - m_1 - 1 + c}{2B} [W_{2F} - (K(m_1, m_2) - K(m_1, 0))] \right] \end{aligned} \tag{10}$$

Here, $\chi \in [0, 1]$ measures how much the government cares about the fate of the fund in the second period. In particular, $\chi = 0$ corresponds to the static benchmark, and $\chi = 1$ corresponds to the case in which the second fund's fate is equally important. Often, $\chi < 1$ because of short-termism of the government. Alternatively, in the context of global economy in which countries' fundamentals are highly correlated, χ captures the extent that one country considers the externality it imposes on other countries.

We are interested in $\frac{\partial m_1^*}{\partial \chi}$, the effect of government myopia on intervention in the first period. The answer generally depends on cost parameters. By Theorem 2.1 in Athey, Milgrom, and Roberts (1998), $m_1^* \equiv \text{argmax}_{m_1} Y(m_1, \chi)$ is non-increasing in χ iff Y has decreasing differences in χ and m_1, and is non-decreasing in χ iff Y has increasing differences

[15] The case of hard budget constraint trivially predicts that the more the government considers the welfare in the second period, the less it would intervene in the first period.

in χ and m_1.

Proposition 6 (Myopic Intervention)

A myopic government may under-or over- intervene initially. In particular,

1. $\frac{\partial m_1^*}{\partial \chi} \geq 0$*, iff either* $m_1^* \geq c$ *and* $m_2^* = m_1^* - c$ *always or* $m_1^* \leq c$ *and* $m_2^* = 0$ *always.*

2. $\frac{\partial m_1^*}{\partial \chi} \leq 0$*, iff either* $m_2^* > m_1^* + 1 - c$ *always or* $m_1^* > c$ *and* $m_2^* < m_1^* - c$ *always.*

Note that this result emphasizes m_1 relative to the case where the intervention externality is absent. A myopic government under-intervenes initially when intervention outcomes are perfectly correlated. This happens when the costs of intervention in the two periods are comparable. When they are both small ($m_1^* \geq c$ and $m_2^* = m_1^* - c$) or both large ($m_1^* \leq c$ and $m_2^* = 0$), the endogenous correlation effect dominates. Hence, increasing the first period's survival probability increases the survival for second period one for one. It is therefore more important to increase the probability of survival by increasing m_1, a fact that a myopic government neglects.

To link Proposition 6 to exogenous parameters, we provide in the next corollary some examples of sufficient conditions that lead to under-intervention. These conditions are neither unique, nor restrictive. For simplicity in exposition, we assume for the remainder of the paper that K is twice-differentiable in a continuous feasible range of intervention $\mathcal{I}$. This specification includes cases of budget constraint and separable quadratic intervention costs. Let K_i denote the partial derivative w.r.t. m_i.

Corollary 2

Myopic government under-intervenes initially if one of the two following conditions holds:

1. $K_1(c, \cdot) > \frac{1-c}{B}$ *and* $K_2(\cdot, 1-c) \geq \frac{1-c}{B}\frac{c\delta}{2c\delta+c-1}$.

2. *For some* $b > c$*, it holds* $K_1(b, \cdot) > \frac{1-c}{B}$, $K_1(c, \cdot) < \frac{1-c}{B}\frac{\delta - c(1+2\delta)}{2\delta - c(1+2\delta)}$, $K_2(\cdot, b-c) \leq \frac{1-c}{2B}$*, and* $K_2(\cdot, 1) \geq \frac{(1-c)c\delta}{B(2c\delta+c-1)}$.

Interestingly, failure to consider dynamic coordination could also result in excessive intervention through the information structure it creates. For example, this happens when the cost for first intervention is sufficiently small such that the initial intervention is large scale, yet the second intervention is sufficiently costly that survival does not always lead to survival. At the same time, a high m_1 reduces the quality of good news, reducing the marginal benefit of m_2. When the costs of intervention in the two periods are rather disproportionate, outcomes are less correlated, and the conditional inference effect dominates.[16] For a myopic government, shading m_1 makes it easier to intervene in the second period no matter the fund survives or fails in the first period. Again, the next corollary gives some illustrating sufficient conditions under which over-intervention occurs.

Corollary 3

A myopic government over-intervenes initially if one of the following conditions holds:

1. *For some $b \geq 0$, it holds $K_1(c, \cdot) > \frac{1-c}{B}$ and $K_2(\cdot, b + 2c\delta) < \frac{1-c}{2B} \frac{\delta}{1+2\delta}$.*
2. *$K_1(b, \cdot) > \frac{1-c}{B}$, $K_1(c, \cdot) < \frac{1-c}{B} \frac{\delta - c(1+2\delta)}{2\delta - c(1+2\delta)}$, $K_2(\cdot, 0) > \frac{1-c}{2B} \frac{2\delta}{2\delta - c(1+2\delta)}$.*

We illustrate the results in Figures **??** and **??**. We also note that when the conditions for increasing or decresing differences do not hold globally, the optimal intervention size in the first period can be non-monotonic in χ. For example, in Figure **??**, m_1^* increases with χ initially and finally decreases.

The above proposition calls for coordinated interventions across governments. For example, since economic fundamentals across EU countries are highly correlated, one member's isolated intervention imposes informational externality on other members. In the case of AMLF and CPFF, because the capacity to intervene using CPFF is comparable to that in AMLF, the later intervention was able to fully captures the benefit from investors' learning of earlier intervention. According to the above proposition, this provides additional justification for the overwhelming scale of AMLF.

[16] The countries could differ in dimensions such as fiscal budget, financial depth, fundamentals, or liquidity. The heterogeneity not only affects interventions during the crisis, but also the safety of assets, as discussed in He, Krishnamurthy, Milbradt, et al. (2015).

4.3 "Too Big to Save First"

In this section, we consider how the government intervenes in two funds of different sizes, given the dynamic coordination effect. In particular, we examine both the size and the sequence of interventions.

Without loss of generality, we normalize the size of fund 1 to 1, and the size of fund 2 to $\lambda > 1$. Here, size simply refers to the total measure of investors. We continue to assume that fund 1 survives if and only if

$$A_1 + m_1 \geq \theta,$$

where A_1, m_1 and θ have the same interpretations as before. Besides, fund 2 survives if and only if

$$\lambda A_2 + m_2 \geq \theta\lambda,$$

where $A_2 = \frac{\int_0^\lambda 1\{a_{2i}=1\}ds}{\lambda} \in [0,1]$ is the fraction of investors who choose to stay and thus λA_2 is the liquidity from remaining investors. Fund 2 survives if and only if the total liquidity is greater than $\theta\lambda$. The threshold is also augmented by λ. The government's choice variables are extended: it chooses not only the intervention plan $\{m_1, m_2\}$, but also which fund to intervene first:

$$\max_{\iota, m_1, m_2} E\left[\int_0^1 V_{1i}di + \int_0^\lambda V_{2i}di\right] - K(m_1, m_2)$$

If $\iota = 1$, the government intervenes fund 1 first. $E\left[\int_0^1 V_{1i}di\right] = \frac{1-c}{2B}\left[1 + B - c(1+\delta) + m_1\right]$, and $E\left[\int_0^\lambda V_{2i}di\right]$ depends on whether $s_1 = S$ or $s_1 = F$. If $\iota = 2$, however, s_2 arrives first, $E\left[\int_0^\lambda V_{2i}di\right] = \lambda \cdot \frac{1-c}{2B}\left[1 + B - c(1+\delta) + m_2\right]$, and $E\left[\int_0^1 V_{1i}di\right]$ depends on whether $s_2 = S$ or $s_2 = F$.

Proposition 7 shows that the government with a budget constraint should save the smaller fund first.

Proposition 7 (Too Big to Save First)

A benevolent government who faces a budget constraint always intervenes to induce perfectly *correlated outcomes across interventions. Besides, it intervenes in the smaller fund first:* $\iota^* = 1$.

Appendix A.7 contains the proof, which has two steps. First, we show that given ι, the optimal intervention plan always satisfies $\frac{m_2}{\lambda} = m_1 - c$, and thus leads to correlated outcomes. Note that if $\lambda = 1$, the result is identical to that in Section 4.1 where $m_2^* = m_1^* - c$. Next, we compare different choices of $\iota \in \{1, 2\}$ and show the optimal intervention sequence features $\iota^* = 1$. Two factors contribute to this result. First, the larger fund benefits more from the resolution of uncertainty due to the revelation of the initial intervention's outcome. Second, it is less costly to intervene into the smaller fund to create the same information structure.

The above result carries through to the case with committed intervention with general cost functions, after a slight modification of the condition in Proposition 5 to $K\left(m_1, \frac{m_2}{\lambda}\right) > K\left(\frac{1}{2}\left[m_1 + \frac{m_2}{\lambda}\right], \frac{1}{2}\left[m_1 + \frac{m_2}{\lambda} - 2c\right]\right)$.

Our result thus relates to the concept of "too big to fail". Rather than emphasizing financial networks and connectedness, we are adding an information-structure perspective to the debate on systemic fragility. Some institutions could be too big to fail, but the best way to save them may entail saving the smaller ones first to better boost market confidence.

5 Discussions and Extensions

5.1 Learning, Endogenous Multiplicity, and Bounded Support

In this section, we relate our paper to Angeletos, Hellwig, and Pavan (2007). We first explain why our baseline model yields unique equilibrium and how equilibrium multiplicity is restored via a mechanism isomorphic to the one in Angeletos, Hellwig, and Pavan (2007). Then, we highlight how equilibrium multiplicity could be endogenized by intervention policy. Finally, we discuss how the key results are robust to distributional assumptions and the role of bounded support in the interaction of private and public information.

Angeletos, Hellwig, and Pavan (2007) show that multiple equilibria emerge under the same conditions that guarantee uniqueness in static global games. The results rely on endogenous learning from regime survivals and exogenous learning from private news that arrives over time. We show that two elements are necessary for this multiplicity result. First, private information interacts with endogenous learning from earlier coordination outcomes. Second, the private information is either very precise or gets very precise as agents continuously receive private signals about the fundamental. Without the first element, the game is equivalent to one in which agents receive only one summary private signal in each period.[17] Our baseline model demonstrates that without the second element, we have an unique equilibrium. Below, we show that multiple equilibria may exist when private signals are very precise. That is, when δ gets very small. [18]

To see this, note that the set of parameters we have examined corresponds to imprecise signals ($2\delta > 1$ and $\frac{1}{1+2\delta} < c < \frac{2\delta}{1+2\delta}$. Moreover, the signal does not get more precise because agents are non-overlapping. If we relax the parameter assumptions, or allow agents' signals to become more precise over time, multiplicity follows. Proposition 8 complements Propositions 1 and 2.[19]

Proposition 8 (Equilibria with general δ and c)

1. *If $s_1 = S$ and $\frac{2\delta}{2\delta+1} < c < 1$,*

   (a) *If $m_2 < m_1 - c$, the unique equilibrium is the Subgame Equilibrium without Dynamic Coordination.*

   (b) *If $m_1 - c < m_2 < m_1 - 2\delta(1-c)$, all three types of equilibria exist. However, in the Equilibrium with Partial Dynamic Coordination, the threshold θ_2^* decreases with m_2,*

[17] The variance of the signal is $Var\left(\frac{\sigma^2}{n}\right)$ with n signals in period-n.

[18] Likewise, Angeletos, Hellwig, and Pavan (2007) show that there always exists an equilibrium in which no attack occurs after the first period, and this would be the unique equilibrium if agents did not receive any private information after the first period. One can easily write a two-period version of Angeletos, Hellwig, and Pavan (2007) and show this is the only equilibrium if the private signal is sufficiently imprecise.

[19] Technically, multiple equilibria resurface because we can apply the argument of iterated deletion of dominated regions only from one end of θ space. Despite this, with slight modifications on the intervention cost functions, the main intuitions for the results from earlier sections still apply as long as we are consistent with equilibrium selection.

(c) *If* $m_1 - 2\delta(1-c) < m_2$*, the unique equilibrium is the Subgame Equilibrium with Dynamic Coordination.*

2. *If* $s_1 = F$ *and* $0 < c < \frac{1}{2\delta+1}$

   (a) *If* $m_2 < m_1 + 2c\delta$*,the unique equilibrium is the Subgame Equilibrium with Dynamic Coordination.*

   (b) *If* $m_1 + 2c\delta < m_2 < m_1 + 1 - c$*, all three types of equilibria exist. However, in the Equilibrium with Partial Dynamic Coordination, the threshold* θ_2^* *decreases with* m_2*.*

   (c) *If* $m_2 > m_1 + 1 - c$*, the unique equilibrium is the Subgame Equilibrium without Dynamic Coordination.*

Our baseline model also differs from Angeletos, Hellwig, and Pavan (2007) in two additional ways: the government's action is endogenous and the private signal is bounded.[20] Government's action therefore affects equilibrium selection and learning. In particular, when the government's intervention induces equilibria with full or no dynamic coordination, it shuts down the interaction between private signal and public learning. Consequently, the equilibrium is unique even if the signal is infinitely precise. In this regard, the government's endogenous intervention can determine the equilibrium multiplicity through the first element.

Next, we discuss the case when private signals follow Normal distribution which are unbounded, i.e., $\varepsilon_i \sim N(0, \delta)$. Other than equilibrium multiplicity, our main intuition carries through. We still assume that investors are non-overlapping to keep matters comparable with our baseline model.[21] We characterize the equilibrium in each period and emphasize that government intervention in period 1 still has a dynamic informational effect on period 2.

Lemma 5 below summarizes equilibrium outcomes in two periods. Detailed analysis can be found in Appendix C.

[20](Uniform $[-\delta, \delta]$) in our model but unbounded support in their model ($N\left(z, \frac{1}{\alpha}\right)$).

[21]The case when investors perfectly overlap can be identically analyzed as one in which $\varepsilon_i \sim N\left(0, \frac{\delta}{2}\right)$. All results in this section are unchanged.

Lemma 5

Equilibrium when signals follow Normal distribution

1. *Given m_1, there exists unique equilibrium thresholds in period 1:*

$$\begin{aligned} \theta_1^* &= 1 + m_1 - c \\ x_1^* &= 1 + m_1 - c - \delta \Phi^{-1}(c). \end{aligned}$$

2. *Given (m_1, m_2) and $s_1 = S$,*

   (a) *When $m_2 > m_1 - c$, $(\theta_2^* = \theta_1^*, x_2^* = \infty)$ consists a threshold equilibrium.*

   (b) *Equilibrium strategies (θ_2^*, x_2^*) which satisfy $\theta_2^* < \theta_1^*$ and $x_2^* < \infty$ may or may not exist. If they exist, they can be non-unique.*

Table 2 presents the local comparative statics when there exists a unique equilibrium strategy. When m_1 increases from 0.7 to 0.9, both θ_2^* and x_2^* decrease, validating the dynamic coordination effect.

Table 2: θ_2^* as a function of m_1 ($s_1 = S$)

m_1	0.7	0.75	0.8	0.85	0.9
θ_2^*	0.7602	0.6981	0.6693	0.6511	0.6384
x_2^*	0.9667	0.8222	0.7565	0.7152	0.6866

Other parameters are $c = 0.5, \delta = 0.5, m_2 = 0.1$.

5.2 Costly Interventions and Their Interactions

In this section, we relate our paper to Goldstein and Huang (2016) which studies costless information design in interventions. We first show that the tradeoff between endogenous correlation effect and conditional inference effect are similar to that in Goldstein and Huang (2016), and ignoring the intervention cost the endogenous correlation effect always dominate. We then argue that when intervention costs (corresonponding to information design cost in their paper) are taken into consideration, the conditional inference effect could become dominant with new implications. We thus add to Goldstein and Huang (2016) by explicitly characterizing the information design tradeoff and analyzing it under general cost functions. Finally, we emphasize that the interaction of interventions also matter in determining which effect would dominate.

Our paper derives the informational tradeoffs present in Goldstein and Huang (2016) in a different setting. The policymaker commits to abandon the regime with a high enough frequency so that a regime maintenance results in no attack, similar to our endogeous correlation effect; but maintaining the regime too often is costly as there is less informational benefit when the regime is maintained, similar to our conditional inference effect. This implies that without information design cost, we always want to choose an informational structure such that survival leads to survival and failure leads to failure, because that's the best information one can provide to the investors (which tells them whether the fund would survive or not). Our setup allows us to derive the thresholds of run and survival, the agents' payoffs, and the optimal policy in closed-form, if we ignore the information design cost, or use some tractable cost functions. By doing so, we hope to provide a more analytical analysis and alternative illustration of the important tradeoffs also discussed in Goldstein and Huang (2016).

That said, our setup differs in that the government designs information structure for the second intervention through a costly initial intervention. This implies that information design is costly, and intervention costs in both coordination games matter. The former implies that the optimal design may not always involve perfectly correlated survival outcomes. The latter has implications when thinking about endogenous intervention policy across countries or across episodes of runs. In particular, survival (policy not to abandon) leads to survival (no subsequent attack) holds only when the intervention costs across the two periods are comparable. If one intervention's cost is so low, but the subsequent intervention's cost is extremely high due to, for example, moral hazard, the initial success as public news is not quite relevant for the subsequent coordination because the high cost prohibits correlated outcomes, but its failure would make investors really negatively update.

5.3 Correlated Fundamentals and Multi-Period Interventions

So far we have assumed $\theta_1 = \theta_2$, what if the fundamentals across the two periods are positively correlated but non-identical? Earlier studies have demonstrated that it is very difficult to obtain analytical solutions if one simply make θ_2 a noisy version of θ_1. Here we show that our results extend to the case of imperfectly-correlated fundamentals. The intuition also applies to multiple-period setup.

Suppose at the beginning of period 2, everyone learns from the period-1 intervention whether period 2 is an extension of period 1's coordination game, or an independent one. In other words, it becomes public with probability π that $\theta_2 = \theta_1$, and with probability $1 - \pi$ that θ_2 is a random draw from $[-B, B]$ independent of θ_1. Because of risk-neutrality, our baseline model corresponds to $q = 1$. In the case that $q = 0$, the intervention problem is symmetric gives our benchmark policy $m_1^* = m_2^*$. More generally, when $q \in (0, 1)$, the intuition for all the implications continues to apply and the previous results are only affected qualitatively.

The economic mechanism and intuition in the baseline model can also be generalized to a multi-period setup. While such an exercise is beyond the scope of this paper and does not add to the insights we bring, we briefly describe the implication of emphasizing early interventions under such settings. For simplicity, suppose that the economy lasts for n periods with the same fundamental θ. In each period, there is a continuum of investors of measure 1 who choose whether to stay or run. The government, equipped with total resources M, aims to maximize the aggregate welfare across n periods. If $n = 2$, the setup returns to the main section.

Suppose the proposed intervention plan equally divides the total resources across n periods, $m_1 = m_2 = \cdots = m_n = \frac{M}{N}$, then it is clear that the intervention outcomes in all periods are perfectly correlated: $s_1 = s_2 = \cdots = s_n$. As a result, the government has incentive to increase m_1, which consequently raises the possibility of $\Pr(s_1 = 1)$, as well as $\Pr(s_i = 1)$ for $i = 2, \cdots n$. Given optimal intervention $m_1^* > \frac{M}{n}$, the government now allocates the remaining resources $M - m_1^*$ across $n - 1$ periods. A similar argument tells us that $m_2^* > \frac{M - m_1^*}{n-1}$. Along the line of the analysis, we could reach the result $m_1^* > m_2^* > \cdots m_n^*$. In other words, optimal intervention plan always emphasizes early intervention.

5.4 Contingent Interventions

In reality, government can sometimes choose the size of later intervention after the outcome of the initial intervention is realized. We analyze this case in this section. The intuition and key tradeoff in earlier discussions still apply. Note that any $m_{2F} \in \big(0, 1+m_1^*-c\big]$ cannot be optimal since if $s_1 = F$, the fund in the second still fails for sure despite for costly intervention. If $m_{2F}^* = 0$, the endogenous correlation effect is even reinforced. If $m_{2F}^* > 1+m_1^*-c$, however, it is possible to have failed initial intervention but successful subsequent intervention, and the endogenous correlation effect is weaker. The overall dynamic coordination still boils down to a tradeoff between the endogenous correlation effect and the conditional inference effect.

Proposition 9 (Emphasis on Initial Intervention (Contingent Case))
When $K_2(0, 1-c) > \frac{(1-c)^2}{B-1}$, then contingent interventions strictly emphasizes initial intervention: $m_1^* > m_{2S_1}^*$.

This result extends Proposition 5 to contingent interventions. If the cost for the subsequent intervention (second period) is big enough, then initial intervention is emphasized. This is just one example of the sufficient conditions under which the endogenous correlation effect dominates the conditional inference effect. Note this result applies to situations in which the initial intervention is more costly than the subsequent intervention.

Proposition 10 (Myopic Intervention (Contingent Case))
The government's initial intervention is weakly increasing in the extent it considers dynamic coordination, i.e., $\frac{\partial m_1^*}{\partial \chi} \geq 0$ if one of the two following conditions hold:

1. $K(\cdot, 1-c) - K(\cdot, 0) > \min\{\frac{(1-c)^2}{2c\delta-1+c}, \frac{c\delta(1-c)}{B-(1-c)}\}$ *and* $K_1(c, \cdot) \geq \frac{1-c}{B}$.

2. $K(\cdot, 1-c) - K(\cdot, 0) > \min\{\frac{(1-c)^2}{2c\delta-1+c}, \frac{c\delta(1-c)}{B-(1-c)}\}$ *and* $1 - c - K(\cdot, 1-c) + K(\cdot, 0) - (2 + B - c)K_2(\cdot, 1-c) > 0$.

The government's contingent intervention is weakly decreasing in the extent it considers dynamic coordination, i.e., $\frac{\partial m_1^*}{\partial \chi} \leq 0$, *when* $K_2(\cdot, 0) > \frac{1-c}{B+c-2}\frac{2\delta}{2\delta-c(1+2\delta)}$ *and* $K_1(c, \cdot) < \frac{1-c}{B}\frac{\delta-c(1+2\delta)}{2\delta-c(1+2\delta)}$.

These sufficient conditions for under- and over-interventions simply correspond to corollaries 2 and 3. With contingent interventions, myopic government still under-or over- intervenes initially. The optimal initial intervention is weakly increasing in the extent it considers dynamic coordination, i.e., $\frac{\partial m_1^*}{\partial \chi} \geq 0$, iff either $m_1^* \geq c$ and $m_{2S}^* = m_1^* - c$ and $m_{2F}^* = 0$, or $m_1^* \leq c$ and $m_{2S}^* = m_{2F}^* = 0$ always. It is weakly decreasing iff $m_1^* > c$ and $m_{2S}^* < m_1^* - c$ and $m_{2F}^* = 0$ always. Because when $m_{2F}^* = 0$, the endogenous correlation effect is the same as in the committed intervention case, and the same intuition carries through.

Finally, regarding the sequence of interventions in funds of different sizes, saving the smaller fund first is still cheaper to create the same learning on the fundamental, and the larger fund still benefits more from the uncertainty reduction. A policy that induces perfectly correlated outcomes and saves the larger fund first cannot be optimal. In order words, the larger fund is still "too big to save first."

Proposition 11 (Too Big to Save First (Contingent Case))
If interventions always lead to perfectly correlated outcomes ($s_1 = s_2$), it is socially efficient to save the small fund first.

5.5 Various Forms of Interventions

In the model, we have interpreted intervention as liquidity injection. We argue below that our model captures a broader array of interventions that are commonly used (Bebchuk and Goldstein, 2011; Diamond and Rajan, 2011).

Direct lending and investing in borrower funds This is exactly the interpretation in our model. During the financial crisis of 2008-2009, the US government directly participated in the commercial paper market through direct purchasing. Our general cost function to a large extent captures investment returns to the government and some inefficiencies discussed in Bebchuk and Goldstein (2011).

Direct capital infusion to investors Governments around the globe have injected capital to both retail and institutional investors. For instance, the U.S. Troubled Asset Relief Program (TARP) provided about US$250 billion to banks, and the UK injected about US$90 billion to its major banks. Tax breaks and related measures represent capital infusion to retail investors directly. To map these policies into our model, suppose the government injects a fraction α of investors' existing capital. This changes the capital of each investor from 1 unit to $1+\alpha$ without altering the investor's optimization problem. Consequently, the one period survival threshold becomes $\theta^* = (1-c)(1+\alpha)$. We can relabel $m = (1-c)\alpha$ and the model solutions are equivalent. Thus, the intervention again increases the probability of survival.

Government guarantees During the financial crisis, governments used guarantees that are similar to FDIC to limit the potential losses of the lenders. Specifically in our model, suppose that the government guarantees a proportion ξ of a lender's or investors losses, then the lender who stays (rolls over) receives the return R when the fund survives, and $-(1-\xi)c$ if it fails. Since our investors are risk neutral, the survival threshold now is $\theta^* = \frac{1-c}{1-c\xi}$. Again, we can relabel $m = \frac{c(1-c)\xi}{1-c\xi}$ and this is equivalent to an intervention that increases the probability of success.

Interest Rate Reduction During the financial crisis, the Fed Reserve Board cut the fed funds rate from 4.25% in Jan 2008 to 1% in Oct 2008. Many other countries took similar measures in the face of a global contraction in lending. In the model, this is equivalent to reducing r, the payoff for not investing. Under risk-neutrality, it is equivalent to increasing the survival probability through changing c, which is exactly the rolf of m in our model.

5.6 Moral Hazard

Moral hazard is a big concern in government bailouts. Indeed, fund managers may divert the capital injected by the government, or gamble by investing in more risky assets. Below, we show that fund managers' moral hazard provides a micro-foundation for the separable intervention cost $K(m_1, m_2) = k(m_1) + k(m_2)$.

To be more specific, assume the fund manager is able to divert a constant fraction $\eta \in (0,1)$ for any amount of liquidity μ injected by the government. Among the diverted capital, the fund manager can consume $u(\eta\mu) < \eta\mu$, and the rest $\eta\mu - u(\eta\mu)$ is inefficiently lost (iceberg costs). We assume standard utility function over consumption with increasing and concave $u(\cdot)$ and $u(0) = 0$. Under this setup, the optimal intervention problem is isomorphic to the problem solved earlier, where intervention incurs a cost $k(m)$.

To see this, note that the government is aware of the diverting technology. Therefore, to effectively inject m to the fund, the government needs to spend μ such that $(1-\eta)\mu = m$. Equivalently, injecting m into the fund costs the government $k(m) = \frac{m}{1-\eta} - u\left(\frac{m}{1-\eta}\right)$. Obviously, the effective cost function $k(m)$ is increasing and convex in m, consistent with our cost specification.

6 Conclusion

How should a benevolent government intervene in a dynamic environment in which agents have strategic complementarity? Through the lens of sequential global games in which governments are large players who mitigate coordination failures, we establish general results on the existence and uniqueness of equilibria, and show that government intervention can affect coordination both contemporaneously and dynamically. Our results suggest that optimal intervention emphasizes initial action, validating the conventional wisdom in broad settings. However, depending on costs across interventions, an initial intervention could have either a positive or negative informational externality on subsequent coordination. Finally, some funds are "too big to save first" because they benefit more from resolution of uncertainty about the fundamentals, and intervening in smaller funds first costs less to generate this informational externality. Our paper thus has policy relevance to various intervention programs, such as the bailout of money market mutual funds during the financial crisis.

The dynamic learning mechanism and thus the information-structure effect also apply to broader contexts, such as interventions in currency attacks, credit market freezes, cross-sector industrialization, regulatory union, and green energy development. Our discussion therefore opens several avenues for future research. For example, how the government signals their private knowledge about economic fundamentals? Moreover, this paper only considers common forms of interventions. Understanding the optimal contingent intervention not only is of theoretical interest, but also provides new insights and guidance to policymakers.

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Appendix

A Derivations and Proofs

A.1 Proof of Lemma 1

We prove the thresholds here.

Suppose there is a threshold $x^* \in \mathbb{R}$ such that each agent invests if and only if $x \leq x^*$. The measure of agents who invest is thus,

$$A(\theta) = \Pr\left(x \leq x^* \middle| \theta\right) = \begin{cases} 0 & \text{if } \theta > x^* + \delta \\ \frac{x^* - (\theta - \delta)}{2\delta} & \text{if } x^* - \delta \leq \theta \leq x^* + \delta \\ 1 & \text{if } \theta < x^* - \delta. \end{cases} \tag{11}$$

It follows that the investment succeeds if and only if $\theta \leq \theta^*$ where θ^* solves

$$A(\theta^*) + m = \theta^*. \tag{12}$$

By standard Bayesian updating, the posterior distribution about θ conditional on the private signal is also uniform distribution with bandwidth 2δ. Therefore, the posterior probability of investment success is

$$\Pr\left(R = 1 \middle| x\right) = \Pr\left(\theta \leq \theta^* \middle| x\right) = \begin{cases} 0 & \text{if } x > \theta^* + \delta \\ \frac{\theta^* - (x - \delta)}{2\delta} & \text{if } \theta^* - \delta \leq x \leq \theta^* + \delta \\ 1 & \text{if } x < \theta^* - \delta. \end{cases} \tag{13}$$

For the marginal investor who is indifferent between investing or not, his signal x^* satisfies

$$\Pr\left(R = 1 \middle| x^*\right) = c \tag{14}$$

Jointly solve equations (12) and (14), we obtain the two thresholds

$$\begin{cases} \theta^* = 1 + m - c \\ x^* = 1 - c + \delta - 2c\delta + m. \end{cases} \tag{15}$$

A.2 Proof of Lemma 2

Proof. "if" $\Leftarrow$

If $m_2 > m_1 - c$, and if all agents know that other agents will adopt a threshold strategy $x_2^* = \infty$, then

$$A_2 + m_2 = 1 + m_2 > 1 + m_1 - c = \theta_1^* > \theta. \tag{16}$$

Therefore, the investment succeeds with probability 1. Therefore, it is individually rational for each agent to set $x_2^* = \infty$.

"only if" $\Rightarrow$

We prove by contradiction. Suppose that an equilibrium in which all agents adopt a threshold $x_2^* = 1 + m_1 - c + \delta$ when $(m_1 - c) - m_2 = \Delta > 0$. Therefore, any agent with a signal $x_2 < \theta_1^* + \delta$ will invest. In other words,

$$\Pr\left(\theta < 1 + m_2 \middle| x_2, \theta < \theta_1^*\right) \geq c$$

holds for any x_2.

Consider an agent who observes $\hat{x}_2 = m_1 + 1 - c + \delta - \frac{\Delta}{2}$. Such an agent exists when $\theta \in \left(m_1 + 1 - c + \delta - \frac{\Delta}{2}, m_1 + 1 - c + \delta\right)$. Apparently,

$$\Pr\left(\theta < 1 + m_2 \middle| x_2 = \hat{x}_2, \theta < \theta_1^*\right) \geq c = 0 < c$$

which violates the assumption that all agents invest irrespective of their signals.

□

A.3 Proof of Lemma 3, 4, Proposition 1, 2, and 8

Here we solve the equilibrium in period 2 under both $s_1 = S$ and $s_1 = F$, and under all parameter values. The solutions directly prove the lemmas and propositions.

Our solutions take two steps. First, we assume a solution pair (θ_2^*, x_2^*) exists and derive the equilibrium values. Second, we check the conditions that these solutions must satisfy and thus derive the parameter ranges such that they indeed constitute a solution.

Case 1: Period 1 fund survives: $s_1 = S$

1. If $\theta_1^* - 2\delta < \theta_2^* < \theta_1^*$, then in equilibrium

$$\theta_2^* - m_2 = A\left(\theta_2^*\right) = \begin{cases} 1 & \text{if } x_2^* - \theta_2^* > \delta \\ \frac{x_2^* - (\theta_2^* - \delta)}{2\delta} & \text{if -}\delta\text{¡}x_2^* - \theta_2^* < \delta \\ 0 & \text{if } x_2^* - \theta_2^* < -\delta \end{cases}$$

and

$$c = \begin{cases} 1 & \text{if } x_2^* < \theta_2^* - \delta < \theta_2^* < \theta_1^* \\ \frac{\theta_2^* - (x_2^* - \delta)}{2\delta} & \text{if } \theta_2^* - \delta < x_2^* < \theta_1^* - \delta \\ \frac{\theta_2^* - (x_2^* - \delta)}{\theta_1^* - \left(x_2^* - \delta\right)} & \text{if } \theta_1^* - \delta < x_2^* < \theta_2^* + \delta \\ 0 & \text{if } \theta_2^* + \delta < x_2^*. \end{cases}$$

Jointly solve the above equations, the solutions are.

   (a) $\theta_2^* = 1 + m_2 - c$ and $x_2^* = 1 + m_2 - c + \delta\left(1 - 2c\right)$. The solution exists if $m_1 - 2\delta < m_2 < m_1 - 2\delta\left(1 - c\right)$.

(b) $\theta_2^* = 1 + m_2 - c + \frac{c[m_2 - m_1 + 2\delta(1-c)]}{2\delta - c(1+2\delta)}$ and $x_2^* = 1 + m_2 - c + \delta(1-2c) + \frac{c(1+2\delta)[m_2 - m_1 + 2\delta(1-c)]}{2\delta - c(1+2\delta)}$. The solution exists in two cases: 1) $m_1 - 2\delta(1-c) < m_2 < m_1 - c$ if $0 < c < \frac{2\delta}{1+2\delta}$; 2) $m_1 - c < m_2 < m_1 - 2\delta(1-c)$ if $\frac{2\delta}{1+2\delta} < c < 1$

2. If $\theta_2^* < \theta_1^* - 2\delta$, then in equilibrium

$$\theta_2^* - m_2 = A(\theta_2^*) = \begin{cases} 1 & \text{if } x_2^* - \theta_2^* > \delta \\ \frac{x_2^* - (\theta_2^* - \delta)}{2\delta} & \text{if -}\delta\text{¡}x_2^* - \theta_2^* < \delta \\ 0 & \text{if } x_2^* - \theta_2^* < -\delta \end{cases}$$

and

$$c = \begin{cases} 1 & \text{if } x_2^* < \theta_2^* - \delta \\ \frac{\theta_2^* - (x_2^* - \delta)}{2\delta} & \text{if } \theta_2^* - \delta < x_2^* < \theta_2^* + \delta \\ 0 & \text{if } \theta_2^* + \delta < x_2^*. \end{cases}$$

Jointly solve the above equations, the solutions are.

(a) $\theta_2^* = 1 + m_2 - c$ and $x_2^* = 1 + m_2 - c + \delta(1-2c)$. The solution exists if $m_2 < m_1 - 2\delta$.

Combine the above results, we prove Lemma 3, 4, Proposition 1, and half of Proposition 8. The next case finishes the rest of the proof.

Case 2: Period 1 fund fails: $s_1 = F$
The analysis is identical. We will just list the results as below.

(a) $\theta_2^* = 1 + m_2 - c - \frac{(1-c)(m_1 + 2c\delta - m_2)}{c(1+2\delta)-1}$ and $x_2^* = 1 + m_2 - c + \delta(1-2c) - \frac{(1-c)(1+2\delta)(m_1 + 2c\delta - m_2)}{c(1+2\delta)-1}$.. The solution exists in two cases: 1) $m_1 + 2c\delta < m_2 < m_1 + (1-c)$ if $0 < c < \frac{1}{1+2\delta}$; 2) $m_1 + (1-c) < m_2 < m_1 + 2c\delta$ if $\frac{1}{1+2\delta} < c < 1$.

(b) $\theta_2^* = 1 + m_2 - c$ and $x_2^* = 1 + m_2 - c + \delta(1-2c)$. The solution exists if $m_1 + 2c\delta < m_2 < m_1 + 2\delta$.

(c) $\theta_2^* = 1 + m_2 - c$ and $x_2^* = 1 + m_2 - c + \delta(1-2c)$. The solution exists if $m_2 > m_1 + 2\delta$.

Combining this result, Proposition 2 and the other parts of 8 naturally follow.

A.4 Proof of Proposition 4

Plugging in the government's budget constraint, we are able to obtain the aggregate social welfare as a function of m_1. As a by-product, we are also able to calculate the net benefit of initial intervention. Lemma 6 summarizes the results.

Lemma 6
Aggregate Social Welfare W and Net benefit of initial intervention $\left.\frac{\partial W}{\partial m_1}\right|_{m_1+m_2=M}$

1. *If* $m_1 > \frac{M+2\delta(1-c)}{2}$,

$$\begin{aligned}
W &= W_1 + \Pr(s_1 = S) W_{2S}^{nc} + \Pr(s_1 = F) W_{2F}^{c} \\
&= \frac{1-c}{2B}\left[2 + 2B - 2c(1+\delta) + M\right] \\
\frac{\partial W}{\partial m_1} &= 0.
\end{aligned}$$

This case only exists for $M > 2\delta(1-c)$.

2. *If* $\frac{M+c}{2} < m_1 < \frac{M+2\delta(1-c)}{2}$,

$$\begin{aligned}
W &= W_1 + \Pr(s_1 = S) W_{2S}^{pc} + \Pr(s_1 = F) W_{2F}^{c} \\
&= \frac{1-c}{2B}\left[2 + 2B - c(2+\delta) + 2m_1 \right. \\
&\quad \left. + \frac{\delta c (c + M - 2m_1)^2 - 2\delta\left[c - 2(1-c)\delta\right](c + M - 2m_1)}{\left[c - 2(1-c)\delta\right]^2}\right] \\
\frac{\partial W}{\partial m_1} &= \frac{(1-c)\left[2c(1+2\delta)\left[c - 2(1-c)\delta\right] - 4c\delta(c + M - 2m_1)\right]}{2B\left[c - 2(1-c)\delta\right]^2} < 0.
\end{aligned}$$

This case only exists for $M > c$.

3. *If* $\frac{M-(1-c)}{2} < m_1 < \frac{M+c}{2}$,

$$\begin{aligned}
W &= W_1 + \Pr(s_1 = S) W_{2S}^{c} + \Pr(s_1 = F) W_{2F}^{c} \\
&= \frac{1-c}{2B}\left[2 + 2B - c(2+\delta) + 2m_1\right] \\
\frac{\partial W}{\partial m_1} &= \frac{1-c}{B} > 0.
\end{aligned}$$

This case always exists.

4. *If* $\frac{M-2c\delta}{2} < m_1 < \frac{M-(1-c)}{2}$,

$$\begin{aligned}
W &= W_1 + \Pr(s_1 = S) W_{2S}^{c} + \Pr(s_1 = F) W_{2F}^{pc} \\
&= \frac{1-c}{2B}\left[2 + 2B - c(2+\delta) + 2m_1 + \frac{c\delta(-1 + c + M - 2m_1)^2}{(-1 + c + 2c\delta)^2}\right] \\
\frac{\partial W}{\partial m_1} &= \frac{1-c}{2B}\left[2 - \frac{4c\delta(c - 2m_1 + M - 1)}{(2c\delta + c - 1)^2}\right].
\end{aligned}$$

This case only exists for $M > 1-c$. *We note that the derivative changes sign from negative to positive exactly once in this region.*

5. If $m_1 < \frac{M-2c\delta}{2}$,

$$\begin{aligned} W &= W_1 + \Pr(s_1 = S) W_{2S}^{c} + \Pr(s_1 = F) W_{2F}^{nc} \\ &= \frac{1-c}{2B}\left[2 + 2B - 2c(1+\delta) + M\right] \\ \frac{\partial W}{\partial m_1} &= 0. \end{aligned}$$

This case only exists for $M > 2c\delta$.

Given that the welfare function is continuous, the maximum welfare in case 3 is higher than case 1 and 5, and how welfare varies with respect to m_1 in region 2 and 4, the result in the proposition follows.

A.5 Proof of Proposition 5

Proof. Suppose the optimal $m_1 < m_2$, we show this leads to a contradiction. Notice welfare $W_1 + E[W_2] - K(m_1, m_2)$ is

$$L = -K(m_1, m_2) + \frac{1-c}{2B}\begin{cases} 2(m_1 + 1 - c + B) - c\delta & \text{if } m_2 < m_1 + (1-c) \\ 2(m_1 + 1 - c + B) - c\delta + \frac{c\delta(-1+c-m_1+m_2)^2}{(-1+c+2c\delta)^2} & \text{if } m_1 + (1-c) < m_2 < m_1 + 2c\delta \\ m_1 + m_2 + 2(1 - c + B - c\delta) & \text{if } m_2 > m_1 + 2c\delta. \end{cases} \tag{17}$$

We want to show that the above is not optimal because it is strictly dominated by the welfare at $\left(m_1', m_2'\right) = \left(\frac{1}{2}[m_1 + m_2], \frac{1}{2}[m_1 + m_2 - 2c]\right)$, which equals

$$L' = \frac{1-c}{2B}\left[m_1 + m_2 + 2(1 - c + B) - c\delta\right] - K\left(\frac{1}{2}(m_1 + m_2), \frac{1}{2}[m_1 + m_2 - 2c]\right) \tag{18}$$

The case when $m_2 < m_1 + (1-c)$ and when $m_2 > m_1 + 2c\delta$ are straightforward. It remains to show that $W' > W$ when $m_1 + (1-c) < m_2 < m_1 + 2c\delta$. Note that

$$\begin{aligned} sgn\left(L' - L\right) &= sgn\left[(m_2 - m_1) - \frac{c\delta(-1 + c - m_1 + m_2)^2}{(-1 + c + 2c\delta)^2}\right] \\ &= sgn\left\{-c\delta\left[(-1+c)^2 + (m_2 - m_1)^2 + 2(-1+c)(m_2 - m_1)\right] + (m_2 - m_1)(-1 + c + 2c\delta)^2\right\} \\ &= sgn\left\{-c\delta(m_2 - m_1)^2 + \left[2c\delta(1-c) + (-1 + c + 2c\delta)^2\right](m_2 - m_1) - c\delta(-1+c)^2\right\}. \end{aligned}$$

It suffices to show $sgn\left\{-c\delta(m_2 - m_1)^2 + \left[2c\delta(1-c) + (-1 + c + 2c\delta)^2\right](m_2 - m_1) - c\delta(-1+c)^2\right\} > 0$ at both $m_2 = m_1 + 2c\delta$ and $m_2 = m_1 + (1-c)$, which is straightforward algebra.

Notice that when the cost is separable and symmetric,

$$\begin{aligned} & K(m_1, m_2) - K\left(\frac{1}{2}[m_1 + m_2], \frac{1}{2}[m_1 + m_2 - 2c]\right) \\ = & \left[K(m_2) - K\left(\frac{1}{2}[m_1 + m_2]\right)\right] - \left[K\left(\frac{1}{2}[m_1 + m_2 - 2c]\right) - K(m_1)\right] > 0 \end{aligned} \tag{19}$$

due to K being weakly convex and increasing. The above also holds when $\frac{1}{2}[m_1 + m_2 - 2c]$ is replaced by $\frac{1}{2}(m_1+m_2)$ and K only depends on m_1+m_2 and $|m_1-m_2|$ and is increasing in $|m_1-m_2|$. Therefore initial intervention is strictly emphasized under those conditions. $\square$

A.6 Proof of Proposition 6 and Corollaries 2 and 3

Proof. The following identity holds

$$
\begin{aligned}
\frac{\partial}{\partial \chi} Y(m_1;\chi) &= \max_{m_2} \left[\mathbb{E}[W_2(m_1,m_2) - (K(m_1,m_2) - K(m_1,0))]\right] \\
&= \left[\mathbb{I}_{\{m_2^*>m_1+1-c\}} + \mathbb{I}_{\{m_1>c\}}\mathbb{I}_{\{0<m_2^*<m_1-c\}}\right] (\mathbb{E}[W_2(m_1,m_2^*)] - (K(m_1,m_2^*) - K(m_1,0))) \\
&\quad + \mathbb{I}_{\{m_1\geq c\}}\mathbb{I}_{\{m_2^*=m_1-c\}} (\mathbb{E}[W_2(m_1,m_1-c)] - (K(m_1,m_1-c) - K(m_1,0))) \\
&\quad + \mathbb{I}_{\{m_1\leq c\}}\mathbb{I}_{\{m_2^*=0\}}\mathbb{E}[W_2(m_1,0)] \qquad (20)
\end{aligned}
$$

Note that fixing m_1, increasing m_2 does not increase welfare $\mathbb{E}[W_2]$ in $[m_1-c, m_1+1-c]$, thus the four indicator products sum to one. This is seen in Figure 5, with the four indicators corresponding to $m_2 > m_1+1-c$, $m_2 < m_1-c$, $m_2 = m_1-c$, and $m_1-c < m_2 \leq m_1+1-c$ respectively. The first term is non-increasing in m_1 while the last two terms are non-decreasing. In general, the overall expression is non-monotone in m_1 because its value could jump up or down when the indicators change values. However, if parameters are such that one indicator function is always one, then the expression is monotone in m_1 and we could draw robust comparative statics. If one of the first two indicators is always 1 as we vary m_1, then Y has decreasing differences in m_1 and χ; if one of the last two indicators is always 1, Y has increasing differences. The conclusions then follow from Theorem 2.1 in Athey, Milgrom, and Roberts (1998).

Corollaries 2 and 3 provide some examples of sufficient conditions that lead to underintervention or overintervention globally when the government is myopic. We prove them below, but note that there are other sufficient conditions, especially ones on cost parameters defined through levels rather than derivatives:

If $K_1(c,\cdot) > \frac{1-c}{B}$, we have $m_1^* < c$ because the maximum marginal benefit of m_1 on investors' welfare is $\frac{1-c}{B}$, $K_1(c,\cdot) \geq \frac{1-c}{B}$ implies $m_1^* \leq c$. Therefore $m_2^* > m_1 - c$ for sure and $W_{2S} = (1-c)$. Given $K_2(\cdot, c(1+2\delta)) < \frac{1-c}{2B}\frac{\delta}{1+2\delta}$, we have $K(\cdot, c(1+2\delta)) < \frac{c(1-c)\delta}{2B}$, then $\frac{B-m_1-1+c}{2B}W_{2F} > K(m_1, m_1+2\delta c) - K(m_1,0)$. Thus the increase in W_{2F} exceeds the intervention cost at $m_2 = m_1 + 2\delta c$, $m_2^* > m_1 + 1 - c$. When this happens we know $\frac{\partial Y}{\partial \chi}$ equals the first term with the first indicator product being one, and is non-increasing in m_1. Thus Y has decreasing differences in m_1 and χ.

Alternatively, if $K_1(b,\cdot) > \frac{1-c}{B}$, we have $m_1^* < b$ for b potentially bigger than c. If the cost in the second intervention is sufficiently small such that $m_2^* > b + 2c\delta > 2c\delta + m_1^*$, we also have the first indicator product being 1 and Y has decreasing differences in m_1 and χ. One sufficient condition is $[W_2(m_1^*, m_1^*+2c\delta) - W_2(m_1^*, m_1^*-c)]/c(1+2\delta) > K_2(b+2c\delta)$, i.e., $K_2(b+2c\delta) < \frac{(1-c)\delta}{2B(1+2\delta)}$.

Next for the last indicator to be one always, $K_1(c,\cdot) > \frac{1-c}{B}$. In addition, $K_2(\cdot, 1-c) \geq \frac{1-c}{B}\frac{c\delta}{2c\delta+c-1}$ is a sufficient condition for $m_2^* = 0$ because this implies the cost exceeds the benefit at both $m_2 = 1-c+m_1$ and $m_2 = m_1 + 2c\delta$, and the convexity of K in m_2 excludes $m_2^* > 1-c+m_1$. Then $\frac{\partial Y}{\partial \chi}$ equals the last term and is non-decreasing in m_1. We could alternatively use $K(\cdot, 1-c) \geq \frac{c(1-c)\delta}{2B}$ as a sufficient condition on cost, rather than on the derivative. The same goes for other sufficient conditions that we provide in this proposition.

Next, if $K_1(b,\cdot) > \frac{1-c}{B}$ and $K_1(c,\cdot) < \frac{1-c}{B}\frac{\delta-c(1+2\delta)}{2\delta-c(1+2\delta)} \leq \frac{\partial Y}{\partial m_1}$ for some $b > c$, we have $b > m_1^* > c$. On the one hand, if in addition the minimum marginal benefit in region $m_2 \leq m_1^* - c$ is bigger than the maximum marginal cost, i.e., $\min\left\{\frac{1-c}{2B}, \frac{1-c}{2B}\frac{2\delta(1-c)}{2\delta-c(1+2\delta)}\right\} \geq \frac{1-c}{2B} \geq K_2(\cdot, b-c) > K_2(\cdot, m_1 - c)$, we have $\mathbb{E}[W_2]-(K(m_1,m_2)-K(m_1,0))$ increasing in the entire region of $[0, m_1-c]$. Moreover, if $K_2(m_1^*, m_1^*+1-c) > K_2(\cdot,1) \geq \frac{(1-c)c\delta}{B(2c\delta+c-1)}$, the lower bound on marginal cost is weakly bigger than the maximum marginal benefit (RHS) in the region $m_2 \geq 1-c+m_1^*$, therefore $m_2^* = m_1^* - c$ and the third indicator product is always one. Y has increasing differences in m_1 and χ.

On the other hand, $K_2(\cdot,0) > \frac{1-c}{2B}\frac{2\delta}{2\delta-c(1+2\delta)}$ implies $\frac{1-c}{2B}\frac{2\delta}{2\delta-c(1+2\delta)} < K_2(m_1, m_1-c)$, which means the maximum marginal benefit in the region $m_2 \leq m_1^* - c$ taken at equality is less than the marginal cost. Thus $m_2^* < m_1^* - c$ and the second indicator product is always one. Y has decreasing differences in m_1 and χ.

These sufficient conditions are stated in the corollaries. We note that instead of directly computing the derivatives for the first term in $\frac{\partial Y}{\partial \chi}$, when m_2^* is interior we can apply envelop theorem to compute the partial derivative in m_1 of $\mathbb{E}[W_2(m_1,m_2)] - (K(m_1,m_2) - K(m_1,0))$. Because $K_{12} = 0$, we know the partial derivative must be negative in the corresponding regions from figure 3. $\square$

A.7 Proof of Proposition 7

Proof. Define $n_1 = m_1$ and $n_2 = \frac{m_2}{\lambda}$. The two inequalities on funds' survival translate into

$$A_1 + n_1 \geq \theta$$
$$A_2 + n_2 \geq \theta$$

For the remaining analysis, we prove that a benevolent government who faces a hard budget constraint always prefers to first intervene into fund 1–the relatively smaller one.

We prove by backward induction. In the first step, we fix the choice of ι to be 1 and study the optimal intervention plan (n_1, n_2) when λ varies. The budget constraint shows as $n_1 + n_2\lambda = M$. We will show that for both $\lambda > 1$ and $\lambda \in (0,1)$, the government will intervene up to $n_2 = n_1 - c$. The case that $\lambda \in (0,1)$ is simply isomorphic to $\iota = 2$. Thus, we establish the result that the government will always induce perfectly correlated intervention outcomes. In step 2, we compare different chocies of $\iota \in \{1,2\}$ and show the optimal intervention order: $\iota = 1$ if $\lambda > 1$ and $\iota = 2$ if $\lambda \in (0,1)$. Thus, the smaller fund should always be bailed out first.

Lemma 7
Suppose $\iota = 1$, $\forall\lambda > 0$, the optimal intervention plan is:

$$n_1^* = \frac{M + c\lambda}{1+\lambda}$$
$$n_2^* = \frac{M - c}{1+\lambda}.$$

Under (n_1^*, n_2^*), intervention always leads to correlated outcomes: $s_1 = s_2$.

Proof. With heterogeneous fund sizes, the aggregate social welfare naturally follows.

1. If $n_1 > \frac{M+2\delta\lambda(1-c)}{1+\lambda}$,

$$\begin{aligned} W &= \frac{1-c}{2B}\left\{\left[1+B-c\left(1+\delta\right)\right]\left(1+\lambda\right)+M\right\} \\ \frac{\partial W}{\partial n_1} &= 0. \end{aligned}$$

2. If $\frac{M+c\lambda}{1+\lambda} < n_1 < \frac{M+2\delta\lambda(1-c)}{1+\lambda}$,

$$\begin{aligned} W &= \frac{1-c}{2B}\left[1+B-c\left(1+\delta\right)+n_1\right] \\ &\quad +\lambda\frac{1-c}{2B}\left[1+n_1-c+B+\frac{\delta c\left(c-n_1+n_2\right)^2+2\delta\left(c-n_1+n_2\right)\left[2\delta-c\left(1+2\delta\right)\right]}{\left[2\delta-c\left(1+2\delta\right)\right]^2}\right] \\ \frac{\partial W}{\partial n_1} &< 0. \end{aligned}$$

3. If $\frac{M-(1-c)\lambda}{1+\lambda} < n_1 < \frac{M+c\lambda}{1+\lambda}$,

$$\begin{aligned} W &= \frac{1-c}{2B}\left[\left(1+B-c+n_1\right)\left(1+\lambda\right)-c\delta\right] \\ \frac{\partial W}{\partial n_1} &> 0. \end{aligned}$$

4. If $\frac{M-2c\delta\lambda}{1+\lambda} < n_1 < \frac{M-(1-c)\lambda}{1+\lambda}$,

$$\begin{aligned} W &= \frac{1-c}{2B}\left[1+B-c\left(1+\delta\right)+n_1\right]+\lambda\frac{1-c}{2B}\left[1+B-c+n_1\right] \\ &\quad +\lambda\frac{1-c}{2B}\frac{c\delta\left(-1+c-n_1+n_2\right)^2}{\left(-1+c+2c\delta\right)^2} \\ \frac{\partial W}{\partial n_1} & \quad \textit{changes from negative to positive exactly once.} \end{aligned}$$

5. If $n_1 < \frac{M-2c\delta\lambda}{1+\lambda}$,

$$\begin{aligned} W &= \frac{1-c}{2B}\left\{\left[1+B-c\left(1+\delta\right)\right]\left(1+\lambda\right)+M\right\} \\ \frac{\partial W}{\partial n_1} &= 0. \end{aligned}$$

It easily establishes that $\frac{\partial W}{\partial n_1} = 0$ in case 1 and 5, $\frac{\partial W}{\partial n_1} > 0$ in case 3, and $\frac{\partial W}{\partial n_1} < 0$ in case 2. Similar to the proof of Proposition 4, the aggregate welfare in case 1 equals that in case 5, the maximal welfare is attained at the right boundary of case 3. That is, when $n_1 = \frac{M+c\lambda}{1+\lambda}$ □

Now that we have established the result on optimal intervention plan (n_1^*, n_2^*) conditional on ι, we can compare the social welfare under different ι.

If $\iota = 1$, the total social welfare directly follows optimal $(n_1^*, n_2^*) = \left(\frac{M+c\lambda}{1+\lambda}, \frac{M-c}{1+\lambda}\right)$

$$W\big|_{\iota=1} = \frac{1-c}{2B}\left[\left(1 + B - c + \frac{M-(1-c)\lambda}{1+\lambda}\right)(1+\lambda) - c\delta\right].$$

If $\iota = 2$, then $(n_1^*.n_2^*) = \left(\frac{M+c}{1+\lambda}, \frac{M-c\lambda}{1+\lambda}\right)$ and the total social welfare is

$$W\big|_{\iota=2} = \frac{1-c}{2B}\left[(1+B-c)(1+\lambda) - c\lambda\delta + (1+\lambda)\frac{M+c}{1+\lambda}\right].$$

Clearly,

$$W\big|_{\iota=1} - W\big|_{\iota=2} = \frac{1-c}{2B}c(1+\delta)(\lambda-1) > 0$$

□

A.8 Proof of Proposition 9

Proof. If $s_1 = S$, increasing m_{2S} beyond $m_1^* - c$ incurs additional cost without increasing $\mathbb{E}[W_2]$, as is clear in Figure 1. Thus, $m_{2S}^* \leq m_1^* - c$. When $s_1 = F$, the condition on the parameter means that the marginal cost of increasing m_{2F} at $1-c$ exceeds the marginal benefit, which is bounded above by $\frac{(1-c)^2}{B-1}$. Thus $m_{2F}^* < 1-c$. Subsequently, $m_{2F}^* = 0$ because increasing m_{2F} does not increase $\mathbb{E}[W_2]$, also clearly seen in Figure 1. □

A.9 Proof of Proposition 10

In general, the government chooses $\{m_1, m_{2S}, m_{2F}\}$ to maximize welfare. For a given m_1, define the objective as

$$\begin{aligned} Y(m_1;\chi) \quad = \quad & W_1 - K(m_1,0) + \chi\left[\frac{B+m_1+1-c}{2B}\max_{m_{2S}}[W_{2S} - (K(m_1,m_{2S}) - K(m_1,0))]\right. \\ & \left. + \frac{B-m_1-1+c}{2B}\max_{m_{2F}}[W_{2F} - (K(m_1,m_{2F}) - K(m_1,0))]\right] \end{aligned} \tag{21}$$

Here, $\chi \in [0,1]$ measures how much the government cares the fate of the second period's fund.

Proof. The proof is very similar to that in Proposition 6 and Corollaries 2 and 3, albeit algebraically more involved. We start with the first half of the proposition. Because the maximum marginal benefit of m_1 on the investors' total welfare is $\frac{1-c}{B}$, $K_1(c,\cdot) \geq \frac{1-c}{B}$ implies $m_1^* \leq c$. Figure 1 implies when $m_{2S} > m_1 - c$, welfare is weakly decreasing in m_2, thus $m_{2S}^* = 0$.

$$
\begin{aligned}
& \frac{\partial}{\partial m_1}\frac{\partial}{\partial \chi} Y(m_1;\chi) \\
= \; & \frac{d}{dm_1}\left[\frac{B+m_1+1-c}{2B}\max_{\{m_{2S}\}}[W_{2S}-(K(m_1,m_{2S})-K(m_1,0))]\right. \\
& \left. +\frac{B-m_1-1+c}{2B}\max_{\{m_{2F}\}}[W_{2F}-(K(m_1,m_{2F})-K(m_1,0))]\right] \\
= \; & \frac{1-c}{2B}+\frac{\partial}{\partial m_1}\left[\frac{B-m_1-1+c}{2B}\max_{\{m_{2F}\}}[W_{2F}-(K(m_1,m_{2F})-K(m_1,0))]\mathbb{I}_{\{m_{2F}>m_1+1-c\}}\right] \\
= \; & \frac{1-c}{2B}+\frac{1}{2B}[K(m_1,m_{2F})-K(m_1,0)]\mathbb{I}_{\{m_{2F}^*>m_1+1-c\}}-\frac{1-c}{2B}\mathbb{I}_{\{m_{2F}^*>m_1+2c\delta\}} \\
& -\frac{1-c}{2B}\frac{2c\delta(-1+c-m_1+m_{2F}^*)}{(-1+c+2c\delta)^2}\mathbb{I}_{\{m_{2F}^*\in(m_1+1-c,m_1+2c\delta]\}} \\
\geq \; & 0
\end{aligned} \tag{22}
$$

the second equality holds by Envelope Theorem and by the fact that if $m_{2F} \leq m_1+1-c$, then taking $m_{2F}=0$ dominates as seen in Figure 1. When $K(\cdot,1-c)-K(\cdot,0) > \frac{c\delta(1-c)}{B-(1-c)}$, $W_{2F}(m_2 = m_1+2\delta c) < K(m_1, M_1+1-c)$, thus $m_{2F}^*=0$; when $K(\cdot,1-c)-K(\cdot,0) > \frac{(1-c)^2}{2c\delta-1+c}$, the last two terms on the RHS of the third equality is dominated by the first two term as $m_{2F}^*=0$. In either case, we have the whole expression being non-negative.

From $1-c-K(\cdot,1-c)+K(\cdot,0)-(2+B-c)K_2(\cdot,1-c)>0$, we have $K_2(\cdot,1-c) < \frac{1-c}{B+2-c}\frac{2\delta}{2\delta-(1+2\delta)c}$, the marginal benefit for increasing m_{2S} in W_{2S} exceeds the cost as long as $m_{2S} < m_1-c$, therefore $m_{2S}^* = [m_1-c]^+$. When $m_1 \leq c$, $m_{2S}^*=0$, the local derivative is the same as above, thus is positive. When $m_1 \geq c$, $m_{2S}^* = m_1-c$, $m_{2F}^*=0$, the local derivative is

$$
\begin{aligned}
& \frac{\partial}{\partial m_1}\frac{\partial}{\partial \chi} Y(m_1;\chi) \\
= \; & \frac{1-c}{2B}-\frac{1}{2B}[K(m_1,m_{2S})-K(m_1,0)]-\frac{m_1+B+1-c}{2B}\frac{\partial}{\partial m_1}[K(m_1,m_1-c)-K(m_1,0)] \\
= \; & \frac{1-c}{2B}-\frac{1}{2B}[K(m_1,m_{2S})-K(m_1,0)]-\frac{m_1+B+1-c}{2B}[K_2(m_1,m_1-c) \\
& +K_1(m_1,m_1-c)-K_1(m_1,0)], \quad \textit{note } K_1(m_1,m_1-c)=K_1(m_1,0) \\
= \; & \frac{1-c}{2B}-\frac{1}{2B}[K(m_1,m_{2S})-K(m_1,0)]-\frac{m_1+B+1-c}{2B}[K_2(m_1,m_1-c)] \\
\geq \; & \frac{1}{2B}[1-c-K(m_1,1-c)+K(m_1,0)-(2+B-c)K_2(m_1,1-c)] \geq 0
\end{aligned} \tag{23}
$$

The last two inequalities come from the fact $m_1 \leq 1$, and the fact $1-c-K(\cdot,1-c)+K(\cdot,0)-(2+B-c)K_2(\cdot,1-c)>0$. Therefore we have Y has increasing differences in (m_1,χ).

Now to prove the second half of the theorem, Note $K_2(\cdot,0) > \frac{1-c}{B+c-2}\frac{2\delta}{2\delta-c(1+2\delta)}$, thus $K(\cdot,1-c) > \frac{1-c}{B+c-2}\frac{1-c}{1-c(1+\frac{1}{2\delta})} > \frac{1-c}{B-2+c}$. And W_{2F} at $m_2 = m_1+2c\delta$ is still less than $K(m_1,m_1+1-c)-K(m_1,0)$. Consequently $m_{2F}^*=0$. $K_2(\cdot,0) > \frac{1-c}{B+c-2}\frac{2\delta}{2\delta-c(1+2\delta)}$ also implies $\frac{1-c}{B+m_1+1-c}\frac{2\delta}{2\delta-c(1+2\delta)} < K_2(m_1,m_1-c)$, which means $m_{2S}^* < m_1 - c$.

$$\begin{aligned}
& \frac{\partial}{\partial m_1}\frac{\partial}{\partial \chi}Y(m_1;\chi) \\
= \quad & \frac{d}{dm_1}\left[\frac{B+m_1+1-c}{2B}\max_{\{m_{2S}\}}[W_{2S}-(K(m_1,m_{2S})-K(m_1,0))]\right. \\
& \left.+\frac{B-m_1-1+c}{2B}\max_{\{m_{2F}\}}[W_{2F}-(K(m_1,m_{2F})-K(m_1,0))]\right] \\
= \quad & \frac{\partial}{\partial m_1}\left[\frac{B-m_1-1+c}{2B}\max_{\{m_{2F}\}}[W_{2S}-(K(m_1,m_{2S})-K(m_1,0))]\right] \\
= \quad & \frac{\partial}{\partial m_1}W_{2S}(m_{2S}^*)+\frac{B-m_1-1+c}{2B}\frac{\partial}{\partial m_1}[K(m_1,m_{2S}^*)-K(m_1,0)] \\
& -\frac{1}{2B}(K(m_1,m_{2S})-K(m_1,0))<0
\end{aligned} \tag{24}$$

The first term is negative as $m_{2S}^* < m_1 - c$. The second term is non-positive as K is weakly increasing in second argument. Finally, the third term is zero as K has zero cross-partials.

Finally, the above argument would not work if $m_1^* \leq c$. But this can be ruled out in that the minimum $\frac{\partial Y}{\partial m_1} = \frac{1-c}{2B}\left[1-\frac{c(1+2\delta)}{2\delta-c(1+2\delta)}\right] = \frac{1-c}{B}\frac{\delta-c(1+2\delta)}{2\delta-c(1+2\delta)}$. Notice we have used the fact that $m_{2F}^* = 0$. This is bigger than the marginal cost $K_1(c,\cdot)$, thus $m_1^* > c$, and we indeed have an interior m_{2S}^*.

□

A.10 Proof of Proposition 11

Proof. We prove the case under separable cost functions to focus on the information channel: $K(m_1,m_2) = k(m_1)+k(m_2)$. Again, let L (L') be the total welfare net the intervention cost if the smaller (larger) fund is saved first.

First, consider the case that $m_1^* \leq c$ and $m_1^{'*} \leq c$. In this case,

$$\begin{aligned}
L \quad &= \quad \max_{m_1}\frac{1-c}{2B}\left[(1+B-c+m_1)(1+\lambda)-c\delta\right]-k(m_1) \\
L' \quad &= \quad \max_{m_1'}\frac{1-c}{2B}\left[\left(1+B-c+\frac{m_1'}{\lambda}\right)(1+\lambda)-c\lambda\delta\right]-k\left(m_1'\right).
\end{aligned}$$

Since $\lambda > 1$, obviously $L > L'$.

Next, consider the case that both $m_1^* > c$ and $m_1^{'*} > c$.

$$\begin{aligned}
L \quad &= \quad \max_{m_1}\frac{1-c}{2B}\left[(1+B-c+m_1)(1+\lambda)-c\delta\right]-k(m_1)-\frac{B+m_1+1-c}{2B}k((m_1-c)\lambda) \\
L' \quad &= \quad \max_{m_1'}\frac{1-c}{2B}\left[\left(1+B-c+m_1'\right)(1+\lambda)-c\lambda\delta\right]-k\left(\lambda m_1'\right)-\frac{B+m_1'+1-c}{2B}k(m_1\lambda-c).
\end{aligned}$$

In this case, even if $m_1^* = \frac{m_1^{'*}}{\lambda}$, $L\big|_{m_1^*=\frac{m_1^{'*}}{\lambda}} - L^{'}_{m_1^{'}=m_1^{'*}}$ equals

$$L\big|_{m_1^*=\frac{m_1^{'*}}{\lambda}} - L^{'}_{m_1^{'}=m_1^{'*}} = c\delta\left(\lambda-1\right) + \left[k\left(m_1^{'*}\right) - k\left(\frac{m_1^{'*}}{\lambda}\right)\right] + \frac{B + \frac{m_1^{'*}}{\lambda} + 1 - c}{2B}\left[k\left(\frac{m_1^{'*}}{\lambda} - c\right) - k\left(\left(\frac{m_1^{'*}}{\lambda} - c\right)\lambda\right)\right].$$

Since $k\left(\cdot\right)$ is convex,

$$\begin{aligned} L\big|_{m_1^*=\frac{m_1^{'*}}{\lambda}} - L^{'}_{m_1^{'}=m_1^{'*}} &> \left[k\left(m_1^{'*}\right) - k\left(\frac{m_1^{'*}}{\lambda}\right)\right] - \left[k\left(\left(\frac{m_1^{'*}}{\lambda} - c\right)\lambda\right) - k\left(\frac{m_1^{'*}}{\lambda} - c\right)\right] \\ &> 0. \end{aligned}$$

□

B Full Analysis of Section 3.2.2

Is there any equilibrium that agents choose to run irrespective of their signals? In other words, the threshold x_2^* that agents in period 2 adopt satisfy $x_2^* \leq \theta_1^* - \delta$. It turns out that such an equilibrium exists if and only if $m_2 < m_1 + 1 - c$. In this type of equilibrium, government intervention in the first period has a dominant effect on coordination among investors in the second period. Therefore, we name it after *Subgame Equilibrium with Dynamic Coordination*.

Lemma 8 describes this type of equilibrium. Since it is common knowledge that $\theta > \theta_1^*$, any equilibrium with $(\theta_2^* < \theta_1^*, x_2^* < \theta_1^* - \delta)$ is equivalent to $(\theta_2^*, x_2^*) = (-\infty, -\infty)$.

Lemma 8

Subgame Equilibrium with Dynamic Coordination

If $s_1 = F$, $(\theta_2^*, x_2^*) = (-\infty, -\infty)$ consists an equilibrium if and only if $m_2 < m_1 + 1 - c$.

Next, we turn to threshold equilibria with $\theta_2^* > \theta_1^*$ so that the fate of the fund in period 2 still has uncertainty. Similar to the analysis when $s_1 = S$, we consider two types of equilibria, depending on whether the marginal investor find the public news useful.

Lemma 9

Subgame Equilibrium without Dynamic Coordination

If $s_1 = F$ and $m_2 > m_1 + 2c\delta$, there exists a equilibrium with thresholds

$$\begin{cases} \theta_2^* = 1 + m_2 - c \\ x_2^* = 1 + m_2 - c + \delta\left(1 - 2c\right). \end{cases} \tag{25}$$

Lemma 10

Subgame Equilibrium with Partial Dynamic Coordination

If $s_1 = F$ and $\min\left\{m_1 + 2c\delta, m_1 + 1 - c\right\} < m_2 < \max\left\{m_1 + 2c\delta, m_1 + 1 - c\right\}$, there exists an equilib-

rium with thresholds

$$\begin{cases} \theta_2^* = 1 + m_2 - c - \frac{(1-c)(m_1+2c\delta-m_2)}{c(1+2\delta)-1} \\ x_2^* = 1 + m_2 - c + \delta\left(1-2c\right) - \frac{(1-c)(1+2\delta)(m_1+2c\delta-m_2)}{c(1+2\delta)-1}. \end{cases} \tag{26}$$

Given any (m_1, m_2) and $s_1 = F$, Proposition 2 clearly follows Lemma 8, 9 and 10.

C Full Analysis of Normally Distributed Signals

The equilibrium outcome in period 1 is characterized by two thresholds (θ_1^*, x_1^*) which satisfy

$$\begin{aligned} A_1\left(\theta_1^*\right) + m_1 &= \theta_1^* \\ \Pr\left(\theta < \theta_1^* \middle| x_1 = x_1^*\right) &= c \end{aligned}$$

where $A_1\left(\theta_1^*\right) = \Pr\left(x_1 < x_1^* \middle| \theta = \theta_1^*\right)$ is the measure of investors who choose to roll over. Simple calculation shows that,

$$\begin{aligned} \theta_1^* &= 1 + m_1 - c \\ x_1^* &= 1 + m_1 - c - \delta\Phi^{-1}\left(c\right) \end{aligned}$$

The equilibrium in period 2 is again, state-independent. We discuss the outcomes when $s_1 = S$ and leave the case $s_1 = F$ to the Appendix. When the intervention in the first period has succeeded, equilibrium in the second period will be either a subgame equilibrium with full dynamic coordination (similar to Lemma 2), or one with partial dynamic coordination (similar to Lemma 4). The case without dynamic coordination vanishes as the support of the noise now spans between (∞, ∞). The first type of equilibrium is denoted as $(\theta_2^*, x_2^*) = (\infty, \infty)$ and any equilibrium with $(\theta_2^* > \theta_1^*, x_2^* = \infty)$ is equivalent. The necessary conditions that $(\theta_2^*, x_2^*) = (\infty, \infty)$ consists an equilibrium are

$$\begin{aligned} \Pr\left(1 + m_2 > \theta \middle| \theta < \theta_1^*\right) &= 1 \\ \Rightarrow m_2 &> m_1 - c. \end{aligned}$$

Likewise, the necessary conditions that an equilibrium with partial dynamic coordination exists is that the solution (θ_2^*, x_2^*) to the equation system

$$\begin{aligned} A_2\left(\theta_2^*\right) + m_2 &= \theta_2^* \\ \Pr\left(\theta < \theta_2^* \middle| x_2^*, \theta < \theta_1^*\right) &= c \end{aligned}$$

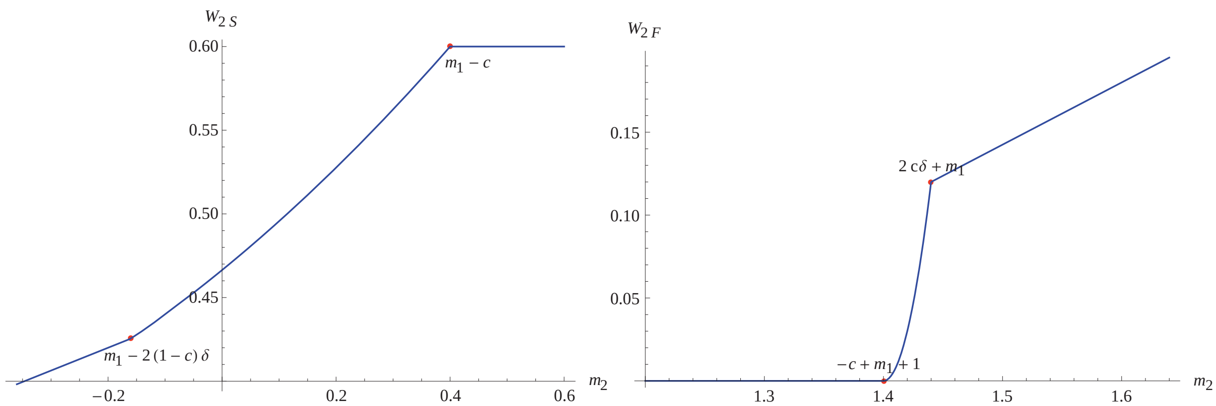
exists and satisfies $\theta_2^* < \theta_1^*$. Equivalently, we are looking for θ_2^* that solves

$$1 - \left(\theta_2^* - m_2\right) = c\Phi\left(\frac{\theta_1^* - \theta_2^* - \delta\Phi^{-1}\left(\theta_2^* - m_2\right)}{\delta}\right) \tag{27}$$

We numerically solve equation (27).

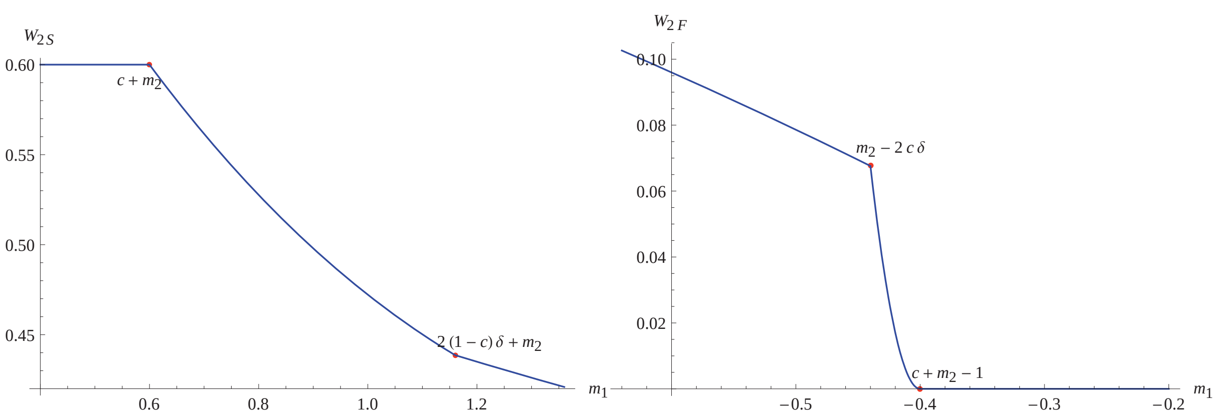
Figures

Figure 1: W_{2S} and W_{2F} as A Function of m_2



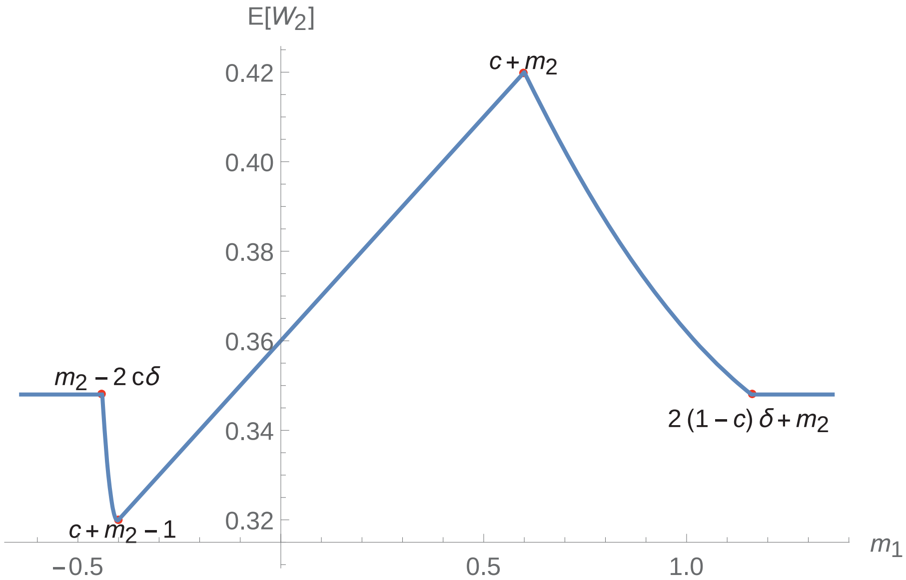
Parameters: $\delta = 0.8,\ c = 0.4,\ B = 3,\ m_1 = 0.8.$

Figure 2: W_{2S} and W_{2F} as A Function of m_1



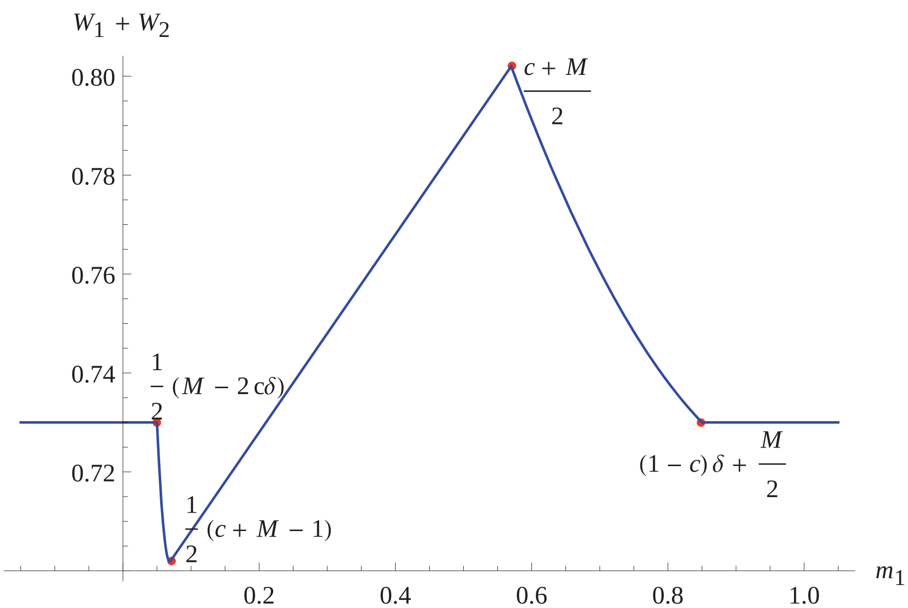
Parameters: $\delta = 0.8,\ c = 0.4,\ B = 3,\ m_2 = 0.2.$

Figure 3: $E[W_2]$ as a function of m_1



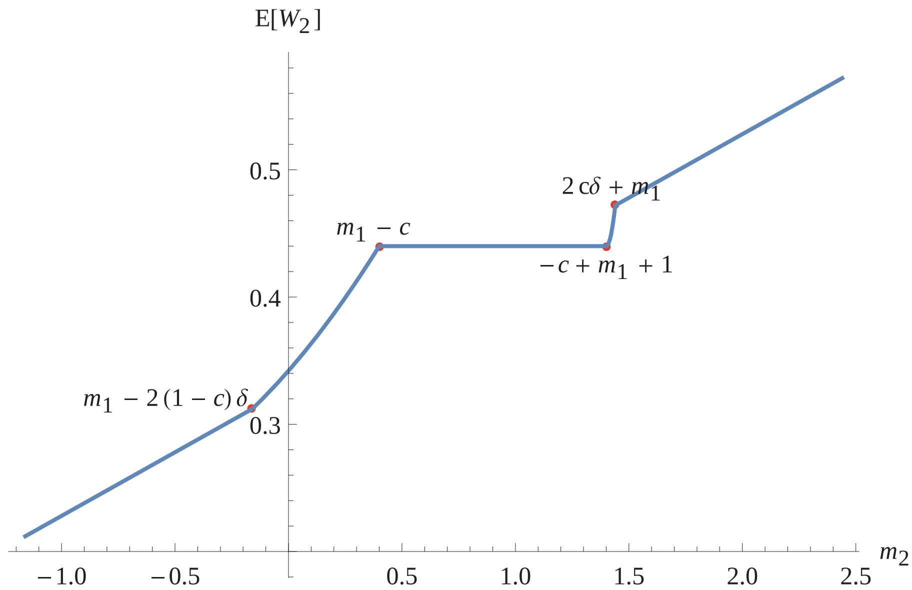
Parameters: $\delta = 0.8,\ c = 0.4,\ B = 3,\ m_2 = 0.2.$

Figure 4: $W_1 + W_2$ as A Function of m_1 $(m_1 + m_2 = M > 2c\delta)$



Parameters: $\delta = 0.8,\ c = 0.4,\ B = 3,\ M = 0.74.$

Figure 5: $E[W_2]$ as A Function of m_2



Parameters: $\delta = 0.8,\ c = 0.4,\ B = 3,\ m_1 = 0.8.$

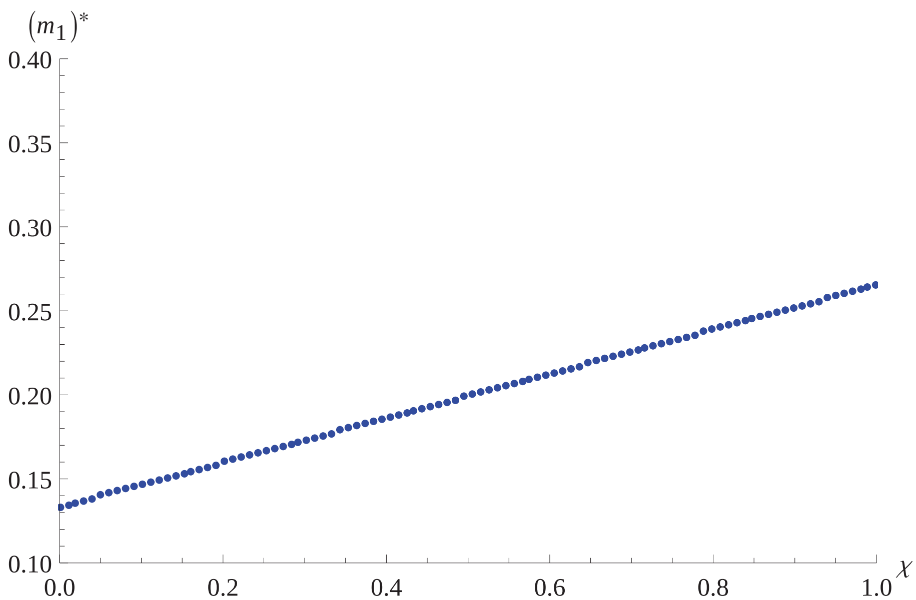


Figure 6: Endogenous initial intervention as a function of the extent the policy-maker considers the subsequent intervention

Parameters: $\delta = 1.2,\ c = 0.6,\ B = 3,\ k_1 = 0.5,\ k_2 = 0.5,\ K(m_1, m_2) = \frac{1}{2}k_1 m_1^2 + \frac{1}{2}k_2 m_2^2.$

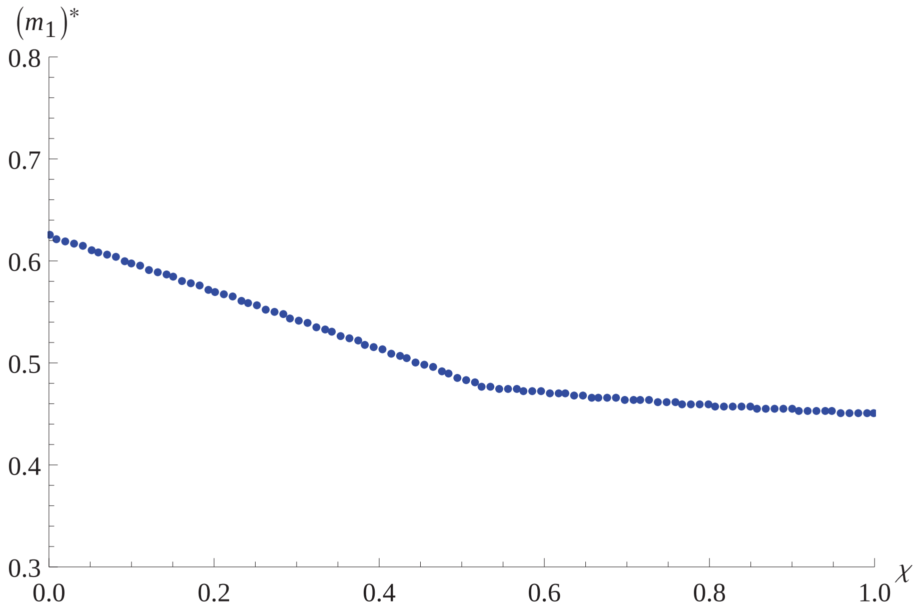


Figure 7: Endogenous initial intervention as a function of the extent the policy-maker considers the subsequent intervention

Parameters: $\delta = 2,\ c = 0.25,\ B = 3,\ k_1 = 0.2,\ k_2 = 0.8,\ K(m_1, m_2) = \frac{1}{2}k_1 m_1^2 + \frac{1}{2}k_2 m_2^2.$

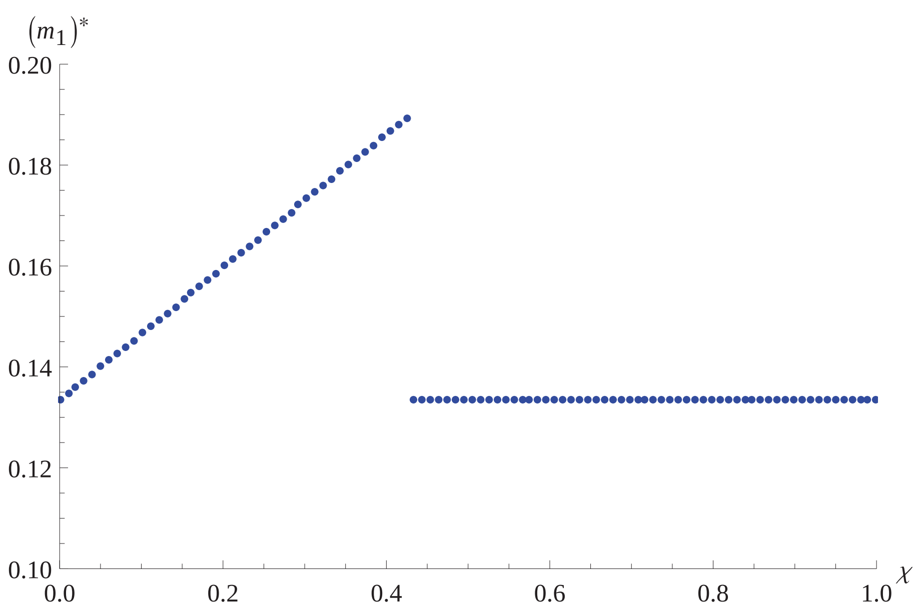


Figure 8: Endogenous initial intervention as a function of the extent the policy-maker considers the subsequent intervention

Parameters: $\delta = 1.2,\ c = 0.6,\ B = 3,\ k_1 = 0.5,\ k_2 = 0.01,\ K(m_1, m_2) = \frac{1}{2}k_1 m_1^2 + \frac{1}{2}k_2 m_2^2.$