SEARCHING HIGH AND LOW: EXTREMAL DEPENDENCE OF INTERNATIONAL SOVEREIGN BOND MARKETS

BOJAN BASRAK
Faculty of Science, Department of Mathematics
University of Zagreb
Croatia
bbasrak@math.hr

PETRA POSEDEL
Zagreb School of Economics and Management
Croatia
pposedel@zsem.hr

MARINA TKALEC
Corresponding author
Institute of Economics Zagreb
Croatia
mtkalec@eizg.hr

MARUŠKA VIZEK
Institute of Economics Zagreb
Croatia
mvizek@eizg.hr

Abstract

This paper examines the degree of interdependence among sovereign bond markets in 24 developed and developing countries during times of stress or crisis using extreme value theory. We discuss tail behavior of individual sovereign bond spreads and compare the shape of that tail to exponential and power-law distributions. We proceed by estimating bivariate tail dependence index χ and search for the evidence of asymptotic tail dependence in sovereign bond spreads series. In order to establish the statistical significance of estimated bivariate tail dependence indices, we construct a bootstrap-based approach to searching for the presence of asymptotic tail dependence derived on the basis of Davis *et al.* (2012). Our empirical findings suggest that the US bond market does not exhibit extreme right tail co-movements with European sovereign bond market turbulences. Even though the UK did not adopt the euro, its sovereign bond market exhibits statistically significant right tail dependencies with a number of Eurozone bond markets, possibly indicating that it is not immune to financial distress originating from the EMU. New EU member states exhibit more frequent right tail dependencies with other new EU member states when compared to old EU members.

Key words: sovereign bond spreads, extreme value theory, tail dependence.

JEL classification: C40, C50, G12, G15.

Introduction[1]

Co-movements in international financial markets have been the subject of intensive empirical examination in the literature. Studies using multivariate GARCH models, regime-switching models, extreme value theory and copulas such as Longin and Solnik (1995; 2001), De Santis and Gerard (1997), Ang and Bekaert (2002), Poon et al. (2004), and Jondeau and Rockinger (2006) evidence the existence of asymmetry in extreme correlations for equity markets: large negative returns are more correlated than large positive returns. Longin and Solnik (2001) also show that in asymptotic terms extreme correlation is zero for very large positive returns and strictly positive for very large negative returns.

Although asymmetric correlation structure is also documented for bond markets, and in particular for sovereign bond markets in studies such as Beber *et al.* (2009), Favero *et al.* (2010), Aßmann and Boysen-Hogref (2012), and Favero (2014), an extensive analysis that focuses on high frequency changes in sovereign bond markets has received far less attention. Instead, the emphasis was placed on the impact of economic news on conditional bond volatility, thus downplaying the importance that rare events such as sovereign debt crises, large changes in investment returns, or even defaults may inflict on sovereign bond yield movements. Our analysis is related to three earlier studies that measure extremal dependence on bond markets, which however focus on both bond and equity markets and assess not only their individual tail characteristics, but also extremal cross-dependence of these markets.

Hartman *et al.* (2004) use extreme value theory to study the likelihood of crashes in equity and sovereign bond markets and extreme co-movements between those two markets. They derive nonparametric estimates for the expected number of market crashes given that at least one market crashes. Their results suggest that simultaneous crashes between equity markets in Germany, France, Japan, the UK, and the US are much more likely than between bond markets of those countries, even though the returns on both markets exhibit statistically significant tail dependence. Cappiello *et al.* (2006) use a Dynamic Conditional Correlation GARCH model to investigate presence of asymmetric volatility in international equity and bond returns for 21 developed countries. They show that national equity return

[1] This work was supported by Croatian Science Foundation under the project 1356.

series exhibit asymmetry in conditional variance, while little evidence is present that would indicate asymmetry in bond returns variance. However, despite the lack of evidence of asymmetric conditional volatilities, bonds (as well as equities) exhibit asymmetry in conditional correlation. Garcia and Tsafack (2011) outline limits of using extreme value theory or bivariate GARCH models in characterizing extremal dependence and propose an alternative regime-switching copula model that includes one normal regime in which tail dependence is symmetric and a second regime characterized by asymmetric dependence and apply it to sovereign bond and equity markets in Canada, France, the US, and the UK. They reaffirm Hartman *et al.* (2004) findings and provide evidence that returns for both markets in both regimes are asymptotically tail dependent, albeit sovereign bond markets in both regimes exhibit smaller propensity for extreme co-movements when compared to equity markets.

Since these three studies suggest that sovereign bond markets are indeed characterized by extreme movements and exhibit tail dependent behavior, the aim of this paper is to provide a comprehensive analysis of extremal dependence of international sovereign bond markets. By applying extreme value theory, we analyze sovereign bond spreads for 23 EU member states and the US. The contribution to the literature of this study is threefold. First, in terms of methodology, our paper is somewhat related to Hartman *et al.* (2004) who develop a novel non-parametric test developed from extreme value theory in order to assess the expected number of market crashes and thus establish tail dependencies between bond and equity markets. Our methodological approach is similar to theirs insofar as it is also grounded in extreme value theory, but differs in terms of the choice of test statistics. We feel that extreme value theory in general is well suited to address tail dependence behavior of financial series than the frequently used conditional correlation analysis which is strongly predisposed towards multivariate normal distribution and thus might underestimate the frequency of rare events in the financial markets. We begin by assessing marginal tail behavior of individual spread changes using standard tools of extreme value theory: qq-plots and Hill estimators. We proceed by estimating the bivariate tail dependence index *chi* (χ) along with the Pearson correlation measure for all country pairs in order to establish the degree of tail dependency. As we are only interested in whether large negative shocks (which we usually describe as sovereign debt crisis and which manifest in rising sovereign spreads) on various sovereign bond markets are interdependent, we only observe what happens in the right tails of both countries. In order to establish the statistical significance of estimated bivariate tail dependence indices, we construct a bootstrap-based approach to searching for the presence of asymptotic tail dependence on the basis of Davis *et al.* (2012). To the best of our knowledge, this is the first paper that applies such a methodological approach.

Second, unlike other studies, the analysis performed in this study covers the turbulent European sovereign debt crisis period. In the light of findings suggesting tail dependence of sovereign bond markets is significantly smaller when compared to equity markets, we feel that the European debt crisis can be viewed as one of those unprecedented tail events that have deeply shaken international sovereign bond markets. It can thus have the potential to significantly upend the nature of extreme co-movements of sovereign bond markets and provide new insight into interdependencies of sovereign bond markets during times of crises.

Third, as extremal dependence of bond markets in developing countries was not studied in the past, our study also contributes to the literature by including ten developing European countries into the analysis. Due to the fact that financial instruments issued by developing countries generally record higher degree of volatility when compared to their developed counterparts, one could expect that developing countries´ sovereign bond spreads might also be characterized by heavier tails and more pronounced tail dependence.

The rest of the paper is organized as follows. In the next section we explain the methodology used to assess the degree of extremal dependence in sovereign bond markets. In the third section we describe the data, while the fourth section discusses empirical findings. We summarize our conclusions in the last section.

Methodology

It is often suggested in the financial econometrics literature that relative returns of stock prices typically follow a distribution of the so-called "power—law type". In statistics, these distributions are also called regularly varying and they represent an extension of the Pareto distributions which are often used in economics. The behavior of exchange rates is also sometimes modeled by distributions in this class. The same modeling framework appears in many other areas of economics, finance, and insurance in particular. The use of such distributions is also justified by theoretical results showing that many standard time series such as GARCH or stochastic volatility models have distributions of that type. On the other hand, understanding tail behavior is of utmost importance for many applications, and risk assessment in particular. Motivated by all this, we explore tail behavior of sovereign spread movements using the regular variation assumption.

Suppose that we have a stationary sequence $X_1, X_2, \dots, X_n$ of sovereign spreads movements with the same marginal distribution $F(x)$. In the case that F exhibits heavy tail of a power-law type, a good indicator for the mass in the tails is the tail index. In order to inference heavy tails for a set of one-dimensional data assumed to be stationary, we need to decide which heavy-tailed model is appropriate and then estimate the tail index of the marginal distribution.

Distribution of a random variable X is called regularly varying at the right tail if

$$P[X > x] = 1 - F(x) = x^{-\alpha} L(x), \qquad x > 0 \tag{1}$$

where L is a so-called slowly varying function, a function such that the $\lim(L(xt)/L(t)) = 1$, for all $x > 0$ (see Embrechts *et al.*, 1997). We begin our analysis by following the semiparametric assumption (1) of regular variation and estimate the tail parameter α. Estimation of the tail parameter α represents the main, but a rather nontrivial step in the statistical analysis of such data sets. The standard estimator of the parameter α>0 in statistical literature is the so-called Hill estimator (Hill, 1975), which is based on a certain number of the largest-order statistics.

For $1 \le i \le n$ denote by $X_{(i)}$ the *i*'th largest value in the sample $X_1, X_2, \dots, X_n$ so that $X_{(1)} \ge X_{(2)} \ge \dots \ge X_{(n)}$. Then the Hill estimator of ${}^{1}\!/_{\alpha}$ based on k upper-order statistics is calculated as

$$H_{k,n} = \frac{1}{k}\sum_{i=1}^{k} log\,\frac{X_{(i)}}{X_{(k+1)}}$$

Alternatively one can use all order statistics above a given level u. Statistical properties of this estimator are quite well understood, as well as many pitfalls in its practical application (for details see Resnick (2007) or Embrechts *et al.* (1997)). They are mostly related to the choice of the number k or equivalently the threshold u. This is typically performed by the exploration of the so-called Hill plot which plots k against $H_{k,n}$. An appropriate k or the threshold u is selected by finding a plateau in such a plot, i.e. an interval of k's where the plot looks approximately stable. This is a somewhat subjective procedure, which can be aided by smoothing or rescaling of the Hill plot. Two alternatives to a Hill plot of this type are smooHill and altHill described in Resnick and Stărică (1997) and Drees *et al.* (2000). The latter turns out to be often useful because it dedicates more of the plot space to the interval around the true tail parameter than the conventional Hill plot. We use the altHill plot estimated as

$$\left\{\left(\theta, H^{-1}_{\lceil n^{\theta}\rceil,n}\right), 0 \leq \theta \leq 1\right\},$$

where we write $\lceil y \rceil$ for the smallest integer greater than or equal to $y \geq 0$.

However, an uncritical application of these procedures to data which do not have a distribution of a regularly varying type is often encountered in the literature. It seems advisable to perform at least some sort of goodness-of-fit procedure to see if assumption (1) actually fits the data at all. One of the standard and most illustrative procedures of this kind is based on the fact that the tail behavior of the data above a large threshold u is actually approximately log exponential whenever assumption (1) holds. We therefore compare the logarithm of sovereign spread changes with the exponential distribution on a qq-plot in order to verify whether sovereign spread series actually fit assumption (1).

From our perspective it is very interesting not only to study the individual distribution of sovereign spread movements, but also their joint behavior and their statistical association. A canonical measure of dependence between two numerical variables in statistics is the (Pearson) correlation coefficient. Although the correlation coefficient can be estimated quite well on the basis of time series data, this coefficient is a rather unreliable measure of dependence, especially when applied to heavy-tailed data such as sovereign spread changes. It is also very interesting to determine the association between countries during the time of crises, i.e. when one or both spreads make strong upward movements.

An alternative measure of statistical dependence at an arbitrary high level u is provided by the coefficient of bivariate tail dependence index *chi* - χ (Coles *et al.*, 1999; Poon *et al.*, 2004). It is defined for two variables X and Y with the same marginal distribution as

$$\chi = \lim_{u \to \infty} P(X_t > u | Y_t > u) \tag{2}$$

where we also assume that the marginal distribution has unbounded support on the right, as it is the case with nearly all commonly used distributions such as the normal, exponential, or power-law. Furthermore, from equation (2) it follows that *chi* is a nonnegative value with values in the interval [0,1]. If the degree of dependence vanishes in the limit, as $u \to \infty$, then $\chi = 0$ and in this case we say that the variables are asymptotically tail independent. Roughly speaking *chi* aims to assess the degree of dependence that may eventually prevail in the limit. The assumption of equality of marginal distributions seems relatively strong, but it can be easily satisfied by transforming individual series to have the same marginal distribution e.g. normal or unit Fréchet as it is commonly done in extreme value theory. Prior to measuring dependence in extreme levels of variables X and Y, representing sovereign spread changes of two countries of interest, the data are converted into an appropriate common scale as for example the unit Pareto margins (see for example Straetmans *et al.* (2008)) to make fair comparisons possible. This can be accomplished by converting the original pair (X, Y) into

$$\left(\tilde{X}, \tilde{Y}\right) = ((1 - F_X)^{-1}, (1 - F_Y)^{-1}) \tag{3}$$

where F_X, F_Y denote marginal distribution functions of variables X, Y. They are typically unknown so that in practice the empirical distribution functions $\hat{F}_X$ and $\hat{F}_X$ are plugged into equation (2). In that case, the order of magnitude of the high quantiles of one variable becomes comparable with those of the other.

As de Carvalho and Rua (2014) point out, *chi* measures joint dependence between two variables under very extreme circumstances. Because of the limiting part in the definition of the tail dependence coefficient *chi*, it is actually not so straightforward to estimate this quantity, although a natural estimator can be obtained by fixing $u = u_n$ at a very high threshold and calculating a nonparametric estimator of *chi* as

$$\hat{\chi} = \frac{\sum_{i=1}^{n} \mathbb{I}X_i > u_n \, \mathbb{I}Y_i > u_n}{\sum_{i=1}^{n} \mathbb{I}X_i > u_n} \tag{4}$$

The properties of the χ estimator are well understood; see Schmidt and Stadtmüller (2006) or Davis *et al.* (2012), where this estimator appears as a special case of the cross-extremogram. Theory developed in Davis *et al.* (2012) allows one even to construct a bootstrap-based procedure for the interval estimation of *chi* that allows one to search for the presence of asymptotic tail dependence between the changes of the two spreads. Observe that one still has to select the threshold u_n in an appropriate way for the practical application algorithm. One can do this again by the plateau finding procedure as in the case of the Hill estimator, as recommended by Schmidt and Stadtmüller (2006). We adopt this approach, but for the purpose of our study, we select u_n as the upper 10 percent empirical quantile of our data (cf. Davis et al. (2012)). In this case $\hat{\chi}$ has a rather natural interpretation as an estimator of the conditional probability of spread changes in a country above the level of the 10 percent quantile, given that the spread in the other country already moved above the corresponding quantile. To test if the *chi* values are significantly different from zero, we use a bootstrap based approach derived on the basis of results in Davis *et al.* (2012). For each pair of countries, using stationary bootstrap algorithm, we generate two independent bootstrap time series of the same length which have similar marginal distributions and time-varying dependence as the two series corresponding to the pair of countries. Repeating this many times and estimating χ for each of these bootstrap samples, we can approximate the *p*-value of the originally estimated χ. Note that this procedure is different from the permutation tests used in Davis *et al.* (2012).

Data

We use weekly sovereign bond spreads for 23 EU countries and the US. Data for the US and the UK span from April 1990 to April 2015 (altogether 1,306 observations), while other countries in the dataset are spanned somewhere in that time period, with Slovenia having the smallest number of observations (214 observations). The data for developed and some developing countries are obtained from Bank of America Merrill Lynch government bond yields collected from Bloomberg. However, for Bulgaria, Croatia, Hungary, Latvia and Lithuania, Emerging Market Bond Index (EMBI) is used, as Bank of Amerika Merrill Lynch database does not include these five countries. The EMBI spread is a typical and widely used proxy for emerging countries' sovereign bond spreads calculated by J.P. Morgan. EMBI spreads and Bank of America Merrill Lynch sovereign bond spreads are expressed in basis points and percentage points respectively.

Prior to conducting extremal analysis of sovereign spreads, we used differencing in order to transform each individual time series $Y_1, Y_2, \ldots$ Such a transformation produces the series of sovereign spread changes $X_n = Y_n - Y_{n-1}$, which to a reasonable extent appear to be stationary. For some countries however, the assumption of stationarity might be questionable even after the transformation, as the volatily in the series appears to change abruptly during and after the 2008 financial crisis. Stationarity might still be justified if one allows for the influence of the unobserved state of the economy as in Markov switching models for instance (Lange and Rahbek, 2009), so we keep this assumption throughout, but caution is advised in the interpretation.

Results

In order to analyze if our sovereign bond spread distributions are heavy-tailed, we use the approach described in the methodology section. First of all, we use graphical representation to detect if our data follow a power-law distribution. Next we evaluate the tail dependencies between different pairs of countries.

We compare the distribution of our observations above a chosen threshold with the exponential and power-law distribution on a qq-plot (see Appendix 1 for more details). It turns out that for at least some countries, the fit to the power–law distribution looks reasonable. These countries are Hungary, Latvia, Lithuania, Slovakia, and Slovenia. For several other countries it seems that the exponential distribution would be a better fit. Countries such as France, Italy, Poland, and Spain, seem to be significantly less heavy-tailed as the movements in sovereign bond spreads in these countries are extremely well described by simple exponential tails. For some countries such as Belgium, Ireland, Portugal, and Romania there is reason to suspect either exponential or power-law tails. More formally, Table 1 that presents summary statistics for the analyzed time series suggests excess kurtosis and greater mass in the tails for all the countries except for the US.

Although some of the countries may exhibit power-law tail behavior only in the very extreme right tail, we keep this assumption for the time being for all our data sets. Even in countries where the power-law tail behavior seems to be hard to justify, movements of spreads appear to have tails significantly heavier than normal. Table 2 presents the results obtained using Hill and altHill plots (see Appendix 2 for more details). Parameter *alpha* for Greece is estimated at 1.5 approximately; on the other hand the *alpha* for Denmark is relatively high at 3.0. Observe that the lower value of alpha indicates a heavier tail of spread movements. It appears from our table that countries with less sustainable public finance (Greece and Hungary for example) have significantly heavier tails in general and therefore more violent upward movements of the spreads. Hence a reasonable econometric model of the movements should take all this into account.

Table 1: Summary statistics

Country	*N*	*Mean*	*Standard deviation*	*Skewness*	*Kurtosis*
Austria	832	0.000	0.060	2.100	32.580
Belgium	911	0.000	0.080	-1.100	41.420
Bulgaria	676	-0.090	14.980	5.070	92.580
Croatia	782	0.110	12.640	1.560	14.090
Czech Republic	424	0.000	0.140	0.640	4.750
Denmark	1,162	0.000	0.070	-2.530	56.690
Finland	1,140	0.000	0.100	-15.700	418.110
France	1,288	0.000	0.060	0.060	4.790
Greece	424	0.030	1.080	-6.700	102.550
Hungary	797	0.120	22.320	1.560	25.250
Ireland	806	0.000	0.240	-6.000	98.530
Italy	806	0.000	0.130	-0.600	14.590
Latvia	797	-0.210	32.530	-0.750	28.500
Lithuania	281	-0.980	21.090	0.840	6.510
Netherlands	832	0.000	0.030	0.660	5.810

Poland	829	0.000	0.200	-4.260	67.350
Portugal	832	0.000	0.280	-0.190	14.820
Romania	250	-0.010	0.190	-2.830	25.990
Slovakia	669	0.000	0.120	1.770	17.800
Slovenia	214	0.000	0.180	0.980	4.950
Spain	1,149	0.000	0.140	-1.060	17.110
Sweden	431	0.000	0.060	-0.190	6.310
United Kingdom	1,306	0.000	0.090	-0.690	6.910
United States	1,306	0.000	0.100	0.110	0.850

Next we calculate Pearson correlations for all possible pairs of countries. Pearson correlation is useful in this exercise since it captures dependence completely in the context of multivariate normal distributions, but it is less useful as a measure of dependence in the context of heavy tailed distributions. Right tail dependence is presented by the right tail *chi* indicator which was calculated using top 10 percent observations in the right tail.

Table 2: Number of tail observations and the value of the Hill tail index

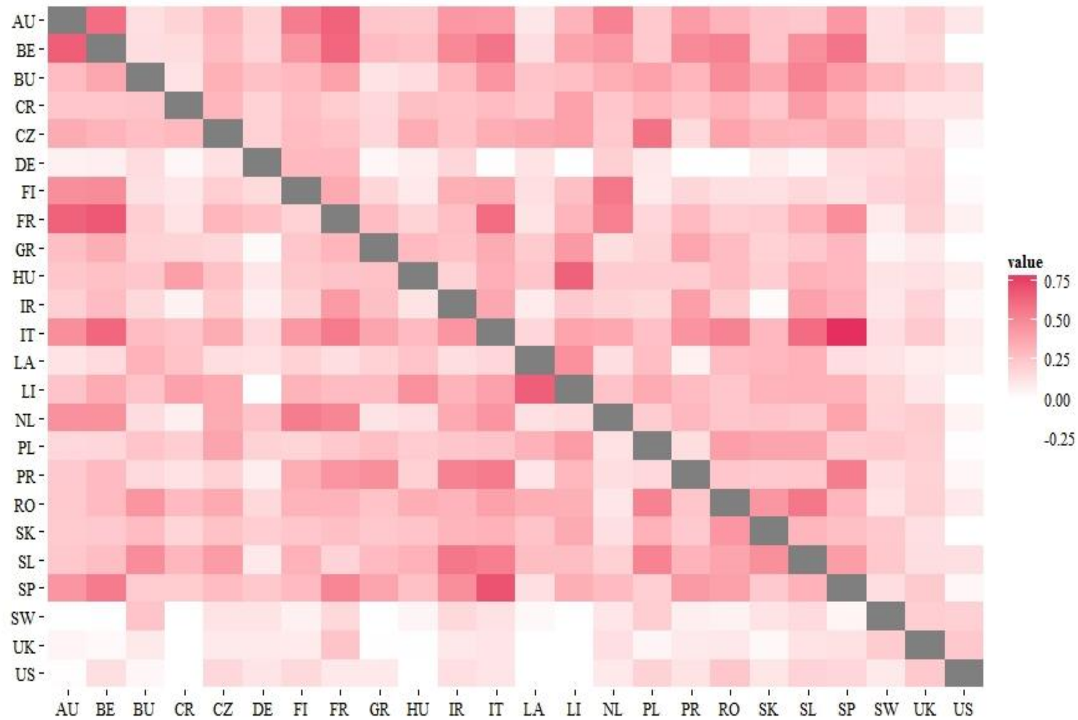
Country	*Sample period*	*alpha*	*Threshold*	*Tail observations*	*Hill (1/alpha)*	*Number of observations*	*Share of observations in the tail (in %)*
Austria	05:99 - 15:04	1.547	0.030	100	0.646	832	12
Belgium	11:97 - 15:04	1.678	0.060	80	0.596	911	9
Bulgaria	05:02 - 15:04	2.066	15.000	43	0.484	676	6
Croatia	05:00 - 15:04	1.888	10.200	95	0.530	782	12
Czech Republic	07:03 - 15:04	2.061	0.110	63	0.485	424	15
Denmark	93:01 - 15:04	3.013	0.130	32	0.332	1,162	3
Finland	93:06 - 15:04	2.053	0.070	70	0.487	1,140	6
France	90:08 - 15:04	2.814	0.075	84	0.355	1,288	7
Greece	03:07 - 15:04	1.506	0.340	86	0.664	424	20
Hungary*	01:00 - 15:04	1.892	15.000	98	0.529	797	12
Ireland	99:11 - 15:04	1.502	0.100	94	0.666	806	12
Italy	11:99 - 15:04	1.945	0.100	91	0.514	806	11
Latvia*	01:00 - 15:04	1.701	15.000	153	0.588	797	19
Lithuania*	12:09 - 15:04	1.870	12.000	56	0.535	281	20
Netherlands	05:99 - 15:04	1.697	0.025	98	0.589	832	12
Poland	99:05 - 15:04	3.095	0.250	46	0.323	829	6
Portugal	05:99 - 15:04	1.283	0.090	134	0.779	832	16
Romania	07:10 - 15:04	2.194	0.100	42	0.456	250	17
Slovakia*	02:06 - 15:04	2.135	0.110	65	0.468	669	10

Slovenia*	11:03 - 15:04	2.207	0.200	19	0.453	214	9
Spain	04:93 - 15:04	1.501	0.060	188	0.666	1,149	16
Sweden	07:01 - 15:04	2.449	0.070	38	0.408	431	9
United Kingdom	90:04 - 15:04	2.899	0.100	105	0.345	1,306	8
United States	04:90 - 15:04	3.216	0.100	189	0.311	1,306	14

Note: Germany is the benchmark country; * represents evidence of heavy-tailed behavior.

Table 3 provides values of the estimated Pearson correlations and the right tail *chi* indicators for all pairs of countries. For easier comparison we also provide a heat map of Pearson correlation and right tail dependence of sovereign bond spreads (Figure 1). Figure 1 shows somewhat darker colors for the right tail dependence indicator when compared to Pearson correlations. More formally, by comparing the two corresponding values of Pearson correlation and right tail *chi*, we see that for 209 out of 276 pairs (or 75.7 percent) right tail dependence is above Pearson correlation, although this is difficult to interpret. The highest right tail *chi* was depicted for Italy and Spain – most probably due to large spillovers from the EU sovereign debt crisis at its peak in 2012. The smallest right tail *chi* was obtained for the Denmark and Greece pair suggesting that the two countries might be experiencing negative dependencies in the tails – possibly because when risk perception increases in Greece, investors turn to safer markets such as the one in Denmark which results in spreads moving upwards in Greece and downwards in Denmark. However, to get a complete picture one would need to check the right to left tail dependencies.

Figure 1: Heat map of Pearson correlation and right tail dependence of sovereign bond spreads



Note: AU - Austria, BE - Belgium, BU - Bulgaria, CR - Croatia, CZ – Czech Republic, DE - Denmark, FI - Finland, FR - France, GR - Greece, HU - Hungary, IR - Ireland, IT - Italy, LA - Latvia, LI - Lithuania, NL - Netherlands, PL - Poland, PR - Portugal, RO - Romania, SK - Slovakia, SL - Slovenia, SP - Spain, SW - Sweden, UK – United Kingdom, US – United States; Pearson correlation is presented below the diagonal, while the right-tail dependence (*chi*) is presented above the diagonal; for two pairs (LI and DE, and LI and the US) the value of *chi* was estimated to be zero.

Table 3: Pearson correlation and right tail dependence of sovereign bond spreads

	FR	UK	IR	SW	FI	DE	CZ	PL	SK	SL	RO	GR	IT	SP	PR	BE	NL	US	AU	BU	HU	CR	LI	LA
FR	-	0.201	0.273	0.084	0.190	0.264	0.308	0.176	0.210	0.328	0.205	0.280	0.606	0.474	0.291	0.677	0.527	0.058	0.643	0.209	0.182	0.121	0.315	0.117
UK	0.248	-	0.097	0.219	0.089	0.092	0.092	0.042	0.027	0.120	0.096	0.006	0.110	0.128	0.091	0.026	0.136	0.232	0.040	0.091	0.005	-0.042	-0.066	-0.003
IR	0.442	0.185	-	0.106	0.194	0.069	0.218	0.168	0.018	0.390	0.217	0.268	0.373	0.325	0.408	0.281	0.182	0.038	0.198	0.158	0.114	0.058	0.224	0.090
SW	0.163	0.225	0.163	-	0.061	0.116	0.118	0.208	0.119	0.156	0.056	-0.060	0.122	0.044	0.071	-0.046	0.107	0.195	-0.011	0.249	0.041	-0.001	-0.066	0.027
FI	0.363	0.220	0.333	0.186	-	0.153	0.204	0.093	0.129	0.167	0.133	0.172	0.346	0.126	0.172	0.483	0.563	0.017	0.470	0.132	0.094	0.109	0.268	0.136
DE	0.299	0.214	0.173	0.163	0.306	-	0.127	0.097	0.082	0.038	-0.079	0.036	0.008	0.146	-0.019	0.066	0.202	-0.046	0.061	0.150	0.075	0.036	-0.063	0.120
CZ	0.256	0.175	0.256	0.233	0.279	0.186	-	0.586	0.313	0.299	0.384	0.173	0.341	0.354	0.155	0.320	0.231	0.035	0.355	0.271	0.346	0.293	0.395	0.372
PL	0.229	0.205	0.235	0.233	0.183	0.195	0.372	-	0.383	0.378	0.407	0.268	0.254	0.211	0.141	0.171	0.124	0.010	0.165	0.247	0.212	0.210	0.420	0.324
SK	0.269	0.138	0.313	0.209	0.246	0.222	0.256	0.328	-	0.307	0.442	0.232	0.328	0.267	0.225	0.224	0.132	-0.069	0.220	0.281	0.252	0.180	0.359	0.253
SL	0.200	0.136	0.545	0.238	0.318	0.091	0.409	0.500	0.455	-	0.378	0.292	0.535	0.410	0.319	0.274	0.195	0.132	0.235	0.476	0.327	0.311	0.279	0.277
RO	0.320	0.200	0.320	0.120	0.348	0.160	0.360	0.542	0.458	0.545	-	0.256	0.399	0.306	0.241	0.287	0.106	0.095	0.232	0.448	0.345	0.288	0.335	0.341
GR	0.302	0.100	0.256	0.047	0.233	0.023	0.163	0.190	0.186	0.227	0.280	-	0.352	0.300	0.375	0.341	0.141	-0.010	0.271	0.193	0.290	0.183	0.427	0.222
IT	0.571	0.222	0.444	0.140	0.438	0.167	0.349	0.259	0.299	0.591	0.520	0.372	-	0.789	0.452	0.626	0.363	0.076	0.469	0.281	0.286	0.248	0.386	0.170
SP	0.509	0.236	0.469	0.140	0.297	0.237	0.279	0.205	0.224	0.318	0.400	0.372	0.691	-	0.425	0.549	0.290	0.039	0.446	0.213	0.266	0.216	0.344	0.135
PR	0.440	0.190	0.519	0.140	0.349	0.072	0.186	0.169	0.224	0.227	0.240	0.465	0.543	0.536	-	0.292	0.138	0.039	0.223	0.158	0.194	0.123	0.304	0.110
BE	0.626	0.176	0.494	0.140	0.455	0.187	0.279	0.229	0.254	0.455	0.520	0.279	0.568	0.571	0.506	-	0.427	-0.014	0.653	0.138	0.265	0.144	0.390	0.139
NL	0.500	0.214	0.358	0.186	0.542	0.253	0.349	0.217	0.254	0.227	0.240	0.116	0.444	0.381	0.301	0.471	-	0.051	0.460	0.147	0.140	0.065	0.155	0.129
US	0.093	0.234	0.136	0.093	0.162	0.114	0.163	0.193	0.104	0.182	0.240	0.093	0.111	0.174	0.120	0.132	0.099	-	0.010	0.037	-0.172	-0.023	-0.255	-0.155
AU	0.631	0.202	0.432	0.140	0.542	0.193	0.302	0.229	0.254	0.227	0.320	0.233	0.420	0.429	0.422	0.600	0.531	0.107	-	0.137	0.233	0.184	0.319	0.100
BU	0.397	0.231	0.299	0.302	0.303	0.281	0.326	0.397	0.373	0.500	0.480	0.116	0.448	0.412	0.313	0.368	0.343	0.162	0.279	-	0.149	0.124	0.270	0.250
HU	0.260	0.127	0.192	0.116	0.228	0.104	0.256	0.212	0.209	0.318	0.280	0.256	0.342	0.300	0.212	0.262	0.212	0.075	0.244	0.235	-	0.403	0.643	0.247
CR	0.208	0.115	0.256	0.163	0.260	0.200	0.302	0.320	0.239	0.409	0.320	0.163	0.273	0.282	0.231	0.231	0.244	0.115	0.231	0.250	0.296	-	0.396	0.243
LI	0.286	0.111	0.321	0.185	0.333	0.000	0.357	0.370	0.333	0.318	0.240	0.286	0.393	0.321	0.286	0.357	0.250	0.000	0.250	0.250	0.423	0.393	-	0.651
LA	0.143	0.076	0.141	0.116	0.190	0.130	0.140	0.275	0.299	0.318	0.280	0.186	0.177	0.138	0.063	0.150	0.138	0.063	0.115	0.324	0.278	0.253	0.481	-

Note: AU - Austria, BE - Belgium, BU - Bulgaria, CR - Croatia, CZ – Czech Republic, DE - Denmark, FI - Finland, FR - France, GR - Greece, HU - Hungary, IR - Ireland, IT - Italy, LA - Latvia, LI - Lithuania, NL - Netherlands, PL - Poland, PR - Portugal, RO - Romania, SK - Slovakia, SL - Slovenia, SP - Spain, SW - Sweden, UK – United Kingdom, US – United States; Pearson correlation is presented above the diagonal, while the right-tail dependence (*chi*) is presented below the diagonal; for two pairs (LI and DE, and LI and the US) the value of *chi* is estimated to be zero; the dark-shaded cells represent statistical significance at the 1, medium-dark-shaded at the 5 and light-shaded at the 10 percent level (obtained by 1,000 bootstrap replications).

As one would expect the lowest right tail *chi*'s in general were depicted for either very liquid markets such as the UK and the US or for economies that are perceived stable in terms of public finance such as Denmark and Sweden. Also interesting is the case of Slovenia – a small Central European country that appears in 25 percent of pairs with highest right tail dependence indicators. Slovenia shows evidence of very high positive tail dependence with Bulgaria, Croatia, the Czech Republic, Slovakia and Romania. This country has experienced significant public finance and banking troubles in the past few years which culminated after the 2013 Cyprus financial crisis when Slovenia was not able to issue new sovereign bonds as the sovereign bond market shut down for that country and it was forced to turn to private placement financing instead.

The shaded areas in Table 3 represent statistical significance of right tail *chi* at the 1, 5 and 10 percent level obtained by applying bootstrap-based approach derived on the basis of theoretical analysis in Davis *et al.* (2012). There are 229 pairs of countries for which the observed right tail dependence is statistically significant at the 10 percent level. This corresponds to 83 percent of all pairs explored here. Among developing sovereign markets there are several interesting findings. The US sovereign bond market exhibits significant and strongest right tail dependence with the UK market and appears to be less prone to extreme spread co-movements with continental EU sovereign bond market.

The UK bond market on the other hand records extreme sovereign bond spread upswings when extreme spread upswings are recorded in French, Irish, German, Finish, Italian, Spanish, Portuguese, Belgian, Dutch, and Austrian sovereign spreads. We can thus conclude that British government bonds might be exposed to turmoil taking place in the European Monetary Union, even though it is not a member of the Monetary Union. Extreme upswings in Swedish bonds also appear to be less frequently correlated with upswings taking place in sovereign bond markets in the EMU countries, as we found evidence of significant and strong right tail dependence with the Bulgarian, Czech, and Polish bond markets. In comparison, right-tail cross-country co-movement structures for other developed European economies are more complex as they include significant extremal dependencies with other developed EMU and non-EMU countries, but also with developing countries. One also has to note the lack of right tail dependencies between the Greek sovereign bond market and sovereign markets of other countries, but this is probably due to the fact that we could only obtain Greek spreads for the most turbulent period (e.g. from 2007 to 2014), which in turn might mean that even though 20 percent of the time Greek sovereign bonds exhibited extremal weekly surges, many extreme changes in Greek spreads possibly did not even end up in the tail.

According to the results of the test of presence of asymptotic tail dependence, developing European countries which are also often referred to as the new EU member states show statistically significant strong positive tail dependence with a number of countries. Extreme increases in sovereign bond spreads in all new member states seem to be equally exposed to corresponding increases in both developed and developing countries. The only exception is Latvia for which extreme increases in sovereign bond markets are not correlated with changes of the similar magnitude in the Czech sovereign bond market, or with similar changes in other nine developed countries. It is also quite interesting to note that bond markets of all new member states except Latvia exhibit significant right tail dependence with the Greek, Italian, Portuguese, and Spanish sovereign bond markets, which might mean that they are vulnerable to adverse sovereign bond developments in the European periphery.

Concluding remarks

The aim of this study is to assess the linkages among sovereign bond markets during crises periods using univariate and bivariate statistics derived from extreme value theory. In the first part of the empirical analysis we show that for at least some countries sovereign bond changes are well described by a regularly varying heavy tailed distribution. For some other countries it seems that the exponential function would be a better fit. Although for some countries there is no sign of power-law-tail behavior, spread changes for each and every one of them have tails significantly heavier than normal. But, as one might expect, they do not belong to the same class of distributions.

Statistical significance of right tail *chi* at the 1, 5, and 10 percent level obtained by applying bootstrap-based approach derived on the basis of theoretical analysis in Davis *et al.* (2012) is detected for 229 pairs of countries. This corresponds to 83 percent of all pairs.

The results suggest that the US sovereign bond market exhibits significant and strong right tail dependence with the UK market and appears not to share much extreme right tail co-movements with sovereign bond markets of continental European countries. The UK bond market on the other hand does exhibit joint right tail co-movements with French, Irish, German, Finish, Italian, Spanish, Portuguese, Belgian, Dutch, and Austrian sovereign spreads, thus suggesting that European debt crisis might have adversely affected British government bonds even though the UK is not a member of the European Monetary Union. Extreme increases in sovereign bond spreads in new EU member states appear in all pairs of these countries instead of the Latvia-Czech Republic pair. Bond markets of all new member states except Latvia exhibit significant right tail dependence with the Greek, Italian, Portuguese, and Spanish sovereign bond markets, which makes them vulnerable to adverse sovereign bond developments in the European periphery.

The results of our investigation suggest that national borders do not seem to matter much for sovereign bond market spillovers. As a result of financial account liberalization and consequent free movement of capital and financial integration, financial turmoil quickly spreads across borders. From the standpoint of national financial stability, extremal dependence of sovereign bond markets can thus be regarded as a drawback of intensified financial integration that requires surveillance that cannot be limited to national borders.

References

Ang, A. and Bekaert , G. , 2002, International Asset Allocation With Regime Shifts, *The Review of Financial Strudies*, 15(4): 1137-1187.

Aßmann, C. and Boysen-Hogref, J., 2012, Determinants of government bond spreads in the euro area: in good times as in bad, *Empirica*, 39(3), 341-356.

Beber, A., Brandt, M. and Kavajecz, K., 2009, Flight-to-quality or flight-to-liquidity? Evidence from the euro-area bond market, *Review of Financial Studies*, 22 (3),925–957.

Cappiello, L., Engle, R. and Shepard, K, 2006, Asymmetric Dynamics in the Correlations of Global Equity and Bond Returns, *Journal of Financial Econometrics*, 4 (4), 537-572.

Coles, S., Heffernan, J. and Tawn, J., 1999, Dependence measures for extreme value analyses, *Extremes*, 2, 339-365.

Davis, R. A., Mikosch, T. and Cribben, I., 2012, Towards estimating extremal serial dependence via the bootstrapped extremogram, *Journal of Econometrics*, 170(1), 142-152.

De Carvalho, M. and Rua, A., 2014, Extremal dependence in International output growth: tales from the tails, *Oxford Bulletin of Economics and Statistics*, 76(4), 605-620.

De Santis, G. and Gerard, B., 1997, International asset pricing and portfolio diversification with time-varying risk, *The Journal of Finance*, 52, 1881-1912.

Drees, H., de Haan, L. and Resnick, S. I., 2000, How to make a Hill plot, *Annals in Statistics*, 28(1), 254-274.

Embrechts, P., Klüppelberg, C., and Mikosch, T., 1997, *Modelling Extremal Events for Finance and Insurance*, Springer, Berlin.

Favero, C., Pagano, M. and von Thadden, E.-Lu., 2010, How does liquidity affect government bond yields? *Journal of Financial and Quantitative Analysis*, 45 (1), 107–134.

Favero, C., 2014, Modelling and forecasting government bond spreads in the euro area: A GVAR model, *Journal of Econometrics*, 177, 343–356.

Garcia, R. and Tsafack, G., 2011, Dependence structure and extreme comovements in international equity and bond markets, *Journal of Banking & Finance*, 35(8), 1954–1970

Hartmann, P., Straetmans, S. and de Vries, C. G. 2004, Asset Market Linkages in Crisis Period, *The Review of Economics and Statistics*, 86(1), 313–326.

Hill, B. M., 1975, A simple General Approach to Inference about the Tail of a Distribution. *Annals of Statistics*, (46), 1163-1173.

Jondeau, E. and Rockinger, M., 2006, The Copula-GARCH model of conditional dependencies: An international stock market application, *Journal of International Money and Finance*, 25(5), 827–853.

Lange, T. and Rahbek, A., 2009, An introduction to regime switching time series models, *Handbook of Financial Time Series*, Springer Berlin Heidelberg, 871-887.

Longin, F. and Solnik, B., 1995, Is the correlation in international equity returns constant:1960-1990?, *Journal of International Money and Finance*, 14, 3-26.

Longin, F. and Solnik, B. 2001, Extreme Correlations of International Equity Markets, *The Journal of Finance*, 56(2), 649-676.

Poon, S. H., Rockinger, M. and Tawn, J., 2004, Extreme Value Dependence in Financial Markets: Diagnostics, Models, and Financial Implications, *The Review of Financial Studies*, 17(2), 581-610.

Resnick, S.I., 2007, Heavy-Tail Phenomena, *Probabilistic and Statistical Modeling*, Springer, New York.

Resnick, S.I. and Stărică, C., 1997, Smoothing the Hill estimator, *Advances in Applied Probability*, 29, 271-293.

Schmidt, R. and Stadtmüller, U., 2006, Non-Parametric Estimation of Tail Dependence, *Scandinavian Journal of Statistics*, 33, 307-335.

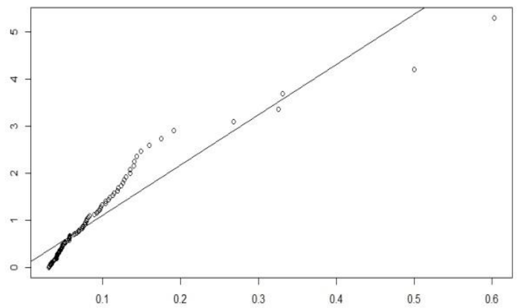
Straetmans, S. T. M., Verschoor, W. F. C. and Wolff, C. C. P., 2008, Extreme US stock market fluctuations in the wake of 9/11, *Journal of Applied Econometrics*, 23, 17-42.

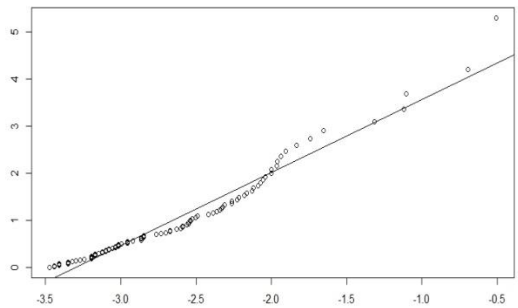
Appendix 1

Qq-plots are presented on the left and a modified version of the qq-plot (one in which the data are in logarithms) are presented on the right. Both plots are designed for threshold data and compared either to the exponential distribution or to the Pareto (or power-law) distribution. The straight line on the graph is shown to help the interpretation of the graph. If the figure on the left shows that the data above a chosen threshold diverge from the straight line in a concave form, there is a reason to suspect heavier tail in the data than in the theoretical exponential model. The figure on the right then compares the data with a power-law distribution. A good fit is again indicated by accumulation of points near the straight line.

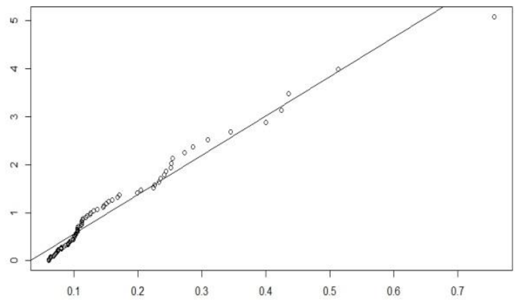
The X axis presents ordered data, while the Y axis for the graph on the left presents exponential quantiles and for the graph on the right it presents exponential quantiles of the logarithm of the time series observed.

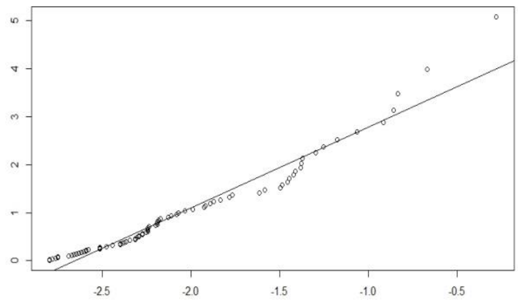
Austria



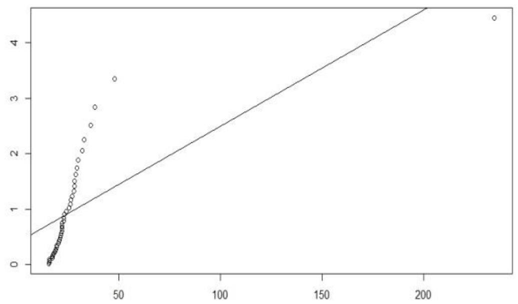


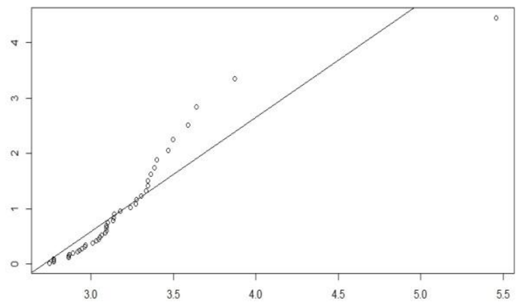
Belgium



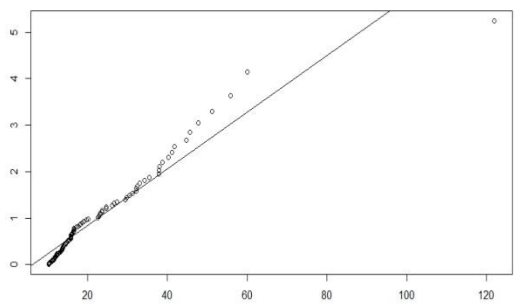


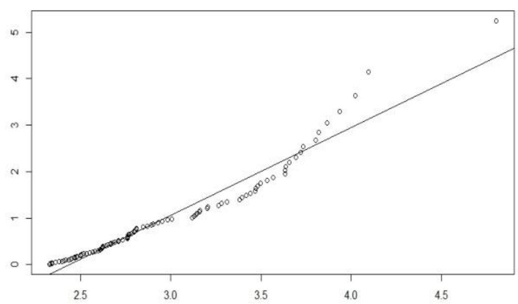
Bulgaria



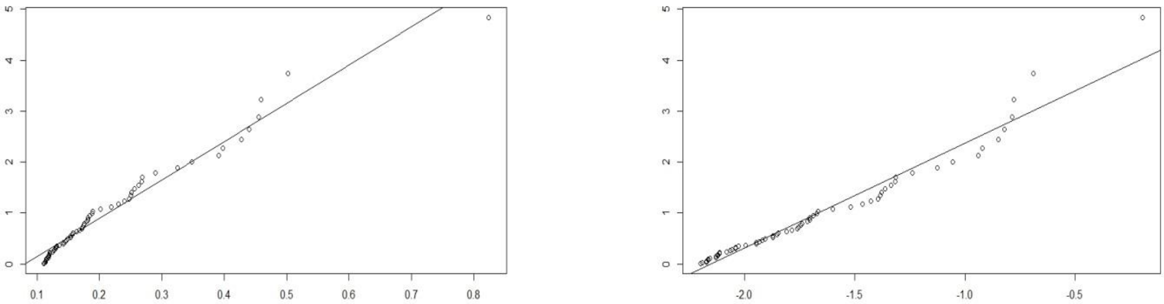


Croatia

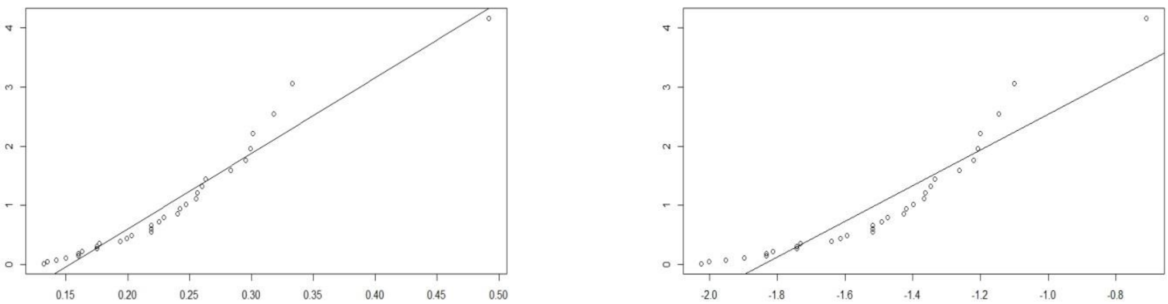




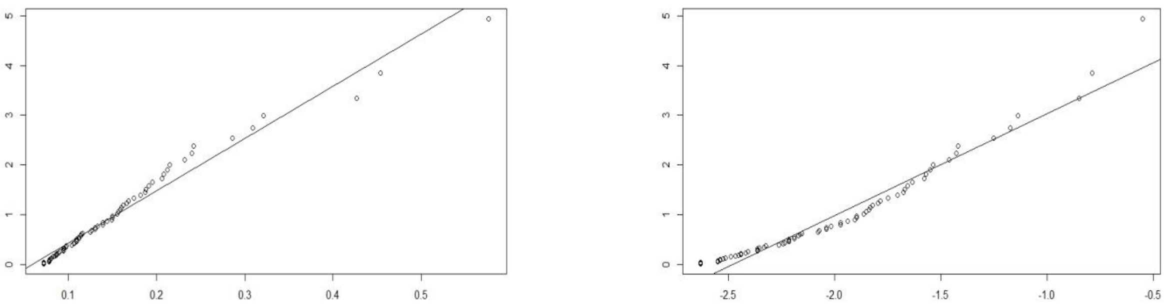
Czech Republic



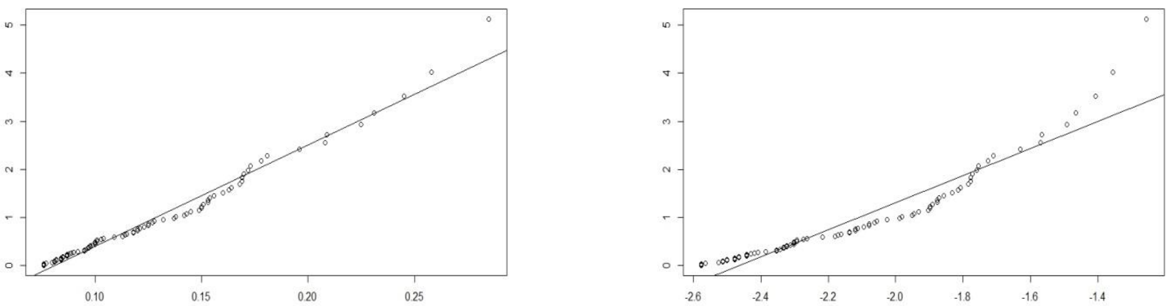
Denmark



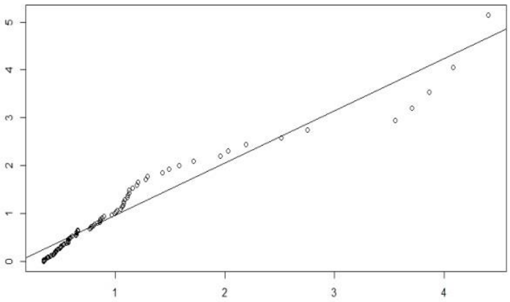
Finland

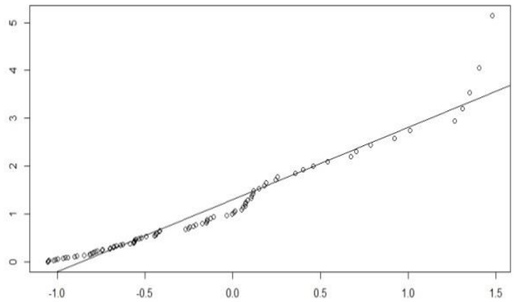


France

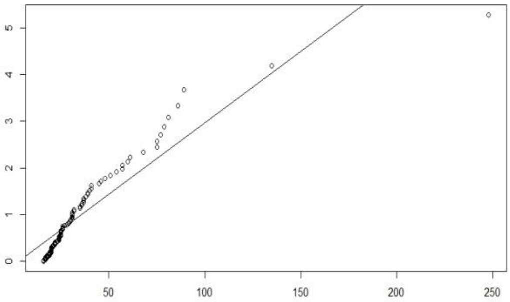


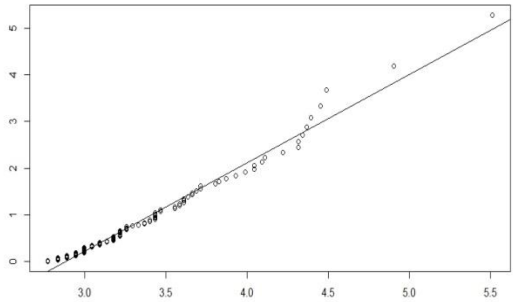
Greece



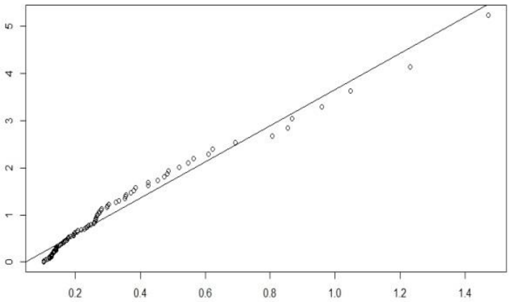


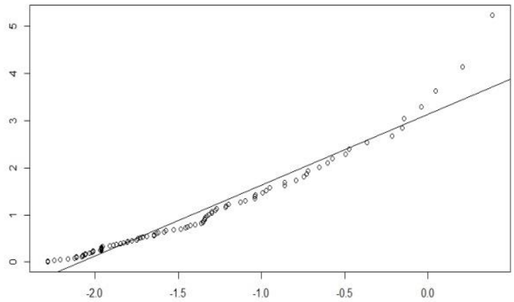
Hungary



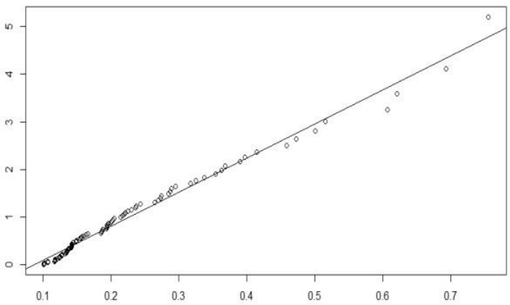


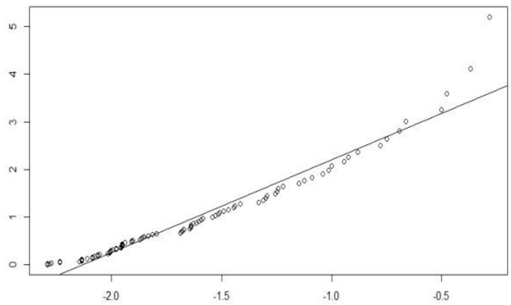
Ireland



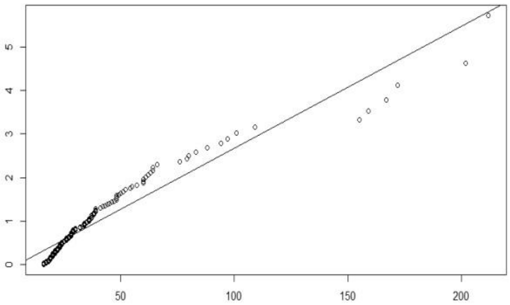


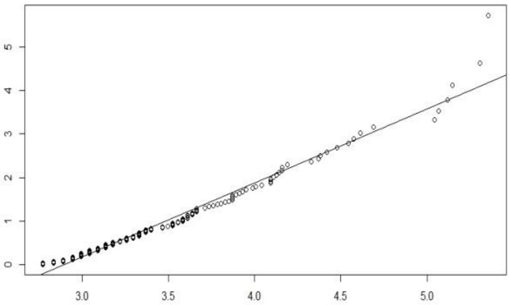
Italy



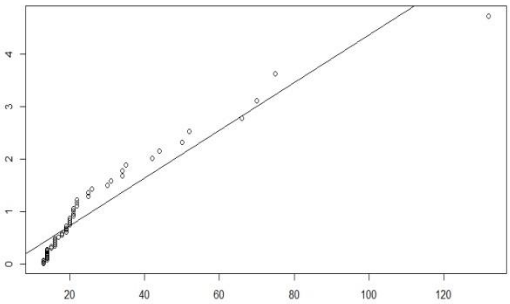


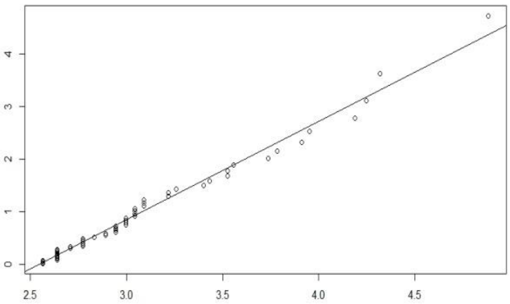
Latvia



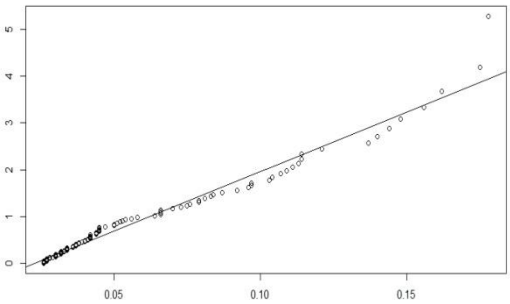


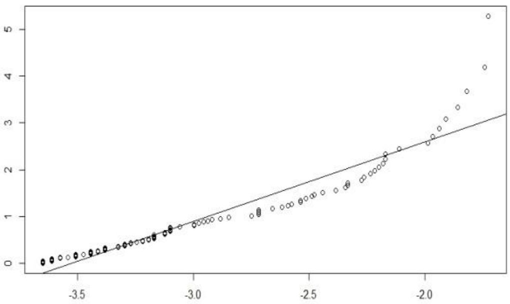
Lithuania



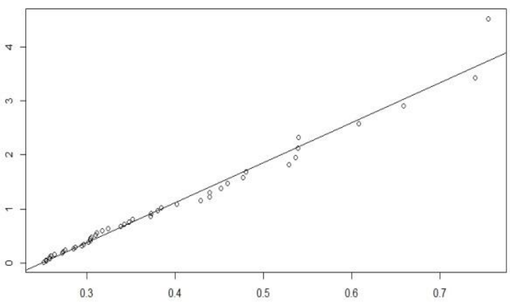


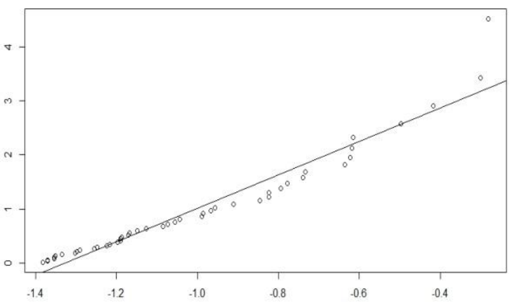
Netherlands



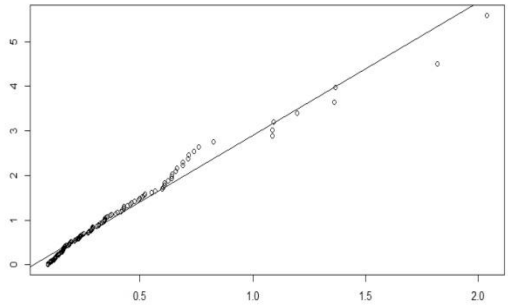


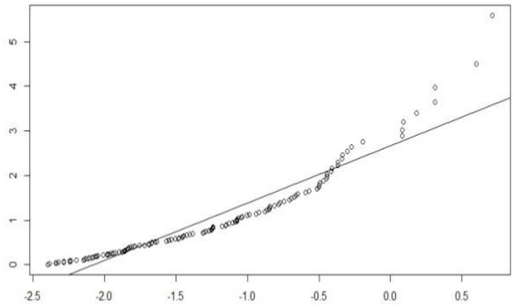
Poland



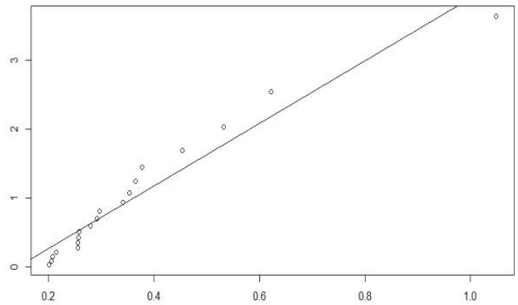


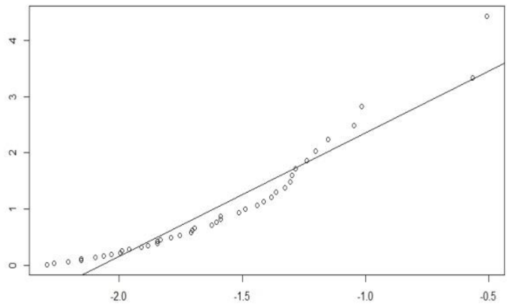
Portugal



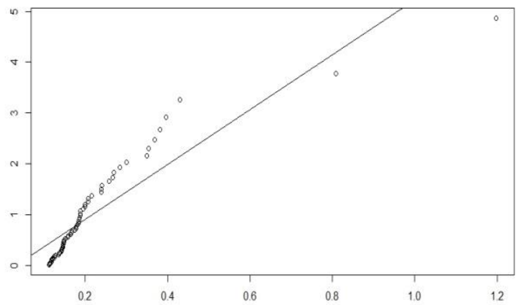


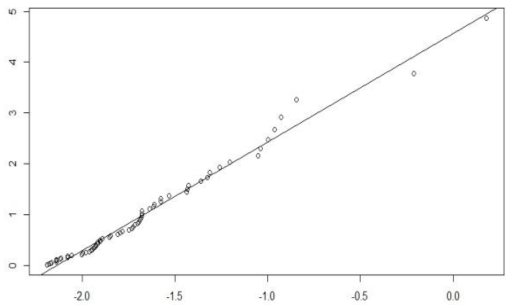
Romania



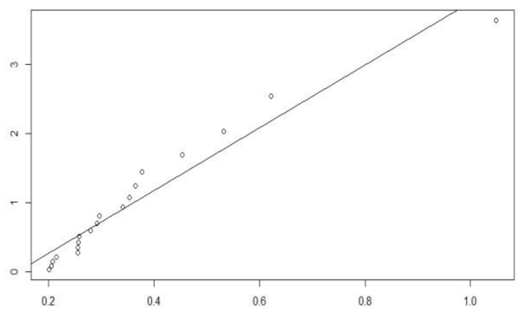


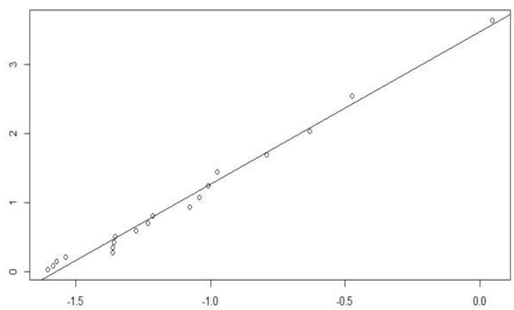
Slovakia



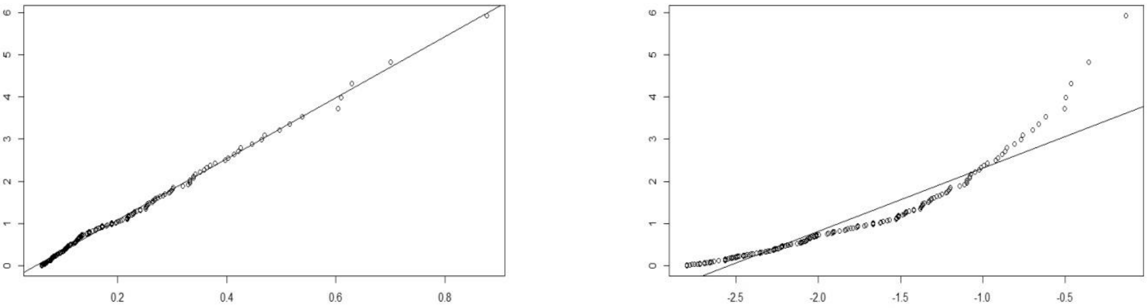


Slovenia

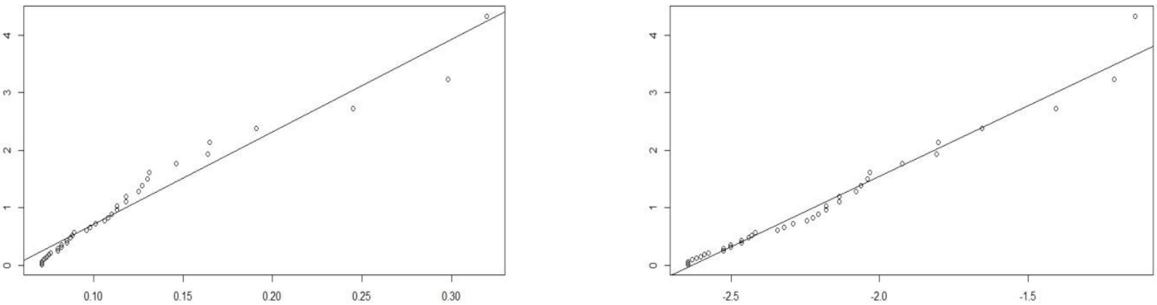




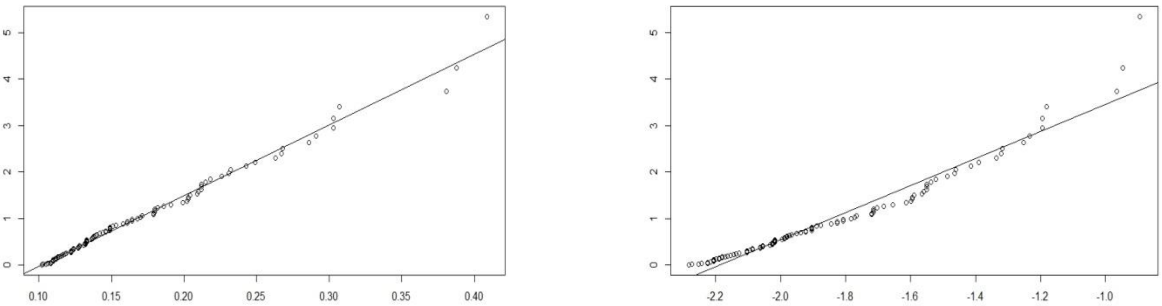
Spain



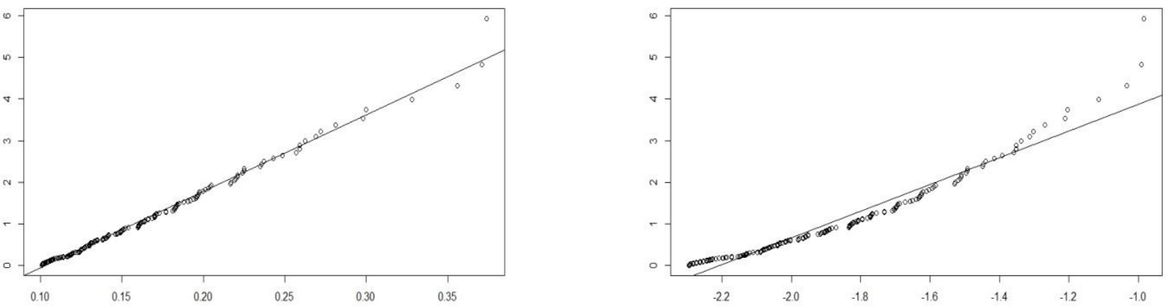
Sweden



United Kingdom



United States

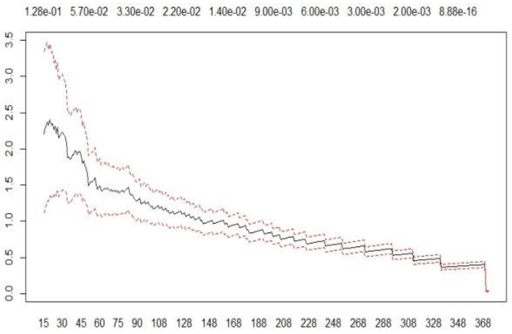


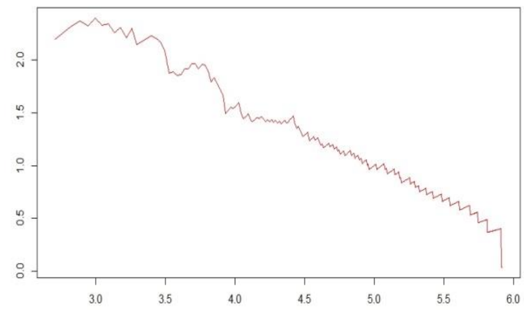
Appendix 2

The figure on the left presents the Hill plot while the figure on the right shows the alternative Hill plot or altHill (with the ordered statistics in logarithms).

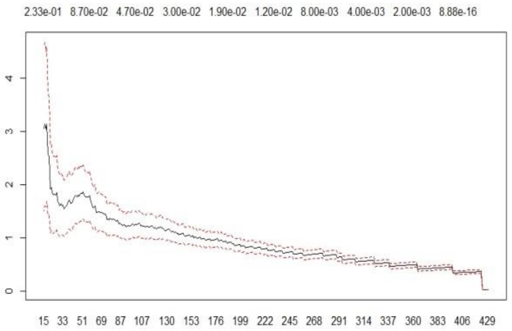
The Y axis presents the tail index (*alpha*) together with a 95 percent confidence interval (the latter available only for the Hill plot), while the X axis for the graph on the left presents order statistics and for the graph on the right it presents the logarithm of order statistics. Numbers above the Hill plot suggest the threshold level.

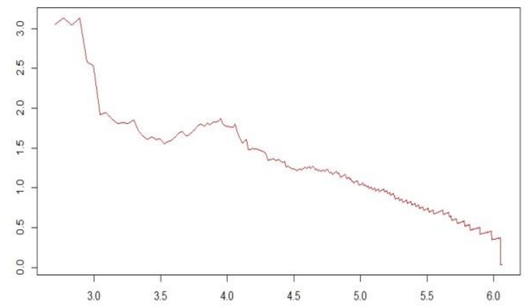
Austria



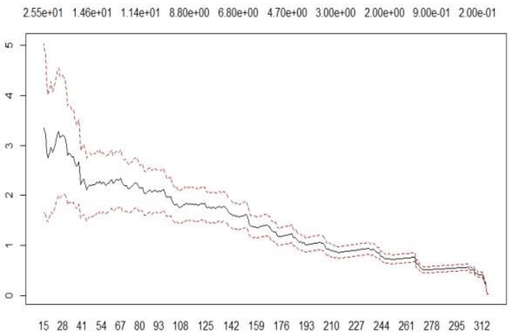


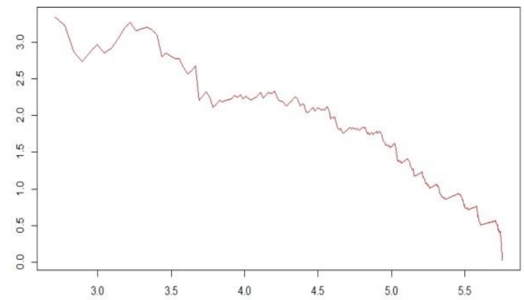
Belgium



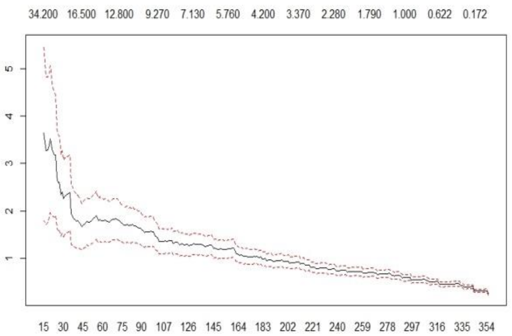


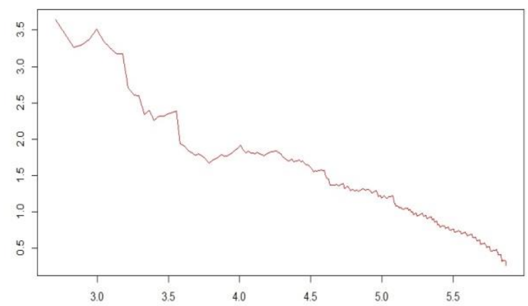
Bulgaria



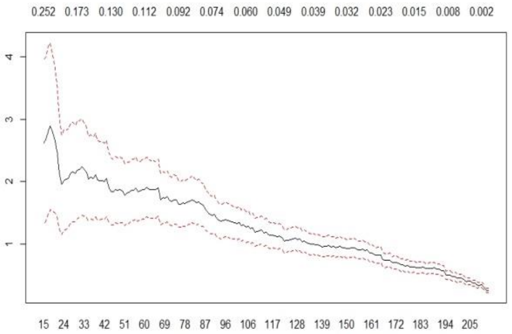


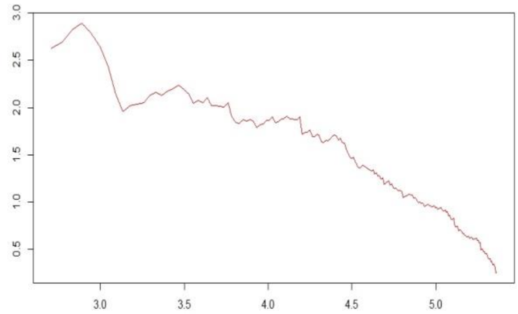
Croatia



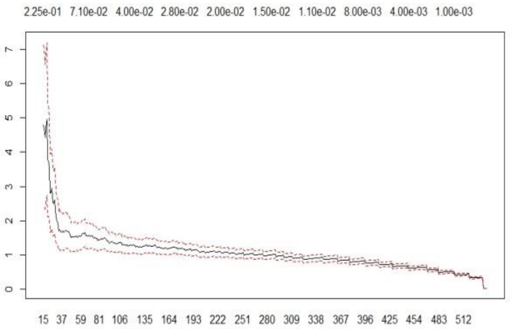


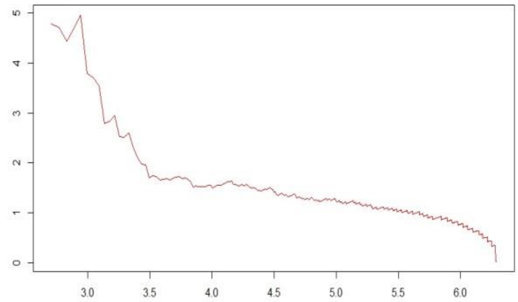
Czech Republic



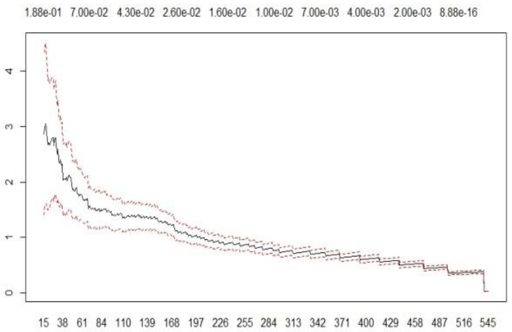


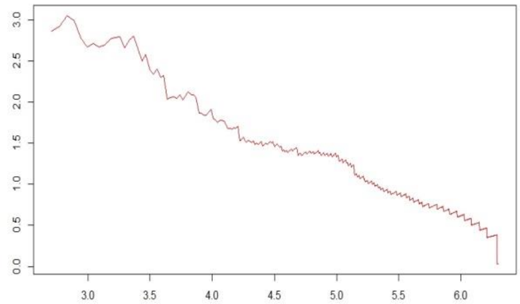
Denmark



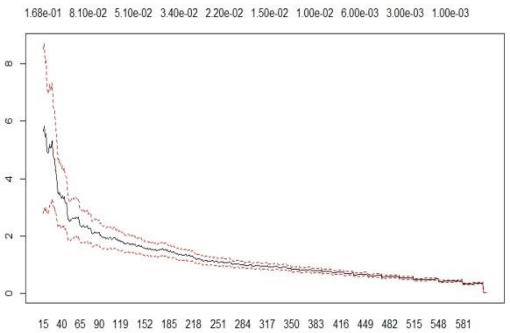


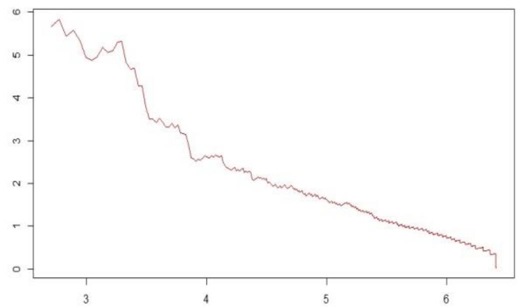
Finland



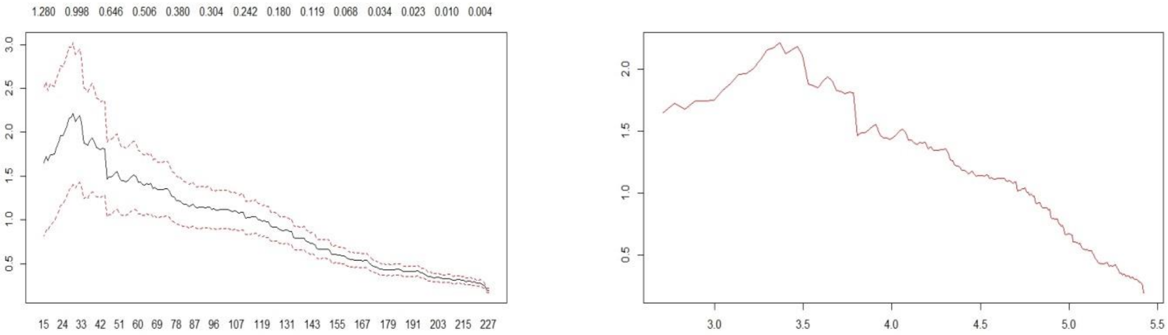


France

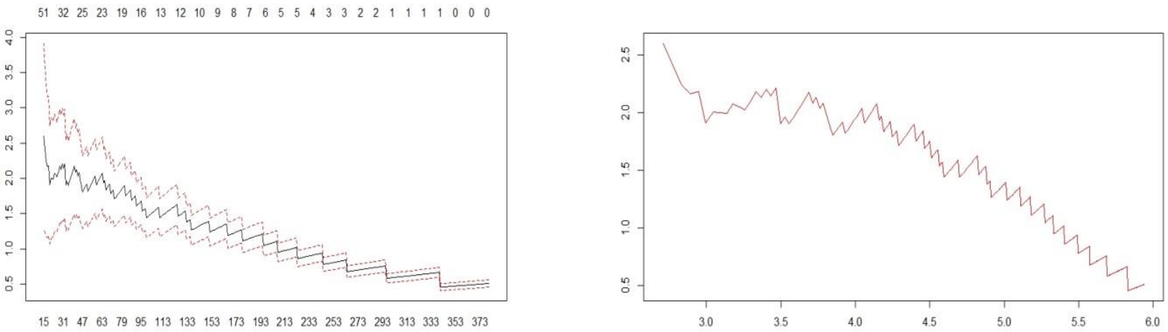




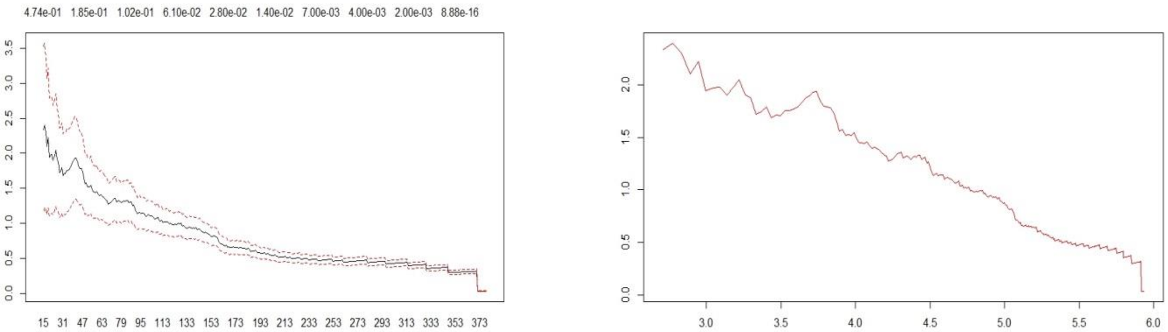
Greece



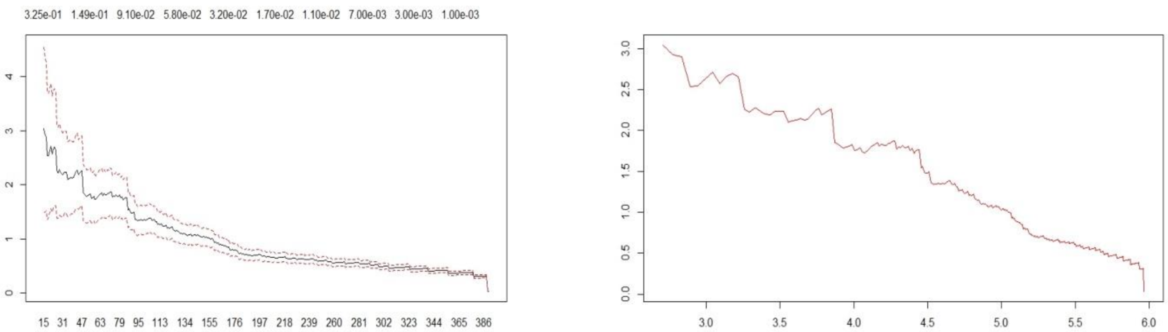
Hungary



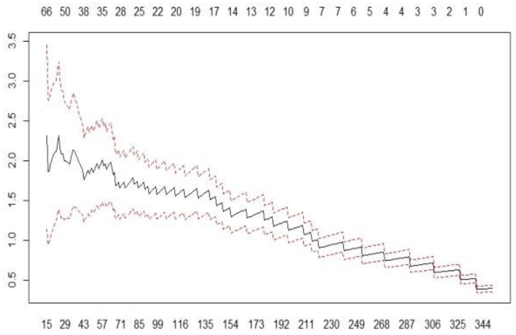
Ireland

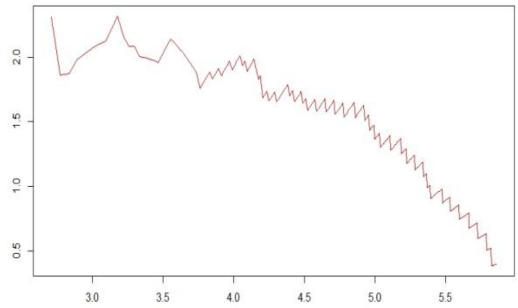


Italy

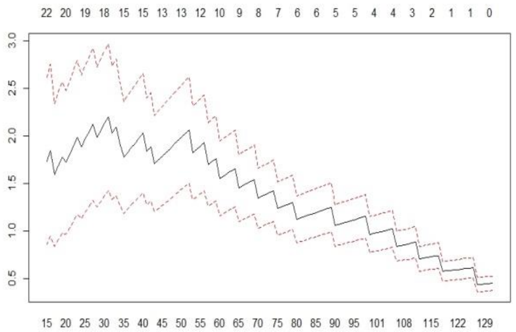


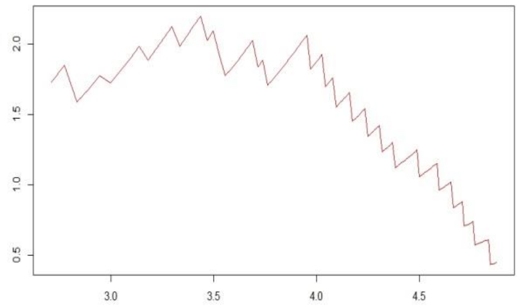
Latvia



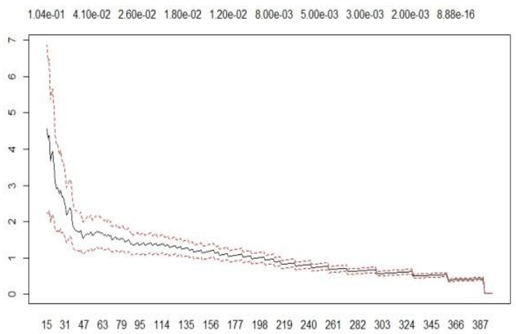


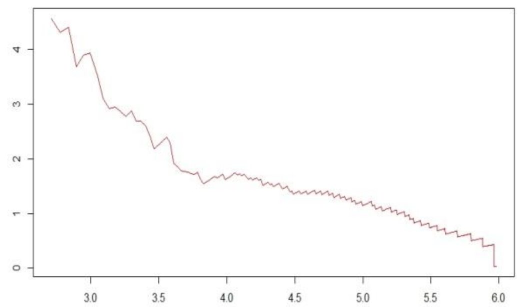
Lithuania



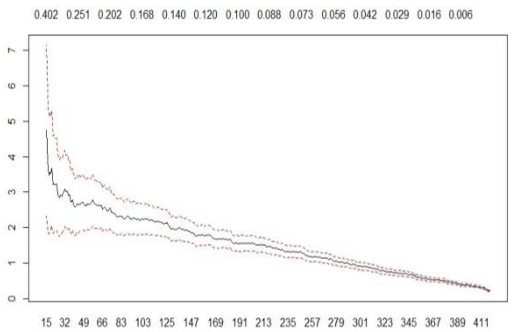


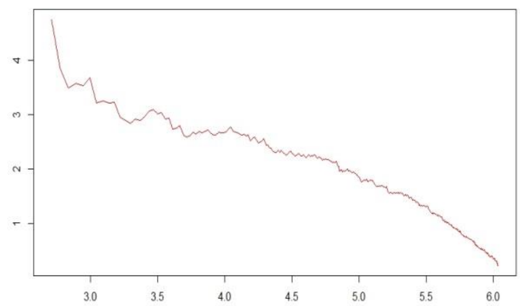
Netherlands



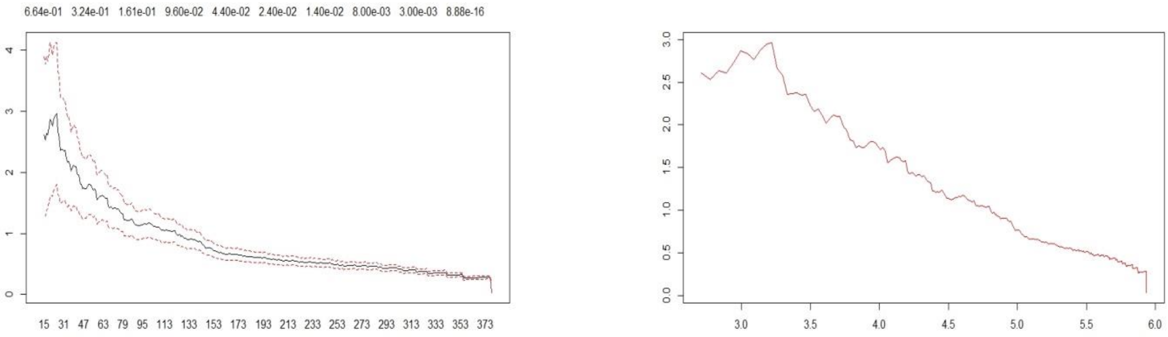


Poland

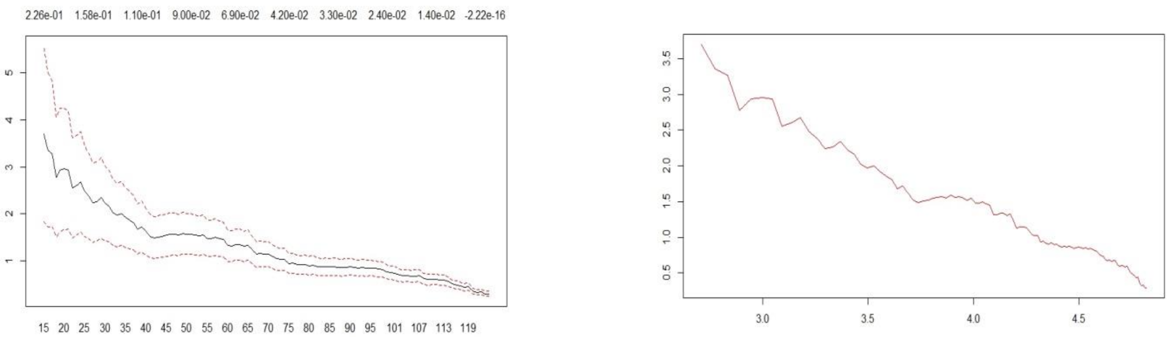




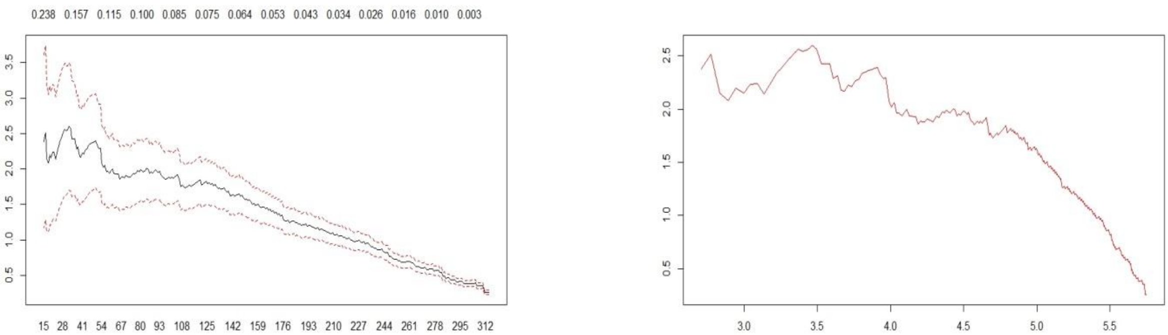
Portugal



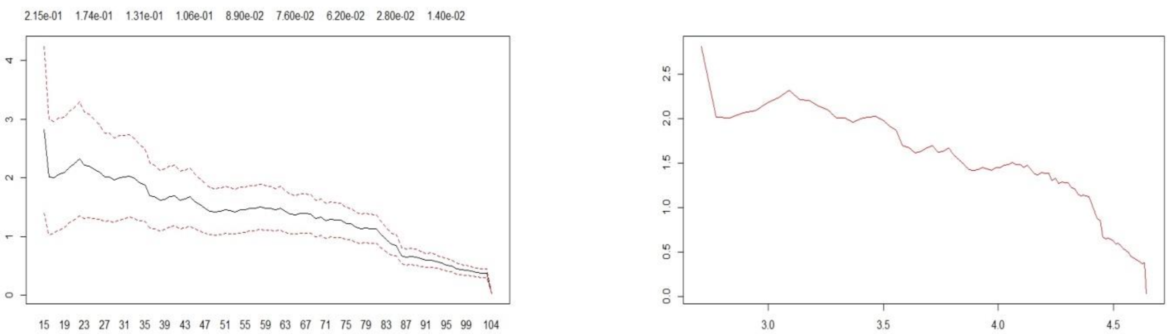
Romania



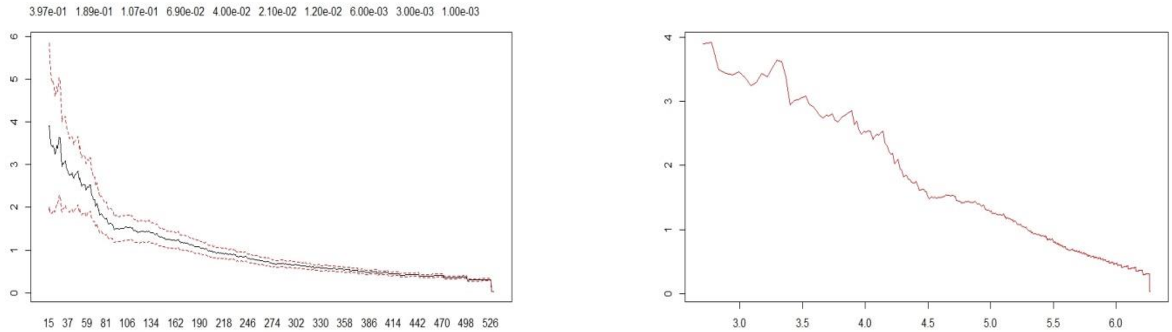
Slovakia



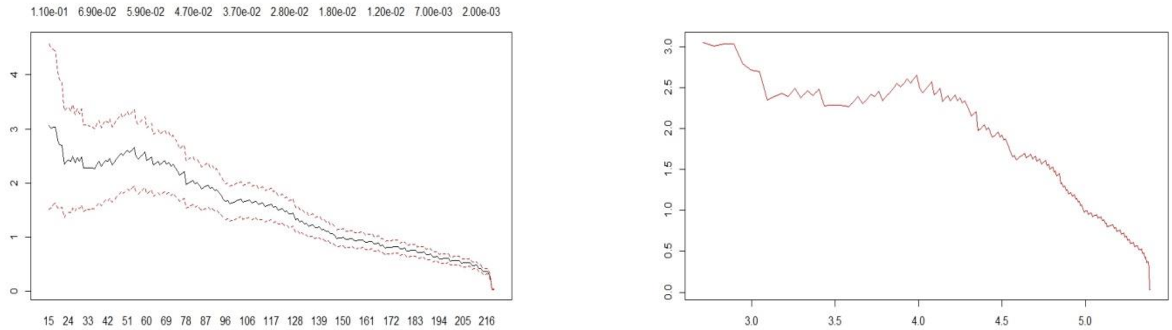
Slovenia



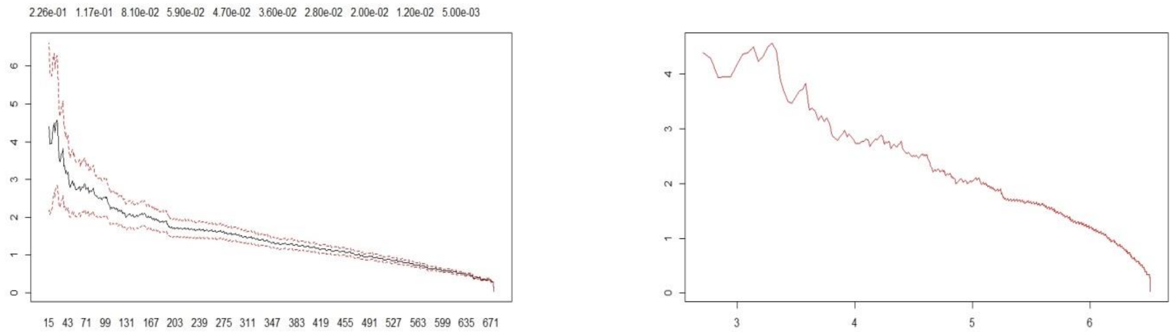
Spain



Sweden



United Kingdom



United States