A Model of Credit Market Sentiment

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Abstract

We present a model of credit market sentiment in which investors form beliefs about future creditworthiness by extrapolating past defaults. Our key contribution is to model the endogenous two-way feedback between credit market sentiment and credit market outcomes. This feedback arises because investors' beliefs depend on past defaults, but beliefs also drive future defaults through investors' willingness to refinance debt at low interest rates. Our model is able to capture many documented features of credit booms and busts, including the link between credit growth and future returns, and the "calm before the storm" periods in which fundamentals have deteriorated but the credit market has not yet turned.

I. Introduction

Recent empirical research in finance and economics has revived the idea that investor sentiment drives credit booms and busts. In their sweeping analysis of sovereign debt cycles over eight centuries, Reinhart and Rogoff (2009) suggest that boom periods are sustained by investors' incorrect perception that "this time is different." In US corporate debt markets, Greenwood and Hanson (2013) show that times of high credit growth are associated with low future returns to credit investors. They interpret their findings to suggest that during periods of elevated sentiment, credit investors are subsequently surprised when firms default or downgrade. López-Salido, Stein, and Zakrajšek (2015) show that elevated credit market sentiment is closely tied to future contractions in aggregate economic activity.

What drives sentiment in the credit markets? In this paper, we present a model in which sentiment is driven by investors' extrapolation of past credit market outcomes. Following periods of calm, with low or no defaults, investors believe that firms will be able to continue to service their debts. Conversely, following periods of turbulence, investors believe that defaults will remain elevated. In making this assumption, we draw on growing evidence across a variety of assets and investor types that expectations of financial markets outcomes are extrapolative (Greenwood and Shleifer 2014, Koijen, Schmeling, and Vrugt 2015), as well as some new evidence that we present here about extrapolation in credit markets.

We consider a firm that invests in a series of short-term projects that require ongoing investment of capital, which the firm finances using debt that pays a fixed coupon until maturity, upon which the firm pays back the principal and issues new debt. Projects generate a random cash flow that varies according to the state of the economy. Investors do not directly observe the payoffs on the project, but may draw inferences about the state of the economy by the firm's ability to continue making payments on the debt. If the firm's debt rises above a level at which a leverage constraint binds, the firm defaults, and a fraction of the debt is written off.

We introduce credit market sentiment into the model through the beliefs of investors, who must estimate the likelihood that the firm will default in the future, resulting in a loss of capital and coupon payments. Investors estimate this probability by extrapolating recent default behavior. We show that investors' extrapolative beliefs lead to an endogenous two-way feedback between

investor sentiment and credit market outcomes. This feedback arises because investors' beliefs depend on past defaults, but beliefs also drive *future* defaults through investors' willingness to refinance debt.
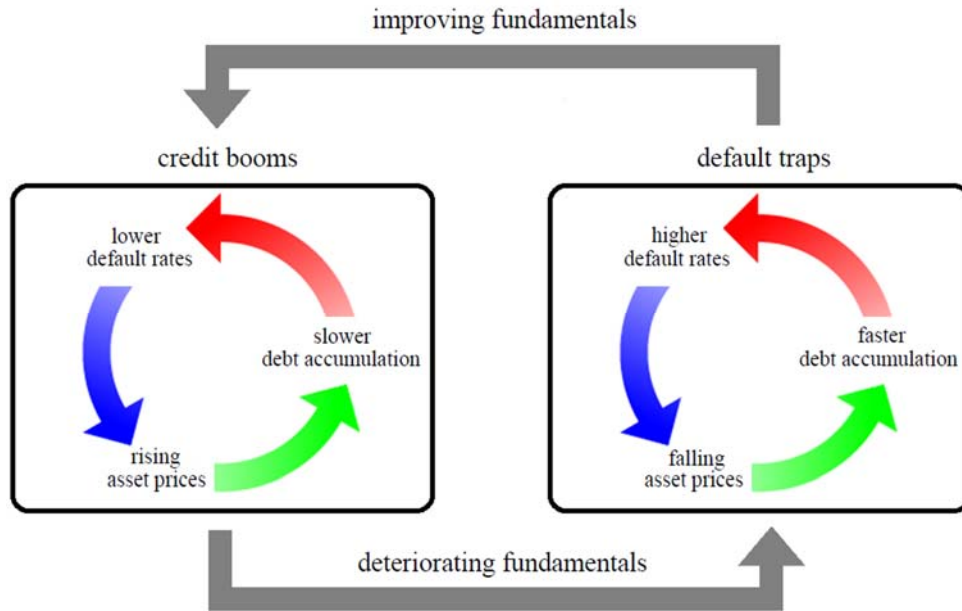

Figure 1. The Credit Cycle.

Figure 1 illustrates the feedback loop. During credit booms, default rates are low and so bond investors believe that future default rates will continue to be low. In the near term, investors' beliefs are self-fulfilling: the perception of low future defaults leads to rising bond prices, which, in turn, makes it easier for the firm to refinance existing debt and issue new debt. Holding constant the firm's "fundamentals", cheaper debt financing leads to slower debt accumulation and a near-term decline in future defaults, which further reinforces investors' beliefs.

Conversely, suppose that the economy has just been through a wave of defaults. Since investors over-extrapolate these recent outcomes, credit market sentiment turns bearish and investors believe that likelihood of future defaults is high. Investors' beliefs can turn out to be self-fulfilling in the short run: bearish credit market sentiment makes it harder for firms to refinance existing debts, leading to an increase in defaults in the short run. In extreme circumstances, this can lead to default spirals in which defaults lead to more pessimism and further defaults.

In our model, transitions between credit booms and default spirals are ultimately caused by changes in fundamentals. However, because investors extrapolate past defaults and not fundamentals, transitions are not fully synchronized with changes in fundamentals, and can be highly path dependent.

We show that during credit booms, elevated credit market sentiment due to low past defaults can enable borrowers to temporarily "paper-over" deteriorations in fundamentals, prolonging credit booms. But, elevated sentiment during these periods of poor fundamentals raises leverage, and thus financial fragility. And, because sentiment is high towards the end of the credit boom, credit markets frequently experience periods of "calm before the storm," in which default rates are low just before they increase rapidly, a phenomenon that is prevalent in many historical accounts of the credit cycle (Klarman 1991, Eichengreen and Mitchener 2003). Overall, extrapolation makes the credit cycle far more persistent than the underlying fundamentals.

The model has natural implications for return predictability emphasized in empirical studies of credit market sentiment (Greenwood and Hanson 2013, Baron and Xiong 2015). In the long term, poor fundamentals *eventually* cause defaults. Thus at long horizons, elevated credit market sentiment predicts high defaults and low returns. The dynamics of prices and returns vary by horizon: elevated credit market sentiment leads risky bonds to outperform in the short term, but to underperform in the long run. Our model also naturally generates the strong correlation between credit growth and low future returns shown in Greenwood and Hanson (2013).

The narrative of credit booms and busts we have described does not hinge on whether debt is short-term or not. However, debt maturity does play a role, with shorter maturity debt enhancing the impact of investor sentiment and increasing fragility. We show that there are two effects. On the one hand, when debt is shorter-term, firm defaults are more exposed to investor sentiment because shorter term debt has to be rolled over more frequently, making the firm highly exposed to changes in investor beliefs. On the other hand, when debt is shorter term, changes in expected default probabilities have less of an impact on the *price* of debt, making it easier for the firm to expand or contract supply, irrespective of the level of sentiment.

Our paper has much in common with Austrian theories of the credit cycle, including von Mises (1924), and von Hayek (1925), as well as Minsky (1986) and Kindleberger's (1978)

accounts of booms, panics, and crashes. More recently, the idea that investors may neglect tail risk in credit markets was developed theoretically by Gennaioli, Shleifer, and Vishny (2012, 2015) and supported by numerous accounts of the financial crisis (Coval, Jurek and Stafford 2009, Krishnamurthy and Muir 2015, among many others). We also draw on growing evidence that investors extrapolate cash flows, past returns, or past crash occurrences (Barberis, Shleifer, and Vishny 1998, Barberis, Greenwood, Jin, and Shleifer 2015, Jin 2015, Greenwood and Hanson 2015). Most related here are Jin (2015) and Bordalo, Gennaioli and Shleifer (2016). Jin (2015) presents a model in which investors' perception of crash risk depends on recent experience. Bordalo, Gennaioli and Shleifer (2016) also provide a model of credit cycles in which investor expectations also play an important role.[1] Their model is similar to ours along some dimensions, but investor expectations in their model are based on the underlying fundamentals rather than credit market outcomes, which in our model are endogenous. In our model, this leads to episodes in which the credit market can become quite disconnected from fundamentals, and path dependent. Relatedly, our model provides a potential explanation for why credit cycles can be of longer duration than underlying fundamentals such as GDP growth, as has been documented by many authors.[2] Overall, our contribution compared to previous work is on the propagation of credit cycles driven by the interplay between expectations and the refinancing nature of credit markets.[3]

Section II summarizes some of the empirical evidence about credit booms and the associated mispricing of credit that our model can help explain. In Section III, we present the model and contrast it with the rational benchmark. Section IV compares simulated results in our model with time-series relationships in the data. Section V develops an extension with multiple firm types. The final section concludes.

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[1] See also Gennaioli, Shleifer, and Vishny (2015) for a precursor to Bordalo, Gennaioli, and Shleifer (2016).

[2] See Taylor (2015) for an overview.

[3] See also Coval, Pan and Stafford (2014) who suggest that in derivatives markets, model misspecification only reveals itself in extreme circumstances, by which time it is too late. Bebchuk and Goldstein (2011) present a model in which self-fulfilling credit market freezes can arise because of interdependence between firms.

II.   Stylized Facts about Credit Booms and Busts

We begin by laying out a set of stylized facts about the evolution of credit market sentiment. We primarily draw on previous work on the corporate credit market in the US, although we present a few new results and update the U.S. time-series evidence through 2014. We present the main results here and leave some details of the data construction for the Internet Appendix.

Observation 1. *Heightened credit market sentiment is associated with strong current credit growth and low subsequent returns*.

We start with the observation that there is a time-varying level of sentiment in credit markets. Greenwood and Hanson (2013) develop a simple measure of credit market overheating based on the composition of corporate debt issuance. Their measure—the share of all corporate bond issuance from speculative grade firms—captures the intuition that when credit markets are overheated, low quality firms increase their borrowing to take advantage. Greenwood and Hanson (2013) show that declines in issuer quality are associated with concurrent growth in *total corporate credit* and that both quantity and quality predict low corporate bond returns. Adopting a similar intuition, Baron and Xiong (2015) show that bank credit expansion also predicts low equity returns in a large panel of countries. López-Salido, Stein and Zakrajšek (2015) show that measures of credit market sentiment also have forecasting power for business cycle activity. Relatedly, Schularick and Taylor (2012) show that elevated credit growth predicts future financial crises, and Mian, Sufi, and Verner (2016) show that growth in household credit is related to future GDP.

Table 1 updates the data from Greenwood and Hanson (2013) through 2014 and also considers a set of additional proxies for credit market sentiment. The table shows regressions of the form:

$$rx_{t+k}^{HY} = a + b \cdot Sent_t + \varepsilon_t, \qquad (1)$$

where *rx* denotes the log excess return on high yield bonds over a 2- or a 3-year horizon, and *Sent* is a proxy for credit market sentiment, measuring using data through the end of year *t*. Excess returns are the difference between the return on the high yield bond index and duration-matched

Treasury bonds. All of our data begin in 1983.[4] Columns (1) and (5) show that the log high yield share significantly predicts reductions in subsequent excess high yield returns. A one standard deviation in the log high yield share is associated with an 8.3 percentage point reduction in log returns over the next two years, or 9.7 percentage points over the next three years.

Columns (2) and (6) of Table 1 show that the same forecasting results hold when credit market sentiment is measured as the growth in total credit. Aggregate corporate credit is the sum of corporate debt securities and loans from Table L103 of the Flow of Funds accounts. A one standard deviation increase in credit growth forecasts a 7.4 percentage point reduction in log returns over the next two years, or 9.3 percentage points over the next three years.

Table 1 supplements these forecasting results with regressions based on two additional measures of credit market sentiment. The first is a measure based on the Federal Reserve's senior loan officer survey, and the second is the expected bond premium *EBP* from Gilchrist and Zakrajšek (2012).

Every quarter, the Fed surveys senior loan officer of major domestic banks concerning their lending standards to households and firms. Officers report whether they are easing or tightening lending standards in the past quarter. We construct a measure of credit market sentiment, *Loansent*, by taking the three-year average percentage of banks that have reported easing credit standards in any given quarter. The idea behind this aggregation is that sentiment captures the *level* of beliefs about future creditworthiness, whereas the quarterly survey measures changes from the previous quarter. The senior loan officer opinion survey begins in the first quarter of 1990, so this measure of sentiment based on this survey begins in December 1992. *Loansent* is 55% correlated with the high yield share and 68% correlated with the growth in aggregate corporate credit.

Gilchrist and Zakrajšek (2012) construct a measure of the price of corporate credit risk by analyzing interest rate spreads on traded corporate bonds. By forming an estimate of the true underlying default risk of the underlying firms, they measure a residual component—the excess bond premium *EBP*—that can be interpreted as a measure of credit market sentiment. *EBP* is −41%

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[4] For results over different time horizons and with additional controls, see Greenwood and Hanson (2013) who compute alternate proxies for issuer quality that extend back as far as 1926.

correlated with the high yield share and –20% correlated with the growth in aggregate corporate credit.

Table 1 shows that both of these additional measures of credit market sentiment forecast corporate bond returns in the expected direction. To summarize, Table 1 confirms that periods of high credit market sentiment are associated with growth in total credit, a loosening of credit standards, and are followed by low subsequent returns.

Observation 2. *Measures of credit market sentiment are correlated with current and lagged defaults; when default rates fall, sentiment rises.*

A key assumption in our model is that market participants extrapolate based on past credit market *outcomes*. While there is abundant evidence of investor extrapolation in other contexts,[5] we provide some preliminary evidence for the assumption of extrapolative expectations in credit markets, by analyzing the correlation between our investor sentiment measures and recent past default rates. Table 2 shows the results of time-series regressions of the form:

$$Sent_t = a + b \cdot Def_t + c \cdot Def_{t-1} + \varepsilon_t, \tag{2}$$

where *Def* denotes the default rate on high yield bonds. We estimate this regression using the same four measures of credit market sentiment from Table 1. Some measures of sentiment (*HYS* and *EBP*) are more highly correlated with *current* default rates, while others are also strongly correlated with lagged defaults rates (Credit Growth and *Loansent*).

Observation 3. *Reductions in credit market sentiment induce future defaults.*

Our third observation is that changes in credit market sentiment can have a causal impact on near-term credit market outcomes. Whatever the fundamentals may be, when credit market sentiment declines, investors are reluctant to roll over debt, making future defaults more likely. We provide some tentative evidence for this observation in Table 3, which displays regressions of the form:

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[5] See Greenwood and Shleifer (2014) for a summary of this evidence. See also Dominguez (1986), Frankel and Froot 1987, 1988), Shiller, Kon-Ya and Tsutsui (1996), Case, Shiller and Thompson (2012), Amromin and Sharpe (2013), and Bacchetta, Mertens and Wincoop (2009).

$$Def_{t+k} = a + b \cdot \Delta Sent_t + \varepsilon_t. \tag{3}$$

The dependent variable is current or future defaults, and the independent variable is the change in sentiment in year t. Again, we draw on the four proxies for credit market sentiment used in Table 1. Table 3 shows that changes in all four credit sentiment measures forecast defaults in the following year. For some of the measures of credit sentiment, changes are also correlated with contemporaneous defaults or defaults in year $t + 2$.

III. The Model

A. Setup and rational benchmark

There are two types of agents, a single representative firm and a continuum of homogenous corporate bond investors.

When the economy begins at time 0, the representative firm begins to invest in short-term projects; each project lasts for a dt period. Projects arrive continuously over time with intensity normalized to one and with a per-unit investment cost of I. Projects generate instantaneous cash flows $\tilde{x}_t$, which follows a regime-switching Markov process

$$\begin{array}{c} \\ \tilde{x}_t = H \\ \tilde{x}_t = L \end{array} \begin{pmatrix} \tilde{x}_{t+dt} = H & \tilde{x}_{t+dt} = L \\ 1 - q_H dt & q_H dt \\ q_L dt & 1 - q_L dt \end{pmatrix}, \tag{4}$$

where $q_H, q_L > 0$ are the intensities for the transition from the high state H to the low state L and vice versa. That is, for a project that lasts from time t to $t + dt$, its payoff is set at time $t + dt$ to $\tilde{x}_{t+dt}$. The flows of benefit and cost satisfy

$$L < I < H. \tag{5}$$

Thus, a time-t project generates a net operating profit when $\tilde{x}_{t+dt} = H$ and generates a net operating loss when $\tilde{x}_{t+dt} = L$.[6]

The firm needs to finance the projects by raising debt from investors. Average maturity of the debt is governed by the parameter γ, as follows. At time t, denote the number of outstanding

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[6] When the payoff becomes L, the firm may choose not to invest in the project. Here we do not allow the firm to do so: it must invest in the projects all the time regardless of their payoffs.

corporate bonds as F_t, the per-unit bond price as p_t, the market value of outstanding debt as $D_t = F_t p_t$. Each bond continuously pays coupons at an exogenous rate of k until the arrival of a Poisson shock with intensity γ. Upon the Poisson shock, the bond contract matures. And in the absence of default, the firm pays off the face value of maturing debt by issuing new debt. Maturity-triggering Poisson shocks are independent across bonds originated at different times, and are also independent of $\tilde{x}_t$. Outside of default, a fraction γdt of outstanding debt matures every dt period. Thus the expected maturity of debt is $1/\gamma$.

Non-maturing bonds are marked at the market price and the net operating profit is also converted to outstanding bonds at the market price.[7] So, in the absence of default, the evolution of D_t is

$$D_{t+dt} = (1 - \gamma dt) D_t \frac{kdt + p_{t+dt}}{p_t} + \frac{\gamma D_t dt}{p_t} + (I - x_{t+dt}) dt. \tag{6}$$

Rearranging terms gives

$$\frac{dD_t}{dt} = \underbrace{\left(\frac{kdt + dp_t}{p_t dt} \right) D_t}_{\substack{\text{rate of return} \\ \text{on non-maturing bonds}}} + \underbrace{\frac{(1 - p_t)}{p_t} \gamma D_t}_{\substack{\text{inflow/outflow} \\ \text{from maturing bonds}}} + \underbrace{I - x_{t+dt}}_{\substack{\text{inflow/outflow} \\ \text{from projects}}} + o(dt). \tag{7}$$

If the quantity on the right hand side of (7) is positive, the firm needs to raise this dollar amount by issuing new bonds; if this quantity is negative, the firm uses this dollar amount to repurchase outstanding bonds from investors.

Upon default, investors are entitled to a fraction $\eta < 1$ of the future coupon payments. In the absence of default, bonds will mature at their face value upon the arrivals of independent Poisson shocks with intensity γ.

There is a single homogenous group of risk neutral investors with a time discount rate of ρ. Investors perceive a default likelihood of λ_t and believe this same likelihood will hold at all future dates. We later describe how investors form their beliefs about λ_t by extrapolating past outcomes. Given these assumptions, the market price per bond p_t is given by

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[7] Due to time homogeneity of Poisson arrivals, all outstanding bonds share the same price.

$$p_t = \mathbb{E}_t^b \left[\underbrace{\int_t^{\tilde{\tau}_\lambda \wedge \tilde{\tau}_\gamma} e^{-\rho(s-t)} k\, ds}_{\substack{\text{coupon payment until} \\ \text{default or debt maturity}}} + \underbrace{e^{-\rho(\tau-t)} \mathbf{1}_{\tau=\tilde{\tau}_\lambda < \tilde{\tau}_\gamma} \eta p_t}_{\substack{\text{partial recovery} \\ \text{of value upon default}}} + \underbrace{e^{-\rho(\tau-t)} \mathbf{1}_{\tau=\tilde{\tau}_\gamma < \tilde{\tau}_\lambda}}_{\substack{\text{payment upon} \\ \text{debt maturity}}} \right]. \tag{8}$$

Solving equation (8), the price of debt is given by

$$p_t = \frac{k+\gamma}{\rho+\gamma+(1-\eta)\lambda_t} < \frac{k+\gamma}{\rho+\gamma}. \tag{9}$$

p_t is always less than $(k+\gamma)/(\rho+\gamma)$ because investors perceive some default risk and hence require a price discount to hold defaultable bonds. Naturally, for any $\eta < 1$, equation (9) shows that p_t decreases in the perceived default likelihood, λ_t. Furthermore, the bond yield in our model is $\rho + (1 - \eta)\lambda_t$ and the credit spread is $(1 - \eta)\lambda_t$. Thus, equation (9) says that the relationship between prices and yields is steeper for longer maturity (lower γ) bonds.

Equation (9) says that, when setting the bond price, investors do not directly take the evolution of F_t into account, nor do they pay direct attention to fundamentals. This is partly due to the fact that investors receive the coupon k instead of the net operating cash flows $\tilde{x}_t - I$ and therefore may not know the fundamentals. Below we explain how prices and credit evolve if investors price credit more rationally.

We now further specify an upper barrier and a lower barrier for F_t. We assume that, if F_t reaches a lower barrier $\underline{F} \geq 0$ and has a tendency of falling further, it is then floored at $\underline{F}$, and the extra net payoffs are distributed to equity holders in the form of dividends. If, on the other hand, F_t reaches an upper barrier $\overline{F} > \underline{F}$ and has a tendency of rising further, a *default* is triggered. In the event of default, a proportional write-down occurs: a fraction $1 - \eta$ of bond holdings is removed from the firm's balance sheet, in which case the remaining number of bonds is $\eta \overline{F}$.

The idea underlying the upper and lower barriers $\overline{F}$ and $\underline{F}$ can be motivated via the pecking order. Firms only raise external finance in the form of debt. And when there is free cash flow available to equity, they first use this cash flow to retire existing debts. However, once the face value of debt reaches a sufficiently low level, $\underline{F} \geq 0$, they choose to pay out all available free cash flow to their equity holders. This lower threshold can be endogenized by assuming that the firm's equity holders trade off the value of receiving dividends today versus the value of further debt

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reduction. Further debt reduction lowers the future probability of default and hence raises the expected value of future dividends. Since the benefits of debt reduction decline with the level of debt, the firm chooses to pay out free cash flow to equity holders once the face value of debt reaches a certain value.

To endogenize the upper threshold $\overline{F}$, we could assume that at any time, equity holders can also default on their outstanding debt and abscond with some fraction of the firm's total enterprise value. Equity holders decide whether or not to exercise this default option by comparing the present value of expected future dividends to the value of this outside default option. Since, all else equal, the present value of expected future dividends is decreasing in the amount of outstanding debt, this means that equity holders will choose to default once the face value of debt reaches a sufficiently high level $\overline{F} > \underline{F}$.

Based on the assumptions outlined above, we summarize the law of motion for the number of bonds:

Assumption 1 (Law of Motion for Debt Financing). If $\underline{F} < F < \overline{F}$

$$dD_t = \left(\frac{kdt + dp_t}{p_t dt}\right) D_t dt + \frac{(1-p_t)}{p_t}\gamma D_t dt + (I - x_{t+dt})dt. \tag{10}$$

Given that $F_t = D_t / p_t$, (10) is equivalent to

$$dF_t = [kF_t + (1-p_t)\gamma F_t + I - x_{t+dt}]dt/p_t. \tag{11}$$

If $F_t = \underline{F}$

$$dF_t = \mathbf{1}_{kF_t + (1-p_t)\gamma F_t + I - x_{t+dt} > 0}[kF_t + (1-p_t)\gamma F_t + I - x_{t+dt}]dt/p_t. \tag{12}$$

If $F_t = \overline{F}$, a default event is triggered and $F_{t+dt} = \eta\overline{F}$. ∎

To complete the model description, we now introduce our key assumption regarding how investors form their beliefs about future defaults. The perceived default likelihood λ_t is a weighted average of past realized default rates, as described below:

Assumption 2 (Endogenous Belief Formation). The default likelihood perceived by investors at time t is

$$\lambda_t = \beta \int_{-\infty}^{t} e^{-\beta(t-s)} f(\lambda_{s-ds}) dN_{s-ds}, \quad \beta > 0, \tag{13}$$

where s is the running variable for the integral, N_t measures the number of default events that have taken place up to time t, the parameter β controls for how far investors extrapolate into the past, and $f(\cdot)$ measures how much impact a default has on the post-default level of λ_t as a function of its pre-default level. For instance, if $F_{t-dt} = \overline{F}$, a default occurs between time $t - dt$ and t, and $dN_{t-dt} \equiv N_t - N_{t-dt} = 1$, and $\lambda_t - \lambda_{t-dt} = \beta f(\lambda_{t-dt})$. The differential representation of (13) is[8]

$$d\lambda_t = \underbrace{-\beta\lambda_t dt}_{\text{memory decay}} + \underbrace{\beta f(\lambda_t) dN_t}_{\text{default impulse}}. \tag{14}$$

∎

Equation (13) says that when a default occurs, it creates an instant contribution of $\beta f(\lambda_t)$ to the perceived default likelihood. As time elapses, investor memory decays exponentially at rate β. Thus, a higher value of β means that investors place greater weight on recent events.

Our specification of beliefs is the primary assumption in the model, and draws on evidence across a variety of settings that expectations of financial markets outcomes are extrapolative (Greenwood and Shleifer 2014, Koijen, Schmeling, and Vrugt 2015). It also matches the empirical evidence, presented earlier, that empirical measures of credit market sentiment seem to correlate strongly with recent default rates. As in Barberis, Greenwood, Jin and Shleifer (2015), we do not take a position on the source of investors' extrapolative expectations, which could be driven by representativeness heuristic (as in Barberis, Shleifer, Vishny 1998 or Rabin 2002) or by experience effects (Malmendier and Nagel 2011).

The formation of beliefs is endogenous, because λ_t is determined by past defaults, which were triggered by past beliefs of investors, past debt levels, and past realizations of fundamentals. At each point in time, our model is fully characterized by three state variables, $\tilde{x}_t, F_t,$ and λ_t.

Our assumption that beliefs are formed entirely in a backward-looking manner is stark, and can be contrasted with those of a more rational forward-looking investor. In the rational benchmark

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[8] The timing of belief update is not crucial: when a default occurs at time $t - dt$, whether it changes the investor belief at time $t - dt$ or at time t does not affect our result. The reason is that in this model, the time-varying bond price only affects the *incremental* debt raised or repurchased by the firm, which, in continuous-time, is infinitesimal.

case, investors do not pay direct attention to past realized defaults because defaults are a function of other variables. Instead, investors are fully aware of the regime-switching process for $\tilde{x}_t$ in (4), the Poisson arrivals of bond maturity with intensity γ, the law of motion for F_t in (11) and (12), as well as the default trigger at $F_t = \overline{F}$ and the dividend trigger at $F_t = \underline{F}$. So instead of (8), investors use all the available information to set the bond price p_t as a function of $\tilde{x}_t$ and F_t.

Proposition 1 below summarizes the solution of bond prices in the rational benchmark.

Proposition 1: Rational Bond Prices. In a fully rational model, the bond prices, $p_H(F_t)$ and $p_L(F_t)$ are jointly determined by a system of two first-order non-linear ordinary differential equations

$$0 = (k+\gamma) - (\rho+\gamma)p_L(F_t) + q_L(p_H(F_t) - p_L(F_t))$$

$$+ \frac{p_L'(F_t)}{p_L(F_t)}(kF_t + (1 - p_L(F_t))\gamma F_t + I - L),$$

$$0 = (k+\gamma) - (\rho+\gamma)p_H(F_t) + q_H(p_L(F_t) - p_H(F_t))$$

$$+ \frac{p_H'(F_t)}{p_H(F_t)}(kF_t + (1 - p_H(F_t))\gamma F_t + I - H). \tag{15}$$

for $\underline{F} < F_t < \overline{F}$, with the following boundary conditions

$$k + \gamma + q_H p_L(\underline{F}) = (\gamma + \rho + q_H)p_H(\underline{F}), \qquad p_L(\overline{F}) = \eta p_L(\eta\overline{F}). \tag{16}$$

Here we assume $k\underline{F} + (1 - p_H(\underline{F}))\gamma\underline{F} + I - H < 0$; otherwise rational bond prices become degenerate. [9,10,11]

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[9] In the rational benchmark, positive bond prices do not always exist. When the speed of debt accumulation is high—for instance, when I is much larger than L—rational investors' self-fulfilling beliefs could drive $p_L(F_t)$ toward zero. In this case, there may not be a sustainable level of positive price that prevents p_L from falling further. When the speed of debt accumulation is high, the default risk is high once the project payoff becomes low. This pushes down the bond price, which makes it more difficult for the firm to raise debt, causing the speed of debt accumulation to become even higher, essentially a downward spiral of the bond price.

[10] If the condition $k\eta\overline{F} + (1 - p_H(\eta\overline{F}))\gamma\eta\overline{F} + I - H < 0$ does not hold, then once F_t falls into the range $[\eta\overline{F}, \overline{F}]$ it will always stay in that range. In the numerical examples we use, $k\eta\overline{F} + (1 - p_H(\eta\overline{F}))\gamma\eta\overline{F} + I - H$ stays negative.

[11] The system of non-linear differential equations cannot be solved analytically. We adopt a projection method with Chebyshev polynomials to solve this system numerically. Details of the numerical procedure are given in the Appendix.

Proof: See the Appendix.   ∎

B. Formal results

We now turn to the main results. We start by describing the evolution of credit sentiment in good times. Second, we examine credit sentiment in bad times. We then take a closer look at model dynamics—transitions between good and bad times and the forecastability of returns—and compare the dynamics documented empirically. Lastly, we study the role of debt maturity.

B.1 Credit sentiment in good times

During credit booms, historical default rates have been low. Since bond investors extrapolate past defaults when forming their expectations about future defaults, they believe that future default rates will continue to be low. In the near term, these beliefs can be self-fulfilling: the perception of low future defaults leads to high bond prices, which, in turn, makes it easy for the firm to refinance existing debts and issue new debts. As a result, cheaper debt financing and slower debt accumulation can lead to a near-term decline in future defaults, and this further reinforces investors' beliefs about future default rates.

Below we elaborate on this narrative of credit sentiment in good times by making two related points. First and most simply, we show how greater optimism prolongs credit booms. Second, we show how longer periods of strong fundamentals allow the economy to sustain longer downturns in fundamentals before entering a credit bust. We describe a particular example of this, in which firms can have brief periods of "papering over losses" during which fundamentals worsen but investor optimism prevents firms from entering a default cycle.

We start with the observation that greater optimism can prolong credit booms. From the bond price in (9) and the law of motion for F_t in (11), we know that investor optimism (low λ_t) pushes the bond price p_t close to one. All else being equal, higher bond price makes it easier for firms to refinance. Investor optimism helps lower F_t and hence keep the firm away from default, which in turn leads to greater investor optimism.

Longer periods of strong fundamentals allow the economy to sustain longer downturns in fundamentals before entering a credit bust. To see this, suppose fundamentals stay at H from time

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0 to t and switch to L thereafter. Under mild conditions, a later switch time (higher t) tends to delay defaults. This is because a longer period with good fundamentals tends to further lower F_t from the default barrier $\bar{F}$. As a result, it takes longer for the number of outstanding bonds to accumulate back to a high level after bad fundamentals arrive. There is also an important multiplier effect arising from beliefs. The longer the economy spends in good times without having a default, the lower is investors' *perceived* likelihood of future defaults. Thus, even as fundamentals switch from H to L, the credit spread continues to fall, slowing down debt accumulation. Of course, because fundamentals have deteriorated, the economy *eventually* reaches the default barrier $\bar{F}$. The period immediately before reaching the upper barrier is one of "calm before the storm" because of simultaneously weak fundamentals but high prices/low credit spreads.

Proposition 2 and Corollary 1 below formalize these results.

Proposition 2: Time Evolution of the Number of Outstanding Bonds. Suppose $\underline{F} < F_0 < \bar{F}$. If $\tilde{x}_0 = H$, then, assuming that $\tilde{x}_t$ does not experience a transition, F_t deterministically evolves as

$$
\begin{aligned}
F(t) &= F_0 \exp\left(\rho t + \tfrac{(1-\eta)\lambda_0}{\beta}(1-e^{-\beta t})\right) + \frac{I-H}{k+\gamma}\left[\exp\left(\rho t + \tfrac{(1-\eta)\lambda_0}{\beta}(1-e^{-\beta t})\right)-1\right] \\
&+ \frac{\gamma(I-H)}{k+\gamma}\exp\left(\rho t - \tfrac{(1-\eta)\lambda_0}{\beta}e^{-\beta t}\right)\int_0^t \exp\left(-\rho\tau + \tfrac{(1-\eta)\lambda_0}{\beta}e^{-\beta\tau}\right)d\tau
\end{aligned}
$$

$$(17)$$

unless it gets capped by $\bar{F}$ or gets floored by $\underline{F}$.

Similarly, if $\tilde{x}_0 = L,$ then, assuming that $\tilde{x}_t$ does not experience a transition, F_t deterministically increases until it reaches the upper barrier $\bar{F}$. Specifically

$$
\begin{aligned}
F(t) &= F_0 \exp\left(\rho t + \tfrac{(1-\eta)\lambda_0}{\beta}(1-e^{-\beta t})\right) + \frac{I-L}{k+\gamma}\left[\exp\left(\rho t + \tfrac{(1-\eta)\lambda_0}{\beta}(1-e^{-\beta t})\right)-1\right] \\
&+ \frac{\gamma(I-L)}{k+\gamma}\exp\left(\rho t - \tfrac{(1-\eta)\lambda_0}{\beta}e^{-\beta t}\right)\int_0^t \exp\left(-\rho\tau + \tfrac{(1-\eta)\lambda_0}{\beta}e^{-\beta\tau}\right)d\tau,
\end{aligned}
$$

$$(18)$$

so long as $F_t < \eta\bar{F}$. Once $F_t = \bar{F},$ a default occurs and $F_{t+dt} = \eta\bar{F}$.

Proof: See the Appendix.   ∎

Corollary 1: Passage Time until Default (Prolonged Credit Bubble). Suppose $\underline{F} < F_0 < \overline{F}$, and suppose the project payoffs stay at H from time 0 to t and switch to L thereafter. Also for simplicity suppose $F_l \in (\underline{F}, \overline{F})$, $\forall 0 \le l \le t$. Then the post-switch passage time until default, denoted as s, is the unique solution of

$$
\left((k+\gamma)\overline{F}+I-L\right)e^{-\rho s} = \left((k+\gamma)F(t)+I-L\right)\exp\left(\frac{(1-\eta)\lambda_0 e^{-\beta t}}{\beta}(1-e^{-\beta s})\right)
$$
$$
+\gamma(I-L)\exp\left(-\frac{(1-\eta)\lambda_0 e^{-\beta t}}{\beta}e^{-\beta s}\right)\int_0^s \exp\left(-\rho\tau + \frac{(1-\eta)\lambda_0 e^{-\beta t}}{\beta}e^{-\beta\tau}\right)d\tau.
$$

(19)

The sensitivity of s with respect to t is

$$
s' = \underbrace{-\frac{(k+\gamma)}{\rho\left((k+\gamma)\overline{F}+I-L\right)}F'(t)\exp\left(\rho s + \frac{(1-\eta)\lambda_0 e^{-\beta t}}{\beta}(1-e^{-\beta s})\right)}_{\text{effect of a longer period of good fundamentals on } s \text{ through affecting the post-switch number of outstanding bonds}}
$$

$$
\underbrace{+\frac{((k+\gamma)F(t)+I-L)(1-\eta)\lambda_0 e^{-\beta t}}{\rho\left((k+\gamma)\overline{F}+I-L\right)}\exp\left(\rho s + \frac{(1-\eta)\lambda_0 e^{-\beta t}}{\beta}(1-e^{-\beta s})\right)(1-e^{-\beta s}) - \frac{(1-\eta)\lambda_0 e^{-\beta(t+s)}}{\rho}s' - \frac{\gamma(I-L)}{\rho\left((k+\gamma)\overline{F}+I-L\right)}s'}_{\text{effect of a longer period of good fundamentals on } s \text{ through affecting the post-switch perceived default likelihood}}
$$

(20)

If $kF_t + (1-p_t)\gamma F_t + I - H < 0$, then $s'(t) > 0$.

Proof: See the Appendix. ∎

     We have noted that credit sentiment allows the firm to sustain longer downturns in fundamentals before entering a credit bust. An interesting example of this arises when elevated credit sentiment can allow the credit market to withstand temporary drops in fundamentals. Specifically, if fundamentals turn low for a short time and then turn high again, the firm can temporarily paper over losses without incurring default. We illustrate this phenomenon in Figure 2 in which fundamentals switch from H to L for a period of time and then switch back to H. Panel A shows the path of fundamentals; Panel B shows the impact on debt growth and prices. Recall that investors do not pay direct attention to fundamentals, so investor optimism continues to rise after the firm fundamentals get worse *until* a default occurs, helping the firm to roll over debt and hence prolonging a credit boom.

     Panel C shows the contrast between this and the rational model, in which forward-looking investors significantly push down the bond price once the firm fundamentals switch from H to L,

well before a default occurs. Lower bond prices make it more difficult for the firm to raise debt and hence cause default to happen earlier.

We note that while investor optimism sometimes allows the firm to paper over losses, it does so at the cost of increased fragility. This is because without inducing a default, the firm's outstanding debt level remains high. Worsened fundamentals imply higher debt payments, contributing to a higher default risk. This finding suggests that even though a credit bubble can be further extended in the behavioral model due to investor optimism, fragility will remain high unless economic fundamentals stay healthy for a long time.
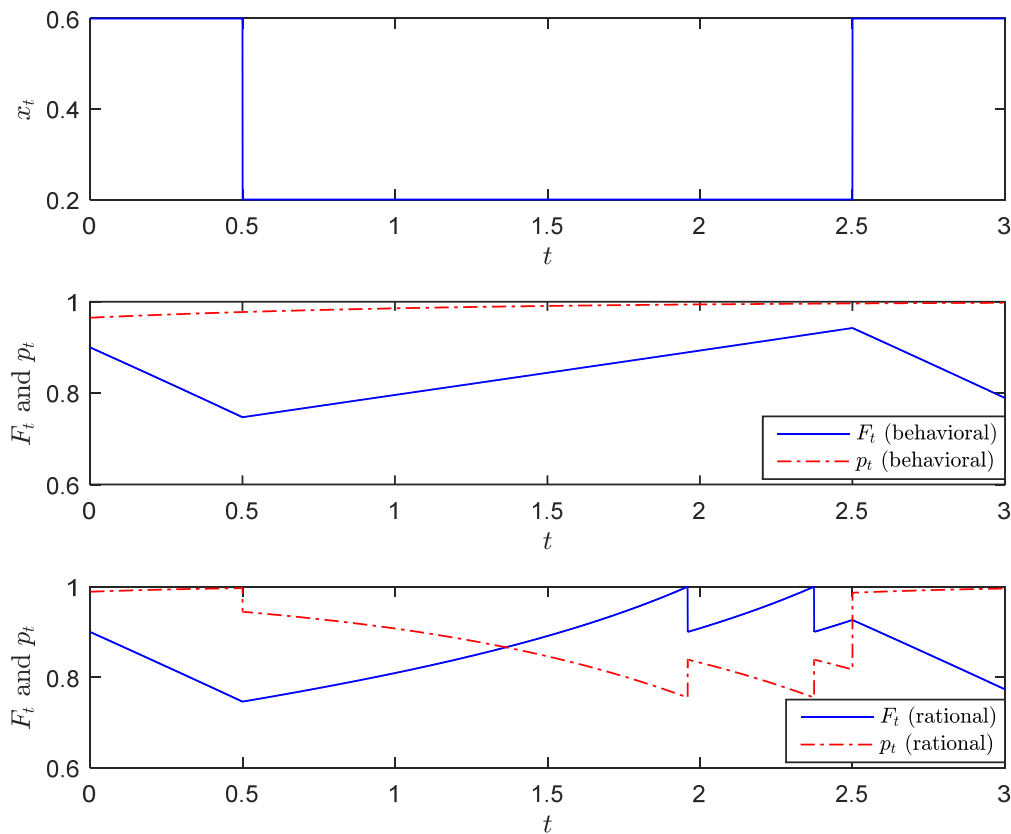

Figure 2. Papering over Losses. Panel A plots the time evolution of fundamentals x_t. In this example, the fundamentals are specified as $x_t = H$ for $0 \leq t < 0.5$ and $t \geq 2.5$ and $x_t = L$ for $0.5 \leq t \leq 2.5$. Panel B plots the number of outstanding bonds F_t and the bond price p_t for the behavioral model. Panel C plots, for comparison, the number of outstanding bonds F_t and the bond price p_t for the rational model. The parameter

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values are $H = 0.6$, $L = 0.2$, $I = 0.25$, $k = 5\%$, $\rho = 5\%$, $\overline{F} = 1$, $\underline{F} = 0.5$, $\beta = 0.9$, $\eta = 0.9$, $\gamma = 1/2$, $q_H = 0.2$, and $q_L = 0.5$. The initial conditions are: $F_0 = 0.9$ and $\lambda_0 = 0.2$.

B.2 Credit sentiment in bad times

Upon default, credit markets enter bad times. Since bond investors extrapolate past defaults, they tend to believe that future default rates will continue to be high. If fundamentals remain poor, investors' pessimism can be self-fulfilling: their perception of high future defaults pushes down bond prices, which makes it more difficult to raise more debt to refinance existing debt and cover the shortfall between the cash inflows and the ongoing investment costs. With accelerating debt accumulation, it shortens the time before the next default occurs. The increasing incidence of default further increases pessimism.

Below we present two results regarding credit sentiment in bad times. First, we describe the conditions under which pessimism can spiral during default cycles. Second, we show that the length of a credit downturn can affect the speed of subsequent market recovery when fundamentals eventually recover.

We start by describing how the economy may enter into a pessimism spiral. As we show below, it depends on how much beliefs update upon the realization of a default event. From the evolution of λ_t in (14), upon default at time t, λ_t jumps up by $\beta f(\lambda_t)$, so the credit spread widens and the bond price drops. If fundamentals stay depressed, then F_t will keep rising again from $\eta \overline{F}$ towards $\overline{F}$. Thus, by the time the next default is triggered, whether or not the perceived default rate will have fallen by $\beta f(\lambda_t)$ (back to the level before last default) depends on the parameter β and the functional form of $f(\cdot)$.

For the remainder of the paper, we adopt a simple form for $f(\cdot)$:

$$f(\lambda_t) = \alpha_1, \tag{21}$$

where α_1 is a constant, so that the contribution of default to beliefs does not depend on the pre-default level of λ_t. Figure 3 shows that when α_1 is small ($\alpha_1 = 0.2$), each "default cycle" is sufficiently long and a default's impact on beliefs λ_t is limited. As a result, when a default event occurs, investors do not become too pessimistic. In this case, default cycles are approximately of equal length. On the other hand, if α_1 is high ($\alpha_1 = 1$), each default has a larger impact on investor

beliefs, so that a series of defaults makes investors increasingly pessimistic. In this case, the length of default cycles decreases over time and spiraling investor pessimism arises.
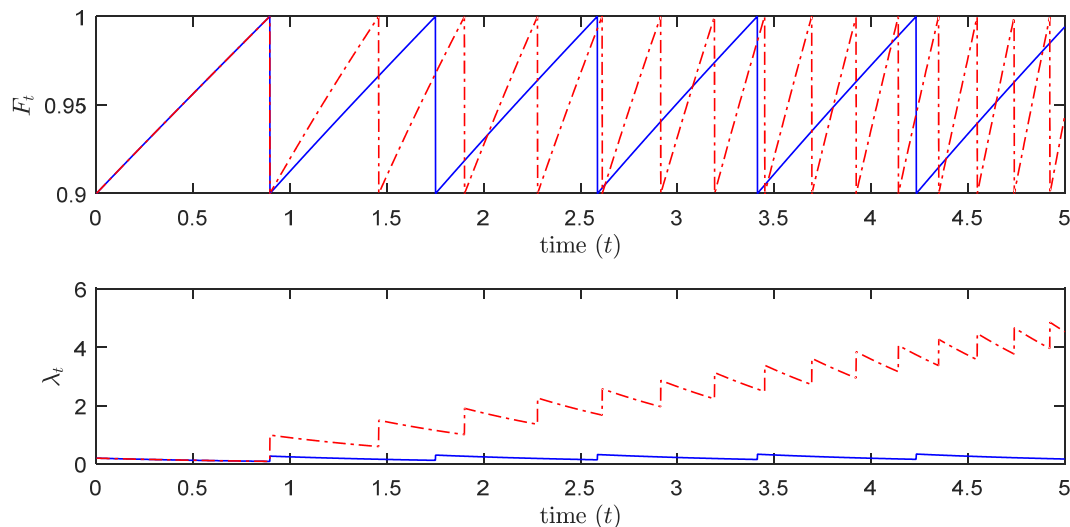

Figure 3. Defaults and Credit Sentiment in Bad Times. The top panel plots the time evolution of F_t with $f(\lambda_t) = 0.2$ (blue line) and with $f(\lambda_t) = 1$ (red dot-dashed line). The bottom panel plots the time evolution of the perceived default likelihood λ_t with $f(\lambda_t) = 0.2$ (blue line) and with $f(\lambda_t) = 1$ (red dot-dashed line). The other parameter values are: $L = 0.2$, $I = 0.25$, $k = 5\%$, $\rho = 5\%$, $\beta = 0.9$, $\eta = 0.9$, $\gamma = 1/2$, $\bar{F} = 1$, and $\lambda_0 = 0.2$. Fundamentals stay at L, and $F_0 = \eta \bar{F}$.

Our second observation about credit sentiment during bad times is that the length of the downturn significantly affects the speed of subsequent market recovery. A longer downturn leads bond investors to be more pessimistic, pushing down the bond price. Since spreads are high, the firm may have to raise more debt even when fundamentals improve. In other words, through the belief channel in our model, a longer credit downturn is followed by a slower market recovery.

Figure 4 formalizes this point by providing an example in which fundamentals switch back from L to H at either $t = 5$ (the top two panels) or $t = 1.5$ (the bottom two panels). When the switch occurs at $t = 5$, continuing investor pessimism causes debt F_t to keep rising after the switch, even though fundamentals have improved. On the other hand, when the switch occurs at $t = 1.5$, the level of investor pessimism is not that high at the switch. As a result, good fundamentals lead to debt reduction right away, and the credit market recovers quickly.
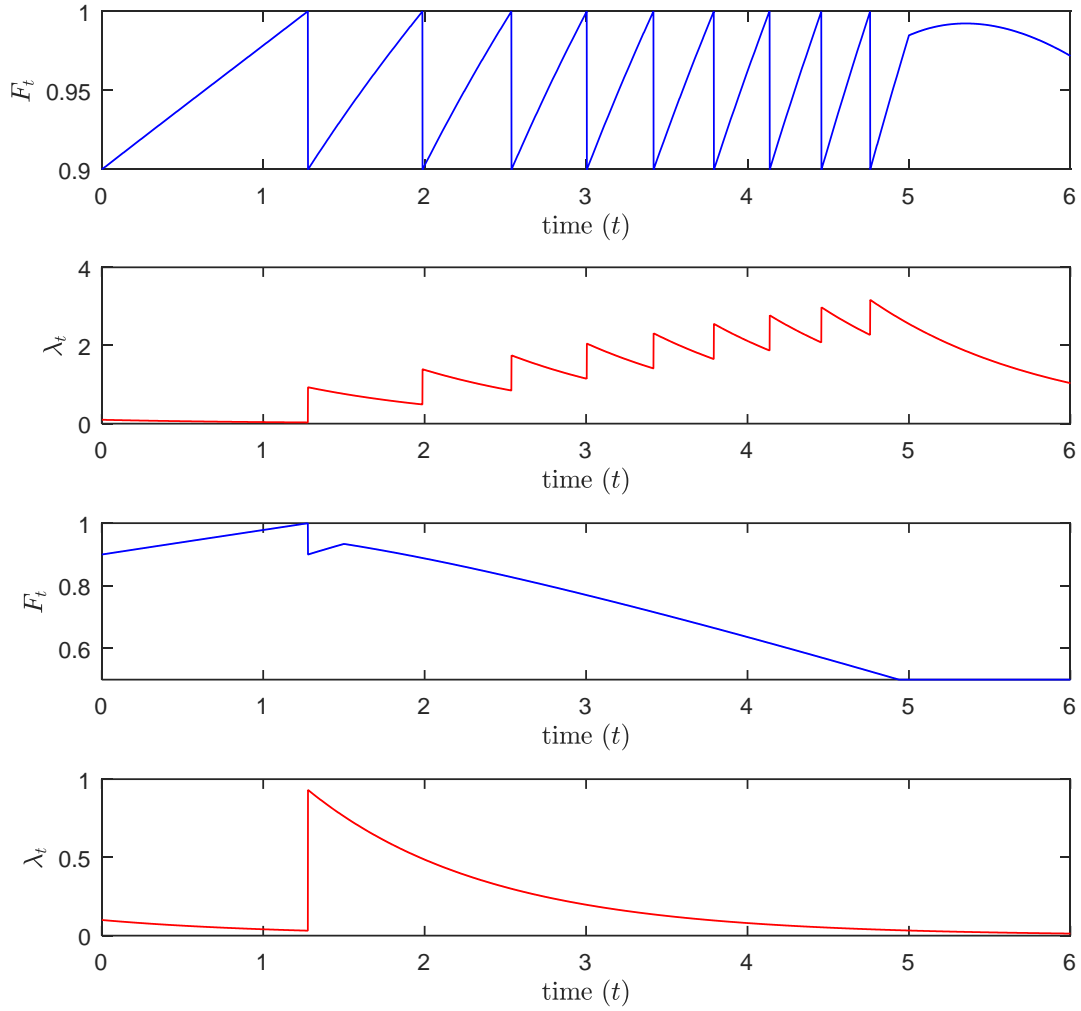
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Figure 4. Market Recovery after Downturns. The figure shows that the length of the downturn in fundamentals affects the speed of eventual market recovery. The top two panels plot the time evolutions of F_t and λ_t when fundamentals recover in period 5. The bottom two panels plot the time evolutions of F_t and λ_t when fundamentals recover in period 1.5. Fundamentals stay at L before the recovery and stay at H thereafter. The parameter values are: $H = 0.3$, $L = 0.1$, $I = 0.125$, $k = 5\%$, $\rho = 5\%$, $\beta = 0.9$, $\eta = 0.9$, $\gamma = 1/2$, $\alpha_1 = 1$, $\bar{F} = 1$, and $\underline{F} = 0.5$. The initial debt level is $F_0 = \eta\bar{F}$ and the initial belief is $\lambda_0 = 0.1$.

Taking stock of the results so far, we have shown that when investors extrapolate, the evolution of the credit market is highly path dependent and seemingly disconnected from

fundamentals. Figure 5 illustrates these features "through the cycle" by comparing debt growth and prices in our model with those that would materialize in the rational benchmark case. The example we consider here, fundamentals start high, fall for a number of periods, and then permanently recover in period 7.5.
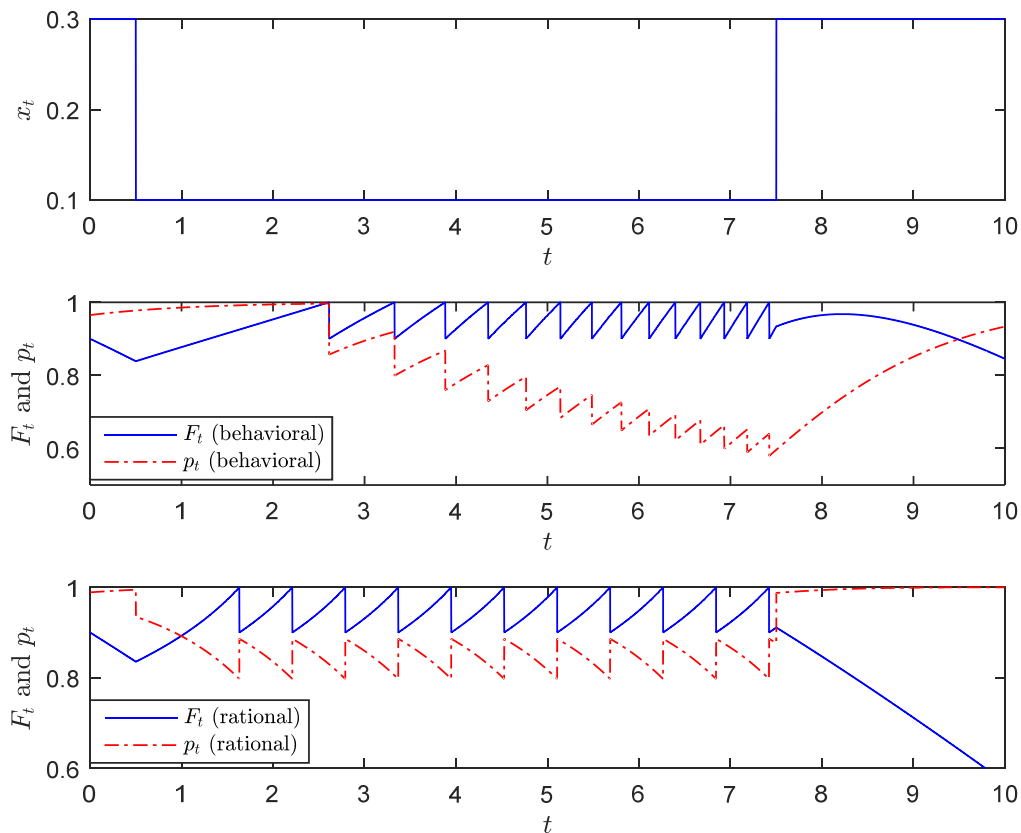

Figure 5. Through the Cycle Comparison between Model and Rational Benchmark. Panel A plots the time evolution of fundamentals x_t. In this example, the fundamentals are specified as $x_t = H$ for $0 \leq t < 0.5$ and $t \geq 7.5$ and $x_t = L$ for $0.5 \leq t \leq 7.5$. Panel B plots the number of outstanding bonds F_t and the bond price p_t for the behavioral model. Panel C plots, for comparison, the number of outstanding bonds F_t and the bond price p_t for the rational model. The parameter values are $H = 0.3$, $L = 0.1$, $I = 0.125$, $k = 5\%$, $\rho = 5\%$, $\bar{F} = 1$, $\underline{F} = 0.5$, $\beta = 0.9$, $\eta = 0.9$, $\gamma = 1/2$, $q_H = 0.2$, $q_L = 0.5$, and $\alpha_1 = 1$. The initial conditions are: $F_0 = 0.9$ and $\lambda_0 = 0.2$.

The figure shows many features of credit booms and busts that we have already noted. Investor optimism prolongs credit booms: after the fundamentals switch from H to L at $t = 0.5$, it

takes *longer* for the first default to occur in the behavioral model than in the rational benchmark. Second, after fundamentals switch to *L*, the bond price continues to rise in the behavioral model because of increasing investor optimism. In the rational model, the bond price drops because rational investors understand that with low fundamentals, the firm needs to raise more debt, and as a result default risks go up. Third, during bad times ($0.5 \leq t \leq 7.5$), the bond price oscillates with a clear downward trend in the behavioral model, but with no trend in the rational model. In the behavioral model, the bond price is path-dependent and can get pushed down further after more defaults with spiraling investor pessimism. Lastly, after fundamentals switch back to *H* at $t = 7.5$, the firm in the rational model immediately begins to retire debt because rational bond price jumps up due to lower default risks. However, in the behavioral model, the firm has to keep raising more debt even in the presence of improved fundamentals: investor pessimism is persistent and it keeps the bond price low. Overall, there are a lot of differences in the path of prices and quantities between Panel B and Panel C, despite being driven by identical paths in fundamentals.

B.3 Sentiment, credit growth, and return predictability

One stark contrast between the rational model and the behavioral model concerns the predictability of bond returns. In the rational model, no observable variable predicts future bond returns. In the behavioral model, many variables do. We summarize our analytical results on predictability in the proposition below.

Proposition 3: Return Predictability. In absence of bond maturity, the instantaneous bond gross return from time t to $t + dt$ is

$$\tilde{R}_{t,t+dt} = (kdt + \tilde{p}_{t+dt}) / p_t. \tag{22}$$

When bond matures, its instantaneous return is $\tilde{R}_{t,t+dt} = 1 / p_t$.

In the rational benchmark, over any finite time period $[t, T]$, the rationally expected continuously compounded gross return is

$$\mathbb{E}_t^r[\tilde{R}_{t,T}] = e^{\rho(T-t)}. \tag{23}$$

In the behavioral model, in the absence of default, the subjective expected gross return perceived by bond investors from time t to $t + dt$ is

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$$\mathbb{E}_t^b[\tilde{R}_{t,t+dt}] = 1 + \rho dt. \tag{24}$$

On the other hand, the objective expected gross return measured by outside econometricians, in the absence of default, is

$$\mathbb{E}_t^r[\tilde{R}_{t,t+dt}] = 1 + \left(\rho + \frac{(1-\eta)\beta\lambda_t}{\rho + \gamma + (1-\eta)\lambda_t} + (1-\eta)\lambda_t \right) dt. \tag{25}$$

Proof: See the Appendix.   ∎

As described in Proposition 3, in the rational benchmark, the expected return—either instantaneous or over any finite horizon—is always independent of any state variables in the economy. Bond prices fully incorporate in any future default risks that are known ex ante. In the behavioral model, behavioral bond investors subjectively believe they have fully priced all future default risks when valuing corporate bonds. However, given their backward-looking belief formation, they make systematic mistakes.

A few observations from (25) are worth noting. First, in the absence of default, the true expected gross return in the behavioral model is always greater than $1 + \rho dt$, for two reasons: 1) behavioral bond investors are concerned about an immediate default with a perceived likelihood λ_t, but a default never actually occurs immediately if $F_t < \overline{F}$ —this effect is captured by the term $(1-\eta)\lambda_t dt$; and 2) behavioral bond investors do not realize that λ_t will decrease over time in the absence of default, and that decrease in λ_t will lead to bond price appreciation—this effect is captured by the term $[(1-\eta)\beta\lambda_t/(\rho + \gamma + (1-\eta)\lambda_t)]dt$.

Second, the wedge between the true expected return and the subjective required return $1 + \rho dt$ monotonically increases in λ_t. Put differently, future returns are inversely related to the level of credit sentiment. This means that the post-default short-term bond returns—short term in the sense that the next default has not arrived—are abnormally high in the behavioral model. This finding mimics the abnormally high equity premium of the stock market after financial crises discussed in Jin (2015) and shown in several empirical studies (Muir 2015, Krishnamurthy and Muir 2015). For comparison, notice that high post-default short-term bond returns do not arise in the rational benchmark; bond price in that model always adjusts so that the expected gross return equals $1 + \rho dt$.

Third, upon default, the instantaneous gross return becomes $\eta' + O(dt)$, where $\eta' < \eta$. Because behavioral bond investors are backward-looking when forecasting future default risks, the bond price jumps *down* upon default, and at the same time investors' ownership of future coupons is reduced to a fraction of η.

Finally, longer-term expected bond returns in the behavioral model may well be lower than those in the rational model. As we see in Figure 5, over-pessimism of bond investors in the behavioral model may well accelerate future defaults, and more frequent defaults can lead to more negative bond returns.

Running forecasting regressions on bond returns over a finite time horizon cannot be done analytically. We implement these forecasting regressions through numerical simulations and then compare our results with the forecasting results we showed earlier in Tables 1, 2 and 3.

Table 4 performs this exercise. The top panels show autocorrelations measured at different horizons; the middle panel shows return forecasting regressions, and the bottom panels show the results of default forecasting regressions. Results are based on the following set of parameter values: $H = 0.6$, $L = 0.2$, $I = 0.25$, $k = 5\%$, $\rho = 5\%$, $\overline{F} = 1$, $\underline{F} = 0.5$, $\beta = 0.9$, $\eta = 0.9$, $q_H = 0.2$, $q_L = 0.5$, and $\alpha_1 = 1$. We show results for average 5-year debt ($\gamma = 1/5$) and 2-year debt ($\gamma = 1/2$).

Table 4 yields the following observations:

Autocorrelation of Returns: In the model, returns are positively autocorrelated at short horizons and negatively autocorrelated at longer horizons. Short-term momentum can arise from the following channel. During a credit boom bond returns are on average high in absence of defaults. At the same time, investors are quite optimistic (λ_t is low) and this helps to slow down debt accumulation and hence delays defaults, leading to high bond returns in the short term. This is an important sense in which credit market sentiment is partially self-fulfilling in our model in the short run. Long-term reversal arises because defaults will eventually occur after a long credit boom when F_t hits M. Also any autocorrelation will eventually get washed out by the randomness of fundamentals.

Defaults and Spreads: Defaults and spreads are positively correlated at all horizons with the autocorrelation decaying with horizon. A high autocorrelation of defaults is a manifestation of the persistence of the underlying state variables in the model. A high F_t, a high λ_t, or a low x_t today suggests, on average, tomorrow F_t will be high, λ_t will be high, and x_t will be low. In other words, there is persistence in default occurrences from the model. The persistence in λ_t drives the autocorrelation in spreads because in our model spread is defined as $(1 - \eta)\lambda_t$.

High levels of credit spreads predict high future returns on risky bonds: High levels of credit spreads coincide with pessimistic beliefs of bond investors (a high λ_t) in the model, and the most typical time for this to occur is when a new credit cycle begins (with a high x_t) after a sequence of defaults. This suggests that subsequent returns are high, both because bond prices will appreciate over time and because the dividend yield (coupon rate divided by bond price) is high.

High levels of debt (F_t) *predict low and then high future returns on risky bonds*: All else being equal, high levels of debt suggest that in the near future the default risk is high and hence bond returns are low. However, once the fundamental switches from *L* to *H*, the credit market tends to recover. With lingering pessimistic but recovering beliefs, the bond returns become high as we discussed above. This explains why high levels of debt predict high future returns over longer horizons.

High levels of past debt growth ($F_t - F_{t-3}$) *predict low future returns on risky bonds*: High levels of debt growth typically occur during the later stage of a credit bubble; during the earlier stage, the fundamental x_t stays high and therefore the level of debt decreases. The late stage of a credit boom is marked by a deterioration in fundamentals and more rapid debt growth. As a result, high levels of debt growth are typically followed by defaults that drive down bond returns. This closely matches empirical findings in Table 1 and Greenwood and Hanson (2013), Baron and Xiong (2015) and López-Salido, Stein, and Zakrajšek (2015).

Reductions in Sentiment (*increases in λ_t*) *predict higher future defaults*: Consistent with the results in Table 3, Table 4 shows that in our model, increases in perceived future defaults lead

to actual defaults. This effect is strongest in the short run, and subsides at longer horizons. The table also shows that the *level* of sentiment predicts defaults.

High levels of debt (F_t) and debt growth predict high future defaults on risky bonds: All else being equal, a higher F_t brings the economy closer to the default barrier so by construction future default risks are high. This effect is sometimes smaller in the short run because initially high debt growth can reflect high sentiment, which serves as an offset.

High levels of operating cash flow (x_t) predict low future defaults on risky bonds: When x_t equals H, firms retire bond and this moves F_t away from M and therefore reduces future default risks.

B.4 The role of debt maturity

Debt maturity plays an important role in our model. Shortening debt maturity can make the provision of credit become more exposed to investor sentiment: shorter-term debt needs to be rolled over more frequently, and hence changes in credit sentiment can have a larger effect on debt financing. At the same time, shortening debt maturity also reduces the life of each bond and hence reduces the likelihood that it defaults.

Below we elaborate on the role of debt maturity by making two points. First, longer debt maturity makes debt financing less imminent, prolonging credit booms, particularly when credit sentiment is high. However, when credit sentiment is low, shorter debt maturity makes debt financing more difficult, causing faster debt accumulation. In this case, shorter debt maturity leads to a slower recovery after credit busts.

We first show that longer debt maturity can prolong credit booms. In Figure 6 below, fundamentals stay high from $t = 0$ to $t = 0.5$ and switch to low thereafter. With short maturity ($\gamma = \frac{1}{2}$, implying an a average maturity of 2 years), default occurs around $t = 2.25$, but with long maturity ($\gamma = 0.1$, implying an average maturity of 10 years), default does not occur until slightly before $t = 2.5$. Longer maturity creates two effects. On the one hand, it helps firms to reduce debt when fundamentals are high. So by the time fundamentals deteriorate from high to low, the debt level is lower and hence further away from its default barrier in the case of long maturity. On the

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other hand, longer maturity slows down debt accumulation after fundamentals become low because refinancing is less frequent, further prolonging the credit boom. Both effects are stronger when credit sentiment is high and bond price is high.
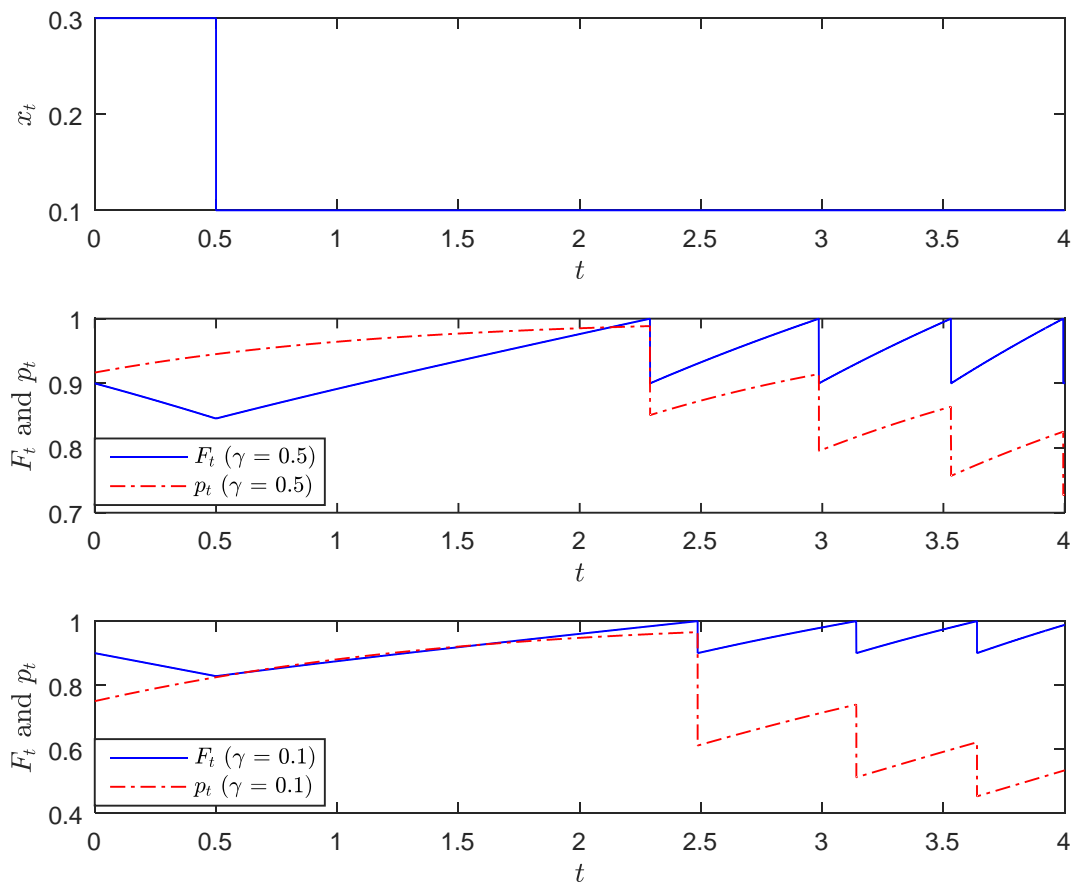

Figure 6. Debt Maturity and Credit Boom. Panel A plots the time evolution of fundamentals x_t. In this example, the fundamentals are specified as $x_t = H$ for $0 \le t < 0.5$ and $x_t = L$ for $t \ge 0.5$. Panel B plots the number of outstanding bonds F_t and the bond price p_t with $\gamma = 0.5$. Panel C plots, for comparison, the number of outstanding bonds F_t and the bond price p_t with $\gamma = 0.1$. The other parameter values are $H = 0.3$, $L = 0.1$, $I = 0.125$, $k = 5\%$, $\rho = 5\%$, $\overline{F} = 1$, $\underline{F} = 0.5$, $\beta = 0.9$, $\eta = 0.9$, and $\alpha_1 = 1$. The initial conditions are: $F_0 = 0.9$ and $\lambda_0 = 0.5$.

Debt maturity plays a particularly important role during downturns, in which case short-term debt can slow down credit market recovery after a sequence of defaults. With short debt maturity, the firm needs to roll over maturing bonds very frequently. When credit sentiment is

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low—that is, when investors believe that future default rates are high—bond price is low and credit spread is high. As a result, frequently rolling over debt becomes particularly expensive. In this case, even after fundamentals improve from L to H, the firm may need to keep accumulating debt, causing more defaults to happen and resulting in a slow recovery. Figure 7 below illustrates this point.
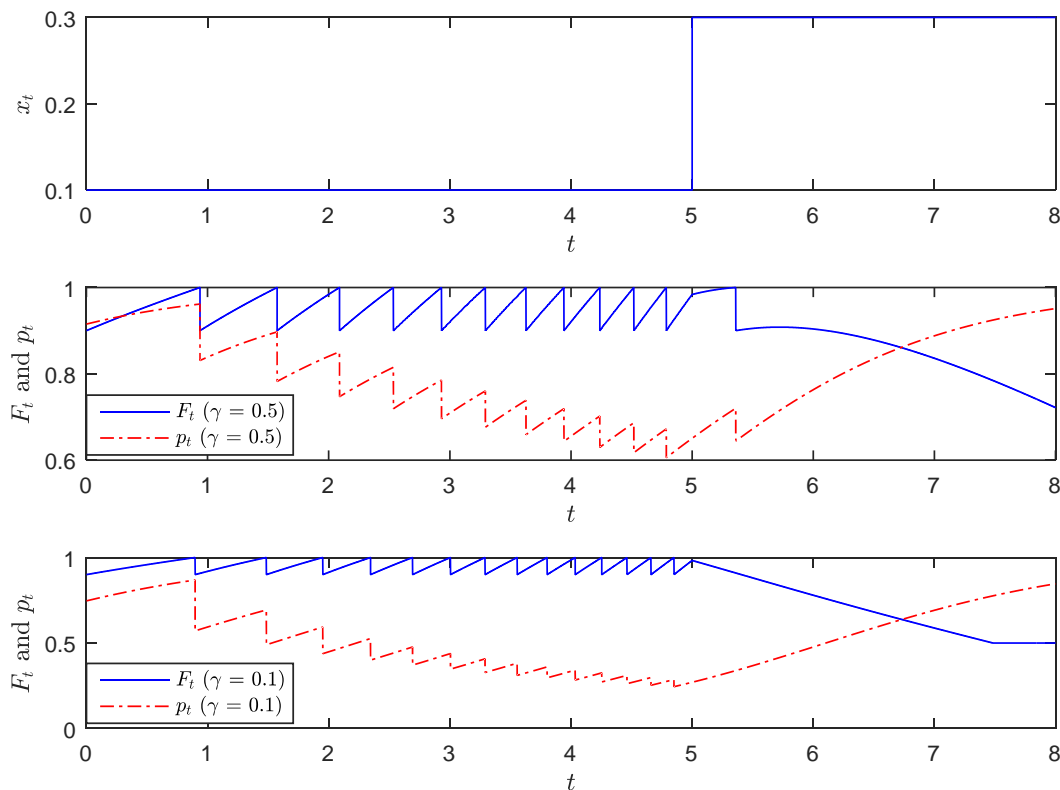
Figure 7. Debt Maturity and Credit Market Recovery. Panel A plots the time evolution of fundamentals x_t. In this example, the fundamentals are specified as $x_t = L$ for $0 \le t < 5$ and $x_t = H$ for $t \ge 5$. Panel B plots the number of outstanding bonds F_t and the bond price p_t with $\gamma = 0.5$. Panel C plots, for comparison, the number of outstanding bonds F_t and the bond price p_t with $\gamma = 0.1$. The other parameter values are $H = 0.3$, $L = 0.1$, $I = 0.125$, $k = 5\%$, $\rho = 5\%$, $\overline{F} = 1$, $\underline{F} = 0.5$, $\beta = 0.9$, $\eta = 0.9$, and $\alpha_1 = 1$. The initial conditions are: $F_0 = 0.9$ and $\lambda_0 = 0.51$.

IV.   Extensions: Multiple Firms and Belief Contagion

One of the limitations of our model is that, because it is based on a single firm, defaults are binary events. In this section, we extend the model to consider multiple firms. Although the qualitative conclusions we draw are largely the same, this extension has the advantage of mapping more closely to the continuous default rates that we observe in the real world. We also use it to explore belief contagion. Belief contagion can arise if investors update their beliefs about future defaults based on the default of any firm in the economy (in the real world, downgrades might serve a similar purpose).

We assume that there are M firms, $i = 1, 2, \ldots, M$. We will focus on the limiting case in which M grows large.

For the cash flow of each firm, we assume it consists of two components: a systematic component that evolves according to (4) and a firm-specific component. Specifically,

$$\tilde{x}_{i,t} = \tilde{x}_t + \tilde{\varepsilon}_{i,t}, \tag{26}$$

where $\tilde{\varepsilon}_{i,t}$ follows

$$
\begin{array}{cc}
 & \tilde{\varepsilon}_{i,t+dt} = \varepsilon \quad \tilde{\varepsilon}_{i,t+dt} = -\varepsilon \\
\begin{array}{c} \tilde{\varepsilon}_{i,t} = \varepsilon \\ \tilde{\varepsilon}_{i,t} = -\varepsilon \end{array} &
\begin{pmatrix} 1 - q_\varepsilon dt & q_\varepsilon dt \\ q_\varepsilon dt & 1 - q_\varepsilon dt \end{pmatrix}
\end{array}. \tag{27}
$$

Thus, the mean of $\tilde{\varepsilon}_{i,t}$ is zero and the parameter q_ε governs the persistence of these firm-specific shocks.

We now impose the key assumption that each firm updates its perceived likelihood of future defaults when *any* firm in the economy defaults. In other words,

$$\lambda_t = \beta \int_{-\infty}^{t} e^{-\beta(t-s)} f(\lambda_{s-ds}) dN_{s-ds}, \quad \beta > 0, \tag{28}$$

where $N_t = M^{-1} \sum_{i=1}^{M} N_{it}$ is the cumulative aggregate-level default rate up to time t. In other words, a default event for firm i between time t and time $t + dt$ contributes $1/M$ to the change of N over the same period. When M grows large, changes of λ_t become smooth because only a small fraction of firms default at each point in time.

The price of debt for all firms is still given by[12]

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[12] In the rational benchmark, with different debt levels, investors will price corporate bonds differently.

$$p_t = \frac{k + \gamma}{\rho + \gamma + (1 - \eta)\lambda_t}. \tag{29}$$

And the outstanding debt for firm i evolves according to

$$dF_{i,t} = [kF_{i,t} + (1 - p_t)\gamma F_{i,t} + I - x_{i,t+dt}]dt/p_t. \tag{30}$$

If $F_{i,t} = \underline{F}$

$$dF_{i,t} = \mathbf{1}_{kF_{i,t} + (1 - p_t)\gamma F_{i,t} + I - x_{i,t+dt} > 0}[kF_{i,t} + (1 - p_t)\gamma F_{i,t} + I - x_{i,t+dt}]dt/p_t. \tag{31}$$

If $F_{i,t} = \overline{F}$, a default event is triggered and $F_{i,t+dt} = \eta\overline{F}$.

We start our analysis by setting a uniform distribution of firms over the range of $(\underline{F}, \overline{F})$ at time 0. That is

$$F_{i,0} = \frac{(M + 1 - i)\underline{F} + i\overline{F}}{M + 1}, \quad i = 1, 2, \ldots, M. \tag{32}$$

Also for each firm i, we randomly draw the initial firm-specific cash flow shock $\tilde{\varepsilon}_{i,0}$ from $\{\varepsilon, -\varepsilon\}$ with equal probabilities.

Figure 8 shows the evolution of credit and defaults in the multiple firm model. The systematic component of cash flow stays high from $t = 0$ to $t = 0.5$ and from $t = 7.5$ to $t = 10$, temporarily switching to low between $t = 0.5$ and $t = 7.5$. We plot the average number of outstanding bonds $\overline{F}_t$, the bond price p_t, the perceived default likelihood λ_t, as well as the *actual* default rate averaged over the past quarter. With multiple firms in the economy ($M = 500$), the perceived default likelihood drops between $t = 0$ and $t = 1.5$; even after fundamentals switch from high to low, λ_t continues to fall and bond prices rise for about a year. During this time, the default rate stays at zero because all the firms retire debt between $t = 0$ and $t = 0.5$ and therefore it takes some time for the debt level to accumulate up to $\overline{F}$ after fundamentals deteriorate. Once fundamentals have fallen, the average debt level gradually climbs up and plateaus at around 0.95 by $t = 6$. The first firm defaults at $t = 1.22$, nearly 9 months after the fundamentals deteriorated. Although both the perceived and the *actual* default rates rise over time, the average debt level

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stops rising because, for the firms which go through default, their debt level drops by fraction of $1 - \eta$, keeping the average debt level stay below $\overline{F}$. [13]
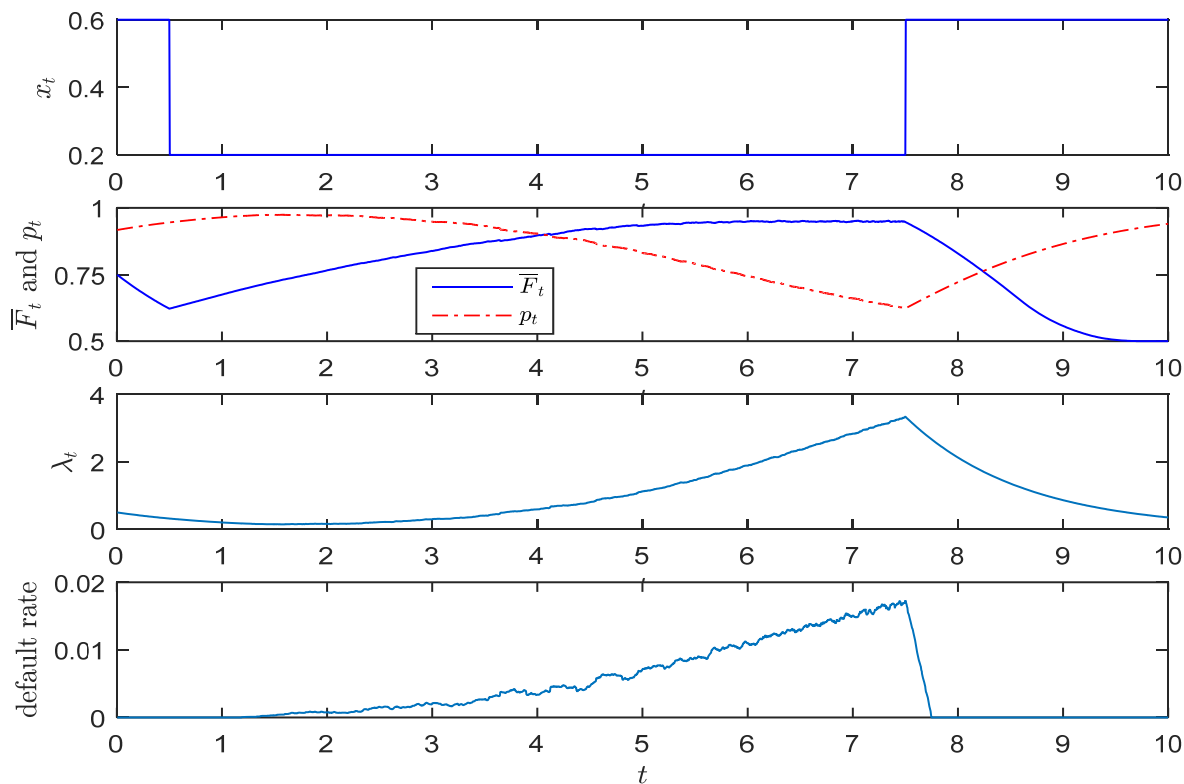

Figure 8. Multiple Firms and Credit Cycle. Panel A plots the time evolution of the systematic component of cash flows x_t. In this example, $x_t = H$ for $0 \le t < 0.5$ and $7.5 \le t \le 10$, and $x_t = L$ for $0.5 \le t < 7.5$. Panel B plots the average number of outstanding bonds $\overline{F}_t$ and the bond price p_t. Panel C plots the perceived default likelihood λ_t. Panel D plots the percentage of defaulting firms averaged over the past quarter. The parameter values are: $H = 0.6$, $L = 0.2$, $I = 0.25$, $k = 5\%$, $\rho = 5\%$, $\overline{F} = 1$, $\underline{F} = 0.5$, $\beta = 0.9$, $\eta = 0.9$, $\gamma = 1/2$, $M = 500$, $\varepsilon = 0.05$, and $q_\varepsilon = 0.5$. The initial conditions are: $F_{i,0}$ is uniformly distributed between $\underline{F}$ and $\overline{F}$, and $\lambda_0 = 0.5$.

Beyond adding realism, the multiple-firms extension allows us to consider belief contagion: if one firm defaults, this adversely impacts the terms of credit for all other firms. Intuitively, belief contagion is most extreme when there is dispersion in the quality of firms borrowing. Specifically, under general conditions, adding a mean-preserving spread to the

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[13] The realized default rate is the percentage of defaulting firms averaged over the past quarter: a default rate of 1.8% (at around $t = 7.5$) indicates that on average, a firm defaults every 0.22 years.

distribution of outstanding bonds across firms tends to add fragility to the economy: When adverse fundamental shocks arrive, a more dispersed distribution of firms' outstanding bonds is more likely to lead to defaults, and firm-specific defaults impose negative externality on the economy as a whole, resulting in quicker transitions to default for the firms that have not yet defaulted.

V.   Conclusions

We present a model of credit market sentiment in which investors extrapolate past defaults. Our key contribution is to model the endogenous two-way feedback between credit market sentiment and credit market outcomes. This feedback mechanism is unique to credit markets because firms must return to the market to refinance their debts.

Our analysis leaves open at least two areas for further analysis. First, we have not allowed for any relationship between conditions in credit markets and the fundamentals of the economy. Such a relationship plays a major role in Austrian accounts of credit cycles: as the credit boom grows, increasing amounts of capital are devoted to poor quality projects. Incorporating this aspect into our model may further enhance the feedback between sentiment and credit market outcomes.

Second, we have been silent on issues of welfare and optimal policy, even though our model suggests a potential role. During credit booms, high sentiment can prevent defaults from occurring in the near future, which can be welfare-improving if fundamentals recover soon enough. Nonetheless, self-fulfilling beliefs during default spirals can be welfare-reducing, both because these deteriorating beliefs accelerate future default realizations and because they lead to a slow recovery in the presence of improving fundamentals. Accepting this at face value suggests a role for policy in moderating investor beliefs.

Appendices

Proof of Proposition 1. If $\tilde{x}_0 = H$ and $\tilde{x}_t$ does not experience a transition, (9), (11) and (14) then imply that before F_t gets capped by $\bar{F}$ or gets floored by $\underline{F}$

$$\frac{dF_t}{dt} = (\rho + (1-\eta)\lambda_0 e^{-\beta t})F_t + \frac{(I-H)(1-\eta)\lambda_0 e^{-\beta t}}{k+\gamma} + (I-H)\frac{\rho+\gamma}{k+\gamma}. \tag{A1}$$

The homogenous solution for (A1) is

$$F_h(t) = K\exp\left(\rho t - \frac{(1-\eta)\lambda_0}{\beta}e^{-\beta t}\right), \tag{A2}$$

where K is a coefficient that needs to be determined by the initial condition. Given (A2), a particular solution for the differential equation (A1) is

$$F_p = \frac{I-H}{k+\gamma}\exp\left(\rho t - \frac{(1-\eta)\lambda_0}{\beta}e^{-\beta t}\right)\int_0^t \left((1-\eta)\lambda_0 e^{-\beta\tau} + \rho + \gamma\right)\exp\left(-\rho\tau + \frac{(1-\eta)\lambda_0}{\beta}e^{-\beta\tau}\right)d\tau. \tag{A3}$$

So the overall solution, given the initial condition that $F(t=0) = F_0$, is

$$F(t) = \exp\left(\rho t - \frac{(1-\eta)\lambda_0}{\beta}e^{-\beta t}\right)\times$$
$$\left[\exp\left(\frac{(1-\eta)\lambda_0}{\beta}\right)F_0 + \frac{I-H}{k+\gamma}\int_0^t \left((1-\eta)\lambda_0 e^{-\beta\tau} + \rho + \gamma\right)\exp\left(-\rho\tau + \frac{(1-\eta)\lambda_0}{\beta}e^{-\beta\tau}\right)d\tau\right]. \tag{A4}$$

Rearranging terms gives

$$F(t) = F_0\exp\left(\rho t + \frac{(1-\eta)\lambda_0}{\beta}(1-e^{-\beta t})\right) + \frac{I-H}{k+\gamma}\left[\exp\left(\rho t + \frac{(1-\eta)\lambda_0}{\beta}(1-e^{-\beta t})\right)-1\right]$$
$$+ \frac{\gamma(I-H)}{k+\gamma}\exp\left(\rho t - \frac{(1-\eta)\lambda_0}{\beta}e^{-\beta t}\right)\int_0^t \exp\left(-\rho\tau + \frac{(1-\eta)\lambda_0}{\beta}e^{-\beta\tau}\right)d\tau. \tag{A5}$$

If $\tilde{x}_0 = L$ and $\tilde{x}_t$ does not experience a transition, then (A5) becomes

$$F(t) = F_0\exp\left(\rho t + \frac{(1-\eta)\lambda_0}{\beta}(1-e^{-\beta t})\right) + \frac{I-L}{k+\gamma}\left[\exp\left(\rho t + \frac{(1-\eta)\lambda_0}{\beta}(1-e^{-\beta t})\right)-1\right]$$
$$+ \frac{\gamma(I-L)}{k+\gamma}\exp\left(\rho t - \frac{(1-\eta)\lambda_0}{\beta}e^{-\beta t}\right)\int_0^t \exp\left(-\rho\tau + \frac{(1-\eta)\lambda_0}{\beta}e^{-\beta\tau}\right)d\tau. \tag{A6}$$

And $F(t)$ in this case monotonically increases until it hits the upper barrier of $\bar{F}$. ∎

Proof of Corollary 1. Combining (17) and (18) gives

$$\left((k+\gamma)\overline{F}+I-L\right)e^{-\rho s} = \left(\begin{array}{l}\left((k+\gamma)F_0 + I - H\right)\exp\left(\rho t + \frac{(1-\eta)\lambda_0}{\beta}(1-e^{-\beta t})\right) + H - L \\[2mm] +\gamma(I-H)\exp\left(\rho t - \frac{(1-\eta)\lambda_0}{\beta}e^{-\beta t}\right)\int_0^t \exp\left(-\rho\tau + \frac{(1-\eta)\lambda_0}{\beta}e^{-\beta\tau}\right)d\tau\end{array}\right)\times$$

$$\exp\left(\frac{(1-\eta)\lambda_0 e^{-\beta t}}{\beta}(1-e^{-\beta s})\right) \tag{A7}$$

$$+\gamma(I-L)\exp\left(-\frac{(1-\eta)\lambda_0 e^{-\beta t}}{\beta}e^{-\beta s}\right)\int_0^s \exp\left(-\rho\tau + \frac{(1-\eta)\lambda_0 e^{-\beta t}}{\beta}e^{-\beta\tau}\right)d\tau.$$

Taking derivatives with respect to t on both sides of (A7) and treating s as a function of t, we obtain

$$s' = -\frac{(k+\gamma)}{\rho\left((k+\gamma)\overline{F}+I-L\right)}F'(t)\exp\left(\rho s + \frac{(1-\eta)\lambda_0 e^{-\beta t}}{\beta}(1-e^{-\beta s})\right)$$

$$+\frac{\left((k+\gamma)F(t)+I-L\right)(1-\eta)\lambda_0 e^{-\beta t}}{\rho\left((k+\gamma)\overline{F}+I-L\right)}\exp\left(\rho s + \frac{(1-\eta)\lambda_0 e^{-\beta t}}{\beta}(1-e^{-\beta s})\right)(1-e^{-\beta s}) - \frac{(1-\eta)\lambda_0 e^{-\beta(t+s)}}{\rho}s' - \frac{\gamma(I-L)}{\rho\left((k+\gamma)\overline{F}+I-L\right)}s' \tag{A8}$$

Notice from (11) that if for some time l, $kF_l + (1-p_l)\gamma F_l + I - H < 0$, then for any subsequent time $h \geq l$, so long as fundamentals stay at H, the number of outstanding debt will decrease. As a result, when $kF_t + (1-p_t)\gamma F_t + I - H < 0$, $F'(t) < 0$. Given that $F'(t) < 0$, if $s'(t) \leq 0$, a contradiction arises: the left hand side of (A8) is less than or equal to zero but the right hand side is strictly positive. As a result, $s'(t) > 0$. ∎

Proof of Proposition 2. For $\underline{F} < F_t < \overline{F}$, the definitions of $p_H(F_t)$ and $p_L(F_t)$ give

$$p_H(F_t) = kdt + e^{-\rho dt}\mathbb{E}[p(F_{t+dt}, \tilde{x}_{t+dt}) \mid \tilde{x}_t = H, F_t],$$
$$p_L(F_t) = kdt + e^{-\rho dt}\mathbb{E}[p(F_{t+dt}, \tilde{x}_{t+dt}) \mid \tilde{x}_t = L, F_t]. \tag{A9}$$

From the regime-switching process for $\tilde{x}_t$ in (4) and the assumption of Poisson arrival of random maturity, we can write (A9) as

$$p_H(F_t) = kdt + (1-\rho dt)\mathbb{E}[(1-\gamma dt)p(F_{t+dt}, \tilde{x}_{t+dt}) + \gamma dt \mid \tilde{x}_t = H, F_t]$$
$$p_L(F_t) = kdt + (1-\rho dt)\mathbb{E}[(1-\gamma dt)p(F_{t+dt}, \tilde{x}_{t+dt}) + \gamma dt \mid \tilde{x}_t = L, F_t]. \tag{A10}$$

Taking expectations and rearranging terms

$$0 = (k+\gamma) - (\gamma+\rho)p_H(F_t) + q_H[p_L(F_t) - p_H(F_t)] + p'_H(F_t)dF_t/dt,$$
$$0 = (k+\gamma) - (\gamma+\rho)p_L(F_t) + q_L[p_H(F_t) - p_L(F_t)] + p'_L(F_t)dF_t/dt. \tag{A11}$$

Substituting (11) into (A11) then gives (15).

When $\tilde{x}_t = H$, $F_t = \underline{F}$, and assume that $k\underline{F} + (1-p_H(\underline{F}))\gamma\underline{F} + I - H < 0$, (12) requires F_t to be floored at $\underline{F}$ so $dF_t = 0$. Hence, the first equation in (A11) implies $k + \gamma + q_H p_L(\underline{F}) = (\gamma + \rho + q_H)p_H(\underline{F})$. When $\tilde{x}_t = L$ and $F_t = \overline{F}$, the default rule requires $p_L(\overline{F}) = \eta p_L(\eta\overline{F})$. ∎

Proof of Proposition 3. In the rational benchmark, (15) implies that for $\underline{F} < F_t < \overline{F}$, in absence of default, the instantaneous expected gross return always equals $1 + \rho dt$. When $F_t = \underline{F}$ and $\tilde{x}_t = H$, the first boundary condition in (16) guarantees that the instantaneous gross return also equals $1 + \rho dt$. When $F_t = \overline{F}$ and $\tilde{x}_t = L$, the second boundary condition in (16) and the second equation in (A9) imply that the expected instantaneous gross return is $1 + \rho dt$. Putting these together, in the rational benchmark, over any finite time period $[t, T]$, the expected continuously compounded gross return is $e^{\rho(T-t)}$.

In the behavioral model, (8) and (9) imply that from the investors' perspective, the subjective instantaneous expected gross return, in absence of default, equals $1 + \rho dt$. That is,

$$\frac{kdt + \mathbb{E}^s[p_{t+dt} \mid \lambda_t]}{p_t} = \frac{kdt + (1 - \lambda_t - \gamma dt)p_t + \gamma dt + \eta \lambda_t p_t dt}{p_t} = 1 + \rho dt. \tag{A12}$$

And bond investors pay no attention to the default trigger rule $F_t = \overline{F}$. However, from outside econometricians' perspective, the objective instantaneous expected gross return is time-varying. For $\underline{F} \le F_t < \overline{F}$, (8) and (9) give

$$\frac{kdt + \mathbb{E}^r_t[p_{t+dt}]}{p_t} = \frac{kdt + \gamma dt + (1 - \gamma dt)p(\lambda_{t+dt})}{p_t}$$

$$= 1 + (\rho + (1 - \eta)\lambda_t)dt + \left(\frac{(1 - \eta)(-d\lambda_t)}{\rho + \gamma + (1 - \eta)\lambda_t} \right) \tag{A13}$$

$$= 1 + \left(\rho + \frac{(1 - \eta)\beta\lambda_t}{\rho + \gamma + (1 - \eta)\lambda_t} + (1 - \eta)\lambda_t \right)dt.$$

∎

Solving the System of Ordinary Differential Equations. The value of F_t ranges from $\underline{F}$ to $\overline{F}$, whereas the domain for Chebyshev polynomials is $[-1, 1]$. So we transform F_t to a new state variable z_t

$$z_t = \frac{2F_t - (\overline{F} + \underline{F})}{\overline{F} - \underline{F}}. \tag{A14}$$

Define $y_H(z) \equiv p_H(F(z))$ and $y_L(z) \equiv p_L(F(z))$. The differential equations in (15) can be rewritten as

$$0 = (k + \gamma) - (\rho + \gamma)y_L(z) + q_L(y_H(z) - y_L(z))$$

$$+ \frac{y'_L(z)}{(\overline{F} - \underline{F})y_L(z)}\big([k + (1 - y_L(z))\gamma][(\overline{F} - \underline{F})z + (\overline{F} + \underline{F})] + 2I - 2L\big),$$

$$0 = (k + \gamma) - (\rho + \gamma)y_H(z) + q_H(y_L(z) - y_H(z)) \tag{A15}$$

$$+ \frac{y'_H(z)}{(\overline{F} - \underline{F})y_H(z)}\big([k + (1 - y_H(z))\gamma][(\overline{F} - \underline{F})z + (\overline{F} + \underline{F})] + 2I - 2H\big).$$

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And the boundary conditions in (16) can be rewritten as

$$k + \gamma + q_H y_L(-1) = (\gamma + \rho + q_H) y_H(-1), \qquad y_L(1) = \eta y_L \left(\frac{2\eta \bar{F} - (\bar{F} + \underline{F})}{\bar{F} - \underline{F}} \right). \qquad \text{(A16)}$$

We apply the projection method to solve $y_H(z)$ and $y_L(z)$. Specifically, we approximate these two functions by

$$\hat{y}_L(z) = \sum_{r=0}^{n} a_r T_r(z), \qquad \hat{y}_H(z) = \sum_{r=0}^{m} b_r T_r(z), \qquad \text{(A17)}$$

where $T_r(z)$ is the rth degree Chebyshev polynomial of the first kind.[14] The projection method chooses the coefficients $\{a_r\}_{r=0}^{n}$ and $\{b_r\}_{r=0}^{m}$ so that the differential equations and the boundary conditions are *approximately* satisfied. One criterion for a good approximation is a minimum weighted sum of squared errors

$$\sum_{i=1}^{N} \frac{1}{\sqrt{1-z_i^2}} \left[\begin{array}{l} (k+\gamma) - (\rho+\gamma)\hat{y}_L(z_i) + q_L(\hat{y}_H(z_i) - \hat{y}_L(z_i)) \\[2mm] + \dfrac{\hat{y}_L'(z_i)}{(\bar{F}-\underline{F})\hat{y}_L(z_i)} \left([k+(1-\hat{y}_L(z_i))\gamma][z_i(\bar{F}-\underline{F})+(\bar{F}+\underline{F})] + 2I - 2L] \right) \end{array} \right]^2$$

$$+\sum_{i=1}^{N} \frac{1}{\sqrt{1-z_i^2}} \left[\begin{array}{l} (k+\gamma) - (\rho+\gamma)\hat{y}_H(z_i) + q_H(\hat{y}_L(z_i) - \hat{y}_H(z_i)) \\[2mm] + \dfrac{\hat{y}_H'(z_i)}{(\bar{F}-\underline{F})\hat{y}_H(z_i)} \left([k+(1-\hat{y}_H(z_i))\gamma][z_i(\bar{F}-\underline{F})+(\bar{F}+\underline{F})] + 2I - 2H \right) \end{array} \right]^2 \quad \text{(A18)}$$

$$+K_1 \left[\hat{y}_L(1) - \eta \hat{y}_L\left(\frac{2\eta \bar{F} - (\bar{F}+\underline{F})}{\bar{F}-\underline{F}} \right) \right]^2 + K_2 [k+\gamma+q_H \hat{y}_L(-1) - (\gamma+\rho+q_H)\hat{y}_H(-1)]^2,$$

where $\{z_i\}_{i=0}^{N}$ are the N zeros of $T_N(z)$ and K_1 and K_2 are positive numbers. By the Chebyshev interpolation theorem, if N is sufficiently larger than n and m, and if the sum of weighted square in (A18) is sufficiently small, the approximated functions $\hat{y}_L(z)$ and $\hat{y}_H(z)$ are very close to the true solutions.

For the numerical results in the main text, we set $m = 20$, $n = 20$, $N = 500$, and $K_1 = K_2 = 10^8$. We then apply the Levenberg-Marquardt algorithm and obtain a minimized sum of squared errors less than 10^{-7}. The small size of errors indicates a good convergence of the numerical solution. The solution is also insensitive to the choice of n, m, N, K_1, or K_2. These findings indicate that the numerical solutions are sufficient approximations for the true y functions.

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[14] See Mason and Handscomb (2003) for detailed discussion of the properties of Chebyshev polynomials.

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Table 1. Credit Market Sentiment.

Time-series regressions of the form

$$rx_{t+k}^{HY} = a + b \cdot Sent_t + \varepsilon_t,$$

where $Sent_t$ denotes investor sentiment in year t. The dependent variable is the cumulative 2-, or 3-year excess return on high-yield bonds. *HYS* is the fraction of nonfinancial corporate bond issuance with a high-yield rating from Moody's. The percentage change in corporate credit is computed using Table L103 from the Flow of Funds. *Loansent* is the three-year average of the percentage of loan officers reporting a loosening of underwriting standards. *EBP* is the expected bond premium from Gilchrist and Zakrajšek (2012). *t*-statistics for *k*-period forecasting regressions (in brackets) are based on Newey-West (1987) standard errors, allowing for serial correlation up to *k*-lags.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	2-year future excess returns				3-year future excess returns			
Log(*HYS*)	-15.95				-18.63			
	[-3.06]				[-3.78]			
Growth of Corporate Credit		-126.50				-158.02		
		[-2.28]				[-2.86]		
Loansent			-0.57				-0.80	
			[-2.12]				[-4.03]	
EBP				19.29				24.72
				[2.43]				[4.79]
Constant	-15.67	11.46	-0.27	3.25	-17.54	14.93	-1.26	4.51
	[-2.18]	[3.85]	[-0.06]	[0.94]	[-2.29]	[4.42]	[-0.21]	[1.15]
N	29	29	20	29	28	28	19	28
R-squared	0.20	0.11	0.20	0.20	0.25	0.16	0.38	0.32

Table 2. Credit Market Sentiment and Current and Past Default Rates.

Time-series regressions of the form

$$Sent_t = a + b \cdot Def_t + c \cdot Def_{t-1} + \varepsilon_t,$$

where *Def* denotes the default rate on speculative grade bonds, and *Sent* is a measure of credit market sentiment. *HYS* is the fraction of nonfinancial corporate bond issuance with a high-yield rating from Moody's. The percentage change in corporate credit is computed using Table L103 from the Flow of Funds. *Loansent* is the three-year average of the percentage of loan officers reporting a loosening of underwriting standards. *EBP* is the expected bond premium from Gilchrist and Zakrajšek (2012). *t*-statistics for *k*-period forecasting regressions (in brackets) are based on Newey-West (1987) standard errors, allowing for serial correlation up to *k*-lags.

	Dependent variable:			
	(1)	(2)	(3)	(4)
		Growth of		
	Log(*HYS*)	Corporate Credit	*Loansent*	*EBP*
Def_t	-0.113	-0.005	-3.425	0.104
	[-2.21]	[-1.22]	[-11.19]	[4.98]
Def_{t-1}	0.009	-0.008	-2.152	-0.021
	[0.44]	[-3.47]	[-4.24]	[-0.96]
Constant	-0.734	0.118	18.076	-0.36
	[-3.47]	[7.92]	[4.58]	[-2.21]
N	31	31	22	30
R-squared	0.400	0.436	0.813	0.426

Table 3. Reductions in Sentiment lead to Increases in Default Rates.

Time-series regressions of the form

$$Def_{t+k} = a + b \cdot \Delta Sent_t + \varepsilon_t,$$

where Def_{t+k} denotes the default rate on speculative grade bonds in year $t+k$, and $\Delta Sent_t$ is the one-year change in credit market sentiment in year t. *HYS* is the fraction of nonfinancial corporate bond issuance with a high-yield rating from Moody's. The percentage change in corporate credit is computed using Table L103 from the Flow of Funds. Loansent is the three-year average of the percentage of loan officers reporting a loosening of underwriting standards. *EBP* is the expected bond premium from Gilchrist and Zakrajšek (2012). *t*-statistics for *k*-period forecasting regressions (in brackets) are based on Newey-West (1987) standard errors, allowing for serial correlation up to *k*-lags.

	(1) Def_t	(2) Def_{t+1}	(3) Def_{t+2}	(4) Def_t	(5) Def_{t+1}	(6) Def_{t+2}	(7) Def_t	(8) Def_{t+1}	(9) Def_{t+2}	(10) Def_t	(11) Def_{t+1}	(12) Def_{t+2}
ΔLog(*HYS*)	-0.793 [-0.75]	-2.853 [-3.67]	-1.289 [-1.97]									
ΔCredit Growth				-54.135 [-5.69]	-37.168 [-3.08]	3.947 [0.34]						
Δ*Loansent*							0.093 [3.08]	0.132 [13.97]	0.044 [1.98]			
Δ*EBP*										-0.380 [-0.51]	4.281 [5.04]	1.757 [1.71]
Constant	4.663 [7.18]	4.751 [8.33]	4.748 [7.22]	4.594 [11.19]	4.648 [8.72]	4.717 [6.65]	4.113 [8.19]	3.486 [13.18]	3.908 [5.54]	4.723 [6.77]	4.762 [8.66]	4.761 [7.26]
Observations	31	30	29	31	30	29	24	23	22	29	29	28
R-squared	0.022	0.283	0.058	0.530	0.250	0.003	0.390	0.859	0.103	0.003	0.420	0.070

Table 4. Simulations and Return Forecasting Results in the Model.

We show 1-5 year autocorrelations and return forecasting regressions for cash flows, the face value of debt, the number of defaults, beliefs, and returns. The second panel shows coefficients from forecasting regressions of $t+k$ returns on variables in the model where $k = 1$ to 5 years. Results are based on the following set of parameter values: $H = 0.6$, $L = 0.2$, $I = 0.25$, $k = 5\%$, $\rho = 5\%$, $\overline{F} = 1$, $\underline{F} = 0.5$, $\beta = 0.9$, $\eta = 0.9$, $q_H = 0.2$, $q_L = 0.5$, and $\alpha_1 = 1$. We show results for average 5-year debt ($\gamma = 1/5$) and 2-year debt ($\gamma = 1/2$).

		Panel A: 5-year debt ($\gamma = 1/5$)					Panel B: 2-year debt ($\gamma = 1/2$)				
	Autocorrelations	**1-yr**	**2-yr**	**3-yr**	**4-yr**	**5-yr**	**1-yr**	**2-yr**	**3-yr**	**4-yr**	**5-yr**
(1)	Cashflow (x_t)	0.52	0.26	0.13	0.06	0.03	0.52	0.26	0.13	0.06	0.03
(2)	Debt face value (F_t)	0.81	0.56	0.37	0.22	0.12	0.83	0.59	0.40	0.26	0.15
(3)	Number of defaults (N_t)	0.79	0.54	0.35	0.21	0.12	0.78	0.54	0.34	0.21	0.12
(4)	Beliefs (λ_t)	0.89	0.68	0.48	0.31	0.19	0.88	0.68	0.48	0.32	0.20
(5)	Returns (R_t)	0.42	0.02	-0.13	-0.16	-0.15	0.35	0.00	-0.16	-0.17	-0.15
	Univariate return forecasting	**1-yr**	**2-yr**	**3-yr**	**4-yr**	**5-yr**	**1-yr**	**2-yr**	**3-yr**	**4-yr**	**5-yr**
(1)	Cashflow (x_t)	0.08	0.01	-0.02	-0.03	-0.03	0.05	0.01	0.00	-0.01	-0.01
(2)	Debt face value (F_t)	-0.12	0.05	0.14	0.19	0.22	-0.05	0.02	0.06	0.09	0.10
(3)	Debt growth ($F_t - F_{t-3}$)	-0.25	-0.14	-0.08	-0.03	0.02	-0.11	-0.07	-0.05	-0.02	0.00
(4)	Beliefs (λ_t)	0.07	0.11	0.10	0.08	0.06	0.04	0.07	0.07	0.05	0.04
(5)	Belief change ($\lambda_t - \lambda_{t-1}$)	-0.16	0.03	0.09	0.10	0.08	-0.10	0.01	0.06	0.06	0.05
	Univariate default forecasting	**1-yr**	**2-yr**	**3-yr**	**4-yr**	**5-yr**	**1-yr**	**2-yr**	**3-yr**	**4-yr**	**5-yr**
(1)	Cashflow (x_t)	-0.79	-0.79	-0.79	-0.78	-0.79	-0.67	-0.67	-0.67	-0.66	-0.66
(2)	Debt face value (F_t)	2.87	2.77	2.56	2.26	1.86	2.36	2.26	2.07	1.80	1.47
(3)	Debt growth ($F_t - F_{t-3}$)	0.49	0.72	0.90	1.01	1.00	0.47	0.66	0.81	0.88	0.86
(4)	Beliefs (λ_t)	0.83	0.56	0.35	0.21	0.11	0.83	0.56	0.35	0.22	0.12
(5)	Belief change ($\lambda_t - \lambda_{t-1}$)	1.20	0.90	0.63	0.42	0.27	1.12	0.88	0.58	0.41	0.26