Forced Retirement Risk and Portfolio Choice

(Preliminary and Not for Circulation)

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Abstract

Literature on the effect of labor income on portfolio choice fails to consider that workers face the risk of being forced to retire before their planned retirement age. Using data from the Health and Retirement Studies (HRS), this paper finds such forced retirement risk to be significant and also highly correlated with stock market fluctuations. A life-cycle portfolio choice model with the estimated forced retirement risk shows that labor income subjects to such a risk becomes stock-like as individuals approach their retirement. Therefore, contrary to the conventional wisdom, those who are still working but close to retirement should have lower share of risky assets in their financial portfolios than retirees do. Given that most of financial assets are held by middle-aged households, this finding gives an alternative explanation to the risk premium puzzle.

Keywords: Forced Retirement, Portfolio Choice, the Risk Premium Puzzle

JEL Classification: D14, E11, G11, G12

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1 Introduction

Given the aging population, there is a rising concern about older Americans' financial well-being in retirement. Most of discussions on this topic focus on whether households build an adequate level of savings to sustain a desirable level of consumption in retirement. What is less studied, however, is how households should manage their financial savings as they approach their retirement. With the transition from defined-benefit to defined-contribution pension system, households become more responsible for managing their own financial assets, but there is surprisingly little guidance on portfolio management for older households that is based on a correct understanding of risks that older households face and a rigorous economic theory. In this paper, we examine how older households should adjust allocations of their financial wealth between risky and safe assets. In particular, we focus on how a specific risk they face at the end of their working life, i.e., the risk of being forced into retirement before planned retirement age, affects their optimal financial portfolio choice.

A long-standing rule of thumb in portfolio adjustment over age is that households should reduce the share of risky assets in financial portfolio as they approach retirement. Most of life-cycle funds in the current financial market are designed based on this principle. An often-cited justification for this strategy relates to changes in the human capital of households (Jagannathan and Kocherlakota, 1996). As households approach retirement, the size of human capital shrinks as they expect less future labor earnings. *If* their human capital is bond-like, i.e., if the size of risk they have in their labor earnings is not large and/or if it is not strongly correlated with stock returns, decrease in human capital justifies a shift toward risk-free assets in financial portfolio because they are losing buffer against negative stock return shocks. But *if* their human capital is stock-like, i.e., if the risk in human capital is large and strongly correlated with stock returns, an adjustment in the opposite direction will be justified. Therefore, estimating the size and characteristics of risk in households' human capital is a crucial task in designing the right portfolio adjustment strategy for households approaching their retirement.

Most of the papers in the literature on the role of human capital on financial portfolio choice model the risk in human capital as uncertainties in the earnings process that households face *before* their retirement. Based on that approach, these papers conclude that human capital is bond-like since the estimated risk is small and not strongly correlated with stock returns (see Viceira, 2001, Cocco, Gomes and Maenhout, 2005, and Hugget and Kaplan, 2016 for example). For older households that are close to retirement, however, there is a much larger uncertainty in their *timing* of retirement rather than in their labor earnings process *before* their retirement. The estimated size of the risk in human capital can be much larger if households face a risk of being forced to retire before their planned retirement age. Such a risk can make human capital much more stock-like if the risk of forced retirement is correlated with the performance of the stock market. Existing papers in this literature fail to capture this risk by assuming either that the retirement timing is fixed or households have a full control over when to retire (see Bodie, Merton and Samuelson, 1992 for an example for the latter).

In this paper, using the Health and Retirement Studies (HRS) data, we first document that older Americans face a significant risk of being forced to retire before their planned retirement age. We use the question on the self-assessed reason of retirement to identify a forced retirement. About a quarter of retirements turn out to be involuntary. They could not work until their planned retirement age due to reasons including health issues and their employers' decision. Every year, on average, about 4 percent (2 percent) of households in ages between 60 and 64 (55 and 59) who wanted to keep working involuntary lose their jobs and are forced to retire. An involuntary early retirement often involves a loss of several years' worth of labor earnings. Most forced retirees do not return to the labor market, and do not rely on unemployment insurance and disability income. These findings imply that households close to retirement face a substantial risk in their human capital. Furthermore, we find that the probability of being forced to retire is negatively correlated with the performance in the stock market. An increase in the probability of forced retirement follows a large negative returns in the stock market.

We then build a life-cycle portfolio choice model with the estimated forced retirement risk to examine the implication of such a risk on the optimal portfolio choice. In this model, households plan to work until a certain age, but they may be forced to retire before reaching that age. The probability of a forced retirement is a function of age and correlated with stock returns, calibrated based on the findings from the HRS data. The result from the model suggests that the forced retirement risk makes the part of human capital that is exposed to this risk stock-like, so it is optimal for households to increase the share of risky assets in financial portfolio as they approach their retirement, opposite to the conventional wisdom. We also show that what is behind the stock-like human capital is not the existence of the forced retirement risk per se but the correlation between stock returns and forced retirement risk. Once we mute this correlation in our model, the effect of having a significant risk in remaining labor earnings is dominated by the effect of having a flow of income not correlated with stock returns, so human capital becomes bond-like.

This paper relates and contributes to literature in three aspects. First, this paper contributes to the literature on household portfolio choice by documenting additional source of human capital risk and examining its implication on the optimal portfolio choice. Existing studies in this literature find that human capital is bond-like even under a counterfactually high correlation between earnings shocks and stock returns (Viceira, 2001; Cocco, Gomes and Maenhout, 2005; and Hugget and Kaplan, 2016). Fagereng, Guiso, and Pistaferri (2016), by showing that firms provide substantial wage insurance to workers, provide one reason why shocks to the earnings process are typically very small. On the other hand, Hugget and Kaplan (2016) show that the left-skewed earnings growth distribution makes human capital more stock-like but its effect is still limited. None of these papers focuses on the role of retirement timing uncertainty that is the most important source of risk in human capital of older households. In existing papers in this literature, retirement timing is either fixed (Cocco, Gomes and Maenhout, 2005, for example) or determined by households (Bodie, Merton and Samuelson, 1992). In the latter case households can use the retirement timing as a buffer against negative asset return shocks. Based on the observation that retirement timing is not a choice variable but rather a shock for a significant fraction of older households, we take the opposite extreme, where retirement timing is purely determined by demand side in the labor market. We are not arguing that no household can use the retirement timing as a buffer against negative asset return shocks at all. We choose this set up to focus on how close human capital gets to a risky asset for households that are exposed to such a risk, which has been neglected in the literature. There are several papers that focus on mechanisms through which human capital can be more stock-like. Heaton and Lucas (2000) resort to entrepreneurial risk and Benzoni, Collin-Dufresne and Goldsten (2007) to cointegration between wage and stock returns to make human capital stock-like. This paper shows a different channel through which human capital becomes a close substitute for a risky asset.

Secondly, this paper contributes to the small but growing literature on the uncertainty in retirement timing. Chan and Stevens (2001) show that in the US involuntary job loss is not rare at old ages and returning to labor market after a job loss becomes significantly more difficult at older ages. Dorn and Sousa-Poza (2010) show that involuntary early retirement is common in European countries. Gorodnichenko, Song and Stolyarov (2013) and Gustman, Steinmeier and Tabatabai (2016) discuss macroeconomic determinants of retirement timing. By using the HRS data, we document that the probability of being forced to retire is fairly high at older ages and negatively correlated with stock returns in the U.S. Some papers examine the economic implications of the uncertainty in retirement timing. Smith (2006) and Dong and Yang (2016) rely on involuntary retirement to explain the "retirement consumption puzzle," i.e., a downward shift of consumption expenditure at retirement (Modigliani and Brumberg, 1954; Friedman, 1957; Heckman, 1974; Haider and Stephens, 2007; Battistin, Brugiavini, Rettore and Weber, 2009). Caliendo, Casanova, Gorry and Slavov (2016) show that uncertainty about the timing of retirement is a major financial risk to individuals' lifetime consumption. This paper relates the uncertainty in retirement timing to household portfolio choice.

Lastly, this paper relates to literature on the age effect and the retirement effect on portfolio choice. Ameriks and Zeldes (2004) document age effect on household portfolio choice. Rosen and Wu (2004), Berkowits and Qiu (2006), Fan and Zhao (2009), Love and Smith (2010), Goldman and Maestas (2013), and Lee (2015) examine how health status changes or health expenditure risks at older ages affect household portfolio choice. This paper focuses on the role of forced retirement risk in understanding the optimal portfolio adjustment over age around retirement. Chen and Nam (2016) provide empirical evidence that retirement contributes positively to households' exposure to financial risks in its portfolio, consistent with the prediction from the current paper.

The remainder of this paper is organized as follows. Section 2 introduces data and defines variables. Section 3 presents empirical evidence on the size of forced retirement risk and its correlation with the stock market returns. Section 4 sets up the life-cycle portfolio choice model with the forced retirement risk. Section 5 presents the optimal portfolio choice under the presence of forced retirement risk. Section 6 concludes.

2 Data

We use the Health and Retirement Study (HRS) data to find empirical evidence of forced retirement risk. The HRS has surveyed more than 20,000 elderly in the United States since 1992. The HRS provides detailed demographic and socioeconomic characteristics of participants. In particular, we take advantage of detailed questions on retirement to study the forced retirement risk. In the following subsection, we provide the description of key variables we use to determine the forced retirement risk and explain our sample selection criteria in detail.

2.1 Key Variables

Retirement Status

The HRS provides the current retirement status of survey respondents. More specifically, the HRS surveys the retirement status from the following question.

> *Q: At this time do you consider yourself to be completely retired, partly retired, or not retired at all?*
>
> *A: 1) not retired; 2) completely retired; 3) partly retired*

Based on answers to this question, we classify respondents who consider themselves as completely retired or partly retired as retirees.[1] In addition to the current retirement status, the HRS also questions the year and month of retirement:

> *Q: In what month and year did you [partly/completely] retire?*

From these questions and age of participant in the survey year, we can determine the year of retirement and age at retirement even though the participants have retired between the survey years. For example, if a participant in 2010 HRS, whose age is 62, answered that he/she retired in 2009, we estimate that his/her retirement age is 61 and retirement year is 2009.

Forced Retirement Indicator

Among respondents who consider themselves partly or completely retired, the HRS gathers additional information whether they were forced into retirement:

[1] We classify the latter group as retirees mainly because they typically have no labor earnings and rarely come back to the labor market.

Q: Thinking back to the time you [partly/completely] retired, was that something you wanted to do or something you felt you were forced into?

A: 1) Wanted to do; 2) Forced into; 3) Part wanted, part forced

We classify respondents who answered the forced retirement question as *2) forced into* as forced retirees.[2] So our definition of a forced retirement is based on self-assessed reason of retirement. Relying on self-assessment is appropriate given that whether the retirement was voluntary or involuntary is a subjective question (Dorn and Sousa-Poza, 2010). Below we also examine how this measurement is correlated with more conventional measure of retirement timing uncertainty, that is the difference between the actual and expected retirement age.

2.2 Sample Selection

While the HRS surveys relatively elderly sample, not all sample in the HRS is relevant to our study. In particular, the forced retirement risk only matters when survey respondents are close to typical retirement ages. So, we first select respondents aged between 55 and 69. We also restrict our sample to male household head.[3] While retirement status of spouse may affect household financial decision including portfolio choice, the retirement of household head has more significant effect as his income is the largest part of the household income. As the Asset and Health Dynamics Among the Oldest-Old (AHEAD) data was merged into the original HRS data in 1998, the sample composition in the HRS changed significantly. To keep our sample size consistent throughout the survey years, we exclude the sample before the 1998 survey. Our sample is further reduced as we exclude retirees who have not been asked about the forced retirement question. After applying our sample selection criteria, we finally obtain 13,724 samples.

[2] From 2000 to 2010, among those with age between 55-70, 383 chose *2)* or *3)*, 303 of which chose *2)* only. Hence there is only slight quantitative difference by treating both *2)* and *3)* as being forced to retire. In our main analysis, we will focus on the narrower definition with *2)* only.

[3] The definition of household head does not exist in the HRS. Alternatively, we define a household head as the member of the household whose earning is the highest among members throughout survey periods (which counts for over 95 percent of our sample households). For households with switched "highest earner", which are very rare in our sample, we label the member with more "higher earning" waves in our sampling period as the household head.

3 Empirical Evidence

This section presents empirical evidence on the presence of forced retirement risk among the elderly and how the forced retirement risk varies with age and year. Statistics show that significant number of older people do retire involuntarily, i.e., are being forced to retire earlier than they planned to. A forced retirement typically means a loss of several years' worth of labor earnings, not mitigated by returning to the labor market later or by relying on unemployment insurance or disability income. The probability of being forced to retire tends to increase after having a downturn in the stock market.

3.1 Prevalence of Forced Retirement

To show that many households in the US do not have a full control over their retirement timing, we first summarize the proportion of retirements that are considered involuntary. Table 1 shows the number of retirements and the proportion of forced retirements by retirement age and year they retired. Overall, the number of forced retirements take non-negligible proportion among total retirements while there are large variations in the number by retirement age and year. About 28 percent of retirees in the entire sample consider themselves as being forced into retirement. The proportion of forced retirements decreases in age: more than 40 percent of retirements between 55 and 59 are forced retirements, while for retirements between 65 and 69, the proportion of forced retirements drops to 23 percent. This simply reflects that there are more voluntary retirements for older age groups. More interestingly, the proportion of forced retirements varies greatly across years. For example, the proportion peaked at the highest value of 45.6 percent in 2009 right after financial crisis. On the other hand, during the stock market boom in 1999, the proportion of forced retirements only takes 20 percent.

3.2 Forced Retirement Risk

While the non-negligible proportion of forced retirements among total retirements establishes that many workers do not have a full control over their retirement timings, it does not reveal how likely a worker will be forced to retire conditional on willing to continue to work. To measure such a risk,

Table 1: Number of Retirements and Forced Retirees (FR) Ratio

	Retirement Age							
	55-59		60-64		65-69		Total	
Retirement Year	# of Retirements	% of FR	# of Retirements	% of FR	# of Retirements	% of FR	# of Retirements	% of FR
1998	86	37.2%	159	18.9%	85	20.0%	330	23.9%
1999	48	29.2%	162	19.1%	82	17.1%	292	20.2%
2000	56	23.2%	128	28.1%	82	19.5%	266	24.4%
2001	36	22.2%	129	20.2%	54	18.5%	219	20.1%
2002	37	40.5%	148	25.0%	62	14.5%	247	24.7%
2003	45	37.8%	85	21.2%	61	29.5%	191	27.7%
2004	39	23.1%	76	22.4%	64	18.8%	179	21.2%
2005	36	50.0%	77	14.3%	73	20.5%	186	23.7%
2006	33	42.4%	47	34.0%	70	25.7%	150	32.0%
2007	56	42.9%	58	27.6%	62	22.6%	176	30.7%
2008	40	55.0%	54	37.0%	48	33.3%	142	40.8%
2009	42	57.1%	59	47.5%	57	35.1%	158	45.6%
2010	50	60.0%	58	43.1%	42	23.8%	150	43.3%
2011	28	50.0%	55	34.5%	29	24.1%	112	35.7%
2012	19	42.1%	49	46.9%	22	40.9%	90	44.4%
Total	651	40.2%	1,344	26.3%	893	23.0%	2,888	28.4%

Notes: This table is based on respondents whose retirement age is between 55 and 69 and retirement year is between 1998 and 2012. Retirees who have been asked about the forced retirement questions are included in this table.

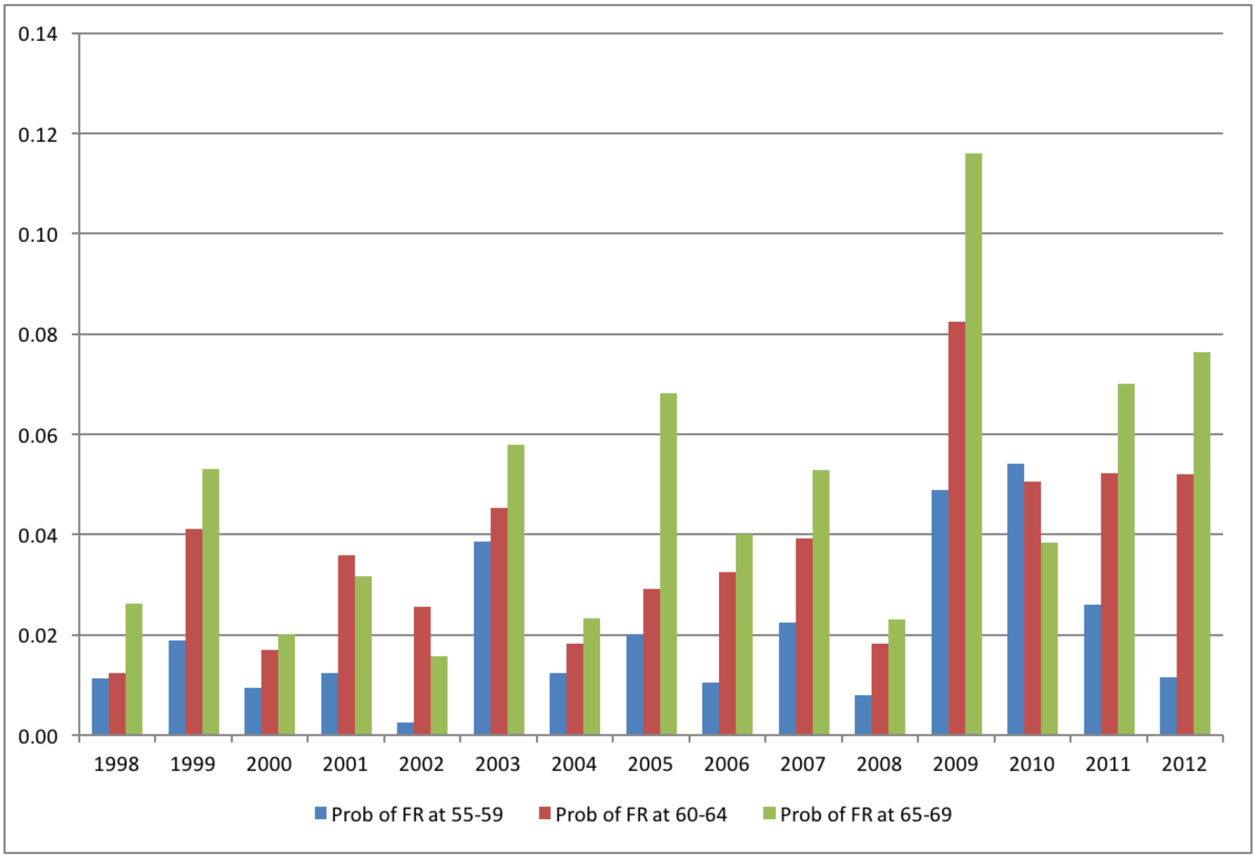
which we call the *forced retirement risk*, we use the following formula:

$$ForcedRetirementRisk_{i,j} = \frac{N(ForcedRetirees_{i,j})}{N(ForcedRetirees_{i,j}) + N(Working_{i,j})} \tag{1}$$

where $N(ForcedRetirees_{i,j})$ is the number of forced retirees at age i in year j, and $N(Working_{i,j})$ is the number of people who are working at age i in year j. The denominator captures all the individuals who were working in year $j-1$ and wanted to keep working in year j. The numerator captures those who could not do so because they are forced to retire.

Based on this definition, we estimate the forced retirement risk by age and year as shown in Figure 1. On average, the risk of being forced to retire is not negligible. In the entire age group considered, the average size of risk is 4 percent, meaning that every year 4 percent of workers who wanted to keep working are forced to retire. The size of risk increases in age. The forced retirement risk in age between 55 and 59 is 2 percent, while it is almost doubled in age between 60 and 64. Additionally, there is an annual variation in forced retirement risk. Below we examine how the size of forced retirement risk is related to the returns to the stock market.

Figure 1: Forced Retirement Risk



3.2.1 Economic significance of the forced retirement risk

The probability of being forced to retire is not a sufficient statistics to reveal the effective size of such a risk. What also matters is, conditional on being forced to retire, what is the difference between the expected retirement age and the actual retirement age. If a household is forced to retire one year earlier than the expected retirement age, that is still a significant loss of financial resources, but a much smaller shock compared to losing five years' worth of labor earnings.

Table 2 shows the distribution of the difference between the expected and actual retirement ages, conditional on being forced to retire.[4] Note that the expected retirement age measure in the HRS is fairly noisy. Many households are not asked this question so the number of observations used in this analysis is small. Also, some households give unrealistically high or low ages as expected retirement ages. Still, the results suggest that a forced retirement risk often involves a loss of labor

[4] In case the same individual answered the expected retirement age questions in multiple waves, we use the most recent one observed before their forced retirement.

earnings for many years compared to their expectations. If forced to retire between age 60 and 64, the median household loses two years' worth of labor earnings, while a quarter of such households lose more than four years' worth of labor earnings. For those who are forced to retire before age 60, these numbers increase to five and seven years, respectively.

Table 2: Expected - actual retirement age

	Percentile					N
	10	25	50	75	90	
55-59	-1	2	5	7	9	198
60-64	-2	0	2	4	6	322

There are possible channels through which households can mitigate the impact of these shocks. One way is to return to the labor market, so that they can have a bridge job before they fully retire. Even if they cannot make similar earnings as before, this will provide some buffer against disastrous earnings shocks. But we find that only about 8 percent of forced retirees come back to the labor market. One reason behind this would be the loss of firm-specific human capital. This is also consistent with the findings from a number of studies that the labor market demand side constraints hinder post-career employment (e.g., Hurd, 1996, Scott, 2004, and Kantarci and van Soest, 2008).

It is also possible that forced retirees rely on unemployment insurance, at least for a short period. It is even possible to 'double dip', i.e., getting retirement benefits such as Social Security income and defined-benefit pension incomes and unemployment insurance benefit at the same time. But that does not turn out to be a common practice. The share of forced retirees that report any unemployment benefit is fairly low (about 20 percent) while the income replacement rate conditional on receiving such benefit is also low (about a quarter). Given that about half of the forced retirements are due to health issues, another possibility is that forced retirees rely on disability income. This also does not turn out to be prevalent. Only about 5 percent of forced retirees (10 percent of those who are forced to retire due to health issues) report any disability income after being forced to retire. Conditional on receiving disability income, it replaces about 40 percent of pre-retirement earnings.

3.2.2 Correlation with stock returns

Having established that older Americans face a significant forced retirement risk, now we turn to the correlation between the size of the forced retirement risk and stock returns. Figure 1 showed that there is a surge in forced retirements during the Great Recession, so we conjecture that an increase in the forced retirement risk follows downturns in the stock market.

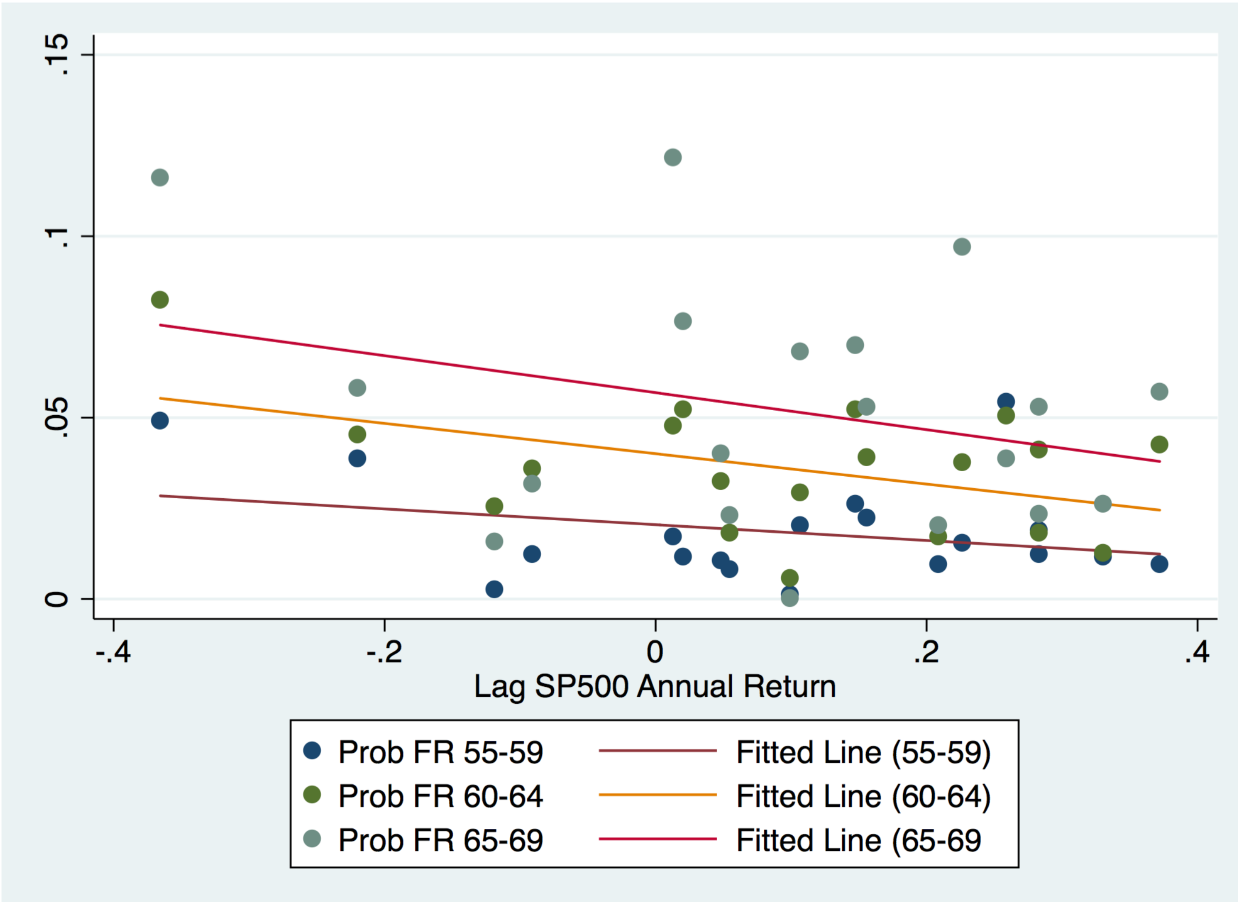
To confirm this conjecture, we regress the probability of being forced to retire in each year on the annual S&P 500 returns from the last year. Admittedly, we only have data for 15 years so we cannot precisely estimate the correlation between the two variables. But the estimated regression lines in Figure 2 suggests that the probability of being forced to retire increases after having negative returns in the stock market. The estimated effect is not small. For example, after having a positive 20 percent return on S&P 500, the probability of a households between age 60 and 64 being forced to retire is about 3 percent, while it goes up to 5 percent after having a negative 20 percent return on S&P 500. The estimated slope for the age group 60-64 turns out to be statistically significant notwithstanding of the small sample size.

The conclusion so far is that the probability of older Americans being forced to retire is not negligible, and once a forced retirement occurs it accompanies significant financial losses. In addition, after having a negative return stock market, households face an increased probability of being forced to retire. The size of the forced retirement risk, and its correlation with stock returns, are the key elements in determining how much the human capital can be stock-like close to retirement. In the following section, we examine whether the human capital that is subject to the forced retirement risk can be stock-like, by incorporating this risk into the household's financial portfolio choice problem.

4 Life-cycle Portfolio Choice Model

We build a life-cycle portfolio choice model to investigate how the forced retirement risk affects the portfolio choice of households. Every period, households choose how to allocate their savings between risky and safe assets as well as how much to consume and save. The model features aggregate stock return shocks, idiosyncratic income shocks, and mortality risk. The main innovation in our model is to incorporate the forced retirement risk estimated in the previous section. The retirement

Figure 2: Forced retirement risk and S&P returns



age is exogenously determined and it is uncertain. This uncertainty may also be correlated with the returns to stocks. Otherwise, the model is close to standard models used in the literature, in particular that in Cocco, Gomes and Maenhout (2005).

4.1 Preference

Households maximize the following objective function:

$$E_1 \sum_{t=1}^{T} \delta^{t-1} \left(\prod_{j=0}^{t-2} P_j \{ P_{t-1} \frac{C_{it}^{1-\gamma}}{1-\gamma} + b(1-P_{t-1}) \frac{D_{it}^{1-\gamma}}{1-\gamma} \} \right), \tag{2}$$

where i is an index for an individual household, C_{it} the consumption in age t, D_{it} is the amount of bequest that it will leave if it dies in age t, δ is the time discount factor, b is the weight that it puts on bequest, γ is the risk aversion coefficient, and P_t is the survival probability between age

$t-1$ and t. This is basically present discounted value sum of flow utility where households face uncertainty in the length of lifetime and have bequest motive.

4.2 Labor income process before retirement

Households that are still working face idiosyncratic risks in their labor income. The labor income process is as following:

$$log(Y_{it}) = f(t, Z_{it}) + \nu_{it} + \varepsilon_{it} \tag{3}$$

$$\varepsilon_{it} \sim N(0, \sigma_{\varepsilon}^2) \tag{4}$$

$$\nu_{it} = \nu_{i,t-1} + u_{it} \tag{5}$$

$$u_{it} \sim N(0, \sigma_u^2). \tag{6}$$

The labor income (Y_{it}) fluctuates around its conditional mean ($f(t, Z_{it})$), where the latter is a function of age and possibly also of other characteristics of households such as education. The deviation between the actual labor income and its conditional mean is determined by both the permanent shocks (ν_{it}) and temporary shocks (ε_{it}), where the former is modeled as a random walk process. The innovation (u_{it}) to the random walk process can be correlated with the stock returns, while temporary shocks are independent.

4.3 Retirement income

For most of defined benefit pension plans and also for Social Security, the retirement income depends on the average of earnings the household made during its working life. Let Ψ denote the average labor income the household had in its working life. While households are working, it evolves according to:

$$\Psi_{it} = \frac{(t-1)\Psi_{i,t-1} + Y_{it}}{t}. \tag{7}$$

If a household is retired at the normal retirement age K, it starts to receive a fixed retirement income every year that is calculated as:

$$log(Y_{it}) = log\lambda + log(\Psi_{iK}),\ \forall t \geq K. \tag{8}$$

This models the social security and private defined benefit pension income of households, where λ is the replacement rate.

If a household is forced to retire at age s that is lower than K, then the retirement income is calculated as:

$$log(Y_{it}) = log\lambda_s + log(\Psi_{iK}),\ \forall t \geq s. \tag{9}$$

where

$$\Psi_{iK} = \frac{s\Psi_{is}}{K}(= \frac{s\Psi_{is} + (K-s)0}{K}). \tag{10}$$

Hence, the forced retirement affects the income stream in two ways. First, it reduces Ψ_{iK} that goes into the formula of the retirement income calculation, by plugging in zero earnings for the years not working before the normal retirement age in the calculation. This captures the fact that for certain defined benefit pension plans (and also for Social Security income up to some point) early retirement negatively affects the pension benefit accrual. Second, given Ψ_{iK}, it also affects the annual income flow by changing the replacement factor λ_s. In the baseline model, we calculate λ_s such that the early retirement does not affect the expected present value sum of total retirement income given Ψ_{iK}. In other words, we allow an actuarially fair early retirement benefit from the age of forced retirement, regardless of when it happens. In reality, how a forced retirement affects retirement income depends on the benefit formula of defined pension benefits as well as their specific work history. In certain cases, either the effect of a forced retirement on Ψ_{iK} is limited (e.g., Social Security income for those who worked more than thirty five years) and/or actuarially fair early retirement benefit is not available before certain ages (e.g., Social Security income is not available before age 62). Later we will also examine the alternative specifications to consider these possibilities.

4.4 Uncertainty in retirement age

In the household portfolio choice literature, retirement age has been considered either to be fixed (e.g., Cocco, Gomes and Maenhout, 2005 and Gomes and Michalides, 2005) or to be a choice of households (Bodie, Merton and Samuelson, 1992). But as we have examined in the previous sections, many households are forced to retire so for them retirement is not a buffer against shocks but rather a shock itself. Furthermore, this uncertainty on retirement age can be correlated with stock returns, which may amplify the implication of the forced retirement risk on portfolio choice.

We incorporate the forced retirement risk into our model, while not allowing households to choose their retirement age. Bodie, Merton and Samuelson (1992) assume that retirement timing is solely determined by households, so they can use this to buffer against negative asset return shocks. Based on the observation that retirement timing is not a choice variable but rather a shock for a significant fraction of older households, we take the opposite extreme, where retirement timing is purely determined by demand side in the labor market. We are not arguing that no household can use the retirement timing as a buffer against negative asset return shocks. We choose this set up to focus on how close human capital gets to a risky asset for households that are exposed to such a risk, which has been neglected in the literature.

We assume that the probability of being forced to retire in the following year, Ω_t, to be zero for those who are not older than 55. For those who are still working in their age between 56 and 63, the probability that they will be forced to retire in the following year is:

$$\Omega_t = \bar{\Omega}_t + \kappa_t \iota_t, \tag{11}$$

where $\bar{\Omega}_t$ is the average value of this probability and κ_t determines how much this probability is affected by aggregate shocks, both specific to age t, and ι_t is an aggregate shock that affects the risk of forced retirement.

4.5 Financial assets

The model has two financial assets, a risk-free asset and a risky asset. The risk-free asset has a fixed gross return $\bar{R_f}$. The return process for the risky asset is:

$$R_{t+1} - \bar{R_f} = \mu + \eta_{t+1} \tag{12}$$

$$\eta_{t+1} \sim N(0, \sigma_\eta^2) \tag{13}$$

$$Corr(\eta_{t+1}, u_{t+1}) = \rho, \tag{14}$$

where μ is the risk premium and η_{t+1} is a shock to the stock return. The stock return shock may be correlated with the permanent income shock.

Households need to choose how to allocate their savings between the two assets. They cannot borrow and they cannot short stocks. Hence, the share of assets invested in stocks, α_{it}, needs to be between 0 and 1.

4.6 Optimization problem

Let X_{it} be the cash-on-hand at the beginning of the period. Then it is determined as:

$$X_{it} = W_{it} + Y_{it} \tag{15}$$

$$W_{i,t+1} = R^P_{i,t+1}(W_{it} + Y_{it} - C_{it}) \tag{16}$$

$$R^P_{i,t+1} \equiv \alpha_{it} R_{t+1} + (1 - \alpha_{it})\bar{R_f} \tag{17}$$

where $W_{i,t}$ is the assets at the beginning of the period which is determined by the amount of savings in the previous period and the performance of the overall portfolio, R^p_{it}.

Using scalability of the problem, we normalize all the variables with respect to $exp(\nu_{it})$. Let $\tilde{C}_t$, $\tilde{X}_t$ and $\tilde{\Psi}_t$ are normalized values of C_t, X_t and Ψ_t. Then the Bellman equation can be expressed as following:

$$V_{it}(\tilde{X}_{it}, \tilde{\Psi}_{it}, Ret_t, RA_t) = Max_{\tilde{C}_{it} \geq 0, 0 \leq \alpha_{it} \leq 1}[U(\tilde{C}_{it}) + \delta P_t E_t exp(\nu_{i,t+1})^{1-\sigma} V_{i,t+1}(\tilde{X}_{i,t+1}, \tilde{\Psi}_{i,t+1}, Ret_{t+1}, RA_{t+1})] \tag{18}$$

under constraints (4) - (16), where Ret is a dummy variable capturing whether the household is retired or not, and RA captures the age of retirement once the household is retired.

4.7 Calibration

Table 3 summarizes the calibration of the parameters. For the parameters that also appear in Cocco, Gomes and Maenhout (2005) we use the same values as in their benchmark model. Conditional probabilities of survival (P_t) are from the mortality tables of the National Center for Health Statistics. The model starts from age 20 and they can live up to age 100.

Table 3: Calibration of parameters

Parameter	Value
Own calibration	
Mean of forced retirement risk ($\bar{\Omega}$) for age 55-59	0.02
Mean of forced retirement risk ($\bar{\Omega}$) for age 60-63	0.035
Variance of forced retirement risk (κ) for age 55-59	0.025
Variance of forced retirement risk (κ) for age 60-63	0.05
From Cocco et al. (2005)	
Normal retirement age (K)	65
Discount factor (δ)	0.96
Risk aversion (γ)	10
Bequest motive (b)	0
Average labor income ($f(t, Z_{it})$)*	
Variance of transitory income shocks (σ_ε^2)	0.0738
Variance of permanent income shocks (σ_u^2)	0.0106
Correlation between (permanent) labor income shocks and stock returns (ρ)	0
Riskless rate ($R_f - 1$)	0.02
Risk premium ($\mu - 1$)	0.04
Std. of stock return (σ_η)	0.157

Notes: Benchmark values used for the model.
* See Table 2 in Cocco, Gomes and Maenhout (2005).

Calibration of Ω_t is one of the most important contributions of this paper. Based on the evidence from the HRS, we calibrate $\bar{\Omega}$ to be 0.02 for age 55-59 and $\bar{\Omega}$ to be 0.035 for age 60-63. Also, based on the observed correlation patterns between the stock returns and the forced retirement risks, we calibrate κ to be 0.025 for age 55-59 and 0.05 for age 60-63, while letting $\iota_t = -\eta_t$. For example, when the return on the risky asset goes up by 10 percentage points, it reduces the forced retirement risk by 0.25 percentage point for age 55-59 and by 0.5 percentage point for age 60-63.

The hazard rate might seem trivial, but it is not. According to the calibrated parameters, the chance of being retired involuntarily before age 60, i.e., losing more than five times of annual earnings, is roughly 10 percent. The chance of being forced to retire before the normal retirement age (65) goes up to about 25 percent. Hence this is indeed a significant risk that older households face before their retirement.

4.8 Computational strategy

We solve this model using backward induction. The last period problem is trivial since it is a static maximization problem (i.e., allocation between its own consumption in the last year and bequest). This gives us the value function in the last year. Using this as the continuation value, we solve the maximization problem of the penultimate year. This repeats until the first period.

In the maximization, we use grid search to determine the optimal combination of consumption and portfolio choice. We use Gaussian quadrature to discretize the distribution of shocks and numerically integrate over them. The continuous state spaces, $\bar{X}_t$ and $\bar{\Psi}_t$, are discretized using 400 and 80 grid points, respectively. Increasing the number of grid points does not affect the results. In evaluating the continuation values off the grid points, we use cubic interpolation.

5 Results

We first compare the policy function for the stock share in financial wealth between those who are still working and those who are forced to retire. This comparison identifies how the part of human capital that is exposed to the forced retirement risk affects the portfolio choice of households. We further investigate what is the mechanism behind the estimated effect. To be specific, we turn off the correlation between the forced retirement risk and stock return risk, to examine whether the impact of the forced retirement risk on the portfolio choice mainly comes from the existence of the risk itself or the correlation. And then we construct age profiles of wealth and stock share by simulating the model to demonstrate that the optimal portfolio adjustment over age under the forced retirement risk is dramatically different from a long-standing consensus that one should reduce investment on risky assets as approaching retirement.

5.1 Portfolio choice under forced retirement risk

Figure 3 plots the optimal stock share over normalized cash-in-hand ($\tilde{X}$). Panel (a) is for age 56 where households face the forced retirement risk for the first time, while Panel (b) is for age 60. The blue curve corresponds to a household who is still working and the red one corresponds to a household that is forced to retire at the age considered in each panel. Under the normalization with respect to $exp(\nu_{it})$, the annual labor earning of a household that is still working is approximately 25. Hence the wealth-to-income ratio range shown in the figure is between 0 and 12, where the most relevant range in this age group is around 6-8 (i.e., $\tilde{X}$ in 150 - 200). One state variable that is not explicitly shown in the figure is the normalized average labor income in the past ($\tilde{\Psi}$). In this figure, we assume $\tilde{\Psi}$ to be 20, which is close to the average value of this variable in this age range.

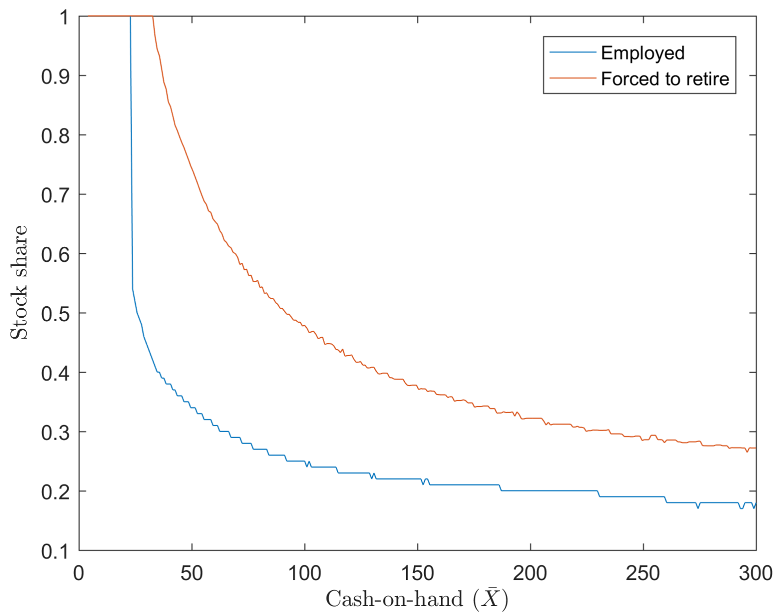
The optimal stock share is a decreasing function of financial wealth regardless of the current working status. At these ages, a large fraction of human capital is composed of retirement income that is affected neither by the performance of the stock market nor the forced retirement.[5] It functions as a close substitute to a risk free asset, as investigated in Cocco, Gomes and Maenhout (2005), so the larger the financial wealth (i.e., the lower the share of 'safe' human capital in the entire portfolio including human capital), the lower the optimal share of risky assets in the financial portfolio.

The remaining part of human capital, i.e., expected labor earnings until retirement and a part of retirement income that is affected be early retirement, is exposed to the forced retirement risk. Comparison between the two households that are identical except for its current labor force participation demonstrates how this part of human capital affects the portfolio choice. For both age 56 and 60, the optimal stock share is much lower for those who are still working. In other words, the part of human capital exposed to the forced retirement risk is considered as a close substitute for the risky asset, so holding this human capital crowds out investment in the risky asset in the financial portfolio. The impact is larger when households have less financial assets (i.e., when share of human capital in the entire portfolio including human capital is higher). The size of impact is similar between age 56 and 60. Compared to age 60, at age 56 the risk of being forced to retire in the next year is lower while the loss in labor earning associated with such an early retirement is

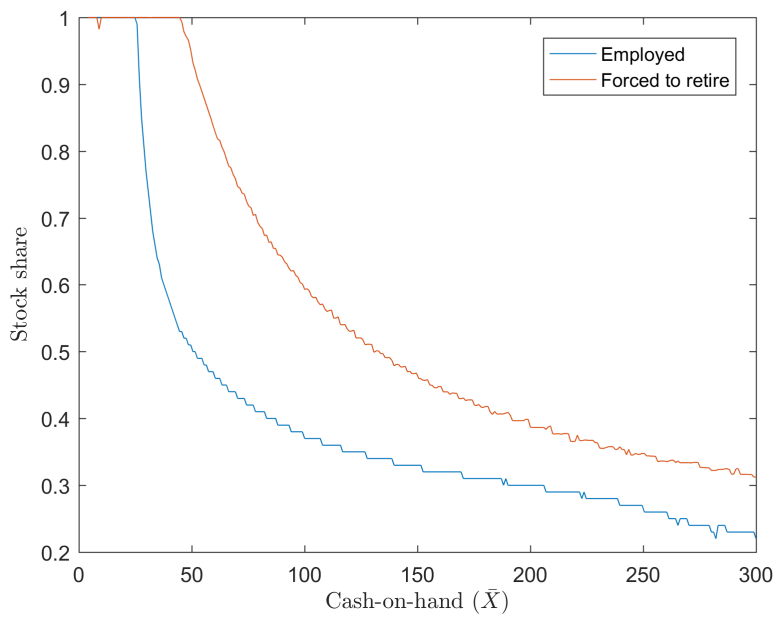
[5] Recall that in the baseline model a forced retirement risk affects the retirement income only through its effect on Ψ_{iK} and the magnitude of its effect is relatively small.

Figure 3: Stock share comparison: workers vs. forced retirees

(a) Age 56



(b) Age 60



Note: Under the normalization with respect to $exp(\nu_{it})$, labor earnings of the employed household is about 25 in this age range. We assume $\bar{\Psi} = 20$, which is average value in this age range.

higher. These two factors seem to cancel out each other.[6]

5.2 Role of correlation between stock returns and forced retirement risk

But what makes the part of human capital exposed to forced retirement risk a close substitute for a risky asset? Is it the existence of forced retirement risk per se, or is it the correlation between this risk and the stock return risk? To investigate the mechanism behind the effect we observed in the previous subsection, we revisit the comparison of the stock share policy function under no such correlation.

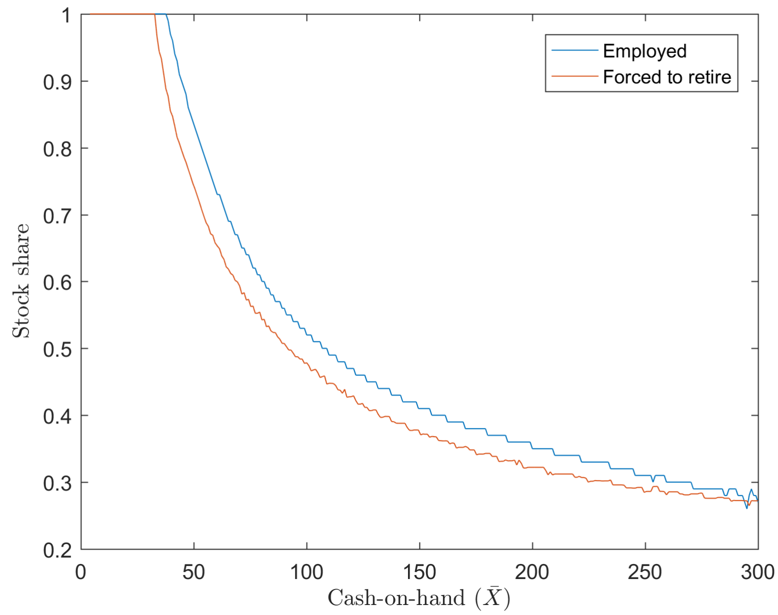
Once we turn off the correlation, we find a qualitatively opposite result (Figure 4). Now the optimal stock share is higher for those who are still working, for both ages considered. Working households still face the forced retirement risk. But as long as that risk is not correlated with stock returns, the effect of the risk in remaining labor earnings is dominated by the effect of having a flow of income that is uncorrelated with stock returns. Quantitatively, the size of effect of having additional income on the optimal stock share is relatively small.

By comparing blue curves in Figure 3 and Figure 4, we can see that the effect of the correlation between the forced retirement risk and stock returns on the portfolio choice is large. One might find it puzzling because the effect of stock returns on the forced retirement risk, according to our calibration, seemed to be rather small. For example, during ages between 55 and 59, a negative stock return shock that corresponds to one standard deviation (i.e., 10 percent loss) increases the probability of being forced to retire only by 0.4 percentage point. However, that is 20 percent increase in the hazard rate (from 2 to 2.4 percentage points). Also, that increases probability of having a large negative stock return conditional on being forced to retire. To be more specific, let us first approximate the stock return process with Gaussian quadrature with three supports, {-0.21, 0.06, 0.27}, where each number is net return on investment in the risky asset. According to the calibrated joint process, conditional on a household being forced to retire in age between 55 and 59, the likelihood that it also experiences a return of negative 21 percent on its stock investment is more than twice of that of having a large positive return of 27 percent (22.4 percent vs. 10.9 percent). Given that it is more likely to make a loss in its investment in stocks when it also loses significant fraction of human capital, a household that faces the calibrated forced retirement risk

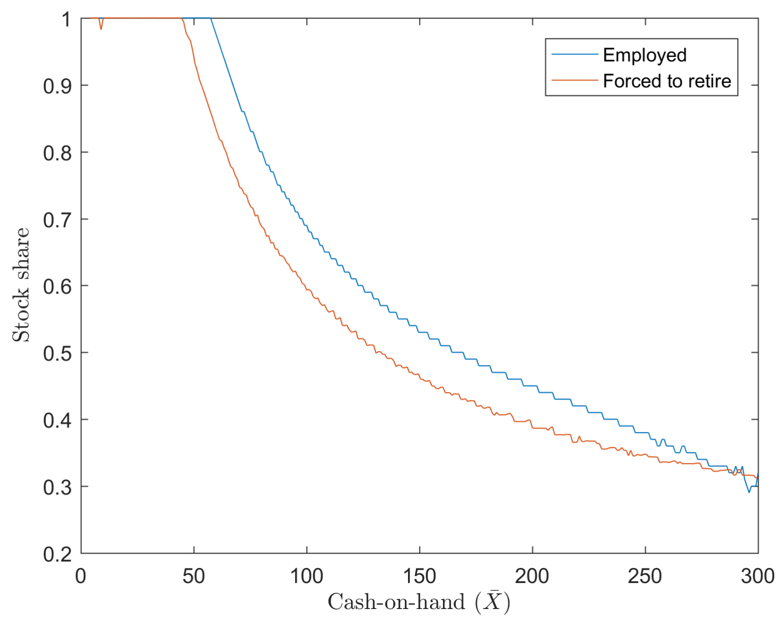
[6] After age 60 the size of impact reduces and it disappears at age 64, where everyone retires next year.

Figure 4: Stock share comparison: under no correlation between forced retirement risk and stock return risk

(a) Age 56



(b) Age 60



Note: Under the normalization with respect to $exp(\nu_{it})$, labor earnings of the employed household is about 25 at these ages. We assume $\tilde{\Psi} = 20$, which is average value in this age range.

hedge this risk by investing more in the safe asset.

Note that in Viceira (2001), retired households almost always have a lower share of risky assets in financial portfolio compared to working households, even under an unrealistically high correlation between permanent labor income shocks and stock return shocks. We show that one can easily overturn his findings by explicitly modeling the forced retirement risk and the correlation between that risk and stock returns. On the other hand, Heaton and Lucas (2000) resort to entrepreneurial income risk to explain risk premium puzzle. We show that even non-entrepreneurs may view (a part of) their human capital as a close substitute for stocks, hence reducing demand for them.

5.3 Age profiles of optimal wealth and stock share

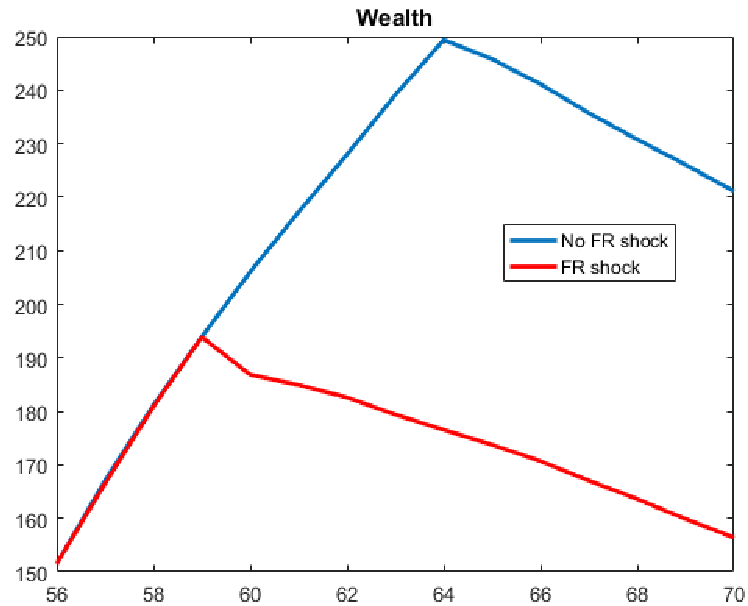
The above policy function comparisons examined how the optimal stock shares are different between those who are forced to retire and those who are still working conditional on the same wealth level. Forced retirement, however, also reduces the level of wealth. To examine how a forced retirement shock affects the optimal portfolio choice through its direct effect on the policy function *and* its indirect effect through changes in wealth, we construct a life-cycle profile of wealth and the optimal stock share around the retirement age (55-70).

Lifecycle profiles from the baseline model are shown in Figure 5. For the wealth profiles, there is nothing surprising. Households accumulate wealth while they are working and then decumulate once they retire. For the stock share, once the households represented in the red curve retire, there appears a wide gap between the two curves. Most of this is driven by the policy function difference shown in Figure 3 while a part of it comes from the fact that forced retirees now have less wealth, which increases the optimal stock share even further. The gap shrinks as they approach the normal retirement age, as the size of the forced retirement (in terms of the expected value of earnings loss conditional on being forced to retire) decreases. The gap that remains after the normal retirement age is solely from the different level of wealth.

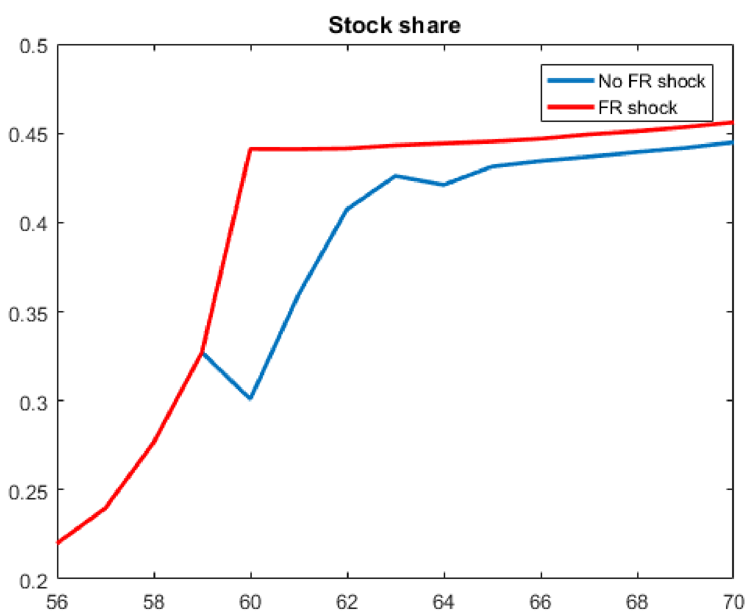
From this figure, we see that the optimal portfolio adjustment around the retirement age is almost the opposite to the conventional wisdom that one has to reduce the share of risky assets as she approaches her retirement. Under the existence of the forced retirement risk, households *increase* their stock share as they approach the normal retirement age and when they are forced to retire. One justification for the conventional wisdom is that younger households have more labor

Figure 5: Life-cycle profiles of wealth and stock share

(a) Wealth



(b) Stock share



Note: The blue curves assume that the households are not forced to retire until the normal retirement age (65) while the red curves assume that the households are forced to retire at age 60. Profiles are constructed as the averages of 1,000 simulations each.

earnings than the older households (see Jagannathan and Kocherlakota, 1996). This justification is based on the assumption that the human capital in bond-like.[7] As long as human capital becomes more stock like, as is the case under the forced retirement risk, that justification is no more valid and we find the opposite to the conventional wisdom to be optimal.

Figure 6 is the life-cycle profiles from the model with no correlation between the forced retirement risk and the stock returns. The wealth profiles are almost the same as those from the baseline model. In this specification households on average accumulate more wealth, as they can invest a larger fraction of their financial savings in the risky asset which provides a higher return on average. As Figure 4 showed, the effect of the additional human capital of those who are still working at a high age on the portfolio choice is fairly small under this specification. As a result, the stock-share profile does not show a significant adjustment around the retirement. Once the households represented in the red curve are forced to retire, their stock share becomes smaller than that of those who are still working due to the policy function difference shown in Figure 4. Later on, the relationship flips, and again it is due to lower wealth level of forced retirees.

5.4 Alternative specifications

In this section, we investigate how alternative assumptions on various elements of the model affect our main result (i.e., policy function comparisons).

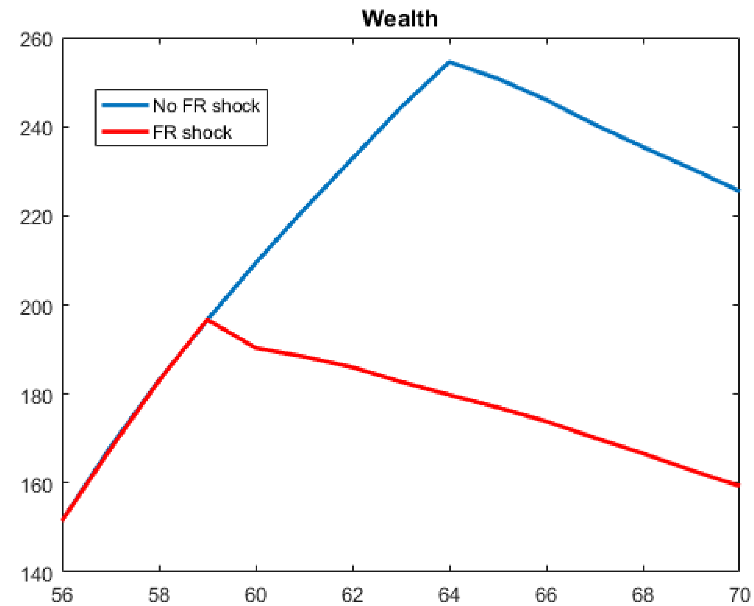
5.4.1 No effect of a forced retirement on Ψ

In the baseline model, we assume that the forced retirement reduces Ψ by including zero earnings to the calculation of this variable for the number of years not working before the normal retirement age. Whether this is a correct description of the defined benefit pensions and Social Security depend on the exact rule of benefit calculations and work history of workers. For example, if the define benefit pension benefit accrual is a function of number of service years and the average of highest certain years of earnings, then the forced retirement can affect Ψ directly by reducing the number of service years (and by reducing the average of highest earnings if his earnings have been increasing

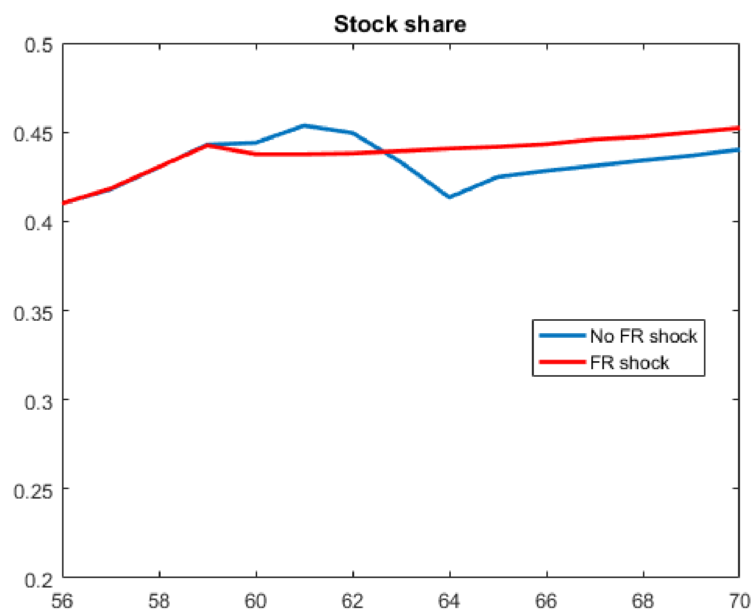
[7] Another justification is that younger households have a longer investment horizon so they have more time to recover their loss if they experience a large loss. As Samuelson (1979) showed, this argument is not correct as long as the stock returns are independent over time.

Figure 6: Life-cycle profiles of wealth and stock share: under no correlation between forced retirement risk and stock return risk

(a) Wealth



(b) Stock share



Note: The blue curves assume that the households are not forced to retire until the normal retirement age (65) while the red curves assume that the households are forced to retire at age 60. Profiles are constructed as the averages of 1,000 simulations each.

over time). On the other hand, if someone has been working for more than 35 years, then the effect of a forced retirement on Ψ can only be marginal for Social Security income.

To examine the opposite extreme to what has been assumed in the baseline model, we calculate the optimal portfolio choices assuming that the forced retirement does not affect Ψ. In other words, if someone is forced to retire at age $s < K$, then we use $\Psi_K = \Psi_s$ in the calculation of retirement income.

Under this specification, we find qualitatively the same results as in the baseline model (Figure 7). Those who are still working should invest less on stocks than those who are forced to retire. Quantitatively, the human capital is less stock-like compared to the baseline. By comparing to Figure 3, we can see that the gap between the blue and red curves in this specification is about 40 percent smaller. Note that in the baseline model a forced retirement affects households' financial resources via two channels: on one hand through the lost labor earnings and on the other hand through the reduced retirement income (through its effect on Ψ). The current specification isolates the effect through the first channel and it shows that the first channel accounts for about 60 percent of the effect in the baseline model.

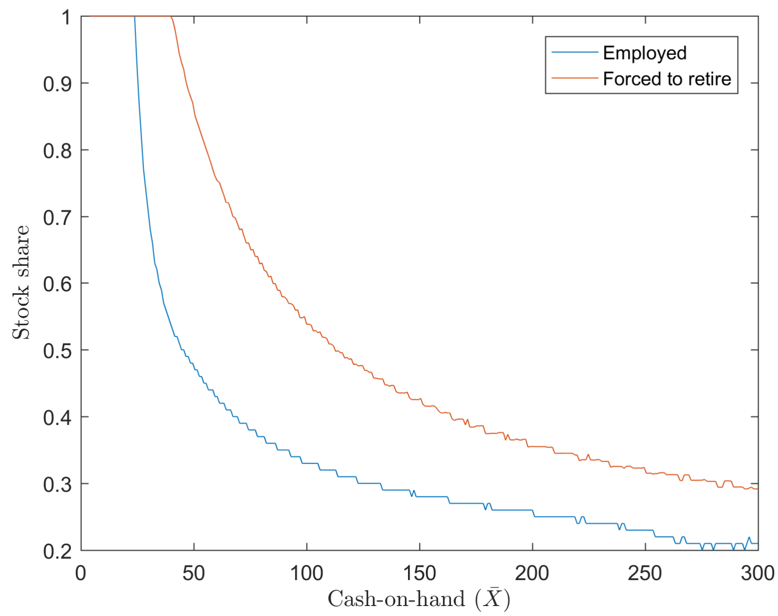
5.4.2 List of other specifications to be implemented

1. Explicitly model health shocks: In the current baseline model, we do not distinguish difference sources of a forced retirement shock (i.e., health shocks vs. employer's choice). If it is due to a health shock, then the forced retirement should be accompanied by a reduced life expectancy and an increase in the expected medical expenditure. The former will function as insurance by reducing the chance of outliving financial resources while the latter will play the opposite role. A priori it is not clear whether incorporating these channels will increase the effect of forced retirement risk or not. We can calibrate these channels using the HRS data and examine their effects quantitatively.

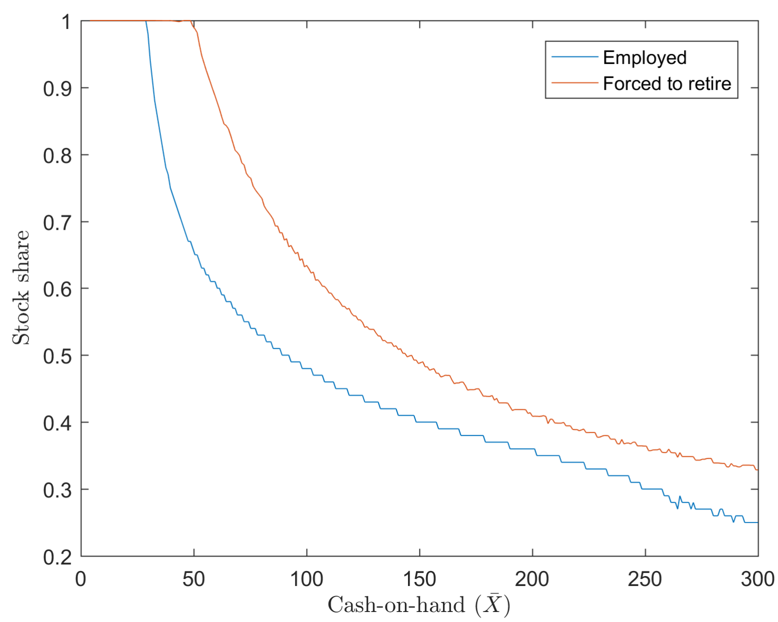
2. Not allowing an actuarially fair early retirement benefits before a certain age: In the baseline model, we assume that the actual retirement age does not affect the (expected) present value sum of retirement benefits conditional on Ψ. This might not be true for certain defined pension benefits, and it is certainly not true for Social Security income. In the latter case,

Figure 7: Stock share comparison: No effect on Ψ

(a) Age 56



(b) Age 60



Note: Under the normalization with respect to $exp(\nu_{it})$, labor earnings of the employed household is about 25 at these ages. We assume $\tilde{\Psi} = 20$, which is average value in this age range.

households need to wait until age 62 to get the first retirement benefits. To reflect this, we can run a specification where only for a certain fraction (say 50 percent) of retirement benefits an actuarially fair adjustment for early retirement is allowed, while for the other part of the benefits they have to wait until a certain age (say 62).

3. Examine the effect of transition from defined benefit to defined contribution pension systems: This can be examined by lowering the replacement factor (λ) while increasing the disposable labor earnings while working.

6 Conclusion

In this paper, we build a life-cycle portfolio choice model with forced-retirement risk and the correlation between retirement risk and stock market performance to examine how retirement risk affects household portfolio choice. We first estimated the retirement risk for different age groups and show that, although varying across different age groups, the risk is significant and strongly correlated with stock returns. Then taking the estimated retirement risk into calibration, we show that such a risk makes a part of human capital stock-like, resulting in a lower optimal stock share for workers than for retirees. By running a counterfactual exercise with no correlation between such a shock and stock returns, we find that the correlation, not the risk per se, is the main driver behind the stock-like human capital.

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