

# Slow Recovery with Uncertainty Shocks and Optimal Firm Liquidation

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## Abstract

I analyze optimal dynamic contracting between risk-averse investors and firm insiders in a dynamic general equilibrium model with heterogeneous firms. Borrowing constraints arise endogenously in this economy because investors do not observe firm output. I quantify the effect of aggregate uncertainty shocks and show that with realistic parameters, such shocks can lead to recessions with a 4% drop in GDP. It takes 8 quarters for GDP to return to pre-crisis levels after uncertainty reverts back to lower levels. This slow recovery is due to a large increase in the severity of the agency problem faced by small and medium-sized firms. A significant fraction of these firms sharply reduce investment both during the recession and several quarters into the recovery phase. In contrast, a negative first moment shock to productivity generates a fast recovery from a similar drop in GDP.

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# 1 Introduction

The US economy experienced a weak recovery following the Great Recession of 2007 – 09. It took nine quarters for GDP to return to its pre-crisis level. Reinhart and Rogoff (2009) document that this phenomenon is not limited to the most recent crisis; financial crises, especially deep ones, are followed by slow recoveries<sup>1</sup>. A second robust feature of recessions is that micro-economic uncertainty increases during these episodes. Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012) documents a sharp increase in the cross-sectional variance of plant level TFP shocks during the recent crisis<sup>2</sup>. In this paper I derive slow recovery as an equilibrium outcome in an economy with a moral hazard problem. An increase in the uncertainty of firm-level productivity worsens the agency friction, and the key result of this paper is to show that moral hazard can have quantitatively large aggregate consequences.

My model describes a continuum of firms which use a decreasing returns to scale technology for production. Firms are exposed to both firm-specific productivity shocks and aggregate shocks. In my baseline model, the aggregate shocks result in changes in the volatility of firm-specific productivity. Following Bloom et al. (2012), I call these “uncertainty” shocks. The Modigliani-Miller theorem does not hold because of an agency friction – investors do not observe firm output and firm insiders could divert it for their private benefit<sup>3</sup>. In equilibrium, investors offer insiders an incentive contract which is subject to constraints induced by the moral hazard friction. Under the optimal contract, firms can sometimes be liquidated. This is necessary to provide ex ante incentives to the insiders, but liquidations are inefficient ex post – investors recover only a fraction of the firm value while insiders receive their outside

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<sup>1</sup>While there is debate on the extent to which financial frictions played a role in the recent crisis, evidence of binding constraints during this period is well documented. Campello, Graham, and Harvey (2009) surveyed chief financial officers of US firms and found that 86% of constrained firms cut back on investing in attractive projects in 2008, compared to only 44% during normal times.

<sup>2</sup>Stock and Watson (2012) find evidence that the Great Recession was characterized by an increase in micro-level uncertainty along with a financial shock.

<sup>3</sup>Insiders in my model are individuals or groups with significantly higher control rights than cash-flow rights.

option. My model also features risk-averse investors. In general equilibrium, variation of the investors marginal utility across aggregate states impacts the cost of providing incentives. The possibility of inefficient liquidations in states with high state prices has a first-order effect on contract valuation and therefore on investment.

I quantify the response of aggregate quantities and risk premium to this aggregate shock. I find that GDP drops by 4% when the economy spends 6 successive quarters in the state with higher uncertainty. More importantly, it takes 8 quarters for GDP to return to pre-crisis levels after uncertainty reverts back to lower levels. In contrast, when the aggregate shocks are shocks to average firm-productivity rather than its volatility, recovery is fast. The economy recovers in a single quarter after 6 successive realizations of a negative first moment shock whose size is calibrated to reduce GDP by 4% at the trough.

The reason for the slow recovery from uncertainty shocks is as follows. The investor rewards firm insiders for good performance by increasing their share of future firm profits. This better aligns the incentives of the insiders relaxing borrowing constraints which allows for higher investment. As a consequence, on average, larger firms have less severe agency problems in this economy. Insiders of small and medium-sized firms, on the contrary, own a small fraction of the firm. The agency problem is severe and the investment policy is quite sensitive to changes in the intensity of the friction. An increase in firm-specific uncertainty worsens the agency problem and results in these firms reducing investment sharply. The total effect of the increase in uncertainty are: (a) a drop in the aggregate capital stock by 1.5% compared to pre-crisis levels, and (b) a reduction in the capital stock of small and medium-sized firms. Each of these effects contribute about half of the drop in output and the ensuing delay in recovery. In contrast, a negative first moment shock of the magnitude sufficient to generate a 4% drop in GDP is not accompanied by reduction of capital stock. Therefore, GDP recovers as soon as average productivity reverts back to its higher level.

The uncertainty shock also reduces the entry rate of new firms. Successive realizations of

the uncertainty shock reduces output. Since capital is only partially irreversible, it does not fully absorb the drop in output. This results in a decline in the investor's consumption, and the rise in her discount rate lowers the value of undertaking new investments.

This paper is the first to use a general equilibrium framework with long-term optimal contracts to analyze endogenous firm liquidations due to uncertainty shocks. There are at least two advantages in using an optimal contract. First, my results do not depend on specific assumptions about borrowing constraints and how they might change across a heterogeneous cross-section of firms as a result of aggregate shocks. A potential worry with an exogenously assumed form of borrowing constraint is that if investors and firms were allowed to optimally respond to the financing friction by using a better contract, the slow recovery following an increase in firm-level uncertainty would not be an equilibrium outcome. Second, alternative forms of financing helps us better connect the models to empirical observations, since in reality firms widely use lines of credit, debt of multiple maturity, interest rate derivatives and other forms of state-contingent financial securities<sup>4</sup>.

## **Related literature**

My paper relates to three existing strands of the literature. The first examines the effect of financial frictions on dynamics of aggregate macro-economic quantities and asset prices. The conventional view is that financial frictions tighten as a result of a shock to the first moment of productivity. This weakens the balance sheet of firms and as a result, their ability to borrow. The influential work of Bernanke and Gertler (1989), Bernanke, Gertler, and Gilchrist (1999), Kiyotaki and Moore (1997), and more recent results by Brunnermeier and Sannikov (2014) examine the impact of a drop in aggregate productivity on the borrowing ability of firms. All of these papers use an assumed form of financing, predominantly single-period debt contracts. The increase in borrowing constraints arise as a result of a drop in collateral value (see also

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<sup>4</sup>Sufi (2009) reports that 85% of firms in his sample obtained a line of credit. This included fully equity financed firms which held no debt.

Khan and Thomas (2013), Khan, Senga, and Thomas (2014)) or a drop in the recovery rate upon default (Gomes and Schmid (2009)). The latter considers multi-period debt contracts.

A more recently studied channel focusses on an increase in micro-uncertainty as the underlying driver leading to tighter financial constraints. This is the channel I consider in this paper. Bloom (2009) and Bloom et al. (2012) find that changes in micro-economic uncertainty has large aggregate effects in the US, although these papers abstract away from financial constraints and instead derive these results assuming non-convex cost of adjustment costs. Closer to my paper, Arellano, Bai, and Kehoe (2016), Christiano, Motto, and Rostagno (2014), and Gilchrist, Sim, and Zakrajsek (2014) consider the effect of an increase in firm level uncertainty on the ability of firms to raise financing. In these models firms are restricted to borrow using single-period, non-state contingent debt. In contrast, firms in my model borrow using optimal, long-term contracts. A big advantage of using optimal contracts is that it alleviates the concern that results arise from the assumed form of borrowing constraints and that if firms were allowed to optimally contract, key features of aggregate dynamics such as the higher rate of firm exits would not be an equilibrium outcome. Dyrda (2016) analyzes a model similar to my paper. However, in his model, firm exits are exogenous. In contrast, since exits play an important role in determining the growth rates of aggregate quantities, I model them as an equilibrium outcome.

The second area of research to which my paper relates is the role of new firms in shaping aggregate dynamics especially growth. Early empirical evidence that young firms have higher growth rates was provided by Evans (1987). Using micro-data in the US, Haltiwanger, Jarmin, and Miranda (2013) provide evidence of the important role played by younger firms in aggregate dynamics. Ibsen and Westergaard-Nielsen (2011) confirm this finding in Danish data. The theoretical literature which attempts to explain differences in productivity with firm age usually assumes ex-ante differences in productivity across firms. An early model is Jovanovic (1982) in which firms learn about their productivity over time. Clementi, Khan,

Palazzo, and Thomas (Clementi et al.) consider a similar model and analyze the effect of a drop in mean productivity accompanied by a simultaneous increase in fixed costs. These shocks lower the number of young firms leading to drop in the growth rate of aggregate quantities. In contrast to this paper, I focus on the effect of an uncertainty shock. Moreover, all firms in my model are ex-ante identical.

The third area of research to which my paper relates is optimal dynamic contracting. The financial contracts in my model are related to the discrete-time models of DeMarzo and Fishman (2007a), DeMarzo and Fishman (2007b), and Biais, Mariotti, Plantin, and Rochet (2007) (see also DeMarzo and Sannikov (2006) for a characterization of the contract in a continuous time setting.) DeMarzo, Fishman, He, and Wang (2012), Hoffmann and Pfeil (2010), and Piskorski and Tchistyi (2010) consider the effects of persistent, publicly observable, productivity shocks on the optimal contract in a partial equilibrium setting. The key difference of my paper with these studies is that while these papers examine the optimal contract in a partial equilibrium setting with an exogenously assumed discount rate, I provide a general equilibrium perspective where the discount rate investors use to value future cash flows is an endogenous equilibrium outcome. The latter arises from consumption smoothing motives of the risk-averse representative investor. Prior literature considers optimal contracts for other sources of financing frictions in a general equilibrium setting. Cooley, Marimon, and Quadrini (2004) analyze the implications of limited contract enforcement, while Dow, Gorton, and Krishnamurthy (2005) and Albuquerque and Wang (2008) model the effects of a free cash-flow problem on investment and state prices. Apart from the difference in the underlying source of financing friction, none of these papers consider the impact of an increase in uncertainty, which is the focus of my paper.

The rest of this paper is organized as follows. In Section 2, I describe the model. In Section 3, I discuss features of the optimal contract and provide a partial equilibrium perspective of the consequences of an increase in the uncertainty of firm-level productivity. In Section 4, I

quantitatively analyze the aggregate dynamics and the cross-sectional behavior of firms in general equilibrium. Section 5 concludes.

## 2 The Model

The economy consists of a continuum of ex-ante identical firms which use capital to produce a single homogenous good. This good can be used both for consumption and investment. The production technology of each firm is operated by a single agent (the “insider”). Insiders, however, lack the capital needed to start a firm and maintain it as an ongoing concern. Financing is provided by a single representative investor who finances all firms in the economy.

An agency problem arises in this economy because the investor does not observe firm output but has to rely on the insider’s reports. The latter could potentially divert a part of the output by under-reporting firm performance. In response to this agency problem, each insider enters into a long-term contract with the representative investor. This contract, which is optimal given the agency problem, gives rise to endogenous borrowing constraints preventing firms from investing at the frictionless first-best level.

Aside from this friction, the model is an otherwise standard stochastic model of endogenous growth. Aggregate shocks in this economy consist of transitions between two states: a “crisis” state characterized by higher variance or lower mean of firm-specific productivity compared to the “normal” state. The model quantifies the effect of aggregate shocks on policies of the optimal contract and the endogenous changes in macro-economic quantities and risk-premium.

### 2.1 Production and Financing

Each firm uses a decreasing returns to scale production technology to produce output

$$y = A(s)k^\nu, \tag{1}$$

where  $k$  is the level of capital stock of the firm,  $0 < \nu < 1$  is the returns to scale parameter. Firms are exposed to both firm-specific and aggregate shocks. Each period, firms draw productivity  $A(s)$  from a two-point distribution which depends on the aggregate state of the economy denoted by  $s$ . The two possible realizations  $A(s) = \{A_1(s), A_2(s)\}$  are independently distributed across firms in the cross-section:

$$A(s) = \begin{cases} A_1(s) & \text{with probability } p \\ A_2(s) & \text{with probability } 1 - p \end{cases} \quad (2)$$

where  $A_1(s) < A_2(s)$ . Mean productivity  $\mu(s) = pA_1(s) + (1 - p)A_2(s)$  and the variance  $\sigma^2(s) = p(1 - p)(A_2(s) - A_1(s))^2$  depend on the aggregate state  $s = \{s_1, s_2\}$ . The aggregate states are persistent and follow a Markov-chain with transition function  $P$  represented by the  $2 \times 2$  matrix  $\Pi = [\pi_{i,j}]$ , where  $\pi_{i,j} = P(s_i, \{s_j\})$ . I denote the crisis state by  $s_1$  and normal times by  $s_2$ . For pure uncertainty shocks, therefore,  $\sigma_1 > \sigma_2$  and  $\mu_1 = \mu_2$ , while my results for first moment shocks use  $\mu_1 < \mu_2$ , but  $\sigma_1 = \sigma_2$ .

The agency friction in this economy arises because the investor does not observe firm output and has to rely on the insider's reports. The latter could steal by under-reporting output. Diversion is not fully efficient – for every unit diverted, the insider consumes a fraction  $\lambda$  where  $0 < \lambda < 1$ . In response to this agency friction, the investor and the insider enter into a long-term financial contract. The two parties commit to following the terms of the contract in every possible state with no possibility of renegotiation<sup>5</sup>. Since output is not observable, the contract is a function of the history of the insider's reports and past realizations of aggregate shocks. It specifies: (i) payments made by the investor to the insider, (ii) conditions under which the contract will be terminated, and (iii) investment decisions. All policies are conditioned on the aggregate state of the economy. I use the dynamic programming approach of Spear and Srivastava (1987) and Green (1987) to solve

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<sup>5</sup>While this is a restrictive assumption, it provides a useful benchmark and can be thought of as a limiting case where renegotiation is extremely costly as would be the case if the investors consist of a large dispersed pool of individuals.



for the optimal contract. In this recursive formulation, the present discounted value of the future promised payments to the insider (his continuation value) is a sufficient statistic for the entire past history of the insider's reports.

I assume the following attributes for the insider: he is risk-neutral, has limited liability, and has a constant outside option which I normalize to zero. The insider neither participates in financial markets nor accumulates savings. Since he cannot diversify firm-specific risk, his valuation of cash flows will be lower than the representative investor who is able to pool cash flows across all firms and diversify firm-specific risk. To capture this, I assume that the insider is more impatient than the investor, so that his time-preference parameter  $\beta_A$  is smaller than the investor's time-preference parameter  $\beta_P$ .

Figure 1 shows timing and is as follows. Each period has three sub-periods: morning, evening, and night. The aggregate state  $s = \{s_1, s_2\}$  is revealed in the morning and the contract is adjusted in a manner I describe below. The evening begins with a public lottery for contract termination. If a contract is terminated, the firm is liquidated with the investor recovering a fraction of the firm's installed capital. The insider receives his outside option. If the contract survives, production takes place. The insider reports output and pays the investor. The period ends at night when the investor determines the amount to be invested in the firm. All agents also consume during this sub-period. All policies and states carry the superscript  $M$ ,  $E$ , or  $N$ , and refer to morning, evening, or night respectively. I denote the present discounted value of payoffs received by the investor from each contract by  $F$  (with appropriate superscript for the sub-period and subscript for the period). The analogous quantity for the agent (his continuation value) is denoted by  $V$ . Since I use backward induction to solve for the contract, I begin by describing the investment decision which takes place at night.

## Night: The intra-period problem

In the morning the investor updates the insider's continuation value *after* observing the realization of the aggregate shock  $s = \{s_1, s_2\}$ . The investor chooses  $V_1^M$  if the aggregate shock  $s_1$  is realized or  $V_2^M$  if  $s_2$  is realized next period so as to maximize her continuation value

$$\begin{aligned}
 F_t^N(k_{t+1}, V_t^N, s^t) &= \max_{V_1^M, V_2^M} E\left[\frac{\pi(s^{t+1})}{\pi(s^t)} F_{t+1}^M(k_{t+1}, V^M(s'), s^{t+1}) \mid s^t\right] \\
 V_t^N &= E[\beta_A V^M \mid s^t] \\
 s^{t+1} &= \{s^t, s_{t+1}\}, \quad s_{t+1} = \Gamma(s) s_t \\
 \left\{V_1^M, V_2^M\right\} &\in \mathbb{R}_+^2.
 \end{aligned} \tag{3}$$

The first line in (3) calculates the investor's payoff at the end of period  $t$ . In this line,  $V^M(s')$  represents the two choice variables  $V_1^M$  and  $V_2^M$ . In calculating the present value of future cash flows, she uses her marginal utility process,  $\pi(s^t)$ , for discounting. I assume the investor to have constant relative risk-aversion  $\gamma_P$ , so her discount rate  $\pi_{t+1}/\pi_t = \beta_P (C_{t+1}^*/C_t^*)^{-\gamma}$ . Her equilibrium consumption  $C^*$  is determined by her consumption and savings decision. Since the investor holds a well-diversified portfolio of contracts, she can perfectly diversify the idiosyncratic component of cash flows from each contract. Therefore,  $C^*$  depends only on correlated, aggregate shocks. The second equation is the promise-keeping constraint and ensures that the insider's continuation value is the expected value of his future payments.

Contract policies  $V_1^M$  and  $V_2^M$ , for adjustment of continuation values following the realization of the aggregate shock in period  $t+1$ , depends on the entire history of realizations of aggregate shocks  $s^{t+1}$ . This is because contract policies for investment and the insider's payments are non-linear functions of his continuation value. This means that the cross-section cannot be aggregated to a representative firm. The investor's consumption  $C$  and hence her discount rate depends on the current cross-sectional distribution of capital stock and

continuation values. Starting from an initial cross-sectional distribution, different realized paths of the aggregate shocks lead to different cross-sectional distributions. In other words, the entire history of aggregate shocks is a state variable of this economy.

The optimal contract policies  $V_1^M$  and  $V_2^M$  which determine the law of motion of the cross-sectional distribution depends on the investor's discount rate. The latter, in turn depends on the law of motion of the distribution. Solving for optimal contract policies therefore, requires solving an infinite dimensional fixed point problem. I tackle this curse of dimensionality by assuming that agents in this economy keep track of a truncated history of past shocks. This turns out to be a good approximation for the parameters I use in the simulation experiments. I describe this approximate scheme in detail in Appendix B.

### **Evening: The investment decision**

Investment is observable and contractible and occurs at night. I denote the investor's continuation value at the end of evening by  $F^E$  and the value at the end of night by  $F^N$ . The corresponding quantities for the insider are  $V^E$  and  $V^N$ , respectively. The investor chooses the level of investment  $i_t = \iota k_t$  ( so that  $\iota$  is the investment rate) to maximize her continuation value

$$\begin{aligned}
 F_t^E(k_t, V_t^E, s^t) &= \max_{\iota} \left[ -\iota k - c(\iota)k_t + F_t^N(k_{t+1}, V_t^N, s^{t+1}) \right] \\
 k_{t+1} &= (1 - \delta)k_t + \iota k_t \\
 V_t^E &= V_t^N.
 \end{aligned} \tag{4}$$

From the first line we see that investment incurs an adjustment cost  $c(i_t)$  per unit of capital. I assume this cost to be quadratic in the investment rate  $c(\iota) = \theta \iota^2$ . The second line of the above equation is capital accumulation equation where  $\delta$  is the depreciation rate. The final equality simply states that the insider's continuation value remains unchanged over this sub-period since he does not receive any payments. From (4), we see that the investment

policy  $\iota$  depends on firm size, the insider's continuation value, and the aggregate state  $s^t$  which is the entire history of realizations of the aggregate shock up to and including the present shock  $s = (s_0, s_1, \dots, s_t)$ . I describe how I handle this curse of dimensionality in Appendix B.

### Morning: Production and payments

The evening begins with a public lottery for termination of the contract followed by production for those firms which survive this lottery. The agency problem arises in this period because the investor does not observe output. She designs the contract to provide sufficient incentives for truthful reporting by the insider. Her problem in this sub-period is to choose (i) the probability of termination of the contract  $\zeta_t$ , (ii) her payments  $d_1$  or  $d_2$  following insider reports of low or high output, respectively, and (iii) the updated continuation value promised to the insider which are  $V_1^E$  or  $V_2^E$  for reports of low or high output, respectively:

$$\begin{aligned}
F^M(k, V^M, s) &= \max_{\zeta, d_1, d_2, V_1^E, V_2^E} \zeta \chi k \\
&+ (1 - \zeta) \left[ p(d_1 + F^E(k, V_1^E, s)) + (1 - p)(d_2 + F^E(k, V_2^E, s)) \right] \\
V^M &= (1 - \zeta) \left[ p(A_1(s)k^\nu - d_1 + V_1^E) + (1 - p)(A_2(s)k_t^\nu - d_2 + V_2^E) \right] \\
A_2(s)k^\nu - d_2 + V_2^E &\geq A_1(s)k^\nu - d_1 + V_1^E + \lambda(A_2(s) - A_1(s))k^\nu, \\
d_1 &\leq A_1(s)k^\nu, \quad d_2 \leq A_2(s)k^\nu \\
\left\{ \zeta, V_1^E, V_2^E \right\} &\in [0, 1] \times \mathbb{R}_+^2.
\end{aligned} \tag{5}$$

In the above equation,  $F^M$  and  $V^M$  are the investor's and insider's continuation values, respectively at the end of morning and prior to the start of this sub-period. To reduce clutter, I have suppressed the dependence on time since all actions above occur completely within period  $t$ . The right-hand side of the first line of (5) lists the investor's payoffs. With probability  $\zeta$ , the contract is terminated. When this happens, the investor recovers a fraction

$0 < \chi < 1$  of the firm's capital. The insider receives his outside option of zero and exits the economy. If, however, the firm survives the lottery, production takes place. The insider pays the investor  $d_1$  (or  $d_2$ ) if he reports low (or high) output. The second equation in (5) is the promise-keeping constraint which ensures that the insider's continuation value is equal to the expected value of his future payments. In this line I have already assumed that the insider receives nothing if the contract is terminated. The third equation is the incentive compatibility constraint and ensures that the insider has no incentive to steal by mis-reporting. In this equation,  $\lambda$  represents the efficiency with which the insider can divert firm output. From this equation we see that the severity of the agency problem depends on the product of the parameter  $\lambda$  and the spread between high and low output realizations  $A_2 - A_1$ . Finally, the fourth pair of inequalities in (5) represent the insider's limited liability condition.

### **Firm entry**

Each period, there is a mass of potential insiders ready to enter the economy<sup>6</sup>. Since I am interested in studying the effect of the agency friction on the endogenous growth rate of the economy, I do not assume an exogenous productivity process. Instead, growth in my model is through the entry of new firms. This is similar to an AK model in which there is growth through accumulation of physical capital. To ensure balanced growth, I assume that the mass of potential entrants,  $H_t$ , is proportional to the total number of existing firms,  $N_t$ , in the morning of that period, i.e.  $H_t = hN_t$ . The constant of proportionality  $h$  is a measure of the investment opportunity set. For simplicity, I assume that  $h$  is constant and does not vary with macro-economic conditions.

Starting a firm requires insiders to pay a random, sunk-cost  $\tilde{e}k_0$ , where the size of an entering firm,  $k_0$ , is a parameter. Except for differences in this cost, the investment

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<sup>6</sup>I borrow this modeling technique from Gomes, Kogan, and Zhang (2003)

opportunity set of every insider is ex-ante identical. The random cost is drawn from a uniform distribution and is revealed to each potential insider in the morning of that period. Insiders however do not have any wealth to pay this cost and have to secure financing from the representative investor. I assume that the latter has all the bargaining power, so she chooses the insider's initial continuation value to maximize her continuation payoff

$$V_0(k_0, s^t) = \arg \max_{V \in \mathcal{R}_+} \{F(k_0, V, s^t)\}. \quad (6)$$

Firms which draw a cost  $\tilde{e}k_0 > V_0$  do not secure funding and expire worthless. The investor's valuation  $V_0$  depends on current macro-economic conditions through her discount rate. In crisis states, when the investor's discount rate is high,  $V_0$  is low, and fewer firms enter the economy as a result. If the insider is able to secure financing, he enters into a long-term contract with the investor. The contract pays for the initial installation cost, and in subsequent periods, conditional on continuation, the investor commits to providing capital for investment, and compensating the insider according to the terms of the contract.

## 2.2 The Representative Investor and Aggregation

There is a single representative investor who derives utility from consumption of the single good  $C_t$  and has standard time-separable power utility

$$E_0 \sum_{t=0}^{\infty} \beta_P^t \frac{C_t^{1-\gamma_P}}{1-\gamma_P}, \quad (7)$$

where  $\gamma_P$  is the investor's constant relative risk-aversion, and  $\beta_P$  is her time preference parameter. She has access to complete financial markets and derives income from accumulated wealth  $W_t$ . She makes consumption and investment decisions to maximize expected lifetime utility subject to her budget constraint

$$E_t \left[ \sum_{s=0}^{\infty} \pi_{t,t+s} C_{t+s} \right] \leq W_t. \quad (8)$$

The investor saves by investing in existing firms and financing new ones. Her optimal consumption-savings decision implies the standard expression for her discount rate process

$$\pi_{t,t+s} = \beta_P \left( \frac{C_{t+s}}{C_t} \right)^{-\gamma_P}. \quad (9)$$

I prove this in Appendix A.

Each period, her net cash flow aggregated over all contracts,  $D_t$ , is total output produced by all existing firms net of investment and payments to insiders

$$D_t = Y_t - I_t - \mathcal{T}_t. \quad (10)$$

The first term on the right-hand side of Equation(10) is total output  $Y_t = \sum_j y_t^j$  where the output of an individual firm is given by Equation(1) and the sum is over firms that survived the lottery held in the morning of period  $t$ . The second term accounts for total investment made by the investor in this period. This has three components. First, total investment in existing firms costs  $I_t^e = \sum_j \theta \iota_j^2 k_j + \iota_j k_j$ . Second, investment along the extensive margin from new firm entry costs a total of

$$I_t^n = \int_0^{\bar{e}} dH = \frac{h}{2} \bar{e}^2 N_t \quad (11)$$

where  $N_t$  is the number of existing firms at the beginning of period  $t$  and  $\bar{e} = V_0/k_0$  is the maximum installation cost which gets financed. Third, disinvestment from firm liquidation amounts to  $I_t^l = \sum_{j \in \text{Liq}} \chi k_{j,t}$ . Total investment is the sum of these three components  $I_t = I_t^e - I_t^l + I_t^n$ . Finally, the last term in (10) aggregates total payments to existing firm insiders  $\mathcal{T}_t = \sum_j y_t^j - d_t^j$ , where  $d_t^j$  is the payment made to insider  $j$ . As discussed in Equation(5), the latter can be either  $d_1$  or  $d_2$  based on the insider's report of low or high output.

In equilibrium, the investor's consumption

$$C_t = D_t \tag{12}$$

This is the goods market clearing condition. I prove this in Appendix A.

### 2.3 General Equilibrium

Given a history of aggregate shocks  $s^t = (s_0, s_1, \dots, s_t)$ , the optimal contract policy of a firm at time  $t$  are functions of the firm's installed capital  $k_t$  and beginning of sub-period continuation values ( $V_t^M$ ,  $V_t^E$ , or  $V_t^N$  for morning, evening, or night, respectively). These policies include the investment rate, the liquidation probability, adjustments made to the insider's continuation values, and payments made to the insider for low (or high) output reports. For each firm-insider with capital  $k_t$  and morning continuation value  $V_t^M$ , his capital stock  $k_{t+1}$  and continuation value in the morning of next period  $V_{t+1}^M$  (conditional on survival of the morning lottery) are completely determined by the realization of  $A_t$  and the aggregate shock  $s_{t+1}$  at the beginning of the period.

I denote the joint distribution of capital stock and continuation values in the cross-section by  $\Psi_t = \psi(k, V^M, s^t)$ . The optimal contract policies for investment and adjustment of continuation values determine the transition probability for an individual insider's states  $(k, V^M)$  for each realization of firm-specific shock and aggregate shock  $s^{t+1}$ . The law of large numbers ensures that we do not need to keep track of individual realizations of the firm-specific shock, but rather, optimal contract policies determine the distribution  $\Psi_{t+1}$ , given the period  $t$  distribution  $\Psi_t$  and  $s^{t+1}$ . I can now define the equilibrium.

**Definition 1** *A recursive competitive equilibrium is defined as a sequence of contract policies for existing firms, the initial capital stock  $k_0$  and initial continuation value  $V_0$  with which new contracts are initiated, the consumption policies of the representative household  $C_t$ , and a*



sequence of distributions for capital and continuation values  $\{\Psi_t\}$  such that, given the initial distribution  $\Psi_0$ :

(i) individual contracts are optimal,

(ii) the initial continuation value of an insider is determined by (Eq. 6),

(iii) the representative investor's policies are optimal according to Eq. 4, Eq. 5, Eq. 3 and subject to her budget constraint, Eq. 8

(iv) the goods market clears  $C_t = D_t$

(v) the market for contracts clears

(vi) contract policies correctly anticipate the law of motion of the future evolution of the distribution  $\Psi_{t+1}$ .

A sketch of the numerical approach used to compute both the steady-state equilibrium and the transition dynamics is described in Appendix B.

### 3 Solution Features

In this section I first discuss policies of the dynamic contract and how they provide incentives to the firm insider. Next, I analyze how contract policies change following an uncertainty shock (increase in the volatility of firm-specific productivity). This partial equilibrium analysis will provide intuition for the general equilibrium results discussed in Section 4.

The investor uses: (a) direct payments, (b) promises of future payments, (c) the threat of liquidation, and (d) higher levels of investment in the firm to provide incentives to the firm insider. These policies depend on the size of the firm, the firm insider's continuation value, and on macro-economic conditions since the latter determines the investor's marginal utility process and therefore her discount rate. For ease of exposition, in the following, in discussing policies, I fix firm-size (median size) and the state of the economy (normal state). Figure 2 plots these policies as a function of the insider's continuation value.

Panel A of Figure 2 shows that at sufficiently high continuation values, the insider receives direct transfers from the investor. At such high continuation values, the agency problem is mitigated because the insider is promised a large portion of future firm output. Since the insider is more impatient than the outside investor, it becomes too costly to delay his payments in this region. He is paid more when he reports high output (shown by the solid, blue line) compared to a low-output report (shown by the dot-dash, red line). The spread between these payments is sufficient to discourage the insider from mis-reporting firm output and stealing.

At lower continuation values, it is sub-optimal to provide incentives by direct payments to the insider. Instead, as shown in Panel B of Figure 2, incentives are provided through future promises. A report of high output results in an increase in his continuation value (solid, blue line), while a low output leads to a decrease (shown by the dot-dash, red line). Once again, the spread provides sufficient incentive to the insider to truthfully report firm output and share it with the investor.

At very low continuation values, as shown in Panel C of Figure 2, the insider's limited liability makes it becomes impossible for the investor to provide the spread in continuation values necessary to discourage the insider from diverting firm output. In this region, incentives are provided through the threat of firm liquidation. A firm with very low continuation value has a non-zero liquidation probability which increases to one as the continuation value drops to zero. Since the continuation value measures how far the firm is from the liquidation region, it can be interpreted as financial slack of the firm. Indeed, implementations of the optimal contract using realistic securities, use measures of financial slack such as unused lines of credit (see DeMarzo and Fishman (2007b)) or cash reserves (Biais et al. (2007)) of a firm to capture the insider's continuation value<sup>7</sup>. Note that although liquidations are ex-post inefficient, they

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<sup>7</sup>The only instance for which I rely on an implementation of the contract is in choosing the value of the product  $\lambda(A_2 - A_1)$ . Since my results do not depend on the particular form of implementation, I do not discuss this.

are necessary to provide incentives.

Finally, incentives are also provided by the investment policy. This is shown in Panel D of Figure 2. The investment rate is, for most of the state space, an increasing function of continuation value<sup>8</sup>. This is because a firm-insider with a higher continuation value has his incentives better aligned with the investor. This relaxes the agency problem and the investment rate is therefore increased to a value closer to the unconstrained, first-best rate.

For a fixed aggregate state, the contract policies discussed above is similar to DeMarzo and Fishman (2007a). Since the focus of this paper is on the effect of an aggregate uncertainty shock, in the remainder of this section, I discuss how contract policies change following the shock. This will provide intuition for the following section in which I focus on the aggregate consequences of the uncertainty shock.

Panel A of Figure 3 shows the cross-sectional distribution of financial slack (continuation value) in the stochastic steady-state. The solid blue line shows this distribution for a firm whose size is at the 50-th percentile of the firm-size distribution in the stochastic steady-state. The dot-dash red line shows the same for a firm in the 25-th percentile. Comparing these two lines we see that smaller firms have a higher mean and higher cross-sectional variance of financial slack per unit of capital than larger firms.

Next, I analyze how the distribution of financial slack changes following an aggregate uncertainty shock when the economy transitions to the “crisis” state<sup>9</sup>. An increase in firm-specific uncertainty makes the agency problem more severe. To discourage the insider from diverting firm-output, the investor now needs to provide a bigger spread in continuation values between high and low output. This causes the cross-sectional distribution of financial slack to spread out. Panel B of Figure 3 shows the steady increase in the standard deviation

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<sup>8</sup>This does not hold for continuation values very close to the liquidation region. In this region, with liquidation highly likely, the investor increases investment since her downside is limited to the liquidation value of the firm’s capital while investing increases the upside.

<sup>9</sup>Note that although the shock increases the volatility of firm-specific productivity, the shock is aggregate in nature because all firms experience a correlated increase in idiosyncratic uncertainty.

of the cross-sectional distribution of financial slack through the duration of the crisis, rising by about 12% over 6 quarters. This means that as successive realizations of the crisis state makes the left-tail of the distribution of financial slack heavier. This has the implication that, a bigger fraction of firms will decrease their investment rate because they are now more constrained. To show this, I focus on firms in the 50-th percentile of the firm-size distribution and examine the fraction of these firms that significantly decrease their investment rate because of depleted financial slack <sup>10</sup>. Panels C and D of Figure 3 shows results for two different measures of the severity of the drop in the investment rate. Panel C reports the fraction of firms which are so severely constrained that their investment rate is zero or negative, while Panel D reports the fraction of firms that are investing at less than 75% of the investment rate of the unconstrained firms. Panel C shows that the fraction of firms which are either not investing or dis-investing increases to more than 4.5%. This is about 1.2% higher than in the stochastic steady-state. Panel D shows that if we focus on firms that are investing at three-quarters the investment rate of unconstrained firms, then this fraction rises to about 11.4% compared to 8.7% in the steady-state. Both Panels C and D show that recovery from this slump in investment by medium-sized firms is slow. It takes a while for this large fraction of constrained firms to diffuse out of this region. Even 12 quarters after the crisis has ended, there are still about 4% of medium sized firms who are either not investing or disinvesting. The consequence of this is much lower accumulation of capital stock of the economy during and after the crisis. This prolongs recovery by a considerable amount.

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<sup>10</sup>I focus on these firms because they make the largest investments. Smaller firms have a higher investment rate, but since they are smaller in size, they invest less than mid-size firms. Large firms, on the other hand, have a lower, or even negative, investment rate.

## 4 Quantitative Analysis

### 4.1 Parameters and Calibration

I calibrate the model at quarterly frequency. I set some of the parameters to standard values used in the literature and calibrate the rest by targeting cross-sectional moments of investment rates and cash-ratios, and the time-series means of aggregate quantities and asset prices. I report all parameters in Table 1 and a comparison of simulation moments along with their data counterparts in Table 2.

I set the decreasing returns-to-scale parameter  $\nu = 0.6$  following Cooper and Haltiwanger (2006). The model implied relative firm size as measured by the inter-quintile range of the size distribution normalized by median firm-size is 0.83. This matches the corresponding quantity estimated from the cross-section of firms covered by COMPUSTAT. I set the depreciation parameter  $\delta = 1.75\%$  to correspond to an annual depreciation rate of 7%. This is in line with commonly used values (for example, Cooper and Haltiwanger (2006) use an annualized depreciation rate of 6.9%). I choose the quadratic adjustment cost parameter  $\theta = 5$ , the mean of the cash-flow process  $E(A) = 0.87\%$  and the product  $\lambda(A_2 - A_1) = 0.3\%$  to target the cross-sectional mean and standard-deviation of investment rates, and the mean cash-ratio in the cross-section. The reason for the last choice is as follows. Since all firms have the same value of the product  $\lambda(A_2 - A_1)$ , this value has a first-order effect on the mean of the cross-sectional distribution of slack. Following the implementation of Biais et al. (2007), who shows that the level of cash is equivalent to the firm's financial slack, I set choose  $\lambda(A_2 - A_1)$  to target the mean cash-ratio in the cross-section. The model implied mean cash ratio of 0.26 is close to 0.23, which is the mean cash ratio of firms in 2006 documented by Bates, Kahle, and Stulz (2009)<sup>11</sup>. Although not targets, the model implied standard deviation of the cross-sectional cash ratio is close to the data counterpart. Likewise, the model implied

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<sup>11</sup>The average cash ratio of COMPUSTAT firms over the entire sample period starting from 1980 is 0.155

standard deviation of the ratio of cash flow to firm assets is also close to the data. I choose the recovery rate upon liquidation  $\chi = 0.85$ . The size of a new firm is set to be 10% the median firm size.

I choose the investor's risk-aversion to be  $\gamma = 4$  and determine her time-preference parameter to be  $\beta_P = 0.992$  by targeting the average risk-free rate in the US<sup>12</sup>. I choose the insider's time preference parameter  $\beta_A = 0.96$  to be lower than the aggregate investor. This is justified by studies such as Silber (1991) and Longstaff (1995) who document that insiders, on average, have a lower valuation of their firm than outsider investors presumably because the former cannot diversify firm-specific risk. These studies estimate the discount to be in the range of 35–40%. I choose a conservative value of 15% for this discount which determines the value of  $\beta_A$ .

## 4.2 Aggregate Effects of an Uncertainty Shock

In this section I quantify the dynamics of aggregate output, investment, consumption, and risk-premia upon impact of the uncertainty shock. In the high uncertainty state, all firms experience an increase in the volatility of firm-specific productivity compared to the low uncertainty state. In particular, in the high uncertainty state, the difference  $A_2 - A_1$  increases, while the mean and the probability  $p$  of realization of low output remains unchanged. I choose the magnitude of increase in firm specific volatility to match the maximum drop in the level of output during the financial crisis of 2007–2009. I find that this requires firm-specific volatility in the high uncertainty state to be 3.5 times that in the low uncertainty state. This is close to the value of 3.1 estimated by Bloom et al. (2012).

The transition matrix for switches between aggregate states,  $\Pi$  are chosen to correspond to the frequency of occurrence of financial crises. I denote the crisis state by  $s_1$  and the normal state by  $s_2$ . The transition probability  $\pi_{1,2} = 0.5$  which means that, on average,

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<sup>12</sup>Results for alternate values of risk-aversion are available upon request.

financial crises last for 2 quarters. The transition probability to switch from a normal state to a crisis state is much lower:  $\pi_{2,1} = 0.01$ .

I report results for a path in which the economy remains in the high uncertainty state for 6 quarters. This matches the duration of the financial crisis. In my simulations, the timing is as follows. The economy starts out in the low uncertainty state at  $t = 1$ . Next period, it transitions to the crisis state which lasts from  $t = 2$  through  $t = 7$ . The economy transitions back to the low uncertainty state in period  $t = 8$ . Details of the numerical implementation is provided in the Appendix.

### **Recession and Slow Recovery**

Figure 4 shows the response of aggregate quantities upon impact of the shock and also during the recovery when the economy transitions back to the low uncertainty state. The solid blue line in Panel A of Figure 4 shows output dropping by 4% at the trough at  $t = 7$ . In the data, the drop over this period was 4.3% as reported by the BEA GDP Tables. Even though the economy transitions back to the low uncertainty state in  $t = 8$ , the model-implied output recovers slowly. It takes output 8 quarters to recover to pre-crisis levels, which is close to the 9 quarters it took for US GDP to recover. There are two reasons for this slow recovery in the model. The first reason is depletion of capital stock due to the drop in investment of existing firms and also from firm exits. The red dot-dash line in Panel A of Figure 4 shows capital stock to be 1.5% lower at the trough compared to pre-crisis levels. This is close to 2% drop of private capital reported by the BEA Fixed Assets Tables. The second reason is due to a change in the size-distribution of firms in the cross section. This effect, which would be missed by a model with a representative firm, is in fact responsible for slightly more than half of the drop in output and even more responsible for the slow recovery than depletion of capital. Let me discuss the reason for each of the above two reasons in detail.

Most of the drop in capital stock comes from medium-sized firms. This is shown in

Figure 5. Panel A shows that immediately after the economy transitions back to the low uncertainty state in  $t = 8$ , the amount of capital stock held by medium sized firms is 2% less than pre-crisis levels. The reason for this depletion of capital stock is a significant increase in the fraction of constrained firms. These firms had their financial slack depleted during the crisis. In Section 3 above, I highlighted that this caused such firms to invest at a lower rate compared to the stochastic steady-state, with some of them even dis-investing. As a result they were unable to replace the depreciated capital stock. In fact, the heavy left-tail of the cross-sectional distribution of financial slack significantly reduces the speed of recovery. As Panel B of Figure 5 shows, even a year and a half after the end of the crisis, 1% of medium-sized firms still have lower capital stock than they had at the onset of the crisis.

The second reason for the weak recovery is a change in the size distribution of firms during such prolonged crisis. The crisis results in an increase in the number of very large firms and a decrease in the number of medium-sized and small firms. With a decreasing returns to scale technology, this results in a drop in average productivity of about 2.5% at the trough of the cycle. It takes more than 20 quarters in my simulation for the average productivity to recover, and as a result the recovery is weaker.

Panel B shows that the drop in aggregate investment in the model is 20%. This is slightly lower than the 23% drop in private investment reported by the BEA GDP Tables. Panel C shows that consumption increases by about 2.5% immediately upon impact of the shock and drops to about 1.8% at the trough. In the data, consumption increased by 0.6% before dropping by 2.8%. Finally, Panel D shows the increase in firm exit rate. This higher rate of exits is due to the heavier left-tail of the financial slack distribution. In the model, a total of 8% of firms exit the economy over the six quarters of the recession. This is lower than the 15% increase in the exit rate of firms reported by Siemer (2012) using Business Dynamics Statistics data that the 2007-2009 recession.



## **Increase in Risk Premium and Lower Firm Entry**

Risk premium endogenously increases during the crisis because of an increase in precautionary savings motive of the representative investor. Output drops as the crisis progresses and the investor wants to disinvest. But because of partial irreversibility of capital, capital cannot absorb the drop and the investor's consumption drops. The resulting increase in the volatility of consumption growth leads to an increase in the investor's discount rate which lowers the value of new investment. Panel A of Figure 6 shows that the value of new investment drops with each successive realization of the uncertainty shock, falling by 7% at the trough. Panel B shows the percentage drop in new investment expenditure. This drop is twice as large as in panel A because the investment expense for new firms consists of the fixed cost of starting a firm which from Equation(11) is proportional to the square of the drop in the value of starting a new firm.

## **Disproportionate Impact on Young Firms**

Young firms are affected the most from during the crisis with increased uncertainty. Figure 7 shows the differential impact of the uncertainty shock for young and mature firms. Both plots show the effect immediately upon arrival of the uncertainty shock.

Panel A of Figure 7 plots the difference in the average investment rate in the crisis state against that in the stochastic steady-state as a function of firm age. From the figure, we see that firms, irrespective of age, cut back investment. It is young firms, however, who cut back the most. For instance, the average 5 year old firm reduces its investment rate by about 9.7% (annualized). The reason that mature firms do not reduce their investment as much is because the distribution of continuation values of these firms has a thinner left-tail and higher mean.

Panel B of Figure 7 plots the change in the dispersion of investment rate within a given cohort. We see that the dispersion of investment rate increases most for young firms. Growth

becomes a lot riskier for such firms as a result of the uncertainty shock.

### 4.3 A Negative First Moment Shock

I conclude this section by comparing the response of aggregate quantities to a drop in aggregate productivity. In this experiment, the economy can be in one of two states. In normal times, average productivity of all firms is high. If, however, the economy is in the “crisis” state, the average productivity of all firms drops by a fixed amount from the prior period. I choose the magnitude of this decline to be 1%, so that the drop in aggregate output following 6 successive realizations of this low aggregate shock results in a 4.3% drop in aggregate output from peak to trough.

Figure 8 shows a key difference in the response of the economy to a negative first moment shock in comparison to an uncertainty shock. Panel A shows that although output drops by 4% from its peak at the height of the crisis, the economy recovers very quickly once it transitions to the higher productivity state. In other words, there is no slow recovery and output surpasses its pre-crisis level immediately when the economy transitions out of the low productivity aggregate state. The reason for this is that neither of the two reasons for the delayed recovery following an uncertainty shock are present here. First, as shown in Panel B of Figure 8 capital stock does not drop. Second, average productivity also does not drop following this shock. In fact, in simulations, I find the length of the recovery to be fairly independent of the duration and severity of the crisis<sup>13</sup>.

## 5 Conclusion

In this paper I analyze the effect of a systematic increase in the dispersion of firm-level productivity in the presence of an agency friction, in a general equilibrium framework. I find that under reasonable calibrations, successive realizations of such uncertainty shocks can lead

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<sup>13</sup>Results available upon request.

to large drops in aggregate quantities. This deep recession is followed by a slow recovery. The recovery time for aggregate output is similar to what the US economy experienced following the Great Recession of 2007–2009. In contrast to such aggregate uncertainty shocks, shocks which lower average productivity lead to fast recoveries. These results do not depend on an assumed form of borrowing constraint faced by firms. The long-term contract between the firm insider and the investor is optimal given the agency problem.

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# Appendix

## A: Proof of Goods Market Clearing condition and Discount rate

The representative household takes the contract prices  $F_t^N$  as given. The household's Bellman equation is

$$U_t(\vec{b}) = \max_{C_t \geq 0, \vec{b}'} [u(C_t) + E_t[\beta_P U_{t+1}(\vec{b}')] ], \quad (\text{A-1})$$

subject to the budget constraint

$$\begin{aligned} \sum_{i \in \text{Continue}} b_{i,t} (F_{i,t}^e + y_{i,t} - \tau_{i,t} - \iota_{1,i,t} k_{j,t} - \theta \iota_{1,i,t}^2 k_{i,t} - \iota_{2,i,t} k_{i,t} - \theta \iota_{2,i,t}^2 k_{i,t}) + \sum_{j \in \text{Exit}} \chi k_{j,t} \\ - \sum_{i \in \text{Entry}} \tilde{e}_i k_{i,t} = C_t + \sum_{i \in \text{Continue}} b_{i,t+1} F_{i,t}^N, \end{aligned} \quad (\text{A-2})$$

where  $\vec{b}$  is the household's holding of the contracts with individual firms and  $F_{i,t}^N$  is the ex-dividend price of contract  $i$  at the end of period  $t$ .  $\tau_t^i$  is the payment made to the insider in firm  $i$  in period  $t$ ,  $\iota_{1,i,t}$  is the investment rate of firm  $i$  in period  $t$ , and  $\iota_{2,i,t}$  is the dis-investment rate of firm  $i$  in period  $t$ . The household consumes  $C_t$  in period  $t$ . The summation over "continue" refers to firms which remain in operation after the lottery for firm liquidation. It excludes new entrants. The summation over "exits" refers to firms which are liquidated as a result of the liquidation lottery. A fraction  $\chi$  of the physical assets  $k_{j,t}$  of the liquidated firm is recovered.

**Market Clearing:** In equilibrium the market for financial contracts clears

$$b_{i,t} = b_{i,t+1} = 1, \quad (\text{A-3})$$

for continuing firms  $i$ . Substituting this market clearing condition into the budget constraint Eq. A-2 results in the contract price  $F_{i,t}^N$  dropping out. The representative household's consumption

$$C_t = Y_t - I_t - \mathcal{T}_t, \quad (\text{A-4})$$

where

$$\begin{aligned} Y_t &= \sum_{i \in \text{Continue}} y_{i,t} \\ I_t &= \sum_{i \in \text{Continue}} (\iota_{1,i,t} k_{j,t} - \theta \iota_{1,i,t}^2 k_{i,t} - \iota_{2,i,t} k_{i,t} - \theta \iota_{2,i,t}^2 k_{i,t}) - \sum_{j \in \text{Exit}} \chi k_{j,t} + \sum_{i \in \text{Entry}} \tilde{e}_i k_{i,t} \\ \mathcal{T}_t &= \sum_{i \in \text{Continue}} \tau_{i,t}, \end{aligned}$$

which is Eq. 10 after setting total investment in existing firms  $I_t^e = \sum_{i \in \text{Continue}} (\iota_{1,i,t} k_{j,t} - \theta \iota_{1,i,t}^2 k_{i,t} - \iota_{2,i,t} k_{i,t} - \theta \iota_{2,i,t}^2 k_{j,t})$ , total dis-investment from liquidated firms  $I_t^l = \sum_{j \in \text{Exit}} \chi k_{j,t}$ , and total investment in new firms  $I_t^n = \sum_{i \in \text{Entry}} \tilde{e}_i k_{i,t}$ . From Eq. 10 and Eq. A-4 above, we have

$$C_t = D_t, \quad (\text{A-5})$$

which is the goods-market clearing condition.

**Optimality condition:** The ex-dividend price of the financial contract is determined by the household's first-order condition

$$U'(C_t) F_{i,t}^N = \beta_P E[U'(C_{t+1})(d_{i,t+1} + F_{i,t+1}^N)],$$

where  $d_{i,t}$  is the investor's dividend from financial contract  $i$  in period  $t$  and equals output of firm  $i$ , net of the insider's payment and investment expense:  $d_{i,t} = y_{i,t} - \tau_{i,t} - \iota_{1,i,t} k_{i,t} - \theta \iota_{1,i,t}^2 k_{i,t} - \iota_{2,i,t} k_{i,t} - \theta \iota_{2,i,t}^2 k_{i,t}$ .

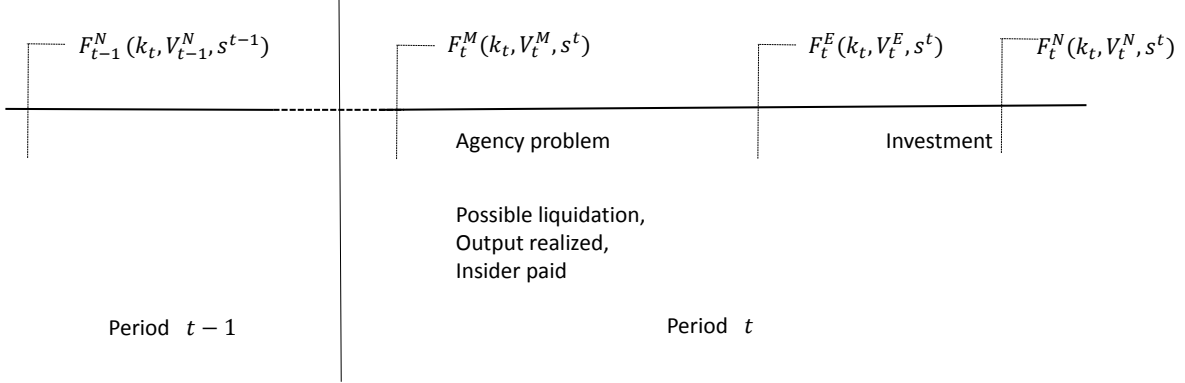
## B: Sketch of the Numerical Solution Approach

Contract policies in this economy depend on the entire past history of aggregate shocks  $s^t = (s_0, s_1, \dots, s_t)$ . However, agents in this economy tackle the curse of dimensionality by behaving as if only the past  $n$  shocks matter. In my simulations, I use  $n = 12$ . I solve for contract policies iteratively in the following way. I initialize the investor's discount rate process to be a constant equal to her time-preference parameter. For the first iteration, I use this discount rate process and solve for the optimal contract assuming no transitions between aggregate states. In subsequent iterations, however, I use the actual Markov transition matrix  $\Gamma$  when solving for the optimal contract. This generates two sets of policies, one for each aggregate state.

1. Starting with the cross-sectional distribution of capital and continuation values corresponding to the aggregate state  $s = 2$ , which is the state in which the economy spends most time, I use the stored contract policies to compute the implied path of the investor's consumption for each of the  $2^n$  path of aggregate shocks.
2. Next, I compute the investor's marginal utility process  $\pi(s^{t+1})/\pi(s^t) = \beta_P \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma}$  for along each path.
3. If the realized marginal utility process is within a pre-specified tolerance of the process assumed in solving for contract policies for each of the  $2^n$  paths, then I stop. Otherwise, I use a weighted average of the realized discount rate process in this iteration and the previous one as the updated process for discount rate. This weighting scheme allows for smooth convergence.



4. I recompute contract policies.



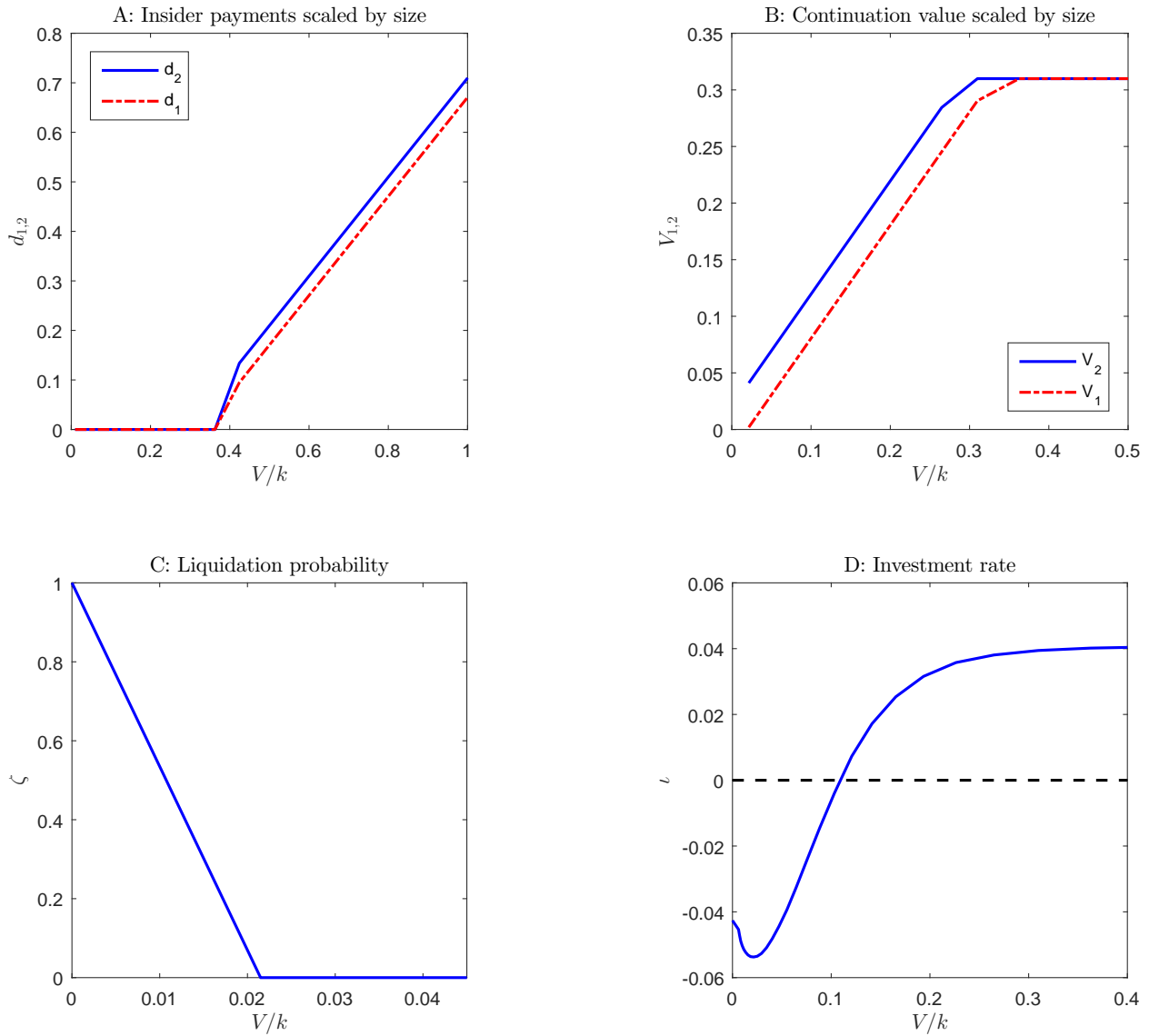
**Figure 1:** Intra-period time-line of the model.  $F^M$ ,  $F^E$ , and  $F^N$  represent the investor’s valuation of the contract in the morning, evening, and night, respectively. The insiders continuation values in each of these sub-periods are  $V^M$ ,  $V^E$ , and  $V^N$ , respectively.

**Table 1:** Parameter values: The model is calibrated at quarterly frequency.  $\nu$  is the decreasing returns-to-scale parameter.  $\theta$  is the quadratic investment cost. Capital depreciates at rate  $\delta$ . The investor recovers a fraction  $\chi$  of a liquidated firm’s capital stock.  $k_0$  is the ratio of the size of a new firm to the size of a median firm. The investor has time preference parameter of  $\beta_P$  and a risk aversion  $\gamma$ . The insider has a time preference parameter  $\beta_A$ . The insider can divert firm output with an efficiency  $\lambda$ . Model predictions depend on the product  $\lambda(A_2 - A_1)$ ; the value below is in the “normal” state. The value in the high uncertainty “crisis” state is 3.5 times higher. The remaining parameters of the cash flow process are  $p$ , which is the probability of realizing a low cash flow  $A_1$ , and the mean  $E[A]$ .

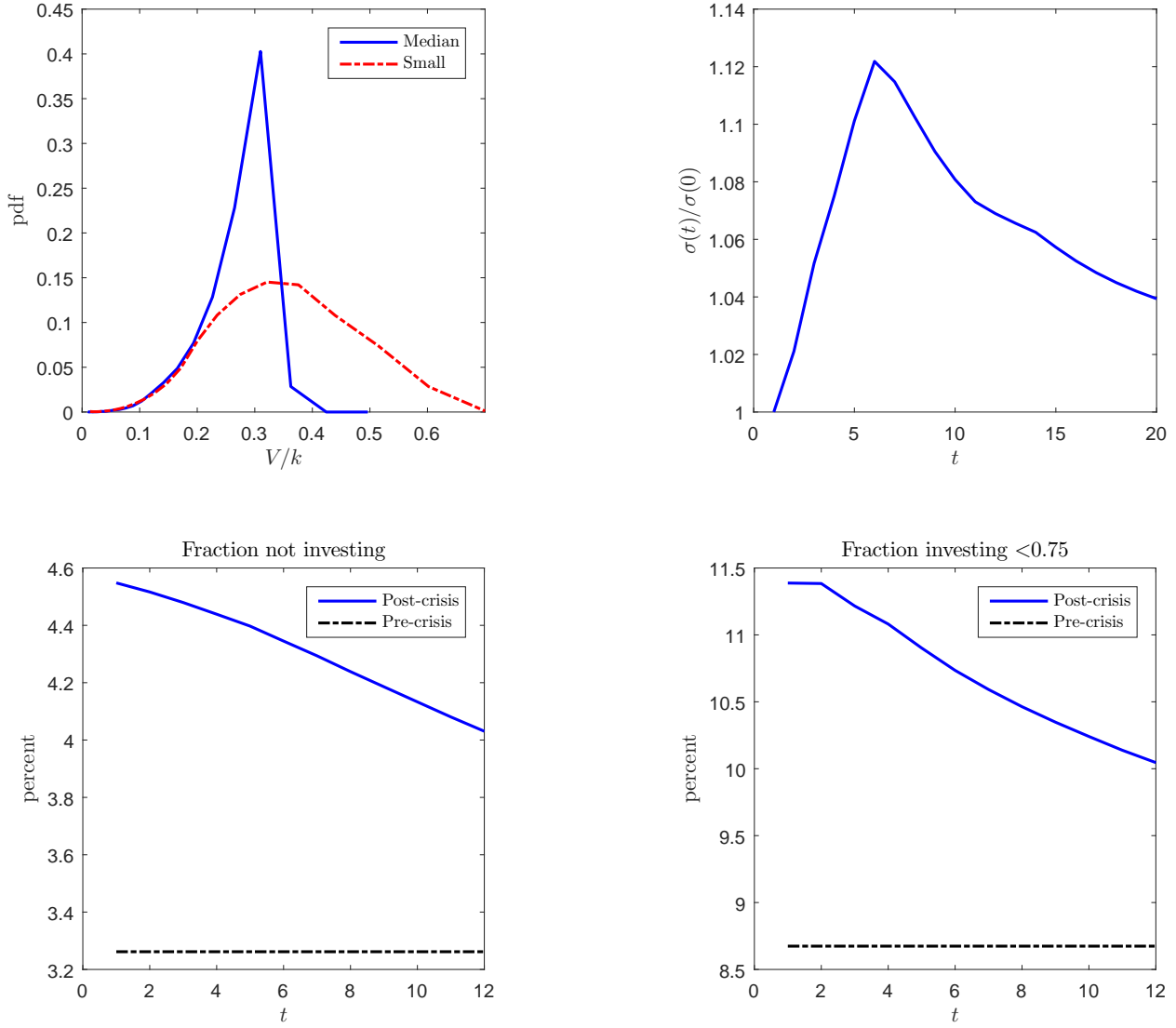
$\nu$	$\theta$	$\delta$	$\chi$	$k_0$	$\beta_P$	$\gamma$	$\beta_A$	$\lambda(A_2 - A_1)$	$p$	$E(A)$
0.6	5	0.0175	0.85	0.10	0.992	4	0.96	0.003	0.5	0.0087

**Table 2:** Calibration Moments: This table compares sample moments in the data (annualized) to those generated by simulated data using parameters in Table 1. Consumption growth and the risk-free rate in the Data column is from Campbell, Lo, and MacKinlay (Campbell et al.). The mean cash ratio is from Bates et al. (2009). The average investment rate, relative firm size, and statistics for Tobin’s Q are estimated using COMPUSTAT data. I report time series averages of the mean and inter-quintile range (IQR).

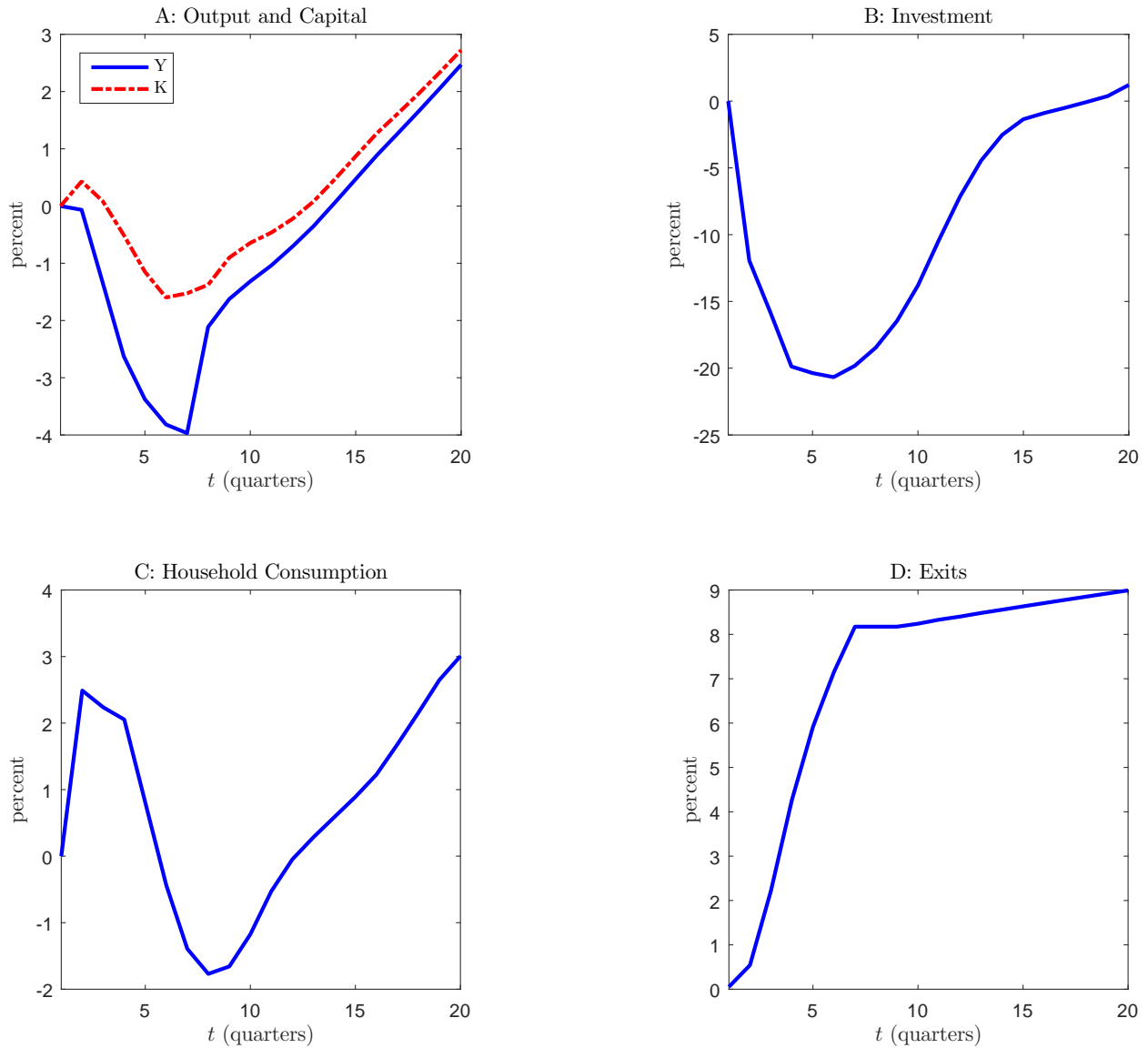
Moment	Data	Model
Aggregate moments		
$C/Y$	0.68	0.71
$E[C_{t+1}/C_t]$	0.017	0.017
$E[r^f]$	0.018	0.018
Cross-sectional moments		
Investment rate (mean)	0.12	0.14
Investment rate (IQR)	0.16	0.24
Relative firm size (IQR)	0.83	0.83
Cash flow/Capital (IQR)	0.23	0.19
Cash ratio (mean)	0.23	0.26
Cash ratio (IQR)	0.22	0.22



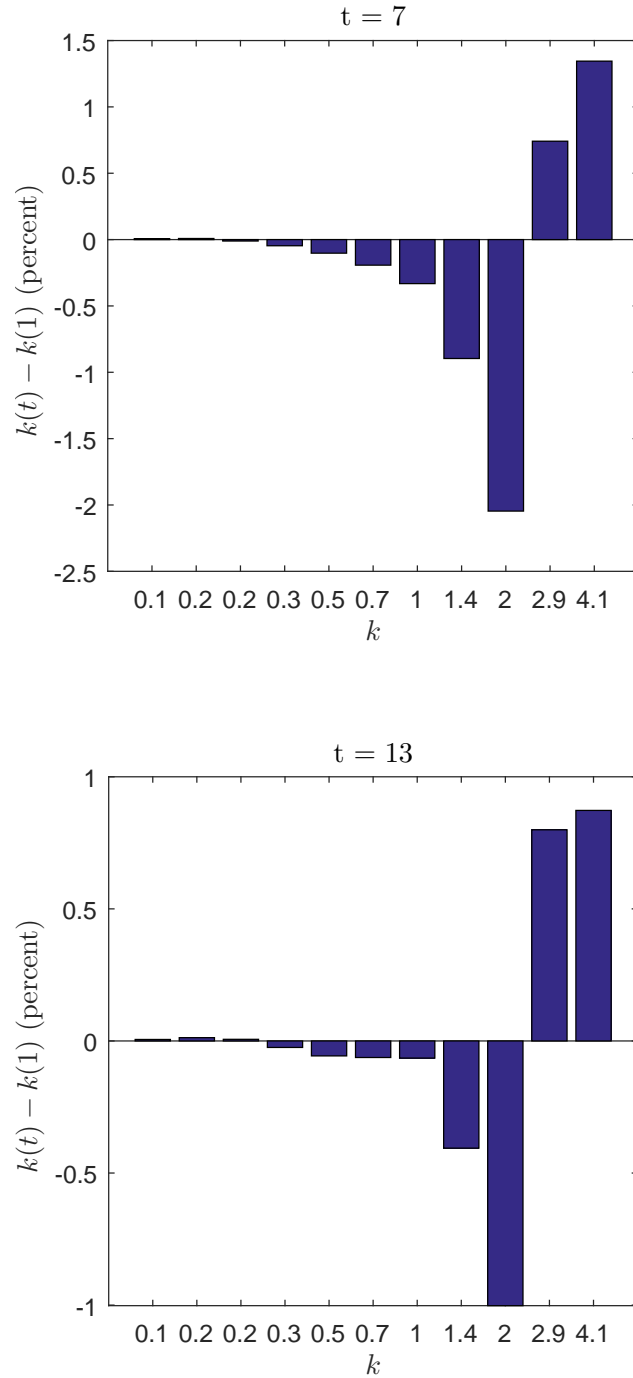
**Figure 2:** Optimal policies of the dynamic contract for fixed firm size and macro-economic state: Panel A shows the insider's payments  $d_1$  and  $d_2$ , as a function of his continuation value following low and high output, respectively. The continuation values in all panels are scaled by the firm's capital stock. Panel B shows the adjustment to his continuation values  $V_1$  and  $V_2$ , for low and high output, respectively. The solid blue line is for high output while the dot-dash red line is for low output reports in both Panels A and B. Panels C and D shows the liquidation probability  $\zeta$  and the investment rate  $\iota$  as a function of the insider's continuation value, respectively.



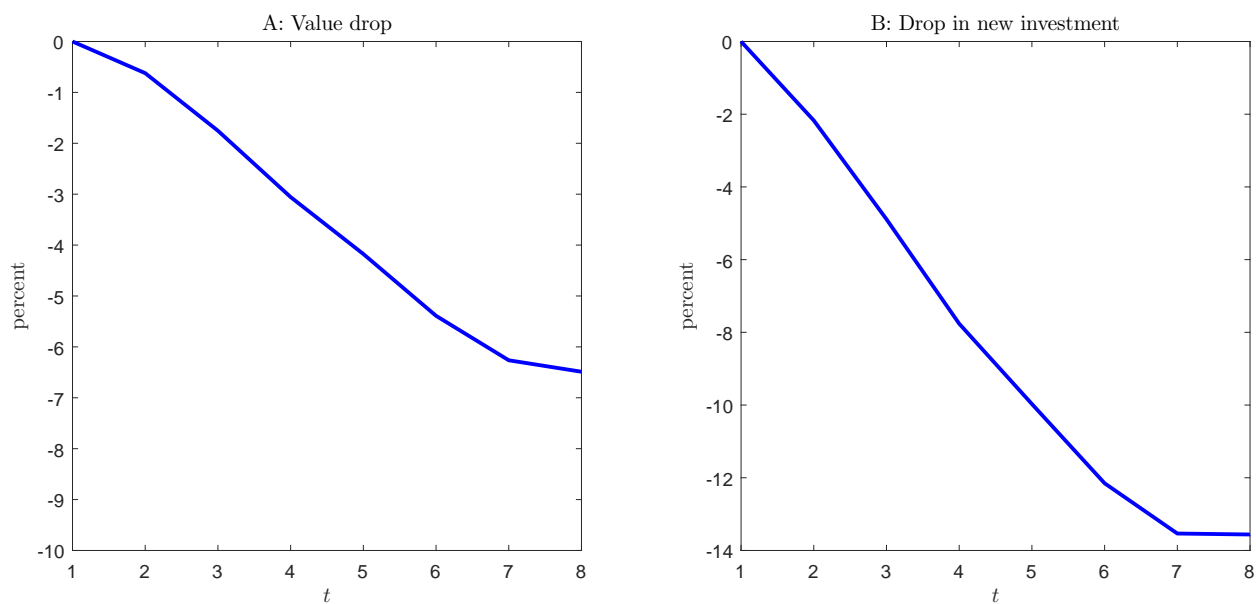
**Figure 3:** Effect of increase in volatility of firm-specific productivity: Panel A shows the distribution of financial slack (continuation values scaled by firm-size) in the stochastic steady-state. The solid blue line shows the distribution of financial slack (continuation values) for a firm whose size is at the 50-th percentile in the stochastic steady-state. The dot-dash red line shows the same for a firm in the 25-th percentile. Panel B shows the increase in the standard deviation of continuation values in the cross-section along a path in which the economy starts out in the lower uncertainty state at  $t = 1$ , transitions to the high uncertainty state and remains there for quarters  $t = (2, 3, \dots, 7)$ , and transitions back to the low uncertainty state and remains there for  $t = (8, 9, \dots, 20)$ . Panels C and D compare the fraction of severely constrained firms in the post-crisis period to that in the pre-crisis period. In both panels C and D, the x-axis measures time since the economy reverts back to the low-uncertainty state.



**Figure 4:** Response of aggregate quantities to uncertainty shock: The shock arrives at  $t = 2$  and lasts for 6 quarters. All the plots show percentage deviation from the initial pre-crisis levels. The solid, blue line in Panel A shows the response of output, while the dot-dash red line shows the path of aggregate capital stock. Panel B shows the change in aggregate investment. Panel C shows the change in the representative investor's consumption, while Panel D shows the total fraction of firms which exit the economy over the crisis.

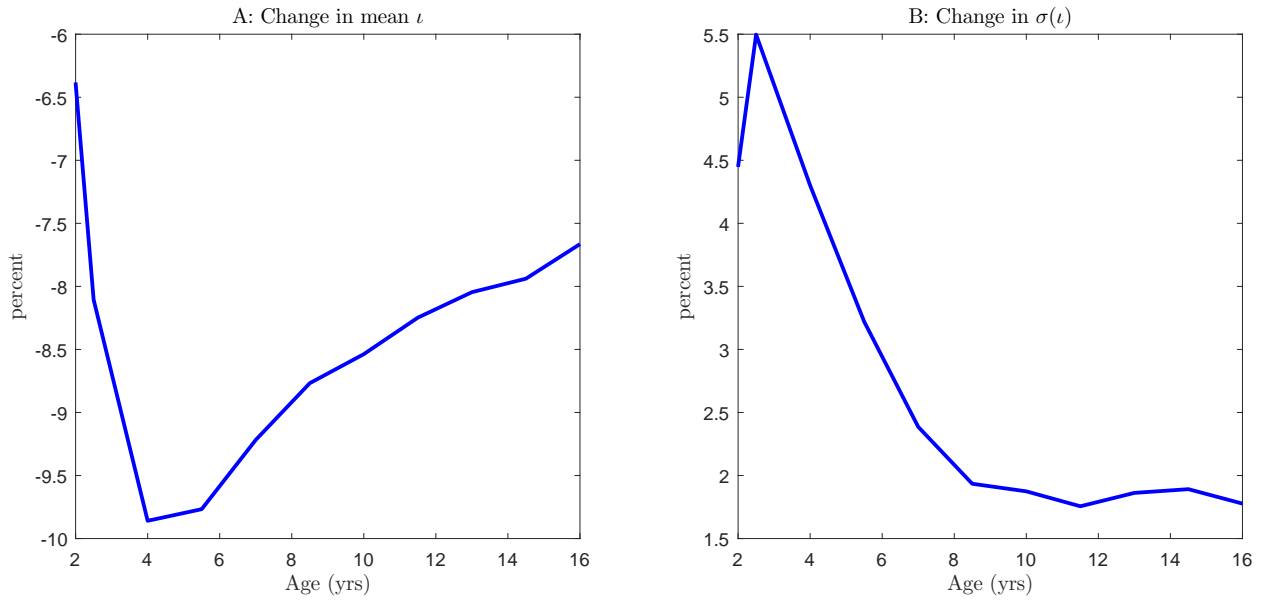


**Figure 5:** Depletion of capital stock in the cross-section: The figure shows the difference in the quantity of capital stock immediately after the end of the crisis from their levels at the start of the crisis for firms of various sizes. The lower panel shows the same distribution a year and a half after the end of the crisis. Firm size is normalized by the size of the median firm. In both panels, the economy starts out in the lower uncertainty state at  $t = 1$ , transitions to the high uncertainty state and remains there for quarters  $t = (2, 3, \dots, 7)$ , and transitions back to the low uncertainty state at  $t = 8$ .

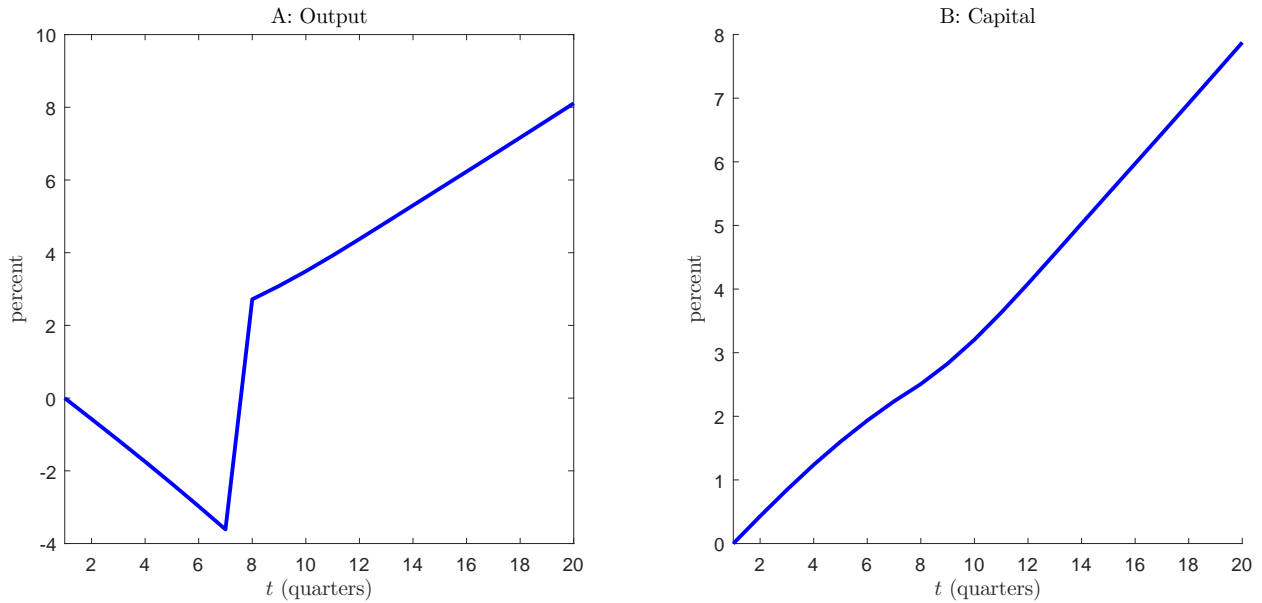


**Figure 6:** Drop in value of new investment: Panel A shows the percentage drop in the investor's valuation of entering into a new contract with a potential insider. Panel B shows the percentage drop in the quantity of new investment. In both panels, the economy starts out in the lower uncertainty state at  $t = 1$ , transitions to the high uncertainty state and remains there for quarters  $t = (2, 3, \dots, 7)$ , and transitions back to the low uncertainty state at  $t = 8$ .





**Figure 7:** Young firms affected most: Panel A shows the (annualized) percentage drop in the investment rate as a function of firm age upon impact of the uncertainty shock where the drop is averaged over all firms in the cross-section of a given vintage. Panel B shows the change in the dispersion of investment rate as a function of firm age, where dispersion is measured by the cross-sectional standard deviation. Both panels show differences with respect to the stochastic steady-state.



**Figure 8:** Quick Recovery following negative first moment shock: The shock arrives at  $t = 2$  and lasts for 6 quarters. The plots show percentage deviation from the initial pre-crisis levels. Panel A shows the response of output, while Panel B shows the path of aggregate capital stock.