Optimal Insurance Demand when Contract Nonperformance Risk is Perceived as Ambiguous

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Abstract

We study the optimal insurance demand of a risk- and ambiguity-averse consumer if contract nonperformance risk is perceived as ambiguous. We find that the consumer's optimal insurance demand is lower compared to a situation without ambiguity and that his degree of ambiguity aversion is negatively associated with the optimal level of coverage. We also determine sufficient conditions for biased beliefs or greater ambiguity to reduce the optimal demand for insurance and discuss wealth effects. Finally, we scrutinize several alternative model specifications to demonstrate the robustness of our main findings and discuss implications of our findings.

Keywords: ambiguity · ambiguity aversion · insurance demand · nonperformance risk

JEL-Classification: D11 · D80 · D81 · G22

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1 Introduction

Insurance allows risk-averse consumers to protect themselves against financial risks. It is well-known that, in a frictionless world with perfect information, full insurance is optimal if the price is actuarially fair. At an unfair price, the consumer trades off expected wealth for a lower riskiness of his wealth distribution and chooses partial insurance (Mossin, 1968; Smith, 1968). Over the past decades, the theory of insurance demand has produced numerous results and some claim it to be *"the purest example of economic behavior under uncertainty"* (Schlesinger, 2013).

It has been well recognized that insurance contracts may fail to perform as intended (Doherty and Schlesinger, 1990). The most salient reason for this to happen is insurer insolvency, which can invalidate otherwise valid claims by insured consumers. Even in the presence of guarantee funds, insurer insolvency leaves uncertainty for the consumer as to the extent and timing of indemnification. Furthermore, the possibility of claims being contested in front of the courts may result in lack of coverage for the policyholder. Besides that, delays in the insurer's claims handling process, subtleties in the contractual language of the insurance policy or probationary periods can leave consumers uncovered despite the fact that they paid a premium to the insurance company.[1]

As stressed by Schlesinger (2013), it is the consumer's perception of nonperformance risk that causes a deviation of his optimal demand relative to a situation without nonperformance risk. We argue in this paper that the consumer will hardly know the precise probability of a given contract not operating as intended at the time the purchase decision is made. Indeed, all the aforementioned examples can create significant uncertainty for consumers when it comes to the performance of insurance contracts. In other words, contract nonperformance risk is likely to be perceived as ambiguous by consumers, and it is precisely the implications of this ambiguity which we study in this paper.[2]

[1] For example, Crocker and Tennyson (2002) show that certain claims that are viewed as being easy to falsify, are less certain to be paid in full; also Tennyson and Warfel (2009) and Asmat and Tennyson (2014) provide evidence of claims underpayment and discuss the effect of the legal framework on the insurers' settlement and verification practices. Bourgeon and Picard (2014) develop an economic model of insurer "nitpicking", which reduces the efficiency of insurance contracts and can lead to substantial uncertainty about the performance of the contract on behalf of the consumer.

[2] Consistent with most of the literature we understand risk as a condition in which the event to be realized

To achieve this goal we employ the model of smooth ambiguity aversion developed by Klibanoff et al. (2005) in the main part of the analysis. It allows to disentangle ambiguity from ambiguity attitude and is particularly amenable for the type of comparative statics analysis which we perform in this paper. Following the definition given by Camerer and Weber (1992), *"ambiguity is uncertainty about probability created by missing information that is relevant and could be known"*. We argue that this applies to contract nonperformance risk in insurance because at the time contracts are bought, consumers are unlikely to know the precise probability of the contract not performing as planned. That individuals are averse towards such ambiguity was first demonstrated in the seminal paper by Ellsberg (1961). Since then, ambiguity aversion has been documented in numerous laboratory experiments (e.g., Einhorn and Hogarth, 1986; Chow and Sarin, 2001), in market settings with educated individuals (see Sarin and Weber, 1993), and in surveys of business owners and managers (see Viscusi and Chesson, 1999; Chesson and Viscusi, 2003).[3] Furthermore, there is a growing literature that studies the effects of ambiguity in insurance (e.g., Alary et al., 2013; Huang et al., 2013; Gollier, 2014; Bajtelsmit et al., 2015) as part of the field of behavioral insurance (see Richter et al., 2014), to which this paper contributes.

Existing studies on the effects of nonperformance risk in insurance have mainly focused on situations without ambiguity. Using an expected utility model, Doherty and Schlesinger (1990) find that most comparative statics do not carry over to situations with nonperformance risk. For example, Mossin's Theorem is violated (see also Mahul and Wright, 2007), increased risk aversion does not necessarily induce an increase in the optimal level of insurance coverage, and insurance is not necessarily an inferior good when preferences exhibit decreasing absolute risk aversion. Against the background of these results one would expect ambiguity to further obfuscate the economic trade-offs associated with the insurance decision. That nonperformance risk has a fundamental impact on the comparative statics of insurance demand

is unknown, but the odds of all possible events are perfectly known, either subjectively or objectively. Ambiguity refers to a condition where also the odds of the possible events are not uniquely assigned, see, for example, Iwaki and Osaki (2014).

[3] For a recent summary of the literature and the boundary conditions of ambiguity aversion in the laboratory, see Kocher et al. (2015). For the prevalence of ambiguity attitudes in a large representative sample, see Dimmock et al. (2015).

not only holds in the consumer-insurer relationship but also between insurers and reinsurers (Bernard and Ludkovski, 2012). Besides that, potential nonreliability of risk management tools is reflected in optimal demands also when it comes to self-insurance and self-protection, see Briys et al. (1991) and Schlesinger (1993). Cummins and Mahul (2003) allow the buyer's and insurer's belief about the probability of insurer default to be different and show how the buyer's optimism or pessimism is reflected in the optimal marginal indemnity schedule. In Biffis and Millossovich (2012), insurer default arises endogenously from the interaction of the insurance premium, the indemnity schedule and the insurer's assets, which has implications for optimal contracting. Huang and Tzeng (2007) explain how tax deductions for net losses allow consumers to offset some of the exposure to contract nonperformance risk. Besides theoretical work, there is some experimental and empirical evidence of the importance of nonperformance risk for insurance demand. Kahneman and Tversky (1979) use the term 'probabilistic insurance' to refer to contract nonperformance risk, and present an experiment that demonstrates aversion against it. Zimmer et al. (2009, 2014) show in an experiment that individuals react to contract nonperformance risk and adjust their insurance purchasing behavior accordingly. Liu and Myers (2016) provide evidence that contract nonperformance risk can reduce insurance demand in the context of microinsurance. Only Biener et al. (2016) allow for nonperformance risk to be ambiguous but they analyze willingness to pay instead of demand and do not provide any comparative statics.

Intuitively, it is not immediate what the implications of an ambiguous perception of nonperformance risk on insurance demand will be. When the risk of loss is perceived as ambiguous, Snow (2011) and Alary et al. (2013) show that, at least in the standard model with two states of the world, ambiguity aversion raises the optimal level of coverage. Our case is quite different; when consumers perceive nonperformance risk as ambiguous, their perception will include probabilistic scenarios in which the insurance contract performs better, suggesting to buy more coverage, and scenarios in which the insurance contract performs worse, suggesting to reduce coverage. The equilibrium effects of ambiguity are not clear a priori. Nevertheless, we are able to show that, if contract nonperformance risk is perceived as ambiguous, this lowers the optimal demand for insurance compared to a situation without ambiguity. The reason is that an ambiguity-averse consumer makes optimal decisions that can be rationalized with the help of a distorted probability distribution. The perception of ambiguity induces him to behave as if he was more pessimistic about the probability of contract nonperformance and to make decisions as if nonperformance was more likely than in the case without ambiguity. Furthermore, his degree of ambiguity aversion is negatively associated with the optimal level of coverage and we derive sufficient conditions for biased beliefs and greater ambiguity to reduce optimal insurance demand further and for a partial wealth effect to be positive. All these results are original and reveal that an ambiguous perception of nonperformance risk has significant demand effects for ambiguity-averse consumers. What's more, we show that our main findings do not depend on whether smooth or nonsmooth ambiguity preferences are used. The fact that an ambiguous perception of nonperformance risk reduces demand is directly attributable to ambiguity aversion and does not depend on its particular implementation.

The rest of the paper is organized as follows. In section 2, we introduce a model of insurance demand under nonperformance risk and enrich it by the consumer's ambiguous perception thereof. In section 3, we compare the optimal level of coverage with and without ambiguity and develop the underlying intuition. We provide several comparative statics results in section 4, including the comparative statics of greater ambiguity aversion, biased beliefs, greater ambiguity and wealth. In section 5, we discuss a variety of extensions and scrutinize the robustness of our main finding. We discuss alternative models of ambiguity aversion, the case of an ambiguity-averse insurer and the joint presence of an ambiguous perception of the risk of loss and of nonperformance risk. A final section concludes.

2 Model and Notations

We consider a consumer with initial wealth W which is subject to a loss of $L \in (0, W)$ that occurs with probability $p \in (0, 1)$. His risk preferences are characterized by a vNM utility function u of final wealth, which is increasing and concave, $u' > 0$ and $u'' < 0$, to reflect non-satiation and risk aversion. The consumer can insure against the risk of loss with $\alpha \in [0, 1]$ denoting the level of coverage and αL the associated indemnity payment in case of a loss. Insurance requires payment of an insurance premium $P(\alpha)$ which is assumed to be an increasing and non-concave function of the level of coverage, $P' > 0$ and $P'' \geq 0$.

As in Doherty and Schlesinger (1990), insurance contracts are imperfect because that they do not perform in some cases. This happens with probability $(1-q) \in (0,1)$ whereas with probability q the contract operates as intended and the indemnity is paid to the full amount. This paper focuses on the consumer's perception of contract nonperformance risk. More specifically, we assume that the policyholder does not know the probability of nonperformance for sure so that nonperformance risk is perceived as ambiguous with $\widetilde{\varepsilon}$ denoting the level of ambiguity. For the time being, the consumer's beliefs about nonperformance risk are assumed to be unbiased, that is $\mathbb{E}\widetilde{\varepsilon} = 0$, and the support of $\widetilde{\varepsilon}$ is such that every possible realization renders proper beliefs, $[\underline{\varepsilon}, \overline{\varepsilon}] \subseteq [-(1-q), q]$.

Three different states of the world are relevant for the analysis. $W_1 = W - P$ is the consumer's final wealth if he does not suffer a loss, in which case contract nonperformance risk is irrelevant. $W_2 = W - P - L + \alpha L$ is the consumer's final wealth when a loss occurs and the insurance contract performs, and $W_3 = W - P - L$ is the consumer's final wealth when he suffers a loss but the contract does not perform. As noted by Doherty and Schlesinger (1990), in hindsight the consumer would have been better off in the latter state had he not purchased insurance to save on premium money. For simplicity, we assume that in case of nonperformance there is no recovery so that no indemnity will be paid at all.[4]

With these specifications, the different levels of the consumer's expected utility can be represented as follows:

$$U(\alpha, \widetilde{\varepsilon}) = (1-p)u(W_1) + p(q - \widetilde{\varepsilon})u(W_2) + p(1 - q + \widetilde{\varepsilon})u(W_3).$$

Ambiguity results in the consumer's expected utility for a given level of coverage to be a random variable whose outcome depends on the realization of $\widetilde{\varepsilon}$.[5] To incorporate the consumer's attitude towards ambiguity, we adopt the approach developed by Klibanoff et al. (2005) who derive the consumer's objective function as a ϕ-weighted expectation of the different expected

[4] This assumption is without loss of generality because the ordering of the different levels of final wealth is given by $W_3 < W_2 \leq W_1$ for any recovery rate. The last inequality is strict for partial insurance coverage.

[5] In our model, ambiguity enters the consumer's objective function as an additive noise term. This specification helps develop the intuition for our results but our main findings are robust to a non-additive specification, see the second paragraph after the proof of Proposition 2.

utilities based on second-order beliefs (see also Neilson, 2010). ϕ is the consumer's ambiguity function, which is increasing and concave, $\phi' > 0$ and $\phi'' < 0$, to reflect non-satiation (in expected utility) and ambiguity aversion. The consumer's ex-ante welfare as a function of the level of coverage is given by

$$V(\alpha) = \mathbb{E}\phi\left[U(\alpha, \widetilde{\varepsilon})\right], \tag{1}$$

where the expectation is taken with respect to the cumulative distribution function F, which describes the consumer's second-order beliefs over the probability of contract nonperformance.

3 The Optimal Level of Coverage

The optimal level of insurance coverage is the one that maximizes the consumer's ex-ante welfare. It is characterized by the associated first-order condition,

$$V'(\alpha) = \mathbb{E}\left\{\phi'\left[U(\alpha, \widetilde{\varepsilon})\right] U_\alpha(\alpha, \widetilde{\varepsilon})\right\} = 0, \tag{2}$$

where subscript α denotes the partial derivative with respect to the level of coverage. The consumer's objective function is globally concave in α,

$$V''(\alpha) = \mathbb{E}\left\{\phi''\left[U(\alpha, \widetilde{\varepsilon})\right] U_\alpha(\alpha, \widetilde{\varepsilon})^2 + \phi'\left[U(\alpha, \widetilde{\varepsilon})\right] U_{\alpha\alpha}(\alpha, \widetilde{\varepsilon})\right\} < 0.$$

Due to ambiguity aversion $\phi''\left[U(\alpha, \varepsilon)\right] < 0$ for every ε, and due to risk aversion and the non-concavity of the premium schedule we obtain that $U_{\alpha\alpha}(\alpha, \varepsilon) < 0$ for every ε. Concavity of the objective function ensures that the first-order condition characterizes a maximum. In the sequel, we assume that it is optimal to purchase at least some insurance and denote the optimal level of coverage by $\alpha^* > 0$.[6]

To disentangle the effects of ambiguity, we decompose $U_\alpha(\alpha, \widetilde{\varepsilon})$ into the marginal utility cost and the marginal utility benefit,

$$MC(\alpha, \widetilde{\varepsilon}) = P'(\alpha)\left[(1-p)u'(W_1) + p(q-\widetilde{\varepsilon})u'(W_2) + p(1-q+\widetilde{\varepsilon})u'(W_3)\right], \text{ and}$$

[6] This is the case as long as $P'(0)$ is less than a threshold that is obtained by rearranging $V'(0) > 0$ and depends on the consumer's risk and ambiguity preferences.

$$MB(\alpha, \widetilde{\varepsilon}) = Lp(q - \widetilde{\varepsilon})u'(W_2).$$

Insurance obligates the consumer to pay a premium which reduces his final wealth in any state. However, the indemnity increases final wealth if the loss occurs and the contract performs as intended. With these notations, the first-order condition can be rewritten as follows:

$$\mathbb{E}\left\{\phi'\left[U(\alpha, \widetilde{\varepsilon})\right] MC(\alpha, \widetilde{\varepsilon})\right\} = \mathbb{E}\left\{\phi'\left[U(\alpha, \widetilde{\varepsilon})\right] MB(\alpha, \widetilde{\varepsilon})\right\}.$$

In the absence of ambiguity ($\widetilde{\varepsilon} \equiv 0$), the consumer's expected utility is non-random and the optimal level of coverage is characterized by:

$$U_\alpha(\alpha, 0) = -MC(\alpha, 0) + MB(\alpha, 0) = 0. \tag{3}$$

It is denoted by α^0 and is the level of coverage identified by Doherty and Schlesinger (1990) for the special case of a linear premium schedule. To develop some intuition, we will investigate how the ambiguity associated with the risk of contract nonperformance affects the marginal utility cost and the marginal utility benefit of insurance. For the former, we derive

$$MC\left(\alpha^0, \widetilde{\varepsilon}\right) - MC\left(\alpha^0, 0\right) = P'(\alpha^0)p\widetilde{\varepsilon}\left(u'(W_3) - u'(W_2)\right).$$

The expression in round brackets is positive due to risk aversion. As a result, the overall sign coincides with the sign of the realization of $\widetilde{\varepsilon}$. If $\widetilde{\varepsilon} = \varepsilon > 0$, the consumer perceives contract nonperformance as more likely relative to a situation without ambiguity. This increases the marginal utility cost of insurance because incurring the insurance premium is most painful in precisely that state of the world where the insurance contract does not perform. Conversely, if $\widetilde{\varepsilon} = \varepsilon < 0$, the consumer perceives contract nonperformance to be less likely relative to a situation without ambiguity, which decreases the marginal utility cost of insurance.

For the marginal utility benefit, we obtain

$$MB\left(\alpha^0, \widetilde{\varepsilon}\right) - MB\left(\alpha^0, 0\right) = -pL\widetilde{\varepsilon}u'(W_2).$$

The sign is the opposite of the sign of the realization of $\widetilde{\varepsilon}$. If $\widetilde{\varepsilon} = \varepsilon > 0$, the consumer perceives contract nonperformance as more likely relative to a situation without ambiguity. This reduces the marginal utility benefit of insurance because the consumer is less likely to receive the indemnity in case of a loss. Conversely, if $\widetilde{\varepsilon} = \varepsilon < 0$, the consumer perceives contract nonperformance to be less likely relative to a situation without ambiguity, which increases the marginal utility benefit of insurance. It becomes clear that any positive realization of $\widetilde{\varepsilon}$ increases the marginal utility cost and decreases the marginal utility benefit of insurance which induces the consumer to lower his demand relative to α^0. On the contrary, any negative realization of $\widetilde{\varepsilon}$ decreases the marginal utility cost and increases the marginal utility benefit of insurance which induces the consumer to increase his demand relative to α^0. The net effect is a priori indeterminate but we are able to obtain a sign-definite answer.

Proposition 1. *The optimal demand for insurance for a risk- and ambiguity-averse consumer is lower when nonperformance risk is perceived as ambiguous than when it is not.*

Proof. We insert the optimal level of coverage without ambiguity into the consumer's first-order condition with ambiguity and determine the sign. We obtain that

$$\begin{aligned} V'(\alpha^0) &= \mathbb{E}\left\{\phi'\left[U(\alpha^0,\widetilde{\epsilon})\right]U_\alpha(\alpha^0,\widetilde{\epsilon})\right\} \\ &= \mathrm{Cov}\left\{\phi'\left[U(\alpha^0,\widetilde{\varepsilon})\right],U_\alpha(\alpha^0,\widetilde{\varepsilon})\right\} + \mathbb{E}\left\{\phi'\left[U(\alpha^0,\widetilde{\varepsilon})\right]\right\}\mathbb{E}\left\{U_\alpha(\alpha^0,\widetilde{\varepsilon})\right\} \end{aligned}$$

from the covariance rule. Expanding expected utility with ambiguity yields that

$$\begin{aligned} U_\alpha(\alpha^0,\widetilde{\varepsilon}) &= -(1-p)P'(\alpha^0)\cdot u'(W_1^0) + p(q-\widetilde{\varepsilon})(L-P'(\alpha^0))\cdot u'(W_2^0) \\ &\quad -p(1-q+\widetilde{\varepsilon})P'(\alpha^0)\cdot u'(W_3^0) \\ &= U_\alpha(\alpha^0,0) + \widetilde{\varepsilon}p\left[P'(\alpha^0)(u'(W_2^0)-u'(W_3^0)) - Lu'(W_2^0)\right], \end{aligned}$$

where superscript 0 indicates final wealth levels when the level of coverage is given by α^0. Now $U_\alpha(\alpha^0,0)=0$ by definition of α^0 and $\mathbb{E}\widetilde{\varepsilon}=0$ due to the assumption that the consumer's perception of ambiguity is unbiased. As a result, we obtain that $\mathbb{E}\left\{U_\alpha(\alpha^0,\widetilde{\varepsilon})\right\}=0$ so that

$$V'(\alpha^0) = \mathrm{Cov}\left\{\phi'\left[U(\alpha^0,\widetilde{\varepsilon})\right],U_\alpha(\alpha^0,\widetilde{\varepsilon})\right\}. \tag{4}$$

To sign the covariance we investigate $\phi'\left[U(\alpha^0,\varepsilon)\right]$ and $U_\alpha(\alpha^0,\varepsilon)$ as functions of ε. The consumer's expected utility strictly decreases in the realization of $\widetilde{\varepsilon}$ because the higher ε, the higher the consumer's belief that the contract will not perform. Due to the concavity of ϕ, the expression $\phi'\left[U(\alpha^0,\varepsilon)\right]$ is then strictly increasing in ε,

$$\frac{\partial\phi'\left[U(\alpha^0,\varepsilon)\right]}{\partial\varepsilon} = \phi''\left[U(\alpha^0,\varepsilon)\right]U_\varepsilon(\alpha^0,\varepsilon) = -\phi''\left[U(\alpha^0,\varepsilon)\right]p(u(W_2^0)-u(W_3^0)) > 0.$$

We also know that

$$U_{\alpha\varepsilon}(\alpha^0,\varepsilon) = p\left[P'(\alpha^0)(u'(W_2^0)-u'(W_3^0)) - Lu'(W_2^0)\right].$$

The expression in square brackets is negative due to diminishing marginal utility and because marginal utility is positive. As a result, $U_\alpha(\alpha^0,\varepsilon)$ is decreasing in ε. The two arguments of the covariance in equation (4) are countermonotonic so that the covariance is negative rendering $V'(\alpha^0)$ negative as well. Due to the concavity of the objective function in the level of coverage, this shows that $\alpha^* < \alpha^0$ which completes the proof. □

To develop some intuition for this result, we return to the first-order condition (2) and evaluate it at the optimal level of coverage without ambiguity, which yields

$$\begin{aligned} V'(\alpha^0) &= \mathbb{E}\left\{\phi'\left[U(\alpha^0,\widetilde{\varepsilon})\right]U_\alpha(\alpha^0,\widetilde{\varepsilon})\right\} \\ &= -\underbrace{\mathbb{E}\left\{\phi'\left[U(\alpha^0,\widetilde{\varepsilon})\right]MC(\alpha^0,\widetilde{\varepsilon})\right\}}_{\text{unconditional marginal cost}} + \underbrace{\mathbb{E}\left\{\phi'\left[U(\alpha^0,\widetilde{\varepsilon})\right]MB(\alpha^0,\widetilde{\varepsilon})\right\}}_{\text{unconditional marginal benefit}}. \end{aligned}$$

From the proof of Proposition 1 we know that $\phi'\left[U(\alpha^0,\varepsilon)\right]$ is increasing in ε if the consumer is ambiguity-averse. Scenarios with $\varepsilon < 0$ receive relatively less weight than scenarios with $\varepsilon > 0$. Said differently, an ambiguity-averse consumer behaves as if he was an expected utility maximizer who overweights scenarios in which the marginal utility cost is higher than the marginal utility benefit of insurance and underweights scenarios in which the marginal utility cost is lower than the marginal utility benefit of insurance. In the spirit of Gollier (2011), we can rewrite the consumer's first-order expression evaluated at α^0 as follows:

$$\begin{aligned} V'(\alpha^0) &= \mathbb{E}\phi'\left[U(\alpha^0,\widetilde{\varepsilon})\right]\mathbb{E}\left\{\frac{\phi'\left[U(\alpha^0,\widetilde{\varepsilon})\right]}{\mathbb{E}\phi'\left[U(\alpha^0,\widetilde{\varepsilon})\right]}U_\alpha(\alpha^0,\widetilde{\varepsilon})\right\} = \\ &= \mathbb{E}\phi'\left[U(\alpha^0,\widetilde{\varepsilon})\right]\mathbb{E}^{Q^0}\left\{U_\alpha(\alpha^0,\widetilde{\varepsilon})\right\}, \end{aligned}$$

where Q^0 denotes a distorted probability measure. The ambiguity-averse consumer behaves in the same way as an expected utility maximizing consumer who has distorted his second-order beliefs.[7] The associated distortion factor $\phi'\left[U(\alpha^0,\widetilde{\varepsilon})\right]/\mathbb{E}\phi'\left[U(\alpha^0,\widetilde{\varepsilon})\right]$ is a Radon-Nikodym derivative that describes the change of measure and $\mathbb{E}^{Q^0}$ is the ambiguity-neutral expectation which corresponds to the risk-neutral expectation in finance. The notation is suggestive of the fact that this distorted measure is endogenous because it depends on the level of coverage chosen by the consumer. Under Q^0, the consumer perceives contract nonperformance as more likely than under the physical probability. The reason is that

$$\mathbb{E}^{Q^0}\left\{1-q+\widetilde{\varepsilon}\right\} = \mathbb{E}\left\{\frac{\phi'\left[U(\alpha^0,\widetilde{\varepsilon})\right]}{\mathbb{E}\phi'\left[U(\alpha^0,\widetilde{\varepsilon})\right]}(1-q+\widetilde{\varepsilon})\right\} = (1-q) + \frac{\mathrm{Cov}\left\{\phi'\left[U(\alpha^0,\widetilde{\varepsilon})\right],\widetilde{\varepsilon}\right\}}{\mathbb{E}\phi'\left[U(\alpha^0,\widetilde{\varepsilon})\right]},$$

with the second summand being positive because $\phi'\left[U(\alpha^0,\varepsilon)\right]$ is increasing in ε under ambiguity aversion. The behavior of an ambiguity-averse consumer can be interpreted as that of an expected utility maximizer who perceives the probability of the contract operating as intended as less likely relative to the situation without ambiguity. As a result, the optimal insurance demand for a risk- and ambiguity-averse consumer is lower when he perceives contract nonperformance risk as ambiguous than when he does not.

Before we proceed, we make some remarks on the optimality of full insurance ($\alpha^* = 1$). In the absence of nonperformance risk, full insurance is optimal if and only if the price of insurance is actuarially fair (Mossin, 1968). As shown by Doherty and Schlesinger (1990), if default is total, less than full coverage is optimal at the fair price, and if default is partial, optimal coverage might be more or less than full at the fair price. Furthermore, in the face of nonperformance risk, consumers might demand more or less coverage at an unfair price as compared to the fair price. We can combine these insights with our result about the effect

[7] Gollier (2011) refers to this interpretation as "observational equivalence".

of ambiguity. If the price is fair so that $P(\alpha) = \alpha pqL$, the optimal level of coverage is less than full under objective nonperformance risk with total default, and even lower when it is perceived as ambiguous (see Proposition 1). As a result, Mossin's Theorem does not hold in our setup. If default is partial or the price is unfair, more than full coverage might be optimal in a situation with objective nonperformance risk, and an ambiguous perception of nonperformance risk might lower the optimal demand to full coverage. Necessary for this to happen is that the premium schedule is sufficiently flat at full coverage ($P'(1) < p(q - \underline{\varepsilon})L$). If the converse holds, it follows that

$$\begin{aligned} U_\alpha(1,\varepsilon) &= \left[-(1-p)P'(1) + p(q-\varepsilon)(L - P'(1))\right] u'(W_1^1) - p(1-q+\varepsilon)P'(1)u'(W_3^1) \\ &< u'(W_1^1)\left[-(1-p)P'(1) + p(q-\varepsilon)(L-P'(1)) - p(1-q+\varepsilon)P'(1)\right] \\ &= u'(W_1^1)\left[-P'(1) + p(q-\varepsilon)L\right] < 0 \end{aligned}$$

for all $\varepsilon > \underline{\varepsilon}$, with superscript 1 indicating terminal wealth levels at full coverage. Then, $V'(1) < 0$ so that less than full coverage is optimal.

4 Some Comparative Statics Results

Doherty and Schlesinger (1990) find that, in the presence of nonperformance risk, it is unclear whether a more risk-averse consumer will buy more or less coverage, even when using a strong increase in risk aversion in the sense of Ross (1981). The reason is that an increase in coverage reduces the spread between W_1 and W_2 but widens the spread between W_2 and W_3.[8] Are better results obtained when it comes to ambiguity aversion? As we will show, the answer to this question is positive.

Proposition 2. *Assume that nonperformance risk is perceived as ambiguous; then, an increase in the consumer's degree of ambiguity aversion will lead to a decrease in the optimal level of insurance coverage.*

[8] Not until recently, Eeckhoudt et al. (2016) have shown that only the notion of a restricted increase in risk aversion, which is even more restrictive than comparative risk aversion in the sense of Ross (1981), yields a clear comparative static result.

Proof. Following Klibanoff et al. (2005), we model an increase in the consumer's degree of ambiguity aversion by replacing his ambiguity function with ψ, where ψ is an increasing and concave transformation of ϕ, $\psi = k(\phi)$ with $k' > 0$ and $k'' < 0$. Let T denote the more ambiguity-averse consumer's objective function; his optimal level of coverage, α^{**}, is then characterized by the corresponding first-order condition,

$$T'(\alpha) = \mathbb{E}\left\{\psi'\left[U(\alpha,\widetilde{\varepsilon})\right]U_\alpha(\alpha,\widetilde{\varepsilon})\right\} = \mathbb{E}\left\{k'\left(\phi\left[U(\alpha,\widetilde{\varepsilon})\right]\right)\phi'\left[U(\alpha,\widetilde{\varepsilon})\right]U_\alpha(\alpha,\widetilde{\varepsilon})\right\} = 0.$$

The second-order condition is satisfied due to the concavity of ψ and u. To compare the optimal level of coverage of the less ambiguity-averse consumer (α^*) with that of the more ambiguity-averse consumer (α^{**}), we insert the former into the first-order condition for the latter and determine the sign. Notice that $k'\left(\phi\left[U(\alpha,\varepsilon)\right]\right)$ is increasing in ε,

$$\begin{aligned}
\frac{\partial k'\left(\phi[U(\alpha,\varepsilon)]\right)}{\partial\varepsilon} &= k''\left(\phi[U(\alpha,\varepsilon)]\right)\phi'[U(\alpha,\varepsilon)]U_\varepsilon(\alpha,\varepsilon) \\
&= -k''\left(\phi[U(\alpha,\varepsilon)]\right)\phi'[U(\alpha,\varepsilon)]p(u(W_2)-u(W_3)) > 0.
\end{aligned}$$

As a result, for any $\varepsilon \in (\underline{\varepsilon},\overline{\varepsilon})$, it holds that

$$k'\left(\phi[U(\alpha,\underline{\varepsilon})]\right) < k'\left(\phi[U(\alpha,\varepsilon)]\right) < k'\left(\phi[U(\alpha,\overline{\epsilon})]\right). \tag{5}$$

The optimal level of coverage of the less ambiguity-averse consumer (α^*) is obtained from his first-order condition,

$$V'(\alpha^*) = \mathbb{E}\left\{\phi'\left[U(\alpha^*,\widetilde{\varepsilon})\right]U_\alpha(\alpha^*,\widetilde{\varepsilon})\right\} = \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \phi'\left[U(\alpha^*,\varepsilon)\right]U_\alpha(\alpha^*,\varepsilon)\mathrm{d}F(\varepsilon) = 0.$$

$\phi'\left[U(\alpha^*,\varepsilon)\right]$ is strictly positive for any $\varepsilon \in [\underline{\varepsilon},\overline{\varepsilon}]$ and $U_\alpha(\alpha^*,\varepsilon)$ is strictly decreasing in ε by the proof of Proposition 1. For the integral to be zero, it follows that $U_\alpha(\alpha^*,\varepsilon)$ must change sign on $[\underline{\varepsilon},\overline{\varepsilon}]$; due to strict monotonicity, this can only happen once. If $\hat{\varepsilon} \in (\underline{\varepsilon},\overline{\varepsilon})$ denotes the null of $U_\alpha(\alpha^*, \varepsilon)$, we obtain that

$$\begin{aligned}
\int_{\underline{\varepsilon}}^{\hat{\varepsilon}} \phi' \left[U(\alpha^*, \varepsilon)\right] U_\alpha(\alpha^*, \varepsilon)\mathrm{d}F(\varepsilon) &> 0, \text{ and} \\
\int_{\hat{\varepsilon}}^{\overline{\varepsilon}} \phi' \left[U(\alpha^*, \varepsilon)\right] U_\alpha(\alpha^*, \varepsilon)\mathrm{d}F(\varepsilon) &< 0.
\end{aligned}$$

Combining this with (5) yields,

$$\begin{aligned}
T'(\alpha^*) &= \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} k'\left(\phi\left[U(\alpha^*, \varepsilon)\right]\right) \phi' \left[U(\alpha^*, \varepsilon)\right] U_\alpha(\alpha^*, \varepsilon)\mathrm{d}F(\varepsilon) \\
&= \int_{\underline{\varepsilon}}^{\hat{\varepsilon}} k'\left(\phi[U(\alpha^*, \varepsilon)]\right) \phi' \left[U(\alpha^*, \varepsilon)\right] U_\alpha(\alpha^*, \varepsilon)\mathrm{d}F(\varepsilon) \\
&\quad + \int_{\hat{\varepsilon}}^{\overline{\varepsilon}} k'\left(\phi[U(\alpha^*, \varepsilon)]\right) \phi' \left[U(\alpha^*, \varepsilon)\right] U_\alpha(\alpha^*, \varepsilon)\mathrm{d}F(\epsilon) \\
&< k'\left(\phi[U(\alpha^*, \hat{\varepsilon})]\right) \int_{\underline{\varepsilon}}^{\hat{\varepsilon}} \phi' \left[U(\alpha^*, \varepsilon)\right] U_\alpha(\alpha^*, \varepsilon)\mathrm{d}F(\varepsilon) \\
&\quad + k'\left(\phi[U(\alpha^*, \hat{\varepsilon})]\right) \int_{\hat{\varepsilon}}^{\overline{\varepsilon}} \phi' \left[U(\alpha^*, \varepsilon)\right] U_\alpha(\alpha^*, \varepsilon)\mathrm{d}F(\epsilon) \\
&= k'\left(\phi[U(\alpha^*, \hat{\varepsilon})]\right) \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \phi' \left[U(\alpha^*, \varepsilon)\right] U_\alpha(\alpha^*, \varepsilon)\mathrm{d}F(\varepsilon) \\
&= k'\left(\phi[U(\alpha^*, \hat{\varepsilon})]\right) V'(\alpha^*) = 0,
\end{aligned}$$

so that $T'(\alpha^*)$ is strictly negative. As a result, it is optimal for the more ambiguity-averse consumer to decrease his level of insurance below α^* such that $\alpha^{**} < \alpha^*$. □

We note that Proposition 1 can be interpreted as a special case of Proposition 2. The reason is that an ambiguity-averse consumer's optimal level of insurance demand in the absence of ambiguity coincides with an ambiguity-neutral consumer's optimal level of insurance demand in the presence of ambiguity under the framework of smooth ambiguity aversion. Proposition 1 states that the optimal level of insurance coverage for an ambiguity-averse consumer is lower than that for an ambiguity-neutral one, which is also obtained from Proposition 2 because the ambiguity-neutral consumer's degree of ambiguity aversion is lower than that of the ambiguity-averse consumer.

As mentioned in Footnote 5, our model incorporates ambiguity as an additive noise term into the consumer's objective function. A more general approach would be to model the probability of contract nonperformance as $(1 - q(\widetilde{\varepsilon}))$ with $\widetilde{\varepsilon}$ distributed according to the cumulative distribution function F. Propositions 1 and 2 continue to hold in such a non-additive model of ambiguity as long as $q'(\varepsilon) < 0$, which can be assumed without loss of generality due to the binary specification of contract nonperformance risk.

To develop the underlying intuition for Proposition 2, we investigate how the consumer's unconditional marginal cost and unconditional marginal benefit of insurance are affected by an increase in his degree of ambiguity aversion. The former is given by

$$\mathbb{E}\left\{\psi'\left[U(\alpha^*,\widetilde{\varepsilon})\right]MC(\alpha^*,\widetilde{\varepsilon})\right\} = \mathbb{E}\left\{k'\left[U(\alpha^*,\widetilde{\varepsilon})\right]\phi'\left[U(\alpha^*,\widetilde{\varepsilon})\right]MC(\alpha^*,\widetilde{\varepsilon})\right\},$$

whereas the latter is obtained as

$$\mathbb{E}\left\{\psi'\left[U(\alpha^*,\widetilde{\varepsilon})\right]MB(\alpha^*,\widetilde{\varepsilon})\right\} = \mathbb{E}\left\{k'\left[U(\alpha^*,\widetilde{\varepsilon})\right]\phi'\left[U(\alpha^*,\widetilde{\varepsilon})\right]MB(\alpha^*,\widetilde{\varepsilon})\right\}.$$

The difference to the less ambiguity-averse consumer is captured by $k'\left[U(\alpha^*,\varepsilon)\right]$, which is increasing in ε. This reinforces scenarios where ε is high so that the consumer perceives contract nonperformance as more likely relative to scenarios where ε is low and the consumer perceives contract nonperformance as less likely. Consequently, for a more ambiguity-averse consumer scenarios where the marginal cost of insurance is high and its marginal benefit is low are reinforced whereas scenarios where the marginal cost of insurance is low and its marginal benefit is high are attenuated. Thus it is optimal to reduce the optimal level of coverage.

This intuition is confirmed when investigating how a more ambiguity-averse consumer distorts the distribution over the different states of the world relative to a less ambiguity-averse consumer. Due to the fact that $k'\left(\phi[U(\alpha^*,\varepsilon)]\right)$ is increasing in ε, the covariance between $k'\left(\phi[U(\alpha^*,\widetilde{\varepsilon})]\right)$ and $\widetilde{\varepsilon}$ is positive. This holds under the physical measure but also with respect to the distorted measure Q^* with Radon-Nikodym derivative $\phi'\left[U(\alpha^*,\widetilde{\varepsilon})\right]/\mathbb{E}\phi'\left[U(\alpha^*,\widetilde{\varepsilon})\right]$. Expanding the covariance between $k'\left(\phi[U(\alpha^*,\widetilde{\varepsilon})]\right)$ and $\widetilde{\varepsilon}$ under Q^* yields

$$\begin{aligned}
& \mathrm{Cov}^{Q^*}\left\{k'\left(\phi[U(\alpha^*,\widetilde{\varepsilon})]\right),\widetilde{\varepsilon}\right\} \\
= \; & \mathbb{E}^{Q^*}\left\{k'\left(\phi[U(\alpha^*,\widetilde{\varepsilon})]\right)\widetilde{\varepsilon}\right\} - \mathbb{E}^{Q^*}\left\{k'\left(\phi[U(\alpha^*,\widetilde{\varepsilon})]\right)\right\}\cdot\mathbb{E}^{Q^*}\left\{\widetilde{\varepsilon}\right\}
\end{aligned}$$

$$
\begin{aligned}
&= \mathbb{E}\left\{\frac{\phi'\left[U(\alpha^*,\widetilde{\varepsilon})\right]}{\mathbb{E}\phi'\left[U(\alpha^*,\widetilde{\varepsilon})\right]}k'\left(\phi[U(\alpha^*,\widetilde{\varepsilon})]\right)\widetilde{\varepsilon}\right\} \\
&\quad -\mathbb{E}\left\{\frac{\phi'\left[U(\alpha^*,\widetilde{\varepsilon})\right]}{\mathbb{E}\phi'\left[U(\alpha^*,\widetilde{\varepsilon})\right]}k'\left(\phi[U(\alpha^*,\widetilde{\varepsilon})]\right)\right\}\cdot\mathbb{E}\left\{\frac{\phi'\left[U(\alpha^*,\widetilde{\varepsilon})\right]}{\mathbb{E}\phi'\left[U(\alpha^*,\widetilde{\varepsilon})\right]}\widetilde{\varepsilon}\right\} \\
&= \frac{1}{\mathbb{E}\phi'\left[U(\alpha^*,\widetilde{\varepsilon})\right]}\mathrm{Cov}\left\{k'\left(\phi[U(\alpha^*,\widetilde{\varepsilon})]\right)\phi'\left[U(\alpha^*,\widetilde{\varepsilon})\right],\widetilde{\varepsilon}\right\} \\
&\quad -\left(\frac{1}{\mathbb{E}\phi'\left[U(\alpha^*,\widetilde{\varepsilon})\right]}\right)^2\mathbb{E}\left\{k'\left(\phi[U(\alpha^*,\widetilde{\varepsilon})]\right)\phi'\left[U(\alpha^*,\widetilde{\varepsilon})\right]\right\}\mathrm{Cov}\left\{\phi'\left[U(\alpha^*,\widetilde{\varepsilon})\right],\widetilde{\varepsilon}\right\}.
\end{aligned}
$$

This is positive if and only if

$$
\frac{\mathrm{Cov}\left\{k'\left(\phi[U(\alpha^*,\widetilde{\varepsilon})]\right)\phi'\left[U(\alpha^*,\widetilde{\varepsilon})\right],\widetilde{\varepsilon}\right\}}{\mathbb{E}\left\{k'\left(\phi[U(\alpha^*,\widetilde{\varepsilon})]\right)\phi'\left[U(\alpha^*,\widetilde{\varepsilon})\right]\right\}} > \frac{\mathrm{Cov}\left\{\phi'\left[U(\alpha^*,\widetilde{\varepsilon})\right],\widetilde{\varepsilon}\right\}}{\mathbb{E}\phi'\left[U(\alpha^*,\widetilde{\varepsilon})\right]},
$$

which, according to the intuition for Proposition 1, states that the behavior of a more ambiguity-averse consumer can be interpreted as that of an expected utility maximizer who distorts the probability of contract nonperformance more compared to the distortion of a less ambiguity-averse consumer. The distorted beliefs consistent with the behavior of the former attribute a higher probability to the event that the contract does not perform compared to the distorted beliefs consistent with the behavior of the latter. Therefore, it is optimal for a more ambiguity-averse consumer to reduce his optimal demand for insurance.

The question dual to the one examined in the previous proposition is that of a change in ambiguity. Before we turn to the comparative statics of ambiguity, we briefly revisit the comparative statics of risk. It is well known that a first-order stochastically dominated shift or an increase in risk in the sense of Rothschild and Stiglitz (1970) do not necessarily reduce the demand for the risky asset in the standard portfolio problem (Rothschild and Stiglitz, 1971). Exploiting the equivalence with the problem of the optimal demand for coinsurance, these changes do not necessarily raise the demand for insurance either. Sufficient conditions that have been proposed in the literature are that relative risk aversion be bounded by unity or that relative prudence be bounded by 2, respectively (Hadar and Seo, 1990; Chiu et al., 2012). In the case of nonperformance risk, Doherty and Schlesinger (1990) show that the relationship between the consumer's optimal demand for insurance and the probability of contract nonperformance is indeterminate. Only when the utility function is quadratic or when preferences exhibit constant absolute risk aversion (CARA), then the optimal level of coverage is monotonically decreasing in the probability of contract nonperformance. To study the comparative statics of ambiguity, we first provide two definitions.

Definition 1.

a) The consumer's beliefs are biased upwards *if his second-order beliefs undergo a first-order stochastic improvement.*

b) The consumer perceives greater ambiguity *if his second-order beliefs undergo an increase in risk in the sense of Rothschild and Stiglitz (1970).*

Under the first definition, the consumer overestimates the probability of contract nonperformance on average and his beliefs are no longer unbiased because the first-order stochastic shift relative to F implies a positive mean. The second definition has been used to study the effects of greater ambiguity on optimal self-insurance and self-protection (Snow, 2011), on the value of information (Snow, 2010), and on the incentives for genetic testing (Hoy et al., 2014). The following proposition analyzes changes in ambiguity according to Definition 1 and presents sufficient conditions for sign-definite comparative statics. They involve an intensity measure of relative ambiguity aversion and of relative prudence in ambiguity preferences, $-z\phi''(z)/\phi'(z)$ and $-z\phi'''(z)/\phi''(z)$, respectively.[9]

Proposition 3.

a) If the consumer's relative ambiguity aversion is bounded by unity, his optimal demand for insurance is lower when his beliefs are biased upwards.

b) Consider a consumer with non-negative ambiguity prudence; if his relative prudence in ambiguity preferences is bounded by 2, his optimal demand for insurance is lower with greater ambiguity.

Proof. We define the consumer's conditional first-order expression evaluated at the optimal level of insurance coverage:

$$g(\varepsilon) = \phi'\left[U(\alpha^*, \varepsilon)\right] U_\alpha(\alpha^*, \varepsilon).$$

[9] For recent studies on prudence in ambiguity preferences, see Berger (2014, 2016) and Baillon (2016).

We recoup the consumer's first-order condition as $\mathbb{E}g(\widetilde{\varepsilon}) = 0$. As shown by Rothschild and Stiglitz (1970) and more generally by Ekern (1980), the sign of subsequent derivatives of $g(\varepsilon)$ informs about how changes in risk in the distribution of $\widetilde{\varepsilon}$ affect this expectation. We obtain the first and second derivative of $g(\varepsilon)$ as

$$\begin{aligned} g'(\varepsilon) &= \phi''\left[U(\alpha^*,\varepsilon)\right]U_\varepsilon(\alpha^*,\varepsilon)U_\alpha(\alpha^*,\varepsilon) + \phi'\left[U(\alpha^*,\varepsilon)\right]U_{\alpha\varepsilon}(\alpha^*,\varepsilon), \text{ and} \\ g''(\varepsilon) &= \phi'''\left[U(\alpha^*,\varepsilon)\right]U_\varepsilon(\alpha^*,\varepsilon)^2U_\alpha(\alpha^*,\varepsilon) + 2\phi''\left[U(\alpha^*,\varepsilon)\right]U_\varepsilon(\alpha^*,\varepsilon)U_{\alpha\varepsilon}(\alpha^*,\varepsilon), \end{aligned}$$

because $U_{\varepsilon\varepsilon}(\alpha^*,\varepsilon) = U_{\alpha\varepsilon\varepsilon}(\alpha^*,\varepsilon) = 0$. We can rewrite these derivatives as follows:

$$\begin{aligned} g'(\varepsilon) &= -\phi'\left[U(\alpha^*,\varepsilon)\right]U_{\alpha\varepsilon}(\alpha^*,\varepsilon)\left\{-U(\alpha^*,\varepsilon)\frac{\phi''\left[U(\alpha^*,\varepsilon)\right]}{\phi'\left[U(\alpha^*,\varepsilon)\right]}\cdot\frac{U_\varepsilon(\alpha^*,\varepsilon)U_\alpha(\alpha^*,\varepsilon)}{U(\alpha^*,\varepsilon)U_{\alpha\varepsilon}(\alpha^*,\varepsilon)} - 1\right\} \\ g''(\varepsilon) &= -\phi''\left[U(\alpha^*,\varepsilon)\right]U_\varepsilon(\alpha^*,\varepsilon)U_{\alpha\varepsilon}(\alpha^*,\varepsilon) \\ &\quad\cdot\left\{-U(\alpha^*,\varepsilon)\frac{\phi'''\left[U(\alpha^*,\varepsilon)\right]}{\phi''\left[U(\alpha^*,\varepsilon)\right]}\cdot\frac{U_\varepsilon(\alpha^*,\varepsilon)U_\alpha(\alpha^*,\varepsilon)}{U(\alpha^*,\varepsilon)U_{\alpha\varepsilon}(\alpha^*,\varepsilon)} - 2\right\}. \end{aligned} \tag{6}$$

Both the sign of $g'(\varepsilon)$ and of $g''(\varepsilon)$ are determined by the sign of the respective curly bracket. We define

$$h(\varepsilon) = \frac{U_\varepsilon(\alpha^*,\varepsilon)U_\alpha(\alpha^*,\varepsilon)}{U(\alpha^*,\varepsilon)U_{\alpha\varepsilon}(\alpha^*,\varepsilon)},$$

which is positive for $\varepsilon < \hat{\varepsilon}$, zero for $\varepsilon = \hat{\varepsilon}$ and negative for $\varepsilon > \hat{\varepsilon}$. For $\varepsilon \neq \hat{\varepsilon}$, we can rewrite it as

$$h(\varepsilon) = \left(-\frac{U_\varepsilon(\alpha^*,\varepsilon)}{U(\alpha^*,\varepsilon)}\right) \Big/ \left(-\frac{U_{\alpha\varepsilon}(\alpha^*,\varepsilon)}{U_\alpha(\alpha^*,\varepsilon)}\right);$$

as such, $h(\varepsilon)$ compares the decay rate of expected utility with respect to the subjective belief of contract nonperformance with the decay rate of the first-order expression with respect to the subjective belief of contract nonperformance. The first derivative of $h(\varepsilon)$ is given by

$$h'(\varepsilon) = \frac{U_\varepsilon(\alpha^*,\varepsilon)U_{\alpha\varepsilon}(\alpha^*,\varepsilon)\Big[U(\alpha^*,\varepsilon)U_{\alpha\varepsilon}(\alpha^*,\varepsilon) - U_\varepsilon(\alpha^*,\varepsilon)U_\alpha(\alpha^*,\varepsilon)\Big]}{U(\alpha^*,\varepsilon)^2U_{\alpha\varepsilon}(\alpha^*,\varepsilon)^2}.$$

Using the fact that the first two terms in the numerator are negative, the sign of $h'(\varepsilon)$ is entirely determined by the square bracket in the numerator. For $\varepsilon < \hat{\varepsilon}$, we obtain the equivalence that $h'(\varepsilon) > 0$ if and only if $h(\varepsilon) > 1$. However, due to continuity of $h(\varepsilon)$, this contradicts with

$h(\hat{\varepsilon}) = 0$ so that $h'(\varepsilon) \leq 0$ and $h(\varepsilon) \leq 1$ must be satisfied for $\varepsilon < \hat{\varepsilon}$. This property together with the assumptions stated in Proposition 3 imply that $g'(\varepsilon) \leq 0$ and $g''(\varepsilon) \leq 0$ for all ε which completes the proof. □

Hence, the comparative statics of changes in ambiguity are structurally isomorphic to the comparative statics of risk in the coinsurance problem. Not all ambiguity-averse consumers with upward biased beliefs or who perceive greater ambiguity reduce their optimal insurance demand. They do so if their preferences meet certain restrictions, but might react in the opposite direction if their preferences do not qualify. Conversely, in situations where nonperformance is perceived as ambiguous, an increase in insurance demand is not necessarily indicative of a downward shift in beliefs or a decrease in ambiguity. This is important for empirical and experimental inference because relative ambiguity aversion and relative prudence in ambiguity preferences emerge as essential control variables.

We develop some intuition for the second result. A threshold value of 2 for a relative intensity measure of prudence is well known in the risk literature, for example when it comes to the effect of an increase in interest rate risk on saving behavior (see Eeckhoudt and Schlesinger, 2008; Chiu et al., 2012). The consumer experiences a substitution effect and a precautionary effect, which are also operative in our setup. The second term of $g''(\varepsilon)$ is strictly negative representing a negative substitution effect because greater ambiguity of nonperformance risk compromises the effectiveness of the insurance contract, which makes it less attractive. The first term of $g''(\varepsilon)$ corresponds to a precautionary effect which may be positive or negative. Greater ambiguity induces an increase in risk in expected utility for a given level of insurance demand. Under non-negative prudence in ambiguity preference, the consumer thus experiences an incentive to adjust the level of coverage in such a way as to increase expected utility for precautionary purposes. If the consumer perceives contract nonperformance to be likely ($\varepsilon > \hat{\varepsilon}$), this is achieved by reducing the level of coverage, consistent with the substitution effect. However, if the consumer perceives contract nonperformance to be unlikely ($\varepsilon < \hat{\varepsilon}$), this is achieved by increasing the level of coverage resulting in a potentially conflicting positive precautionary effect. The restriction on ambiguity preferences developed in Proposition 3 ensures that the precautionary effect never dominates in those cases where it is positive.

Further intuition can be developed by analyzing how greater ambiguity affects the distorted beliefs that rationalize an ambiguity-averse consumer's optimal behavior. The corresponding distorted probability of contract nonperformance risk is given by

$$\mathbb{E}^{Q^*}\{1-q+\widetilde{\varepsilon}\} = (1-q) + \frac{\mathbb{E}\{\widetilde{\varepsilon}\phi'[U(\alpha^*,\widetilde{\varepsilon})]\}}{\mathbb{E}\phi'[U(\alpha^*,\widetilde{\varepsilon})]}. \tag{7}$$

If the consumer has non-negative prudence in ambiguity preferences (i.e., $\phi''' \geq 0$), it holds that $\mathbb{E}\phi'\left[U(\alpha^*,\widetilde{\zeta})\right] \geq \mathbb{E}\phi'[U(\alpha^*,\widetilde{\varepsilon})]$ if $\widetilde{\zeta}$ denotes an increase in risk of $\widetilde{\varepsilon}$ in the sense of Rothschild and Stiglitz (1970) so that the denominator in (7) increases. The reason is that the second derivative of $\phi'[U(\alpha^*,\varepsilon)]$ with respect to ε is given by $\phi'''[U(\alpha^*,\varepsilon)]\,U_\varepsilon(\alpha^*,\varepsilon)^2$, which is non-negative. But also the numerator in (7) increases with greater ambiguity. The second derivative of $\varepsilon\phi'[U(\alpha^*,\varepsilon)]$ with respect to ε is given by

$$\begin{aligned} &2\phi''[U(\alpha^*,\varepsilon)]\,U_\varepsilon(\alpha^*,\varepsilon) + \varepsilon\phi'''[U(\alpha^*,\varepsilon)]\,U_\varepsilon(\alpha^*,\varepsilon)^2 \\ =\; &-U_\varepsilon(\alpha^*,\varepsilon)\phi''[U(\alpha^*,\varepsilon)]\left\{-U(\alpha^*,\varepsilon)\frac{\phi'''[U(\alpha^*,\varepsilon)]}{\phi''[U(\alpha^*,\varepsilon)]}\cdot\frac{\varepsilon U_\varepsilon(\alpha^*,\varepsilon)}{U(\alpha^*,\varepsilon)} - 2\right\}. \end{aligned} \tag{8}$$

The two terms outside the curly bracket are both negative. Drawing on $U(\alpha^*,\varepsilon) = U(\alpha^*,0) + \varepsilon U_\varepsilon(\alpha^*,\varepsilon)$, which is a restatement of the fact that expected utility is linear in ε, allows us to rewrite

$$\frac{\varepsilon U_\varepsilon(\alpha^*,\varepsilon)}{U(\alpha^*,\varepsilon)} = \frac{U(\alpha^*,\varepsilon) - U(\alpha^*,0)}{U(\alpha^*,\varepsilon)} < 1,$$

for any value of ε. As a result, the curly bracket in (8) is negative as soon as the consumer's relative prudence in ambiguity preferences is bounded by 2. Together with the previous observation, the overall sign of (8) is positive so that $\mathbb{E}\left\{\widetilde{\zeta}\phi'\left[U(\alpha^*,\widetilde{\zeta})\right]\right\} > \mathbb{E}\{\widetilde{\varepsilon}\phi'[U(\alpha^*,\widetilde{\varepsilon})]\}$. From Proposition 3 we know, however, that the increase in the numerator must outweigh the increase in the denominator for greater ambiguity to induce a lower level of coverage to be optimal. In such a case, the consumer's behavior with greater ambiguity can be rationalized as that of an expected utility maximizer whose distorted belief is that nonperformance is more likely compared to the case with a smaller level of ambiguity.

Finally, we come to the comparative statics of the consumer's initial wealth. In the coinsurance problem, the comparative statics of risk aversion allow to derive clear wealth effects. If the consumer's absolute risk aversion is decreasing (constant, increasing) in wealth, then the optimal level of coverage decreases (stays constant, increases) when the consumer's level of initial wealth increases, see Mossin (1968) and Schlesinger (1981). In the face of nonperformance risk, these clear comparative statics results are not recouped, see Doherty and Schlesinger (1990). For example, when absolute risk aversion decreases with wealth, insurance might or might not be an inferior good. This indeterminacy will only be exacerbated if we introduce ambiguity. We can still investigate whether ambiguity reinforces or attenuates the wealth effect.

According to the implicit function rule we differentiate the consumer's first-order condition (2) with respect to wealth:

$$\mathbb{E}\left\{\phi''[U(\alpha^*,\widetilde{\varepsilon})]U_W(\alpha^*,\widetilde{\varepsilon})U_\alpha(\alpha^*,\widetilde{\varepsilon})\right\} + \mathbb{E}\left\{\phi'[U(\alpha^*,\widetilde{\varepsilon})]U_{\alpha W}(\alpha^*,\widetilde{\varepsilon})\right\}.$$

The second term, which is governed by the sign of $U_{\alpha W}$, measures the effect of a change in initial wealth on the consumer's expected utility trade-off. This effect is known to be indeterminate from Doherty and Schlesinger (1990). The first term measures how a change in initial wealth affects the consumer's behavioral response to ambiguity and, more specifically, how an increase in wealth induces him to adjust his level of coverage in order to react to ambiguity. We will focus on this effect to answer the question whether ambiguity has a positive or negative effect on the comparative statics of wealth. Our result involves the consumer's index of absolute ambiguity aversion, denoted by $A_\phi(z) = -\phi''(z)/\phi'(z)$, and his relative ambiguity aversion as defined previously.

Proposition 4. *Consider a consumer with non-increasing absolute ambiguity aversion and relative ambiguity aversion bounded by unity. Then, ambiguity reinforces the wealth effect.*

Proof. We conclude that

$$\begin{aligned}
& \mathbb{E}\left\{\phi''[U(\alpha^*,\widetilde{\varepsilon})]U_W(\alpha^*,\widetilde{\varepsilon})U_\alpha(\alpha^*,\widetilde{\varepsilon})\right\} \\
= \; & -\mathbb{E}\left\{-\frac{\phi''[U(\alpha^*,\widetilde{\varepsilon})]}{\phi'[U(\alpha^*,\widetilde{\varepsilon})]}U_W(\alpha^*,\widetilde{\varepsilon})\phi'[U(\alpha^*,\widetilde{\varepsilon})])U_\alpha(\alpha^*,\widetilde{\varepsilon})\right\} \\
= \; & -\mathrm{Cov}\left\{A_\phi(U(\alpha^*,\widetilde{\varepsilon}))U_W(\alpha^*,\widetilde{\varepsilon}), \phi'[U(\alpha^*,\widetilde{\varepsilon})]U_\alpha(\alpha^*,\widetilde{\varepsilon})\right\},
\end{aligned}$$

where the second equality follows from the consumer's first-order condition (2). Under non-increasing absolute ambiguity aversion, the first term in the covariance is an increasing function of ε. To see this, note that

$$U_W(\alpha^*, \varepsilon) = (1-p)u'(W_1^*) + p(q-\varepsilon)u'(W_2^*) + p(1-q+\varepsilon)u'(W_3^*) > 0,$$

and that

$$U_{W\varepsilon}(\alpha^*, \varepsilon) = p\left(u'(W_3^*) - u'(W_2^*)\right) > 0,$$

where the asterisk indicates terminal wealth levels when the level of insurance coverage is given by α^*. Consequently, we obtain that

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}\varepsilon}\left(A_\phi(U(\alpha^*, \varepsilon))U_W(\alpha^*, \varepsilon)\right) \\ = \; &\underbrace{A'_\phi(U(\alpha^*, \varepsilon))}_{\leq 0}\underbrace{U_\varepsilon(\alpha^*, \varepsilon)}_{<0}\underbrace{U_W(\alpha^*, \varepsilon)}_{>0} + \underbrace{A_\phi(U(\alpha^*, \varepsilon))}_{>0}\underbrace{U_{W\varepsilon}(\alpha^*, \varepsilon)}_{>0} > 0. \end{aligned}$$

Whether the second term in the covariance is increasing or decreasing in ε depends on the sign of the function $g'(\varepsilon)$, which was introduced in the proof of Proposition 3 and shown to be non-positive if relative ambiguity aversion is bounded by unity. Consequently, the covariance is negative and the overall sign of the partial wealth effect is positive. □

The last proposition shows that insurance is less likely to be an inferior good when contract nonperformance risk is perceived as ambiguous, the consumer's absolute ambiguity aversion is non-increasing and his relative ambiguity aversion is bounded by unity. The underlying intuition is very simple. If the consumer's initial wealth increases, his expected utility is higher for any level of insurance coverage. With non-increasing absolute ambiguity aversion the consumer is less ambiguity-averse when expected utility is high than when it is low. Proposition 2 informs us that in such a situation it is optimal for the consumer to increase his level of coverage because the ambiguity associated with contract nonperformance risk is less painful.

5 Extensions and Robustness

5.1 Weighted maxmin Expected Utility

The model developed by Klibanoff et al. (2005) has the advantage that it disentangles ambiguity and ambiguity attitude. Nevertheless, it is by far not the only model of decision-making under ambiguity. In this subsection, we use the model developed by Ghirardato et al. (2004), in which the consumer's objective function is formed as a weighted average of the worst case and the best case. If all weight is attached to the worst case, maxmin expected utility emerges as a special case (Gilboa and Schmeidler, 1989).

Let $\beta \in [0,1]$ denote the consumer's degree of pessimism with respect to the perceived ambiguity of contract nonperformance $\widetilde{\varepsilon}$. Recall that the support of the consumer's beliefs is given by $[\underline{\varepsilon}, \overline{\varepsilon}]$ so that his objective function becomes

$$V(\alpha) = \beta \min_{\varepsilon \in [\underline{\varepsilon}, \overline{\varepsilon}]} U(\alpha, \varepsilon) + (1-\beta) \max_{\varepsilon \in [\underline{\varepsilon}, \overline{\varepsilon}]} U(\alpha, \varepsilon) = \beta U(\alpha, \overline{\varepsilon}) + (1-\beta) U(\alpha, \underline{\varepsilon}) = U(\alpha, \theta).$$

$\theta = \beta\overline{\varepsilon} + (1-\beta)\underline{\varepsilon}$ combines the effects of ambiguity and ambiguity aversion, and it is immediate that we recoup the interpretation of the consumer's optimal behavior as that of an expected utility maximizer with distorted beliefs of contract nonperformance. The objective function V is a concave function of the level of coverage. For ambiguity to have no effect on the consumer, it must be that $\theta = 0$, which is the case of ambiguity neutrality. Then, the optimal level of insurance coverage is given by α^0 as characterized in equation (3). The case of ambiguity aversion is that of $\theta > 0$, which is behaviorally equivalent to the optimal decisions made by an expected utility maximizer whose perceived probability of nonperformance is elevated. It follows immediately from our previous results that $\alpha^* < \alpha^0$ in such a case. In the weighted maxmin model, the optimal demand for insurance for a risk- and ambiguity-averse consumer is lower when nonperformance risk is perceived as ambiguous than when it is not. Furthermore, an increase in β leads to an increase in θ, and insurance demand decreases further, so in a sense, the more ambiguity-averse consumer demands less coverage. When it comes to the comparative statics of ambiguity, the weighted maxmin model does not produce clear predictions. The reason is that a change in risk can leave the support of the distribution unchanged, expand it on one end, or expand it on both ends. Adapting the terminology developed by Meyer and Ormiston (1985), a change in risk can be weak, "semi-strong", or strong depending on how it affects the support of the associated distribution. Conditional on β, the effect of such stochastic changes on insurance demand is nil in the first case, positive (negative) in the second case if it is the lower (upper) end of the support which is expanded, or indeterminate in the third case.[10] Wealth effects are those derived in Doherty and Schlesinger (1990) under a distorted probability distribution.

5.2 A Probability Weighting Model

Snow and Warren (2005) and Snow (2011) use a probability weighting model to analyze the effects of ambiguity on optimal decision making. Probability weighting models include rank dependent expected utility (Quiggin, 1982), the decision weighting model of Kahn and Sarin (1988), and cumulative prospect theory (Tversky and Kahneman, 1992). Let ξ denote the consumer's probability weighting function, which is an increasing function defined on cumulative probabilities and such that $\xi(1) = 1$. We exclude overinsurance (i.e., $\alpha \leq 1$) so that $W_3 < W_2 \leq W_1$ with the last inequality strict as soon as insurance is partial (see also Footnote 4). Conditional on $\widetilde{\varepsilon} = \varepsilon$, the consumer's welfare is

$$\xi(p(1-q+\varepsilon))u(W_3) + \left[\xi(p) - \xi(p(1-q+\varepsilon))\right] u(W_2) + \left[\xi(1) - \xi(p)\right] u(W_1),$$

so that his expected ex-ante welfare is given by the following utility objective:

$$V(\alpha) = u(W_1) + \xi(p)\left[u(W_2) - u(W_1)\right] + \mathbb{E}\xi\left(p(1-q+\widetilde{\varepsilon})\right)\left[u(W_3) - u(W_2)\right].$$

Ambiguity neutrality is obtained if ambiguity does not affect the consumer's objective function. For this to hold for any level of ambiguity, ξ must be linear. Now consider an ambiguity-averse consumer; for ambiguity to reduce the value of his objective function, ξ must be convex so that $\mathbb{E}\xi\left(p(1-q+\widetilde{\varepsilon})\right) < \xi(p(1-q))$.[11] Then, it is straightforward to show that, if the con-

[10] Notice that the interpretation of β is not independent of the support of the distribution of beliefs. See Huang and Tzeng (2016) for a recent application to portfolio choice.

[11] In the probability weighting model, when ambiguity is absent, the ambiguity-averse consumer's preferences

sumer is ambiguity-averse, ambiguous nonperformance risk reduces the optimal insurance demand relative to the case without ambiguity. Furthermore, an increase in the consumer's degree of ambiguity aversion, operationalized as a convex transformation of ξ, lowers the optimal demand for insurance under the additional assumption that the increase in ambiguity is such that optimal behavior in the absence of ambiguity is unaffected, see also Snow (2011). The changes in ambiguity defined in Definition 1 reduce the demand for insurance in any case and wealth effects correspond to those derived in Doherty and Schlesinger (1990) when evaluated at a distorted probability distribution.

5.3 Some Remarks on the Supply Side of the Market

Our analysis is based on the assumption that the consumer faces a premium schedule that is increasing and non-concave in the level of coverage and does not depend on the consumer's perception of ambiguity. Such a setting is quite general and includes the obvious case of a risk- and ambiguity-neutral insurer on a competitive market. In such a situation, the premium would be based on the actuarial value of the policy such that $P(\alpha) = \alpha pqmL + K$ with $m \geq 1$ being a loading factor and $K \geq 0$ a fixed cost.

A natural question is whether such a premium schedule is still obtained if the insurer perceives nonperformance risk as ambiguous and is averse to ambiguity (see Cabantous, 2007; Cabantous et al., 2011). We will investigate this question based on the three criteria of decision-making under ambiguity that were presented thus far. The insurer maximizes its profit which is determined by the premium of the policy (P), the indemnity payment in case of a loss (αL), the probability of loss (p), the probability that the contract performs ($q - \widetilde{\delta}$), the administrative costs as measured by the loading factor (m) and the fixed cost (K). For a risk-neutral insurer, the expected profit is given by

$$\widetilde{\Pi} = \Pi(\widetilde{\delta}) = P - \alpha p(q - \widetilde{\delta})mL - K,$$

where we allow for the insurer's perception of nonperformance risk to deviate from the con-

coincide with the ambiguity-neutral consumer's preferences only if their weighting functions coincide on the relevant objective probabilities. In the smooth model, this property holds generically.

sumer's, as represented by $\widetilde{\delta}$ (instead of $\widetilde{\varepsilon}$). If the insurer is ambiguity-neutral and the market is perfectly competitive, the premium reduces to the expression at the end of the previous paragraph as long as beliefs are unbiased. To incorporate ambiguity aversion, let χ denote the insurer's ambiguity function with $\chi' > 0$ and $\chi'' < 0$. If the insurer has other assets in place denoted by A and the market is perfectly competitive, the premium is implicitly defined by

$$\mathbb{E}\chi(A + P^* - \alpha p(q - \widetilde{\delta})mL - K) = \chi(A), \tag{9}$$

because the insurer is just indifferent between offering and not offering the policy. The premium can be written as $P^* = \alpha pqmL + K + \rho$, where $\rho > 0$ is a strictly positive ambiguity premium: An ambiguity-averse insurer charges a higher premium for the same policy than an ambiguity-neutral insurer would do.

To determine slope and curvature of the premium schedule, we apply the implicit function rule to (9). This yields

$$\frac{\mathrm{d}P^*}{\mathrm{d}\alpha} = pmL\left\{q - \frac{\mathbb{E}\widetilde{\delta}\chi'}{\mathbb{E}\chi'}\right\} > 0,$$

where the argument of χ' is suppressed to simplify notation. The premium schedule is increasing. Its second derivative with respect to the level of coverage is given by

$$\frac{\mathrm{d}^2P^*}{\mathrm{d}\alpha^2} = \left(\frac{pmL}{\mathbb{E}\chi'}\right)^2\left\{-\mathbb{E}\widetilde{\delta}^2\chi''\mathbb{E}\chi' + 2\mathbb{E}\widetilde{\delta}\chi''\mathbb{E}\widetilde{\delta}\chi' - \frac{\mathbb{E}\chi''\left(\mathbb{E}\widetilde{\delta}\chi'\right)^2}{\mathbb{E}\chi'}\right\}.$$

The first and the third term in the curly bracket are positive. A sufficient condition for the middle term to be non-negative is for the insurer not to be prudent in ambiguity preferences ($\chi''' \leq 0$). Another way to analyze the curly bracket is to rewrite it as follows:

$$\mathbb{E}\widetilde{\delta}\chi''\mathbb{E}\widetilde{\delta}\chi' - \mathbb{E}\widetilde{\delta}^2\chi''\mathbb{E}\chi' + \frac{\mathbb{E}\widetilde{\delta}\chi'}{\mathbb{E}\chi'}\left(\mathbb{E}\chi'\mathbb{E}\widetilde{\delta}\chi'' - \mathbb{E}\chi''\mathbb{E}\widetilde{\delta}\chi'\right).$$

Similar techniques as in Appendices 1 and 2 of Peter et al. (2016) show that the sign of the sum of the first and the second term is governed by the slope of relative ambiguity aversion whereas the sign of the round bracket is governed by the slope of absolute ambiguity aversion.[12] If absolute ambiguity aversion is non-increasing and relative ambiguity aversion is non-decreasing, the overall sign is negative resulting in a concave premium schedule. As such, a necessary condition for a non-concave premium schedule is that either absolute ambiguity aversion is increasing, in which case relative ambiguity aversion is too, or that relative ambiguity aversion is decreasing, in which case absolute ambiguity aversion is too.

In case of the weighted maxmin expected utility model, let $\gamma \in [0,1]$ denote the insurer's degree of pessimism and $[\underline{\delta}, \overline{\delta}]$ the support of the insurer's beliefs. The premium is determined by requiring that

$$\gamma \min_{\delta \in [\underline{\delta}, \overline{\delta}]} (A + \Pi(\delta)) + (1-\gamma) \max_{\delta \in [\underline{\delta}, \overline{\delta}]} (A + \Pi(\delta)) = A + \Pi(\eta) = A,$$

with $\eta = \gamma \underline{\delta} + (1-\gamma)\overline{\delta}$, so that $P^* = \alpha p(q-\eta)mL + K$. From the insurer's perspective, $\eta < 0$ corresponds to ambiguity aversion because scenarios with a higher probability of having to pay the indemnity yield lower expected profits. The premium is higher than the premium charged by an ambiguity-neutral insurer but it is still linearly increasing in the level of coverage.[13]

Finally, we can apply the probability weighting model to the insurer's pricing decision. For the insurer there are only two relevant states of the world, the one where it pays the indemnity and the one where it does not. The insurer's profit is lower in the first one, which occurs with probability $p(q - \widetilde{\delta})$, than in the second one, which occurs with probability $(1+p) + p(1 - q + \widetilde{\delta})$. If λ denotes the insurer's probability weighting function, the premium is determined by requiring that

$$\begin{aligned} & \mathbb{E}\lambda \left(p(q-\widetilde{\delta})\right)(A + P - \alpha mL - K) + \left(1 - \mathbb{E}\lambda\left(p(q-\widetilde{\delta})\right)\right)(A + P - K) \\ = \quad & A + P - \alpha mL \cdot \mathbb{E}\lambda\left(p(q-\widetilde{\delta})\right) - K = A, \end{aligned}$$

so that $P^* = \alpha mL \cdot \mathbb{E}\lambda\left(p(q-\widetilde{\delta})\right) + K$. For ambiguity to reduce the insurer's expected profit, λ needs to be convex resulting in a higher premium charged by the ambiguity-averse insurer

[12] $\mathbb{E}\widetilde{\delta}\chi' < 0$ follows from ambiguity aversion.

[13] Depending on their informational endowment, consumers might be able to back out information about the insurer's perceived level of ambiguity from the observed market prices. Then, to be consistent the consumer's perception of ambiguity should be such that it is compatible with the one implied by the insurer's pricing.

than by the ambiguity-neutral one.[14] The premium is a linear function of the level of coverage and our results on the effects of the consumer's perceived ambiguity of nonperformance risk continue to hold. The same qualification as in Footnote 13 applies. Overall, ambiguity aversion of the insurer can result in concavity of the premium schedule under the smooth model but never under the other two models. Notice that non-concavity of the premium schedule was only a sufficient condition for the validity of the first-order approach but not necessary.

5.4 Ambiguity Associated with the Risk of Loss

To isolate the effects of an ambiguous perception of nonperformance risk, we assumed the probability of loss to be known by the consumer. A natural question is to wonder to what extent our results carry over to situations in which both the risk of loss and the risk of nonperformance are perceived as ambiguous. When the probability of contract nonperformance is known, ambiguity associated with the risk of loss no longer has clear effects on insurance demand, contrary to the findings in Alary et al. (2013). Intuitively, the behavior of an ambiguity-averse consumer under ambiguity is observationally equivalent to that of an expected-utility maximizer who is more pessimistic about the probability of loss. This increases the marginal benefit of insurance because the loss state in which the insurance contract performs is perceived as more likely. There is a negative effect on the marginal cost of insurance because the no-loss state becomes less likely but there are also two positive effects on the marginal cost because both the state in which the loss happens and insurance performs and in which the loss happens and insurance does not perform are perceived as more likely. The sum of the two positive effects preponderates the negative effect because marginal utility in the latter two cases is higher than in the first one so that overall the marginal cost of insurance increases. As a consequence, the net effect of an ambiguous perception of the risk of loss on insurance demand is indeterminate when contract nonperformance is possible.

However, we are still able to say something about the effects of an ambiguous perception of nonperformance risk. Conditional on the risk of loss being perceived as ambiguous, ambiguity associated with the risk of nonperformance lowers the demand for insurance when

[14] This holds under the additional assumption that the weighting function of an ambiguity-neutral and an ambiguity-averse insurer coincide on the relevant objective probabilities.

both sources of ambiguity are independent. Also, the sufficient conditions for upward biased beliefs and for greater ambiguity to lower the optimal demand for insurance and for a positive partial wealth effect remain unaltered. This shows that Propositions 1, 3 and 4 are robust to the consideration of ambiguity associated with the risk of loss as long as both sources of ambiguity are unrelated. Only Proposition 2 does no longer hold because an increase in ambiguity aversion results in a tension between a negative effect of ambiguity associated with nonperformance risk and an indeterminate and potentially positive effect of ambiguity associated with the risk of loss on insurance demand. This resembles the same tension that impedes signing the comparative statics of risk aversion in the Doherty and Schlesinger (1990) model.

6 Conclusion

In this paper, we analyze optimal insurance demand if consumers face contract nonperformance risk. Insurance offers protection against financial risks, but sources of nonperformance abound including insurer default, contested claims, procedural delays, contractual uncertainty and probationary periods. Whereas existing literature assumes the level of nonperformance risk to be known by the consumer, we study the case of ambiguity and show how this ambiguity affects optimal demand. Despite the fact that most of the comparative statics of nonperformance risk are indeterminate (Doherty and Schlesinger, 1990), we are able to show that the comparative statics of ambiguity are much better behaved. Using the smooth model of ambiguity aversion, we find that ambiguity lowers the optimal demand for insurance. Furthermore, greater ambiguity aversion lowers the demand for insurance and so do upward biased beliefs and greater ambiguity if the consumer's relative ambiguity is bounded by unity or his relative prudence in ambiguity preferences is bounded by 2, respectively. Finally, we analyze a partial wealth effect and determine sufficient conditions under which insurance is less likely to be an inferior good when contract nonperformance risk is perceived as ambiguous. Our main results are robust to the specification of ambiguity preferences, which we scrutinize by investigating a weighted maxmin expected utility model and a probability weighting model. Furthermore, depending on the implementation of ambiguity preferences, our findings continue to hold when the insurer is ambiguity-averse, or when the risk of loss is perceived as ambiguous.

The reason why ambiguity associated with contract nonperformance risk has clear comparative statics whereas nonperformance risk itself does not is related to the mechanism through which both operate. In Doherty and Schlesinger (1990), nonperformance risk is reflected in the odds of the contract performing but also reduces the premium of the insurance contract. The latter is a wealth effect which undermines the comparative statics and renders most of the results indeterminate. In our paper, the focus is on the consumer's perception of contract nonperformance risk so that the insurer's pricing rule is taken as given. As a consequence our model does not contain the conflicting wealth effect that the original Doherty and Schlesinger (1990) model does. This assumption appears natural in our context since most insurer are unlikely to be aware of consumers' perception of nonperformance risk.

Our results show that ambiguity associated with contract nonperformance risk has very clear demand effects, which are not obtained by a mere focus on nonperformance risk and risk aversion. Due to the fact that optimal demand is lower, ambiguous nonperformance risk is a potential cause of uninsurability. Some have argued that this might be particularly relevant in less developed insurance markets where trust in financial institutions might be limited and insurers are less stringently regulated (e.g., Biener et al., 2016). The findings in this paper suggest that informational policies that eliminate ambiguity associated with contract nonperformance risk increase the ex-ante welfare of ambiguity-averse consumers and stimulate demand on the market. However, for the more likely case of a partial reduction in ambiguity, ex-ante welfare still increases whereas demand might rise or fall, depending on the consumer's relative prudence in ambiguity preferences. Furthermore, if one believes that ambiguity aversion is a psychological tendency that inhibits "good" decision-making and that expected utility represents the appropriate normative framework, our results show that ambiguity-averse consumers purchase less insurance than they should as soon as they perceive nonperformance risk as ambiguous. In such a situation policy interventions that reduce the consumer's degree of ambiguity aversion enhance welfare as do policy interventions that eliminate ambiguity.

There are several avenues for further research. A natural question is whether similar demand effects arise when the reliability of other risk management instruments like self-insurance or self-protection is perceived ambiguous. Whereas self-insurance and (market) insurance appear to behave similarly in most contexts, suggesting that similar demand effects as discussed in this paper would arise for self-insurance, the contrary is true for self-protection and insurance (see Ehrlich and Becker, 1972), leaving this an interesting topic for future research. Furthermore, our paper assumes ambiguity as a primitive of the consumer's decision-making environment. An important source of nonperformance risk is correlation between the risks of different insurees. In such a situation, nonperformance risk and the perception thereof are endogenously determined by insurance demand, which might generate further interesting implications. Lastly, applications of nonperformance risk in asymmetric information environments with and without ambiguity are largely unexplored.

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