Parental Education Investment Decision with Imperfect Talent Signal

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Abstract

This paper builds up a theoretical parental education investment model which employs discontinuous utility function and introduces uncertainty into parental education investment decision. With some basic and reasonable assumptions, this paper proves that the correlation between talent signal and optimal education investment level is not monotone. In most situations, the correlation will be positive. However, when parents' utility function is discontinuous at some thresholds, The bonus can motivate parents to add more investment on children so that they can reach the thresholds. In this case, the optimal education investment will decrease as talent increases.

This paper also introduces uncertainty into the decision process. With uncertainty, parents will maximize the expected utility. the optimal education investment curve will be smoothed out by the expectation process. The larger the standard deviation is, the smoother the curve becomes.

Moreover, the paper can also help us to understand the drop-out issue. The model predicts that the drop-out rate will be lower for students whose current years of schooling are close to graduation. The reason is that degrees have values which can motivate students to finish their degrees.

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Introduction

How do parents invest on children's education is always an interesting topic in labor economics. The classic parental education investment model treats parental human capital investment as intergenerational transfer and parents are willing to invest in children's education until the point where the marginal increase in the children's earnings from one more dollar invested equals the market interest rate (Glomm, 1997). Therefore, education investment on children with high ability should be higher (Raut and Tran, 2005).

However, the existence of remedial programs shows that students with poor performance gain additional education investment (Jacob and Lefgren, 2002). Parents of disadvantaged children also invest more on the education of their children (Heckman, 2006). There is also empirical evidence shows that parents invest more in children with lower birth weight. There are already numerous of papers which prove a positive correlation between birth weight and performance in school. Therefore, we can conclude that parents are willing to invest in children with poor performance, which conflicts with the conclusion from the classic model. Dizon-Ross also finds evidence from data that sometimes parents allocate more remedial education investments on children with relatively poor performance (Dizon-Ross, 2013). Although some scholars detect the phenomenon, the explanations given in their papers are still based on the classic model and insist parents invest more on poor performance children as a kind of compensation or due to inaccurate information (Dizon-Ross, 2013; Kinsler and Pavan, 2016). This paper constructs a theoretical model for parental education investment decision. Instead of assuming a smooth utility function which only depends on children's performance, this model allows parents' utility function to be discontinuous at some threshold points. The reason is that parents might get additional bonus when the child's school performance reach certain thresholds, say finish a degree. When the bonus for reaching the threshold is large enough, parents are willing to invest on the child so that he/she can reach the threshold even when the marginal benefit is lower than the marginal cost. With the assumption that marginal benefit is increasing in talent, parental education investment is decreasing in children's ability if parents prefer to invest until their child reach the threshold. When the bonus is not large enough, parents will still prefer the education investment level at which the marginal cost is equal to marginal benefit. In this case, their education investment behavior will follow the prediction of the classic model and invest more when they think their children is talented.

This paper enriches the literature in the way that it employs discontinuous parental utility function. It allows the model to explain the phenomenon which cannot be explained in the previous models. To the best of my knowledge, existing literatures always assume parental utility function to be smooth and continuous in children's performance.

The paper also adds to the literature on parental education investment by emphasizing the importance of parents' belief of their children's ability in parental education investment decision. In the classic model, parental education investment is assumed to depend on children's real ability. However, in most cases, the true ability is unobservable. Parents need to make education investment based on their belief of child's ability based on some signals. There are already some literatures which argue parents' belief is the factor that really matters (Dizon-Ross, 2013; Kinsler and Pavan, 2016). Some papers also investigate how parental beliefs are determined and conclude these beliefs are influenced by children's performance in school and also the performance of children in the same school (Kinsler and Pavan, 2016). This paper explores how parents make education investment decisions when perfect signal is available as well as when uncertainty exists.

The first part discusses the parental education investment decision in perfect signal setting; in the second part, uncertainty is introduced and I discuss how the predictions in part 1 change when the signal is imperfect; the third part is an example to illustrate the predictions in a more intuitive way; the last part is a conclusion.

Theoretical Model

Setting

Parents have certain amount of endowment I and they need to allocate the amount of money into two potential parts: consumption C and education investment on a child EI. Assume child's talent is t. With certain parental education investment, the child's school performance (eg. year of schooling, collage ranking, and SAT grades) is $R = R(t, EI)$. Parents' utility function is in the following form:

$$u = U(C) + V(t, EI)$$

It contains two parts, the utility from consumption $U(C)$ and the utility from child's education $V(t, EI)$.

Parents need to optimize the allocation of the endowments based on a signal $\hat{t} = t + \Delta t$ observed.

Assumptions

To discuss this issue, I made the following reasonable assumptions.

Assumption 1 $U' > 0, U'' < 0$

The utility from consumption is increasing and concave.

Assumption 2 $V(t, EI)) = R(t, EI) + k \cdot 1\{R(t, EI) > Th\}$

where Th is a threshold which parents want their child to reach. They will get additional bonus k when the child successfully reach the threshold. $1\{R(t, EI) \geq Th$ is defined as below:

$$1\{R(t, EI) \geq Th\} = \begin{cases} \text{1, if R(t,EI)}\geq \text{Th} \\ \text{0, Otherwise} \end{cases}$$

The utility from child's education contains two parts: 1. a "years of schooling function" and 2. a bonus which exist only if the child's years of schooling reach certain threshold.

Assumption 3 $\frac{\partial R}{\partial t} > 0, \frac{\partial R}{\partial EI} > 0, \frac{\partial R^2}{\partial t^2} < 0, \frac{\partial R^2}{\partial EI^2} < 0, \frac{\partial R^2}{\partial EI \partial t} > 0$

The "years of schooling" function is assumed to be increasing and concave in both talent and education investment. Moreover, individuals with higher talent will have higher marginal benefit at any given education investment level.

Part 1 Perfect signal

In this part, I discuss the case when parents receive full information of the kid's talent. That is to say, I add the additional assumption that $\hat{t} = t$.

The marginal cost of additional investment on kid's education is the forgone consumption, $\frac{\partial U}{\partial C}$. The marginal benefit of education investment is the marginal utility from additional years of schooling caused by additional education investment. When it's not at the threshold point, the marginal benefit can be represented as $\frac{\partial R}{\partial EI}$.

At the threshold point, however, the marginal benefit will be infinite. The marginal benefit and marginal cost curves are plotted in Figure 1. Here the y axis is the education investment and x axis represents costs/benefits. Point A is when marginal cost is equal to marginal benefit and the education investment level at Point A is referred as $EI_{MC=MB}$. EI_{Th} represents the education investment needed to reach the threshold for a kid with talent t.

For any given t and Th, parents can find the optimal education investment level which maximizes their utility. Depending on the value of parameters and function forms, there are three possible cases:

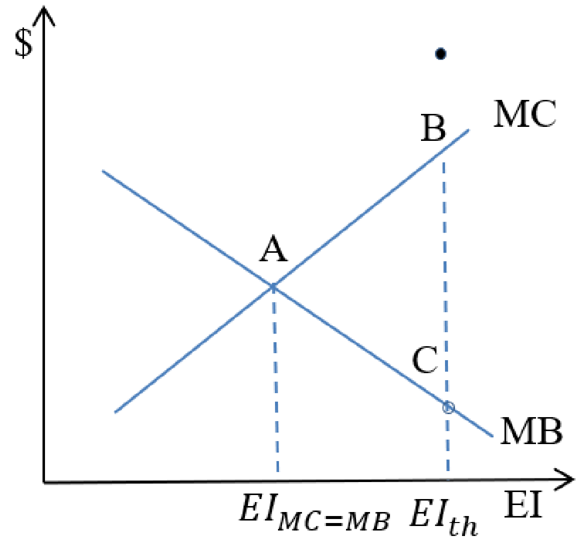


Figure (1) Marginal Cost VS. Marginal Benefit

a) $EI_{MC=MB} > EI_{Th}$, parents' utility will always be maximized at the $EI_{MC=MB}$ regardless of the magnitude of the bonus;

b)$EI_{MC=MB} < EI_{Th}$ but $k < area_{ABC}$[1], the optimal education investment level is still $EI_{MC=MB}$;

c)$EI_{MC=MB} < EI_{Th}$ and $k < area_{ABC}$, here the bonus of the child reaching the threshold is so large that parents will always be willing to invest on the child until she/he reach it. The first two cases can be combined into one as the equilibrium is the same.

Now I want to discuss how the change of talent t affects the level of optimal education investment. One way to interpret this question is that "will parents be more willing to invest on their child's education if the child is more talented?"

Binary Discussion

In the binary setting, the "talent" variable only has two classes. In another word, a child is either a low-talent type or high-talent type. The low and high-type individuals are allowed to be different in talents (t^L and t^H), thresholds (Th^L and Th^H)[2], and bonuses for reach the thresholds ($k^L$and k^H). The education investment levels ensuring marginal cost equal to marginal benefit are referred as $EI^L_{MC=MB}$ and $EI^H_{MC=MB}$ respectively, the education investments needed to reach the thresholds are represented as EI^L_{Th} and EI^H_{Th} respectively, and the optimal education investment levels are referred as EI^{L*} and EI^{H*} respectively.

As each type has two possible equilibria (the marginal benefit equal to marginal cost level and the level ensure child just reach the threshold), there are four potential cases.

Case 1: Both types choose their MC=MB points When the bonuses for reaching the thresholds are small enough, both types prefer to stay at their MC=MB points (Figure 2 a).

As the Assumption 2 assumes that $\frac{\partial R^2}{\partial EI \partial t} > 0$, the marginal benefit for the high-talent type is always higher than the one for the low-talent type for any given education investment level. At the $EI^L_{MC=MB}$ point, the marginal benefit of education investment for the high-type is larger than the marginal cost, so parents will be willing to investment more on them. Therefore, the optimal

[1]The $area_{ABC}$ represents the size of the triangle region ABC in Figure 1.

[2]Although the two types are allowed to have different thresholds, I still assume the thresholds are exogenous.

education investment level for the high-talent type will be higher than the one for the low-talent type.

$$EI^{L*} = EI^{L}_{MC=MB} < EI^{H}_{MC=MB} = EI^{H*}$$

Case 2: High type chooses threshold point, low type chooses its MC=MB point This case is likely to be true when the bonus for the low-type is small and the threshold for the low-type is high, whereas the bonus for the high-type is large and the threshold for the high-type is low (Figure 2 b).

With Assumption 2, it's true that $EI^{L}_{MC=MB} < EI^{H}_{MC=MB}$. Moreover, the high type only goes to the threshold ranking if $EI^{H}_{MC=MB} < EI^{H}_{Th}$. Therefore,

$$EI^{L*} = EI^{L}_{MC=MB} < EI^{H}_{MC=MB} < EI^{H}_{Th} = EI^{H*}$$

The optimal education investment level for the high-talent type is higher than the one for the low-talent type.

Case 3: High type chooses its MC=MB point, low type chooses threshold point

This case is likely to be true when the bonus for the low-type is large and the threshold for the low-type is low, whereas the bonus for the high-type is small and the threshold for the high-type is high (Figure 2 c).

In this case, it's hard to compare the two optimal education investments without additional assumptions. When the threshold for the low-type person is higher or equal to the one for the high-type, $Th^{L} \geq Th^{H}$, the optimal education investment for the low type will be higher than the high type for sure as $EI^{L*} = EI^{L}_{Th} > EI^{H}_{Th} > EI^{H*}$. However, the assumption made here is very odd since it is assuming that parents are setting harder goal for less able child.

Normally, the threshold and the talent are more likely to be positively correlated, which means the threshold for the low type person is lower than the one for the high type one. In this situation, the difference between the marginal effect of education investment on years of schooling $\frac{\partial R(t^{L},EI)}{\partial EI}$ and $\frac{\partial R(t^{H},EI)}{\partial EI}$ and the gap between the $EI^{L}_{MC=MB}$ and $EI^{H}_{MC=MB}$ will play important roles.

The optimal education investment for the low type can be higher than the one for the high type if the gap between the marginal effect of education investment on years of schooling is large, $\frac{\partial R(t^{L},EI)}{\partial EI} << \frac{\partial R(t^{H},EI)}{\partial EI}$. In another word, if the education investment on the low type person is actually very inefficient, the amount of education investment for the child to reach the threshold will be really large. In this case, the fact that low-talent type still prefer the threshold point shows that the bonus for the low-type reaching the threshold is very large.

Another potential case for the optimal education investment for the low type to be higher than the one for the high type is when $EI^{H}_{MC=MB} - EI^{L}_{MC=MB}$ is small. As $EI^{L*} = EI^{L}_{Th} > EI^{L}_{MC=MB}$, when the difference between $EI^{H}_{MC=MB}$ and $EI^{L}_{MC=MB}$ is smaller than the difference between EI^{L}_{Th} and $EI^{L}_{MC=MB}$, $EI^{L*} = EI^{L}_{Th} > EI^{H}_{MC=MB} = EI^{H*}$.

Case 4: Both types choose their threshold points

With the odd assumption that $Th^{L} > Th^{H}$, the optimal education investment for the low type will be higher than the high type for sure as $EI^{L*} = EI^{L}_{Th} > EI^{H}_{Th} = EI^{H*}$. However, when $Th^{L} < Th^{H}$, it becomes very ambiguous (Figure 2 d).

When the difference between thresholds is relatively small (the extreme case will be the threshold is the same for both types) and the education investment is much more efficient for the high type ($\frac{\partial R(t^L,EI)}{\partial EI} << \frac{\partial R(t^H,EI)}{\partial EI}$), the optimal education investment of the low type child will exceed the one for the high type child. Otherwise, the education investment for the high-talent type will be higher.

Continuous Discussion

In the real life, child's talent is more likely to change continuously rather than include just two types. In the binary case, I can only analyze non-trivial change of talent. To estimate the marginal effect of talent change on education investment, in this section, I will discuss the case when *talent* is a continuous variable rather than a binary one.

Based on the utility function it's clear that the marginal cost of education investment is $\frac{\partial U}{\partial EI}$ and the marginal benefit is $\frac{\partial R}{\partial EI}$. The education investment at the MC=MB point will satisfy

$$\frac{\partial U}{\partial EI_{MC=MB}} = \frac{\partial R}{\partial EI_{MC=MB}}$$

Taking derivative of the both sides of the equation with respect to t, I can get

$$\frac{\partial U}{\partial EI_{MC=MB}} \cdot \frac{\partial EI_{MC=MB}}{\partial t} = \frac{\partial R}{\partial EI_{MC=MB}} \cdot \frac{\partial R}{\partial t} + \frac{\partial^2 R}{\partial EI_{MC=MB}\partial t}$$

With Assumption 1 and 3, $\frac{\partial EI_{MC=MB}}{\partial t}$ is greater than 0. This is to say, the education investment at the MC=MB point will increase as the talent increases.

Moreover, if I represent the education investment at the threshold point Th[3] as EI_{Th}, the following equation should be true.

$$Th = R(t, EI_{th})$$

Taking derivative of the both sides of the equation with respect to t, I can get

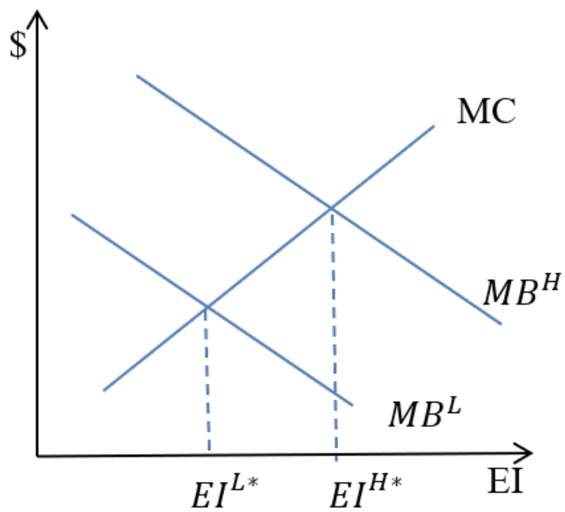
$$\frac{\partial EI_{Th}}{\partial t} = (\frac{\partial Th}{\partial t} - \frac{\partial R}{\partial t}) \cdot (\frac{\partial R}{\partial EI_{Th}})^{-1}$$

As the Assumption 3 made before ensure $\frac{\partial R}{\partial EI_{Th}} > 0$, how EI_{Th} changes when talent changes really depends on the relationship between $\frac{\partial Th}{\partial t}$ and $\frac{\partial R}{\partial t}$. If $\frac{\partial Th}{\partial t} > \frac{\partial R}{\partial t}$, which means the adjusting rate of threshold is faster than the changing rate of years of schooling function (parents lower their threshold a lot when they believe the child is less able), the education investment ensuring child to reach the threshold will be positively correlated with talent. Otherwise, the correlation will be negative.

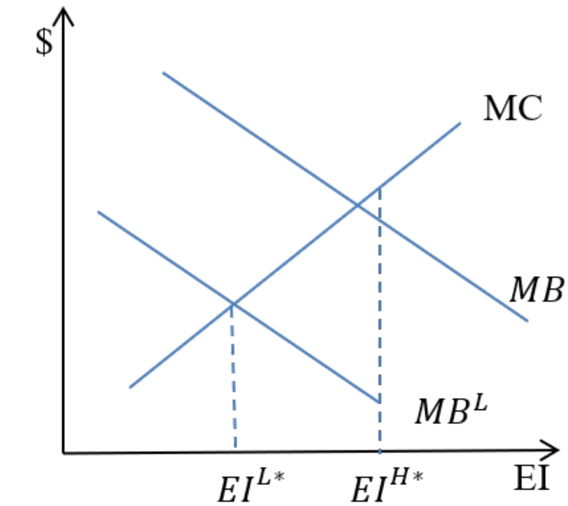
Based on the discussion above, I can conclude how the optimal education investment changes with respect to marginal talent changes. When the optimal point is at the MC=MB point, a marginal increase of talent will increase the optimal education investment. Whereas when the optimal point is at the threshold point, the correlation between the marginal change of talent and the marginal change of optimal education investment depends on the values of $\frac{\partial Th}{\partial t}$ and $\frac{\partial R}{\partial t}$. When the adjusting rate of threshold is faster than the changing rate of years of schooling function, the education investment ensuring child to reach the threshold will be positively correlated with talent. Otherwise, the correlation will be negative. When the change of the talent is non-trivial, the optimal strategy might shift from MC=MB point to threshold point (or the reverse direction). In these cases, it's back to the case 2 and case 3 in the binary discussion section.

The flexibility of the thresholds is somehow problematic and result in ambiguous predictions. To overcome this issue, I will make one additional assumption.

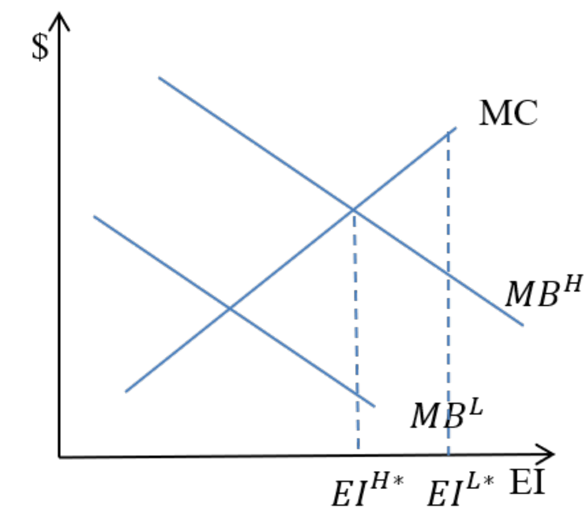
[3] Here I assume the threshold Th to be exogenous but allow the it to be a function of talent t.



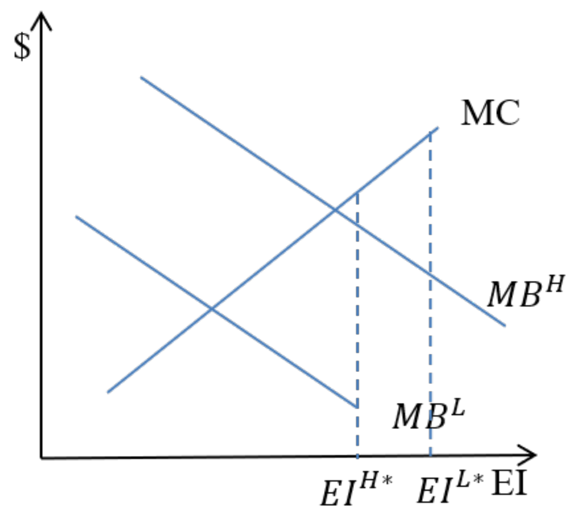
(a) Both types choose MC=MB points



(b) High type chooses threshold point, low type chooses MC=MB point



(c) High type chooses MC=MB point, low type chooses threshold point



(d) Both types choose threshold points

Figure (2) Perfect Signal & Binary *talent*

Assumption 4 $\frac{\partial Th}{\partial t} = 0$

This is to say, from now on, I assume the threshold is always the same for everyone. This is a reasonable assumption based on the real life. For example, a lot of empirical literature find significant evidence that people with high school degree will earn higher wage than the ones who drop out. a part of the difference can be explained by the education, but not all of it. For the rest which cannot be explained by the education, I define it as "the value of degree". It's a type of bonus gained when reaching certain threshold, getting a high school degree[4]. In this case, the Assumption 4 is true.

With the Assumption 4, the previous equation can be transferred into

$$\frac{\partial EI_{Th}}{\partial t} = -\frac{\partial R}{\partial t} \cdot (\frac{\partial R}{\partial EI_{Th}})^{-1}$$

Therefore, the education investment needed to reach certain threshold Th is decreasing as the talent increases.

Figure 3 shows the correlation between talent and optimal education investment. When the child's talent is at a very low level, the additional cost of investing on the child until he/she reaches the threshold is greater than the additional benefits and bonus. Parents will always stop investment at the MC=MB point, so the curve of the optimal education investment will be the same as the curve of the $EI_{MC=MB}$. As $\frac{\partial EI_{MC=MB}}{\partial t}$ is greater than 0, the optimal education investment will be increasing in t. As the talent level increasing, the marginal benefit curve and optimal education investment increase, the difference between the costs of investing the child to the threshold point and the MC=MB point also becomes smaller and smaller. At $t = t_1$, the additional cost of investing the child to the threshold point, S_{ABC} is the same as the additional bonus k. Therefore, child with talent t_1 will get education investment which is just sufficient for him/her to reach the threshold Th. For $t = t_1 + \Delta t$ where $\Delta t \to 0_+$, the marginal benefit becomes greater, so the cumulative difference between the marginal cost and marginal benefit will be smaller than the bonus. The child with talent $t_1 + \Delta t$ will also get education investment which is just sufficient for him/her to reach the threshold Th. When talent is greater than t_1 and the marginal cost is higher than the marginal benefit at the threshold point, parents will always invest on the child until he/she just reach the threshold. The curve of the optimal education investment will be the same as the curve of the $EI_{MC=MB}$. $\frac{\partial EI_{Th}}{\partial t}$ is negative, so the optimal education investment will decrease as talent increases. For $t > t_2$, however, the marginal benefit is higher than the marginal cost at the threshold point, parents will keep increasing their investment until the marginal cost is equal to marginal cost. Therefore, for $t > t_2$, the correlation between optimal education investment and talent is back to positive again.

Figure 4 shows the potential curve when there are multiple thresholds: elementary school degree, high school degree, and college degree. this finding can help us to understand the drop-out issue. Based on this prediction, the drop-out rate should be lower for students whose current years of schooling are close to graduation. For example, a student at his/her last year of elementary school/high school/college is less likely to drop out. The reason is that degrees have values which are bonuses motivating students to finish their degree.

Part 2 Imperfect Signal

In Part 1 I discussed the case when parents know exactly the true talent of their children. This assumption helps simplify the question in some ways, but it's unlikely to be true in real life. Now I will discuss the situation when parents receive imperfect signal. That is to say, the signal of talent received by the parents does not represent the true value of the talent but parents know the conditional distribution of the true talent.

Continuous Discussion

In this section, the *talent* variable is allowed to be change continuously.

[4]Here I ignore the quality issue and assume the year of schooling is the only thing matters.

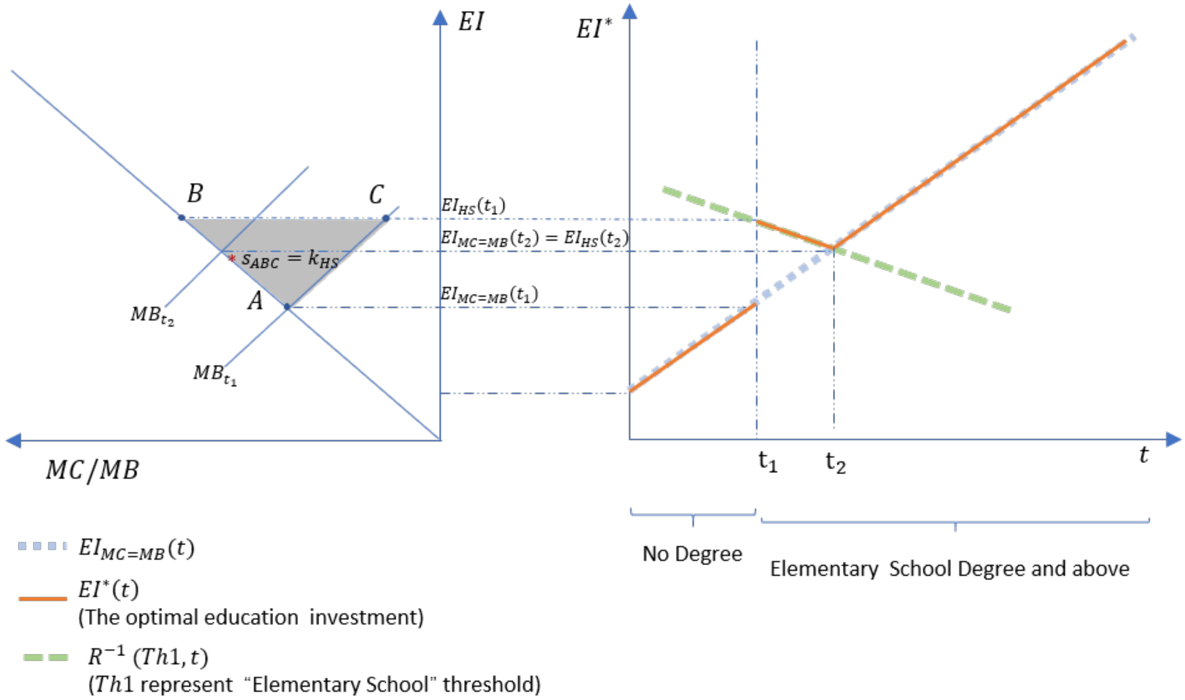


Figure (3) The correlation between t and EI^* with one threshold

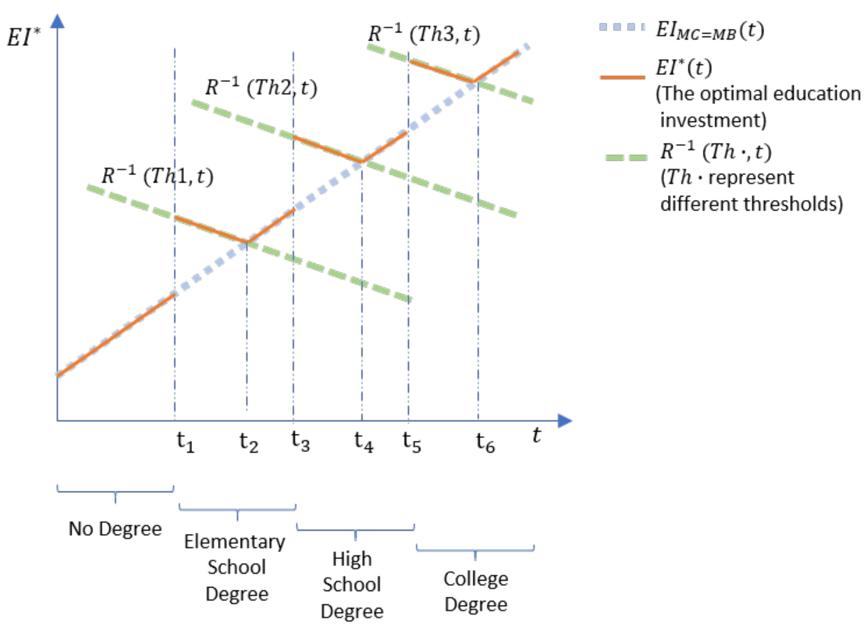


Figure (4) The correlation between t and EI^* with multiple thresholds

Now the parents need to maximize their expected utility as they only know the conditional distribution of the talent rather than the true talent. The optimization equation is as below:

$$\max_{EI} E[u|\hat{t}] = U(I - EI) + \int R(t, EI) \cdot f(t|\hat{t})dt + k \cdot [1 - F(R^{-1}(Th, EI)|\hat{t})]$$

where $F(\cdot|\hat{t})]$ is the conditional cumulative distribution function of the true talent given the signal $\hat{t}$. $R^{-1}(Th, EI)$ is the inverse function of $R(\cdot)$ and it represents the lowest talent able to reach the threshold with education investment EI. Therefore, $R[R^{-1}(Th, EI), EI] = Th$.

The first order derivative (FOC) is as below:

$$\text{FOC:} \qquad \frac{\partial E[u|\hat{t}]}{\partial EI} = -U^{'} + \int \frac{\partial R}{\partial EI} \cdot f(t|\hat{t})dt - k \cdot f(R^{-1}(Th, EI)|\hat{t}) \cdot \frac{\partial R^{-1}}{\partial EI}$$

I can also get the second order derivative (SOC):

$$\begin{aligned} \text{SOC:} \qquad \frac{\partial^2 E[u|\hat{t}]}{\partial^2 EI} &= U^{''} + \int \frac{\partial^2 R}{\partial^2 EI} \cdot f(t|\hat{t})dt \\ &- k \cdot [f^{'}(R^{-1}(Th, EI)|\hat{t}) \cdot (\frac{\partial R^{-1}}{\partial EI})^2 + f(R^{-1}(Th, EI)|\hat{t}) \cdot \frac{\partial^2 R^{-1}}{\partial^2 EI}] \end{aligned}$$

The sign of the second order derivative is ambiguous as I cannot sign all the terms in the SOC based on current assumptions. However, I know that SOC will always be negative at the global maximum point.

Comparative Statics

Here I discuss the Comparative Statics of marginal deviations from the optimal points.

a. For change in bonus

$$\frac{\partial EI^*}{\partial k} = -\frac{\frac{\partial FOC}{\partial k}}{SOC} = -\frac{-f(R^{-1}(Th, EI)|\hat{t}) \cdot \frac{\partial R^{-1}}{\partial EI}}{SOC}$$

As I am discussing trivial changes at maximum point, SOC will be negative for sure. Therefore, $\frac{\partial EI^*}{\partial k} \geq 0$. This is to say, when the bonus increases, parents are always willing to invest more on child's education. This conclusion is true as long as $\frac{\partial R^{-1}}{\partial EI} > 0$, which is automatically ensured by the Assumption 2.

b. For change in talent signal

$$\frac{\partial EI^*}{\partial \hat{t}} = -\frac{\frac{\partial FOC}{\partial \hat{t}}}{SOC} = -\frac{\int \frac{\partial R}{\partial EI} \cdot \frac{\partial f(t|\hat{t})}{\partial \hat{t}}dt - k \cdot \frac{\partial f(R^{-1}|\hat{t})}{\partial \hat{t}} \cdot \frac{\partial R^{-1}}{\partial EI}}{SOC}$$

In the case when there is no bonus, that is to say $k = 0$, the equation can be simplified into

$$\frac{\partial EI^*}{\partial \hat{t}} = -\frac{\int \frac{\partial R}{\partial EI} \cdot \frac{\partial f(t|\hat{t})}{\partial \hat{t}}dt}{SOC}$$

As I am discussing at the global maximizing point, I know the $SOC < 0$. Therefore, the sign of $\frac{\partial EI^*}{\partial \hat{t}}$ is the same as the sign of $\int \frac{\partial R}{\partial EI} \cdot \frac{\partial f(t|\hat{t})}{\partial \hat{t}}dt$. However, without further assumption on the conditional probability distribution function, I cannot get a clear prediction of the sign. Therefore, I made the following reasonable assumption on the conditional probability distribution function.

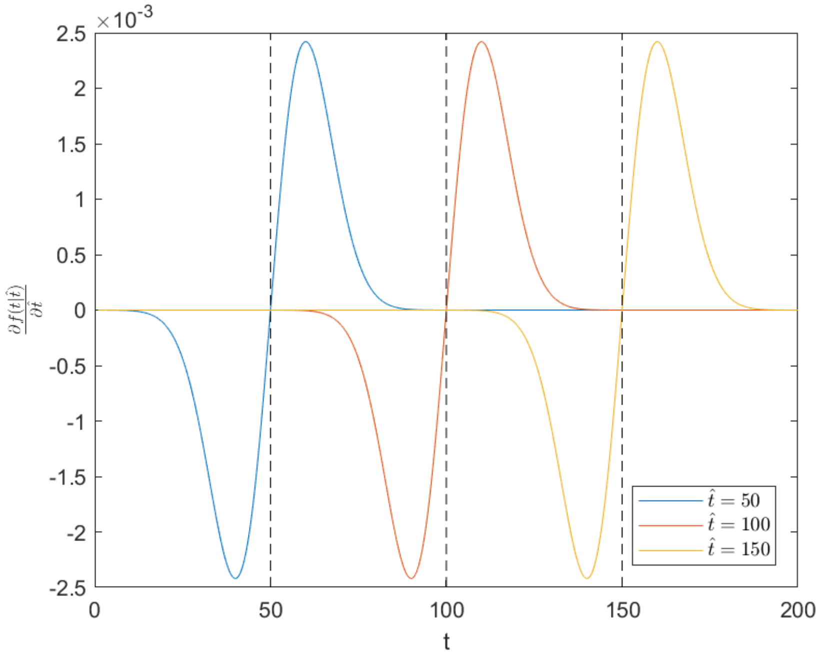


Figure (5) Shape of $\frac{\partial f(t|\hat{t})}{\partial \hat{t}}$ when $\hat{t} = 50,\ 100,\ 150$

Assumption 6 $f(t|\hat{t}) = \frac{\phi(\frac{t-\hat{t}}{\sigma})}{\sigma[\Phi(\frac{\bar{t}-\hat{t}}{\sigma})-\Phi(\frac{\underline{t}-\hat{t}}{\sigma})]} = \frac{\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(t-\hat{t})^2}{2\sigma^2}}}{\int_{\underline{t}}^{\bar{t}}\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\hat{t})^2}{2\sigma^2}}dx}$

where $\bar{t}$ and $\underline{t}$ represent the upper bound and lower bound of the talent respectively, $\phi(\cdot)$ and $\Phi(\cdot)$ represent the pdf and cdf of standard normal distribution respectively.

Here I assume the conditional probability distribution of the true talent is a truncated normal distribution with mean $\hat{t}$ and variance σ^2.

As shown in Figure 5, $\frac{\partial f(t|\hat{t})}{\partial \hat{t}}$ is negative for all t smaller than $\hat{t}$ and it's positive when t is larger than $\hat{t}$.

As $\frac{\partial R}{\partial EI}$ is positive and increasing in t, I know $\int \frac{\partial R}{\partial EI} \cdot \frac{\partial f(t|\hat{t})}{\partial \hat{t}} dt > 0$. That is to say, when there is no additional bonus for education achievement, the correlation between the optimal education investment and the signal observed is positive. In another word, parents are more willing to invest on child's education when they observe a positive signal (eg. high grades, good school performance).

In the case when parents will get additional bonus if child reach certain threshold (for example, parents get additional bonus if the child get college degree), the sign of $\frac{\partial EI^*}{\partial \hat{t}}$ will also depends on the sign and magnitude of $k \cdot \frac{\partial f(R^{-1}|\hat{t})}{\partial \hat{t}} \cdot \frac{\partial R^{-1}}{\partial EI}$.

$R^{-1}(Th, EI)$ represents the talent level which will just reach the threshold level Th with education investment EI. As EI increases, the marginal talent needed to reach the threshold Th will decreases. Therefore, $\frac{\partial R^{-1}}{\partial EI}$ is negative. The mathematical proof is as below.

$$R[R^{-1}(Th, EI), EI] = Th$$

Take derivative of both sides with respective to EI, I can get

$$\frac{\partial R^{-1}}{\partial EI} = -\frac{\partial R}{\partial EI} \cdot \frac{\partial R}{\partial t} < 0$$

Therefore, the sign of $-k \cdot \frac{\partial f(R^{-1}|\hat{t})}{\partial \hat{t}} \cdot \frac{\partial R^{-1}}{\partial EI}$ will be positive if $\frac{\partial f(R^{-1}|\hat{t})}{\partial \hat{t}} > 0$, in another word, when $R^{-1}(Th, EI^*) > \hat{t}$, or when $EI^* < R^{-1}(Th, \hat{t})$. Here the $R^{-1}(Th, \hat{t})$ represents the education investment needed to ensure a child with talent $\hat{t}$ to reach threshold Th.

From Part a I know that $\frac{\partial EI^*}{\partial k} > 0$. When k is large enough, the EI^* curve and the $R^{-1}(Th, \hat{t})$ will cross once at the $R^{-1}(Th, EI^*)$ point. The sign of $-k \cdot \frac{\partial f(R^{-1}|\hat{t})}{\partial \hat{t}} \cdot \frac{\partial R^{-1}}{\partial EI}$ is positive if $\hat{t} < R^{-1}(Th, EI^*)$ and is negative when $\hat{t} > R^{-1}(Th, EI^*)$. Moreover, the $\frac{\partial f(R^{-1}|\hat{t})}{\partial \hat{t}}$ will be 0 when $\hat{t}$ is too far away from the crossing point. Therefore, I predict the slope of the curve to be $-\frac{\int \frac{\partial R}{\partial EI} \cdot \frac{\partial f(t|\hat{t})}{\partial \hat{t}} dt}{SOC}$ for $\hat{t}$ far away from the crossing point. For $\hat{t}$ slightly smaller than $R^{-1}(Th, EI^*)$, the slope is greater than $-\frac{\int \frac{\partial R}{\partial EI} \cdot \frac{\partial f(t|\hat{t})}{\partial \hat{t}} dt}{SOC}$; for $\hat{t}$ slightly larger than $R^{-1}(Th, EI^*)$, the slope will be lower than $-\frac{\int \frac{\partial R}{\partial EI} \cdot \frac{\partial f(t|\hat{t})}{\partial \hat{t}} dt}{SOC}$. At some $\hat{t}$, the slope will become negative.

Part 3 Simulation

Here is an example to show the predictions above in a more intuitive way.

The specifications of the U, R functions are as below.

$$U = a \cdot log(I - EI)$$

$$R(t, EI) = b \cdot t^{\alpha} EI^{1-\alpha}$$

The parameter vector is as below.

$$(a, b, \alpha, I, k, \sigma, \underline{t}, \bar{t}, Th1, Th2, Th3) = (30, 3.5, 0.6, 100, 30, 5, 0, 200, 212, 265, 364)$$

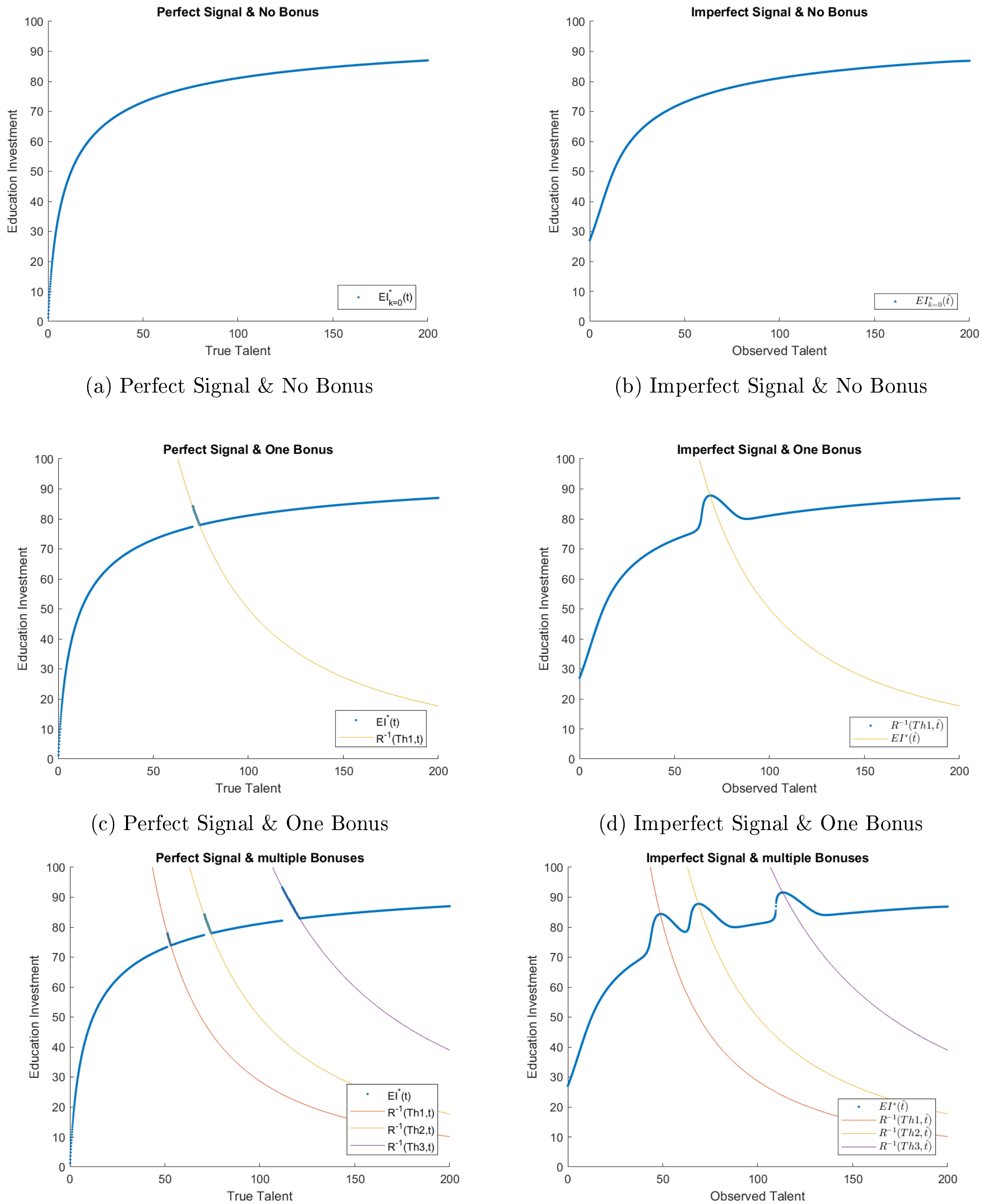
Figure 6 shows the correlation between talent signal and optimal education investment level in six different settings: a. $k = 0$ and perfect signal; b. $k = 0$ and imperfect signal; c. one threshold and perfect signal; d. one threshold and imperfect signal; e. multiple thresholds and perfect signal; multiple thresholds and imperfect signal.

When there is perfect signal, the curve of the optimal education investment behave exactly like the prediction in Part 1. When talent level is low, parents will always prefer the MC=MB point. When talent increases to certain level that the cost of reaching threshold Th is lower than the bonus, the parents will invest until the child reach the threshold so there will be a jump of the optimal investment. After that, the optimal education investment will decrease as talent increases. The reason is that education investment on more talented child is more efficient and then the amount of investment needed to reach the threshold is lower. Moreover, when the school years of schooling at the MC=MB point is greater than the threshold, the parents will go back to the MC=MB point and the correlation between optimal education investment and talent is back to positive again.

When the signal is imperfect, the general trend is similar, except now parents need to maximize expected utility, so the trend is smoother than the perfect signal case. The larger the variance is, the smoother the curve is.

Conclusion

This paper builds up a theoretical parental education investment model which employs discontinuous utility function and introduces uncertainty into parental education investment decision. With some basic and reasonable assumptions, this paper proves that the correlation between talent signal and optimal parental education investment level is not monotone. In most situations, the correlation will be positive. Parents are more willing to invest on a child if he/she is more talented. However, if parents will receive some additional bonuses when the child's years of schooling reach certain thresholds, parents' utility function will be discontinuous at these places. The bonus can motivate parents to add more investment on children who are close to the threshold even if the marginal benefit is actually lower than the marginal cost. In this case, parents will invest until the child just reach the threshold. Education investment on talented child is more efficient so the



(a) Perfect Signal & No Bonus

(b) Imperfect Signal & No Bonus

(c) Perfect Signal & One Bonus

(d) Imperfect Signal & One Bonus

(e) Perfect Signal & Multiple Bonuses

(f) Imperfect Signal & Multiple Bonuses

Figure (6) Correlation between optimal education investment and talent signal in different settings education investment needed to reach the threshold is lower for him/her. Therefore, the optimal education investment is decreasing as talent increases. The model can explain those “irrational” education investment behaviors which cannot be explained by the classic education investment model.

This paper also introduces uncertainty into the decision process. It analyzes parental education investment decision in two settings: perfect signal and imperfect signal. The analyses show that parental education investment with imperfect signal is similar to the perfect signal case. However, now parents will maximize the expected utility, so the optimal education investment curve will be smoothed out by the expectation process. The larger the standard deviation is, the smoother the curve becomes. In the imperfect signal case, the correlation between optimal education investment and talent signal can also be negative in certain situations.

Moreover, the paper can also help us to understand the drop-out issue. The model predicts that the drop-out rate will be lower for students whose current years of schooling are close to graduation. For example, a student at his/her last year of elementary school/high school/college is less likely to drop out. The reason is that degrees have values which motivates students to finish their degree.

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