Empirical Tests of Asset Pricing Models with Individual Assets: Resolving the Errors-in-Variables Bias in Risk Premium Estimation

by

Narasimhan Jegadeesh, Joonki Noh,
Kuntara Pukthuanthong, Richard Roll, and Junbo Wang

September, 2017

Abstract

To attenuate an inherent errors-in-variables bias, portfolios are widely employed to test asset pricing models; but portfolios might diversify and mask relevant risk- or return-related features of individual assets. We propose an instrumental variables approach that allows the use of individual assets yet delivers consistent estimates of ex-post risk premiums. This estimator yields unbiased estimates and well-specified tests in small samples. The market risk premium under the CAPM and the liquidity-adjusted CAPM, premiums on risk factors under the Fama-French three- and five-factors models and the Hou, Xue, and Zhang (2015) four-factor model are all insignificant after controlling for asset characteristics.

Co-Author	Affiliation	Voice	E-Mail
Jegadeesh	Emory University Atlanta GA 30322	404-727-4821	Jegadeesh@ Emory.Edu
Noh	Case Western Reserve University Cleveland OH 44106	216-368-3737	Joonki.Noh@ Case.Edu
Pukthuanthong	University of Missouri Columbia MO 65211	619-807-6124	PukthuanthongK@ Missouri.Edu
Roll	California Institute of Technology Pasadena CA 91125	626-395-3890	RRoll@Caltech.Edu
Wang	Louisiana State University Baton Rouge LA 70808	781-258-8806	junbowang@lsu.edu

Acknowledgements

For insightful and constructive comments, we thank Yakov Amihud, Francisco Barillas, Hank Bessembinder, Michael Brennan, Tarun Chordia, John Cochrane, Wayne Ferson, Chris Jones, Raymond Kan, Cheng-Few Lee, Jay Shanken, Georgios Skoulakis, Avanidhar Subrahmanyam, Guofu Zhou, and seminar participants at CalTech, Case Western Reserve University, Emory University, KAIST, UCLA, University of Melbourne, University of Missouri, University of New South Wales, University of South Florida, University of Technology at Sydney, and Yonsei University, York University, Financial Management Association 2014, and Northern Finance Association 2017.

Empirical Tests of Asset Pricing Models with Individual Assets:

Resolving the Errors-in-Variables Bias in Risk Premium Estimation

Abstract

To attenuate an inherent errors-in-variables bias, portfolios are widely employed to test asset pricing models; but portfolios might diversify and mask relevant risk- or return-related features of individual assets. We propose an instrumental variables approach that allows the use of individual assets yet delivers consistent estimates of ex-post risk premiums. This estimator yields unbiased estimates and well-specified tests in small samples. The market risk premium under the CAPM and the liquidity-adjusted CAPM, premiums on risk factors under the Fama-French three- and five-factors models and the Hou, Xue, and Zhang (2015) four-factor model are all insignificant after controlling for asset characteristics.

Key Words: Risk Premium Estimation, Errors-in-Variables Bias, Instrumental Variables, Individual Stocks, Asset Pricing Models

1. Introduction

A fundamental precept of financial economics is that investors earn higher average returns by bearing systematic risks. While this idea is well accepted, there is little agreement about the identities of systematic risks or the magnitudes of the supposed compensations. This is not due to a lack of efforts along two lines of enquiry. First, numerous candidates have been proposed as underlying risk factors. Second, empirical efforts to estimate risk premiums have a long and varied history.

Starting with the single-factor CAPM (Sharpe, 1964; Lintner, 1965) and the multi-factor APT (Ross, 1976), the first line of enquiry has brought forth an abundance of risk factor candidates. Among others, these include the Fama and French size and book-to-market factors, human capital risk (Jagannathan and Wang, 1996), productivity and capital investment risk (Cochrane, 1996; Eisfeldt and Papanikolaou, 2013; Hou, Xue and Zhang, 2015), different components of consumption risk (Lettau and Ludvigson, 2001; Ait-Sahalia, Parker, and Yogo, 2004; Li, Vassalou, and Xing, 2006), cash flow and discount rate risks (Campbell and Vuolteenaho, 2004) and illiquidity risks (Pastor and Stambaugh, 2003; Acharya and Pedersen, 2005).

The second line of enquiry has produced empirical estimates of risk premiums for many among, what Cochrane (2011) terms as, a "zoo" of risk factors. Most estimation methods have followed those originally introduced by Black, Jensen and Scholes (1972), (BJS), and refined by Fama and Macbeth (1973), (FM). Their most prominent feature is the use of portfolios rather than individual assets in testing asset pricing models. This has long been considered essential because of an error-in-variables (EIV) problem inherent in estimating risk premiums.

The EIV problem is best appreciated by tracing through the BJS and FM methods. They involve two-pass regressions: the first pass is a time-series regression of individual asset returns on the proposed factors. This pass provides estimates of factor loadings, widely called "betas" in the finance literature.[1] The second pass regresses asset returns cross-sectionally on the betas obtained from the first-pass regression. Since the explanatory variables in the second pass are estimates, rather than the true betas, the resulting risk premium estimates are biased and inconsistent; and the directions of the biases are unknown when there are multiple factors involved in the two-pass regressions.

[1] Hereafter, we will adopt the shorthand nomenclature "Beta" to mean "factor sensitivity" or "factor loading."

With a large number (N) of individual assets, the EIV bias can be reduced by working with portfolios rather than individual assets. This process begins by forming diversified portfolios classified by some individual asset characteristics such as a beta estimated over a preliminary sample period. It then estimates portfolio betas on the factors using data for a second period. Finally it runs the cross-sectional regressions on estimated portfolio betas using data for a third period. BJS, Blume and Friend (1973), and FM note that portfolios have less idiosyncratic components; so the errors-in-variables bias is reduced (and can be entirely eliminated as N grows indefinitely).

But using portfolios, rather than individual assets, has its own shortcomings. There is an immediate issue of test power since the dimensioniality is reduced; i.e., average returns vary with fewer explantory variables across portfolios than across individual assets. Perhaps more troubling is that diversification into portfolios can mask cross-sectional phenomena in individual assets that are unrelated to the portfolio grouping procedure. For example, advocates of fundamental indexation (Arnott, Hsu and Moore, 2005) argue that assets with high market values are overpriced and vice versa, but any portfolio grouping by an attribute other than market value itself could diversify away such potential mispricing, rendering it undetectable.

Another disquieting result of portfolio masking involves the cross-sectional relation between average returns and factor exposures ("betas"). Take the single-factor CAPM as an illustration (though the same effect is at work for any linear factor models). The cross-sectional relation between expected returns and betas holds exactly if and only if the market index used for computing betas is on the mean/variance frontier of the individual asset universe. Errors from the beta/return line, either positive or negative, imply that the index is not on the frontier. But if the individual assets are grouped into portfolios sorted by beta, any asset pricing errors across individual assets not related to beta are unlikely to be detected. Therefore, this procedure could lead to a mistaken inference that the index is on the efficient frontier.

Test portfolios are typically organized by firm characteristics related to average returns, e.g., size and book-to-market. Sorting on characteristics that are known to predict returns helps generate a reasonable variation in average returns across test assets. But Lewellen, Nagel, and Shanken (2010) point out sorting on characteristics also imparts a strong factor structure across test portfolios. Lewellen et al. (2010) show as a result that even factors weakly correlated with the sorting characteristics could explain the differences in average returns across test portfolios, regardless of the economic merits of the theories that underlie the factors.

Finally, the statistical significance and economic magnitudes of risk premiums are likely to depend critically on the choice of test portfolios. For example, the Fama and French size and book-to-market risk factors are significantly priced when test portfolios are sorted based on the corresponding characteristics, but they do not command significant risk premiums when test portfolios are sorted only on momentum.

In an effort to overcome the deficiencies of portfolio grouping while avoiding the EIV bias, we develop a new procedure to estimate risk premiums and to test their statistical significance using individual assets. Our method adopts the instrumental variables technique, a standard econometric solution to the EIV problem. We define a particular set of well-behaved instruments and hereafter refer to our approach as the IV method.

To be specific, our IV method first estimates betas for individual assets from a portion of the observations available in the data sample. These become the "independent" variables for the second-stage cross-sectional regressions. Then, we re-estimate betas using non-overlapping observations, which become the "instrumental" variables in the second-stage cross-sectional regressions. Since we use non-overlapping observations to estimate the independent and instrumental variables, while returns are only weakly autocorrelated, if at all, the measurement errors in beta estimates should be virtually uncorrelated cross-sectionally with their instruments.[2]

The IV estimator we propose is consistent for ex-post risk premium, i.e., N-consistent in Shanken (1992). Since consistency is a large sample property, it is important to examine the small sample performances of various estimators for practical applications. To do so, we conduct a number of simulation experiments. We choose simulation parameters matched to those in the actual data. Simulation results verify that the IV method produces unbiased risk premium estimates even with a relatively short time-series for beta estimation. In contrast, the standard approach that fits the the second-stage regressions using OLS (hereafter we will refer to this standard approach as the OLS method) suffers from severe EIV biases. For example, in simulations with a single factor model, we find that the OLS estimator, if used with individual stocks, is significantly biased toward zero even when betas are estimated with 2520 time-series observations. In contrast, the IV estimator yields nearly unbiased risk premium estimates when

[2] Some of our empirical tests also use stock characteristics as additional instruments for betas.

only 252 time-series observations are available to estimate betas.

In terms of test size (i.e., type I error) and power (i.e., type II error), we find that the conventional *t*-tests based on the IV estimator are well specified and they are reasonably powerful, even in small samples. We find similar results for the Fama-French three-factor model. We also show analytically that our IV estimator is consistent even if betas of individual stocks vary over time as long as they follow covariance stationary processes.[3] We find that even with time-varying betas, the IV estimator is unbiased in small sample simulations.

With actual data, we apply the IV method to examine whether the risk factors proposed by the CAPM, the three-factor and five-factor models of Fama and French (1993 and 2014), the q-factor asset pricing model of Hou, Xue, and Zhang (2015), and the liquidity-adjusted capital asset pricing model (LCAPM) of Acharya and Pedersen (2005) command positive risk premiums. These risk factors have been successful when they were tested with portfolios, but these tests potentially suffer from the low dimensionality problems that Lewellen et al. (2010) discussed. In contrast to the original papers, when controlling for corresponding non-β characteristics, we find that none of these factors is associated with a significant risk premium in the cross-section of individual stock returns.

This failure to find significant risk premiums is not due to the lack of test power of the IV method. Our simulation evidence indicates the t-tests based on the IV method provide reasonably high power under the alternative hypotheses that the true risk premiums equal the sample means of factor realizations observed in the data. For example, when the true HML risk premium equals the sample risk premium (4.36% per year), the probability of detecting it, which is the test power, is 91.5%.

In addition, when analyzing real data, in the absence of non-β characteristics as control variables, we find some evidence that SMB and HML betas command significant risk premiums in the cross-section of individual stocks returns. However, when we include corresponding non-β characteristics in the cross-sectional regressions, we find that the risk premiums are not significantly different from zero for any of tested betas.

[3] The assumption that betas follow a covariance stationary process is sensible from an economic perspective. Asset pricing models show that expected returns are linearly related to betas. If betas were to follow a non-stationary process, they can go to infinity, which would imply that expected returns also go to infinity. Infinite expected returns would not be economically meaningful for any reasonable risk aversion parameter.

Several papers in the literature, including Berk et al. (1999), Carlson et al. (2004 and 2006), Zhang (2005), and Novy-Marx (2013) argue that firm characteristics may appear to be priced because they may serve as proxies for betas. For example, consider firms A and B that are identical except for their risk. If firm A were riskier than firm B, then firm A would have bigger book-to-market ratio than firm B because the market would discount its expected cash flows at a bigger discount rate. If error-ridden betas were used in an attempt to account for risk, book-to-market ratios might appear, incorrectly, to be explaining at least part of the observed risk difference.

We develop a method to investigate this alternative explanation. Specifically, we allow for time-varying betas and characteristics, and we let the characteristics anticipate future changes in betas. We show analytically that this IV estimator provides consistent risk premium estimates when the second-stage cross-sectional regression employs the average returns over a long sample period as dependent variable while both betas and characteristics serve as independent variables. Our empirical results are robust with respect to this modified IV approach.

Our paper also contributes to a large literature on testing asset pricing models. As the length of time-series grows indefinitely, Shanken (1992) shows that the EIV bias becomes negligible because the estimation accuracy of betas improves. He also derives an asymptotic adjustment for the FM standard errors of the OLS method. Jagannathan and Wang (1998) extend Shanken's asymptotic analysis to the case of conditionally heterogeneous errors in time-series regression. Shanken and Zhou (2007) and Kan, Robotti and Shanken (2013) extend the result to misspecified models. However, the evidence and analyses in those papers mainly focus on portfolios. Our paper focuses on individual stocks as test assets and proposes the IV method to mitigate the EIV bias in testing asset pricing models.

Using individual stocks in testing asset pricing models is a recent development in the literature. Kim (1995) corrects the EIV bias using lagged betas to derive a closed-form solution for the MLE estimator of market risk premium. The solution proposed by Kim is based on the adjustment by Theil (1971). Other methods proposed by Litzenberger and Ramaswamy (1979), Kim and Skoulakis (2014), and Chordia et al. (2015) are similar, producing N-consistent risk premium estimators. To avoid the EIV bias, Brennan et al. (1998) advocate risk-adjusted returns as dependent variable in the second-stage regressions. However, the method that Brennan et al. use does not estimate the risk premiums of factors.

2. Risk-Return Models and IV Estimation

A number of asset pricing models predict that expected returns on risky assets are linearly related to their covariances with certain risk factors. A general specification of a *K*-factor asset pricing model can be written as:

$$E(\mathrm{r_i}) = \gamma_o + \sum_{k=1}^{K} \beta_{i,k}\gamma_k \tag{1}$$

where $E(\mathrm{r_i})$ is the expected excess return on stock i, $\beta_{i,k}$ is the sensitivity of stock i to factor k, and γ_k is the risk premium on factor k. γ_o is the excess return on the zero-beta asset. If riskless borrowing and lending are allowed, then the zero-beta asset earns the risk-free rate and its excess return is zero, i.e. $\gamma_o = 0$.

The CAPM predicts that only the market risk is priced in the cross-section of average returns. Several recent papers propose multifactor models based on empirical evidence of deviations from the CAPM. For example, Fama and French (1992) propose a three-factor model with size and book-to-market risks as additional priced factors.

Many empirical tests of asset pricing models employ the Fama-MacBeth (FM) two-stage regression procedure to evaluate whether the betas of risk factors are priced in the cross-section. The first-stage estimates factor sensitivities using the following time-series regressions with T periods of data:

$$\mathrm{r}_{i,t} = \mathrm{a}_i + \sum_{k=1}^{K} \beta_{i,k} \mathrm{f}_{k,t} + \varepsilon_{i,t}, \tag{2}$$

where $\mathrm{f}_{k,t}$ is the realization of factor k in time t. The time series estimates of factor sensitivities, say $\hat{\beta}_{i,k}$ for factor k, are the independent variables in the following second-stage cross-sectional regressions used to estimate factor risk premiums: For given time t,

$$\mathrm{r}_{i,t} = \gamma_{o,t} + \sum_{k=1}^{K} \hat{\beta}_{i,k}\gamma_{k,t} + \xi_{i,t}, \tag{3}$$

where realized excess return $\mathrm{r}_{i,t}$ is the dependent variable. The standard FM approach fits OLS regression to estimate the parameters of Eq. (3). These OLS estimates are biased due to the EIV problem since $\hat{\beta}_{i,k}$s are estimated with errors. To mitigate such bias, the portfolios are typically used as test assets, rather than individual assets, because portfolio betas are estimated more precisely than individual betas.

Our empirical tests use individual stocks as test assets to avoid the shortcomings that we discussed earlier when using portfolios as test assets. We propose an instrumental variable estimator to avoid EIV-induced biases. To describe our estimator, rewrite Eq. (3) as $\mathbf{r}_t = \boldsymbol{\gamma}\hat{\mathbf{B}} + \xi_t$ where $\mathbf{r}_t$ is a $1\times N$ row vector of realized excess returns in month t, $\hat{\mathbf{B}}$ is the $(K+1)\times N$ matrix containing the unit vector and K factor loadings, and γ is a $1\times(K+1)$ vector of factor risk premiums (including the excess return of zero-beta asset). We propose the following instrumental variables estimator (IV):

$$\hat{\boldsymbol{\gamma}}_{IV,t}' = (\hat{\mathbf{B}}_{IV}\hat{\mathbf{B}}_{EV}')^{-1}(\hat{\mathbf{B}}_{IV}\mathbf{r}_t') \tag{4}$$

where $\hat{\mathbf{B}}_{IV}$ and $\hat{\mathbf{B}}_{EV}$ are the matrices of instrumental and explanatory variables, respectively. We estimate betas within odd months and even months separately. Then we use odd-month betas as instrumental variables and even-month betas as explanatory variables when month t is even and vice versa when month t is odd.[4] We use daily data within odd and even months to estimate betas so that the measurement errors in the instrumental variables and explanatory variables are not correlated cross-sectionally, but in principle, one could use any non-overlapping intervals to estimate instrumental and explanatory betas. We fit the cross-sectional regressions each month using the IV estimator.

The IV estimator has been widely used in the literature to address the EIV problem, and it is well known that the estimator is consistent under mild regularity conditions. In our context, the IV estimator converges to the ex-post risk premium even for finite T when the number of stocks in the cross-section is sufficiently large. The proposition below formally states the N-consistency[5] of the IV estimator:

Proposition 1: *Suppose stock returns follow an approximate factor structure with K common factors. Under mild regularity conditions, the IV estimator given by Eq. (4) is N-consistent when the number of stocks in the cross-section increases without bound.*

Proof: See Online Appendix E.

[4] The EV and IV betas are computed using half the number of observations that one would use to compute OLS betas and hence they are noisier. However, this does not affect the consistency of the IV estimator. Our simulation results indicate that the IV estimator yields unbiased risk premium estimates even with a fairly short time-series for beta estimation.

[5] Shanken (1992) defines N-consistency.

We can frame the IV estimator as a two-stage least square (2SLS) cross-sectional regression to gain the underlying intuition. The first-stage regresses the explanatory variables against the instrumental variables. The matrix of the first-stage regression slope coefficients is:

$$\hat{\boldsymbol{\lambda}} = (\hat{\mathbf{B}}_{\mathrm{IV}}\hat{\mathbf{B}}_{\mathrm{IV}}')^{-1}(\hat{\mathbf{B}}_{\mathrm{IV}}\hat{\mathbf{B}}_{\mathrm{EV}}'). \tag{5}$$

The second-stage regression uses the fitted values from the first-stage regression as explanatory variables and the OLS estimator of this second-stage regression is the IV estimator. After substituting the relation in Eq. (5) and rearranging the terms, the second-stage regression estimator can be written as:

$$\hat{\boldsymbol{\gamma}}_{\mathrm{IV,t}}' = \hat{\boldsymbol{\lambda}}^{-1}\{(\hat{\mathbf{B}}_{\mathrm{IV}}\hat{\mathbf{B}}_{\mathrm{IV}}')^{-1}(\hat{\mathbf{B}}_{\mathrm{IV}}\mathbf{r}_{\mathrm{t}}')\}. \tag{6}$$

The expression within braces is the OLS estimates of the risk premiums when IV betas are used as regressors. These OLS estimates are pre-multiplied by the inverse of scaling matrix $\hat{\boldsymbol{\lambda}}$ to adjust for the EIV bias.

In the case of a single factor model, λ is the scalar slope coefficient obtained from regressing the explanatory variable on the instrumental variable. Since both explanatory and independent variables measure true beta with uncorrelated errors, λ is less than one. The noisier the errors, the smaller λ, which is the ratio of the IV and OLS slope coefficients; it thus magnifies the OLS estimate to account for the EIV bias. λ also correspondingly magnifies the standard error, and hence the t-statistics would be the same for both OLS and IV risk premium estimates in large samples.

In addition, note that the EIV scaling under a single factor model suggests that the IV method essentially shrinks OLS betas toward their cross-sectional mean of their instruments. Such shrinkage is reminiscent of Vasicek (1973)-style betas that move estimated betas towards the market beta of 1. In the case of multifactor models, the shrinkage depends on the cross-sectional correlation of beta estimates as well.

3. Small Sample Properties of the IV Method - Simulation Evidence

To evaluate the small sample properties of the IV method, we conduct a battery of simulations using the parameters matched to real data. We first investigate the bias and the root-mean-squared error (RMSE) of the IV estimator and then we examine the size and power of the associated *t*-test, which we refer to as the IV test.

3.1. Bias and RMSE of IV Estimator

We set the simulation parameters to equal the corresponding parameters in the actual data during the sample period of January 1956 through December 2012. The Center for Research in Security Prices (CRSP) value-weighted index provides the market return and the one-month T-bill rate is the risk-free rate. For each stock, a market model regression produces the beta and residual returns. Table 1 reports the simulation parameters.

We conduct simulations with the cross-sectional size of N=2000 stocks. We randomly generate daily returns using the following procedure:

1) For each stock, we randomly generate a beta and a standard deviation of return residuals $\sigma_{i,\varepsilon}$ from normal distributions with means and standard deviations equal to the corresponding sample means and standard deviations from the real data.[6] We generate betas and $\sigma_{i,\varepsilon}$ s in the beginning of each simulation and keep them constant across 1000 repetitions.
2) For each day, we randomly generate a market excess return draw from a normal distribution with mean and standard deviation equal to the sample mean and standard deviation from the data.
3) For each stock and each day, we then randomly generate residual returns $\varepsilon_{i,\tau}$ from independent normal distributions with mean zero and standard deviation equal to the value generated in step (1).

For stock i, we compute the excess return on day τ as

$$r_{i,\tau} = \beta_i r_{MKT,\tau} + \varepsilon_{i,\tau} \tag{7}$$

where $r_{MKT,\tau}$ is the market excess returns.

For the first-stage regression in the simulation, we estimate betas using the following market model regression with daily excess returns for each stock:[7]

$$r_{i,\tau} = a_i + \beta_i r_{MKT,\tau} + \varepsilon_{i,\tau}. \tag{8}$$

---

[6] If the random draw of σ_{ε}^{i} is negative, we replace it with its absolute value.

[7] We employ daily returns rather than monthly returns to obtain more precise beta estimates in the first-stage regression. We also experiment with monthly data to estimate betas. In untabulated results, when T=120, 180, and 240 months, we find that the IV estimator with monthly data has similar small sample properties to that with daily data, which are reported in Tables 2 and 3. All of our results are based on the truncated IV method.

Each "month" in the simulation has 21 trading days and we use three years of daily returns (T=756 days) to fit the time-series regression in Eq. (8). For the IV method, we use daily returns from odd and even months during a rolling three-year estimation period to compute independent and instrumental variables, respectively.

We fit the second-stage regression with monthly returns, following the common practice in the literature. We could have fit the second-stage regression with daily returns as well, but this would not improve the precision of the second-stage estimates. To see this intuitively, compare fitting one cross-sectional regression for month t with fitting 21 separate daily regressions for the month and averaging the daily regression slope coefficients over the month. With the same set of firms in both regressions and same betas for the month, the slope coefficient of the monthly regression would be exactly 21 times the average slope coefficient of the daily regressions and the standard error of the monthly regression would also be 21 times the standard error of average daily regression coefficient. As a result, both specifications would yield exactly the same t-statistic for the slope coefficient. There would be some differences between the two specifications if daily returns are compounded to compute monthly returns but such differences are likely small.

We compound daily stock and factor returns to compute corresponding monthly returns. We fit the cross-sectional IV regression in Eq. (4) for each month t to estimate $\gamma_{0,t}$ and $\gamma_{1,t}$. We then roll the three-year estimation window forward by one month and repeat the IV estimation procedure over 660 months (=55 years). Finally, we take the time-series averages of $\gamma_{0,t}$ and $\gamma_{1,t}$.

We conduct the three-factor model simulations analogously, but in addition to market returns and market betas, additional factors and betas are chosen to correspond to the Fama-French SMB and HML factors and betas. We match the means and standard deviations of the simulation parameters to those of actual data, then carry out the IV estimation procedure to estimate γ_0, γ_{MKT}, γ_{SMB} and γ_{HML}. Table 1 presents the simulation parameters in more detail.

One of the issues that often arises with IV estimators is that for any finite N, there is a very small chance that the cross-products of $\hat{\mathbf{B}}_{IV}$ and $\hat{\mathbf{B}}_{EV}$ might be close to non-invertible; this could result in an unreasonably large value of parameter estimates (see Kinal, 1980). To avoid such a potentially ill-behaved IV estimator for finite N, we treat any monthly risk premium estimate that deviates six standard deviations of the corresponding factor realizations from their sample average as a missing value, i.e., the exclusion cutoff is six.[8] In our empirical analyses in Section 4, for any given risk factor, we adjust the exclusion cutoffs to maintain the chances of exclusion binding to below 3% of the number of all available months.

The average differences between the risk premium estimates and the corersponding true simulation parameters over the 1000 replications are the ex-ante biases relative to the true risk premiums. Since all risk premium estimates within a sample are conditional on a particular set of factor realizations, we also report the biases relative to the average realized risk premiums in that particular sample, which are the ex-post biases as defined by Shanken (1992).

Panel A of Table 2 presents the ex-ante and ex-post biases, as percentages of the true market premium.[9] The OLS estimate is biased towards zero by 20% relative to the ex-ante risk premium and by 21% relative to the ex-post risk premium, respectively, which are statistically significantly different from zero, because of the EIV problem. In contrast, the average differences between IV estimates and the ex-ante and ex-post risk premiums are about 1%, which is statistically insignificant.[10]

The next two columns in Panel A present the ex-ante and ex-post RMSEs. The RMSE is a function of both the bias and the standard deviation of the estimation error. The OLS estimator has a smaller standard deviation than the IV estimator, but the former is biased, while the latter is unbiased. The ex-ante RMSE for the IV estimator is slightly smaller than that for the OLS estimator. The ex-post RMSE is .125 for the OLS estimator, compared with .080 for the IV estimator. These results indicate that because of the bias, the accuracy (assessed by RMSE) of the IV estimator is better than that of the OLS estimator for the parameters used in our simulations.[11]

Figure 1 plots the ex-ante and ex-post biases of the IV and OLS estimators as a function of

---

[8] Shanken and Zhou (2007) also similarly truncate their maximum likelihood estimates of risk premiums with portfolios as test assets to avoid undue influene of outliers.

[9] We know the ex-ante or the "true" risk premiums in simulations, but we only observe ex-post realizations in practice. Ex-post biases measure the biases conditional on particular factor realizations and would likely be more relevant in practice although both ex-ante and ex-post measures are conceptually interesting.

[10] Based on the standard errors across the 1,000 repetitions, the t-statistics of the ex-ante and ex-post biases of OLS estimate are -31.30 and -58.14, respectively. Therefore, OLS estimates are significantly biased at any conventional significance level. In contrast, the t-statistics of ex-ante and ex-post biases of IV estimate are insignificant at -0.46 and -0.22, respectively.

[11] The magnitude of the bias in the OLS estimator would be smaller if the true risk premium is smaller than what we assume in the simulations since the EIV bias is proportional to the magnitude of the true risk premium. In untabulated results, we find that the ex-post RMSE for the OLS estimator would be smaller than that for the IV estimator if the true market risk premium were smaller than about 2% per annum (for comparison, the sample risk premium is 5.8%).

the number of days (=T) in the rolling window to estimate the market betas with N=2000 stocks under the single-factor CAPM. The vertical axis reports the ex-ante and ex-post biases as percentages of the true market risk premium. The bias of the OLS estimator is fairly large, -44% for T=252 days.[12] The magnitude of the bias is greater than 5% even for T=2520 days, or 10 years. In contrast, the bias is fairly close to zero for the IV estimator even for T=252 days, or 1 year.

Panel B of Table 2 presents the results for the Fama-French three-factor model. The EIV problem always biases OLS risk premium estimates towards zero in univariate regressions, but in theory the bias could be in any direction in multivariate regressions. The results in Panel B indicate that the OLS risk premium estimates for the Fama-French three-factor model are all biased towards zero. For example, the ex-ante biases of the OLS estimates are -54.4% and -50.6% for SMB and HML, respectively. We find that all ex-ante and ex-post biases of the OLS estimates are significantly different from zero. In contrast, the magnitudes of the biases of IV estimates are all less than 2.1%, and these biases are statistically indistinguishable from zero.[13]

3.2. Size and Power of IV Test

Our tests follow the Fama-MacBeth approach to test whether the risk premiums associated with various common factors are reliably different from zero. For example, in the case of a single factor model, the test statistic is defined as:

$$t_\gamma = \frac{\hat{\gamma}}{\hat{\sigma}_\gamma}, \tag{9}$$

where $\hat{\gamma}$ is the time-series average of monthly IV risk premium estimates and $\hat{\sigma}_\gamma$ is the corresponding Fama-MacBeth standard error (FMSE).[14]

To examine the small sample properties of the t-statistic in Eq. (9) under the null hypotheses, we follow the same steps as above to generate simulated data, but we set all true risk premiums equal to zero. We then examine the percentage of repetitions (out of 1000 total

[12] Since our simulation assumes 21 days per month, T=252 days corresponds to one year.

[13] Based on the standard errors across the 1,000 repetitions, the t-statistics of the ex-ante and ex-post biases of OLS estimate for the three Fama-French factors are smaller than -45, and hence highly significant. In contrast, the t-statistics of ex-ante and ex-post biases of IV estimates for the three Fama-French factors range from -1 to 0, and they are all insignificant statistically.

[14] An earlier version of our paper analytically derived the asymptotic distribution of the IV estimator, which could also be employed in our empirical tests. However, we use the Fama-MacBeth standard errors because they are fairly straightforward to compute and more commonly used in the literature. Since the monthly IV estimates are serially uncorrelated, the usual intuition behind the FM approach goes through.

repetitions) when the t-statistics are positively significant at the various levels (one-sided) using critical values based on the standard normal distribution.

Panel A of Table 3 presents the test sizes of the IV tests under the CAPM and the Fama-French three-factor model for N=2000 stocks, respectively. The results indicate that the IV tests are well specified when T=756 days (=three years of daily data) are used for rolling beta estimation. For example, the test sizes for all risk premiums at the 5% significance level are between 5.0% and 5.2% and those at the 10% significance level are between 9.8% and 10.2%. In unreported results, we find that the distribution of the test statistic becomes closer to the theoretical distribution as we increase T. We also find similar results in simulations with N=1500 stocks.

We now investigate the power of the IV tests to reject the null hypotheses when the alternative hypotheses are true. To evaluate the power, we modify the simulation experiments by adding risk premiums equal to the average risk premiums that we observe from real data. All the other simulation parameters are the same as in the simulations under the null hypotheses. We fix the size of IV tests at the 5% significance level.

Panel B of Table 3 shows that the power of the IV test to reject the null hypothesis under the single-factor CAPM is 85.6%. Under the Fama-French three-factor model, we find that the frequency of rejection of the null of zero market risk premium is 83.8% and that of zero HML risk premium is 91.5%. The test power is somewhat weaker in detecting the positive SMB risk premium but it is still 51.8%. We also find that in 99.6% of the simulations, at least one of the IV risk premiums for the Fama-French three factors is significantly different from zero.

For comparison, Table 3 also presents the power of OLS tests. Under the CAPM, we reject the null hypothesis that the market risk premium equals zero in 84.2% of the simulations with OLS tests, compared with 85.6% with the IV tests. Although, the OLS estimates are biased towards zero, the OLS tests are almost as powerful as the IV tests because of smaller standard errors. We find similar power results for the Fama-French three-factor model as well although the OLS tests are generally less powerful than the IV tests.

We also examine the power of the IV and OLS as the true market risk premium varies (under the CAPM.) Figure 2 presents these results. If the market risk premium equals 2.9% per annum, which is 50% of the ex-post risk premium observed from real data, then the power of the IV tests is about 40% and the power of the OLS test is slightly smaller. Therefore, low power would be a concern during a sample period when the true risk premium is fairly small.

3.3. Time-varying betas

Our simulations so far assume that betas are constant over time. Appendix A proves that the IV estimator provides a consistent estimate of ex-post risk premium even with time-varying betas and risk premiums. We also conduct simulations to investigate the small sample properties of the IV estimator and associated tests with time-varying betas. When betas follow AR(1) processes, we find that the small sample properties of the IV estimator and the size and power of the IV tests are similar to what we report with constant betas in Tables 2 and 3.

4. IV Risk Premium Estimates for Selected Asset Pricing Models

This section employs the IV method to estimate the premiums for risk factors proposed by prominent asset pricing models.

4.1. Data

We obtain stock return, trading volume, and market capitalization data from CRSP and financial statement data from COMPUSTAT for the sample period of January 1956 through December 2012. We include all common stocks (CRSP share codes of 10 or 11).[15] The sample for month t excludes all stocks priced below $1 or stocks with market capitalizations less than $1,000,000 at the end of month t-1. Since daily returns are used to estimate betas, we restrict the sample to stocks with returns in month t, with at least 200 days of return data during each of the three years prior to month t.[16]

Table 4 presents summary statistics for the included stocks. A total of 14,058 distinct stocks enter the sample at different points in time; 2,425 stocks are present in an average month.

4.2. The CAPM and the Fama-French Three-Factor Model

This section first tests whether estimated risk premiums under the CAPM and the Fama-French three-factor models are significantly different from zero using the IV method with individual stocks. It then estimates the risk premiums after controlling for stock characteristics.

Early empirical tests of the CAPM by Fama and MacBeth (1973) and others find strong

[15] We exclude American depository receipts (ADRs), shares of beneficial interest, American Trust components, close-end funds, preferred stocks, and real estate investment trusts (REITs).

[16] We find similar results of asset pricing tests when the sample includes all stocks with at least 100 or 150 return observations per year instead of 200 observations.

support for the CAPM. However, several subsequent papers find that market betas are not priced after controlling for other characteristics. For instance, Jegadeesh (1992) and Fama and French (1993) conclude that the market risk premium is not reliably different from zero after controlling for firm size.

The inability of the CAPM to account for any of the cross-sectional differences in average returns reinvigorates the search for alternative asset pricing models. The arbitrage pricing theory proposed by Ross (1976) provides the general framework for multi-factor asset pricing models. The Fama-French three-factor model is perhaps the most widely used, which identifies size and book-to-market risk factors in addition to the market factor.

We employ individual stocks as test assets in the asset pricing tests and avoid the low dimensionality problem inherent in the tests that use characteristics-sorted portfolios. We use daily rolling windows from month t-36 to month t-1 to estimate betas for month t. In untabulated tests, we find similar test results when we estimate betas with daily rolling windows over the past 12, 24, and, 60 months.

To account for non-synchronous trading effects, beta estimation is supplemented with a one-day lead and lag of the independent variables (Dimson, 1979). For example, the following regression estimates market betas under the CAPM: for firm i and day τ,

$$r_{i,\tau} = a_i + \sum_{k=-1}^{1} \beta_{i,MKT,k} r_{MKT,\tau\text{-}k} + \varepsilon_{i,\tau} \quad (10)$$
$$\hat{\beta}_{i,MKT} = \hat{\beta}_{i,MKT,-1} + \hat{\beta}_{i,MKT,0} + \hat{\beta}_{i,MKT,1}.$$

We estimate odd- and even-month betas separately using returns on days belonging to odd and even months, respectively. Because of the non-synchronous trading adjustment in Eq. (10), the first and the last days of each month are excluded to avoid any potential biases due to overlap.[17] An analogous multivariate regression estimates the three betas under the Fama-French three-factor model.

For each stock and month, SIZE is the natural logarithm of market capitalization at the end of the previous month. BM is the book value divided by the market value where book value is the sum of book equity value plus deferred taxes and credits minus the book value of preferred stock. We compute cross-sectional correlations between each pair of firm-specific variables each month

[17] We find almost identical results while including the first and last days of each month. Also, the results are qualitatively similar when no adjustment is used for non-synchronous trading.

and Table 5 presents the average cross-sectional correlations among betas and characteristics. The CAPM beta estimated using the market model exhibits negative correlation with both SIZE and BM. In the Fama-French three-factor model, the correlations between market beta and the SMB and HML betas are positive. The correlation between SIZE and SMB betas is negative, and the correlation between BM and HML betas is positive.

For comparison, Table 5 also presents the average cross-sectional correlations for 25 Fama-French SIZE and BM sorted portfolios that the literature typically uses as tests assets. For each portfolio and each month, we compute SIZE and BM as the value-weighted averages across all stocks that belong to the portfolio. The magnitudes of correlations among portfolio betas and characteristics are much larger; between the SMB beta and SIZE it is -.97 and between the HML beta and BM it is .88.

Table 6 presents the risk premium estimates using the IV method and individual stocks as test assets. We first test the CAPM using betas estimated from the univariate regression. The market risk premium estimate is -.246%, which is not reliably different from zero (Column (1).) Therefore, there is no empirical support for the CAPM with individual stocks.

For the Fama-French three-factor model, the betas come from multivariate time-series regressions with all three factors. In Column (2), the market risk premium estimate is -.288% and insignificant and the SMB and HML risk premiums are .301% and .344%, respectively. The risk premiums of SMB and HML betas are statistically significant at the 5% level.

The significance of SMB and HML risk premium estimates suggests that these factor risks may be priced in the cross-section, but it is also possible that these significant estimates might be due to an omitted variable bias because the second-stage cross-sectional regressions in Column (2) do not include SIZE and BM, the characteristics that underlie SMB and HML factors, as control variables. To examine this possibility, we include SIZE and BM as additional independent variables in the second-stage cross-sectional regressions. Under the CAPM, in Column (3), the slope coefficients of SIZE and BM are -.120% and .196%, respectively, and both are statistically significant at the 1% level. The market risk premium estimate is -.090%, which is still not significantly different from zero. Under the Fama-French three-factor model in Column (4), none of the risk premiums is significant at the 5% level in the presence of SIZE and BM, including the previously significant SMB and HML risk premiums. In contrast, SIZE and BM remain highly significant. We also find similar results when we use the logarithm of BM (logBM) instead of BM in Column (5).

Table 6 also presents the test results using OLS regression estimates. As in the IV tests, we find that the SMB and HML risk premiums are statistically significant when we do not use SIZE and BM as control variables. However, they become insignificant when SIZE and BM are included. The OLS test results are similar to what we find with the risk premium estimates using the IV method and they also indicate that the factor risks under the CAPM or Fama-French three-factor model are not priced in the cross-section of individual stock returns.

Table 6 also reports the results on two roughly equal subperiods. The factor risk premium estimates are insignificant in the both subperiods when SIZE and BM characteristics are included. The slope coefficients of SIZE and BM are significant in the both subperiods at conventional levels. For the two subperiods, the IV risk premium estimates are similar to their OLS counterparts except for the SMB risk premium in the first subperiod under the Fama-French three-factor model; see Columns (2) and (7).

Given that the IV method works very well in simulations, there are several possible interpretations concerning these empirical results. First, something in the real data compromises the IV method; i.e., something that is missing from the simulations. For example, although our simulation evidence indicates that the IV tests are reasonably powerful to detect positive risk premiums under the CAPM and Fama-French three-factor models, they might not be in the real data. This interpretation does not seem convincing due to the following observation: Without controlling for SIZE and BM, Panel A in Table 6 finds that the SMB and HML risk premiums are significant.

It is also possible that SIZE and BM measure the “true” future SMB and HML betas better than the SMB and HML betas estimated from past data under the Fama-French three-factor model. Consequently, the significant slope coefficients on SIZE and BM might actually represent the risk premiums for SMB and HML factors. We evaluate this possibility in greater detail in Section 5, and we find weak support for this alternative explanation.

4.3. The Fama-French Five-Factor Model

Novy-Marx (2013) and Aharoni, Grundy, and Zeng (2013) among others find that stock returns are significantly related to profitability and investment after controlling for Fama and French’s three factors. Fama and French (2014) propose the following five-factor model that adds two new factors to capture these effects:

$$E(r_{i,t}) = \beta_{i,MKT}\gamma_{MKT} + \beta_{i,SMB}\gamma_{SMB} + \beta_{i,HML}\gamma_{HML} + \beta_{i,RMW}\gamma_{RMW} + \beta_{i,CMA}\gamma_{CMA} \quad (11)$$

where $\beta_{i,MKT}$, $\beta_{i,SMB}$, $\beta_{i,HML}$, $\beta_{i,RMW}$, and $\beta_{i,CMA}$ are the betas with respect to market, size, book-to-market, profitability, and investment factors, and γ_{MKT}, γ_{SMB}, γ_{HML}, γ_{RMW}, and γ_{CMA} are the corresponding risk premiums. The RMW factor is based on the difference between the returns on diversified portfolios of stocks with robust and weak operating profitability and the CMA factor is based on the difference between the returns on diversified portfolios of the stocks with conservative and aggressive investment. We obtain the daily data for the five factors from Ken French's website. As in Fama and French (2014), the sample period for the asset pricing tests in this subsection is from January 1964 through December 2012.

Panel A of Table 7 presents the asset pricing test results for the Fama-French five-factor model. For the entire sample period, we find that the risk premium estimates for the five factors are insignificant at the 5% level regardless of controlling for the corresponding characteristics: SIZE, BM, operating profitability (OP), and investment (INV).[18] Specifically, similar to Panel A of Table 6, the SMB and HML risk premium estimates have the highest t-statistics without controlling for the four characteristics (Column (3).) But they become fairly weak when those four characteristics are included (Column (6).) In contrast, the slope coefficients of SIZE, BM, OP, and INV are highly significant with their usual signs (Column (6).) For the two subperiods whose lengths are roughly equal, we find qualitatively similar results as well.

4.4. The q-factor Asset Pricing Model

Cochrane (1991) and Liu, Whited and Zhang (2009) present production-based asset pricing models in which productivity shocks are tied to the changes in the investment opportunity set, which is consistent with Merton's (1973) ICAPM framework. Since the shocks to productivity are difficult to measure accurately, Hou, Xue, and Zhang (2015) (henceforth HXZ) propose an

[18] We follow Fama and French (2014) to construct OP and INV. As described in French's data library: "OP for June of year t is annual revenues minus cost of goods sold, interest expense, and selling, general, and administrative expenses divided by book equity for the last fiscal year end in t-1." The slope coefficient in a univariate regression of stock returns against this measure of OP is insignificant, as in column (4) of Table 7. Novy-Marx (2013) defines gross profit as revenue minus cost of goods, scaled by current period total assets. In untabulated results, we find significantly positive slope coefficient in the univariate regression with Novy-Marx's definition of OP. Apparently; the definition of OP affects the univariate relation. Because we use Fama-French factors, we follow their definition.

empirical q-factor model where an investment factor and a ROE factor capture productivity shocks. Their asset pricing model is specified as:

$$E(\mathrm{r}_{\mathrm{i,t}}) = \beta_{\mathrm{i,MKT}}\gamma_{\mathrm{MKT}} + \beta_{\mathrm{i,ME}}\gamma_{\mathrm{ME}} + \beta_{\mathrm{i,I/A}}\gamma_{\mathrm{I/A}} + \beta_{\mathrm{i,ROE}}\gamma_{\mathrm{ROE}}, \quad (12)$$

where $\beta_{\mathrm{i,MKT}}$, $\beta_{\mathrm{i,ME}}$, $\beta_{\mathrm{i,I/A}}$, and $\beta_{\mathrm{i,ROE}}$ are the betas with respect to market, size, investment, and ROE factors, respectively, and γ_{MKT}, γ_{ME}, $\gamma_{\mathrm{I/A}}$, and γ_{ROE} are the corresponding risk premiums.

The investment factor captures the level of investments and the ROE factor captures the return on investments, i.e., profitability. The investment factor is constructed as the return difference between firms with low and high levels of investments and the ROE factor is constructed as the return difference between firms with high and low levels of profitability. Intuitively, the investments and rates of return on investments are likely to reflect the sensitivity to unanticipated productivity shocks, and these factors are supposed to capture the price impact of such shocks. HXZ argue that their factors better explain the cross-sectional return differences across portfolios based on various anomalies, e.g., BM, SIZE, momentum, and earnings surprise than the Fama-French three-factor model and the Carhart four-factor model.

HXZ use a variety of different test portfolios for their asset pricing tests. For instance, their test of SIZE and BM uses the 25 Fama-French SIZE and BM sorted portfolios, their test of momentum uses 10 portfolios based on momentum, and their test of the standardized earnings surprises (SUE) uses 10 SUE sorted portfolios. Since all the tests employ selected sets of tests portfolios, they are also subject to the low dimensionality problems. We examine whether the HXZ factors are priced in the cross-section of individual stock returns. We obtain daily HXZ factors from HXZ.

Table 8 presents the test results of the q-factor asset pricing model using the IV method. To facilitate comparison, this table uses the sample period of January 1972 through December 2012, which HXZ use in their empirical tests. Columns (1) to (4) report the risk premium estimates for each of the four betas under the HXZ model in univariate regressions. None of risk premiums are statistically significant at conventional levels. In Columns (6) and (7), when testing each of the investment and ROE factor betas with their corresponding characteristics, we find that only the slope coefficients of the characteristics are reliably different from zero.

Column (5) presents the risk premium estimates for all four betas under the HXZ model together, none of which are reliably different from zero. In Column (8), when adding the three characteristics as control variables: SIZE, OP, and INV, to the HXZ model, we find that the slope coefficients of these characteristics are all statistically significant with usual signs, while none of risk premiums under the HXZ model is reliably different from zero. Therefore, for the entire sample period, we find no empirical support for the HXZ model when using individual stocks as test assets. For the subperiods whose lengths are equal, we obtain qualitatively similar results of asset pricing tests as well.

4.5. Liquidity-Adjusted CAPM

Next, we examine the liquidity-adjusted capital asset pricing model (LCAPM) proposed by Acharya and Pedersen (2005), which accounts for the impact of illiquidity-based trading frictions on asset pricing.[19] According to the LCAPM, the level of illiquidity and the covariances of return and illiquidity innovation with the market-wide return and illiquidity innovation affect expected return. The unconditional expected return in excess of the risk-free rate under the LCAPM is defined as:

$$E\left(r_{i,t}\right) = E(c_{i,t}) + \lambda(\beta_{i,1} + \beta_{i,2} - \beta_{i,3} - \beta_{i,4}), \tag{13}$$

where $c_{i,t}$ is the illiquidity cost, the risk premium is the market excess return minus aggregate illiquidity cost (i.e., $\lambda = E(r_{MKT,t} - c_{MKT,t})$), and the betas are

$$\beta_{i,1} = \frac{cov(r_{i,t}, r_{MKT,t} - E_{t-1}(r_{MKT,t}))}{var(r_{MKT,t} - E_{t-1}\left(r_{MKT,t}\right) - [c_{MKT,t} - E_{t-1}(c_{MKT,t})])}, \tag{14}$$

$$\beta_{i,2} = \frac{cov(c_{i,t} - E_{t-1}(c_{i,t}), c_{MKT,t} - E_{t-1}(c_{MKT,t}))}{var(r_{MKT,t} - E_{t-1}\left(r_{MKT,t}\right) - [c_{MKT,t} - E_{t-1}(c_{MKT,t})])},$$

$$\beta_{i,3} = \frac{cov(r_{i,t}, c_{MKT,t} - E_{t-1}(c_{MKT,t}))}{var(r_{MKT,t} - E_{t-1}\left(r_{MKT,t}\right) - [c_{MKT,t} - E_{t-1}(c_{MKT,t})])},$$

$$\beta_{i,3} = \frac{cov(c_{i,t}, r_{MKT,t} - E_{t-1}(r_{MKT,t}))}{var(r_{MKT,t} - E_{t-1}\left(r_{MKT,t}\right) - [c_{MKT,t} - E_{t-1}(c_{MKT,t})])}.$$

The term $E\left(c_{i,t}\right)$ is the reward for firm-specific illiquidity level, which is the compensation for holding an illiquid asset as in Amihud and Mendelson (1986). Acharya and Pedersen define illiquidity-adjusted net beta as:

$$\beta_{i,LMKT} = \beta_{i,1} + \beta_{i,2} - \beta_{i,3} - \beta_{i,4}. \tag{15}$$

[19] Several other papers, e.g., Pastor and Stambaugh (2003), also propose models where a stock's return sensitivity to market-wide (il)liquidity is priced in the cross-section. Since we do not have daily Pastor and Stambaugh's liquidity factor, we do not examine their model here.

The LCAPM implies that the linear relation between risk and return applies for the liquidity-adjusted market beta, but not for the standard market beta under the CAPM. The LCAPM also implies that the linearity between risk and return applies to excess returns net of firm-specific illiquidity cost ($c_{i,t}$).

Acharya and Pedersen test the LCAPM using two sets of test portfolios sorted on the average and standard deviation of illiquidity. They sort stocks based on Amihud (2002) illiquidity measures during each year and form 25 value-weighted illiquidity portfolios for the subsequent year. They also form 25 value-weighted σ (illiquidity) portfolios similarly by sorting based on the standard deviation of illiquidity.

We examine the correlations between $\beta_{i,LMKT}$ and the value-weighted averages of SIZE and BM for those portfolios used by Acharya and Pedersen. The average cross-sectional correlations between $\beta_{i,LMKT}$ with SIZE for illiquidity and σ (illiquidity) portfolios are -.96 and -.97, and those with BM are .71 and .74, respectively. Such high correlations between liquidity-adjusted market beta, i.e., $\beta_{i,LMKT}$, and SIZE suggest that it would be particularly hard to determine empirically whether average returns differ across test portfolios due to SIZE or illiquidity-adjusted market betas. This situation parallels that in Chan and Chen (1988) who use 20 size-sorted portfolios as test assets and find strong support for the standard CAPM. The correlations between the standard market beta and SIZE across Chan and Chen's test portfolios range from -.988 to -.909 over different sample periods, and the corresponding correlations in the cases of illiquidity and σ (illiquidity) portfolios are within this range. Jegadeesh (1992) shows that when test portfolios are constructed so that SIZE and standard market beta have low correlations (in absolute value), the market beta is not priced and that the significant market risk premium found across size-sorted portfolios is due to the high correlation (in absolute value) between SIZE and market beta.

To avoid such ambiguity, we employ the IV method with individual stocks to investigate whether $\beta_{i,LMKT}$ under the LCAPM is priced in the cross-section. To facilitate comparison, we follow the same procedure as in Acharya and Pedersen (2005) in all other respects. Because of the differences in the market structures of the NYSE/AMEX and NASDAQ, the trading volumes reported in these two markets are not comparable and hence NASDAQ stocks are excluded for this test. In addition to existing screening criteria, following Acharya and Pedersen, we exclude stocks that do not trade for at least 100 days per year, which can suppress noisy illiquidity measures.

Acharya and Pedersen define the illiquidity cost as follows:[20]

$$\mathrm{ILLIQ}_{i,\tau} = \frac{|r_{i,\tau}|}{v_{i,\tau}}, \tag{16}$$

$$c_{i,\tau} = \min\left(0.25 + 0.3\mathrm{ILLIQ}_{i,\tau} P_{MKT,t-1}, 30\right), \tag{17}$$

where $r_{i,\tau}$ is the return on day τ in month t, $v_{i,\tau}$ is the dollar volume (in millions) and $P_{MKT,t-1}$ is the month $t-1$ value of $1 invested in the market portfolio as of the end of July 1962. Eq. (16) is based on Amihud's (2002) illiquidity measure. Acharya and Pedersen use Eq. (17) as a measure of illiquidity cost where $P_{MKT,t-1}$ is used to adjust for inflation and the illiquidity cost is capped at 30% to avoid an obviously unreasonable value for it. The market-wide illiquidity cost $c_{MKT,\tau}$ is the value-weighted average of individual illiquidity costs using market capitalization in month t-1.

As in Acharya and Pedersen (2005), we estimate the innovations in illiquidity costs using AR models and then estimate each individual component of betas in Eq. (15) using a time-series GMM approach and Dimson-type corrections.[21] We then fit the following cross-sectional regression each month t:

$$r_{i,t} = \alpha_t + \gamma_{ILLIQ,t} c_{i,t} + \gamma_{LMKT,t} \hat{\beta}_{i,LMKT} + \varepsilon_{i,t} \quad . \tag{18}$$

where $c_{i,t}$ is the average illiquidity for stock i in month t.[22]

The IV estimator in month t is:

$$\hat{\boldsymbol{\gamma}}_t{}' = (\hat{\boldsymbol{\Psi}}_{IV,t} \hat{\boldsymbol{\Psi}}'_{EV,t})^{-1} \hat{\boldsymbol{\Psi}}_{IV,t} \mathbf{r}_t{}',$$

where $\hat{\boldsymbol{\Psi}}_{IV,t}$ is $\hat{\boldsymbol{\Psi}}_{even,t}$ when month t is odd and it is $\hat{\boldsymbol{\Psi}}_{odd,t}$ when month t is even, and

$\hat{\boldsymbol{\Psi}}_{even,t} \equiv 3 \times N$ matrix with unit vector as the first row, $c_{i,t}$, and estimated even-month LMKT betas for N stocks as the second and third rows, respectively. We estimate the even-month LMKT betas using daily data in even months in the rolling estimation window of month t-36 to month t-1.

$\hat{\boldsymbol{\Psi}}_{odd,t} \equiv$ Analogous to $\hat{\boldsymbol{\Psi}}_{even,t}$ estimated using all daily data in odd months.

[20] Acharya and Pedersen use illiquidity costs at monthly frequency but we use them at daily frequency.
[21] Appendix B presents the AR models that we use to estimate expected and unexpected components of illiquidity.
[22] As in Acharya and Pedersen (2005), 30% capping is applied after taking monthly average.

For the IV estimator in month t+1, we move the three-year rolling estimation window forward by one month. We repeat this estimation procedure until all available observations are exhausted. Then computing the average IV risk premium estimate under the LCAPM and its standard error is conducted in the same way as the other asset pricing models tested above.

Table 9 presents the risk premium estimates for the LMKT betas when using individual stocks as test assets. The slope coefficient on the Amihud illiquidity measure is .220%, and it is positive and highly significant. However, the risk premium estimates for β_{LMKT} are .150% and .085%, respectively, without and with controlling for Amihud illiquidity. These premiums are not reliably different from zero. These results indicate that firm-specific illiquidity, which is a firm characteristic, is positively related to average returns, but liquidity-adjusted market beta, which is the systematic risk under the LCAPM, does not command a risk premium. Table 9 also shows that the liquidity-adjusted market beta is not priced in either of subperiods, while Amihud illiquidity affects average returns significantly in both subperiods.

In comparison, Acharya and Pedersen (2005) report a liquidity-adjusted market risk premium estimate of about 2.5% per month using the value-weighted index (see Panel B of Table 5 in Acharya and Pedersen, 2005), which is about 30% per year.[23] The equity risk premium puzzle literature argues that even an annual risk premium of about 6% observed in the data is hard to justify with realistic levels of risk aversion, and larger risk premiums would be harder to justify. The large risk premium estimate obtained with test portfolios seems likely to be the result of high correlations between $\beta_{i,LMKT}$ and portfolios characteristics rather than a true depiction of the compensation for a systematic risk.

5. Additional Tests

This section examines the robustness of our findings to a number of variations in the test specification and evaluates the strength of the instruments.

5.1. Do Characteristics Proxy for True Betas?

Our results indicate that many of the systematic risk factors proposed in the literature do not command a premium after controlling for stock characteristics. However, it is possible that

[23] The liquidity-adjusted market risk premium equals the market risk premium minus expected market-wide illiquidity cost and hence it is smaller than the unadjusted market risk premium.

because beta estimates contain measurement errors, characteristics may serve as better proxies for true betas and the slope coefficients on characteristics may actually reflect the risk premiums of underlying systematic risk factors. In other words, it is possible that the characteristics measure the "true" future betas better than the betas estimated from past data, which we refer to as the past betas. Several papers in the literature, including Berk et al. (1999), Carlson et al. (2004 and 2006), Zhang (2005), and Novy-Marx (2013), present variations of such an interpretation.

The fact that "true" betas are unobservable makes it difficult to evaluate the tenability of this interpretation directly. However, there are several empirical implications of this "risk-proxy" hypothesis that we can test. One implication of this hypothesis is that characteristics should be more highly correlated with the betas estimated from future data, which we refer to as the future betas, than past betas. However, under the CAPM and Fama-French three-factor model, we find that the average cross-sectional correlations between past betas and future betas are slightly larger than those between characteristics and future betas.

The average cross-sectional correlation between past betas and future betas increases as we increase the sample periods over which we estimate them. For example, under the Fama-French three-factor model, the average correlation between past SMB betas and future SMB betas increases from 0.395 to 0.497 and the average correlation between past HML betas and future HML betas increases from 0.233 to 0.276 as we increase the length of rolling estimation window from 1.5 years to 2.5 years.

We repeat our asset pricing tests by employing the past SMB and HML betas as instrumental variables and their future betas as independent variables. We find that the risk premiums for SMB and HML betas in the second-stage regressions are all insignificant, while the slope coefficients on SIZE and BM remain significant, similar to the results in Table 6. We also apply the same past and future betas specification to the other asset pricing models tested in Section 4 and find that our conclusions stay the same as before.

Finally, we develop and implement asset pricing tests that directly examine whether characteristics are priced because they contain forward-looking information about future betas that is not contained in the corresponding past betas. Formally, suppose that the dynamics of beta is described by the following AR(1) process: for stock i in time t,

$$\beta_{i,t} = \theta + \rho\times\beta_{i,t-1} + u_{i,t}. \tag{19}$$

Suppose that the market is aware of $u_{i,t}$ at the end of time t-1. Then, under the CAPM expected return is[24]:

$$E_{t\text{-}1}(r_{i,t}) = \alpha + \gamma\times\beta_{i,t}. \tag{20}$$

Let the corresponding characteristic also follow an AR(1) process given by:

$$C_{i,t} = a + b\times C_{i,t-1} + e_{i,t}, \tag{21}$$

where, to capture the idea that the characteristic anticipates the innovation in beta, we assume that $e_{i,t} = u_{i,t+1} + \delta_{i,t}$.

We can obtain an unbiased estimate of $\beta_{i,t-1}$ using the past returns available up to time t-1, but these data do not contain the information about $u_{i,t}$. In contrast, $C_{i,t-1}$ contains the information about $u_{i,t}$. Therefore, the expected return under the CAPM can be written as a function of $\beta_{i,t-1}$ and $C_{i,t-1}$ as follows: [25] $E_{t\text{-}1}(r_{i,t}) = \alpha_0 + \gamma_\beta\times\beta_{i,t-1} + \gamma_c\times C_{i,t-1}$. (22)

When we use the estimates of $\beta_{i,t-1}$ and $C_{i,t-1}$ as independent variables in the second-stage regressions, the slope coefficient on characteristic, i.e., γ_c, would be significant because the characteristic anticipates the innovation in the future beta and because the measurement error in $\beta_{i,t-1}$ estimate would allow the characteristic to capture a part of the true risk premium as well. Therefore, we cannot obtain a consistent risk premium estimate.

Accordingly, we develop a modified IV methodology to address this issue and the proposition below shows that this modified IV methodology yields consistent risk premium estimates:

Proposition 2: *Suppose that the CAPM is true as in Eq. (20). Let the time-series dynamics of beta and characteristic be given by Eqs. (19) and (21), respectively. The time-series average of the risk premium estimates computed using estimator in Eq. (23) is consistent when* N, T *and* T_m *converge to infinity:*

$$\hat{\gamma}_t' = \left(\frac{1}{N} X_{IV,t-1}(X_{EV,t-1})'\right)^{-1} \left(\frac{1}{N} X_{IV,t-1}\bar{r}_t'\right), \tag{23}$$

[24] In the main text of the paper, we assume that the true risk premium is constant. We relax this assumption in the Online Appendix and provide a formal and more general proof for Proposition 2.

[25] We can derive the following relation between the parameters in Eqs. (20) and (22): $\alpha_0 = (\alpha + \gamma\theta)\text{-}\gamma(a + bC_{i,t-2} + \delta_{i,t-1})$, $\gamma_\beta = \gamma\rho$ and $\gamma_c = \gamma$.

$$\text{where } X_{IV,t-1} = \begin{pmatrix} 1 & \cdots & 1 \\ \hat{\beta}_{IV,i,t-1} & \cdots & \hat{\beta}_{IV,N,t-1} \\ \overline{C}_{1,t-1} & \cdots & \overline{C}_{N,t-1} \end{pmatrix}, X_{EV,t-1} = \begin{pmatrix} 1 & \cdots & 1 \\ \hat{\beta}_{EV,1,t-1} & \cdots & \hat{\beta}_{EV,N,t-1} \\ \overline{C}_{1,t-1} & \cdots & \overline{C}_{N,t-1} \end{pmatrix}, \quad (24)$$

$\bar{r}_t = [\bar{r}_{1,t}, \cdots, \bar{r}_{N,t}]$, $\bar{r}_{i,t} = \frac{1}{T_m}\sum_{\tau=0}^{T_m-1} r_{i,t+\tau}$ *, and* $\bar{C}_{i,t} = \frac{1}{T_m}\sum_{\tau=1}^{T_m} C_{i,t-\tau}$. *The betas for both explanatory variable ($\hat{\beta}_{EV,i,t-1}$) and instrumental variable ($\hat{\beta}_{IV,i,t-1}$) are estimated from past data. We refer to the risk premium estimator based on the time-series average of Eq. (23) as the IV mean-estimator.*

Proof: See Appendix C.

For the ease of exposition, we present Proposition 2 only for the CAPM, assuming that both beta and characteristic follow AR(1) processes. The proposition also holds for multi-factor models when we allow time-varying risk premium, and betas and characteristics to follow any stationary and ergodic processes. We present the proposition and proof for the general case in the Online Appendix G.

The IV mean-estimator in Proposition 2 differs from the previous IV estimator in Eq. (4) in two important ways. First, the IV mean-estimator uses characteristics as well as $\hat{\beta}_{IV,i,t-1}$ as the instruments for $\hat{\beta}_{EV,i,t-1}$. In contrast, our previous IV estimator employs the only betas estimated from a non-overlapping sample period as the instruments and uses the characteristics only as control variables; see, e.g., Columns (3) to (5) in Panel A of Table 6. Second, the IV mean-estimator uses the time-series averages of current (in month t) and future returns as the dependent variable, and past betas and time-series averages of characteristics as independent variables, while our previous IV estimator uses the returns in month t as the dependent variable and past betas and one-month lagged characteristics as independent variables.

Intuitively, the IV mean-estimator method increases the time lag between estimation of the characteristics and observations of future returns. For example, if we employ a characteristic averaged over the past 12 months as an independent variable and the average return over the next 12 months as dependent variable, the time lag between the two variables is 12 months. As we increase the time lag between independent and dependent variables, under the null hypothesis that expected returns are associated with characteristics indirectly through betas, the informational advantage of the characteristics over beta estimates should diminish and disappear in the limit.

In the additional simulations that we report in Appendix C, we examine the small sample properties of the IV mean-estimator in Proposition 2. For the IV mean-estimator, we find that the ex-ante biases of the slope coefficients on betas range from -6.5% to -4.5% and the biases of the slope coefficients on characteristics are virtually zero under the CAPM and Fama-French three-factor model. In contrast, for the OLS estimator, biases of betas range from -67.6% to -29.1% and those of characteristics range from 4.1% to 18.1%, which indicates that the IV mean-estimates of the slope coefficients on betas and characteristics are substantially less biased than their corresponding OLS estimates. We also investigate the size and power of the asset pricing tests based on the IV mean-estimator. The size of this IV test is close to its theoretical percentile. Under the CAPM, the power of the IV test to detect a positive market risk premium is virtually 100%. Under the Fama-French three-factor model, the power of the IV test for positive market, SMB, and HML risk premiums is also quite close to 100%. In contrast, the power of the OLS test is substantially lower, especially for detecting a positive SMB risk premium, than that of the IV test under the CAPM and Fama-French three-factor model.

With actual (not simulated) data, based on Proposition 2, we run the following cross-sectional regression in month t, i.e., to obtain the IV mean-estimates of risk premiums for all the models that we examined above:

$$\bar{r}_{i,t} = \gamma_{0,t} + \sum_{k=1}^{K} \gamma_{k,t}\hat{\beta}_{i,k,t-1} + \sum_{j=1}^{J} \delta_{j,t}\bar{C}_{i,j,t-1} + \xi_{i,t} \quad (25)$$

where $\bar{r}_{i,t}$ is the average return over month t to month t+11 and the independent and instrumental betas are estimated over the past 36 months as we did earlier. $\bar{C}_{i,i,t-1}$ denotes the average of characteristic *j* over the past 12 months. Table 10 reports the time-series averages of monthly slope coefficients of betas and characteristics. Because the dependent variable in Eq. (25) is the average return over the current and future twelve months (columns labelled as T_m=12), we use the Hansen and Hodrick (HH) standard errors with 12 lags. For comparison, Table 10 also reports the results of asset pricing tests when we use the returns in month t on the left-hand side of Eq. (25) and the characteristics in month t-1 on the right-hand side (columns labeled as T_m=1).

The results in Table 10 are similar to those in the corresponding earlier tables using characteristics only as control variables. Specifically, we find that none of the factor risk premiums is significant for both T_m=1 and T_m=12 at conventional significance levels. Most of characteristics still remain statistically significant, although the effects of some characteristics become weaker than before since portions of them are now captured by the corresponding factor betas. We also find similar results of asset pricing tests when we increase the length of averaging, i.e., T_m in Proposition 2, up to 36 months with 12-month increments. Therefore, the pattern of the results with the IV mean-estimator does not support the notion that characteristics are significant because they are better proxies for true future betas.

5.2. Robustness with Respect to Changes in Beta Estimation Procedure

We carry out a number of tests to examine the robustness of our asset pricing results to changes in how we estimate betas. For the IV estimator, the test results that we report so far employ a 36-month estimation window to obtain betas (i.e., 18 months each for independent and instrumental betas). We also experiment with increasing this estimation interval to 48 and 60 months. In addition, we employ alternate quarters instead of alternate months to estimate independent and instrumental betas. From all these experiments, we find qualitatively similar results of asset pricing tests to those already reported.

We also carry out the following additional experiments to examine whether our test results are sensitive to the changes in the Dimson-type corrections for betas: (i) increasing the number of daily lags up to five days and (ii) using weekly returns to estimate independent and instrumental betas and allowing for up to five weeks of lagged returns.[26] With all these experiments, we find qualitatively similar test results.

5.3. On the Strength of Instrumental Variables

An important issue to consider in IV regressions is the strength of instrumental variables for the corresponding independent variables. The cross-product matrix of instrumental variables and independent variables could be close singular if the correlations between independent and instrumental variables are too low, i.e., if the instruments are too weak. Nelson and Startz (1990) show that if the instruments are sufficiently weak, then the expected value of the IV estimator may not exist. A univariate regression with a weak instrument can provide the intuition behind this result. If the covariance between the independent and instrumental variables is close to zero, then the sample covariance could be small and be either negative or positive, resulting in large variations in both the sign and magnitude of the slope coefficients of IV regressions in finite

[26] We use a rolling window of past 260 weeks (about five years) to estimate these betas.

samples. However, if the covariance and the number of samples are sufficiently large, then the probability that the sample covariance is close to zero becomes negligibly small, and the IV estimator is well behaved.

Nelson and Startz (1990) show that weak instruments would be a concern if

$$\frac{1}{\hat{\rho}_{xz}^2} \gg \mathrm{N}, \tag{26}$$

where $\hat{\rho}_{xz}$ is the correlation between the independent variable (x) and the corresponding instrument (z), and N is the number of individual stocks. In Section 4.2, e.g., when testing the CAPM and Fama-French three-factor model, we had 2425 stocks per month on average and the minimum number of stocks over the entire sample period was 309. From Eq. (26), there would be a weak instrument concern based on the minimum (average) number of stocks, if the correlation were less than .057 (.020) in absolute value.

Table 11 presents the average cross-sectional correlations between the odd- and even-month beta estimates for all asset pricing models that we tested above. The correlation for market beta under the CAPM is .72. The market beta of the Fama-French three-factor (five-factor) model is less precisely estimated and its average correlation is smaller at .59 (.43). The market beta in the q-factor asset pricing model by HXZ and the net beta under the LCAPM also exhibit similar levels of average cross-sectional correlation to that of the market beta under the Fama-French three-factor model. The average correlations for SMB, HML, RMW, CMA, I/A, and ROE betas range from .13 to .48. Although these correlations are smaller than those for market betas, they are all comfortably above the critical value suggested by Nelson and Startz (1990).

Nelson and Startz (1990) and Staiger and Stock (1997) show that the conventional IV standard error estimator based on the asymptotic theory would not be reliable in small samples if the instruments are weak. However, this concern is not relevant in our applications since we use the Fama-MacBeth approach to estimate the standard errors of IV estimator and do not rely on the asymptotic distribution of the IV estimator in our empirical analyses. Nevertheless, we find, in the tests proposed by Staiger and Stock (1997), that our instrumental betas give no cause for concern.[27]

To provide further insights into the strength of the instruments in our IV method, we also

[27] Staiger and Stock (1997) regress the independent variables against their instrumental variables and develop a test based on the goodness of fit for the regression. In unreported results, we find that their test statistics in our applications are well above the critical values for all of our instrumental betas.

estimate the correlations between the instrumental betas and the corresponding true but unobservable betas. Although the true betas are unobservable, we can estimate these correlations based on the correlations between odd- and even-month betas as we show in the following proposition:

Proposition 3*: Let* $\beta_{i,k}$ *be stock* i*'s true but unobservable sensitivity to factor* k *and let* $\hat{\beta}_{odd,i,k}$ *and* $\hat{\beta}_{even,i,k}$ *be the odd- and even-month estimates of the true beta, respectively. Then: we can show that*

$$\text{correlation}\left(\beta_{i,k},\hat{\beta}_{even,i,k}\right)=\text{correlation}\left(\beta_{i,k},\hat{\beta}_{odd,i,k}\right)$$
$$=\sqrt{\text{correlation}\left(\hat{\beta}_{odd,i,k},\hat{\beta}_{even,i,k}\right)}.$$

Proof: See Appendix D.

Table 11 also presents the average cross-sectional correlations between estimated betas and true but unobservable betas.[28] Under the CAPM, the average correlation between even- and odd-month market betas is .72 and thus the average correlation between estimated market betas and unobserved true market betas is .85. Under the Fama-French three- and five-factor models, we find smaller correlations for SMB, HML, RMW, and CMA betas, but even for CMA, the average correlation between estimated CMA betas and unobservable true CMA betas is .36. Under the q-factor asset pricing model, the correlations for the investment and ROE betas are about the same as that for the HML beta under the Fama-French five-factor model. All these correlation estimates are significantly above the cutoff prescribed by Nelson and Startz (1990).

6. Conclusion

We propose a method for estimating risk premiums using individual stocks as test assets. It overcomes concerns about risk premiums estimated with test portfolios, which have been employed in almost all previous researches to mitigate an inherent errors-in-variables problem in testing asset pricing models. Estimated betas from one sample period can serve as effective

[28] To compute the average correlation between estimated betas and true betas, we first compute the square root of the correlation between odd- and even-month betas each month and then compute the average across months. Because the variability of correlation between odd- and even-month betas is relatively small, the square root of average correlation is about the same as the average of the square root of the monthly correlations.

instruments for estimated betas from a non-overlapping sample period that serve as the explanatory variables in cross-sectional regressions. We prove the consistency and provide the asymptotic theory of the proposed IV risk premium estimator when the number of individual stocks and the length of time-series grow simultaneously without bounds. In simulations, our IV method yields unbiased estimates of risk premiums even for relatively short time-series and also provides valid tests for statistical inference. Our simulations also indicate that IV tests are reliable under time-varying betas.

We use the IV method to test whether the premiums for risk factors proposed by several popular asset pricing models are reliably different from zero. The tested asset pricing models are the standard CAPM, the Fama-French three- and five-factor models, the q-factor asset pricing model proposed by Hou, Xue, and Zhang (2015), and the liquidity-adjusted CAPM proposed by Acharya and Pedersen (2005). Previous empirical research, which employed portfolios as tests assets, found strong empirical support for these asset pricing models. But Lewellen, Nagel and Shanken (2010) suggest caution in interpreting those results because of the low dimensionality issue with portfolios. We find that none of the factors from those asset pricing models is associated with a significant risk premium in the cross-section of individual stock returns after controlling for corresponding firm characteristics. The evidence in simulations and empirical analyses indicates that this empirical failure is unlikely to be due to the lack of test power, so it represents a puzzle that calls for further research.

Figure 1
Biases versus Number of Days Used to Estimate Betas

This figure presents the ex-ante and ex-post biases using the ordinary least squares (OLS) and instrumental variables (IV) estimators of the market risk premium under the CAPM, as a function of the number of days in the rolling window to estimate the market betas. The simulations use the market risk premium of 5.80% per annum and 2000 individual stocks in the cross-section. The total sample period for the simulations is 660 months. The y-axis denotes the bias as a percentage of the true market risk premium and the x-axis denotes the number of days (=T) in the rolling window to estimate the market betas. These results are based on 1,000 repetitions for each T.

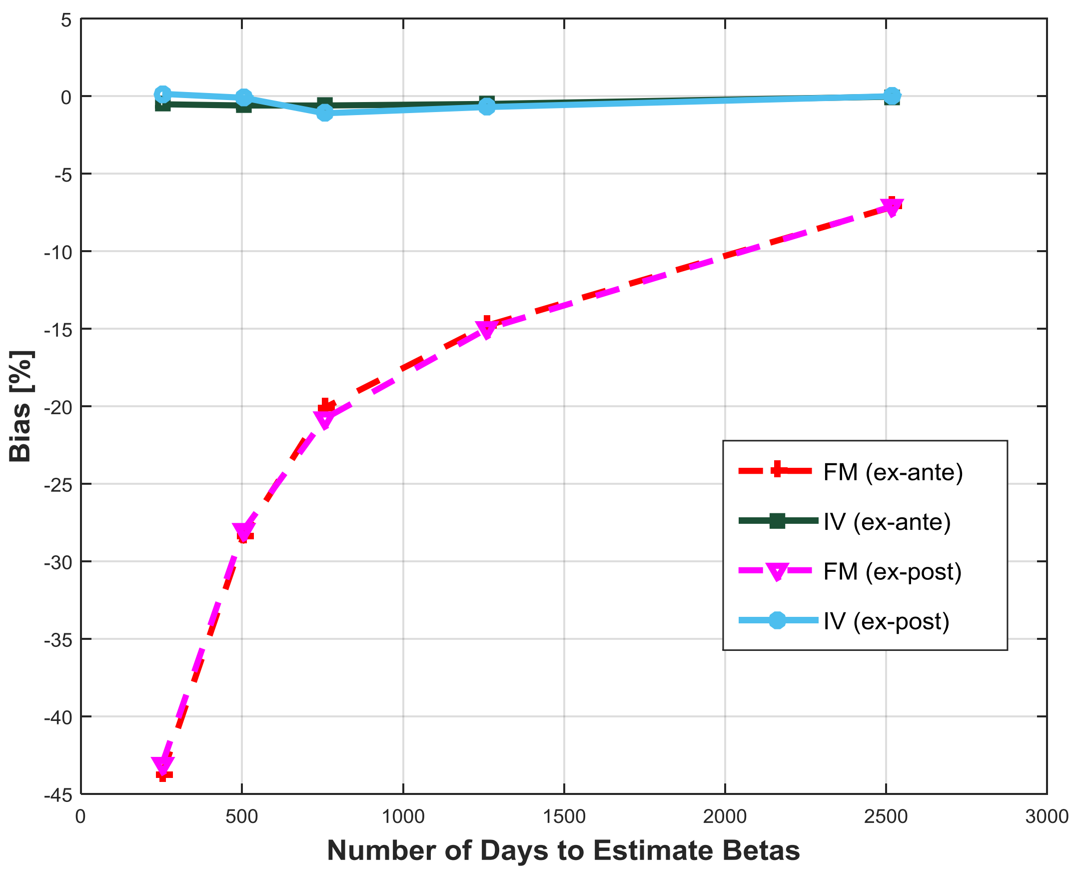

Figure 2
Power of IV and OLS Tests for Varying Levels of Market Risk Premium

This figure presents the power of tests using the ordinary least squares (OLS) and instrumental variables (IV) estimators as a function of the market risk premium. The simulations use 2000 individual stocks in the cross-section and the sample period is 660 months. For each month, rolling betas are estimated using daily return data over the previous 36 months, with data over 18 months to estimate the independent variables (betas) and data over the other non-overlapping 18 months to estimate the corresponding instrumental betas. The power of a test denotes the rejection frequency of the null hypothesis based on 1,000 repetitions. The market risk premiums are in percent per annum.

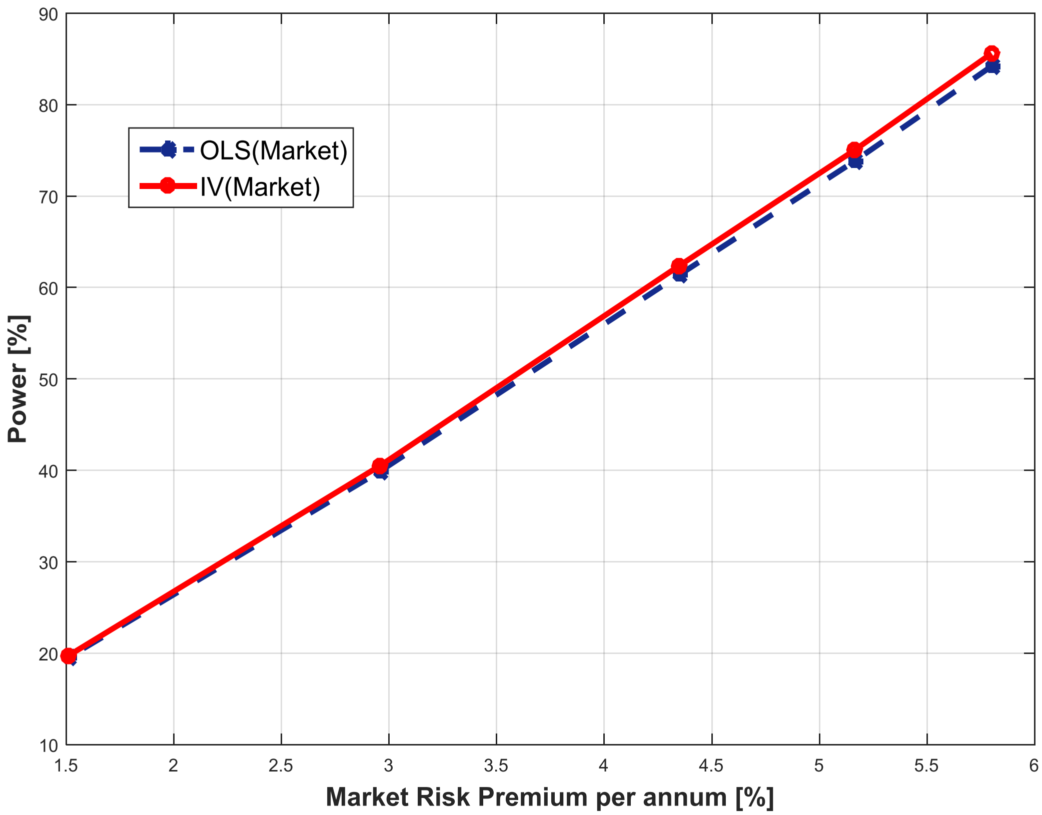

Table 1

Simulation Parameters

This table presents the parameters that are used in the simulations. We set the risk premiums of the common factors and their covariance structure in the simulations equal to the corresponding sample values during the sample period January, 1956 through December, 2012. We maintain the historical covariances among the common factors under the Fama-French three-factor model. The means and standard deviations of the common factors and idiosyncratic volatility are annualized and reported in percentages.

Panel A: Common Factors

		Single-factor CAPM		Fama-French Three-factor Model	
		Mean	Std. Dev.	Mean	Std. Dev.
Factors	MKT	5.80	15.69	5.80	15.69
	SMB			2.64	7.89
	HML			4.36	7.56

Panel B: Betas and Idiosyncratic Volatility

		Single-factor CAPM		Fama-French Three-factor Model	
		Mean	Std. Dev.	Mean	Std. Dev.
Betas	MKT	0.95	0.42	0.95	0.42
	SMB			0.80	0.50
	HML			0.19	0.51
Idiosyncratic Volatility		58.73	23.81	58.73	23.81

Table 2
Small Sample Properties of IV Risk Premium Estimates

Panel A presents the biases and root-mean-squared errors (RMSEs) in risk premium estimates when the second-stage regressions are fitted using the OLS and Instrumental Variable (IV) methods under the single-factor CAPM. Panel B presents those results under the Fama-French three-factor model. The simulation uses 2000 stocks in the cross-section, and the results are based on 1,000 repetitions. The sample period for the simulations is 660 months. For each month, rolling betas are estimated using daily return data over the previous 36 months. Among those 36 months, the IV method uses data over 18 months to estimate the independent variables (betas) and data over the other non-overlapping 18 months to estimate the corresponding instrumental betas. Ex-ante bias is the difference between the mean risk premium estimate and the corresponding true risk premium. Ex-post bias is the difference between the mean risk premium estimate and the sample mean of the corresponding risk factor realizations in that particular simulation. Ex-ante and ex-post biases are expressed as percentages of the true risk premium.

Panel A: Single-factor CAPM

Risk Factor	Estimator	Ex-ante Bias (%)	Ex-post Bias (%)	Ex-ante RMSE	Ex-post RMSE
MKT	OLS	-20.1	-20.8	0.184	0.125
	IV	-0.6	-1.1	0.174	0.080

Panel B: Fama-French Three-factor Model

Risk Factor	Estimator	Ex-ante Bias (%)	Ex-post Bias (%)	Ex-ante RMSE	Ex-post RMSE
MKT	OLS	-28.7	-29.4	0.199	0.158
	IV	1.2	0.5	0.189	0.084
SMB	OLS	-54.4	-55.2	0.136	0.135
	IV	-1.4	-2.1	0.126	0.096
HML	OLS	-50.6	-51.2	0.194	0.193
	IV	1.6	1.0	0.124	0.092

Table 3
Size and Power of the IV Test

Panel A presents the sizes of the OLS and IV tests under the null hypotheses that the risk premiums equal zero for the single-factor CAPM and the Fama-French three-factor model. The t-statistics are computed using Fama-MacBeth standard errors. The number of stocks in the cross-section is set to N=2000, and the results are based on 1,000 repetitions. The sample period for the simulations is 660 months. For each month, rolling betas are estimated using daily return data over the previous 36 months, with data over 18 months to estimate the independent variables (betas) and data over the other non-overlapping 18 months to estimate the corresponding instrumental betas. Panel B presents the power of the OLS and IV tests to reject the null hypothesis when we set the market (MKT) risk premium equal to 5.80% per annum for the single-factor CAPM and when set the MKT, SMB and HML risk premiums equal to 5.80%, 2.64% and 4.36% per annum, respectively, for the Fama-French three-factor model. Panel B presents the percentage of simulations that reject the null hypothesis that the respective factor risk premium is less than or equal to zero at the 5% significance level. The row labeled “MKT or SMB or HML” presents the percentage of 1,000 repetitions that reject the null hypothesis that all of the risk premiums are less than or equal to zero at the 5% significance level.

Panel A: Test Size

Risk	Test	Theoretical Percentiles				
Factor	Based on	1%	2.5%	5%	7.5%	10%
		Single-factor CAPM				
MKT	OLS	1.5%	2.9%	4.9%	8.0%	10.5%
	IV	1.3%	2.8%	5.1%	7.6%	10.2%
		Fama-French Three-factor Model				
MKT	OLS	1.5%	2.8%	5.2%	8.0%	10.5%
	IV	1.3%	2.4%	5.2%	7.3%	9.8%
HML	OLS	1.7%	2.2%	5.1%	8.0%	10.4%
	IV	1.3%	2.7%	5.2%	7.8%	9.9%
SMB	OLS	1.6%	2.9%	4.8%	7.9%	10.3%
	IV	1.1%	2.7%	5.0%	7.7%	10.2%

Panel B: Test Power

Risk Factor	Test Based on OLS	IV
Single-factor CAPM		
MKT	84.2%	85.6%
Fama-French Three-factor Model		
MKT	78.7%	83.8%
SMB	47.3%	51.8%
HML	89.0%	91.5%
MKT or SMB or HML	99.1%	99.6%

Table 4
Summary Statistics

This table presents summary statistics for various characteristics of stocks in the sample. Capitalization is price multiplied by the number of shares outstanding. Book-to-market ratio is computed as in Davis et al. (2000). Excess return is relative to the one-month T-bill rate. Return volatility is the standard deviation of daily returns. The sample period is from January, 1956 through December, 2012.

	Mean	Median	Standard Deviation	Q1	Q3
Number of stocks each month	2,425	2,903	1,286	1,456	3,390
Capitalization, $ billion	0.876	0.094	4.718	0.028	0.383
Book-to-market ratio	0.854	0.719	0.661	0.425	1.089
Excess return (%)	0.535	0.856	5.610	-0.201	1.770
Return volatility (%)	3.140	1.941	6.366	1.018	3.408

Table 5
Correlations Among Estimated Betas
and SIZE and Book-to-Market Characteristics

This table presents the average cross-sectional correlations among betas, SIZE, and BM. Betas are estimated for each month using daily returns data from the previous 36 months. SIZE is the natural logarithm of market capitalization and BM is the book-to-market ratio. Panel A reports the correlations under the CAPM and Panels B and C report the correlations under the Fama-French three-factor model. The sample period is from January, 1956 through December, 2012.

Panel A: Single-factor CAPM

		SIZE	BM
Individual Stocks	MKT	-0.20	-0.24
25 Fama-French portfolios	MKT	-0.56	-0.44

Panel B: Fama-French Three-factor Model: Individual Stocks

	MKT	SMB	HML	SIZE	BM
MKT	1				
SMB	0.34	1			
HML	0.10	0.14	1		
SIZE	0.18	-0.45	-0.13	1	
BM	-0.13	0.06	0.29	-0.34	1

Panel C: Fama-French Three-factor Model: 25 SIZE and BM sorted Portfolios

	MKT	SMB	HML	SIZE	BM
MKT	1				
SMB	-0.08	1			
HML	-0.08	-0.15	1		
SIZE	0.19	-0.97	-0.01	1	
BM	0.07	-0.03	0.88	-0.08	1

Table 6
Risk Premium Estimates with Individual Stocks: CAPM and Fama-French Three-Factor Model

The IV method estimates risk premiums, in percent per month, using individual stocks as test assets. Panel A reports the test results using the IV method in Columns (1) to (5) and those using the OLS method in Columns (6) to (10) for comparison. Panels B and C report the asset pricing results using the IV method for two subperiods. Rows labeled MKT, SMB and HML are the risk premiums for the market, SMB and HML factors, respectively, and the corresponding t-statistics are in parentheses (bold if significant at the 5% level). SIZE is the natural logarithm of market capitalization. BM is the book-to-market ratio at the end of the previous month and logBM is the natural logarithm of BM. Betas for each month are estimated using daily returns data over the previous 36 months and cross-sectional regressions are fitted using the IV and OLS methods. The sample period is from January 1956 through December 2012. N is the average number of stocks in the monthly cross-sections.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	Panel A: 1956-2012, N=2425									
	IV					OLS				
Const	**1.028**	**0.725**	**2.906**	**2.729**	**3.007**	**0.938**	**0.720**	**3.264**	**2.971**	**3.202**
	(7.89)	**(6.00)**	**(4.31)**	**(4.55)**	**(4.92)**	**(7.40)**	**(5.97)**	**(4.86)**	**(4.86)**	**(5.13)**
MKT	-0.246	-0.288	-0.090	-0.018	0.019	-0.102	-0.285	0.040	0.073	0.092
	(-1.36)	(-1.60)	(-0.51)	(-0.10)	(0.11)	(-0.50)	(-1.59)	(0.17)	(0.42)	(0.53)
SMB		**0.301**		-0.043	-0.019		**0.323**		-0.042	-0.007
		(2.20)		(-0.42)	(-0.19)		**(2.22)**		(-0.39)	(-0.07)
HML		**0.344**		0.242	0.185		**0.333**		0.198	0.141
		(2.55)		(1.88)	(1.46)		**(2.37)**		(1.47)	(1.08)
SIZE			**-0.120**	**-0.118**	**-0.122**			**-0.141**	**-0.131**	**-0.133**
			(-3.49)	**(-3.93)**	**(-3.94)**			**(-3.98)**	**(-4.28)**	**(-4.21)**
BM			**0.196**	**0.180**				**0.195**	**0.178**	
			(4.40)	**(4.50)**				**(4.19)**	**(4.41)**	
logBM					**0.177**					**0.169**
					(4.31)					**(4.03)**
	Panel B: 1956-1985, N=1379									
	IV					OLS				
Const	**1.242**	**0.720**	**3.341**	**3.088**		**1.078**	**0.691**	**3.378**	**3.295**	
	(6.62)	**(4.06)**	**(3.30)**	**(3.36)**		**(6.15)**	**(3.94)**	**(3.45)**	**(3.57)**	
MKT	-0.410	-0.399	-0.200	-0.055		-0.265	-0.375	-0.232	-0.007	
	(-1.78)	(-1.69)	(-0.87)	(-0.23)		(-1.04)	(-1.59)	(-0.88)	(-0.03)	
SMB		**0.447**		0.095			0.343		0.101	
		(2.19)		(0.67)			(1.74)		(0.73)	
HML		**0.400**		0.257			**0.405**		0.226	
		(2.03)		(1.41)			**(2.06)**		(1.20)	
SIZE			**-0.139**	**-0.139**				**-0.140**	**-0.150**	
			(-2.67)	**(-2.98)**				**(-2.75)**	**(-320)**	
BM			**0.230**	**0.207**				**0.245**	**0.209**	
			(3.44)	**(3.33)**				**(3.57)**	**(3.35)**	

	Panel C: 1986 to 2012, N=3583							
	IV				OLS			
Const	**0.853**	**0.712**	**2.320**	**2.558**	**0.782**	**0.717**	**3.104**	**2.722**
	(4.97)	**(4.37)**	**(2.74)**	**(3.26)**	**(4.21)**	**(4.43)**	**(3.41)**	**(3.43)**
MKT	-0.084	-0.129	0.060	0.142	0.076	-0.142	0.334	0.144
	(-0.32)	(-0.47)	(0.21)	(0.56)	(0.24)	(-0.52)	(0.85)	(0.57)
SMB		0.116		-0.216		0.114		-0.182
		(0.57)		(-1.42)		(0.53)		(1.09)
HML		0.331		0.264		0.273		0.167
		(1.71)		(1.44)		(1.37)		(0.87)
SIZE			**-0.091**	**-0.108**			**-0.140**	**-0.116**
			(-2.15)	**(-2.77)**			**(-2.84)**	**(-2.95)**
BM			**0.154**	**0.153**			**0.139**	**0.143**
			(2.75)	**(3.14)**			**(2.23)**	**(2.86)**

Table 7
Risk Premium Estimates with Individual Stocks:
Fama-French Five-Factor Model

This table reports risk premium estimates from the IV method, in percent per month, using individual stocks as test assets, and corresponding t-statistics in parentheses (bold if significant at the 5% level). Rows labelled MKT, SMB, HML, RMW, and CMA are the risk premiums for the market, SMB, HML, RMW, and CMA factors, respectively. SIZE is the natural logarithm of market capitalization and BM is the book-to-market ratio at the end of the previous month. OP and INV are the operating profitability and investment/total asset, respectively. Betas for each month are estimated using daily returns data over the previous 36 months and cross-sectional regressions are fitted using the IV method. The sample period is from January, 1964 through December, 2012. N is the average number of stocks in the cross-sections. Panel A reports the test results for the entire sample period, while Panels B and C report those for two subperiods.

	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: 1964-2012, N=2811						
Const	**0.739**	**0.817**	0.541	**0.753**	**0.919**	**3.079**
	(3.05)	**(3.40)**	(1.92)	**(2.97)**	**(3.90)**	**(4.19)**
MKT			-0.198			0.367
			(-0.57)			(1.06)
SMB			0.453			-0.095
			(1.85)			(-0.50)
HML			0.766			0.354
			(1.83)			(0.87)
RMW	0.121		-0.237	0.207		-0.051
	(0.59)		(-0.46)	(1.07)		(-0.11)
CMA		0.030	0.159		-0.043	0.049
		(0.13)	(0.29)		(-0.18)	(0.11)
SIZE						**-0.153**
						(-4.38)
BM						**0.178**
						(4.12)
OP				-0.001		**0.649**
				(-0.00)		**(6.10)**
INV					**-0.963**	**-0.709**
					(-6.62)	**(-9.25)**

Panel B: 1964-1988, N=1992						
Const	0.640	**0.767**	0.304	0.692	**0.894**	**2.670**
	(1.69)	**(2.04)**	(0.65)	(1.73)	**(2.42)**	**(2.28)**
MKT			-0.213			0.217
			(-0.41)			(0.41)
SMB			0.496			-0.097
			(1.37)			(-0.41)
HML			**1.701**			0.891
			(2.90)			(1.63)
RMW	0.218		-0.910	0.142		-0.460
	(0.66)		(-1.32)	(0.45)		(-0.87)
CMA		0.180	0.021		0.123	-0.084
		(0.42)	(0.03)		(0.29)	(-0.14)
SIZE						**-0.149**
						(-2.67)
BM						**0.291**
						(4.34)
OP				-0.186		**0.994**
				(-0.64)		**(5.36)**
INV					**-1.197**	**-0.909**
					(-4.39)	**(-6.65)**
Panel C: 1989-2012, N=3588						
Const	**0.845**	**0.903**	**0.770**	**0.793**	**0.979**	**3.434**
	(2.79)	**(3.01)**	**(2.38)**	**(2.51)**	**(3.31)**	**(3.79)**
MKT			-0.200			0.513
			(-0.44)			(1.11)
SMB			0.409			-0.012
			(1.23)			(-0.04)
HML			-0.123			-0.395
			(-0.21)			(-0.63)
RMW	0132		0.396	0.081		0.721
	(0.45)		(0.52)	(0.32)		(0.84)
CMA		-0.403	0.280		-0.300	0.294
		(-1.30)	(0.37)		(-1.00)	(0.42)
SIZE						**-0.157**
						(-3.70)
BM						0.072
						(1.30)
OP				0.147		**0.331**
				(1.03)		**(3.09)**
INV					**-0.738**	**-0.535**
					(-6.57)	**(-7.24)**

Table 8
Risk Premium Estimates with Individual Stocks:
The q-factor Asset Pricing Model

This table reports the risk premium estimates from the IV method, in percent per month, using individual stocks as test assets, and the corresponding t-statistics in parentheses (bold if significant at the 5% level). Rows labeled MKT, ME, I/A, and ROE report the risk premium estimates for the market, size, investment, and ROE factors, respectively. SIZE is the natural logarithm of market capitalization at the end of the previous month. OP and INV are the operating profitability and investment/total asset, respectively. Betas for each month are estimated using daily returns over the previous 36 months. The sample period is January, 1972 through December, 2012. N is the average number of stocks in the cross-sections.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
				Panel A: 1972-2012, N=3162				
Const	**1.206**	**0.893**	**0.907**	**0.890**	**0.856**	**1.120**	**0.809**	**4.268**
	(8.30)	**(5.00)**	**(4.03)**	**(4.01)**	**(3.68)**	**(4.56)**	**(3.55)**	**(5.39)**
MKT	-0.061				-0.247			0.437
	(-0.28)				(-0.86)			(1.20)
ME		0.110			0.222			-0.118
		(0.61)			(0.67)			(-0.28)
I/A			-0.066		0.001	0.247		-0.547
			(-0.28)		(0.01)	(0.89)		(-0.83)
ROE				-0.087	-0.400		-0.100	-0.632
				(-0.36)	(-0.77)		(-0.46)	(-0.83)
SIZE								**-0.202**
								(-5.41)
OP							**2.699**	**3.734**
							(3.74)	**(4.97)**
INV						**-0.651**		**-0.579**
						(-5.16)		**(-6.54)**
				Panel B: 1972-1992, N=2579				
Const	**1.040**	**0.900**	**1.133**	**0.778**	**0.710**	**1.125**	0.687	**4.116**
	(5.17)	**(3.63)**	**(2.84)**	**(2.09)**	**(2.37)**	**(2.91)**	(1.88)	**(3.38)**
MKT	0.179				-0.076			0.501
	(0.59)				(-0.21)			(1.07)
ME		0.258			0.697			0.719
		(1.07)			(1.56)			(1.21)
I/A			0.729		-0.174	0.862		-0.527
			(1.72)		(-0.29)	(1.46)		(-0.50)
ROE				-0.252	-0.336		**-0.698**	-0.140
				(-0.87)	(-0.46)		**(-2.20)**	(-0.13)
SIZE								**-0.213**

								(-3.58)
OP							**6.11**	**8.012**
							(4.32)	**(5.76)**
INV						**-0.845**		**-0.779**
						(-3.31)		**(-4.88)**

Panel C: 1993-2012, N=3694

Const	**1.234**	**0.936**	**1.102**	**0.838**	**0.926**	**1.117**	**0.843**	**4.305**
	(5.98)	**(3.92)**	**(3.36)**	**(3.10)**	**(2.59)**	**(3.56)**	**(3.06)**	**(4.17)**
MKT	-0.329				-0.278			0.139
	(-0.87)				(-0.57)			(0.25)
ME		-0.021			-0.983			-0.977
		(-0.08)			(-1.59)			(-1.68)
I/A			-0.191		-0.837	-0.084		-1.050
			(-0.68)		(-1.16)	(-0.29)		(-1.30)
ROE				0.193	-1.823		0.074	-1.788
				(0.53)	(-1.68)		(0.22)	(-1.72)
SIZE								**-0.182**
								(-3.88)
OP							0.086	0.001
							(0.15)	(0.00)
INV						**-0.498**		**-0.402**
						(-4.65)		**(-4.59)**

Table 9
Risk Premium Estimates with Individual Stocks:
Liquidity-adjusted CAPM (LCAPM)

This table reports the risk premium estimates from the IV method, in percent per month, using individual stocks as test assets, and the corresponding t-statistics in parentheses (bold if significant at the 5% level). The row labeled LMKT reports the premium estimates for liquidity-adjusted market risk under the liquidity-adjusted CAPM (LCAPM), and the row labeled Amihud illiquidity reports the slope coefficient on firm-specific Amihud illiquidity measure. Liquidity-adjusted market betas for each month are estimated using daily returns over the previous 36 months. The sample period is from January, 1956 through December, 2012. Following the literature, only NYSE/AMEX-listed stocks are included in the analyses. N is the average number of stocks in the cross-sections.

	Sample Period					
	1956-2012, N=1283		1956-1985, N=1204		1986-2012, N=1368	
Constant	**0.559**	**0.503**	**0.833**	**0.719**	0.350	0.312
	(3.85)	**(3.48)**	**(4.32)**	**(3.71)**	(1.64)	(1.44)
LMKT	0.150	0.085	-0.110	-0.165	0.346	0.344
	(0.66)	(0.38)	(-0.34)	(-0.52)	(1.11)	(1.10)
Amihud		**0.220**		**0.361**		**0.061**
Illiquidity		**(4.21)**		**(3.70)**		**(2.78)**

Table 10
Risk Premium Estimates with Characteristics
as Additional Instruments: IV Mean-estimator

This table reports the slope coefficients of the following regression, fitted using the IV mean-estimator in Eq. (25):

$$\bar{r}_{i,t} = \gamma_{0,t} + \sum_{k=1}^{K} \gamma_{k,t}\hat{\beta}_{i,k,t-1} + \sum_{j=1}^{J} \delta_{j,t}\bar{C}_{i,j,t-1} + \xi_{i,t}$$

where $\bar{r}_{i,t}$ is the average return over month t to month t+11 and the independent and instrumental betas are estimated using daily returns over the past 36 months. $\bar{C}_{j,i,t-1}$ denotes the average of characteristic *j* over the past 12 months. The columns titled "T_m=1" and "T_m=12" report the slope coefficients when the averages are taken over the corresponding numbers of months, respectively. Using the IV mean-estimator, the table reports the risk premium estimates for the CAPM, Fama-French three- and five-factors models, q-factor asset pricing model by HXZ, and liquidity-adjusted CAPM. MKT, SMB, HML, RMW, CMA, ME, I/A, and, ROE are the risk premiums for the market, SMB, HML, RMW, CMA, size, investment, and ROE factors, respectively. SIZE is the natural logarithm of market capitalization, BM is the book-to-market ratio, OP is the operating profitability, and INV is investment/total asset at the end of the previous month. LMKT is the liquidity-adjusted market risk premium under the LCAPM, and Amihud illiquidity is the slope coefficient on firm-specific Amihud illiquidity measure. We use Fama-MacBeth standard errors for T_m=1 and Hansen-Hodrick standard errors with 12 lags for T_m=12 to compute the t-statistics reported in parentheses (bold if significant at the 5% level). The sample period for the CAPM, Fama-French three-factor model (FF3M), and liquidity-adjusted CAPM (LCAPM) is from January, 1956 through December, 2012. The sample periods for the Fama-French five-factor model (FF5M) and the q-factor asset pricing model (APM) are from January, 1964 through December, 2012 and from January, 1972 through December 2012, respectively.

	CAPM		FF3M		FF5M		q-factor APM		LCAPM	
	T=1	**T=12**	**T=1**	**T=12**	**T=1**	**T=12**	**T=1**	**T=12**	**T=1**	**T=12**
Intercept	**3.255**	**2.565**	**3.102**	**2.479**	**3.079**	**2.084**	**4.268**	**2.696**	**0.503**	**0.433**
	(4.85)	**(3.49)**	**(5.21)**	**(3.69)**	**(4.19)**	**(3.04)**	**(5.39)**	**(3.77)**	**(3.48)**	**(2.17)**
MKT	0.001	0.056	0.032	0.053	0.367	0.165	0.437	0.335		
	(0.01)	(0.29)	(0.19)	(0.34)	(1.06)	(0.88)	(1.20)	(1.57)		
LMKT									0.085	0.021
									(0.38)	(0.93)
SMB			-0.072	0.042	-0.095	0.123				
			(-0.71)	(0.49)	(-0.50)	(1.41)				
HML			0.196	0.106	0.354	0.141				
			(1.55)	(0.86)	(0.87)	(0.77)				
RMW					-0.051	0.090				
					(-0.11)	(0.42)				
CMA					0.049	0.653				
					(0.11)	(1.88)				
ME							-0.118	0.140		
							(-0.28)	(0.94)		
I/A							-0.547	-0.242		
							(-0.83)	(-0.69)		
ROE							-0.632	-0.354		
							(-0.83)	(-0.94)		
SIZE	**-0.139**	**-0.103**	**-0.136**	**-0.101**	**-0.153**	**-0.089**	**-0202**	**-0.111**		
	(-4.05)	**(-2.78)**	**(-4.55)**	**(-3.03)**	**(-4.38)**	**(-2.61)**	**(-5.41)**	**(-3.40)**		
BM	**0.166**	**0.191**	**0.153**	**0.173**	**0.178**	**0.139**				
	(3.73)	**(2.91)**	**(3.82)**	**(2.95)**	**(4.12)**	**(2.05)**				
OP					**0.649**	0.258	**3.734**	-1.783		
					(6.10)	(1.35)	**(4.97)**	(-1.37)		
INV					**-0.709**	**-0.615**	**-0.579**	-0.324		
					(-9.25)	**(-3.61)**	**(-6.54)**	(-1.84)		
Amihud									**0.220**	**0.152**
Illiquidity									**(4.21)**	**(4.34)**

Table 11
Strength of Instruments

This table presents the average cross-sectional correlations between odd- and even-month betas (in the columns labeled Corr(Odd,Even)) for the CAPM, Fama-French three- and five-factor models, q-factor asset pricing model by HXZ, and liquidity-adjusted CAPM by Acharya and Pedersen. The critical value for the weak instruments test proposed by Nelson and Startz (1990) is .057, based on the smallest number of stocks in any month of the sample period. The square root of the correlation of odd- and even-month betas is the correlation between the unobservable "true" betas and the corresponding beta estimates based on the IV method (in the columns labeled Corr(True Beta, Beta Est.)).

Panel A: CAPM

Sample period	Corr(Odd, Even)	Corr(True Beta, Beta Est.)
	MKT	MKT
1956-2012	0.72	0.85

Panel B: Fama-French Three-factor Model

Sample period	Corr(Odd, Even)			Corr(True Beta, Beta Est.)		
	MKT	SMB	HML	MKT	SMB	HML
1956-2012	0.59	0.48	0.33	0.77	0.69	0.57

Panel C: Fama-French Five-factor Model

Sample period	Corr(Odd, Even)					Corr(True Beta, Beta Est.)				
	MKT	SMB	HML	RMW	CMA	MKT	SMB	HML	RMW	CMA
1964-2012	0.43	0.37	0.21	0.16	0.13	0.66	0.61	0.46	0.40	0.36

Panel D: q-factor Asset Pricing Model

Sample period	Corr(Odd, Even)				Corr(True Beta, Beta Est.)			
	MKT	ME	I/A	ROE	MKT	ME	I/A	ROE
1972-2012	0.49	0.37	0.19	0.21	0.70	0.61	0.44	0.46

Panel E: Liquidity-adjusted CAPM (LCAPM)

Sample period	Corr(Odd, Even)	Corr(True Beta, Beta Est.)
	LMKT	LMKT
1956-2012	0.58	0.76

Appendix

Appendix A. Consistency of the IV Estimator with Time-varying Betas and Simulation Evidence

The proposition below shows that the IV estimator is also consistent when betas are time-varying. For ease of exposition, we present the special case for a single factor model where time-variation in beta is captured by white noise. Online Appendix F contains the proposition and its proof for multi-factor models with time-varying betas that follow any covariance stationary process.

Proposition: Suppose that asset returns follow a one-factor model, and betas of stocks vary over time. Let $\beta_{i,t}$ *be the beta of stock* i *at time* t *and* β_i *be its unconditional mean. Let*

$$\beta_{i,t} = \beta_i + u_{i,t}, \tag{27}$$

where $u_{i,t}$ *is white noise, uncorrelated across stocks. Let* γ_t *be the ex ante risk premium at time t. Under mild regularity conditions, the estimator in Eq. (4),* $\hat{\gamma}_t{}'$*, is N-consistent for any T.*

Proof: Under the one factor model, expected return is given by:

$$E(r_{i,t}) = \gamma_0 + \beta_{i,t}\Upsilon_t = \gamma_0 + (\beta_i + u_{i,t})\Upsilon_t = \gamma_0 + \beta_i\Upsilon_t + u_{i,t}\Upsilon_t. \tag{28}$$

Here, $\Upsilon_t = \gamma_t + f_t$ is the exposed risk premium.

Suppose both β_i and $u_{i,t}$ are observable. We can fit the following model:

$$r_{i,t} = a + \Upsilon_t\beta_i + cu_{i,t} + \varepsilon_{i,t} \tag{29}$$

The slope coefficient on beta is the factor risk premium, i.e.

$$\Upsilon_t = \frac{cov(r_{i,t}, \beta_i)}{var(\beta_i)} \tag{30}$$

In practice β_i and $u_{i,t}$ are not observable. We can obtain an estimate of beta from past data, say $\hat{\beta}$, but $u_{i,t}$ cannot be estimated from returns data since it is a time t innovation. Consider the following IV regression:

$$r_{i,t} = a + \Upsilon_t \hat{\beta}_{EV,i} + \xi_{i,t} \tag{31}$$

where $\hat{\beta}_{EV,i}$ is estimated using half the sample period, and $\xi_{i,t}$ (which can be written as $\varepsilon_{i,t} + u_{i,t}\Upsilon_t - \Upsilon_t(\hat{\beta}_{EV,i} - \hat{\beta}_i)$ based on Eqs. (28) and (29)) is the error in the regression. Let $\hat{\beta}_{IV,i}$ be the instrumental variable, which is estimated using the other half of the sample period. We can express estimates of betas as:

$$\hat{\beta}_{EV,i} = \beta_i + \eta_{EV,i} \tag{32}$$

$$\hat{\beta}_{IV,i} = \beta_i + \eta_{IV,i.} \tag{33}$$

Since we estimate betas using linear regressions, we can express η's as linear combinations of u and ε. Specifically,

$$\eta_{EV,i} = \frac{2}{T}\left(\sum_{t \in EV} \omega_{1,i,t} u_{i,t} + \sum_{t \in EV} \omega_{2,i,t}\, \varepsilon_{i,t}\right) \tag{34}$$

$$\eta_{IV,i} = \frac{2}{T}\left(\sum_{t \in IV} \omega_{1,i,t} u_{i,t} + \sum_{t \in IV} \omega_{2,i,t}\, \varepsilon_{i,t}\right) \tag{35}$$

where $\omega_{1,i,t}$ and $\omega_{2,i,t}$ are finite weights and T/2 is the number of observations used to estimate $\hat{\beta}_{EV,i}$ and $\hat{\beta}_{IV,i}$ Since $\hat{\beta}_{EV,i}$ and $\hat{\beta}_{IV,i}$ are estimated over non-overlapping periods,and betas are uncorrelated with $u_{i,t}$ and $\varepsilon_{i,t}$ cross-sectionally,

$$cov(\eta_{EV,i}, \eta_{IV,i}) = 0, cov(\beta_i, \eta_{IV,i}) = 0, cov(\beta_i, \eta_{EV,i}) = 0,$$
$$cov(\beta_i, u_{i,t}) = 0, cov(\beta_i, \varepsilon_{i,t}) = 0 \tag{36}$$

Also, because both regression residuals in return process, $u_{i,t}$ are white noise, and Υ_t is fixed at time t,

$$cov(r_{i,t}, \eta_{IV,i}) = 0. \tag{37}$$

Therefore,

$$\hat{\gamma}_{IV} \rightarrow \frac{cov(r_{i,t}, \hat{\beta}_{IV,i})}{cov(\hat{\beta}_{EV,i}, \hat{\beta}_{IV,i})} = \frac{cov\left(r_{i,t}, \beta_i\right)}{var(\beta_i)} = \frac{cov\left(a + \Upsilon_t\beta_i + \Upsilon_t u_{i,t} + \varepsilon_{i,t}, \beta_i\right)}{var(\beta_i)} = \Upsilon_t \qquad (38)$$

as N converges to infinity.

If in Eq. (27) $u_{i,t}$ follows any stationary and ergodic process, we prove in Online Appendix F shows that the IV estimator is consistent when we allow T to converge to infinity first, and then allow N to converge to infinity.

Simulations with Time-varying Betas

This section describes our procedure for the simulations with time-varying betas discussed in Section 3.3. The simulation assumes that $\beta_{i,t}$, the beta of stock i in month t, follows an AR(1) process. Specifically:

$$\beta_{i,t}\text{-}\beta_i = \rho\left(\beta_{i,t-1}\text{-}\beta_i\right) + u_{i,t} \qquad (39)$$

where $u_{i,t}$ is the shock to beta, and β_i is the unconditional mean of beta. We first estimate ρ from real data, which we then use in the simulation. Specifically, we first estimate the three-year rolling betas for each stock, producing monthly time-varying betas. We then trim these beta estimates at the 2.5% and 97.5% levels, and shrink them by applying a simple adjustment rule: adjusted beta = 2/3×beta estimate + 1/3. We then compute the average autocorrelation of the betas across all stocks, which equals .96. Table A1 presents the average time-series standard deviations for single-factor and three-factor betas.

To generate time-varying betas, we first randomly generate the time-series mean of each beta as we did for the constant-beta simulations. We next draw $u_{i,t}$ from a normal distribution with mean zero and standard deviation equal to $\sqrt{1-\rho^2}$ times the average time-series standard deviation of the corresponding beta. We then compute $\beta_{i,t}$ through the AR(1) specification above. We assume that $\beta_{i,t}$ stays constant for 21 trading days for a given month. Finally, using this time-varying betas, we generate daily returns by following the same simulation procedure described in

Section 3.1. We conduct the same IV estimation procedure for risk premiums as in the simulations with constant betas. This simulation procedure is used for the single factor CAPM and the Fama-French three-factor model. Table A2 presents the biases and RMSEs of IV risk premium estimates with time-varying betas and Table A3 presents the size and power of IV tests with time-varying betas. The results here are similar to the corresponding results in Tables 2 and 3.

We also experiment with less persistent time-varying betas that have different values of ρ ranging from 0.56 to 0.96. The results from these additional simulations with different ρs are similar to the corresponding results in Tables A2 and A3. For example, with ρ=0.56, under the single factor CAPM, we find that the ex-ante and ex-post biases of IV market risk premium estimates are -1.0% and -1.5%, respectively, while those biases of OLS market risk premium estimates are -28.2% and -28.6%, respectively. With ρ=0.56, under the Fama-French three-factor model, the ex-ante and ex-post biases of IV risk premium estimates range from -3.5% to 0.0%, while those biases of OLS risk premium estimates range from -64.7% to -38.1%. The magnitudes of these biases of IV and OLS risk premium estimates are similar to those with ρ=0.96 in Table A2. When ρ=0.76 is used, under the single factor CAPM, we find that the ex-ante and ex-post biases of IV market risk premium estimates are -2.0% and -4.2%, respectively, while those biases of OLS market risk premium estimates are -28.4% and -30.5%, respectively. With ρ=0.76, under the Fama-French three-factor model, the ex-ante and ex-post biases of IV risk premium estimates range from -5.7% to -2.1%, while the biases of OLS risk premium estimates range from -65.4% to -37.9%. We thus conclude that the different degree of persistence in time-varying betas does not affect the small sample properties of IV risk premium estimator significantly.

Table A1

Time-varying Beta Parameters

Average time-series standard deviations of time-varying betas and their AR(1) Coefficients (= ρ)

		Single Factor Model		Fama-French Three-Factor Model	
		ρ	StdDev	ρ	StdDev
Betas	MKT	0.96	0.15	0.96	0.15
	SMB			0.96	0.19
	HML			0.96	0.21

TABLE A2

Small Sample Properties of IV Risk Premium Estimates
with Time-varying Betas

Panel A: Single-factor CAPM

Risk Factor	Estimator	Ex-ante Bias (%)	Ex-post Bias (%)	Ex-ante RMSE	Ex-post RMSE
MKT	OLS	-25.7	-27.0	0.184	0.147
	IV	-1.5	-2.8	0.182	0.078

Panel B: Fama-French Three-factor Model

Risk Factor	Estimator	Ex-ante Bias (%)	Ex-post Bias (%)	Ex-ante RMSE	Ex-post RMSE
MKT	OLS	-35.4	-36.1	0.212	0.189
	IV	-1.5	-2.2	0.176	0.086
SMB	OLS	-61.0	-60.1	0.146	0.144
	IV	-4.2	-3.4	0.126	0.094
HML	OLS	-56.8	-57.6	0.213	0.216
	IV	-1.9	-2.7	0.112	0.084

TABLE A3
Size and Power of the IV Tests
with Time-varying Betas

Risk Factor	Estimator	Theoretical Percentiles				
		1%	2.5%	5%	7.5%	10%
Panel A: Size for Single-factor CAPM						
MKT	OLS	1.8%	3.2%	4.1%	8.2%	10.9%
	IV	1.3%	2.7%	4.7%	7.9%	10.1%
Panel B: Size for Fama-French Three-factor Model						
MKT	OLS	1.9%	3.1%	4.1%	8.3%	11.1%
	IV	1.4%	2.8%	4.6%	7.7%	10.3%
SMB	OLS	1.7%	3.3%	4.9%	7.1%	9.5%
	IV	1.2%	2.9%	5.2%	7.3%	9.7%
HML	OLS	1.6%	2.9%	4.7%	7.8%	10.5%
	IV	1.4%	2.6%	4.9%	7.2%	9.8%

Risk Factor	Test Power	
	OLS	IV
Panel C: Power for Single-factor Model CAPM		
MKT	85.2%	85.4%
Panel D: Power for Fama-French Three-factor Model		
MKT	79.0%	84.2%
SMB	45.4%	52.9%
HML	91.0%	91.7%
MKT or SMB or HML	99.5%	99.7%

Appendix B. Innovations in Illiquidity Costs

We follow Acharya and Pedersen (2005) and fit the following time-series regression to estimate expected and unexpected components of market-wide illiquidity cost ($\tilde{c}_{MKT,t} = c_{MKT,t}$-$E_{t-1}(c_{MKT,t})$):

$$0.25 + 0.3\overline{ILLIQ}_{MKT,t}P_{MKT,t-1} = \alpha_0 + \sum_{l=1}^{L} \alpha_l \left(0.25 + 0.3\overline{ILLIQ}_{MKT,t-1}P_{MKT,t-1}\right) + \tilde{c}_{MKT,t},$$

where $\overline{ILLIQ}_{MKT,t}$is the value-weighted average of min ($\overline{ILLIQ}_{i,t}, \frac{30-0.25}{0.30P_{MKT,t-1}}$), which Acharya and Pedersen define as un-normalized illiquidity, truncated for outliers. These variables are defined based on Eqs (16) and (17). We cannot reject the hypothesis that the residuals are white noise based on the Durbin-Watson tests for L=2. The results we report in the text are based on the application of the AR(2) model to estimate expected and unexpected components of illiquidity cost for the market as well as for individual stocks. We repeat the tests with L ranging from 2 to 6 and find that the results are not sensitive to the choice of L.

Appendix C. Proof of Proposition 2

Proof: Let $X_{IV,t-1} = \begin{pmatrix} 1 & \cdots & 1 \\ \hat{\beta}_{IV,i,t-1} & \cdots & \hat{\beta}_{IV,N,t-1} \\ \overline{C}_{1,t-1} & \cdots & \overline{C}_{N,t-1} \end{pmatrix}$, $X_{EV,t-1} = \begin{pmatrix} 1 & \cdots & 1 \\ \hat{\beta}_{EV,1,t-1} & \cdots & \hat{\beta}_{EV,N,t-1} \\ \overline{C}_{1,t-1} & \cdots & \overline{C}_{N,t-1} \end{pmatrix}$,

the estimator is $\hat{\gamma}_t^{'} = (\frac{1}{N}X_{IV,t-1}(X_{EV,t-1})')^{-1} \left(\frac{1}{N}X_{IV,t-1}\bar{r}_t^{'}\right)$, where $\bar{r}_t = \left[\bar{r}_{1,t}, \cdots, \bar{r}_{N,t}\right]$.

Denote $\hat{\gamma}_t = [\hat{\gamma}_{0,t}, \hat{\gamma}_{\beta,t}, \hat{\gamma}_{C,t}]$, where $\hat{\gamma}_{0,t}, \hat{\gamma}_{\beta,t}, \hat{\gamma}_{C,t}$ are the estimated coefficient of constant, beta and characteristics in the cross-sectional regression. We will show that the coefficient of beta in the regression converges to the exposed factor risk premium Υ. The proof for coefficients of constant and characteristics converge to zero is similar (we provide a more general proof in the Online Appendix G).

With some algebra, we can show that $\hat{\gamma}_{\beta,t} = \frac{\text{num}}{\text{den}}$, where

num $= (\widehat{cov}(\hat{\beta}_{EV,i,t-1}, \bar{C}_{i,t-1})\hat{E}(\bar{C}_{i,t-1})$-$\widehat{var}(\bar{C}_{i,t-1})\hat{E}(\hat{\beta}_{EV,i,t-1}))\hat{E}(\bar{r}_{i,t})$+ $\widehat{var}(\bar{C}_{i,t-1})\hat{E}(\hat{\beta}_{IV,i,t-1}\bar{r}_{i,t})$-$\widehat{cov}(\hat{\beta}_{EV,i,t-1}, \bar{C}_{i,t-1})\hat{E}(\bar{C}_{i,t-1}\bar{r}_{i,t})$, and
den $= \widehat{cov}(\hat{\beta}_{IV,i,t-1}, \hat{\beta}_{EV,i,t-1})\widehat{var}(\bar{C}_{i,t-1})$-$\widehat{cov}(\bar{C}_{i,t-1}, \hat{\beta}_{EV,i,t-1})\widehat{cov}(\bar{C}_{i,t-1}, \hat{\beta}_{EV,i,t-1})$ where $\hat{E}$, $\widehat{cov}$ and $\widehat{var}$ denote the cross-sectional sample mean, covariance and variance.

Assume that $\beta_{i,t-1} = \beta_i + u_{i,t-1}$, and for each i, $u_{i,t-1}$ follows a stationary and ergodic process with mean zero. Hence, $\hat{\beta}_{EV,i,t-1} \rightarrow \beta_i$ and $\hat{\beta}_{IV,i,t-1} \rightarrow \beta_i$ as T converges to infinity. Similarly, when T_m converges to infinity, $\bar{C}_{i,t-1} \rightarrow C_i$, and $\bar{r}_{1,t} \rightarrow \beta_i \Upsilon$. Therefore, num $\rightarrow (cov(\beta_i, C_i)E(C_i)$-$var(C_i)E(\beta_i))E(\beta_i)\Upsilon + var(C_i)E((\beta_i)^2)\Upsilon - cov(\beta_i, C_i)E(C_i\beta_i)\Upsilon$ = $(var(\beta_i)var(C_i)$-$(cov(C_i, \beta_i))^2)\Upsilon$ when N, T and T_m converge to infinity. Similarly, num $\rightarrow$ ($var(\beta_i)var(C_i)$-$(cov(C_i, \beta_i))^2$ when N, T and T_m converge to infinity. This implies that the estimated risk premium $\hat{\gamma}_{\beta,t} \rightarrow \Upsilon$ when N, T and T_m converge to infinity.

Details of Simulations with Time-varying Betas and Characteristics

This section describes the procedure that we use for the simulations with time-varying betas and time-varying characteristics discussed in Section 5.1. We assume that $\beta_{i,t}$ and $c_{i,t}$, i.e., the beta and corresponding characteristic of stock i in month t, follow AR(1) processes:

$$\beta_{i,t}\text{-}\beta_i = \rho(\beta_{i,t-1}\text{-}\beta_i) + u_{i,t}$$

$$c_{i,t}\text{-}\beta_i = \rho(c_{i,t-1}\text{-}\beta_i) + e_{i,t}$$

where $u_{i,t}$ and $e_{i,t}$ are the shocks to beta and characteristic, respectively, the shock to the characteristic is defined as $e_{i,t} = u_{i,t+1} + \delta_{i,t}$, and β_i is the unconditional mean of beta. We set ρ to 0.96, i.e., beta and characteristic have the same persistence. Note that $\beta_{i,t}$ and $c_{i,t}$ are correlated through β_i cross-sectionally and $\beta_{i,t}$ and $c_{i,t}$ are correlated through $u_{i,t}$ cross-sectionally.

We generate the time-varying betas and daily and monthly returns by following the same simulation procedure described in Appendix A. To generate the time-varying characteristic, we independently generate the shock to characteristic $\delta_{i,t}$ and combine it with the shock to future beta $u_{i,t}$. The standard deviation of $\delta_{i,t}$ is determined based on the time-series average of cross-sectional correlation between $\beta_{i,t}$ and $c_{i,t}$, whose information is presented in Panel B of Table 5.[29] For the IV mean-estimator, we use the average characteristic over the past twelve months as additional instrumental and control variables, and we use the average return over the sample period from month t to month t+11 as the dependent variable.

The same simulation procedure as before is used for the single-factor CAPM and the Fama-French three-factor model. For the IV mean-estimator, Table A4 presents the ex-ante biases and RMSEs of risk premium estimates and slope coefficients of characteristics and Table A5 presents the size and power of the corresponding IV tests, where the theoretical percentile for the size is 5%. For comparison, the results for the OLS estimator are also provided in Tables A4 and A5.

[29] To simulate time-varying market beta and corresponding characteristic, the average of cross-section correlation is set to 0.37, which is the average of absolute value of Corr(SMB beta, SIZE) and Corr(HML beta, BM).

TABLE A4
Small Sample Properties of the IV Mean-estimator
with Time-varying Betas and Characteristics

Panel A: Single-factor CAPM

Risk Factor	Estimator	Ex-ante Bias (%)	Ex-ante RMSE	Ex-ante Bias (%)	Ex-ante RMSE
		Risk Premium		Characteristic	
MKT	OLS	-29.1	0.196	4.1	0.027
	IV Mean	-5.1	0.058	0.1	0.007

Panel B: Fama-French Three-factor Model

Risk Factor	Estimator	Ex-ante Bias (%)	Ex-ante RMSE	Ex-ante Bias (%)	Ex-ante RMSE
		Risk Premium		Characteristic	
MKT	OLS	-37.6	0.222	5.5	0.033
	IV Mean	-5.2	0.059	0.1	0.007
SMB	OLS	-67.6	0.158	18.1	0.048
	IV Mean	-4.5	0.043	-0.2	0.012
HML	OLS	-60.6	0.227	6.7	0.028
	IV Mean	-6.5	0.042	0.1	0.005

TABLE A5

Size and Power of the Tests by the IV Mean-estimator
with Time-varying Betas and Characteristics

Panel A: Single-factor CAPM

Risk Factor	Estimator	Size (Theoretical: 5%)	Power	Size (Theoretical: 5%)
		Risk Premium	Risk Premium	Characteristic
MKT	OLS	4.0%	83.7%	4.5%
	IV Mean	5.4%	99.9%	4.7%

Panel B: Fama-French Three-factor Model

Risk Factor	Estimator	Size (Theoretical: 5%)	Power	Size (Theoretical: 5%)
		Risk Premium	Risk Premium	Characteristic
MKT	OLS	5.7%	77.6%	4.5%
	IV Mean	5.5%	99.9%	4.9%
SMB	OLS	5.8%	36.7%	4.8%
	IV Mean	4.6%	99.1%	4.9%
HML	OLS	5.6%	82.8%	5.4%
	IV Mean	5.3%	99.9%	4.7%

Appendix D. Proof of Proposition 3

For expositional convenience, we assume that the even-month beta is the independent variable and odd-month beta is its instrument. We need to show that the correlation of true beta (x) and estimated beta (x^*) from even months is the square root of the correlation of estimated beta (x^*) and its instrument (z), i.e.,

$$\mathrm{correlation}(x, x^*) = \sqrt{\mathrm{correlation}(x^*, z)}$$

where $x^* = x + u_{even}$ $z = x + u_{odd}$

and, x u_{even} and u_{odd} are mutually independent and $\sigma_u^2 = \sigma_{u_{odd}}^2 = \sigma_{u_{even}}^2$

By the definition of correlation, we have

$$\mathrm{correlation}(x^*, z) = \frac{\mathrm{cov}(x^*, z)}{\sqrt{\mathrm{var}(x^*)\mathrm{var}(z)}} = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_u^2}$$

$$\mathrm{correlation}(x^*, x) = \frac{\mathrm{cov}(x^*, x)}{\sqrt{\mathrm{var}(x^*)\mathrm{var}(x)}} = \frac{\sigma_x^2}{\sqrt{\sigma_x^2(\sigma_x^2 + \sigma_u^2)}}$$

$$= \sqrt{\mathrm{correlation}(x^*, z)}.$$

References

Acharya, V., Pedersen, L.H., 2005. Asset pricing with liquidity risk. Journal of Financial Economics 77, 375-410.

Aharoni, G., Bruce, G., Zeng, Q., 2013. Stock returns and the Miller Modigliani valuation formula: Revisiting the Fama French analysis. Journal of Financial Economics 110(2), 347-357.

Ait-Sahalia, Y., Parker, J.A., Yogo, M., 2004. Luxury goods and the equity premium. Journal of Finance 59, 2959-3004.

Amihud, Y., 2002. Illiquidity and stock Returns: Cross-section and time-series effects. Journal of Financial Markets 5, 31-56.

Amihud, Y., Mendelson, H., 1986. Asset pricing and the bid-ask spread. Journal of Financial Economics 17, 223-249.

Arnott, R., Hsu, J., Moore, P., 2005. Fundamental indexation. Financial Analysts Journal 61, 83-99.

Berk, J.B., Green, R.C., Naik, V., 1999. Optimal investment, growth options and security returns. Journal of Finance 54, 1153-1607.

Black, F., Jensen, M.C., Scholes, M., 1972. The capital asset pricing model: Some empirical tests. Michael C. Jensen, ed: Studies in the Theory of Capital Markets, 79–121.

Blume, M., Friend, I., 1973. A new look at the Capital Asset Pricing Model. Journal of Finance 28, 19-34.

Brennan, M., Chordia, T., Subrahmanyam, A., 1998. Alternative factor specifications, security characteristics, and the cross-section of expected returns. Journal of Financial Economics 49, 345-373.

Campbell, J., Vuolteenaho, T., 2004. Bad beta, good beta. American Economic Review 94, 1249-1275.

Carlson, M., Fisher, A., Giammarino, R., 2004. Corporate investment and asset price dynamics: Implications for the cross-section of returns. Journal of Finance 59, 2577–2603.

Carlson, M., Fisher, A., Giammarino, R., 2006. Corporate investment and asset price dynamics: Implications for SEO event studies and long-run performance. Journal of Finance 61 (3), 1009–1034.

Chan, K.C., Chen, N., 1988. An unconditional asset pricing test and the role of firm size as an instrumental variable for risk. Journal of Finance 43, 309-325.

Chordia, T., Goyal, A., Shanken, J., 2015. Cross-sectional asset pricing with individual stocks: Betas versus characteristics. Working paper, Emory University.

Cochrane J., 1991. Production-based asset pricing and the link between stock returns and economic fluctuations. Journal of Finance 46, 209-237.

Cochrane J., 1996. A Cross-sectional test of an investment-based asset pricing model. Journal of Political Economy 104, 572-621.

Cochrane, J., 2011. Presidential Address: Discount rates. Journal of Finance 66, 1047–1108.

Dimson E., 1979. Risk measurement when shares are subject to infrequent trading. Journal of Financial Economics 7, 197-226.

Eisfeldt A., Papanikolaou, D., 2013. Organization capital and the cross-section of expected returns. Journal of Finance 68, 1365-1406.

Fama E., French, K.R., 1992. The cross-section of expected stock returns. Journal of Finance 47, 427-465.

Fama E., French, K., 1993. Common risk factors in the returns on stocks and bonds. Journal of Financial Economics 33, 3–56.

Fama, E., French, K., 2014. A five-factor asset pricing model. Journal of Financial Economics 116, 1-22.

Fama, E.F., MacBeth, J.D., 1973. Risk, return and equilibrium: Empirical tests. Journal of Political Economy 81, 607-636.

Hou, K., Xue, C., Zhang, L., 2015. Digesting anomalies: An investment approach. Review of Financial Studies, 28 (3), 650-705.

Jagannathan R., Wang, Z., 1996. The conditional CAPM and the cross-section of expected returns. Journal of Finance 51, 3-53.

Jagannathan R., Wang, Z., 1998. An asymptotic theory for estimating beta-pricing models using cross-sectional regression. Journal of Finance 53, 1285-1309.

Jegadeesh N., 1992. Does market risk really explain the size effect? Journal of Financial and Quantitative Analysis 27, 337-351.

Kan R., Robotti, C., Shanken, J., 2013. Pricing model performance and the two-pass cross-sectional regression methodology. Journal of Finance 68, 2617–2649.

Kim D., 1995. The errors-in-variables problem in the cross-section of expected stock returns. Journal of Finance 50, 1605-1634.

Kim S., Skoulakis, G., 2014. Estimating and testing linear factor models using large cross sections: The regression-calibration approach. Working paper, Georgia Institute of Technology.

Kinal, T. W., 1980. The existence of moments of k-class estimators. Econometrica 48, 241-49.

Lettau M., Ludvigson, S., 2001. Resurrectng the (C)CAPM: A cross-sectional test when risk premia are time-varying. Journal of Political Economy 109, 1238-1287.

Lewellen J., Nagel, S., Shanken, J., 2010. A skeptical appraisal of asset pricing tests. Journal of Financial Economics 96, 175-194.

Li Q., Vassalou, M., Xing, Y., 2006. Sector investment growth rates and the cross section of equity returns. Journal of Business 89, 1637-1665.

Lintner J., 1965. The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. Review of Economics and Statistics 47 (1), 13–37.

Litzenberger, R.H., Ramaswamy, K., 1979. The effect of personal taxes and dividends of capital asset prices: The theory and evidence. Journal of Financial Economics 7, 163-196.

Liu L.X., Whited, T., Zhang, L., 2009. Investment-based expected stock returns. Journal of Political Economy 117, 1105-1139.

Merton R., 1973. An intertemporal asset pricing model. Econometrica 41, 867-888.

Nelson, C., Startz, R., 1990. The distribution of the instrumental variable estimator and its t ratio when the instrument is a poor one. Journal of Business 63, S125-S140.

Novy-Marx, R., 2013. The other side of value: The gross profitability premium. Journal of Financial Economics 108, 1–28.

Pastor, L., Stambaugh, R., 2003. Liquidity risk and expected stock returns. Journal of Political Economy 111, 642-685.

Ross, S.A., 1976, The arbitrage theory of capital asset pricing. Journal of Economic Theory 13, 341-360.

Shanken, J., 1992. On the estimation of beta-pricing models. Review of Financial Studies 5, 1-33.

Shanken, J., Zhou, G., 2007. Estimating and testing beta pricing models: Alternative methods and their performance in simulations. Journal of Financial Economics 84, 40-86.

Sharpe, W., 1964. Capital asset prices: A theory of market equilibrium under conditions of risk. Journal of Finance 19 (3), 425–442.

Staiger D., Stock, J., 1997. Instrumental variables regression with weak instruments. Econometrica 65, 557–586.

Theil H., 1971. Principles of Econometrics, 1st Edition, John-Wiley & Sons, New York.

Vasicek, O., 1973. A note on using cross-sectional information in Bayesian estimation of security betas. Journal of Finance 28, 1233–1239.

Zhang, L., 2005. The value premium. Journal of Finance 60(1), 67–103.

Online Appendix

to

Empirical Tests of Asset Pricing Models with Individual Assets: Resolving the Errors-in-Variables Bias in Risk Premium Estimation

by

Narasimhan Jegadeesh, Joonki Noh,
Kuntara Pukthuanthong, Richard Roll, and Junbo Wang

Appendix E. Proof of Proposition 1

Let $\boldsymbol{\beta}$ be matrix of true values of betas for all factors and assets. When there are K factors and N assets, it is a $K \times N$ matrix. Next, let $\hat{\boldsymbol{\beta}}_{IV}$ and $\hat{\boldsymbol{\beta}}_{EV}$ ($K \times N$ matrix) be estimated betas. "IV" subscript indicates the beta instruments and the "EV" subscript denotes the corresponding explanatory variables, respectively. We define IV and EV periods as the periods of data used to estimate IV and EV betas. The symbol "^" indicates an estimate. Define $\hat{\mathbf{B}}_{IV} \equiv [\mathbf{1}; \hat{\boldsymbol{\beta}}_{IV}]$ and $\hat{\mathbf{B}}_{EV} \equiv [\mathbf{1}; \hat{\boldsymbol{\beta}}_{EV}]$, "$\mathbf{1}$" denotes a $1 \times N$ vector of ones and the operator ";" stacks the first row vector on top of second matrix. Hence, $\hat{\mathbf{B}}_{IV}$ and $\hat{\mathbf{B}}_{EV}$ are $(K+1) \times N$ matrices that contain the intercept and K estimated factor loadings. Without loss of generality, we assume that there are total T+1 periods, we use first T periods to estimate IV and EV beta, and run cross-sectional regression using returns at time T+1. Define $\mathbf{r}_t$ as the $1 \times N$ vector of excess returns at t, and denote $\hat{\boldsymbol{\gamma}}$, a $1 \times (K+1)$ vector, as the estimates of zero beta rate and K risk premiums. With these matrix notations, Eq. (3) in the paper can be written as

$$\mathbf{r}_{T+1} = \hat{\boldsymbol{\gamma}} \hat{\mathbf{B}}_{EV} + \boldsymbol{\xi}_{T+1}$$

where and $\boldsymbol{\xi}_{T+1}$ denotes the $1 \times N$ vector of return residuals, and its formula will be defined on next page.

We then propose the following IV estimator for risk premiums at month T+1:

$$\hat{\boldsymbol{\gamma}}_{T+1}' = (\hat{\mathbf{B}}_{IV} \hat{\mathbf{B}}_{EV}')^{-1} (\hat{\mathbf{B}}_{IV} \mathbf{r}_{T+1}'). \qquad \text{(Eq. (4) in paper)}$$

In this paper, $\hat{\mathbf{B}}_{IV}$ ($\hat{\mathbf{B}}_{EV}$) can be either $\hat{\mathbf{B}}_e$ (even-month betas) or $\hat{\mathbf{B}}_o$ (odd-month betas) in our implementation (define $\hat{\mathbf{B}}_e \equiv [\mathbf{1}; \hat{\boldsymbol{\beta}}_{even}]$ and $\hat{\mathbf{B}}_o \equiv [\mathbf{1}; \hat{\boldsymbol{\beta}}_{odd}]$ where $\hat{\boldsymbol{\beta}}_{even}$ and $\hat{\boldsymbol{\beta}}_{odd}$ ($K \times N$ matrix) are estimated betas using data in even and odd months, respectively.). For example, if $\hat{\mathbf{B}}_{IV}$ is $\hat{\mathbf{B}}_o$, $\hat{\mathbf{B}}_{EV}$ is $\hat{\mathbf{B}}_e$, and vice-versa.

E.1. Consistency of the IV estimator

In this section, we prove the consistency of the IV estimator. Define $\hat{\mathbf{B}}_{sample} \equiv [\mathbf{1}; \hat{\boldsymbol{\beta}}_{sample}]$ where sample =odd or even. In addition, $\mathbf{B} \equiv [\mathbf{1}; \boldsymbol{\beta}]$ is the matrix that contains the vector of ones and true value (or unconditional expected value) of betas.

Let $\mathbf{f}_t$ denote the factor realization in period t; it is a $1 \times K$ vectors. And let $\boldsymbol{\varepsilon}_t$ be the vectors of regression residuals at time t for all assets in Eq. (2) of the paper; it is a $1 \times N$ vector. Without loss of generality, assume that factors have zero means (for example, demeaned factors). The estimation error in the first stage regression is $\hat{\boldsymbol{\beta}}_{sample} - \boldsymbol{\beta} = \left(\mathbf{F}^d_{sample}{}'\mathbf{F}^d_{sample}\right)^{-1}\mathbf{F}^d_{sample}{}'\boldsymbol{\Omega}^d_{sample} = \mathbf{u}_{sample}$, where $\mathbf{F}^d_{sample} = \mathbf{F}^d_o \equiv [\mathbf{f}^d_1; \mathbf{f}^d_3 \cdots; \mathbf{f}^d_{T-1}]$ when sample contains odd periods and $\mathbf{F}^d_{sample} = \mathbf{F}^d_e \equiv [\mathbf{f}^d_2; \mathbf{f}^d_4; \cdots; \mathbf{f}^d_T]$ when sample contains even periods. $\boldsymbol{\Omega}^d_{sample} = [\boldsymbol{\varepsilon}^d_1; \boldsymbol{\varepsilon}^d_3; \cdots; \boldsymbol{\varepsilon}^d_{T-1}]$ when sample contains odd periods and $\boldsymbol{\Omega}^d_{sample} = [\boldsymbol{\varepsilon}^d_2; \boldsymbol{\varepsilon}^d_4; \cdots; \boldsymbol{\varepsilon}^d_T]$ when sample contains even periods (for simplicity, we assume that T is even). The superscript d indicates a demeaned factor or residual, where average values of factors or residuals are taken over sample periods (such as odd and even periods). With these notations, $\hat{\mathbf{B}}_{sample} - \mathbf{B} = [\mathbf{0}; \left(\mathbf{F}^d_{sample}{}'\mathbf{F}^d_{sample}\right)^{-1}\mathbf{F}^d_{sample}{}'\boldsymbol{\Omega}^d_{sample}]$, where "$\mathbf{0}$" denotes a $1 \times N$ vector of zeros.

The dependent variable in the second stage cross-sectional regression is $\mathbf{r}_{T+1}$. The regression in the second stage (the cross-sectional regression) can be written as $\mathbf{r}_{T+1} = \hat{\boldsymbol{\gamma}}\hat{\mathbf{B}}_{EV} + \boldsymbol{\xi}_{T+1}$. Since the true model is $\mathbf{r}_{T+1} = (\mathbf{f}_{T+1} + \boldsymbol{\gamma})\boldsymbol{\beta} + \boldsymbol{\varepsilon}_{T+1}$[30], the cross-sectional residuals are

[30] Rewriting Eqs. (1) and (2) in Section 2 of main text in matrix notation, we have $E(\mathbf{r}_{T+1}) = \boldsymbol{\gamma}\boldsymbol{\beta}$ (1') and $\mathbf{r}_{T+1} = \boldsymbol{\alpha} + \mathbf{f}_{T+1}{}'\boldsymbol{\beta} + \boldsymbol{\varepsilon}_{T+1} = \boldsymbol{\alpha} + \boldsymbol{\varepsilon}_{T+1}$ (2') where $\boldsymbol{\alpha} = [\alpha^1, \cdots, \alpha^N]$ and $\boldsymbol{\varepsilon}_t = [\varepsilon^1_{T+1}, \cdots \varepsilon^N_{T+1}]$ (assume that

$$\boldsymbol{\xi}_{T+1} = (\mathbf{f}_{T+1} + \boldsymbol{\gamma})(\boldsymbol{\beta} - \hat{\boldsymbol{\beta}}_{EV}) + \boldsymbol{\varepsilon}_{T+1} = -(\mathbf{f}_{T+1} + \boldsymbol{\gamma})\mathbf{u}_{EV} + \boldsymbol{\varepsilon}_{T+1}.$$

The estimated risk premium for this regression can be written as $\hat{\boldsymbol{\gamma}}_{T+1}' = (\hat{\mathbf{B}}_{IV}\hat{\mathbf{B}}_{EV}')^{-1}(\hat{\mathbf{B}}_{IV}\mathbf{r}_{T+1}')$. In order to show the N-consistency and derive asymptotic distribution of the above estimator, we need to make the following assumptions: (1) The residual process $\boldsymbol{\varepsilon}_t = [\varepsilon_t^1, \varepsilon_t^2, \cdots, \varepsilon_t^N]$[31] is stationary. The elements in $\boldsymbol{\varepsilon}_s$ are cross-sectionally uncorrelated, and $\boldsymbol{\varepsilon}_s$ and $\boldsymbol{\varepsilon}_t$ are uncorrelated when s is not equal to t. Moreover, assume that the variances (second moments) and fourth moments of all residuals exist. Let $\boldsymbol{\Sigma}$ be the covariance matrix for the residuals, then the above assumption implies that it is a diagonal matrix with finite elements. (2) The factor process $\mathbf{f}_t$ is stationary, with finite variance, and is independent with residuals. With these assumptions and several regularity conditions (described in the Theorem), N-consistent is presented in Theorem E1.

Theorem E1 (Consistency, Proposition 1 in paper) Assume that for any t, elements in $B^1(\varepsilon_t^1)', \cdots, B^N(\varepsilon_t^N)'$ (where $[B^1, \cdots, B^N] = \mathbf{B}$[32]) have finite variances, and when N→∞, $\mathbf{BB}'/N$ converges to invertible matrice (denote the matrices by $\mathbf{bb}'$), then the estimated risk premiums $\hat{\boldsymbol{\gamma}}_{T+1}' \equiv (\frac{1}{N}\hat{\mathbf{B}}_{IV}\hat{\mathbf{B}}_{EV}')^{-1}(\frac{1}{N}\hat{\mathbf{B}}_{IV}\mathbf{r}_{T+1}')$, converges to $(0, \boldsymbol{\gamma} + \mathbf{f}_{T+1})'$ when N converges to infinity.

Proof of the consistency: Note that:

$$\hat{\boldsymbol{\gamma}}_{T+1}' - (0, \boldsymbol{\gamma} + \mathbf{f}_{T+1})' = (\frac{1}{N}\hat{\mathbf{B}}_{IV}\hat{\mathbf{B}}_{EV}')^{-1}(\frac{1}{N}\hat{\mathbf{B}}_{IV}\boldsymbol{\xi}_{T+1}').$$

The consistency is established based on Markov's Law of Large Numbers: (1) Since elements in $B^1(\varepsilon_t^1)', \cdots, B^N(\varepsilon_t^N)'$ have finite variances, and factors are stationary with finite variances, and factors are independent with regression residuals, elements in $B^1(\xi_{T+1}^1)', \cdots, B^N(\xi_{T+1}^N)'$ have finite

---

$E(\mathbf{f}_t') = 0$). Take the expectation of (2'), we have $E(\mathbf{r}_{T+1}) = \boldsymbol{\alpha}$ (2''). Combining the three Eqs. ((1'), (2') and (2'')) above, the true return process can be written as $\mathbf{r}_{T+1} = (\mathbf{f}_{T+1} + \boldsymbol{\gamma})\boldsymbol{\beta} + \boldsymbol{\varepsilon}_{T+1}$.

[31] Note that we are using superscript i to indicate the i'th stock in the Online Appendix.

[32] With this notation, when there are K factors, $B^i = [1; \beta_1^i; \cdots; \beta_K^i]$.

variances. Similarly, elements in $B^1((\mathbf{F}^d_{IV}'\mathbf{F}^d_{IV})^{-1}\mathbf{F}^d_{IV}'\boldsymbol{\varepsilon}^{d,1}_{IV})',\cdots,B^N((\mathbf{F}^d_{IV}'\mathbf{F}^d_{IV})^{-1}\mathbf{F}^d_{IV}'\boldsymbol{\varepsilon}^{d,N}_{IV})'$ have finite variances, and elements in $B^1((\mathbf{F}^d_{EV}'\mathbf{F}^d_{EV})^{-1}\mathbf{F}^d_{EV}'\boldsymbol{\varepsilon}^{d,1}_{EV})',\cdots,B^N((\mathbf{F}^d_{EV}'\mathbf{F}^d_{EV})^{-1}\mathbf{F}^d_{EV}'\boldsymbol{\varepsilon}^{d,N}_{EV})'$ have finite variances ($\boldsymbol{\varepsilon}^i_{IV}$ is the residuals vector for stock i in IV periods, and $\boldsymbol{\varepsilon}^i_{EV}$ is the residuals vector for stock i in EV periods; superscript d indicates demeaned residual where average values are taken in IV or EV periods); (2) for any i, $E(B^i(\xi^i_{T+1})')=\mathbf{0}$, $E(B^i((\mathbf{F}^d_{IV}'\mathbf{F}^d_{IV})^{-1}\mathbf{F}^d_{IV}'\boldsymbol{\varepsilon}^{d,i}_{IV})')=\mathbf{0}$, and $E(B^i((\mathbf{F}^d_{EV}'\mathbf{F}^d_{EV})^{-1}\mathbf{F}^d_{EV}'\boldsymbol{\varepsilon}^{d,i}_{EV})')=\mathbf{0}$; (3) regression residuals ε^i_t are not cross-sectional and time-sereies correlated so that $(\mathbf{F}^d_{IV}'\mathbf{F}^d_{IV})^{-1}\mathbf{F}^d_{IV}'\boldsymbol{\varepsilon}^{d,i}_{IV}$ and $(\mathbf{F}^d_{EV}'\mathbf{F}^d_{EV})^{-1}\mathbf{F}^d_{EV}'\boldsymbol{\varepsilon}^{d,i}_{EV}$ are uncorrelated, and $(\mathbf{F}^d_{IV}'\mathbf{F}^d_{IV})^{-1}\mathbf{F}^d_{IV}'\boldsymbol{\varepsilon}^{d,i}_{IV}$ is uncorrelated with ξ^i_{T+1}; (4) Given that regression residuals have finite variances and fouth moments, factors have finite variances, and factors are independent with regression residuals, then elements in $(((\mathbf{F}^d_{IV}'\mathbf{F}^d_{IV})^{-1}\mathbf{F}^d_{IV}'\boldsymbol{\varepsilon}^{d,1}_{IV})(\xi^1_{T+1})',\cdots,((\mathbf{F}^d_{IV}'\mathbf{F}^d_{IV})^{-1}\mathbf{F}^d_{IV}'\boldsymbol{\varepsilon}^{d,N}_{IV})(\xi^N_{T+1})'$ have finite variances, and elements in $(((\mathbf{F}^d_{IV}'\mathbf{F}^d_{IV})^{-1}\mathbf{F}^d_{IV}'\boldsymbol{\varepsilon}^{d,1}_{IV})((\mathbf{F}^d_{EV}'\mathbf{F}^d_{EV})^{-1}\mathbf{F}^d_{EV}'\boldsymbol{\varepsilon}^{d,1}_{EV})',\cdots,((\mathbf{F}^d_{IV}'\mathbf{F}^d_{IV})^{-1}\mathbf{F}^d_{IV}'\boldsymbol{\varepsilon}^{d,N}_{IV})((\mathbf{F}^d_{EV}'\mathbf{F}^d_{EV})^{-1}\mathbf{F}^d_{EV}'\boldsymbol{\varepsilon}^{d,N}_{EV})'$ also have finite variances; apply Markov's Law of Large Numbers,

$$\frac{1}{N}\hat{\mathbf{B}}_{IV}\boldsymbol{\xi}_{T+1}'=\frac{1}{N}\sum_{i=1}^{N}(B^i+[0;(\mathbf{F}^d_{IV}'\mathbf{F}^d_{IV})^{-1}\mathbf{F}^d_{IV}'\boldsymbol{\varepsilon}^{d,i}_{IV}])(\xi^i_{T+1})'\to 0, \text{ and}$$

$$\frac{1}{N}(\hat{\mathbf{B}}_{IV}\hat{\mathbf{B}}_{EV}')=\frac{1}{N}((\mathbf{B}+[\mathbf{0};(\mathbf{F}^d_{IV}'\mathbf{F}^d_{IV})^{-1}\mathbf{F}^d_{IV}'\boldsymbol{\Omega}^d_{IV}])(\mathbf{B}+[\mathbf{0};(\mathbf{F}^d_{EV}'\mathbf{F}^d_{EV})^{-1}\mathbf{F}^d_{EV}'\boldsymbol{\Omega}^d_{EV}])')\to\mathbf{bb}'.$$

This implies that $\hat{\boldsymbol{\gamma}}_{T+1}'-(0,\boldsymbol{\gamma}+\mathbf{f}_{T+1})'$ is N-consistent.■

One key assumption in this proof is that elements in $B^1(\varepsilon^1_t)',\cdots,B^N(\varepsilon^N_t)'$ have finite variances. To satisfy this condition, besides finite variances of residuals, we also impose regularity condition for beta. For example, maximum values of betas among all stocks are finite.

This Theorem can be extended to case that the regression residuals are not highly correlated (Shanken (1992) call it "weakly correlated"). The intuition is that when these correlations are small enough so that $(\frac{1}{N}\hat{\mathbf{B}}_{IV}\boldsymbol{\xi}_{T+1}')$ converges to zero and $\frac{1}{N}(\hat{\mathbf{B}}_{IV}\hat{\mathbf{B}}_{EV}')$ is finite, it can be shown that the estimator is still N-consistency. We also derive the conditional (on factors) and unconditional asymptotic distribution of the estimator. These Theorems and proofs can be requested from the authors.

Appendix F. Consistency of IV estimator with time-varying betas

Theorem E1 requires that factor loading betas are constant. In this subsection, we relax this assumption and analyze the properties of the estimated risk premiums.

Proposition: *In addition to the assumptions in Theorem E1, suppose betas of stocks vary over time. Let* $\boldsymbol{\beta}_t^i$ *be the beta of stock* i *at time* t *and* $\boldsymbol{\beta}^i$ *be its unconditional mean. Let*

$$\boldsymbol{\beta}_t^i \equiv \boldsymbol{\beta}^i + \mathbf{u}_t^i \qquad (1)$$

In addition, true risk premium $\boldsymbol{\gamma}_t$ *is time-varying.*

(1) When $\mathbf{u}_t^i$ and $\mathbf{u}_s^j$ *are uncorrelated for any* $i \neq j$ and $s \neq t$, *and any* $\mathbf{u}_t^i$'s *are uncorrelated with factors and regression residuals,* $\hat{\boldsymbol{\gamma}}_{T+1}' - (0, \boldsymbol{\gamma}_{T+1} + \mathbf{f}_{T+1})'$ *converges to zero in probability when* N *converges to infinity, under mild regularity conditions.*

(2) Assume that for each time t, $\mathbf{u}_t^i$ and $\mathbf{u}_t^j$ *are uncorrelated for any* $i \neq j$. *Moreover, for each* i, $\mathbf{u}_t^i$ *follows a stationary and ergodic process. In addition,* $\mathbf{u}_t^i$'s *are uncorrelated with factors and regression residual. With mild regularity conditions,* $\hat{\boldsymbol{\gamma}}_{T+1}' - (0, \boldsymbol{\gamma}_{T+1} + \mathbf{f}_{T+1})'$ *converges to zero in probability when we take probability limit as T converges to infinity first, and then take probability limit as N converges to infinity.* ■

Proof of proposition:

(1) In this proposition, factor loadings are assumed to be time-varying. Thus, the model becomes:

$$E_t(r^i) = \gamma_0 + \sum_{k=1}^{K} \beta_{t,k}^i \times \gamma_{t,k}.$$

The first stage regression is the following: $r_t^i = \alpha^i + \sum_{k=1}^{K} \beta_{t,k}^i \times f_{k,t} + \varepsilon_t^i$, where $\beta_{t,k}^i \equiv \beta_k^i + u_{t,k}^i$. With the above equation, we can rewrite return dynamics as follows:

$$r_t^i = \alpha^i + \sum_{k=1}^{K} \beta_k^i \times f_{k,t} + (\varepsilon_t^i + \sum_{k=1}^{K} u_k^i \times f_{k,t}).$$

Define $\nu_t^i = (\varepsilon_t^i + \sum_{k=1}^{K} u_k^i \times f_{k,t})$, the above equation becomes

$$r_t^i = \alpha^i + \sum_{k=1}^{K} \beta_k^i \times f_{k,t} + \nu_t^i.$$

It is clear that regression residuals ν_t^i is stationary and it is not correlated across stocks or auto correlated when $\mathbf{u}_t^i$'s are not correlated across stocks and not auto correlated, $\mathbf{u}_t^i$'s are uncorrelated with factors and regression residuals, and the factors are stationary processes with finite variances. Therefore, if we replace ε_t^i with ν_t^i, we can apply Theorem E1 with the same assumptions, regularity conditions, and the proof is the same.

(2) Following the same deviation as in Appendix E, the second stage regression is $\mathbf{r}_{T+1} = \hat{\boldsymbol{\gamma}}\hat{\mathbf{B}}_{EV} + \boldsymbol{\xi}_{T+1}$. Since the true model is $\mathbf{r}_{T+1} = (\mathbf{f}_{T+1} + \boldsymbol{\gamma}_{T+1})\boldsymbol{\beta} + \mathbf{v}_{T+1}$, the cross-sectional residuals are

$$\boldsymbol{\xi}_{T+1} = (\mathbf{f}_{T+1} + \boldsymbol{\gamma}_{T+1})(\boldsymbol{\beta} - \hat{\boldsymbol{\beta}}_{EV}) + \mathbf{v}_{T+1}.$$

$$\hat{\boldsymbol{\gamma}}_{T+1}' - (0, \boldsymbol{\gamma}_{T+1} + \mathbf{f}_{T+1})' = (\frac{1}{N}\hat{\mathbf{B}}_{IV}\hat{\mathbf{B}}_{EV}')^{-1}(\frac{1}{N}\hat{\mathbf{B}}_{IV}\boldsymbol{\xi}_{T+1}').$$

Take probability limit as T converges to infinity, $\hat{\mathbf{B}}_{IV} \to \mathbf{B}$, $\hat{\mathbf{B}}_{IV} \to \mathbf{B}$ and $\boldsymbol{\xi}_{T+1} \to \mathbf{v}_{T+1}$. Hence,

$$(\frac{1}{N}\hat{\mathbf{B}}_{IV}\hat{\mathbf{B}}_{EV}')^{-1}(\frac{1}{N}\hat{\mathbf{B}}_{IV}\boldsymbol{\xi}_{T+1}') \rightarrow (\frac{1}{N}\mathbf{BB}')^{-1}(\frac{1}{N}\mathbf{B}\boldsymbol{\nu}_{T+1}') .$$

Next, take probability limit as N converges to infinity, $(\frac{1}{N}\mathbf{BB}')^{-1}(\frac{1}{N}\mathbf{B}\boldsymbol{\nu}_{T+1}') \rightarrow 0$ following Markov's law of large numbers, with the assumption that the variances of elements in $B^1(\nu^1_{T+1})', \cdots, B^N(\nu^N_{T+1})'$ (where $[B^1, \cdots, B^N] = \mathbf{B}$ and $[\nu^1_{T+1}, \cdots, \nu^N_{T+1}] = \boldsymbol{\nu}_t$) converge to zero as N converges to infinity.■

Appendix G. Time-varying characteristics

In this section, we incorpate time-varying characteristics in the cross-sectionally regression: i.e. in the second stage regression, the independent variables are estimated betas and characteristics of stocks. The key structure is that both estimated betas and characteristics are proxies for the true factor loading (true betas). Therefore, they should be correlated. Moreover, we assume that the processes for betas and characteristics are autocorrelated. We propose a new estimator: mean estimator, and prove its NT-consistency. The mean estimator in this appendix is more general, with the estimator in Proposition 2 in Section 5 as a special example.

Denote that C^i_t as characteristics for stock i at time t. Assume that there are L characteristics, so C^i_t is a L by 1 column vector. The dependent variable in the second stage cross-sectional regression is the average return $\bar{\mathbf{r}}_{NV} = \frac{1}{T_m}\sum_{t \in NV}\mathbf{r}_t$ over the T_m periods not in the sample (we call them NV periods) to construct $\hat{\boldsymbol{\beta}}_{IV}$ and $\hat{\boldsymbol{\beta}}_{EV}$. Without loss of generality, assume that we construct IV and EV betas using observations from time 1 to time T, and NV periods are from $T+1$ to $T+T_m$. In addition, we take average of characteristics from $T-T_c$ to $T-1$, i.e. define $\overline{\mathbf{C}} = \frac{1}{T_c}\sum_{t=1}^{T_c}\mathbf{C}_{T-t}$, with $\mathbf{C}_t = [C^1_t, \cdots, C^N_t]$ an $L \times N$ matrix[33]. With the notations above, we run the following cross-sectional regression with characteristics: $\bar{\mathbf{r}}_{NV} = \hat{\boldsymbol{\gamma}}\hat{\mathbf{B}}_{EV} + \hat{\boldsymbol{\kappa}}\overline{\mathbf{C}} + \boldsymbol{\xi}_{NV}$. The

[33] In Section 5, we assume that $T_c = T_m$, but we relax this assumption here.

estimated coefficient of characteristics, denoted by $\hat{\boldsymbol{\kappa}}$, is a 1 by L vector. Since the true model is $\mathbf{r}_t = (\mathbf{f}_t + \boldsymbol{\gamma}_t)\boldsymbol{\beta}_t + \boldsymbol{\varepsilon}_t$ (i.e. true value of $\boldsymbol{\kappa}$ is zero), the cross-sectional regression residuals are

$$\xi_{NV} = (\frac{1}{T_m}\sum_{t\in NV}(\mathbf{f}_t + \boldsymbol{\gamma}_t))(\bar{\boldsymbol{\beta}}_{NV} - \hat{\boldsymbol{\beta}}_{EV}) + (\frac{1}{T_m}\sum_{t\in NV}(\mathbf{f}_t + \boldsymbol{\gamma}_t))(\boldsymbol{\beta}_t - \bar{\boldsymbol{\beta}}_{NV}) + \bar{\boldsymbol{\varepsilon}}_{NV}$$

where $\bar{\boldsymbol{\beta}}_{NV} = \frac{1}{T_m}\sum_{t\in NV}\boldsymbol{\beta}_t$ and $\bar{\boldsymbol{\varepsilon}}_{NV} = \frac{1}{T_m}\sum_{t\in NV}\boldsymbol{\varepsilon}_t$.

Assume that the process of beta follows Eq. (1) in Appendix F. Hence $\xi_{NV} = (\frac{1}{T_m}\sum_{t\in NV}(\mathbf{f}_t + \boldsymbol{\gamma}_t))(\boldsymbol{\beta} - \hat{\boldsymbol{\beta}}_{EV}) + (\frac{1}{T_m}\sum_{t\in NV}(\mathbf{f}_t + \boldsymbol{\gamma}_t))\frac{1}{T_m}\sum_{t\in NV}\mathbf{u}_t + (\frac{1}{T_m}\sum_{t\in NV}(\mathbf{f}_t + \boldsymbol{\gamma}_t))\mathbf{u}_t + \bar{\boldsymbol{\varepsilon}}_{NV}$. Here, the error term $\mathbf{u}_t = [u_t^1, \cdots, u_t^N]$ is the deviation of true beta at time t from its unconditional expected value, and it is stationary and are auto-correlated. Denote $\mathbf{B} = [B^1, \cdots, B^N]$ the matrix of unconditional expected value of betas.

Similarly, assume that characteristics $C_t^i = C^i + \upsilon_t^i$. For each i, the process υ_t^i is stationary ergodic. Moreover, for any t1 and t2, υ_{t1}^i and υ_{t2}^j are uncorrelated for any $i \neq j$. Denote $\mathbf{C} = [C^1, \cdots, C^N]$ the unconditional expected value of characteristics. In addition, assume that υ_t^i and u_t^i can be correlated, but not perfectly correlated. With these assumptions, the estimated coeffiicents in this regression are defined as

$\hat{\boldsymbol{\gamma}}' = (\frac{1}{N}[\hat{\mathbf{B}}_{IV}, \bar{\mathbf{C}}][\hat{\mathbf{B}}_{EV}, \bar{\mathbf{C}}]')^{-1}(\frac{1}{N}[\hat{\mathbf{B}}_{IV}, \bar{\mathbf{C}}]\frac{1}{T_m}\sum_{t\in NV}\mathbf{r}_t')$. We will prove the NT-consistency through the following Theorem.

Theorem G1 Suppose that the assumptions in Theorem E1, proposition of Appendix E and in this subsection hold. Define

$\delta_s^i = -(\frac{1}{T_m}\sum_{t\in NV}(\mathbf{f}_t + \boldsymbol{\gamma}_t))[\left(\frac{1}{T}\mathbf{F}_{EV}^{d}{}'\mathbf{F}_{EV}^{d}\right)^{-1}(\mathbf{f}_s^{d}{}'\varepsilon_s^{d,i} + u_s^{d,i})]$, for any s in the sample periods used to construct $\hat{\boldsymbol{\beta}}_{IV}$ or $\hat{\boldsymbol{\beta}}_{EV}$. Define $\pi_s^i = (\frac{1}{T_m}\sum_{t\in NV}(\mathbf{f}_t + \boldsymbol{\gamma}_t))u_s^i + (\mathbf{f}_s + \boldsymbol{\gamma}_s)u_s^i + \varepsilon_s^i$ for any s in NV periods.

Define $\vartheta_s^i = [\left(\frac{1}{T}\mathbf{F}_{sample}^d{}'\mathbf{F}_{sample}^d\right)^{-1}(\mathbf{f}_s^d{}'\varepsilon_s^{d,i} + u_s^{d,i})]$ with sample as either IV and EV periods, and s in the sample periods. Assume that the variances of elements in $\frac{1}{TN}(\sum_{i=1}^{N}\sum_{t\in EV}\vartheta_t^i)$ and $\frac{1}{TN}(\sum_{i=1}^{N}\sum_{t\in IV}\vartheta_t^i)$ converge to zero when both T and N converge to infinity, variances of elements in $\frac{1}{TN}(\sum_{i=1}^{N}\sum_{t\in EV}B^i\delta_t^1)$ and $\frac{1}{TN}(\sum_{i=1}^{N}\sum_{t\in EV}C^i\delta_t^1)$ converge to zero when both T and N converge to infinity, variances of elements in $\frac{1}{TN}(\sum_{i=1}^{N}\sum_{t\in EV}B^i(\vartheta_t^i)')$, $\frac{1}{TN}(\sum_{i=1}^{N}\sum_{t\in IV}B^i(\vartheta_t^i)')$, $\frac{1}{TN}(\sum_{i=1}^{N}\sum_{t\in EV}C^i(\vartheta_t^i)')$, and $\frac{1}{TN}(\sum_{i=1}^{N}\sum_{t\in IV}C^i(\vartheta_t^i)')$ converge to zero when both T and N converge to infinity, variances of elements in $\frac{1}{T_cN}(\sum_{i=1}^{N}\sum_{t=T-T_c}^{T-1}B^i(\upsilon_t^i)')$, and $\frac{1}{T_cN}(\sum_{i=1}^{N}\sum_{t=T-T_c}^{T-1}C^i(\upsilon_t^i)')$ converge to zero when both T_c and N converge to infinity, and variances of elements in $\frac{1}{T_mN}(B^1\pi_{T+1}^1 + \cdots + B^N\pi_{T+1}^N + \cdots + B^N\pi_{T+T_m}^N)$ and $\frac{1}{T_mN}(C^1\pi_{T+1}^1 + \cdots + C^N\pi_{T+1}^N + \cdots + C^N\pi_{T+T_m}^N)$ converge to zero when both T_m and N converge to infinity, then $\hat{\boldsymbol{\gamma}}'$ converges to $(0, \frac{1}{T_m}\sum_{t\in NV}(\mathbf{f}_t + \gamma_t), \mathbf{0}_L)'$ (where $\mathbf{0}_L$ is the 1 by L vector of zeros) in probability when N, T_m, T_c and T converge to infinity■

Proof: The estimated risk premium satisfies:

$$\hat{\boldsymbol{\gamma}}' - (0, \frac{1}{T_m}\sum_{t\in NV}(\mathbf{f}_t + \gamma_t), \mathbf{0}_L)' = (\frac{1}{N}[\hat{\mathbf{B}}_{IV}, \overline{\mathbf{C}}][\hat{\mathbf{B}}_{EV}, \overline{\mathbf{C}}]')^{-1}(\frac{1}{N}[\hat{\mathbf{B}}_{IV}, \overline{\mathbf{C}}]\frac{1}{T_m}\sum_{t\in NV}\xi_t').$$

To prove NT-consistency, we need to show:

$$\frac{1}{N}(\hat{\mathbf{B}}_{IV}\frac{1}{T_m}\sum_{t\in NV}\xi_t') = \frac{1}{NT_m}\sum_{i=1}^{N}\sum_{t=T+1}^{T+T_m}(B^i + [0; \left(\mathbf{F}_{IV}^d{}'\mathbf{F}_{IV}^d\right)^{-1}(\mathbf{F}_{IV}^d{}'\varepsilon_{IV}^{d,i} + u_{IV}^{d,i})])\pi_t^i +$$

$$\frac{1}{NT}\sum_{i=1}^{N}\sum_{t\in EV}(B^i + [0; \left(\mathbf{F}_{IV}^d{}'\mathbf{F}_{IV}^d\right)^{-1}(\mathbf{F}_{IV}^d{}'\varepsilon_{IV}^{d,i} + u_{IV}^{d,i})])\delta_t^i \to 0,$$

$$\frac{1}{N}(\overline{\mathbf{C}}\frac{1}{T_m}\sum_{t\in NV}\boldsymbol{\xi}_t') = \frac{1}{NT_m}\sum_{i=1}^{N}\sum_{t=T+1}^{T+T_m}(\mathbf{C}+\frac{1}{T_c}\sum_{t=T-T_c}^{T-1}\boldsymbol{\upsilon}_t)\pi_t^i +$$

$$\frac{1}{NT_m}\sum_{i=1}^{N}\sum_{t\in NV}(\mathbf{C}+\frac{1}{T_c}\sum_{t=T-T_c}^{T-1}\boldsymbol{\upsilon}_t)\delta_t^i \to 0,$$

and $(\frac{1}{N}([\hat{\mathbf{B}}_{IV},\overline{\mathbf{C}}][\hat{\mathbf{B}}_{EV},\overline{\mathbf{C}}]'))^{-1}$ is bounded.

Since the variances of elements in $\frac{1}{TN}(\sum_{i=1}^{N}\sum_{t\in EV}B^i\delta_t^1)$ converge to zero when N and T converge to infinity, and $\left(\mathbf{F}_{IV}^{d}{}'\mathbf{F}_{IV}^{d}\right)^{-1}\mathbf{F}_{IV}^{d}{}'(\varepsilon_{IV}^{d,i}+u_{IV}^{d,i}) \to 0$ as T converges to infinity, by Markov's Law of the Large Numbers,

$$\frac{1}{NT}\sum_{i=1}^{N}\sum_{t\in EV}(B^i+[0;\left(\mathbf{F}_{EV}^{d}{}'\mathbf{F}_{EV}^{d}\right)^{-1}\mathbf{F}_{EV}^{d}{}'(\varepsilon_{EV}^{d,i}+u_{EV}^{d,i})])\delta_t^i \to 0.$$

Similarly, $\frac{1}{NT_m}\sum_{i=1}^{N}\sum_{t=T+1}^{T+T_m}(B^i+[0;\left(\mathbf{F}_{IV}^{d}{}'\mathbf{F}_{IV}^{d}\right)^{-1}\mathbf{F}_{IV}^{d}{}'(\varepsilon_{IV}^{d,i}+u_{IV}^{d,i})])\pi_t^i$ converges to zero since variances of elements in $\frac{1}{T_mN}(B^1\pi_{T+1}^1+\cdots+B^N\pi_{T+1}^N+\cdots+B^N\pi_{T+T_m}^N)$ converge to zero when both N and T_m converge to zero, and $\left(\mathbf{F}_{IV}^{d}{}'\mathbf{F}_{IV}^{d}\right)^{-1}\mathbf{F}_{IV}^{d}{}'(\varepsilon_{IV}^{d,i}+u_{IV}^{d,i}) \to 0$ as T converge to zero. For the same reason, we can show that $\frac{1}{N}(\overline{\mathbf{C}}\frac{1}{T_m}\sum_{t\in NV}\boldsymbol{\xi}_t') \to 0$.

Moreover,

$$\frac{1}{N}(\hat{\mathbf{B}}_{IV}\hat{\mathbf{B}}_{EV}') = \frac{1}{N}(\mathbf{BB}') + \frac{1}{TN}(\sum_{i=1}^{N}\sum_{t\in EV}B^i(\vartheta_t^i)') + (\frac{1}{TN}(\sum_{i=1}^{N}\sum_{t\in IV}B^i(\vartheta_t^i)'))' + \frac{1}{N}(\sum_{i=1}^{N}(\frac{1}{T}\sum_{t\in IV}(\vartheta_t^i)\frac{1}{T}\sum_{t\in EV}(\vartheta_t^i)'))$$

With the assumptions that the variances of elements in $\frac{1}{TN}(\sum_{i=1}^{N}\sum_{t\in EV}B^i(\vartheta_t^i)')$ and $\frac{1}{TN}(\sum_{i=1}^{N}\sum_{t\in IV}B^i(\vartheta_t^i)')$ converge to zero when both T and N converge to infinity and

$\frac{1}{T}\sum_{t\in IV}(\vartheta_t^i)' \to 0$ $\frac{1}{T}\sum_{t\in EV}(\vartheta_t^i)' \to 0$, it is clear that $\frac{1}{N}(\hat{\mathbf{B}}_{IV}\hat{\mathbf{B}}_{EV}') \to \mathbf{bb}'$ when both T and N converge to infinity. Similarly, $\frac{1}{N}(\hat{\mathbf{B}}_{IV}\overline{\mathbf{C}}') \to \mathbf{bC}'$, $\frac{1}{N}(\hat{\mathbf{B}}_{EV}\overline{\mathbf{C}}') \to \mathbf{bC}'$, and $\frac{1}{N}(\overline{\mathbf{C}}\overline{\mathbf{C}}') \to \mathbf{CC}'$; hence, it is clear that $(\frac{1}{N}([\hat{\mathbf{B}}_{IV},\overline{\mathbf{C}}][\hat{\mathbf{B}}_{EV},\overline{\mathbf{C}}]'))^{-1}$ is bounded since beta and characteristics are not perfectly correlated (based on the assumption that υ_t^i and u_t^i are correlated, but not perfectly correlated).■

In above theorem, we assume that betas and characteristics can be correlated stationary and ergodic processes; hence, Proposition 2 in paper is a special case. The key conditions for the above theorem are variances of elements in $\frac{1}{TN}(\sum_{i=1}^{N}\sum_{t\in EV}\vartheta_t^i)$ and $\frac{1}{TN}(\sum_{i=1}^{N}\sum_{t\in IV}\vartheta_t^1)$ converge to zero when both T and N converge to infinity, variances of elements in $\frac{1}{TN}(\sum_{i=1}^{N}\sum_{t\in EV}B^i\delta_t^1)$ and $\frac{1}{TN}(\sum_{i=1}^{N}\sum_{t\in EV}C^i\delta_t^1)$ converge to zero when both T and N converge to infinity, variances of elements in $\frac{1}{TN}(\sum_{i=1}^{N}\sum_{t\in EV}B^i(\vartheta_t^i)')$, $\frac{1}{TN}(\sum_{i=1}^{N}\sum_{t\in IV}B^i(\vartheta_t^i)')$, $\frac{1}{TN}(\sum_{i=1}^{N}\sum_{t\in EV}C^i(\vartheta_t^i)')$, and $\frac{1}{TN}(\sum_{i=1}^{N}\sum_{t\in IV}C^i(\vartheta_t^i)')$ converge to zero when both T and N converge to infinity, variances of elements in $\frac{1}{T_cN}(\sum_{i=1}^{N}\sum_{t=T-T_c}^{T-1}B^i(\upsilon_t^i)')$, and $\frac{1}{T_cN}(\sum_{i=1}^{N}\sum_{t=T-T_c}^{T-1}C^i(\upsilon_t^i)')$ converge to zero when both T_c and N converge to infinity, and variances of elements in $\frac{1}{T_mN}(B^1\pi_{T+1}^1+\cdots+B^N\pi_{T+1}^N+\cdots+B^N\pi_{T+T_m}^N)$ and $\frac{1}{T_mN}(C^1\pi_{T+1}^1+\cdots+C^N\pi_{T+1}^N+\cdots+C^N\pi_{T+T_m}^N)$ also converge to zero when both T_m and N converge to infinity. These conditions are satisfied when cross-sectional correlation of regression residuals, error terms in betas and characteristics among all assets are weak (as in Shanken (1992)), processes for regression residuals, error terms in betas and characteristics of each asset are stationary and ergodic, together with some regularity conditions for the uncondional mean of betas and characteristics (for example, maximum values of the unconditional means of betas and characteristics among all stocks are finite).