When Good News Is Not That Good: The Asymmetric Effect of Correlation Uncertainty

Junya Jiang*
University of North Carolina at Charlotte.

Abstract

This paper examines how aversion to uncertainty about the information transfer across firms affects asset prices in an equilibrium. I show that a firm's stock price reacts more strongly to the bad news than the good news from its *economically linked* firms, and there is *price inertia* if the news is not strong enough. Moreover, I show that equilibrium prices do not always fully incorporate relevant firm-specific news. The stock price movement displays overreaction and underreaction, depending on the magnitude of the news, the information quality, the strength of the economic link, the firm size, and the firm risk. The model further explains the asymmetric pattern of financial time series, including the expected stock return and volatility, and the correlation and covariance. The model offers several testable predictions, which are consistent with recent empirical studies on how asset prices and returns are affected by the firm-specific news.

*Belk College of Business, University of North Carolina at Charlotte, Charlotte, NC 28223. Email: jjiang6@uncc.edu.

1 Introduction

Firms do not exist as independent entities in financial market, but are linked to each other through many types of relationships.[1] The information transfers literature finds that the news about one firm affects the valuation of its economically linked firms in a nontrivial way.[2] The aim of this paper is to examine how an investor's aversion to *uncertainty* about the information transfer affects the asset prices in an equilibrium.

In this paper, the investor views the signal about one firm's future cash flows as *imprecise* or *uncertain* signal to the other firm: good or bad news for a firm does not always indicate good or bad news for the economic-related firms. I develop an equilibrium model to investigate how the uncertainty about the effect of information transfer contributes to each firm's stock price as well as the asset pricing implications.[3] Previous empirical studies find that the non-announcing firm's stock price movements can either over- or underreact (Ramnath, 2002; Thomas and Zhang, 2008; Ramalingegowda et al., 2012; and Chen and Eshleman, 2014). I expand this literature and show that the stock price's over-or underreaction is characterized by several ingredients, including the sign and the magnitude of the news, the information quality, the strength of the economic link, the level of uncertainty, the firm size, and the firm risk. In addition, I present several testable predictions on stock price reaction to news. Compared to most of the previous theoretical models (Daniel, et al., 1998, Barberis, et al., 1998, and Hong and Stein, 1999) that consider only one risky asset in the economy, my model is able to explain the stock price underreaction and overreaction to news *across* firms. Moreover, it provides new insights to understand the pervasive asymmetric patterns of financial time-series, including the correlation, the covariance, the expected returns and volatilities.

[1] The economic links considered in this paper include the customers-suppliers relation in a supply chain (Pandit, Wasley, and Zach, 2011; Cheng and Elsman, 2014), peer firms in the same industry (Ramnath, 2002; Thomas and Zhang, 2008), or a firm and its blockholder (Ramalingegowda et al., 2012).

[2] The information transfer phenomenon has been studied extensively in accounting and finance literature. Firth (1976), Foster (1981), Clinch and Sinclair (1987), and Freeman and Tse (1992) study the effect of earning announcement of one firm to the other firms in the same industry. Han and Wild (1997), Kim et al (2008), and Glesason et al (2008) study the information transfer effect of management earning forecast. Even though I focus on firm-specific news in this paper, the information transfer around specific firm-specific events are also studied in literature.

[3] To the best of my knowledge, this paper is one of the first to investigate the information transfer effect in an equilibrium model.

Specifically, a representative investor observes a piece of news about the future payoff of the "*announcing*" firm, and this news conveys information about the "*non-announcing*" firm indirectly through the correlation channel between the two firms.[4] However, the investor is unable to precisely estimate the impact of news transferred from a related firm and averse to Knightian uncertainty[5]. Due to the uncertainty originated from the information transfer, the investor cautiously processes the news effect across firms, and considers a set of plausible correlation structures in the prior distribution of firms' payoffs. The investor evaluates the outcome regarding to each correlation structure and makes the investment decisions based on the correlation structure that yields the lowest expected utility. This max-min approach of decision-making under Knightian uncertainty is axiomatized by Gilboa and Schmeidler (1989) and its dynamic extension is developed in Epstein and Schneider (2008). The validity of this investor preference facing Knightian uncertainty is consistent with experimental evidence by Ellsberg (1961) and more recent portfolio choice experiments such as Ahn, Choi, Gale, and Kariv (2011) and Bossaerts, Ghirardato, Guarnaschelli, and Zame (2010).[6]

The equilibrium characterization of the information transfer under uncertainty is intuitive and straightforward. Facing the correlation uncertainty, the investor tends to consider the worst-case scenario to determine the news effect across the firms as well as the firm valuations. Suppose the two firms are positively correlated, when there is bad news about the announcing firm, the investor would think this is also bad news to the non-announcing firm, and believe the news would affect the non-announcing firm in a similar way. In other words, the worst case scenario is when the two firms are highly correlated to each other and the news about one firm is highly relevant to the other. Therefore, the equilibrium prices are determined by the highest plausible correlation coefficient. On the other hand, when there is good news about the announcing firm, the investor would think this good news is not that

[4]For illustration purpose, I denote the announcing firm as the one that receives the news concerning the future payoff about itself. It is not required that the news has to be actually announced by the announcing firm, but can come from the financial analysts' reports, or a specific event that directly affects the announcing firm's future payoff. The non-announcing firm is just a firm that is economically related to the announcing firm, of which the future payoff is indirectly affected by the piece of news.

[5]Knightian uncertainty, or ambiguity, is defined as uncertainty about the probabilities over payoffs. Ambiguity is distinguished from risk, which is uncertainty over payoffs (Savage 1954). Another way to think about the difference between risk and ambiguity is risk is when one does not know the outcome but understands the odds of each outcome. Ambiguity on the other hand is a situation where one does not have enough information to understand the odds of each outcome.

[6]This behavior is also consistent with recent research in neuroeconomics that finds that when subjects are faced with decisions under ambiguity, the areas of the brain associated with fear and survival instincts are activated (Hsu, Bhatt, Adolphs, Tranel, and Camerer 2005; Smith, Dickhaut, McCabe, and Pardo 2002).

good to the non-announcing firm. In such cases, the equilibrium prices are determined by the lowest plausible correlation coefficient. Overall, the endogenous correlation structure in the equilibrium corresponds to the *highest* correlation coefficient under bad news, and the *lowest* correlation coefficient under good news. When the news is not strong enough, the endogenous correlation is negatively determined by the magnitude of the news.

Based on the decreasing endogenous correlation structure conditional on the news, I present several important asset pricing implications of the information transfer. *First of all*, I show that the stock price reacts more strongly to the bad news than the good news from a related firm, that is, there is an asymmetric effect of information transfer. When the news from the related firm is not strong enough (to convey whether this is good or bad news), the stock price shows no reaction, and a "*price inertia*" feature is obtained. Intuitively, if the news decreases, an investor requires a lower price as compensation for the lower posterior mean in order to hold the risky assets. However, the aversion to uncertainty dictates the investor to revise his belief about the correlation upwards if the signal drops. The news effect from two directions counterbalances each other. The lower posterior mean that would require a drop in the equilibrium price is exactly offset by the lower risk premium that would require an increase in the price. As a result, the price does not change. Condie, Gauguli and Illeditsch (2015) demonstrate that the stock price shows a lack of reaction, when the investor has concern about the predictability of news regarding the firm *itself*, in a single period model with only one risky asset. Instead, I provide a dynamic model to show how the stock price can display lack of reaction towards news from a related firm. Furthermore, I show that both the asymmetric effect and the price inertia effect are more significant when the ambiguity increases.

Secondly, I show that the firm's stock price could under- or overreact to the news about the related firm, and this over- and under-reaction is determined by several important factors, including the strength of the economic link, the firm capitalization, the information quality, and the level of correlation uncertainty. I show that the price change displays underreaction when the economic link is strong, and overreaction otherwise. This model offers alternative explanations about individual firm's stock price reaction in information transfer literature. Cheng and Eshleman (2014) proposes a moderated confidence hypothesis that, psychologically, investors have difficulty judging the precision of signals, therefore systematically bias their estimates of signal precision toward the unconditional mean. As a result, the investors overweight imprecise signals, resulting in non-announcing firm's stock prices overreaction to the news (as in Thomas and Zhang, 2008). On the other hand, the investors underweight precise signals so the non-announcing firm's stock prices underreact to the news, as documented in Ramnath (2002). My model explains the empirical evidences in Cheng and Eshleman (2014) from an uncertainty perspective. When the signal is precise, it is shown that the autocorrelation of the non-announcing firm is positive, thus the stock price displays underreaction; and the autocorrelation of the non-announcing firm is negative if the signal is very imprecise. Specifically, I characterize the condition under which the autocorrelation of the non-announcing is positive (negative), based on the strength of the economic link, the firm capitalization, the information quality and the level of correlation uncertainty.

Thirdly, the model also demonstrates the information transfer effect on the announcing firm's stock price and the price movement. Intuitively, the better the news the higher the firm's stock price, but the information transfer effect on the announcing firm is quite different from that on the non-announcing firm. The marginal effect of the good news and the bad news on the stock price is symmetric, because the news conveys direct information about the announcing firm. However, when the news is not strong enough, the announcing firm's stock price is more sensitive to the marginal change of news than that when the news is strong. This is because in the equilibrium, when the news about the announcing firm is not strong, the non-announcing firm is lack of reaction, resulting in a larger investor demand for the announcing firm's stock. Moreover, I show that the announcing firm's stock price generates predictability of the non-announcing firm's stock price by examining the cross-correlation under certain conditions.

In addition to the individual firm effect, I also investigate the information transfer effect on the portfolio with all firms. The entire portfolio can also overreact or underreact to the firm-specific news. If the signal is precise, the entire portfolio under-react to the news. If the non-announcing firm is viewed as a representative of all other firms in the market, my model explains the well-documented stock market underreaction (see for instance Jadedeesh and Titman, 1993, 2001). In general, if either the signal is precise or the economic link is strong, the model implies an under-reaction of the entire portfolio. I also derive precise conditions under which the portfolio overreacts to news (DeBondt and Thaler, 1985).

My model is helpful to understand the price momentum and reversal in the financial market from the information transfer perspective. Jadadeesh and Titman (1993, 2001), Lo and Mackinlay (1998), among many others, document positive serial correlation. Previous literatures explain that the momentum of short-term stock continuation because of investor's underreaction to new information (Chen et al, 1996; Barberis, Sheifer and Vishny, 1998; Daniel et al 1998, 2001), investor inattention (Hong and Stein 1999), and investor's information uncertainty (Zhang, 2006). My model documents that the information transfer effect also contributes to the short-term stock market continuation under certain circumstances. On the other hand, the auto-correlation of individual firm's stock price can be positive or negative, and the cross-correlation is helpful to explain the largely undereaction of the stock market (Lo and Mackinlay, 1990). My model offers several new testable predictions in this regard. I present concrete conditions on some fundamental elements - the strength of economic link, firm capitalization, information quality and the level of correlation uncertainty - about the positiveness or negativeness of the auto-correlation of each firm and the cross-correlation between firms. Moreover, the underreaction or overreaction increases with the risk of the asset and ambiguity aversion in the model, which is consistent with the empirical findings in Williams (2015).

Fourth, I examine the risk premium and the expected stock returns. The excess risk premium is generated due to the correlation uncertainty. Similar to the stock price reaction to the news, I also show that the conditional expected stock return of each firm displays different patterns with respect to the news, depending on how the news predicts the assets' future payoffs. The conditional expected return of the announcing firm's stock price always decreases with respect to the news. But the conditional expected return of the non-announcing firm's stock price is not monotonic in general except for a relatively weak economic link.

Lastly, the model also provides new insights to understand the asymmetric pattern of the financial time series, including correlation, covariance, and volatility. Since a high correlation is always associated with the arrival of bad news and a low correlation corresponds to a piece of good news instead, the model explains the asymmetric volatility patterns of the stock price return. The asymmetric volatility is robust and persistent for the announcing firm's stock. The asymmetric property of the non-announcing firm's stock return volatility holds largely, however, due to the price inertia, it may display the opposite asymmetric feature when the news is not strong enough. The asymmetric pattern of the covariance pattern is also consistent with the asymmetric property of the correlation and volatility. I further quantify the measures for the asymmetries conditional on the news and show that the asymmetric pattern of financial time series is more pronounced when the news is strong.

This paper contributes to the literature which explores the asset pricing implications of the firm-specific news. Bernard and Thomas (1990), and Abarnamell and Bernard (1992) report that investors do not seem to completely adjust their earnings expectations based on the error in their earnings expectation, and this underreaction to earnings information leads to predictable stock returns. Sloan (1996) shows that the stock price fails to reflect fully information contained in the accrual and cash flow components of current earnings. Zhang (2006) explains the short-term stock underreaction by the information uncertainty factor. Caskey (2009) develops an equilibrium model with heterogeneous ambiguity-averse investors, and shows that prices underreact to overall aggregate signal but overreact to some signal components. Therefore, Caskey (2009) can explain the stock price overreaction to the non-cash portion of profits and underreaction to the cash portion. By contrast, I consider the firm-specific news instead of aggregative signals, and develop an equilibrium model of information transfer across firms when the investor is ambiguity-averse to the relevance of the information. More importantly, the news impact on the valuation of the announcing firm is jointly determined by the news impact on the valuation of the other relevant firms in equilibrium.

The paper is closely related to a strand of literature on economic links. Cohen and Frazzini (2008) find evidence of return predictability across economically linked firms and stock prices do not promptly incorporate relevant firm-specific news. Patton and Verardo (2012) investigate the announcing firm's stock beta with the release of firm-specific news. They found that when the earning announcements have larger positive or negative surprise, investor can extract more information from the other firms and the aggregate economy, and the stock beta is larger. Cohen and Lou (2002) document substantial return predictability from the set of easy-to-analyze firms to other set of complicated firms, which requires more sophisticated analysis to incorporate the information into prices. To some extent my model is similar to Cohen and Lou (2012), in which the same piece of information affects two sets of firms. My model provides explanations for their findings of return predictability across firms if we view the easy-to-analyze firm as the announcing firm and the other complicated firm as the non-announcing firm. My model also contributes to the economic link literature by investigating the information transfer effect on stock comovement (correlation and covariance) in addition to the expected stock return and volatility. [7]

Since this paper focuses on the information transfer under uncertainty, my model is starkly different from the theoretical models proposed in the behavioral finance literature. Daniel et al. (1998) develop a model based on overconfidence and self attribution bias, in which investors hold too strong beliefs about their own information, thus overreact to the private signals and underreact to public signals. Barberis, Shleifer, and Vishny (1998) suggest that stocks react more strongly to bad news than to good news mainly because investors change their sentiments based on the past streams of realizations, and discount recent information. Hong and Stein (1999) consider a model of information diffusion, in which some investors underreact to the news and other trend followers overreact to the news. By contrast, the firm-specific news in my model is public and the public news can be virtually testable. I show that the stock price overreaction or underreaction can be generated by the level of the correlation uncertainty and other firm-specific elements, instead of purely relying on the psychological bias. The behavioral finance literature also document the asymmetric phenomenon of financial time series. For instance, Hong and Stein (1999) argues that investor heterogeneity is central to the asymmetric phenomenon. Ang, Bakaert, and Liu, (2005), and Ang, Chen, and Xing (2006) study loss aversion and disappointment aversion preference, in which investors care differently about downside losses than the upside gains. My model provides an alternative explanation for the asymmetric pattern of the financial time series from the uncertainty perspective. I further demonstrate that the asymmetry effect is persistent under all market conditions and become more significant as the ambiguity increases. Conrad, Cornell, and Landsman (2002) find that individual stocks do indeed react more strongly to bad earnings announcements versus good earnings announcements in good times, as measured by the equity market valuation, but not in bad times.

[7]Kelsey, Kozhan and Pang (2011), Peng and Johnstone (2016) also find the asymmetric pattern in price continuation and implied volatility.

To empirically test the model predictions in this paper requires a proxy for the information transfer uncertainty, alternatively, correlation uncertainty. Inspired by Zhang (2006) and Bloom (2009) that study the information uncertainty and the macroeconomic uncertainty, the correlation uncertainty in this paper can be tested empirically using the dispersion among analyst forecasts, or the volatility of correlation between the dividends to measure. Zhang (2006) suggests several measures of information uncertainty for the announcing firm, and a similar methodology can be applied to measure the correlation uncertainty. For instance, the ratio of the firm size of the announcing firm to the non-announcing firm, and the ratio of the firm ages can be used as a proxy to test my model prediction. Since my model predictions document the effect of the uncertainty about information transfer on the stock prices and asset returns, I can also empirically examine the changes of those proxies.[8]

This paper draws from many important contributions of asset pricing under ambiguity literature and adds some new contributions in this area. Epstein and Schneider (2002), Caskey (2009), and Illeditsch (2011) address the conditional distribution of signals in an information ambiguity setting.[9] My model departs from the information ambiguity literature in the sense that the information quality is known, instead, the correlation structure among risky assets is ambiguious. To examine the joint distribution for multiple assets' random payoffs, many previous research have investigated the ambiguity on the marginal distribution.[10] In this regard, Jiang and Tian (2016) might be the most relevant study in which the authors study the correlation uncertainty and its asset pricing implications by fixing the marginal distribution. But my model is remarkably different from Jiang and Tian (2016) in that the current paper focuses on the effect of economic shocks and its implications for conditional asymmetric properties, whereas Jiang and Tian (2016) characterize an equilibrium model

[8] A complete test of my model predictions is beyond the scope of this paper and is left for future study. Some relevant empirical evidences are presented in Section 4.

[9] Caskey (2009) considers an ambiguous-averse investor who follows Klibanoff, Marinacci, and Mukerji's (2005) smooth ambiguity aversion preference and a Savage investor who has expected utility with respect to a unique prior belief. Each investor observes informative signals on one risky firm (asset) and the uncertainty on the information quality allows the ambiguity-averse investor prefer to trade based on aggregated signals that reduce ambiguity at the cost of a loss in information. Similar to Caskey (2009), Illeditsch (2011) considers a setting of an ambiguity-averse investor with a random payoff on one risky asset subject to an uncertain shock. Illeditsch (2011) shows that the desire to hedge the information uncertainty leads to excess volatility. In my model, there are two risky assets and the representative investor is ambiguity-averse to the correlation estimation.

[10] For example, Boyle, Garlappi, Uppal, and Wang (2012), Cao, Wang, and Zhang (2005), Easley and O'Hara (2009), and Garlappi, Uppal, and Wang (2007) investigate expected return parameter uncertainty. Easley and O'Hara (2010) and Epstein and Ji (2013) discuss volatility parameter uncertainty.

with heterogeneous correlation ambiguity among investors to explain under-diversification and limited participation puzzle, and flight-to-quality and flight-to-safety.

The rest of this paper is organized as follows. In Section 2, I introduce the model in a dynamic framework of correlation uncertainty. Section 3 characterizes the equilibrium. In Section 4 I present model predictions and supporting empirical evidences. Section 5 concludes and the proof details are provided in Appendixes.

2 Model with Correlation Uncertainty

There are two time periods. Investors trade at time $t = 0$ and $t = 1$ and consumption occurs at the terminal time $t = 2$. There are two risky assets and one risk free asset. The risk-free rate is set to be zero. Each risky asset denotes a stock of a full-equity firm which pays dividend $\tilde{d}_i$ at the terminal time. The total supply of asset $i = 1, 2$ is denoted by $\overline{\theta}_i$.

The dividends are revealed at the terminal time. The marginal distribution of $\left(\tilde{d}_1, \tilde{d}_2\right)$ is known and $\tilde{d}_i \sim \mathcal{N}(\overline{d}_i, \sigma_i^2), i = 1, 2$. A piece of public news about the first firm (the announcing firm) arrives at time $t = 1$, and this news is interpreted as [11]

$$\tilde{s} = \tilde{d}_1 + \epsilon, \tag{1}$$

where ϵ has a normal distribution with zero mean and variance σ_ϵ^2. ϵ is independent of $\tilde{d}_1$ and $\tilde{d}_2$.

A representative investor makes use of the news $\tilde{s}$ for the valuation of the non-announcing firm's stock price. For instance, the investor performs a regression as follows

$$\tilde{d}_2 = \alpha + \beta \times \tilde{s} + \epsilon_2. \tag{2}$$

But the investor is *uncertain* about the impact of news on the announcing firm. In other words, β is a plausible set, rather than a precise number. Since $\beta = \rho \frac{\sigma_1 \sigma_2}{\sigma_s^2}$, where ρ is the unconditional correlation coefficient between $\tilde{d}_1$ and $\tilde{d}_2$, a range $\beta_a \leq \beta \leq \beta_b$ corresponds

[11] Equivalently, this news can be used to forecast the future payoffs of the announcing firm such as $\tilde{d}_1 = a \times \tilde{s} + \epsilon_1$, where ϵ_1 is independent of the news, and $a = \sigma_1^2/\sigma_s^2$.

to a set of unconditional correlation coefficients $\rho_a \leq \rho \leq \rho_b$, where $\beta_a = \rho_a \frac{\sigma_1 \sigma_2}{\sigma_s^2}$ and $\beta_b = \rho_b \frac{\sigma_1 \sigma_2}{\sigma_s^2}$. Therefore, the uncertainty about information transfer is the same as the *correlation uncertainty* betweens firms.

Specifically,

$$\begin{bmatrix} \tilde{d}_1 \\ \tilde{d}_2 \\ \tilde{s} \end{bmatrix} \sim \left(\begin{bmatrix} \bar{d}_1 \\ \bar{d}_2 \\ \bar{d}_1 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 & \sigma_1^2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 & \rho\sigma_1\sigma_2 \\ \sigma_1^2 & \rho\sigma_1\sigma_2 & \sigma_s^2 \end{bmatrix} \right), \rho_a \leq \rho \leq \rho_b. \tag{3}$$

$\mathcal{M}$ is a set of distribution of $(\tilde{d}_1, \tilde{d}_2, \tilde{s})$ given in (3) for all $\rho_a \leq \rho \leq \rho_b$. Given the uncertainty about the information transfer effect, or equivalently, the correlation uncertainty, the investor is ambiguity-averse in the sense of having multiple-prior utility in Epstein and Schneider (2007) and Wang (2003) as follows,

$$U_t = \min_{m_t \in \mathcal{M}_t} \mathbb{E}_{m_t}[u(C_t) + \alpha U_{t+1}], \tag{4}$$

where $u(\cdot), C_t$, and α are the standard utility function, consumption at t and the subjective discount factor respectively. For simplicity I assume that $u(W) = -e^{-\gamma W}$, $\alpha = 1$ and there is no consumption prior to the terminal time. Let $\mathcal{M}_t$ and m_t denote the set of models considered by the investor at time t and a specific model within that set, respectively. $E_{m_t}[\cdot]$ is the expectation given the beliefs generated by model m_t.

Precisely, the investor at time $t = 0$ is aware of the news coming and the set of models is

$$\mathcal{M}_0 = \left\{ m_\rho : (\tilde{d}_1, \tilde{d}_2) \text{ has a Gaussian distribution via (3), written as } m_\rho, \rho \in [\rho_a, \rho_b] \right\}. \tag{5}$$

The set of models $\mathcal{M}_1$ at time $t = 1$ is

$$\mathcal{M}_1 = \left\{ m(s) : m(s) \text{ is the conditional distribution of } (\tilde{d}_1, \tilde{d}_2) \text{ under } m \in \mathcal{M} \text{ given } s \right\}. \tag{6}$$

The model significantly differs from the previous studies about information ambiguity. Epstein and Schneider (2008), and subsequent studies such as Caskey (2009), Illeditsch (2011), Kelsey, Kozhan and Pang (2011), and Zhou (2015), all investigate the ambiguity about the news quality in the sense that variance of the signal, σ_ϵ, moves within a plausible range, while the correlation structure of asset payoffs is given as exogenous.[12] In contrast, the investor in my setting has no ambiguity about the news quality. In fact, the ambiguity is about the relevance of news across firms; alternatively, the ambiguity about the asset payoffs' correlated structure. By its very construction, $\mathcal{M}_0$ and $\mathcal{M}_1$ together satisfy the dynamic consistency condition in Epstein and Schneider (2007) and Wang (2008).

3 Characterization of Equilibrium

In this section I first characterize the equilibrium at $t = 1$. Before doing so, I first solve the optimal portfolio choice problem for the representative investor, by characterizing the optimal demand and the worst-case correlation coefficient between the asset payoffs when the asset prices are given exogenously. The characterization of the equilibrium at $t = 0$ is presented afterwards.

3.1 Optimal Portfolio Choice

By abuse of notation I use p_i to represent the price at time $t = 1$ in this section. Under the CARA utility assumption, the optimal portfolio choice problem under consideration is

$$\max_{\theta} \min_{\rho \in [\rho_a, \rho_b]} \mathbb{E}_\rho \left[u(W_2) | \tilde{s} = s \right] = u \left(\max_{\theta} CE(\theta) \right) \tag{7}$$

where $W_2 = W_1 + \theta_1(\tilde{d}_1 - p_1) + \theta_2(\tilde{d}_2 - p_2)$ and $\theta = (\theta_1, \theta_2)$ is the demand vector on the risky assets, and $CE(\theta) = \min_{\rho \in [\rho_a, \rho_b]} CE(\rho, \theta)$ is the certainty equivalent of the multi-prior expected utility (MEU) investor for a given demand vector θ. $CE(\rho, \theta) = \mathbb{E}_\rho \left[W_2 | \tilde{s} = s \right] - \frac{\gamma}{2} Var_\rho \left[W_2 | \tilde{s} = s \right]$ denotes the certainty equivalent of a standard expected utility (SEU) investor with the belief that the correlation structure of the asset payoff is ρ. For a SEU investor, there is no uncertainty about the effect of information transfer.

Let us start with the computation of the certain equivalent of a MEU investor. If there is no holdings on the second risky asset ($\theta_2 = 0$), then any correlation coefficient $\rho \in [\rho_a, \rho_b]$

[12] See also Mele and Sangiorgi (2015), Condie and Ganguli (2011), and Condie, Ganguli and Illeditsch (2015) for the ambiguity about information quality in a rational equilibrium model.

solves $CE(\theta)$. On the other hand, if $\theta_2 \neq 0$, let $\phi = \frac{\sigma_1^2}{\sigma_1^2+\sigma_\epsilon^2}$, and

$$\hat{\rho}(s;\theta) = \frac{\sigma_1\theta_1}{\sigma_2\theta_2}\frac{1-\phi}{\phi} - \frac{1}{\gamma\theta_2}\frac{s-\overline{d}_1}{\sigma_1\sigma_2}, \tag{8}$$

then [13]

$$CE(\theta) = \begin{cases} CE(\rho_a, \theta), \text{if } \hat{\rho}(s;\theta) < \rho_a \\ CE(\rho_b, \theta), \text{if } \hat{\rho}(s;\theta) > \rho_b \\ CE\left(\hat{\rho}(s;\theta), \theta\right), \text{if } \rho_a \leq \hat{\rho}(s;\theta) \leq \rho_b. \end{cases} \tag{9}$$

The intuition of (9) is as follows. Without loss of generality I assume a positive holding on the second risky asset ($\theta_2 > 0$), the worst-case correlation coefficient depends on the trade-off between the effect of news on the portfolio mean and the portfolio variance. For the portfolio mean, the correlation coefficient has a positive effect if and only if the signal is greater than its expected value, which indicates good news for the first firm.

$$argmin_{\rho\in[\rho_a,\rho_b]}\mathbb{E}_\rho[W] = \begin{cases} \rho_a, \text{if } s > \overline{d}_1 \\ \rho_b, \text{if } s < \overline{d}_1, \end{cases}$$

When $s = \overline{d}_1, \mathbb{E}_\rho[W]$ is independent of the correlation coefficient. For the portfolio variance, it depends on the correlation structure. It is easy to see that,[14]

$$argmax_{\rho\in[\rho_a,\rho_b]}Var_\rho[W] = \mathcal{L}\left(\rho_a, \rho_b; \frac{\sigma_1\theta_1}{\sigma_2\theta_2}\frac{1-\phi}{\phi}\right).$$

Put it together, the overall effect of the correlation on $CE(\rho,\theta)$ depends on both the news and the correlation structure. Specifically,

$$argmin_{\rho\in[\rho_a,\rho_b]}CE(\rho,\theta) = \mathcal{L}\left(\rho_a, \rho_b; \hat{\rho}(s;\theta)\right).$$

For the MEU investor, the correlation structure used to compute the certain equivalent is *negatively* determined by the news s. I will show the same insight in equilibrium in the next subsection.

[13] See Appendix A for its proof.

[14] $\mathcal{L}(\rho_a, \rho_b; x)$ is x truncated by ρ_a and ρ_b on both sides.

Let $S_i = (\overline{d}_i - p_i)/\sigma_i$ be the unconditional Sharpe ratio of asset $i = 1, 2$. In solving the optimal portfolio choice problem for the MEU investor, I use

$$\tau(x,y) = \begin{cases} \min\left\{\frac{x}{y}, \frac{y}{x}\right\}, & \text{if } xy > 0, \\ \max\left\{\frac{x}{y}, \frac{x}{y}\right\}, & \text{if } xy < 0, \\ 0, & \text{if } xy = 0. \end{cases} \tag{10}$$

to describe the dispersion between x and y.

Proposition 1 *Let $\theta(\rho)$ denote the optimal demand when the correlation coefficient between asset payoff is ρ for a SEU investor, i.e.,*

$$\theta(\rho) = \frac{1}{\gamma}\Sigma_\rho^{-1} \times \begin{bmatrix} \overline{d}_1 + \phi(s - \overline{d}_1) - p_1 \\ \overline{d}_2 + z_\rho\phi(s - \overline{d}_1) - p_2 \end{bmatrix}, \tag{11}$$

where

$$\Sigma_\rho = \begin{bmatrix} \sigma_1^2(1-\phi) & \rho\sigma_1\sigma_2(1-\phi) \\ \rho\sigma_1\sigma_2(1-\phi) & \sigma_2^2(1-\rho^2\phi) \end{bmatrix}, \tag{12}$$

$z_\rho = \rho\frac{\sigma_2}{\sigma_1}$. *Assume that at least one unconditional Sharpe ratio is not zero, $\rho^* = \mathcal{L}\left(\rho_a, \rho_b; \tau(S_1, S_2)\right)$, then $\theta(\rho^*)$ is the optimal demand of the representative investor under correlation uncertainty and ρ^* is its corresponding worst-case correlation coefficient.*

To explain its intuition, I assume that the investor has no knowledge at all about the correlation coefficient. When two Sharpe ratios are close to each other, it indicates that both assets offer very similar investment opportunities, thus the higher correlation the smaller the diversification benefits. The worst-case scenario is associated with the highest possible correlation coefficient. On the other hand, when two risky assets generate fairly opposite investment opportunities, the diversification benefit is increasing with respect to the asset correlation, therefore, the worst-case scenario of a mean-variance utility is obtained at the lowest correlation coefficient. Therefore, the worst-case correlation coefficient must be $\tau(S_1, S_2)$, a *similarity measure* of Sharpe ratios, as documented in Proposition 1.

In the optimal portfolio choice problem of a MEU investor, the worst-case correlation structure depends on the dispersion of the unconditional Sharpe ratios, since the unconditional Sharpe ratios are given exogenously. In equilibrium, the stock prices depend on the news so as to the unconditional Sharpe ratio, as a consequence, the worst-case correlation coefficient relies on the news. This is the objective of the next subsection.

3.2 Equilibrium at $t = 1$

Let $n \equiv \frac{\sigma_1 \overline{\theta}_1}{\sigma_2 \overline{\theta}_2} \frac{1-\phi}{\phi}$. The number n can be written as $\frac{\sigma_\epsilon^2}{\sigma_1 \sigma_2 \overline{\theta}}$ or alternatively $\left(\frac{\sigma_\epsilon}{\sigma_1}\right)^2 \frac{\sigma_1 \overline{\theta}_1}{\sigma_2 \overline{\theta}_2}$, where $\overline{\theta} = \frac{\overline{\theta}_2}{\overline{\theta}_1}$ denotes the ratio of firm 2's share to the firm 1's share. Notice that σ_i is the asset price volatility, a product of the return volatility and the stock price. Therefore, $\sigma_i \overline{\theta}_i$ equals a product of the firm capitalization and its return volatility. Consequently, $\frac{\sigma_2 \overline{\theta}_2}{\sigma_1 \overline{\theta}_1}$ is the ratio of the firm capitalization times the ratio of return volatility.

Proposition 2 *1. (*The Endogenous Correlation*) The endogenous correlation coefficient between the asset payoffs conditional on $\tilde{s} = s$ is $\rho(s)\sqrt{\frac{1-\phi}{1-\rho(s)^2\phi}}$, where $\rho(s)$ is the worst-case correlation coefficient that is determined explicitly as follows.*

- *For all* bad news $s < s^L \equiv \overline{d}_1 + \gamma \sigma_1 \sigma_2 \overline{\theta}_2 (n - \rho_b)$, $\rho(s) = \rho_b$;
- *for all* good news $s > s^H \equiv \overline{d}_1 + \gamma \sigma_1 \sigma_2 \overline{\theta}_2 (n - \rho_a)$, $\rho(s) = \rho_a$;
- *for all moderate news $s \in [s^L, s^H]$,*

$$\rho(s) = \frac{1}{\sigma_1 \sigma_2 \overline{\theta}_2} \left\{ \overline{\theta}_1 \sigma_\epsilon^2 - \frac{s - \overline{d}_1}{\gamma} \right\}. \tag{13}$$

2. (The Endogenous Asset Prices) *The endogenous stock price is given by*

$$p_i(s) = \mathbb{E}_{\rho(s)} \left[\tilde{d}_i | \tilde{s} = s \right] - \gamma Cov_{\rho(s)}(\tilde{d}_i, \tilde{d}), i = 1, 2, \tag{14}$$

where $\tilde{d} = \overline{\theta}_1 \tilde{d}_1 + \overline{\theta}_2 \tilde{d}_2$.

The intuition of Proposition 2 follows from the above calculation of certainty equivalent for a MEU investor. Since the market demand must be the market supply $\overline{\theta}$, the worst-case correlation coefficient in the equilibrium has the same expression as the solution to the certainty equivalent, by replacing θ with $\overline{\theta}$ in Equation (9).

As explained above, the endogenous correlation structure in the equilibrium is influenced by the nature of the news. If the signal conveys bad news about the announcing firm, the MEU investor will interpret this news as highly relevant to the non-announcing firm, and the worst case scenario is when the correlation is the highest. On the other hand, if the signal conveys good news about the announcing firm, the investor will interpret that this good news has nothing to do with the non-announcing firm, so the endogenous correlation structure corresponds to the lowest plausible one. When the news is not strong enough, which falls in $[s^L, s^H]$, the endogenous correlation coefficient is negatively determined by the magnitude of the news s due to the worst-case consideration of the investor. Overall, the worst-case correlation structure between the asset payoffs has a negative relationship with the news in the equilibrium.

Remarkably, the range of the moderate news, $s^H - s^L$, is a proportion of $\rho_b - \rho_a$, which measures the degree of ambiguity about the news. A higher degree of the correlation uncertainty indicates a wider range of the moderate news, and a more significant decreasing shape of the endogenous correlation coefficient.

The equilibrium is obtained by examining the role of the signal and how ambiguity aversion revises the investor's belief in interpreting the relevance of news. To illustrate, first consider a situation when the news is extremely useless; then $\sigma_\epsilon = \infty$, $\phi = 0$ and $s^L = \infty$. The worst-case correlation coefficient should always correspond to the highest plausible estimation ρ_b that minimizes the equilibrium utility of the representative investor.[15] As a result, each stock price is given by $\overline{d}_i - \gamma Cov_{\rho_b}\left(\tilde{d}_i, \tilde{d}\right)$ as in a standard CAPM model (Cochrane, 1992).

After a piece of news $\tilde{s} = s$ about the first firm is revealed on the market, the investor evaluates the trade-off between the diversification benefit and the correlation uncertainty. When the news is good, the impact of news indicating a low correlation dominates the impact of the ambiguity concern indicating a high correlation, therefore the correlation structure in equilibrium corresponds to the lowest estimation. On the other hand, a piece of bad news intensifies the investor's concern on the correlation estimation, thus compounds her worst

[15] A similar result is reached by Jiang and Tian (2016) in their equilibrium analysis. However, they derive the endogenous correlation structure for heterogeneous investors under the setting of Knightian uncertainty on correlation without signaling.

case belief to the highest correlation structure. Therefore, $\rho(s)$ is decreasing with respect to the news s.

By similar intuition, the endogenous correlation coefficient $\rho(s)$ decreases, as presented in Equation (13),

1. if the signal has a better quality, in the sense that σ_ϵ is smaller;
2. if the firm 2's capitalization is larger relative to the firm 1; or
3. if the firm 1's volatility is larger.

The stock price in Proposition 2 is written as $p_i(s) = \mathbb{E}_{\rho(s)}[m_{1,2}\tilde{d}_i|\tilde{s} = s]$, where $m_{1,2}$ is the stochastic discount time factor from time $t = 1$ to $t = 2$,

$$m_{1,2} = \frac{e^{-\gamma\tilde{d}}}{\mathbb{E}_{\rho(s)}[e^{-\gamma\tilde{d}}|\tilde{s} = s]} \tag{15}$$

is the marginal utility of the representative (MEU) investor on the portfolio $\tilde{d}$. Compared with the model of the SEU investor, the correlation structure between asset payoffs depend on the news.

Finally, it is important to compare Proposition 2 with Proposition 1. Assuming ρ^* is given in Proposition 1, by equation (14), the unconditional Sharpe ratios are

$$S_1 = \gamma\left\{\sigma_1(1-\phi)\overline{\theta}_1 + \rho^*\sigma_2(1-\phi)\overline{\theta}_2\right\} - \frac{\phi}{\sigma_1}(s - \overline{d}_1), \tag{16}$$

and

$$S_2 = \gamma\left\{\rho^*\sigma_1(1-\phi)\overline{\theta}_1 + \sigma_2(1-\rho^{*2}\phi)\overline{\theta}_2\right\} - \frac{\rho^*\phi}{\sigma_2}(s - \overline{d}_1). \tag{17}$$

By Proposition 1, the worst-case correlation coefficient ρ^* must satisfy

$$\rho^* = \mathcal{L}\left(\rho_a, \rho_b; \tau(S_1, S_2)\right), \tag{18}$$

which is a highly nonlinear equation since S_1, S_2 depend on ρ^* in Equation (16) and Equation (17). If the representative investor chooses any number either smaller or larger than the fixed point in Equation (18), the investor scarifies her expected (multi-prior) preferences by Proposition 1. Therefore, in equilibrium, the endogenous correlation coefficient must be the fixed point of Equation (18). By solving the fixed point problem in Equation (18), $\rho(s) = \rho^*$ is obtained in Proposition 2.

Proposition 3 (The Decreasing Correlation Principle) *The endogenous correlation coefficient between the asset payoffs conditional on $\tilde{s} = s$,*

$$corr(\tilde{d}_1, \tilde{d}_2 | \tilde{s} = s) = \rho(s) \sqrt{\frac{1 - \phi}{1 - \rho(s)^2 \phi}}, \tag{19}$$

is decreasing *with respect to the magnitude of the news $\tilde{s} = s$.*

As will be shown later, the decreasing correlation principle is the central result that generates several important model predictions. It states the correlation structure between firms' payoff is asymmetric conditional on the news, and this asymmetric correlation structure further yields asymmetric effects on the stock prices and the returns.

To illustrate the decreasing correlation principle numerically, Figure 1 depicts the worst-case correlation coefficient $\rho(s)$ (top panel) and the endogenous conditional correlation $corr(\tilde{d}_1, \tilde{d}_2 | \tilde{s} = s)$ (bottom panel) with respect to the news s for $\rho_a = 0.4 - \epsilon, \rho_b = 0.4 + \epsilon$, for $\epsilon = 0.05$, and $\epsilon = 0.1$. Other parameters are $\sigma_1 = 3$, $\sigma_2 = 2$, $\sigma_\epsilon = 1\%$; $\overline{d}_1 = 0$, $\overline{d}_2 = 0$, $\overline{\theta}_1 = 1$, $\overline{\theta}_2 = 1$, and $\gamma = 1$. Since ϵ measures the level of uncertainty, the higher the investor's uncertainty about the impact of news, the more significant the decreasing pattern of the correlation.

To summarize, the endogenous correlation coefficient between asset payoffs decreases,

- if the signal has better quality, in the sense that σ_ϵ decreases;
- if the firm 2's capitalization is larger relative to the firm 1; or
- if the firm 1's volatility is larger.

3.3 Equilibrium Prices at time $t = 1$

In this section I investigate how the news and the correlation uncertainty jointly affect stock prices. For illustration purpose, I consider a positively correlated structure (that is, $\rho_a \geq 0$).[16]

Proposition 4 *1. The better of the news, the higher the price of each risky asset.*

2. The price of the non-announcing firm reacts more strongly to the bad news than the good news. Moreover, when the news is moderate, the price stays constant.

3. With other parameters being fixed, for the good news, the better the quality of news the higher the stock prices. However, for the bad news, the better the quality of the news the lower the stock prices.

Propositions 4 (1) is intuitive. The better the news about the future payoff of the announcing firm, the higher the stock price of both firms. However, the price reaction of the announcing firm and the non-announcing firm is significantly different. Precisely,

$$\frac{\partial p_1(s)}{\partial s} = \begin{cases} \phi, & \text{if } s < s^L, \\ 1, & \text{if } s^L \leq s \leq s^H, \\ \phi, & \text{if } s > s^H. \end{cases} \tag{20}$$

$$\frac{\partial p_2(s)}{\partial s} = \begin{cases} \rho_b \frac{\sigma_2}{\sigma_1} \phi, & \text{if } s < s^L, \\ 0, & textif s^L \leq s \leq s^H, \\ \rho_a \frac{\sigma_2}{\sigma_1} \phi, & \text{if } s > s^H. \end{cases} \tag{21}$$

Intuitively, since the investor is not sure how to interpret the news from one firm to the other firms, the ambiguity aversion leads the investor to react more strongly to a signal which conveys bad news than a signal that conveys good news. Thus the impact of the news is *asymmetric* given a piece of good news versus bad news. As a consequence, the price effect on the non-announcing firm is stronger for bad news than good news. To illustrate from a hedging perspective, let us assume the "true" correlation coefficient is ρ_0, but the investor

[16] In a negatively correlated structure, the results of the second stock price can be modified easily. I discuss the negatively economic-linked firms in Section 4.4.

only knows that $\rho_a \le \rho_0 \le \rho_b$, without knowing the distribution of the correlation coefficient. The right delta hedging ratio for the second risky asset using the first risky asset is $\rho_0 \frac{\sigma_2}{\sigma_1}$ (See Anderson and Danthine, 1981). Clearly, $\rho_a \frac{\sigma_2}{\sigma_1} \le \rho_0 \frac{\sigma_2}{\sigma_1} \le \rho_b \frac{\sigma_2}{\sigma_1}$. Hence, the investor's stronger (weaker) reaction to the bad (good) news is consistent with the under-hedge (over-hedge) of the risk in the non-announcing firm against the announcing firm.

A striking result of the information transfer under uncertainty is that the non-announcing firm's stock price stays constant when the news is not strong enough. The intuition is as follows. When the signal is not strong enough, conveying neither good nor bad news, the investor does not know how to interpret the news to the non-announcing firm; hence, the price shows no response to the news. Precisely, within the moderate range, $s^L \le s \le s^H$, the stock price stays unchanged, resulting from a counterbalance between the impact of news and the impact of correlation uncertainty. In fact, by straightforward calculation,

$$p_2(s) = \overline{d}_2 - \gamma \sigma_2^2 \phi, \forall s \in [s^L, s^H]. \tag{22}$$

Equation (22) demonstrates an important "inertia" property on the risky asset under the ambiguity environment with a piece of news. Using the incomplete preference of Bewley (2002), Easley and O'Hara (2009) identify the portfolio inertia. Cao, Wang and Zhang (2005), Epstein and Schneider (2007) demonstrate that portfolio inertia occurs in risk-free portfolio. Epstein and Wang (1995), Illeditsch (2011), and Jiang and Tian (2016) prove the portfolio inertia for risky portfolios under different frameworks of ambiguity. Condie, Ganguli and Illeditsch (2015) identify *inertia to information* in an economy with one risky asset. The authors show that the stock price stay constant when there is uncertainty for this firm's *own* information. In my setting, I show that the stock price could stay constant facing the news about its *related firm*, which I call "price inertia".

To illustrate the intuition behind the price inertia, first consider a SEU investor whose correlation belief about the asset payoffs is exactly ρ. The equilibrium asset prices are given as $p_i^{SEU} = \mathbb{E}_\rho[\tilde{d}_i | \tilde{s} = s] - \gamma Cov_\rho(\tilde{d}_i, \tilde{d}), i = 1, 2$. Clearly, the SEU investor under the bad news requires a lower price as compensation for the lower posterior mean in order to hold the risky assets. However, this is no longer true for the MEU investor since ρ becomes a plausible range of numbers instead of a fixed number. The MEU investor revises her belief (estimation) about the correlation upwards if the signal drops. The effect of correlation on volatility counterbalances the effect of news on the mean. As a result, the price does not change because the lower posterior mean that would require a drop in the equilibrium price is exactly offset by the lower risk premium that would require an increase in the price.

The price effect to the announcing firm is also remarkable in equilibrium. For the announcing firm, since the signal conveys direct information about its future payoff, the impact of the news on the asset price is *symmetric* give a piece of good news versus bad news. However, the investor demand is stronger on the announcing firm, resulting from the non-announcing firm's lack of reaction facing moderate news, so the supply-demand equation enforces a stronger marginal price effect on the announcing firm.

Figure 2 presents the above results about endogenous stock prices graphically with regard to the news impact. The announcing firm's stock price is increasing with the news all the time. For the second firm, when $s < s^L = 6.55$, and $s > s^H = 9.05$, the stock price is always increasing; however, the stock price keeps constant as the magnitude of news s is within $[6.55, 9.05]$.

Proposition 4 (3) highlights the effect of the news quality joint with the magnitude of the news. The good news that is precise leads to a larger price increase, while the bad news that is precise leads to a larger price decrease.

I summarize my model predictions as follow.

Model Prediction I.

1. *When the news conveys direct information about the future payoff, the stock price is more sensitive to a piece of moderate news than the profound news (good or bad). The stock price reaction to the good news and the bad news is symmetric.*

2. *When the news conveys indirect information about the future payoff, the stock price reacts more<u>strongly</u> to the bad news than the good news. The stock price shows* **lack of reaction** *when the news is moderate.*

3.4 Equilibrium at $t = 0$

To finish the characterization of the equilibrium, I derive the equilibrium price at $t = 0$. By the dynamic consistency property of the multi-prior expected utility, the dynamic optimal portfolio choice problem is

$$\max_{D} \min_{\rho \in [\rho_a, \rho_b]} \mathbb{E}[J(W_1, s)]$$

where D is the number of stocks at time $t = 0$ and $J(W_1, s)$ is the derived expected utility conditional on $\tilde{s} = s$ at time $t = 1$,

$$J(W_1, s) = \max_{\theta} \min_{\rho \in [\rho_a, \rho_b]} \mathbb{E}_{\rho}[u(W_2)|\tilde{s} = s].$$

The equilibrium asset prices at time $t = 0$ are given by the next result.

Proposition 5 *The stock price of firm i at time $t = 0$ is $p_i = \mathbb{E}[m_{0,1} p_i(s)]$, where*

$$m_{0,1} = \frac{e^{-\gamma(p_1(s)\overline{\theta}_1 + p_2(s)\overline{\theta}_2 + \frac{\gamma}{2}\overline{\theta}'\Sigma_{\rho(s)}\overline{\theta})}}{\mathbb{E}\left[e^{-\gamma(p_1(s)\overline{\theta}_1 + p_2(s)\overline{\theta}_2 + \frac{\gamma}{2}\overline{\theta}'\Sigma_{\rho(s)}\overline{\theta})}\right]}. \tag{23}$$

is the stochastic discount factor in the first time period, $\rho(s)$ is given in Proposition 2, and $p_i(s)$ is the asset price at time $t = 1$ given in Proposition 4. Moreover, $m_{0,1}$ is strictly decreasing *with respect to the news s.*

By Proposition 5, the log of the price kernel, $Log(m_{0,1})$, is in essence (up to a constant) the mean-variance utility of the portfolio, $\mathbb{E}_{\rho}(s)[\tilde{d}|\tilde{s} = s] - \frac{\gamma}{2}Var_{\rho(s)}(\tilde{d}|\tilde{s} = s)$. Moreover, the pricing kernel is *log-convex* with respect to $\tilde{s} = s$. By contrast, the log of the price kernel in the first time period in a standard dynamic equilibrium model is a linear function of the news. Gollier (2011) also demonstrates the non-linear feature of the log of the pricing kernel in a discrete version of the smooth ambiguity model.

Model Prediction II.

The price increases on average in each time period. Precisely, $p_i < \mathbb{E}[p_i(s)|\tilde{s} = s]$ and $p_i(s) < \mathbb{E}_{\rho(s)}[\tilde{d}_i]$ for each $i = 1, 2$.

4 Model Implications

This section presents further model implications. I first present the model prediction for the stock prices. Next I discuss the implications for the risk premium and conditional risk premium. In the end, I examine the conditional correlation and covariance between two stock returns as well as the conditional return volatility.

4.1 The stock price reaction

I first study the stock price reaction by examining the autocorrelation of stock price changes.

Proposition 6 *1. For the announcing firm, the price change in two consecutive time periods is <u>negatively</u> correlated.*

2. For the non-announcing firm, the autocorrelation of the price changes is <u>positive</u> if $\rho_a \geq \frac{n}{2}$; <u>negative</u> when $\rho_b \leq \frac{n}{2}$.

To understand Proposition 6, we first consider the situation of a SEU investor who has the correlation coefficient belief about asset payoffs as ρ, in which each stock price is $P_i^{SEU} = \mathbb{E}[\tilde{d}_i|\tilde{s} = s] - \gamma Cov_\rho(\tilde{d}_i, \tilde{d})$. And $corr(\Delta P_1^{SEU}, \Delta P_2^{SEU}) = 0$, the stock price changes of each firm between the first two periods are *independent*, a weak form of market efficiency. In other words, the firm-specific news has been fully incorporated in the stock prices.

By contrast, the stock prices do not fully reflect the relevant news, given the information transfer effect under uncertainty, implying stock predictability in a rational equilibrium model.[17] There are several remarkable aspects in Proposition 6. First of all, the price changes of the announcing firm in consecutive time periods are *NOT* independent anymore due to the correlation uncertainty in equilibrium. Precisely, the announcing firm has a short-term overreaction but reversal in the nest time period, as the autocorrelation of the price changes is negative and the correlation between the short-term price changes with the long-term price change is positive.[18] This short-term reversal property of the announcing firm is remarkable

[17] Other rational equilibrium models explain the stock predicability includes Johnson (2005), Vaynos and Wooley (2012).

[18] It means that $corr\left(p_1(s) - P_1, \tilde{d}_1 - P_1\right) > 0$. Its proof is given in the proof of Proposition 6 in Appendix.

because there is no concern about the information quality for this firm itself, rather, it follows from its information transfer concern to other firms through an equilibrium mechanism. Indeed, the short-term overreaction of the announcing firm is associated with the lack of reaction of the non-announcing firm and the overreaction of the announcing firm when the news is not strong enough.

Second, the autocorrelation of the price changes for the non-announcing firm has different sign as the announcing firm and different predictability implications due to the correlation uncertainty. When ρ_a is large enough, it means that the economic link between two firms is relatively strong, the price changes in the first two time periods are *positive* correlated, thus, there is a underreaction for the non-announcing firm. Moreover, there are positive correlation between the price changes in the short-term period and in the long-term period. Accordingly, the model explains price momentum for both risky assets in this situation.[19] That is, a good (bad) investment in the first time period continues to be good (bad) in the second time period.

However, it is not always the case that the non-announcing firm displays a momentum. For instance, when the economic link is virtually weak (for a small ρ_b), the model implies a negative autocorrelation of the price changes for the non-announcing firm. This negative autocorrelation (overreaction) does not guarantee a reversal at time $t = 2$ since the prices at future time periods $t = 1$ and $t = 2$ not necessarily move in the same direction, which is different from the short-term reversal to the announcing firm. Moreover, when $\rho_a < \frac{n}{2} < \rho_b$, the autocorrelation for the non-announcing firm can be either positive or negative, depending on other model parameters. Overall, there is rich predictability structure for the non-announcing firm due to the correlation uncertainty.

Proposition 6 is helpful to study whether the stock price overreacts or underreacts. Thomas and Zhang (2008), Ramnath (2002) report stock price can be either overreaction or underreaction for peer firms in the same industry, and Cheng and Eskhmena (2014) document similar findings for firms in the supply-chain. Based on the moderate confidence hypothesis, Cheng and Eskhemna (2014) suggest price underreaction and the post-earnings announcement drift when the signal is precise; and price overreaction when the signal is

[19]See Daniel, Hirshleifer and Subrahmanyam (1998), Barberis, Shleifer and Vishny (1988), and Hong and Stein (1999) for an explanation of the momentum from a behaviorial finance perspective such as overconfidence, investor sentiment and gradual response to the information.

imprecise. In fact, their findings are two special cases in Proposition 6. When the signal is extremely precise, ϕ is close to one and $n \sim 0$; thus, $\rho_a \geq \frac{n}{2}$ holds, and the positive autocorrelation indicates a stock price underreaction. On the other hand, if the signal is sufficiently imprecise, ϕ is close to zero, $n \sim \infty$ and $\rho_b \leq n$ holds naturally. Hence, by Proposition 6, the non-announcing firm's stock price displays a negative autocorrelation, and stock price overreacts. Figure 3 explains the price momentum and reversal under the presented conditions. If the economic link is relatively strong, in the sense that the lowest correlation is a relatively large number, it indicates a larger region of the very good news, where the stock price reacts less strongly to the news. Therefore, on average, the non-announcing firm's stock price underreacts (momentum). The bottom panel explains the reversal when the economic link is weak. Moreover, as shown in Figure 4, the higher the correlation uncertainty, the stronger underreaction or overreaction of the stock price.

The next proposition is about the predictability across firms (or the portfolio).

Proposition 7
1. *The correlation between the price changes of the announcing firm with the price changes of the non-announcing firm in the subsequent time period is* positive *when $\rho_a \geq \frac{n}{2}$ and* <u>*negative*</u> *when $\rho_b \leq \frac{n}{2}$.*

2. *The autocorrelation of the price changes of the portfolio $\tilde{d}$ is* <u>*positive*</u> *if $\rho_a \geq n$;* <u>*negative*</u> *when $\rho_b \leq n$.*

3. *The correlation between the price changes of the announcing firm with the price changes of the portfolio in the subsequent time period is* <u>*positive*</u> *when $\rho_a \geq n$ and* <u>*negative*</u> *when $\rho_b \leq n$.*

Proposition 7 (1) reports the cross-correlation between the announcing firm's stock price changes with the non-announcing firms's stock price changes. It shows that the announcing's stock price has predictability about the non-announcing stock price if the economic link satisfies certain conditions. Proposition 7 (1) is related to recent empirical evidences in Cohen and Frazzini (2008), and Chen and Lou (2012). They document strong predictability from one firm to another firm when the firm-specific news is revealed.

I further investigate how the price changes of the portfolio $\tilde{d}$ is affected by the correlation uncertainty. Because of the predictability component on each firm, naturally, the autocorrelation between the price changes of the portfolio is non-zero. In fact, for $\rho_a \geq n$, there is an under-reaction on the whole portfolio since the underreaction on the non-announcing firm dominates the overreaction on the announcing firm; similarly, for $\rho_b \leq n$, there is an over-reaction on the portfolio.

Since the price changes of the portfolio is equivalent to the price changes in each firm, the autocorrelation between the price changes of the portfolio also depends on the cross-autocorrelation between two firms, in addition to the first order autocorrelation in each firm. Similar to Lo and MacKinlay (1990), the cross-autocorrelation between two firms is given in Proposition 7 (1). In fact, if we consider the price changes of the announcing firm in the latter time period, this cross-autocorrelation is negative because of the overreaction of the announcing firm. On the other hand, the another cross-autocorrelation becomes positive for $\rho_a \geq \frac{n}{2}$ by using the same insight on the underreaction of the non-announcing firm. When the firm-specific news is precise, Proposition 7 (2) implies an underreaction of the market portfolio (Jadedeesh and Titman, 1993, 2001; Lo and Mackinlay, 1988); but the imprecise firm-specific news could lead an overreaction of the market (DeBondt and Thaler, 1985).

Proposition 7 (3) demonstrates the predictability of the portfolio under specific news. When $\rho_a \geq n$ ($\rho_b \leq n$), we see that the cross-correlation between the announcing firm's stock price and the portfolio is positive (negative). Therefore, the announcing firm's stock price is useful to predict the portfolio price. In this regard, Proposition 7 (3) is related to Patton and Verardo (2012), which shows that the specific firm news yields market predictability.

In addition to the information quality, Proposition 6 - 7 state concrete conditions about other elements that explain stock price over- or underreaction. For instance, when the announcing-firm is significantly smaller than the non-announcing firm, the non-announcing firm's stock price displays underreaction (n is close to zero in this situation). By contrast, if the non-announcing firm is significantly smaller than the announcing firm, the non-announcing firm's stock price displays overreaction.

I present the model predictions for the stock price reactions to news as follow.

Model Prediction III.

1. *There is short-term underreaction (momentum) for the non-announcing firm's stock price, if one of the following conditions holds:*

    - *the news is very precise;*
    - *the announcing firm is very risky;*
    - *the non-announcing firm's size is much larger than the announcing firm.*

    In particular, there exists short-term momentum of the market (while the non-announcing firm denotes all other firms and its size is much larger than the announcing firm).

2. *There exists short-term overreaction for the non-announcing firm's stock price, if one of the following conditions holds:*

    - *the news is very imprecise;*
    - *the announcing firm's size is much larger than the non-announcing firm.*

4.2 Risk premium and Conditional risk premium

This subsection discusses about the risk premium and the conditional risk premium of each asset. Let $\tilde{R}_i = \frac{p_i(s)-p_i}{p_i}$ be the return in the first time period, and $\tilde{R}_i(s) = \frac{\tilde{d}_i-p_i(s)}{p_i(s)}$ be the returns of asset i, conditional on the news $\tilde{s} = s$.

Proposition 8 *1. Each risky asset has a <u>positive</u> excess risk premium due to correlation uncertainty. Moreover, the higher the correlation uncertainty the higher the excess risk premium.*

2. *The conditional risk premium of the announcing firm is always decreasing with respect to the news. The conditional risk premium of the non-announcing firm is also decreasing with respect to the news when $\rho_b \leq \frac{n}{2}$, but the news effect to non-announcing firm's conditional risk premium in not monotonic in general.*

The first part of Proposition 8 is consistent with vast uncertainty literature to demonstrate excess risk premium. I provide another source of excess risk premium for each firm under from the uncertain information transfer perspective, even though the quality of the news is certain. The model also implies that the higher uncertain on the information transfer the higher the excess risk premium for each firm.

Proposition 8 (2) reports the effect of the news on the conditional risk premium in each firm. Intuitively, the better the news the higher the price, and thus the smaller the conditional risk premium. The second part of Proposition 8 justifies this intuition for the announcing firm always, and for the non-announcing firm largely.

4.3 Asymmetric effects to asset returns

In this subsection I explain the asymmetric properties of asset return. For simplicity I assume that both the expected value of the asset payoffs are reasonable large such that

$$\frac{d_1}{\gamma} > \sigma_1^2(1-\phi)\overline{\theta}_1 + \rho_b\sigma_1\sigma_2(1-\phi)\overline{\theta}_2; \frac{d_1}{\gamma} > \rho_b\sigma_1^2\overline{\theta}_2, \tag{24}$$

$$\frac{d_2}{\gamma} > \rho\sigma_1\sigma_2(1-\phi)\overline{\theta}_1 + \sigma_2^2(1-\rho^2\phi)\overline{\theta}_2, \rho \in \{\rho_a, \rho_b\}; \frac{d_2}{\gamma} > \rho_b\sigma_2^2\overline{\theta}_2. \tag{25}$$

Assumptions (24) - (25) are minor conditions which ensure positive asset prices in equilibrium at the absence of firm-specific news.

Proposition 9 *1. The conditional correlation, corr* $\left(\tilde{R}_1, \tilde{R}_2|\tilde{s} = s\right)$ *is <u>decreasing</u> with respect to the news* s *except for the region* $min(s_1, s_2) < s < max(s_1, s_2)$*, where* s_1 *and* s_2 *be the unique solution of* $p_1(s_1) = 0$ *and* $p_2(s_2) = 0$*.*

2. The conditional volatility $Var(\tilde{R}_1|\tilde{s} = s)$ *is decreasing for* $s \geq s_1$*.*

3. The conditional volatility $Var(\tilde{R}_2|\tilde{s} = s)$ *is decreasing for* $s \geq s_2$ *except for the region* $s^L \leq s \leq s^H$*.*

4. The conditional covariance $Cov(\tilde{R}_1, \tilde{R}_2|\tilde{s} = s)$ *is decreasing with respect to* s *for all* $s \geq max(s_1, s_2)$*.*

Proposition 9 demonstrates a robust asymmetric pattern of asset correlation and it follows from the deceasing correlation principle of the endogenous correlation coefficient. The correlation of asset returns is larger under a bad news than a good news. Therefore, assets are more likely to comove under very bad firm-specific news. Specifically, the conditional correlation between asset returns is

$$corr\left(\tilde{R}_1, \tilde{R}_2 | \tilde{s} = s\right) = corr\left(\tilde{d}_1, \tilde{d}_2 | \tilde{s} = s\right) sign(p_1(s)p_2(s)). \tag{26}$$

By assumption (24) - (25), we have $p_1(s^L) > 0, p_2(s^L) > 0$, so $s_1, s_2 < s^L$. The product of these two asset prices is always positive except for the region $min(s_1, s_2) \leq s \leq max(s_1, s_2)$.

Similar to asymmetric correlation discussed above, Proposition 9 (2)-(3) present a robust asymmetric stock volatility pattern conditional on the firm-specific news. For the announcing firm, its conditional stock volatility conditional on the bad news is always higher than the good news. Note that the region $s \geq s_1$ includes all signals which lead to positive stock prices, thus the conditional volatility is always decreasing as long as the stock price is positive.

For the non-announcing firm, the model also implies the asymmetric property of its stock volatility. The information transfer channel also affects the stock volatility in addition to the stock price and its return. Proposition 9 (3) states that the conditional volatility is decreasing with respect to the news, except for a small hump due to the price inertia feature under the moderate news. Therefore, the better the news the smaller the non-announcing form's stock volatility, vice versa

Likewise, the model predicts an asymmetric pattern for the covariance. Proposition 9 (4) shows that the covariance under bad signals is always higher than under good ones. The asymmetric property of the covariance is largely consistent with the asymmetric property of the correlation and volatility.

The model predictions about the conditional correlation, conditional covariance and volatility are summarized below.

Model Prediction VI.

1. *The better the news, the <u>smaller</u> the conditional correlation of stock returns for almost all types of news.*

2. *The better the news, the <u>smaller</u> the conditional covariance of the stock returns.*

3. *For the announcing firm, the better the news, the <u>smaller</u> the conditional volatility of the stock return.*

4. *For the non-announcing firm, the better the news, the <u>smaller</u> the conditional volatility of the stock return, except for a particular range of the news.*

Kroner and Ng (1993) investigate the conditional covariance between a large-firm and a small-firm time series. By calibrating a M-GARCH model, the authors find the asymmetric pattern of the conditional covariance conditional on information including firm-specific news. Brooks and Del Negro (2006) document the asymmetric pattern of the conditional correlation between international stocks. See also Conrad et al (1991), Campbell and Hanschel (1992).

Since the model does not specific the characteristics of the firms, to some extent these two risky assets can be also used to represent equity portfolios or industry-sectors, and the industry-specific news in one industry can be transferred into another industry. In this way, my model predictions include the asymmetric pattern of the conditional condition/covariance between portfolios. Indeed, Ang and Chen (2002) document the asymmetric property of conditional correlation between US portfolios, Hong, Tu and Zhou (2007) find the asymmetric property of conditional covariance and betas for US portfolios, Bakaert and Wu (2000) also find the asymmetric conditional variance between Japanese portfolios. Even though these authors do not compute the conditional statistics based on the specific news as I proposed in the model, I argue that these conditional events used in calculation are related to some industry news, and good (bad) industry news are associated with high (low) asset return. Therefore, Proposition 9 are also supported by these empirical findings at the portfolio level. Other relevant empirical studies are presented in Table 1.

4.4 Measurements of Asymmetric Patterns

In this subsection I investigate further about the asymmetric pattern of financial time series. Inspired by the previous studies, such as Longin and Solnik (2001), Ang and Chen (2002), and Ang and Bekaert (2002), I also use the exceedance level, for all $c \geq c_0$,[20] to measure the asymmetric pattern of conditional covariance and conditional volatility, where

$$c_0 = max\left\{\gamma\sigma_1\left(\sigma_1\overline{\theta}_1\frac{1-\phi}{\phi} - \rho_a\sigma_2\overline{\theta}_2\right), \gamma\sigma_1\left(\rho_b\sigma_2\overline{\theta}_2 - \sigma_1\overline{\theta}_1\frac{1-\phi}{\phi}\right), 0\right\}.$$

Proposition 10 *Assume that* $\rho_a + \rho_b \geq \frac{1-\phi}{\phi}\frac{\sigma_2\overline{\theta}_2}{\sigma_1\overline{\theta}_1}$. c^* *denotes a specific number that is greater than* c_0 *given in Appendix B. Then,*

1. $Var(\tilde{R}_1|\tilde{s} \geq \overline{d}_1 + c) < Var(\tilde{R}_1|\tilde{s} \leq \overline{d}_1 - c), \forall c \geq c^*$
2. $Var(\tilde{R}_2|\tilde{s} \geq \overline{d}_1 + c) > Var(\tilde{R}_2|\tilde{s} \leq \overline{d}_1 - c), \forall c \geq c^*$.
3. $Cov(\tilde{R}_1, \tilde{R}_2|\tilde{s} = \overline{d}_1 + y) < Cov(\tilde{R}_1, \tilde{R}_2|\tilde{s} = \overline{d}_1 - y), \forall y \geq c^*$.

So far we consider the information transfer effect for positive economic link. I want to point out that in some particular situations the economic link can be negative in the sense that the plausible correlation coefficients between asset payoffs are negative. For instance, if two firms are competitors in the same industry, good news for one firm may indicate bad news for another. It is also well-documented that gold as well as bond market is often negatively correlated with the equity market. Then, it is also interesting to consider the negative economic link in my model.

To finish my discussion in this section, I briefly explain the main results of information transfer under uncertainty for negative economic link. Both the optimal portfolio and the characterization of the equilibrium are given the same as in Proposition 1- 8. My model implications largely hold and can be easily modified to reflect the negative correlated environment.

As an illustration, I present one result on the asymmetric patterns in a negatively correlated economic link situation. For simplicity, I assume that both firms contribute comparable risks to the market in the sense that

$$\frac{1-\rho_a^2\phi}{|\rho_a|} < \frac{\sigma_2\overline{\theta}_2}{\sigma_1\overline{\theta}_1} \leq \frac{2}{|\rho_a + \rho_b|}. \tag{27}$$

[20]The results hold for any positive exceedance level c with relatively involved technical arguments. The proof for any $c \geq c_0$ is simpler but the main insights of the model are preserved.

Proposition 11 *Consider the information transfer in a negative correlated situation, and (27). c^* is a specific positive constant such that $c^* \geq c_0$.*

1. *For the first asset,* $\mathbb{E}[\tilde{R}_1|\tilde{s} > \overline{d}_1 + c] > \mathbb{E}[\tilde{R}_1|\tilde{s} < \overline{d}_1 - c]$*, but for the second asset,* $\mathbb{E}[\tilde{R}_2|\tilde{s} > \overline{d}_1 + c] < \mathbb{E}[\tilde{R}_2|\tilde{s} < \overline{d}_1 - c], \forall c \geq c^*$.

2. $Var(\tilde{R}_i|\tilde{s} > \overline{d}_1 + c) < Var(\tilde{R}_i|\tilde{s} < \overline{d}_1 - c), i = 1, 2, \forall c \geq c^*$.

3. $Cov(\tilde{R}_1, \tilde{R}_2|\tilde{s} > \overline{d}_1 + c) > Cov(\tilde{R}_1, \tilde{R}_2|\tilde{s} > \overline{d}_1 - c), \forall c \geq c^*$ *if and only if the following condition holds:*

$$\rho_b(\overline{d}_2 - \alpha_b) + \rho_a(\overline{d}_2 - \beta_b) + \frac{\sigma_2}{\sigma_1}\rho_a\rho_b(2\overline{d}_1 - \alpha_a - \alpha_b) > 0. \tag{28}$$

where $\{\alpha_a, \alpha_b, \beta_a, \beta_b\}$ *are defined in Appendix B.*

Proposition 11 presents the asymmetric pattern of the expected return, the stock volatility and the return covariance. As regard to the expected return, the right tail is heavier than the left tail. This asymmetric feature is intuitive because $\tilde{s}$ coveys direct information about the first risky asset, thus good news always leads to a higher expected return. Since the second asset is negatively correlated with the first asset, it will display the opposite pattern.

For the volatility of returns, the left tail is always heavier than the right tail on the first risky asset, which is consistent with what have been documented empirically in literature. It is interesting to examine the volatility of the second risky asset in the negatively correlated economy. In contrast to Proposition 10, the model shows that the volatility on the right tail for the second risky asset is heavier than the left tail under certain condition. Together by Proposition 10 and Proposition 11, the volatility pattern of the second risky asset is complicated, resulting from the price inertia. Due to the same reason, the asymmetric pattern of the covariance is also complicated. I find that the covariance on the right tail is not necessarily smaller than the covariance on the left tail, under certain economic condition such as Equation (28). The intuition is as follows. Under the correlation uncertainty, the second risky asset price overreacts to the bad news; and the overreaction on the second risky asset is so high that it yields a higher right tail covariance in a negatively correlated environment.

After comparing all the conditional asymmetric measures, I present the last prediction below.

Model Prediction V.

The asymmetric pattern of conditional variance, conditional covariance and conditional correlation is pronounced facing a higher degree of uncertainty.

My model predictions provide theoretical grounding for Williams (2015), which empirically examines the role of news to macro-uncertainty in shaping the responses of stock market participants to firm-specific earnings news. The investors' uncertainty increases under stronger macro-uncertainty environment, thus deeply affects their behaviors facing good and bad firm-specific news. Williams (2015) documents that the asymmetric effects are more pronounced for firms whose prior returns are more correlated with macro-uncertainty.

5 Conclusion

In this paper, I develop a theory of information transfer under uncertainty framework, to study how the stock prices respond to relevant firm news in an equilibrium. Assuming the investor are averse to the uncertainty about news impact across firms, my model suggests that the level of uncertainty contributes to the stock price comovement, and the information transfer effect is significant. The non-announcing firm's stock price reacts more strongly to bad news than good news, and shows a lack of reaction when the news is not strong enough. Moreover, the non-announcing firm's stock price movement underreacts when (1) the quality of the news about the announcing firm is good, or (2) the announcing firm's size is significantly smaller than the non-announcing firm's size, or (3) the non-announcing firm is very risky, or (4) the economic link is relatively strong. The non-announcing firm's stock price movement overreacts otherwise. The model offers several testable predictions about stock price momentum and reversal for individual firm's price as well as the stock market. The model provides alternative explanation on the stock market anomalies from the correlation uncertainty perspective.

My model also explains the persistent asymmetric pattern of conditional correlation and covariance between firms or equity portfolios through the transfer of firm-specific news or industry news. Specifically, the conditional correlation and conditional variance are larger under bad news than good news. This paper also presents similar asymmetric pattern of the conditional volatility or conditional stock return. Furthermore, a larger uncertainty about the information transfer leads to a more pronounced asymmetric pattern of the financial time series.

The analysis in this paper demonstrates that the information transfer under uncertainty has a significant impact on the stock prices and stock price movements, which enable us to understand stock momentum and reversal at both the firm-level and the market-level. The information transfer under uncertainty also has a substantial effect on the correlation and covariance structure of stock returns, thus further helps in understanding the excess comovement. Both the stock return and stock volatility are influenced by the information transfer channel. Further empirical tests for my model predictions are left for future work.

Appendix A: Equilibrium

To prove Proposition 1, we first state a simple lemma on the function $\tau(x, y)$ as below.

Lemma 1 *For any $t \in [-1, 1]$, $t \leq \tau(x, y)$ if and only if $(xt - y)(yt - x) \geq 0$; $t < \tau(x, y)$ if and only if $(xt - y)(yt - x) > 0$.*

Proof of Equation (9).

Conditional on the news $\tilde{s} = s$, the posterior joint distribution for the random payoffs $\tilde{d}_1, \tilde{d}_2$ is normal. Moreover, the conditional expected payoffs is

$$\mathbb{E}_\rho[\tilde{d}|\tilde{s} = s] = \begin{bmatrix} \overline{d}_1 + \phi(s - \overline{d}_1) \\ \overline{d}_2 + z_\rho \phi(s - \overline{d}_1) \end{bmatrix}, \tag{A-1}$$

and the conditional covariance matrix is given by (12). It is easy to obtain

$$\begin{aligned} CE(\rho, \theta) &= W_0 + \left(\overline{d}_1 + \phi(s - \overline{d}_1) - p_1\right)\theta_1 + \left[\overline{d}_2 + \phi z(s - \overline{d}_1) - p_2\right]\theta_2 \\ &\quad - \frac{\gamma}{2}\sigma_1^2\theta_1^2(1 - \phi) - \frac{\gamma}{2}\sigma_2^2\theta_2^2(1 - \rho^2\phi) - \gamma\rho\sigma_1\sigma_2\theta_1\theta_2(1 - \phi). \end{aligned}$$

Without loss of generality we assume that $W_0 = 0$ in the proofs below. When $\theta_2 = 0$, $CE(\rho, \theta)$ is clearly independent of ρ. The maximum certainty equivalent $CE(\theta)$ among $\theta_2 = 0$ is

$$\max_{\theta_2 = 0} CE(\theta) = \frac{1}{2\gamma}\frac{\left(\overline{d}_1 + \phi(s - \overline{d}_1) - p_1\right)^2}{\sigma_1^2(1 - \phi)}. \tag{A-2}$$

If $\theta_2 \neq 0$, as a function of ρ, $CE(\rho, \theta)$ is a quadratic and convex function with a global minimal value at $\hat{\rho}(s; \theta)$, where

$$\hat{\rho}(s; \theta) = \frac{\sigma_1}{\sigma_2}\frac{1 - \phi}{\phi}\frac{\theta_1}{\theta_2} - \frac{1}{\gamma\theta_2}\frac{s - \overline{d}_1}{\sigma_1\sigma_2}. \tag{A-3}$$

Hence $CE(\theta)$ is given by Equation (9). Moreover, the maximin value of B is reduced to be the maximum of the following four values

$$B = \max\{B_0, B_1, B_2, B_3\} \tag{A-4}$$

where $B_0 \equiv \frac{1}{2\gamma}\frac{\left(\overline{d}_1+\phi(s-\overline{d}_1)-p_1\right)^2}{\sigma_1^2(1-\phi)}$, $B_1 \equiv \max_{\hat{\rho}(s;\theta)<\rho_a,\theta_2\neq 0} CE(\rho_a,\theta)$, $B_2 \equiv \max_{\hat{\rho}(s;\theta)>\rho_b,\theta_2\neq 0} CE(\rho_b,\theta)$, and $B_3 \equiv \max_{\rho_a\leq\hat{\rho}(s;\theta)\leq\rho_b,\theta_2\neq 0} CE(\hat{\rho}(s;\theta),\theta)$.

$\square$

Proof of Proposition 1.

The proof is divided into several steps.

Step 1. We apply a dual approach to the optimal portfolio choice problem.

By direct computation,

$$\frac{\partial CE(\rho,\theta)}{\partial \rho} = \phi\frac{\sigma_2}{\sigma_1}(s-\overline{d}_1)\theta_2 + \gamma\sigma_2^2\theta_2^2\rho\phi - \gamma\sigma_1\sigma_2\theta_1\theta_2(1-\phi),$$

and

$$\frac{\partial^2 CE(\rho,\theta)}{\partial \rho^2} = \gamma\sigma_2^2\theta_2^2\phi > 0.$$

Then, $CE(\rho,\theta)$ is quasi-convex with respect to ρ for each demand vector θ.

On the other hand, given a ρ, the Hessian matrix of $CE(\rho,\theta)$ with respect to θ is

$$H \equiv \begin{bmatrix} -\gamma\sigma_1^2(1-\phi) & -\gamma\rho\sigma_1\sigma_2(1-\phi) \\ -\gamma\sigma_1\sigma_2(1-\phi) & -\gamma\sigma_2^2(1-\rho^2\phi) \end{bmatrix}$$

For any $x = (x_1, x_2) \in \mathbb{R}^2$,

$$xHx' = -\gamma\left\{\sigma_1^2(1-\phi)x_1^2 + 2\rho\sigma_1\sigma_2(1-\phi)x_1x_2 + \sigma_2^2(1-\rho^2\phi)x_2^2\right\} < 0$$

because the determinant is

$$4\rho^2\sigma_1^2\sigma_2^2(1-\phi)^2 - 4\sigma_1^2\sigma_2^2(1-\phi)(1-\rho^2\phi) = 4\sigma_1^2\sigma_2^2(1-\phi)(\rho^2-1) < 0.$$

Therefore, H is negative definite; thus, $CE(\theta,\rho)$ is quasi-concave with respect to θ for each ρ. Hence, we are readily to apply the Sion's minimax theorem, yielding

$$B = C \equiv \min_{\rho}\max_{\theta}\left[\mathbb{E}_\rho(W_1|\tilde{s}=s) - \frac{\gamma}{2}Var_\rho(W_1|\tilde{s}=s)\right] \tag{A-5}$$

Moreover,

$$\max_{\theta}\left[\mathbb{E}_{\rho}(W_1|\tilde{s}=s)-\frac{\gamma}{2}Var_{\rho}(W_1|\tilde{s}=s)\right]=\frac{1}{2\gamma}b'\Sigma_{\rho}^{-1}b$$

where b is $\mathbb{E}_{\rho}[\tilde{d}|\tilde{s}=s]-p$ in Equation (A-1) and Σ_{ρ} is the conditional covariance matrix stated in Equation (12).

Step 2. Derive the value B.

Let $G(\rho)\equiv b'\Sigma_{\rho}^{-1}b$ and $\rho^*\equiv argmix_{\rho\in[\rho_a,\rho_b]}G(\rho)$. Then $B=\frac{1}{2\gamma}\min_{\rho}G(\rho)=\frac{1}{2\gamma}G(\rho^*)$. We derive B explicitly and show that $\mathcal{L}(\rho_a,\rho_b;\tau(S_1,S_2))=argmix_{\rho\in[\rho_a,\rho_b]}G(\rho)$.

By direct calculation, we obtain

$$G(\rho)=\frac{A_0+A_1\rho+A_2\rho^2}{1-\rho^2} \tag{A-6}$$

where

$$A_0 \;=\; \sigma_2^2\left(\overline{d}_1-p_1+\phi(s-\overline{d}_1)\right)^2+\sigma_1^2(1-\phi)\left(\overline{d}_2-p_2\right)^2,$$

$$A_1 \;=\; -2\sigma_1\sigma_2(1-\phi)\left(\overline{d}_2-p_2\right)\left(\overline{d}_1-p_1\right)=-2(1-\phi)\sigma_1^2\sigma_2^2S_1S_2,$$

and

$$A_2=-\left\{\sigma_2^2\phi^2(s-\overline{d}_1)^2+\phi\sigma_2^2\left(\overline{d}_1-p_1\right)^2+2\phi\sigma_2^2\left(s-\overline{d}_1\right)\left(\overline{d}_1-p_1\right)\right\}.$$

It follows that

$$A_0+A_2=(1-\phi)\{\sigma_1^2(\overline{d}_2-p_2)^2+\sigma_2^2(\overline{d}_1-p_1)^2\}=(1-\phi)\sigma_1^2\sigma_2^2(S_1^2+S_2^2)\geq 0. \tag{A-7}$$

Moreover,

$$A_0+A_2\geq |A_1|. \tag{A-8}$$

By simple calculation, we obtain

$$G'(\rho)=\frac{A_1+2\rho(A_0+A_2)+\rho^2A_1}{(1-\rho)^2} \tag{A-9}$$

Let $\Delta \equiv 4(A_0 + A_2)^2 - 4A_1^2$ and $\Delta \geq 0$ by virtue of Equation (A-8). If $S_1 = S_2 = 0$, we see that $G(\rho) = A_0 = \sigma_2^2\phi^2(s - \overline{d}_1)^2$ for all $\rho \in [\rho_a, \rho_b]$ and

$$B = \frac{1}{2\gamma}\sigma_2^2\phi^2(s - \overline{d}_1)^2, \text{ if } S_1 = S_2 = 0. \tag{A-10}$$

We consider four different cases.

Case 1. $\Delta = 0$ *and* $|A_1| > 0$.

When $\Delta = 0$, then $|S_1| = |S_2|$. If $A_1 > 0$, that is, $S_1S_2 < 0$, then $G'(\rho) > 0$ always, thus $\rho^* = \rho_a$. Since $\tau(S_1, S_2) = -1$ in this case, $\mathcal{L}(\rho_a, \rho_b; \tau(S_1, S_2)) = \rho_a$. If $A_1 < 0$, or equivalently, $S_1S_2 > 0$, then $G'(\rho) < 0$ always, then $\rho^* = \rho_b$. Furthermore, in this case $\tau(S_1, S_2) = 1$. Hence $\mathcal{L}(\rho_a, \rho_b; \tau(S_1, S_2)) = \rho_b$.

Case 2. $A_1 = 0$.

$A_1 = 0$ if and only if $S_1S_2 = 0$. Hence $\tau(S_1, S_2) = 0$. Since $A_1 = 0$, then

$$G'(\rho) = \frac{2(A_0 + A_2)\rho}{(1-\rho)^2}$$

Recall that $A_0 + A_2 \geq 0$, and $A_0 + A_2 = 0$ if and only if $S_1 = S_2 = 0$. By assumption, either $S_1 \neq 0$ or $S_2 \neq 0$, thus $A_0 + A_2 > 0$. Then $G(\rho)$ increases when $\rho \geq 0$; decreases when $\rho \leq 0$. Hence $\rho^* = \mathcal{L}\left(\rho_a, \rho_b; 0\right)$. Then we have proved that when $A_1 = 0, \Delta > 0$, $\rho^* = \mathcal{L}\left(\rho_a, \rho_b; \tau(S_1, S_2)\right)$.

From now on we assume that $\Delta > 0$ and $A_1 \neq 0$. Let

$$\kappa = \frac{A_0 + A_2}{A_1} = -\frac{S_1^2 + S_2^2}{2S_1S_2},. \tag{A-11}$$

and by (A-8,)$|\kappa| > 1$. Let $\alpha = -\kappa - \sqrt{\kappa^2 - 1}, \beta = -\kappa + \sqrt{\kappa^2 - 1}$. Then

$$G'(\rho) = \frac{A_1}{(1-\rho)^2}(\rho - \alpha)(\rho - \beta). \tag{A-12}$$

Case 3. $A_1 > 0$.

In this case, $S_1S_2 < 0$. Since $\kappa > 0$ we have $\kappa > 1$. Then $\alpha = -\kappa - \sqrt{\kappa^2 - 1} < -1$ and $-1 < \beta < 1$. By Equation (A-12), $\rho^* = \mathcal{L}(\rho_a, \rho_b; \beta)$. Moreover, we verify that

$$\beta = -\kappa + \sqrt{\kappa^2 - 1} = \frac{-(S_1^2 + S_2^2)}{-2S_1S_2} + \frac{|S_1^2 - S_2^2|}{-2S_1S_2} = \tau(S_1, S_2).$$

Case 4. $A_1 < 0$.

In this case, $S_1S_2 > 0$. Moreover, $\kappa < 0$ so $\kappa < -1$. We can easily check that $\beta > 1$ and $-1 < \alpha < 1$. Hence, by Equation (A-12), $\mathcal{L}(\rho_a, \rho_b; \alpha) = argmin_\rho G(\rho)$, and

$$\alpha = -\kappa - \sqrt{\kappa^2 - 1} = \frac{S_1^2 + S_2^2}{2S_1S_2} - \frac{|S_1^2 - S_2^2|}{2S_1S_2} = \tau(S_1, S_2).$$

To summarize, we have proved that $\rho^* \equiv argmin_{\rho \in [\rho_a, \rho_b]} G(\rho) = \mathcal{L}(\rho_a, \rho_b; \tau(S_1, S_2))$, and the maximum value is

$$B = \frac{1}{2\gamma} G(\rho^*) = CE(\rho^*, \theta(\rho^*)). \tag{A-13}$$

Step 3. Given the demand vector $\theta(\rho)$, we investigate $\hat{\rho}(s, \theta(\rho))$.

By using the formula (11), the demand on the second risky asset is

$$\theta(\rho)_2 = \frac{S_2 - \rho S_1}{\gamma(1 - \rho^2)\sigma_2}$$

and the demand on the first risky asset is

$$\theta(\rho)_1 = \frac{(S_1 - \rho S_2) + \rho\phi(S_2 - \rho S_1)}{\gamma(1 - \rho^2)(1 - \phi)\sigma_1} + \frac{1}{\gamma}\frac{\phi}{1 - \phi}\frac{s - \bar{d}_1}{\sigma_1^2}.$$

Clearly, $\theta(\rho)_2 = 0$ if and only if $S_2 = \rho s_1$. Moreover, for $S_2 \neq \rho S_1$, we obtain

$$\begin{aligned}
\frac{\theta(\rho)_1}{\theta(\rho)_2} &= \frac{\sigma_1\sigma_2^2\{S_1(1 - \rho^2\phi) - S_2\rho(1 - \phi)\}}{\sigma_1^2\sigma_2(1 - \phi)(S_2 - \rho S_1)} + \frac{\sigma_2^2\phi(1 - \rho^2)}{\sigma_1^2\sigma_2(1 - \phi)(S_2 - \rho S_1)}(s - \bar{d}_1) \\
&= \frac{\sigma_2}{\sigma_1(1 - \phi)}\frac{S_1 - \rho S_2 + \rho\phi(S_2 - \rho S_1)}{S_2 - \rho S_1} + \frac{\sigma_2}{\sigma_1^2}\frac{\phi}{1 - \phi}\frac{1 - \rho^2}{S_2 - \rho S_1}(s - \bar{d}_1).
\end{aligned}$$

Then

$$\frac{\sigma_1}{\sigma_2}\frac{1-\phi}{\phi}\frac{\theta(\rho)_1}{\theta(\rho)_2} \quad = \quad \rho+\frac{1}{\phi}\frac{S_1-\rho S_2}{S_2-\rho S_1}+\frac{1-\rho^2}{S_2-\rho S_1}\frac{s-\overline{d}_1}{\sigma_1}.$$

Furthermore,

$$\frac{1}{\gamma\sigma_1\sigma_2}\frac{s-\overline{d}_1}{\theta(\rho)_2}=\frac{1-\rho^2}{S_2-\rho S_1}\frac{s-\overline{d}_1}{\sigma_1}. \tag{A-14}$$

Therefore, by the definition of $\hat{\rho}(s,\theta)$, we obtain

$$\hat{\rho}(s;\theta(\rho))=\rho+\frac{1}{\phi}\frac{S_1-\rho S_2}{S_2-\rho S_1}. \tag{A-15}$$

Step 4. The characterization of the optimal demand.

Assuming first that $\theta(\rho^*)_2=0$, then $S_2\rho^*-S_1=0$. By assumption, either $S_1\neq 0$ or $S_2\neq 0$, we see that $S_2\neq 0$. Therefore $\rho^*=\frac{S_1}{S_2}\in[\rho_a,\rho_b]$ if $\theta(\rho^*)_2=0$ holds. By the proof in Step 3, $B=CE(\rho^*,\theta^*)$. Since $\theta_2^*=0$, $CE(\rho,\theta^*)$ is independent of ρ. Hence $B=\min_\rho CE(\rho^*,\theta^*)$ and thus $\max_\theta\min_\rho CE(\rho,\theta^*)=CE(\rho,\theta^*)$. Therefore, θ^* is the optimal demanding vector.

We next assume that $S_2\rho^*-S_1\neq 0$, that is, $\theta_2^*\neq 0$. There are three cases about ρ^* because of the characterization of ρ^* in Step 2.

Case 1. $\rho^*=\tau(S_1,S_2)\in[\rho_a,\rho_b]$.

In this case, $\rho^*=\frac{S_1}{S_2}$ since $\rho^*\neq\frac{S_2}{S_1}$. By Equation (A-15), $\hat{\rho}(s;\theta(\rho^*))=\rho^*$. Then, we have $B=CE(\rho^*,\theta(\rho^*))=CE\left(\hat{\rho}(s;\theta(\rho^*)),\theta(\rho^*)\right)=CE\left(\theta(\rho^*)\right)$ by Equation (9). Since $CE\left(\theta(\rho^*)\right)=\max_\theta CE(\theta)$, then $\theta(\rho^*)$ is the optimal demanding vector.

Case 2. $\rho^*=\rho_a>\tau(S_1,S_2)$.

By Lemma 1, $\rho_a>\tau(S_1,S_2)$ is equivalent to $(S_1-\rho_a S_2)(S_2-\rho_a S_1)<0$, thus $\frac{S_1-\rho_a S_2}{S_2-\rho_a S_1}<0$. By Equation (A-15), we have

$$\hat{\rho}(s,\theta(\rho_a))=\rho_a+\frac{1}{\phi}\frac{S_1-\rho_a S_2}{S_2-\rho_a S_1}<\rho_a. \tag{A-16}$$

Then, by Step 1,

$$CE\left(\theta(\rho_a)\right) = CE\left(\rho_a, \theta(\rho_a)\right) = B = \max_{\theta} CE(\theta). \tag{A-17}$$

Thus, $\theta(\rho_a)$ is the optimal demanding vector.

Case 3. $\rho^* = \rho_b < \tau(S_1, S_2)$.

By Lemma 1, since $\rho_b < \tau(S_1, S_2)$, then $(S_1 - \rho_b S_2)(S_2 - \rho_b S_1) > 0$; thus

$$\frac{S_1 - \rho_b S_2}{S_2 - \rho_b S_1} > 0.$$

By using Equation (A-15) again,

$$\hat{\rho}(s, \theta(\rho_b)) = \rho_b + \frac{1}{\phi}\frac{S_1 - \rho_b S_2}{S_2 - \rho_b S_1} > \rho_b. \tag{A-18}$$

Hence

$$CE\left(\theta(\rho_b)\right) = CE\left(\rho_b, \theta(\rho_b)\right) = B = \max_{\theta} CE(\theta), \tag{A-19}$$

so $\theta(\rho_b)$ is the optimal demanding vector.

The proof of Proposition 1 is finished. □

Proof of Proposition 2.

In equilibrium, the optimal demand $\theta(\rho^*)_2 = \overline{\theta}_2 > 0$. Hence by the characterization of the optimal demand, and assuming the endogenous Sharpe ratios are not equal to zero simultaneously and the endogenous correlation is ρ^* (which is determined in equilibrium below), the asset prices are determined by Equation (11) for $\theta(\rho^*) = \overline{\theta}$, yielding Equation (**??**) and (**??**). Then, the endogenous Sharpe ratio, given the endogenous correlation coefficient ρ^*, is

$$S_1 = T_1(s, \rho^*) \equiv -\frac{\phi}{\sigma_1}(s - \overline{d}_1) + \gamma\left\{\sigma_1(1-\phi)\overline{\theta}_1 + \rho^*\sigma_2(1-\phi)\overline{\theta}_2\right\} \tag{A-20}$$

and

$$S_2 = T_2(s, \rho^*) \equiv -\rho^*\frac{\phi}{\sigma_1}(s - \overline{d}_1) + \gamma\left\{\rho^*\sigma_1(1-\phi)\overline{\theta}_1 + \sigma_2(1-\rho^{*2}\phi)\overline{\theta}_2\right\}. \tag{A-21}$$

With this characterization, we first show that one of the (endogenous) Sharpe ratios must be non-zero. Otherwise, both $S_1 = S_2 = 0$ in Equations (A-20) and (A-21) imply that

$$0 = S_2 - \rho^* S_1 = \gamma\sigma_2\overline{\theta}_2[1 - \rho^{*2}],$$

which contradicts to the assumption that $\overline{\theta}_2 > 0$ and $|\rho^*| < 1$. Therefore, we are able to apply Proposition 1 for the following characterization of the endogenous correlation coefficient in the subsequent proof.

To proceed, we define

$$J(s,\rho^*) = \tau\left(T_1(s,\rho^*), T_2(s,\rho^*)\right) \tag{A-22}$$

to represent the dispersion of (endogenous) Sharpe ratios. By the characterization of the worst-cast correlation coefficient in Proposition 1, there are three different situations we investigate in details below. Notice that $J(s,\rho^*)$ depends on the endogenous correlation coefficient.

- If $\rho_b < J(s,\rho^*)$, then $\rho^* = \rho_b$.
- If $\rho_a > J(s,\rho^*)$, then $\rho^* = \rho_a$.
- If $\rho_a \leq J(s,\rho^*) \leq \rho_b$, then $\rho^* = J(s,\rho^*)$.

Case 1. $\rho_b < J(s,\rho^*)$

By Lemma 1, $\rho_b < J(s,\rho^*)$ if and only if

$$(T_1(s,\rho_b)\rho_b - T_2(s,\rho_b)) \times (T_2(s,\rho_b)\rho_b - T_1(s,\rho_b)) > 0. \tag{A-23}$$

By straightforward calculation,

$$T_1(s,\rho_b)\rho_b - T_2(s,\rho_b) = \gamma\sigma_2\overline{\theta}_2(\rho_b^2 - 1) < 0, \tag{A-24}$$

and

$$\frac{T_2(s,\rho_b)\rho_b - T_1(s,\rho_b)}{1-\rho_b^2} = \frac{\phi}{\sigma_1}(s - \overline{d}_1) + \gamma\{\rho_b\sigma_2\overline{\theta}_2\phi - \sigma_1\overline{\theta}_1(1-\phi)\}$$

Then $\rho_b < J(s, \rho^*)$ holds if and only if $T_2(s, \rho_b)\rho_b - T_1(s, \rho_b) < 0$, alternatively, $s < s^L$.

Therefore, for any news $s < s^L$, the endogenous correlation coefficient for the representative investor with correlation uncertainty is the highest plausible correlation ρ_b.

Case 2. $\rho_a > J(s, \rho_a)$

By Lemma 1 again, $\rho_a > J(s, \rho_a)$ holds if and only if

$$(T_1(s, \rho_a)\rho_a - T_2(s, \rho_a)) \times (T_2(s, \rho_a)\rho_a - T_1(s, \rho_a)) < 0. \quad \text{(A-25)}$$

Similarly, we have

$$T_2(s, \rho_a) - T_1(s, \rho_a)\rho_a = \gamma\sigma_2\overline{\theta}_2(1 - \overline{\rho}_a^2) > 0$$

and

$$\frac{T_1(s, \rho_a) - T_2(s, \rho_a)\rho_a}{1 - \rho_a^2} = -\frac{\phi}{\sigma_1}(s - \overline{d}_1) + \gamma\{\sigma_1\overline{\theta}_1(1 - \phi) - \rho_a\sigma_2\overline{\theta}_2\phi\}.$$

Therefore, $\rho_a > J(s, \rho_a)$ holds if and only if $s > s^H$. Hence, for any news $s > s^H$, the endogenous correlation coefficient for the representative investor with correlation uncertainty is the smallest plausible correlation ρ_b.

Case 3. $\rho^* = J(s, \rho^*)$.

We examine the fixed point problem of the Equation $y = J(s, y)$ for $y \in (-1, 1)$, in which $J(s, y)$ is defined similarly as in Equation (A-36) by simply replacing ρ^* by the variable y. By Lemma 1, $y = J(s, y)$ holds if and only if

$$(yT_1(s, y) - T_2(s, y)) \times (yT_2(s, y) - T_1(s, y)) = 0. \quad \text{(A-26)}$$

By calculation,

$$T_2(s, y) - yT_1(s, y) = \gamma\sigma_2\overline{\theta}_2(1 - y^2) \neq 0, \forall |y| < 1. \quad \text{(A-27)}$$

and for $|y| < 1$, we have

$$\frac{T_1(s, y) - yT_2(s, y)}{y^2 - 1} = \frac{\phi}{\sigma_1}(s_1 - \overline{d}_1) - \gamma\sigma_1\theta_1(1 - \phi) + \gamma y\sigma_2\overline{\theta}_2\phi.$$

Therefore, the solution of the Equation $y = J(s, y)$ in the range $(-1, 1)$ is

$$y(s) \equiv \frac{\sigma_1}{\sigma_2}\frac{\overline{\theta}_1}{\overline{\theta}_2}\frac{1-\phi}{\phi} - \frac{1}{\gamma}\frac{1}{\sigma_1\sigma_2\overline{\theta}_2}\left(s - \overline{d}_1\right). \tag{A-28}$$

Moreover, this solution $y(s) \in [\rho_a, \rho_b]$ if and only

$$\gamma\sigma_1\left(\sigma_1\overline{\theta}_1\frac{1-\phi}{\phi} - \rho_b\sigma_2\overline{\theta}_2\right) \le s - \overline{d}_1 \le \gamma\sigma_1\left(\sigma_1\overline{\theta}_1\frac{1-\phi}{\phi} - \rho_a\sigma_2\overline{\theta}_2\right). \tag{A-29}$$

That is, $s^L \le s \le s^H$.

Therefore, we have proved that for any news $s \notin \mathcal{U}\bigcup\mathcal{V}$, the investor chooses the endogenous correlation coefficient $\rho^* = y(s)$ in Equation (A-28), which is the same as $\hat{\rho}(s; \overline{\theta})$.

To summarize, we have determined $\rho(s)$ as claimed. Since $\rho(s)$ is clearly decreasing with respect to $\tilde{s} = s$, so is the conditional correlation between asset payoffs, $corr(\tilde{R}_1, \tilde{R}_2|\tilde{s} = s)$. Finally, the asset price are derived in Proposition 1 given $\rho(s)$ in equilibrium.

$\square$

Proof of Proposition 4.

(1) For $s \in [s^L, s^H]$, and $\frac{\partial\rho^*}{\partial s} = -\frac{1}{\gamma\sigma_1\sigma_2\overline{\theta}_2}$, it follows from Theorem 2 that

$$\frac{\partial p_1(s)}{\partial s} = \phi - \gamma\sigma_1\sigma_2\overline{\theta}_2(1-\phi)\frac{\partial\rho^*}{\partial s} = 1.$$

It suffices to consider the intermediate region. By Theorem 2, we obtain

$$\begin{aligned} p_2(s) &= \overline{d}_2 - \gamma\sigma_2^2\overline{\theta}_2 \\ &\quad + \rho\frac{\sigma_1}{\sigma_2}\phi(s - \overline{d}_2) + \gamma\sigma_2^2\phi\overline{\rho}^2 - \gamma\rho\sigma_1\sigma_2(1-\phi)\overline{\theta}_1 \\ &= \overline{d}_2 - \gamma\sigma_2^2\overline{\theta}_2 \end{aligned}$$

in which the Equation (13) is used.

(2) By Proposition 2, a direct computation yields

$$\frac{\partial p_1(s)}{\partial \phi} = \begin{cases} s - \overline{d}_1 + \gamma\sigma_1(\sigma_1\overline{\theta}_1 + \rho_b\sigma_2\overline{\theta}_2), \text{ if } s < s^L, \\ \frac{\gamma\sigma_1^2\overline{\theta}_1}{\phi^2}, \text{ if } s^L \leq s \leq s^H, \\ s - \overline{d}_1 + \gamma\sigma_1(\sigma_1\overline{\theta}_1 + \rho_a\sigma_2\overline{\theta}_2), \text{ if } s > s^H. \end{cases} \tag{A-30}$$

and

$$\frac{\partial p_2(s)}{\partial \phi} = \begin{cases} \rho_b\frac{\sigma_2}{\sigma_1}(s - \overline{d}_1) + \gamma\sigma_2\rho_b(\sigma_1\overline{\theta}_1 + \rho_b\sigma_2\overline{\theta}_2), \text{ if } s < s^L, \\ 0, \quad \text{if } s^L \leq s \leq s^H, \\ \rho_a\frac{\sigma_2}{\sigma_1}(s - \overline{d}_1) + \gamma\sigma_2\rho_a(\sigma_1\overline{\theta}_1 + \rho_a\sigma_2\overline{\theta}_2), \text{ if } s > s^H. \end{cases} \tag{A-31}$$

□

Proof of Proposition 5.

By Proposition 1, and straightforward calculation, we see that

$$J(W_1, s) = u\left(W_1 + \frac{1}{2\gamma}(\mathbb{E}_{\rho^*}[\tilde{d}|\tilde{s} = s] - p(s))' \times \Sigma_{\rho^*}^{-1} \times (\mathbb{E}_{\rho^*}[\tilde{d}|\tilde{s} = s] - p(s))\right) \tag{A-32}$$

where ρ^* is given in Proposition 1. Moreover, the second term in $J(W_1, s)$ is independent of the initial wealth W_1 at time $t = 1$.

The optimal portfolio choice problem at time $t = 0$ can be written as

$$U_0 = \max_D \mathbb{E}\left[u(W_0 + (p(s) - p) \cdot D + \cdots)\right] \tag{A-33}$$

where $\cdots$ represents the second term in $J(W_1, s)$ which depends on s, but independent of D. Therefore, the first-order condition in solving U_0 with respect to D yields

$$\mathbb{E}\left[e^{-\gamma(W_0 + (p(s)-p)\cdot D + \ldots)}(p_i(s) - p_i)\right] = 0, i = 1, 2. \tag{A-34}$$

We make use of Equation (A-34) to derive the equilibrium price at $t = 0$. Since in the representative investor setting, the market demand at time $t = 1$ is $\overline{\theta}$, and by using Proposition 2, we have the conditional asset payoff in equilibrium is

$$\mathbb{E}_{\rho(s)}[\tilde{d}|\tilde{s} = s] - p(s) = \gamma \times \begin{bmatrix} Cov(\tilde{d}_1, \tilde{d}) \\ Cov(\tilde{d}_2, \tilde{d}) \end{bmatrix}, \tag{A-35}$$

then by direct calculation, we obtain

$$J(W_1, s) = W_1 + \frac{\gamma}{2}\overline{\theta}'\Sigma_{\rho(s)}\overline{\theta} \tag{A-36}$$

where $\rho(s)$ is given in Proposition 2.

Since in equilibrium at time $t = 0$, the optimal demand $D = \overline{\theta}$, and the investor's initial endowment is θ_i units of asset i for $i = 1, 2$, then Equation (A-34) implies that

$$p_i = \frac{\mathbb{E}\left[e^{-\gamma(p(s)\cdot\overline{\theta}+\frac{\gamma}{2}\overline{\theta}'\Sigma_{\rho(s)}\overline{\theta})}p_i(s)\right]}{\mathbb{E}\left[e^{-\gamma(p(s)\cdot\overline{\theta}+\frac{\gamma}{2}\overline{\theta}'\Sigma_{\rho(s)}\overline{\theta})}\right]}. \tag{A-37}$$

By straightforward calculation, we have

$$-\frac{1}{\gamma}\frac{\partial Log(m_{0,1})}{\partial s} = \begin{cases} \phi\overline{\theta}_1 + \phi\rho_b\frac{\sigma_2}{\sigma_1}\overline{\theta}_2, \text{if } s < s^L \\ \overline{\theta}_1 - \frac{1}{\gamma}\frac{\phi}{\sigma_1^2}(s - \overline{d}_1), \text{if } s^L \le s \le s^H \\ \phi\overline{\theta}_1 + \phi\rho_a\frac{\sigma_2}{\sigma_1}\overline{\theta}_2, \text{if } s > s^H. \end{cases} \tag{A-38}$$

Therefore, $m_{0,1}$ is decreasing with respect to s. Moreover, the pricing kernel is log-convex with respect to $\tilde{s} = s$.

□

Lemma 2 *Assume Y is an arbitrary random variable, $M, N : \mathbb{R} \to \mathbb{R}$ are two increasing functions, then $Cov(M(Y), N(Y)) \ge 0$. Moreover, $Cov(M(Y), N(Y)) > 0$ if both $M(\cdot)$ and $N(\cdot)$ are strictly increasing on a subset $B \subseteq \mathbb{R}$ such that $Y^{-1}(B)$ has a positive measure.*

Proof. Without loss of generality we assume that $N(x) = x$. Choosing an independent copy Y' of the random variable Y, then because of the increasing property of $M(\cdot)$, we obtain $\mathbb{E}[(Y - Y')(M(Y) - M(Y'))] \ge 0$. Therefore, $Cov(Y, M(Y)) \ge 0$. Moreover, if $M(\cdot)$ is strictly increasing on $B \subseteq \mathbb{R}$ such that $Y^{-1}(B)$ has a positive measure, we obtain $\mathbb{E}[(Y - Y')(M(Y) - M(Y'))] > 0$, so $Cov(Y, M(Y)) > 0$.

Proof of Proposition 6.

First notice that $Cov(X,Y) = Cov(X, \mathbb{E}[Y|\mathcal{F}])$ for any random variable $X \in \mathcal{F}, Y \in \mathcal{G}$ and $\mathcal{F} \subseteq \mathcal{G}$. To simplify $a \sim b$ denotes $a = bc$ for a positive number c.

(1). By Proposition 2

$$\begin{aligned}
corr\,(\Delta p_{01}, \Delta p_{11}) &\sim Cov(\Delta p_{01}, \Delta p_{11}) \sim Cov\left(p_1(s), \tilde{d}_1 - p_1(s)\right) \\
&\sim Cov\left(p_1(s), \mathbb{E}[\tilde{d}_1|\tilde{s} = s] - p_1(s)\right) \\
&\sim Cov\left(p_1(s), Cov_{\rho(s)}(\tilde{d}_1, \tilde{d})\right).
\end{aligned}$$

Because of the decreasing correlation principle, $Cov_{\rho(s)}(\tilde{d}_1, \tilde{d})$ is decreasing with respect to s. On the other hand, the asset price $p_1(s)$ is increasing with respect to s (Proposition 4), therefore, Lemma 2 implies that $corr\,(\Delta p_{01}, \Delta p_{11}) < 0$. Moreover,

$$corr\left(p_1(s), \tilde{d}_1\right) \sim Cov\left(p_1(s), \tilde{d}_1\right) \sim Cov\left(p_1(s), \mathbb{E}[\tilde{d}_1|\tilde{s} = s]\right).$$

Since both $p_1(s)$ and $\mathbb{E}[\tilde{d}_1|\tilde{s} = s]$ are increasing with respect to s, a positive property of $corr\left(p_1(s), \tilde{d}_1\right)$ follows from Lemma 2.

(2) By straightforward calculation, we have

$$\frac{\partial}{\partial \rho(s)} Cov_{\rho(s)}\left(\tilde{d}_2, \tilde{d}\right) \equiv \begin{cases} 0, & \text{if } s < s^L, \\ \frac{2(s-\bar{d}_1)}{\gamma\sigma_1\sigma_2\bar{\theta}_2} - n, & \text{if } s^L \leq s \leq s^H, \\ 0, & \text{if } s > s^H. \end{cases} \tag{A-39}$$

where $n = \frac{\sigma_1}{\sigma_2}\frac{\bar{\theta}_1}{\bar{\theta}_2}\frac{1-\phi}{\phi}$. By using the decreasing correlation principle again, we have

- If $\rho_a \geq \frac{n}{2}$, then $Cov_{\rho(s)}\left(\tilde{d}_2, \tilde{d}\right)$ is increasing with respect to s.
- If $\rho_b \leq \frac{n}{2}$, then $Cov_{\rho(s)}\left(\tilde{d}_2, \tilde{d}\right)$ is decreasing with respect to s.

I first assume that $\rho_a \geq \frac{1}{2}n$. By a direct calculation, $\rho(s)(s - \overline{d}_1)$ is increasing with respect to s. Then by the same idea in (1), we obtain

$$corr\left(p_2(s) - P_2, \tilde{d}_2 - p_2(s)\right) \;\sim\; Cov\left(p_2(s), \rho(s)(s - \overline{d}_1)\right) > 0,$$

as the asset price is also increasing when s moves in a positive economy. Moreover,

$$corr\left(p_2(s), \tilde{d}_2\right) \sim Cov\left(p_2(s), Cov_{\rho(s)}(\tilde{d}_2, \tilde{d})\right)$$

which is positive by Lemma 2.

Finally, assuming $\rho_b \leq \frac{1}{2}n$, then Lemma 2 yields

$$corr(p_2(s) - P_2, \tilde{d}_2 - p_2(s)) \sim Cov\left(p_2(s), Cov_{\rho(s)}(\tilde{d}_2, \tilde{d})\right) < 0. \tag{A-40}$$

However, the function $\rho(s)(s - \overline{d}_1)$ is decreasing over the region $s^L \leq s \leq s^H$ but increasing otherwise. Therefore, the sign of $Cov\left(p_2(s), \rho(s)(s - \overline{d}_1)\right)$ could be positive or negative depending on model parameters (market situations). $\square$

Proof of Proposition 7.

(3) Notice that $\overline{\theta}_1 p_1(s) + \overline{\theta}_2 p_2(s)$ is the time 1 value of the portfolio. By using the same idea as in (1), the autocorrelation of the price changes of the portfolio has the same sign as the covariance between $\overline{\theta}_1 p_1(s) + \overline{\theta}_2 p_2(s)$ and $Var_{\rho(s)}(\tilde{d})$. By straightforward calculation, the conditional variance $Var_{\rho(s)}(\tilde{d})$ is a positive linear transformation of $2n\rho(s) - \rho(s)^2$, which is decreasing in a range of good news, $s > \overline{d}_1$, and increasing in a range of bad news, $s < \overline{d}_1$.

Assuming $\rho_a \geq n$, then $Var_{\rho(s)}(\tilde{d})$ is increasing with respect to s. Since $\overline{\theta}_1 p_1(s) + \overline{\theta}_2 p_2(s)$ is also an increasing function of the news, Lemma 2 yields the positive autocorrelation as desired. On the other hand, if $\rho_b \leq n$, then $Var_{\rho(s)}(\tilde{d})$ is decreasing with respect to s. We apply Lemma 2 again to obtain the negative autocorrelation.

(4) We consider the cross-autocorrelation when the non-announcing firm price changes first. This cross-autocorrelation has the same sign as the covariance between $p_2(s)$ and the conditional covariance $corr_{\rho(s)}\left(\tilde{d}_1, \tilde{d}\right)$. Since both are increasing with respect to s,

this cross-autocorrelation must be positive. The another cross-autocorrelation when the announcing firm price changes first has the same sign as the covariance between $p_1(s)$ and the conditional covariance $Cov_{\rho(s)}\left(\tilde{d}_2, \tilde{d}\right)$, and this conditional covariance is increasing for $\rho_a \geq \frac{n}{2}$ and decreasing for $\rho_b \leq \frac{n}{2}$ as shown above. Then we apply Lemma 2 to finish the proof.

□

Proof of Proposition 8.

The first part is the same as Model Prediction II, which follows from Proposition 2 and Proposition 5.

For the second part, the conditional expected return is

$$\mathbb{E}[\tilde{R}_i | \tilde{s} = s] = \frac{Cov_{\rho(s)}(\tilde{d}_i, \tilde{d})}{p_i(s)}. \tag{A-41}$$

Because each stock price is increasing with respect to s, thus the decreasing principle yields decreasing property of $Cov_{\rho(s)}(\tilde{d}_1, \tilde{d})$. The remaining proof follows from the pattern of $Cov_{\rho(s)}(\tilde{d}_2, \tilde{d})$ as shown in Proposition 7. □

Proof of Proposition 9.

We first prove the property for a positively correlated economy. In this case, it is east to see that $s_1, s_2 < s^L$ under the property of the endogenous asset price. By the definition of the asset return and Equation (12), the conditional correlation coefficient between asset return is

$$corr\left(\tilde{R}_1, \tilde{R}_2 | \tilde{s} = s\right) = sgn(\rho_1(s)\rho_2(s)) corr\left(\tilde{R}_1, \tilde{R}_2 | \tilde{s} = s\right) = sgn(\rho_1(s)\rho_2(s)) \frac{\rho(s)\sqrt{1-\phi}}{\sqrt{1-\phi\rho(s)^2}},$$

where $sgn(y)$ represents the sign function. By Proposition 4, $p_1(s) > 0$ if and only $s < s_1$, $p_2(s) < 0$ if and only if $s > s_2$. Therefore, $p_1(s)p_2(s) > 0$ as long as s does not belong to a small region $[\min(s_1, s_2), \max(s_1, s_2)]$.

Next, we consider the negatively correlated economy. By Proposition 4, $p_1(s)$ is increasing, thus $s_1 < s^L$. By Proposition 4 again, $p_2(s)$ is decreasing and $p_2(s^H) = p_2(s^L) > 0$, thus $s^H < s_2$. Then $p_1(s) > 0, p_2(s) > 0$ for $s \in [s_1, s_2]$, thus the conditional correlation of asset returns is decreasing in the range $s_1 < s < s_2$. In the region $s \leq s_1$, the conditional correlation is a constant since $\rho(s) = \rho_a$; in the region $s \geq s_2$, $\rho(s) = \rho_b$. The proof is completed. The proof follows from the properties of the asset prices in Proposition 4 and the decreasing correlation principle. $\square$

Appendix B: Measuring the Asymmetric Dependence

Before proving Proposition 10, we need a couple of lemmas first.

The first lemma is well known.

Lemma 3 *For any pair of random variables X, Y and a real number c, and $f(y)$ be the density (marginal) distribution of the random Variable Y, we have*

$$\mathbb{E}\left[X|_{Y\leq c}\right] = \frac{\int_{-\infty}^{c} \mathbb{E}\left[X|Y=y\right] f(y)dy}{\int_{-\infty}^{c} f(y)dy}, \tag{B-1}$$

and

$$\mathbb{E}\left[X|_{Y\geq c}\right] = \frac{\int_{c}^{\infty} \mathbb{E}\left[X|Y=y\right] f(y)dy}{\int_{c}^{\infty} f(y)dy}. \tag{B-2}$$

The next lemma, which is interesting in its own right, is about the conditional covariance and conditional variance between two random variables, conditional on one event defined by another random variable.

Lemma 4 *Given a pair of two random variables X_1, X_2, a random variable Y with values in real numbers and a positive number c, let $f(y)$ be the density (marginal) density function of Y, then*

$$\begin{aligned}
Cov(X_1, X_2|_{Y\geq c}) &= \frac{\int_{c}^{\infty} Cov\left(X_1, X_2|Y=y\right) f(y)dy}{\int_{c}^{\infty} f(y)dy} \\
&+ \frac{1}{2}\frac{\int_{c}^{\infty}\int_{c}^{\infty}\{\mathbb{E}\left[X_1|Y=y\right]-\mathbb{E}\left[X_1|Y=z\right]\}\{\mathbb{E}\left[X_2|Y=y\right]-\mathbb{E}\left[X_2|Y=z\right]\}f(y)f(z)dydz}{\left(\int_{c}^{\infty} f(y)dy\right)^2} \\
&= \frac{1}{2}\frac{\int_{c}^{\infty}\int_{c}^{\infty} h(y,z)f(y)f(z)dydz}{\left(\int_{c}^{\infty} f(y)dy\right)^2},
\end{aligned}$$

where

$$\begin{aligned}
h(y,z) \equiv\ & Cov(X_1, X_2|Y=y) + Cov(X_1, X_2|Y=z) \\
&+ \left(\mathbb{E}[X_1|Y=y]-\mathbb{E}[X_1|Y=z]\right)\left(\mathbb{E}[X_2|Y=y]-\mathbb{E}[X_2|Y=z]\right).
\end{aligned}$$

In particular, the conditional variance of X_i *conditional on* $Y \geq c$

$$Var(X_i|_{Y\geq c}) = \frac{\int_c^\infty Var\left(X_i|Y=y\right)f(y)dy}{\int_c^\infty f(y)dy} + \frac{1}{2}\frac{\int_c^\infty\int_c^\infty \left(\mathbb{E}[X_i|Y=y]-\mathbb{E}[X_i|Y=z]\right)^2 f(y)f(z)dydz}{\left(\int_c^\infty f(y)dy\right)^2}.$$

If Y *is a symmetric random variable in the sense that* $f(y)=f(-y)$*, then*

$$Cov(X_1,X_2|_{Y\leq -c}) = \frac{1}{2}\frac{\int_c^\infty\int_c^\infty h(-y,-z)f(y)f(z)dydz}{\left(\int_c^\infty f(y)dy\right)^2}$$

The conditional variance of X_i *conditional on* $Y \leq -c$

$$\begin{aligned} Var(X_i|_{Y\leq -c}) &= \frac{\int_c^\infty Var\left(X_i|Y=-y\right)f(y)dy}{\int_c^\infty f(y)dy} \\ &\quad + \frac{1}{2}\frac{\int_c^\infty\int_c^\infty \left(\mathbb{E}\left[X_i|Y=-y\right]-\mathbb{E}\left[X_i|Y=-z\right]\right)^2 f(y)f(z)dydz}{\left(\int_c^\infty f(y)dy\right)^2}. \end{aligned}$$

Proof:

By using Lemma 3 for $\mathbb{E}\left[X_i|_{Y\geq c}\right]$ and $\mathbb{E}\left[X_1X_2|_{Y\geq c}\right]$, we have

$$\begin{aligned} \mathbb{E}\left[X_1X_2|_{Y\geq c}\right] &= \frac{\int_c^\infty \mathbb{E}\left[X_1X_2|Y=y\right]f(y)dy}{\int_c^\infty f(y)dy} \\ &= \frac{\int_c^\infty \left\{\mathbb{E}\left[X_1|Y=y\right]\mathbb{E}\left[X_2|Y=y\right]f(y) + Cov(X_1,X_2|Y=y)f(y)\right\}dy}{\int_c^\infty f(y)dy} \end{aligned}$$

and

$$\mathbb{E}\left[X_1|_{Y\geq c}\right]\mathbb{E}\left[X_2|_{Y\geq c}\right] = \frac{\int_c^\infty \mathbb{E}\left[X_1|Y=y\right]f(y)dy\int_c^\infty \mathbb{E}\left[X_2|Y=z\right]f(z)dz}{\left(\int_c^\infty f(y)dy\right)^2}.$$

Then

$$Cov\left(X_1,X_2|_{Y\geq c}\right) = \frac{\int_c^\infty Cov(X_1,X_2|Y=y)f(y)dy}{\int_c^\infty f(y)dy} + I$$

where

$$I \equiv \frac{\int_c^\infty \int_c^\infty \mathbb{E}\left[X_1|Y=y\right]\mathbb{E}\left[X_2|Y=y\right]f(y)f(z) - \mathbb{E}\left[X_1|Y=y\right]\mathbb{E}\left[X_2|Y=z\right]f(y)f(z)dydz}{(\int_c^\infty f(y)dy)^2}.$$

In the expression of I above, we interchange the variable between y and z, thus the numerator of I is

$$\begin{aligned}
&\int_c^\infty \int_c^\infty \mathbb{E}\left[X_1|Y=y\right]\{\mathbb{E}\left[X_2|Y=y\right] - \mathbb{E}\left[X_2|Y=z\right]\} f(y)f(z) \\
&= \int_c^\infty \int_c^\infty \mathbb{E}\left[X_1|Y=z\right]\{\mathbb{E}\left[X_2|Y=z\right] - \mathbb{E}\left[X_2|Y=y\right]\} f(y)f(z) \\
&= \int_c^\infty \int_c^\infty -\mathbb{E}\left[X_1|Y=z\right]\{\mathbb{E}\left[X_2|Y=y\right] - \mathbb{E}\left[X_2|Y=z\right]\} f(y)f(z) \\
&= \frac{1}{2}\int_c^\infty \int_c^\infty \{\mathbb{E}\left[X_1|Y=y\right] - \mathbb{E}\left[X_1|Y=z\right]\}\{\mathbb{E}\left[X_2|Y=y\right] - \mathbb{E}\left[X_2|Y=z\right]\} f(y)f(z)dydz
\end{aligned}$$

where the last Equation follows from the average of the integrands on the above two Equations. By the same idea, we have

$$\frac{\int_c^\infty Cov(X_1, X_2|Y=y)f(y)dy}{\int_c^\infty f(y)dy} = \frac{\int_c^\infty \int_c^\infty Cov(X_1, X_2|Y=y)f(y)f(z)dydz}{(\int_c^\infty f(y)dy)^2}$$

and

$$\begin{aligned}
&\int_c^\infty \int_c^\infty Cov(X_1, X_2|Y=y)f(y)f(z)dydz \\
&= \int_c^\infty \int_c^\infty Cov(X_1, X_2|Y=z)f(y)f(z)dydz \\
&= \frac{1}{2}\int_c^\infty \int_c^\infty \left(Cov(X_1, X_2|Y=y) + Cov(X_1, X_2|Y=z)\right) f(y)f(z)dydz.
\end{aligned}$$

We have thus proved the formula for the conditional covariance conditional on $Y \geq c$.

By the same above argument, we have

$$\begin{aligned}
Cov(X_1, X_2|_{Y\le -c}) &= \frac{\int_{-\infty}^{-c} Cov\left(X_1, X_2|Y=y\right) f(y)dy}{\int_c^\infty f(y)dy} \\
&+ \frac{1}{2}\frac{\int_{-\infty}^{-c}\int_{-\infty}^{-c}\{\mathbb{E}[X_1|Y=y]-\mathbb{E}[X_1|Y=z]\}\{\mathbb{E}[X_2|Y=y]-\mathbb{E}[X_2|Y=z]\}f(y)f(z)dyd}{\left(\int_c^\infty f(y)dy\right)^2}
\end{aligned}$$

By changing the variable y by $-y$, z by $-z$ in the last Equation, and $f(y)=f(-y), f(z)=f(-z)$, then we obtain the formula of $Cov(X_1, X_2|_{Y\le -c})$ as required. $\square$

Lemma 5 *Given three variables $X_1, X_2,$ and Y, if $h(-y,-z) > h(y,z), \forall y,z \ge c$, then $Cov(X_1, X_2|_{Y\ge c}) < Cov(X_1, X_2|_{Y\le -c})$. Moreover, if for each $y \ge c$, $Cov(X_1, X_2|Y=-y) > Cov(X_1, X_2|Y=y)$, and*

$$\left(\mathbb{E}\left[X_i|Y=-y\right]-\mathbb{E}\left[X_i|Y=-z\right]\right)^2 > \left(\mathbb{E}\left[X_i|Y=y\right]-\mathbb{E}\left[X_i|Y=z\right]\right)^2,$$

then $Var(X_i|_{Y\ge c}) < Var(X_i|_{Y\le -c})$ for $i=1,2$.

The result holds if all inequality "$<$" is replaced by "$>$".

Proof: This lemma follows from Lemma 4. $\square$

Proof of Proposition 10.

By Lemma 5, it suffices to check several conditions for $X_1=\tilde{R}_1, X_2=\tilde{R}_2$ and $Y=\tilde{s}-\overline{d}_1$.

Step 1. We show that $Var(\tilde{R}_1|\tilde{s}=\overline{d}_1+y) < Var(\tilde{R}_1|\tilde{s}=\overline{d}_1-y), \forall y\ge c_1$, where c_1 is a positive constant that is greater than c_0.

By Equation (12), it equivalents to show that $p_1(\overline{d}_1+y)^2 > p_1(\overline{d}_1-y)^2, \forall y \ge c_1$. By Proposition 2, $y\ge c_0$ implies that $p_1(\overline{d}_1+y)=\overline{d}_1+\phi y-\alpha_a, p_1(\overline{d}_1-y)=\overline{d}_1-\phi y-\alpha_b$, where

$$\alpha_a \equiv \gamma\sigma_1(1-\phi)\left\{\sigma_1\overline{\theta}_1+\rho_a\sigma_2\overline{\theta}_2\right\}, \alpha_b \equiv \gamma\sigma_1(1-\phi)\left\{\sigma_1\overline{\theta}_1+\rho_b\sigma_2\overline{\theta}_2\right\}.$$

By Assumption (24), we have $\overline{d}_1>\alpha_a, \overline{d}_1>\alpha_b$. Then it is easy to show that $p_1(\overline{d}_1+y)^2 > p_1(\overline{d}_1-y)^2, \forall y\ge c_1$ where

$$c_1 \equiv \max\left\{c_0, \frac{(\overline{d}_1-\alpha_b)^2-(\overline{d}_1+\alpha_a)^2}{2\phi(2\overline{d}_1-\alpha_a-\alpha_b)}\right\}.$$

Step 2. We show that $\rho_a^2 p_2(\overline{d}_1 - y)^2 \leq \rho_b^2 p_2(\overline{d}_1 + y)^2$, for all $y \geq c_2$ where c_2 is a positive constant that is greater than c_0.

By Proposition 2, for any $y \geq c_0$, $p_2(\overline{d}_1 + y) = \overline{d}_2 + \rho_a \phi y \frac{\sigma_2}{\sigma_1} - \beta_a, p_2(\overline{d}_1 - y) = \overline{d}_2 - \rho_b \phi y \frac{\sigma_2}{\sigma_1} - \beta_b$, where

$$\beta_a \equiv \gamma\sigma_2 \left\{ \rho_a \sigma_1 (1-\phi)\overline{\theta}_1 + \sigma_2(1 - \rho_a^2 \phi)\overline{\theta}_2 \right\},$$

and

$$\beta_b \equiv \gamma\sigma_2 \left\{ \rho_b \sigma_1 (1-\phi)\overline{\theta}_1 + \sigma_2(1 - \rho_b^2 \phi)\overline{\theta}_2 \right\}.$$

By assumption (25)), $\overline{d}_2 > \beta_a, \overline{d}_2 > \beta_b$, then for all $y \geq c_2$, it is easy to see that $\rho_a^2 p_2(\overline{d}_1 - y)^2 \leq \rho_b^2 p_2(\overline{d}_1 + y)^2$, where

$$c_2 = \max\left\{ c_0, \frac{1}{2\phi\rho_a z_b} \frac{\rho_a^2(\overline{d}_2 - \beta_b)^2 - \rho_b^2(\overline{d}_2 - \beta_a)^2}{\rho_a(\overline{d}_2 - \beta_a) + \rho_b(\overline{d}_2 - \beta_b)} \right\}.$$

Step 3. We show that for all $y \geq c_3$, $Cov(\tilde{R}_1, \tilde{R}_2 | \tilde{s} = \overline{d}_1 + y) < Cov(\tilde{R}_1, \tilde{R}_2 | \tilde{s} = \overline{d}_1 - y)$, where c_3 is a specific positive number that is greater than c_0.

By Step 1 and Step 2, $p_1(\overline{d}_1 - y)^2 \leq p_1(\overline{d}_1 + y)^2$ and $\rho_a^2 p_2(\overline{d}_1 - y)^2 \leq \rho_b^2 p_2(\overline{d}_1 + y)^2, \forall y \geq \max(c_1, c_2)$. Then $\rho_a^2 p_1(\overline{d}_1 - y)^2 p_2(\overline{d}_1 - y)^2 \leq \rho_b^2 p_1(\overline{d}_1 + y)^2 p_2(\overline{d}_1 + y)^2$. Let

$$c_3 = \max\left\{ c_1, c_2, \frac{\overline{d}_1 - \alpha_b}{\phi}, \frac{\overline{d}_2 - \beta_b}{z_b \phi} \right\}.$$

Then for $y \geq c_3$, $p_1(\overline{d}_1 - y) < 0$ and $p_2(\overline{d}_2 - y) < 0$, thus $p_1(\overline{d}_1 - y)p_2(\overline{d}_2 - y) > 0$. Therefore,

$$\rho_a p_1(\overline{d}_1 - y) p_2(\overline{d}_1 - y) < \rho_b p_1(\overline{d}_1 + y) p_2(\overline{d}_1 + y),$$

yielding $Cov(\tilde{R}_1, \tilde{R}_2 | \tilde{s} = \overline{d}_1 + y) < Cov(\tilde{R}_1, \tilde{R}_2 | \tilde{s} = \overline{d}_1 - y)$.

Step 4. We investigate the function $\mathbb{E}[\tilde{R}_1 | \tilde{s} = \overline{d}_1 + y] - \mathbb{E}[\tilde{R}_1 | \tilde{s} = \overline{d}_1 + z]$ and show that $Var(\tilde{R}_1 | \tilde{s} > d_1 + c) < Var(\tilde{R}_1 | \tilde{s} < d_1 - c), \forall c \geq c_3$.

For any $y, z \geq c_3$, a direct computation yields

$$\begin{aligned} \mathbb{E}[\tilde{R}_1|\tilde{s} = \overline{d}_1 + y] - \mathbb{E}[\tilde{R}_1|\tilde{s} = \overline{d}_1 + z] &= \frac{\overline{d}_1 + \phi y}{p_1(\overline{d}_1 + y)} - \frac{\overline{d}_1 + \phi z}{p_1(\overline{d}_1 + z)} \\ &= \frac{\alpha_a \phi(z - y)}{p_1(\overline{d}_1 + y)p_1(\overline{d}_1 + z)}. \end{aligned}$$

Similarly,

$$\mathbb{E}[\tilde{R}_1|\tilde{s} = \overline{d}_1 - y] - \mathbb{E}[\tilde{R}_1|\tilde{s} = \tilde{d}_1 - z] = \frac{\alpha_b \phi(y - z)}{p_1(\overline{d}_1 - y)p_1(\overline{d}_1 - z)}$$

By Step 1, we have $p_1(\overline{d}_1 - y)^2 < p_1(\overline{d}_1 + y)^2, p_1(\overline{d}_1 - z)^2 < p_1(\overline{d}_1 + z)^2$, then, by using $\alpha_a < \alpha_b$, for all $y, z \geq c_3$, we have

$$\left(\mathbb{E}[\tilde{R}_1|\tilde{s} = \overline{d}_1 + y] - \mathbb{E}[\tilde{R}_1|\tilde{s} = \overline{d}_1 + z]\right)^2 < \left(\mathbb{E}[\tilde{R}_1|\tilde{s} = \overline{d}_1 - y] - \mathbb{E}[\tilde{R}_1|\tilde{s} = \overline{d}_1 - z]\right)^2. \quad \text{(B-3)}$$

By using the last formula, Step 1, and Lemma 5, we prove

$$Var(\tilde{R}_1|\tilde{s} > d_1 + c) < Var(\tilde{R}_1|\tilde{s} < d_1 - c), \forall c \geq c_3. \quad \text{(B-4)}$$

Step 5. We show that $\rho_a p_2(\overline{d}_1 - y)^2 > \rho_b p_2(\overline{d}_1 + y)^2$, and $(1 - \rho_a^2\phi)p_2(\overline{d}_1 - y)^2 > (1 - \rho_b^2\phi)p_2(\overline{d}_1 + y)^2$, for any $y \geq c_4$ where c_4 is a number that is greater than c_0.

In fact, for any $y \geq c_0$, we the express $\rho_a p_2(\overline{d}_1 - y)^2 - \rho_b p_2(\overline{d}_1 + y)^2$ as a quadratic function of y with the leading term being $\rho_a z_b^2\phi^2 - \rho_b z_a^2\phi^2 = \rho_a\rho_b\left(\frac{\sigma_2}{\sigma_1}\right)^2(\rho_b - \rho_a) > 0$. Similarly, the leading term of the quadratic function of $(1 - \rho_a^2\phi)p_2(\overline{d}_1 - y)^2 - (1 - \rho_b^2\phi)p_2(\overline{d}_1 + y)^2$, as a function of y, is $(z_b^2 - z_a^2)\phi^2 > 0$. Then there exists a $c_4 \geq c_0$ such that $\rho_a p_2(\overline{d}_1 - y)^2 - \rho_b p_2(\overline{d}_1 + y)^2 > 0$, and $(1 - \rho_a^2\phi)p_2(\overline{d}_1 - y)^2 - (1 - \rho_b^2\phi)p_2(\overline{d}_1 + y)^2 > 0$.

Step 6. We investigate the function of $\mathbb{E}[\tilde{R}_2|\tilde{s} = \overline{d}_1 + y] - \mathbb{E}[\tilde{R}_2|\tilde{s} = \tilde{d}_1 + z]$ and show that $Var(\tilde{R}_2|\tilde{s}_1 > \overline{d}_1 + c) > Var(\tilde{R}_2|\tilde{s}_1 > \overline{d}_1 - c), \forall c \geq c_4$.

First, by Step 5, and Equation (12), $Var(\tilde{R}_2|\tilde{s} = \overline{d}_1 + y) > Var(\tilde{R}_2|\tilde{s} = \overline{d}_1 - y), \forall y \geq c_4$.

Second, for any $y, z \geq c_3$, by Proposition 2, we obtain

$$\mathbb{E}[\tilde{R}_2|\tilde{s} = \overline{d}_1 + y] - \mathbb{E}[\tilde{R}_2|\tilde{s} = \tilde{d}_1 + z] = \frac{\beta_a z_a \phi(z-y)}{p_2(\overline{d}_1 + y)p_2(\overline{d}_1 + z)},$$

and

$$\mathbb{E}[\tilde{R}_2|\tilde{s} = \overline{d}_1 - y] - \mathbb{E}[\tilde{R}_2|\tilde{s} = \tilde{d}_1 - z] = \frac{\beta_b z_b \phi(y-z)}{p_2(\overline{d}_1 - y)p_2(\overline{d}_2 - z)}.$$

Therefore,

$$\Big(\mathbb{E}[\tilde{R}_2|\tilde{s} = \overline{d}_1 + y] - \mathbb{E}[\tilde{R}_2|\tilde{s} = \tilde{d}_1 + z]\Big)^2 = \frac{\beta_a^2 z_a^2 \phi^2 (z-y)^2}{p_2(\overline{d}_1 + y)^2 p_2(\overline{d}_1 + z)^2}$$

and

$$\Big(\mathbb{E}[\tilde{R}_2|\tilde{s} = \overline{d}_1 - y] - \mathbb{E}[\tilde{R}_2|\tilde{s} = \tilde{d}_1 - z]\Big)^2 = \frac{\beta_b^2 z_b^2 \phi^2 (z-y)^2}{p_2(\overline{d}_1 - y)^2 p_2(\overline{d}_1 - z)^2}.$$

By Step 5, $\rho_a p_2(\overline{d}_1 - y)^2 > \rho_b p_2(\overline{d}_1 + y)^2$, and $\rho_a p_2(\overline{d}_1 - z)^2 > \rho_b p_2(\overline{d}_1 + z)^2$, $\forall y, z \geq c_4$, then

$$\rho_a^2 p_2(\overline{d}_1 - y)^2 p_2(\overline{d}_1 - z)^2 > \rho_b^2 p_2(\overline{d}_1 + y)^2 p_2(\overline{d}_1 + z)^2.$$

Since $\rho_a + \rho_b \geq \frac{1-\phi}{\phi}\kappa$, $\beta_a \geq \beta_b$, thus

$$\Big(\mathbb{E}[\tilde{R}_2|\tilde{s} = \overline{d}_1 + y] - \mathbb{E}[\tilde{R}_2|\tilde{s} = \tilde{d}_1 + z]\Big)^2 > \Big(\mathbb{E}[\tilde{R}_2|\tilde{s} = \overline{d}_1 - y] - \mathbb{E}[\tilde{R}_2|\tilde{s} = \tilde{d}_1 - z]\Big)^2.$$

Then by Lemma 5, we have proved that $Var(\tilde{R}_2|\tilde{s}_1 > \overline{d}_1 + c) > Var(\tilde{R}_2|\tilde{s}_1 > \overline{d}_1 - c)$, for any $c \geq c_4$. □

Proof of Proposition 11.

(1) We apply Lemma 3 for the asymmetric expectation of the first risky asset. Under assumption (27), $\alpha_a + \alpha_b > 0$, thus we can prove that $\mathbb{E}[\tilde{R}_1|\tilde{s} = \overline{d}_1 + y] > \mathbb{E}[\tilde{R}_1|\tilde{s} = \overline{d}_1 - y]$ for any $c \geq c_0$.

For the asymmetric variance of the first risky asset, by the same argument of Step 1 and Step 4 in the proof of Proposition 10, and using $\alpha_a^2 < \alpha_b^2$, we can prove that $Var[\tilde{R}_1|\tilde{s} > \overline{d}_1 + c] < Var[\tilde{R}_1|\tilde{s} > \overline{d}_1 - c]$.

(2). By the condition in this part, we know that $\beta_a > 0, \beta_b > 0$. Since $\rho_a < \rho_b \leq 0$, we can show that $\mathbb{E}[\tilde{R}_2|\tilde{s} = \overline{d}_1 + y] < \mathbb{E}[\tilde{R}_2|\tilde{s} = \overline{d}_1 - y]$ by a similar method as in Proposition 11 (1). We also note that $\beta_a < \beta_b$ because of $\rho_b \leq 0$. Then $\beta_a^2 < \beta_b^2$.

For the asymmetric variance, we first show that $Var(\tilde{R}_2|\tilde{s} = \overline{d}_1 + y) < Var(\tilde{R}_2|\tilde{s} = \overline{d}_1 - y)$, that is, $(1-p_a^2\phi)(p_2(\overline{d}_1-y)^2 < (1-p_b^2\phi)(p_2(\overline{d}_1+y)^2$. Since $\rho_b^2 < \rho_a^2$ in the negatively correlated environment, we see that the above inequality holds for any $y \geq c_1$ which is a specific number that is greater than c_0.

By using the same proof in Proposition 10, and $\beta_a^2 < \beta_b^2$, it suffices to show that

$$\frac{\rho_a^2}{p_2(\overline{d}_1 + y)^2 p_2(\overline{d}_1 + z)^2} < \frac{\rho_b^2}{p_2(\overline{d}_1 - y)^2 p_2(\overline{d}_1 - z)^2}.$$

By a similar argument as in the positive environment, we can show that $\rho_a p_2(\overline{d}_1 - y)^2 > \rho_b p_2(\overline{d}_1 + y)^2$, and $\rho_a p_2(\overline{d}_1 - z)^2 > \rho_b p_2(\overline{d}_1 + z)^2$. Since $\rho_a < \rho_b \leq 0$, we obtain

$$\rho_a p_2(\overline{d}_1 - y)^2 \rho_a p_2(\overline{d}_1 - z)^2 < \rho_b p_2(\overline{d}_1 + y)^2 \rho_b p_2(\overline{d}_1 + z)^2.$$

Therefore, we have completed the proof of $Var(\tilde{R}_2|\tilde{s} > \overline{d}_1 + c) < Var(\tilde{R}_2|\tilde{s} < \tilde{d}_1 - c)$ for all $c \geq c_2$, c_2 is a specific number that is greater than c_0.

(3). Notice that for large enough y, $p_2(\overline{d}_1 + y) < 0, p_2(\overline{d}_1 - y) > 0$. Under the condition (28), we can show that

$$\frac{\rho_a}{p_1(\overline{d}_1 + y) p_2(\overline{d}_1 + y)} < \frac{\rho_b}{p_1(\overline{d}_1 - y) p_2(\overline{d}_1 - y)}, \forall y \geq c^*.$$

Therefore, $Cov(\tilde{R}_1, \tilde{R}_2|\tilde{s} = \overline{d}_1 + y) < Cov(\tilde{R}_1, \tilde{R}_2|\tilde{s} = \overline{d}_1 - y), \forall y \geq c^*$. Moreover, the inequality on the conditional covariance is just opposite if $\frac{\sigma_2}{\sigma_1}\rho_a\rho_b(2\overline{d}_1 - \alpha_a - \alpha_b) + \rho_b(\overline{d}_2 - \alpha_b) + \rho_a(\overline{d}_2 - \beta_b) > 0$.

To proceed, we show that $\alpha_a\beta_a\rho_b < \alpha_b\beta_b\rho_a$ under condition (27). By direct calculation, $\alpha_a\beta_a\rho_b - \alpha_b\beta_b\rho_a$ equals to

$$\kappa(\rho_b - \rho_a)\left\{1 + \rho_a\rho_b(2\phi - 1) + \rho_a\rho_b\kappa\phi(\rho_a + \rho_b)\right\}.$$

Since $2+(\rho_a+\rho_b)\kappa > 0$, $1+\rho_a\rho_b(2\phi-1)+\rho_a\rho_b\kappa\phi(\rho_a+\rho_b)$ is increasing with respect to ϕ, and it equals to $1 - \rho_a\rho_b > 0$ for $\phi = 0$, then $1 + \rho_a\rho_b(2\phi - 1) + \rho_a\rho_b\kappa\phi(\rho_a + \rho_b) > 0, \forall\phi \in (0, 1)$. Therefore, $\alpha_a\beta_a\rho_b > \alpha_b\beta_b\rho_a$.

By using the fact that $\alpha_a\beta_a\rho_b < \alpha_b\beta_b\rho_a$, we can show that

$$\sqrt{\alpha_a\beta_a(-\rho_a)}p_1(\bar{d}_1 - y)p_2(\bar{d}_1 - y) > \sqrt{\alpha_b\beta_b(-\beta_b)}p_1(\bar{d}_1 + y)p_2(\bar{d}_1 + y), \forall y \geq c^*.$$

Then for all $y, z \geq c^*$, $\alpha_a\beta_a(-\rho_a)p_1(\bar{d}_1 - y)p_2(\bar{d}_1 - y)p_1(\bar{d}_1 - z)p_2(\bar{d}_1 - z)$ is smaller than $\alpha_b\beta_b(-\rho_b)p_1(\bar{d}_1 + y)p_2(\bar{d}_1 + y)p_1(\bar{d}_1 + z)p_2(\bar{d}_1 + z)$. It implies that the cross dispersion of the expected returns of risky assets on two good news, y and z, is greater than the cross dispersion of the expected returns of risky assets on two mirror bad news, $-y$ and $-z$. Therefore, Lemma 5 derives the asymmetric covariance as required. $\square$

References

[1] Anderson, R. and J-P, Danthine, (1981), "Cross hedging," *Jurnal of Politica Economy*, 89, 1182-1196.

[2] Andrei D., and M. Hasler, (2015), "Investor Attention and Stock Market Volatility," *Review of Financial Studies*, 28, 22- 72.

[3] Ang, A., Bekaert, G., and J. Liu (2005): "Why Stocks May Disappoint" *Journal of Financial Economics* 76, 471-508.

[4] Ang, A., and J. Chen (2002): "Asymmetric Correlations of Equity Portfolios," *Journal of Financial Economics* 63, 443-494.

[5] Ang, A., J. Chen., and Y. Xing (2006):"Downside Risk", *Review of Financial Studies* 19, 1191-1129.

[6] Bekaert, G., and G. Wu (2000): "Asymmetric Volatility and Risk in Equity Market," *Review of Financial Studies* 13, 1-42.

[7] Bernard, V., and J. Thomas (1989): "Post-earning-announcement Dridt-Delayed Price Reponse or Risk Premium?" *Journal of Accounting Research* 27, 1-36.

[8] Bernard, V., and J. Thomas (1990): "Evidence that Stock Prices Do Not Fullu Reflect the Implications of Current Earnings for Future Earnings," *Journal of Accounting and Economics* 13, 305-340.

[9] Bewley, T (2002): "Knightian Decision Theory: Part I," *Decisions in Economics and Finance* 25, 79-110.

[10] Bloom, N (2009): "The Impact of Uncertainty Shock," *Econometrica*, 77, 623-85.

[11] Bossarts, P., P. Ghirardato, S. Guarnaschelli, and W. Zame (2010): "Ambiguity in Asset Markets: Theory and Experiment," *Review of Financial Studies* 23, 1325-1359.

[12] Brooks, R., and M. Del Negro 2006. "Firm-level evidence on international stock market comovement." *Review of Finance* 10(1):69-98.

[13] Campbell, J., and L. Henschel (1992): "No News is Good News: An Asymmetric Model of Changing Volatility in Stock Returns," *Journal of Financial Economics* 31, 281-318.

[14] Caskey, J (2009): "Information in Equity Markets with Ambiguity-Averse Investors," *Review of Financial Studies* 22 (9), 3595-3627.

[15] Cao, H., T. Wang, and H. Zhang (2005): "Model Uncertainty, Limited Market Participation, and Asset Prices," *Review of Financial Studies* 18(4), 1219-1251.

[16] Conrad, J., Gultekin, M., Kaul, G., (1991). "Asymmetric predictability of conditional variances." *Review of Financial Studies* 4, 597-622.

[17] Cheng, C. S. ., and J. Eshleman (2014): "Does the Market Overweight Imprecise Information? Evidence from Customer Earnings Announcements", *Review Accounting Studies* 19, 1125-1151.

[18] Clinch, G., and N. Sinclair (1987): "Intra-industry Information Releases: A Recursive System Approach," *Journal of Accounting and Economics* 24, 3-37.

[19] Condie, S., and J. Ganguli (2011): "Ambiguity and Rational Expectations Equilibria", *Review of Economic Studies*, 78, 821-845.

[20] Condie, S., J. Ganguli, and Philipp K. Illeditsch (2015): "Information Inertia", *Working paper.*

[21] Cohen L., and A. Frazzini (2008): "Economic Links and Predictable Returns." *Journal of Finance* 63 (4), 1977-2011.

[22] Cohen L., and D. Lou (2012): "Complicated Firms." *Journal of Financial Economics* 104 (2), 383-400.

[23] David, A. and P. Veronesi, (2013), "What Ties Return Volatilities to Price Valuations and Fundamentals?" *Journal of Political Economy*, Vol. 121, No. 4, pp. 682-746.

[24] Daniel, K., D. Hirshleifer., and A. Subrahmanyam, (1998): "Investor Psychology and Security Market Under- and Overreactions," *Journal of Finance*, 53, 1839-1884.

[25] DeBondt, W., and R. Thaler(1985): "Does the Stock Market Overreact?" *Journal of Finance*, 40, 793-805.

[26] DellaVigna, S., and I. M. Pollet (2009): "Investor Inattention and Friday Earnings Announcements." *Journal of Finance* 64 (2), 709-749.

[27] Easley, D., and M. O'Hara (2009): "Ambiguity and Nonparticipation: The Role of regulation," *Review of Financial Studies* 22, 1817-1843.

[28] Easley, D., and M. O'Hara (2010): "Microstructure and Ambiguity," *Journal of Finance* 65, 1817-1846.

[29] Epstein, L., and S. Ji (2014): "Ambiguity Volatility and Asset Pricing in Continuous Time," *Review of Financial Studies* 26, 1740-1786.

[30] Epstein, L., and M. Schneider (2007): "Learing under Ambiguity," *Review of Economic Studies* 74, 1275-1304.

[31] Epstein, L., and M. Schneider (2008): "Ambiguity, Information Quality, and Asset Pricing," *Journal of Finance* 63, 197-228.

[32] Fama, E., and K. French (2008): "Dissecting Anomalies," *Journal of Finance* 63, 1653-78.

[33] Firth, M (1976): "The Impact of Earnings Announcements on the Share Price Behaviorial of Similar Types Firms," *Economic Journal* 86, 296-306/

[34] Foster, G (1981): "Intra-industry Information Transfers Associated with Earnings Releases." *Journal of Accounting and Economics* 3(3), 201-232.

[35] Freeman, R., and S. Tse (1992): "An Earnings Prediction Approach to Examining Intercompany Information Transfer." *Journal of Accounting and Economics* 15 (4), 509-523.

[36] Garlappi, L., R. Uppal, and T. Wang (2007): "Portfolio Selection with Parameter and Model Uncertainty: A Multi-Prior Approach," *Review of Financial Studies* 20, 41-81.

[37] Gilboa, I., and D. Schmeidler (1989): "Maxmin Expected Utility with Non-unique Prior," *Journal of Mathematical Economics* 18, 141-153.

[38] Gleason, C.,N. Jenkins., and W. Johnson (2008): "The Contagion Effect of Accounting Restatements," *The Accounting Review* 83(1), 83-110.

[39] Gollier, C (2011): "Portfolio Choices and Asset Pricies: The Comparative Statics of Ambiguity Aversion," *Review of Economic Studies* 78, 1329-1344.

[40] Han, J.C.Y., and J. Wild (1997): "Tinelines of Reporting and Earnings Information Transfers." *Journal of Business, Finance, and Accounting* 24 (3/4). 527-540.

[41] Hong, Y., J. Tu. and G. Zhou (2007): "Asymmetric Correlation of Stock Returns: Statistical Tests and Economic Evaluation," *Review of Financial Studies* 20, 1547-1581.

[42] Hong, H., and J. C. Stein (1999): " A Unified Theory of Underreaction, Momentum Trading, and Overreaction in Asset Markets," *Journal of Finance* 54, 2143-2184.

[43] Illeditsch, P (2011): "Ambiguous Information, Portfolio Inertia, and Excess Volatility," *Journal of Finance* 66, 2213-2247.

[44] Jedadeesh, N., and S. Titman (1993): "Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency." *Journal of Finance* 48 (1), 65-91.

[45] Jedadeesh, N., and S. Titman (2001): "Profitability of Momentum Strategies: An Evaluation of Allternative Explanations." *Journal of Finance* 56 (2), 699-720.

[46] Jiang, J., and W. Tian (2016): "Correlation Uncertainty, Heterogeneous Beliefs, and Asset Prices", working paper, University of North Carolina at Charlotte.

[47] Johnson, T (2002): "Ratinal Momentum Effect," *Journal of Finance*, 585-608.

[48] Kim, Y., M. Lacina., and M. Park (2008): "Positive and Negative Information Transfers from Management Forecasts," *Journal of Accounting Research* 46 (4), 885-908.

[49] Klibanoff, P, M. Marinacci, and S. Mukerji (2005): "A Smooth Model of Decision Making under Ambiguity," *Econometrica*, 73 (6), 1849-92.

[50] Lo, A., and C. MacKinlay (1988): "Stock Market Prices Do Not Follow Random Walks: Evidences from a Simple Specification Test", *Review of Financial Studies* 1, 41-66.

[51] Lo, A., and C. MacKinlay (1990): "When Are Contrarian Profits Due to Stock Market Overreaction?", *Review of Financial Studies* 3, 175-205.

[52] Longin, F., and B. Solnik (2001): "Extreme Correlation of International Equity Markets," *Journal of Finance* 56, 649-676.

[53] Mele, A., and F. Sangiorgi (2015): "Uncertainty, Information Acquisition, and Price Swings in Asset Markets," *Review of Economic Studies*, 82, 1533-1567.

[54] Pandit, S., C. E. Wasley., ad T. Zach (2011): "Information Externalities along the Sypply Chain: the Economic Determinants of Suppliers 'Stock Price Reaction to heir Customers' Earnings Announcements", *Contemporary Accounting Research* 28 (4), 1304-1343.

[55] Patton, A., and M. Verardo (2012): "Does Beta Move with News? Firm-Specific Information Flows and Learning about Profitability", *Review of Financial Studies*, 25, 2789-2839.

[56] Ramnath, S (2002): "Investor and Analysy Reactions to Earnings Announcements of Related Firms: An Empirical Analysis," *Journal of Accounting Research* 40, 1351-1376.

[57] Ramalingegowda, S., T. Shu., and E. Yeung (2012): "Moderated Confidence and Under- and Overreactions to Related Firm's News", working paper.

[58] Sion, M (1958): "On General Minimax Theorems," *Pacific Journal of Mathematics* 8, 171-176.

[59] Sloan, R (1996): "Do Stock Prices Fully Reflect Information in Accurals and Cash Flows abut Future Earnings?" *The Accounting Review* 71, 289-315.

[60] Thomas, J., and F. Zhang (2008): "Overreacton to intra-industry information transfers?" *Journal of Accounting Research* 46 (4).

[61] Vayanos, D., and P. Wooley (2013): "An Institutional Theory of Momentum and Reversal." *Review of Financial Studies*, 26, 1087-1145.

[62] Williams, C (2015): "Asymmetric Responses to Earning News: A Case for Ambiguity," *Accounting Review*, 90, 785-817.

[63] Zhou, J (2015): "The Good, the Bad, and the Ambiguous: The Aggregate Stock Market Dynamics around Macroeconomic News," working paper, Havard University.

Table 1: An Overview of empirical studies

This table lists a sample of studies on the asymmetries of conditional correlation, conditional variance/covariance and beta. These studies typically use regime-switching GARCH and copula models to measure correlation and volatility. Most of the explanations in the literature are statistical. Typically they test pairwise correlation or covariance between U.S. and international market indices or focus on testing the asymmetric reaction to shock at firm level.

Study	Asymmetry Measure	Presence of Asymmetry	Models/Explanations
Ang and Chen (2002)	Conditional correlation	US Portfolios	GARCH-M
Ang and Bekaert (2002)	Conditional correlation	US-UK-Germany	Regime-switching GARCH
Bae, Karolyi, and Stulz (2003)	Counts of extreme return coincidence	International Finance Corporation indices	Multinomial logistic regression approach
Baele, Bekaert and Inghelbrecht (2010)	Correlation	Stock-bond return	Dynamic factor model
Longin and Solnik (1995, 2001)	Conditional correlation	US-UK, France, Germany, Japan Equity Index	Extreme Value Theory
Patton (2006)	Conditional correlation	Mark-Dollar, Yen-Dollar	Copula
Cappiello, Engle and Sheppard (2006)	Conditional correlation	World-wide Equity and Bond Index	DCC-GARCH
Kroner and Ng (1998)	Conditional covariance	A large-firm and a small-firm return series	Multivariant GARCH
Bekaert and Wu (2000)	Conditional covariance	Japanese Portfolios (Nikkei 225)	Volatility feedback effect
Moskowitz (2003)	Conditional covariance	US portfolio	Unspecified
Brooks and Del Negro (2006)	Correlation& Beta	Firm-level international stocks	Unspecified
Christoffersen, Errunza, Jacobs and Langlois (2012)	Correlation & Covariance	Developed and Emerging Market	Dynamic Copula
Hong, Tu and Zhou (2007)	Conditional correlation, Variance and Beta	US Portfolios	Mixed Copula

Figure 1: Asymmetric correlation in equilibrium

This figure demonstrates the asymmetric correlation between the asset returns in a positively correlated economy. A higher correlation is associated with the bad news. The top panel displays the worst-case correlation coefficient $\rho(s)$ in the equilibrium, and the bottom panel displays the endogenous correlation $corr(\tilde{d}_1, \tilde{d}_2 | \tilde{s} = s) = \rho(s)\sqrt{\frac{1-\phi}{1-\rho(s)^2\phi}}$, with respect to the news s. The parameters in this figure are $\rho_a = 0.4 - \epsilon, \rho_b = 0.4 + \epsilon, \epsilon \in \{0.05, 0.1\}$. Other parameters are $\sigma_1 = 3\%$, $\sigma_2 = 2\%$, $\sigma_\epsilon = 1\%$; $\bar{d}_1 = 0$, $\bar{d}_2 = 0$, $\bar{\theta}_1 = 1$, $\bar{\theta}_2 = 1$.

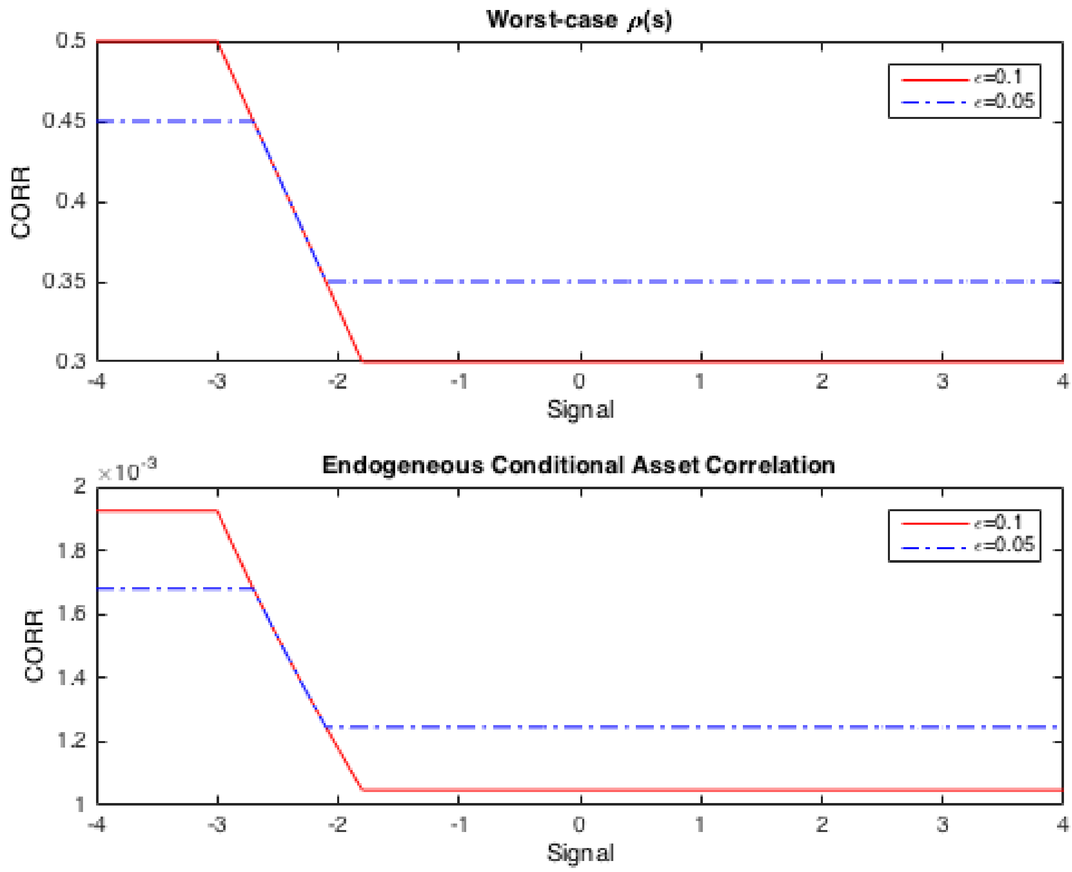
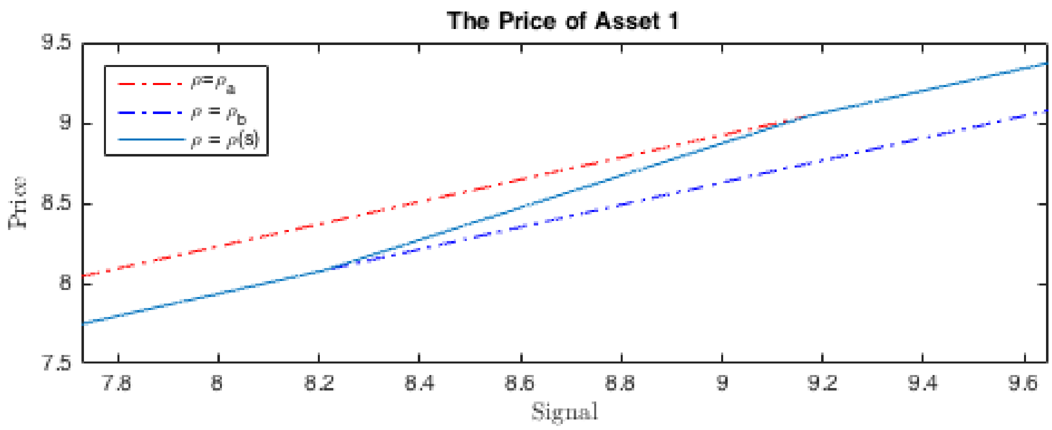


Figure 2: The endogenous asset prices in equilibrium

This figure demonstrates how news affects the endogenous asset price in equilibrium. The parameters in this figure are $\rho_a = 0.4$, $\rho_b = 0.8$, $\sigma_1 = 12\%$, $\sigma_2 = 10\%$, $\sigma_\epsilon = 8\%$; $\overline{d}_1 = 10$, $\overline{d}_2 = 2$, $\overline{\theta}_1 = 100$, $\overline{\theta}_2 = 10$, $\gamma = 2$. $s^L = 8.20$, $s^H = 9.17$.



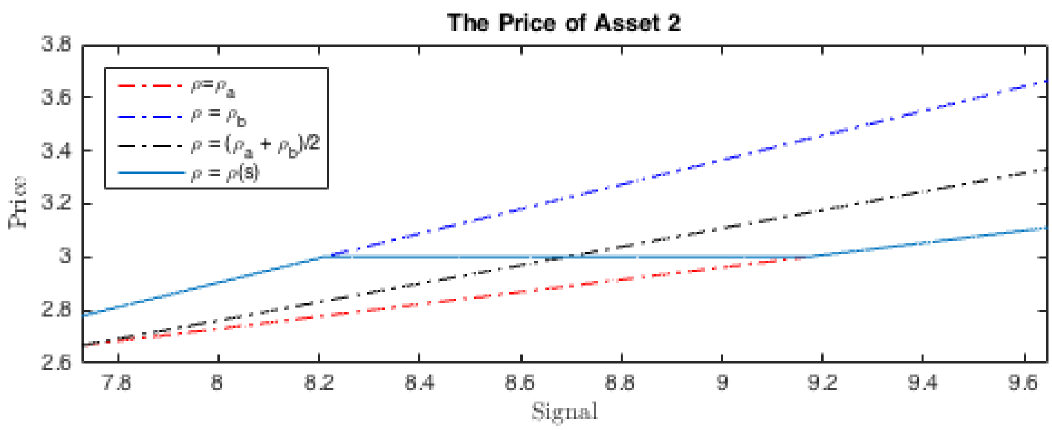
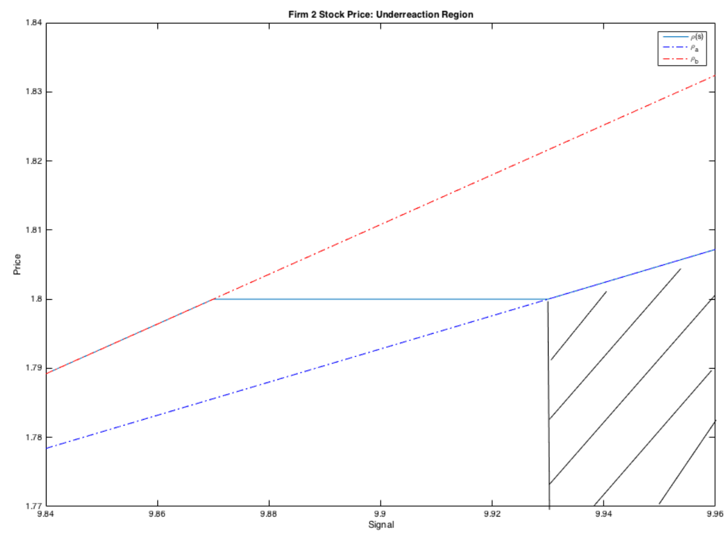


Figure 3: Under and Over-reaction of Stock Prices

This figure demonstrates how stock price momentum (underreaction) and reversal (overreaction) is generated under certain conditions in the model. In the top panel, when ρ_a is large enough, that is when s^L is small, it is more likely to enter the shaded area (a lower price sensitivity with respect to ρ_a); therefore, the stock prices underreact on average. The idea is the same for the bottom panel when ρ_b is small enough.



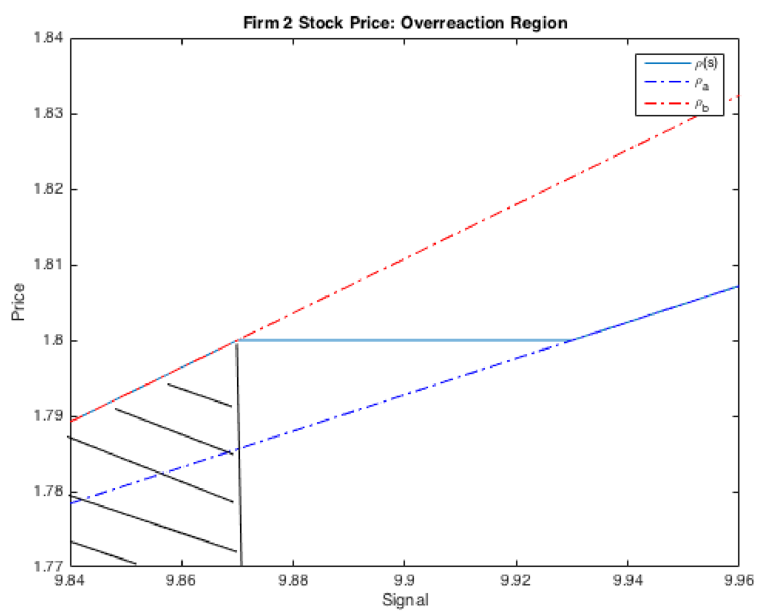


Figure 4: Autocorrelation

This figure demonstrates the effect of correlation uncertainty on the price autocorrelation of the two firms. As shown, the higher the correlation uncertainty the more significant the autocorrelation for each firm. The parameters in this figure are $\rho_a = 0.4 - \epsilon, \rho_b = 0.4 + \epsilon, \epsilon \in \{0.05, 0.1\}$. Other parameters are $\sigma_1 = 3$, $\sigma_2 = 2$, $\sigma_\epsilon = 1\%$; $\overline{d}_1 = 0$, $\overline{d}_2 = 0$, $\overline{\theta}_1 = 1$, $\overline{\theta}_2 = 1$. Notice that in this situation $n = 0.0016\%$, which is extremely small compared with the plausible unconditional correlation coefficient.

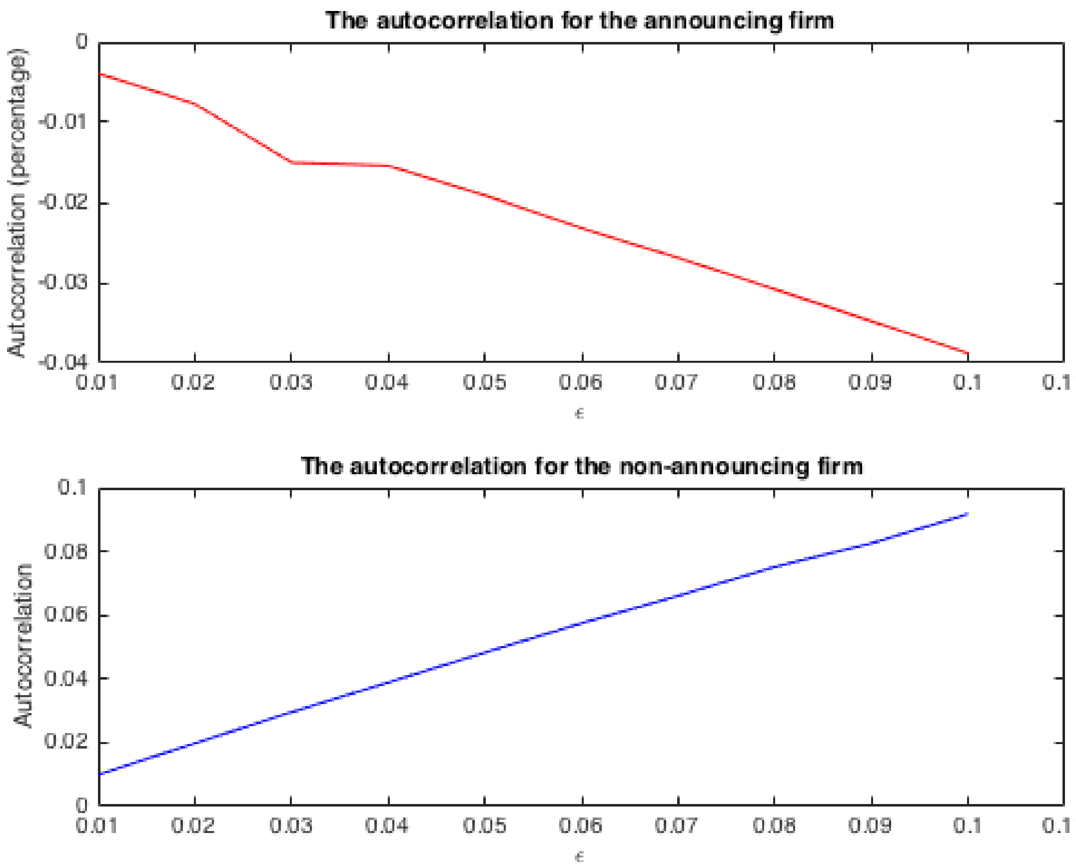


Figure 5: The asymmetric volatility in equilibrium

This figure demonstrates the asymmetric volatility of risky assets in equilibrium. The top panel displays the asymmetric volatility of the first risky asset, that is, a higher volatility under bad news than in good news. The bottom panel displays a complicated asymmetric volatility pattern of the second risky asset. The parameters in this figure are $\rho_a = 0.2$, $\rho_b = 0.7$, $\sigma_1 = 25\%$, $\sigma_2 = 10\%$, $\sigma_\epsilon = 5\%$; $\bar{d}_1 = 10$, $\bar{d}_2 = 5$, $\bar{\theta}_1 = 10$, $\bar{\theta}_2 = 100$, $\gamma = 2$. $s^L = 8.20$, $s^H = 9.17$.

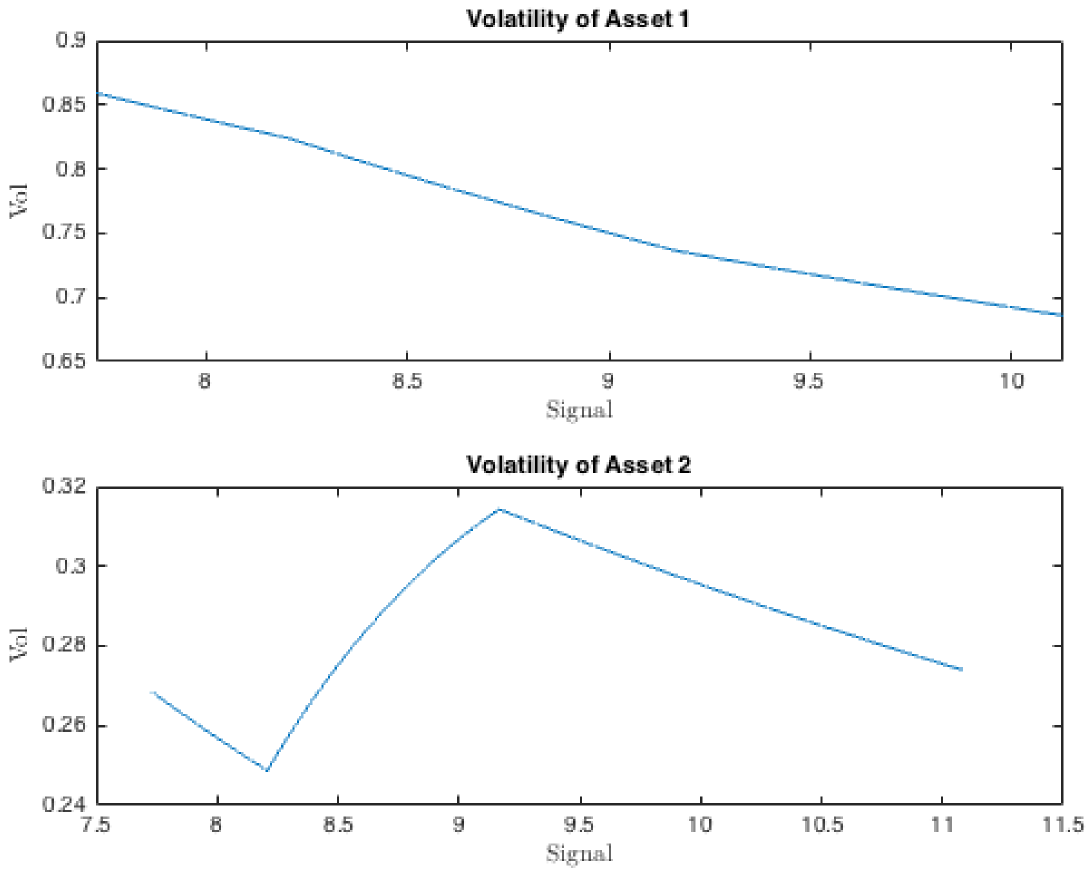


Figure 6: Measures of asymmetric patterns of asset prices

This figure displays the comparison between the asset prices and the expected returns conditional on a piece of good news, $y = \tilde{s} - \overline{d}_1 > 0$, and a piece of bad news $-y$ correspondingly. The parameters in this figure are $\rho_a = 0.2$, $\rho_b = 0.7$, $\sigma_1 = 25\%$, $\sigma_2 = 10\%$, $\sigma_\epsilon = 5\%$; $\overline{d}_1 = 10$, $\overline{d}_2 = 5$, $\overline{\theta}_1 = 10$, $\overline{\theta}_2 = 100$, $\gamma = 2$. $s^L = 6.55$, $s^H = 9.05$. In this case $s^L - \overline{d}_1 = -3.45, s^H - \overline{d}_1 = -0.95$. I plot graphs along positive signals of $\tilde{s} - \overline{d}_1$, which are greater than -0.95, thus $\rho(s) = \rho_a = 0.2$.

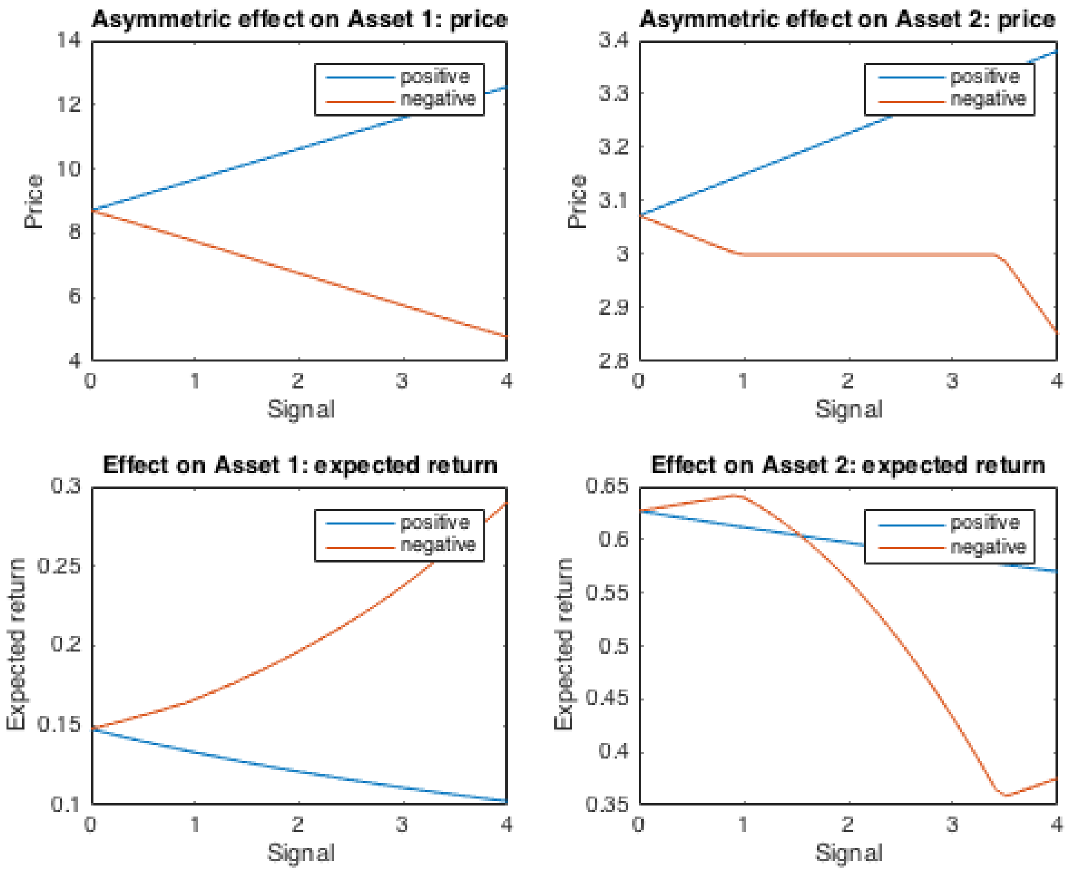


Figure 7: Measures of asymmetric patterns of asset returns

This figure displays the comparison of the conditional statistics - volatility, covariance, correlation, and beta, conditional on a piece of good news, $y = \tilde{s} - \overline{d}_1 > 0$, and a piece of bad news $-y$ correspondingly. The parameters in this figure are $\rho_a = 0.2$, $\rho_b = 0.7$, $\sigma_1 = 25\%$, $\sigma_2 = 10\%$, $\sigma_\epsilon = 5\%$; $\overline{d}_1 = 10$, $\overline{d}_2 = 5$, $\overline{\theta}_1 = 10$, $\overline{\theta}_2 = 100$, $\gamma = 2$. $s^L = 6.55$, $s^H = 9.05$. In this case $s^L - \overline{d}_1 = -3.45$, $s^H - \overline{d}_1 = -0.95$. I plot graphs along positive signals of $\tilde{s} - \overline{d}_1$, which are greater than -0.95, thus $\rho(s) = \rho_a = 0.2$.