Learning about the Neighborhood

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Abstract

We develop a model of neighborhood choice to analyze information aggregation and learning in residential and commercial real estate markets. In the presence of pervasive informational frictions, housing prices serve as important signals to households and commercial real estate developers about the economic strength of a neighborhood. Through this learning channel, noise from supply and demand shocks can propagate from housing prices to real activity, distorting not only migration into the neighborhood, but also the supply of commercial real estate as it is an input to production. Our analysis helps to rationalize the commercial real estate boom that accompanied the recent U.S. housing boom, even though commercial real estate was not subject to the household credit expansion that had contributed to the housing boom.

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In the aftermath of the U.S. housing cycle of the 2000s, much attention has been devoted in academic and policy circles to understanding its causes and consequences. A widely-held view is that the housing boom that occurred to different cities across the U.S. before 2006 was not driven by the economic fundamentals underlying these cities, but rather by an expansion of credit to households (e.g. Mian and Sufi (2009)) or housing speculation (e.g. Shiller (2009) that led to an unexpected and unsustainable rise in housing demand. Interestingly, this housing cycle was accompanied by a similar boom and bust in commercial real estate. Though also characterized by a dramatic run-up and collapse in prices, this second boom and bust, and its relation to local economic outcomes attributed to the recent housing cycle, have received less attention in the literature. Gyourko (2009a), for instance, documents that commercial real estate equity REITs saw a price appreciation of 118% in real terms between 1999 and 2006, and fell 45% by 2008. Office prices also rose relative to construction costs in 14 major MSAs, on average, more than 40% between 2003 and 2008, with only about half of this growth being attributable to stronger rent fundamentals. While residential and commercial real estate markets historically respond in tandem to local economic fundamentals, to the extent that the recent housing boom was not driven by local economic fundamentals, it is difficult to explain the associated commercial real estate boom and bust.

As noted by Glaeser (2013) and other commentators, areas such as Las Vegas and Phoenix experienced more dramatic housing price booms and busts during the recent housing cycle than would be predicted by theories based on supply elasticity. Along with this housing boom, Las Vegas also saw a pronounced expansion in its casinos and resorts, including the construction of the Cosmopolitan, the CityCenter, and the Echelon Place, that continued into 2007.[1] Anchoring, in part, on signals of strong fundamentals from the housing market, such as the influx of maturing baby boomers fueling demand for condominiums, real estate developers built new commercial properties, such that Las Vegas, which already had more guest rooms than even Orlando or NYC in 2007, had more hotel rooms in development than any other city in the country. Phoenix, similarly, constructed about 86.5 million square feet of new commercial space, including offices, shopping centers, industrial buildings, hotels, and apartments, from 2005 to 2009.[2]

In this paper, we develop a theoretical framework to highlight a novel mechanism for

[1] See the April 2007 NYT article "In Las Vegas, Too Many Hotels Are Never Enough".

[2] See the March 2010 NYT article "Phoenix Meets the Wrong End of the Boom Cycle".

noise in housing markets to impact both residential and commercial real estate demands through a learning channel. In our model, home buyers and commercial developers observe neither the economic strength of a neighborhood, which ultimately determines the demand for housing and commercial infrastructure in the neighborhood, nor the supply of housing. Home buyers can choose whether to live in the neighborhood by purchasing a house, and how much labor to supply and commercial infrastructure to buy once in the neighborhood, while home builders and commercial developers choose how many residential units and commercial infrastructure to build to sell to households. Households need commercial infrastructure to produce output, and this complementarity in production links the commercial and residential real estate markets.

In the presence of pervasive informational frictions, local housing markets provide a useful platform for aggregating private information about the economic strength of the neighborhood. It is intuitive that traded housing prices reflect the net effect of demand and supply factors, in a similar spirit of the classic models of Grossman and Stiglitz (1980) and Hellwig (1980) for information aggregation in asset markets. Different from the linear equilibrium in these models, our model features an important neighborhood selection, through which only households with private signals above a certain equilibrium threshold choose to live in the neighborhood. This selection makes our model inherently non-linear. Nevertheless, we are able to derive the equilibrium in analytical forms, building on the cutoff equilibrium framework developed by Goldstein, Ozdenoren, and Yuan (2013) and Albagli, Hellwig, and Tsyvinski (2014, 2015) for asset markets. Furthermore, our model allows us to analyze how informational frictions affect not only the residential housing market but also the households' labor and production decisions, which, in turn, determine their demand in the local commercial real estate market.

Specifically, we first present the model in Section 1. The model features a continuum of households in an open neighborhood, which can be viewed as a county. Each household has a choice of whether to move into the neighborhood by purchasing a house, and has a Cobb-Douglas utility function over its consumption of its own good and its aggregate consumption of the goods produced by other households in the neighborhood. This complementarity in households' consumption motivates each household to learn about the unobservable economic strength of the neighborhood, which determines the common productivity of all households and, consequently, their desire to live in the neighborhood. Each household requires both labor, which it supplies, and commercial infrastructure, to produce its good according to a Cobb-Douglas production function of these two inputs. Since the price of commercial infrastructure depends on its marginal product across households in the neighborhood, competitive commercial developers must form expectations about the unobservable economic strength of the neighborhood when determining how much commercial infrastructure to develop.

We derive the equilibrium in Section 2. Despite each household's housing demand being non-linear, the Law of Large Numbers allows us to aggregate their housing demand, and to derive a cutoff equilibrium for the housing market. Each household possesses a private signal regarding the neighborhood common productivity. By aggregating the households' housing demand, the housing price aggregates their private signals. The presence of unobservable supply shocks, however, prevents the housing price from perfectly revealing the neighborhood strength and acts as a source of informational noise in the housing price. As the housing price also affects commercial developers' expectations, noise in the housing price, originated from either demand or supply side, may also distort their development decisions, leading to correlated cycles between residential and commercial real estate markets.

In Section 3, we examine the effects of supply shocks and demand shocks on the equilibrium cutoff of households' entry decision to the neighborhood, the housing price, commercial real estate price, and commercial real estate development across two dimensions: 1) elasticity of housing supply in the neighborhood and 2) the degree of consumption complementarity in the utility of households. Our analysis highlights that the reaction of a neighborhood to housing market speculation can differ depending on whether its source is a demand or supply side shock, and this gives rise to testable differences in the cross-section when sorting areas by supply elasticity or the degree of complementarity of its industries. Furthermore, our analysis highlights that speculation in housing not only has long-term consequences for the housing market, but also for the commercial real estate market as well. In this sense, there is a learning externality in that households, when choosing their housing demand, do not internalize that it impacts expectations of commercial developers. In addition to a misallocation of resources ex-ante, our learning channel also gives rise to persistent distortions to local economies ex-post. To the extent that any overbuilding of offices and commercial infrastructure is difficult to reverse in the short or medium-term, any excess supply will lower the marginal product of other factors of production, such as labor, and may have amplified, and prolonged, some of the adverse consequences of the housing market in the recent recession. Gao, Sockin, and Xiong (2017), for instance, find consistent evidence that supply overhang in housing markets helped transmit the adverse impact of housing speculation to the real economy during the recent bust.

The existing literature has emphasized the importance of accounting for home buyers' expectations in understanding dramatic housing boom and bust cycles, e.g., Case and Shiller (2003); Glaeser, Gyourko, and Saiz (2008); and Piazzesi and Schneider (2009). Much of the analyses and discussions, however, are made in the absence of a systematic framework that anchors home buyers' expectations to their information aggregation and learning process. In this paper, we help fill this gap by developing a model for analyzing information aggregation and learning in housing markets, and its spillover to commercial real estate. By doing so, we are able to uncover a novel interaction in which distortions to housing prices can impact the supply of commercial real estate through expectations about future rents. We also demonstrate that supply elasticity, beyond its role in driving housing supply, determines the informational content of the housing price, and how households weight the price signal compared to other public signals of demand when learning. This learning effect implies non-monotonic patterns in housing price volatility and real activity.

The literature has offered several other explanations for the comovement of residential and commercial real estate cycles. Rosen (1979) and Roback (1982) link housing and commercial real estate in spatial equilibrium settings in which land can either be developed for residential or commercial use. As a consequence, both markets are driven by similar local fundamentals, as prices, on the margin, reflect both the value of amenities to households and productivity to firms. Our analysis links these two markets instead through the production technology with which households produce output and, as a result, both housing and commercial real estate are driven by expectations about future neighborhood productivity. Gyourko (2009a) emphasizes the role of irrational investor and lender optimism, citing that much of the recent run-up in commercial real estate prices was driven by investors competing over the same income rents, rather than rising cash flows, who had easier access to capital because of declining loan underwriting standards. Levitin and Wachter (2013) explain this recent parallel boom in construction real estate as a change in investor demongraphics and a deterioration in securitization underwriting standards. To the extent that more accommodative lending and securitization standards enabled optimistic commercial developers to act on their expectations of future rents, we view these two channels as complementary to our learning channel.

Our model also differs from Burnside, Eichenbaum, and Rebelo (2013), which offers a model of housing market booms and busts based on the epidemic spreading of optimistic or pessimistic beliefs among home buyers through their social interactions. Our learning-based mechanism is also different from Nathanson and Zwick (2017), which studies the hoarding of land by home builders in certain elastic areas as a mechanism to amplify price volatility in the recent U.S. housing cycle. Glaeser and Nathanson (2017) presents a model of biased learning in housing markets, building on current buyers not adjusting for the expectations of past buyers, and instead assuming that past prices reflect only contemporaneous demand. This incorrect inference gives rise to correlated errors in housing demand forecasts over time, which in turn generate excess volatility, momentum, and mean-reversion in housing prices. In contrast to this model, informational frictions in our model anchor on the interaction between the demand and supply sides, and feedback to both housing price and real outcomes. This key feature is also different from the amplification to price volatility induced by dispersed information and short-sale constraints featured in Favara and Song (2014).

By focusing on information aggregation and learning of symmetrically informed households with dispersed private information, our study differs in emphasis from those that analyze the presence of information asymmetry between buyers and sellers of homes, such as Garmaise and Moskowitz (2004) and Kurlat and Stroebel (2014). Neither does our model emphasize the potential asymmetry between in-town and out-of-town home buyers, which is shown to be important by Chinco and Mayer (2015).

Our work features a tractable cutoff equilibrium framework, similar to that in Goldstein, Ozdenoren, and Yuan (2013) and Albagli, Hellwig, and Tsyvinski (2014, 2015), which employ risk-neutral agents, normally distributed asset fundamentals, and position limits to deliver tractable nonlinear equilibria. Goldstein, Ozdenoren, and Yuan (2013) investigate the feedback to the investment decisions of a single firm when managers, but not investors, learn from prices. Albagli, Hellwig, and Tsyvinski (2014, 2015) focus on the role of asymmetry in security payoffs in distorting asset prices and firm investment incentives when future shareholders learn from prices to determine their valuations. In contrast, we focus on the feedback induced by learning from housing prices to household neighborhood choice and labor decisions in an equilibrium production setting, and its spillover to the investment decisions of commercial developers. The interaction between learning and supply elasticity can potentially help explain why relatively unconstrained areas recently experienced more pronounced house price boom-bust cycles, as documented in, for instance, Davidoff (2013), Glaeser (2013), and Nathanson and Zwick (2017).

In addition, there are extensive studies in the housing literature highlighting the roles played by both demand-side and supply-side factors in driving housing cycles. On the demand side, Himmelberg, Mayer, and Sinai (2005) focus on interest rates, Poterba, Weil, and Shiller (1991) on tax changes, Mian and Sufi (2009) on credit expansion, and DeFusco, Nathanson, and Zwick (2017) and Gao, Sockin and Xiong (2017) on investment home purchases. On the supply side, Glaeser, Gyourko, Saiz (2008) emphasize supply as a key force in mitigating housing bubbles, Haughwout, Peach, Sporn and Tracy (2012) provide a detailed account of the housing supply side during the U.S. housing cycle in the 2000s, and Gyourko (2009b) systematically reviews the literature on housing supply. By introducing informational frictions, our analysis shows that supply-side and demand-side factors are not mutually independent. Supply shocks can affect housing and commercial real estate demand by acting as informational noise in learning, and influence households' and commercial developers' expectations of the strength of the neighborhood.

1 The Model

The model has two periods $t \in \{1,2\}$. There are three types of agents in the economy: households looking to buy homes in a neighborhood or elsewhere, home builders, and commercial real estate developers. Suppose that the neighborhood is new and all households purchase houses from home builders in a centralized market at $t = 1$ after choosing whether to live in the neighborhood. Households choose their labor supply and demand for commercial facilities, such as offices and warehouses, to complete production, and consume consumption goods at $t = 2$. Our intention is to capture the decision of a generation of home owners to move into a neighborhood, and we view the two periods as representing a long period in which they live together and share amenities, as well as exchange their goods and services.

1.1 Households

There is a continuum of households, indexed by $i \in [0,1]$. A household can choose to live in a neighborhood or elsewhere, and we can divide the unit interval into the partition $\{\mathcal{N},\mathcal{O}\}$, with $\mathcal{N} \cap \mathcal{O} = \varnothing$ and $\mathcal{N} \cup \mathcal{O} = [0,1]$. Let $H_i = 1$ if household i chooses to live in the neighborhood, i.e., $i \in \mathcal{N}$, and $H_i = 0$ if it chooses to live elsewhere.[3] If household i at $t = 1$ chooses to live in the neighborhood, it must purchase one house at price P. This reflects, in part, that housing is an indivisible asset and a discrete purchase, consistent with the insights of Piazzesi and Schneider (2009).

Household i in the neighborhood has a Cobb-Douglas utility function over consumption of its own good C_i and its consumption of the goods produced by all other households in the neighborhood $\{C_j\}_{j \in \mathcal{N}}$:

$$U\left(\{C_j\}_{j \in \mathcal{N}}; \mathcal{N}\right) = \left(\frac{C_i}{1-\eta_c}\right)^{1-\eta_c} \left(\frac{\int_{\mathcal{N}/i} C_j dj}{\eta_c}\right)^{\eta_c}. \tag{1}$$

The parameter $\eta_c \in (0,1)$ measures the weights of different consumption components in the utility function. A higher η_c means a stronger complementarity between the consumption of household i and its consumption of the composite good produced by the other households in the neighborhood. As we will discuss later, this utility specification implies that each household cares about the strength of the neighborhood, i.e., the productivities of other households in the neighborhood. This assumption is motivated by the empirical findings of Ioannides and Zabel (2003), and leads to strategic complementarity in each household's housing demand.[4]

The production function of household i is also Cobb-Douglas $e^{A_i} K_i^{\alpha} l_i^{1-\alpha}$, where l_i is the household's labor choice, A_i is its productivity, K_i is the commercial facility rented from commercial developers, and $\alpha \in (0,1)$ the commercial facility share. Household i's productivity, A_i, is comprised of a component, A, common to all households in the neighborhood and an idiosyncratic component, ε_i:

$$A_i = A + \varepsilon_i,$$

where $A \sim \mathcal{N}\left(\bar{A}, \tau_A^{-1}\right)$ and $\varepsilon_i \sim \mathcal{N}\left(0, \tau_\theta^{-1}\right)$ are both normally distributed and independent of each other. Furthermore, we assume that $\int \varepsilon_i d\Phi(\varepsilon_i) = 0$ by the Strong Law of Large Numbers. The common productivity, A, represents the strength of the neighborhood, as a higher A implies a more productive neighborhood. As A determines the households'

[3] See Van Nieuwerburgh and Weill (2010) for a systematic treatment of moving decisions by households across neighborhoods.

[4] There are other types of social interactions between households living in a neighborhood, which are explored, for instance, in Durlauf (2004) and Glaeser, Sacerdote, and Scheinkman (2003).

aggregate demand for housing, it represents the demand-side fundamental. One can view τ_θ as a measure of household diversity.

As a result of realistic informational frictions, A is not observable to households at $t = 1$ when they need to make the decision of whether to live in the neighborhood. Instead, each household observes its own productivity A_i, after examining what it can do if it chooses to live in the neighborhood. Intuitively, A_i combines the strength of the neighborhood A and the household's own attribute ε_i. Thus, A_i also serves as a noisy private signal about A at $t = 1$, as the household cannot fully separate its own attribute from the opportunity provided by the neighborhood. The parameter τ_θ governs both the diversity in the neighborhood, or dispersion in productivity, and the precision of this private signal. As $\tau_\theta \to \infty$, the households' signals become infinitely precise and the informational frictions about A vanish. Households care about the strength of the neighborhood because of complementarity in their demand for consumption. Since households want to have similar amenities to their neighbors, they need to learn about A because it affects their neighbors' consumption decisions. Consequently, while a household may have a fairly good understanding of its own productivity when moving into a neighborhood, complementarity in consumption demand motivates it to pay attention to housing prices to learn about the average level A for the neighborhood.

We start with each household's problem at $t = 2$ and then go backwardly to describe its problem at $t = 1$. At $t = 2$, we assume that A is revealed to all agents. Furthermore, we assume that each household experiences a disutility for labor $\frac{l_i^{1+\psi}}{1+\psi}$, and that a household in the neighborhood $\mathcal{N}$ maximizes its utility at $t = 2$ by choosing labor l_i, commercial facility K_i, and its consumption demand $\{C_j\}_{j\in\mathcal{N}}$:

$$U_i = \max_{\left\{\{C_j\}_{j\in\mathcal{N}}, l_i, K_i\right\}} U\left(\{C_j\}_{j\in\mathcal{N}}; \mathcal{N}\right) - \frac{l_i^{1+\psi}}{1+\psi} \tag{2}$$

$$\text{such that } \; p_i C_i + \int_{\mathcal{N}/i} p_j C_j(i)\, dj + P + RK_i = p_i e^{A_i} K_i^{\alpha} l_i^{1-\alpha} + \Pi_i,$$

where p_i is the price of the good it produces, P is the housing price in the neighborhood, and R is the price of commercial facilities. Households behave competitively and take the prices of their goods as given. For simplicity, we assume that each home builder and commercial developer is part of a household in the neighborhood, similar to the Lucas household paradigm, and that the builder and developer bring home their profit $\Pi_i = P + RK_i$ to the household after construction has taken place. This allows us to focus on distortions to behavior from informational frictions at $t = 1$ in the absence of the mechanical impact on wealth from the purchase of a house. As a result, at $t = 2$, household i's budget constraint simplifies to $p_i C_i + \int_{\mathcal{N}/i} p_j C_j(i)\,dj = p_i e^{A_i} K_i^{\alpha} l_i^{1-\alpha}$. We further normalize the interest rate from $t = 1$ to $t = 2$ to be zero.

At $t = 1$, before choosing its consumption, commercial facility usage, and labor supply, a household need to decide whether to live in the neighborhood. In addition to their private signals, all households and commercial developers observe a noisy public signal, Q, about the strength of the neighborhood A :

$$Q = A + \tau_Q^{-1/2} \varepsilon_Q,$$

where $\varepsilon_Q \sim \mathcal{N}(0, 1)$ independent of all other shocks. As τ_Q becomes arbitrarily large, A becomes common knowledge to all agents.

In addition to the utility flow U_i at $t = 2$ from final consumption, we assume that households have quasi-linear expected utility at $t = 1$, and incur a linear utility penalty equal to the housing price P if they choose to live in the neighborhood and thus have to buy a house. Given that households have Cobb-Douglas preferences over their consumption, they are effectively risk-neutral at $t = 1$, and their utility flow is then the value of their final consumption bundle less the cost of housing. Households make their neighborhood choice subject to a participation constraint that their expected utility from moving into the neighborhood $E[U_i|\mathcal{I}_i] - P$ must (weakly) exceed a reservation utility, which we normalize to 0. One can interpret the reservation utility as the expected value of getting a draw of productivity from another potential neighborhood less the cost of search. Household i makes its neighborhood choice while taking the transfer from the home builder in its family as given:

$$\max_{H_i} \{E[U_i|\mathcal{I}_i] - PH_i, 0\} \tag{3}$$

The choice of neighborhood is made at $t = 1$ subject to each household's information set $\mathcal{I}_i = \{A_i, P, Q\}$, which includes its private productivity signal A_i, the public signal Q, and the housing price P.[5]

[5] We do not include the volume of housing transactions in the information set as a result of a realistic consideration that, in practice, people observe only delayed reports of total housing transactions at highly aggregated levels, such as national or metropolitan levels.

1.2 Commercial Developers

In addition to households, there is a continnum of risk-neutral commercial real estate developers that develop commercial facilities at $t = 1$, and sells them to households for their production at $t = 2$. The representitive developer cares about R, the price of commercial facilities at $t = 2$, which depend on the marginal productivity of the facilities. This, in turn, depends on the strength of the neighborhood, and which households choose to live in the neighborhood. The housing price in the neighborhood serves as a useful signal to the developer when deciding how much commercial facilities to develop at $t = 1$.

To simplify our analysis, and distinguish our mechanism from that of Rosen (1979) and Roback (1982), we decouple the supply of residential housing from the supply of commercial real estate. We assume that land available for commercial facility is in elastic supply, and a plot of land of size K can be developed into a commercial facility by incurring a convex effort cost $\frac{1}{\lambda}K^{\lambda}$, where $\lambda > 1$, that is increasing in how much land is developed.

We assume that households rent commercial facility from commercial developers when production occurs at $t = 2$, and that commercial developers must forecast this demand when choosing how much commercial land K to develop at $t = 1$. The representative commercial developer takes the commercial facility price schedule R as given, and chooses K subject to the maximize its expected profits per unit of commercial land:

$$\Pi_c = \sup_K E\left[RK - \frac{1}{\lambda}K^{\lambda} \middle| \mathcal{I}^c \right] \tag{4}$$

where $\mathcal{I}^c = \{P, Q\}$ is the public information set, which include the housing price P and the public signal Q. We interpret this commercial facility K in this context broadly as commercial real estate, infrastructure, and amenities necessary for households to be productive in a neighborhood.[6] It then follows that the optimal choice of commercial facility sets the marginal cost, $K^{\lambda-1}$, equal to the marginal benefit, $E\left[R \middle| \mathcal{I}^c\right]$:

$$K = E\left[R \middle| \mathcal{I}^c\right]^{\frac{1}{\lambda-1}}.$$

The choice of commercial facility is influenced by the expectations of the commercial developer about future neighborhood productivity, which is affected by the realization of the

[6] One can extend our analysis to consider K to be a public good, in which case the rental rates are the taxes a local government that faces a balanced budget can raise to offset the costs of construction. Our model then has implications for how housing markets impact local government fiscal policy.

housing price P. Market-clearing in the market for commercial facility at $t = 2$ requires that:

$$\int_{\mathcal{N}} K_i di = K \int_{\mathcal{N}} di, \tag{5}$$

where $\int_{\mathcal{N}} di$ represents the population of households that live in the neighborhood. As we mentioned before, we assume that each developer is part of a household and rebates the profits of its sale to the household to which it belongs.

The commercial developers' decision to develop commercial facility at $t = 1$ gives another source of amplification for informational frictions. In addition to distorting neighborhood choice of potential household entrants, informational frictions in housing markets also distort costly investment in resources that foster economic growth in the neighborhood. In the absence of informational frictions, housing prices may distort neighborhood choice but not expectations about neighborhood productivity.

1.3 Home Builders

There is a population of home builders, indexed on a continuum $[0, 1]$, in the neighborhood. Home builders also face uncertainty about the aggregate strength of the neighborhood and the ability of the supply side to respond to the demand. Specifically, builder i builds a single house subject to a disutility from labor:

$$e^{-\frac{1}{1+k}\omega_i} S_i,$$

where $S_i \in \{0, 1\}$ is the builder's decision to build and

$$\omega_i = \xi + e_i$$

is the builder's productivity, which is correlated across builders in the neighborhood through ξ. We assume that $\xi = k\zeta$, where $k \in (0, \infty)$ is a constant parameter, and ζ represents an unobserved, common shock to building cost in the neighborhood. From the perspective of households and builders, $\zeta \sim \mathcal{N}\left(\bar{\zeta}, \tau_\zeta^{-1}\right)$. Then $\xi = k\zeta$ can be interpreted as a supply shock with normal distribution $\xi \sim \mathcal{N}\left(\bar{\xi}, k^2\tau_\zeta^{-1}\right)$, with $\bar{\xi} = k\bar{\zeta}$. Furrthermore, $e_i \sim \mathcal{N}\left(0, \tau_e^{-1}\right)$ such that $\int e_i d\Phi\left(e_i\right) = 0$ by the Strong Law of Large Numbers.

Builders in the neighborhood at $t = 1$ maximize their revenue:

$$\Pi_s\left(S_i\right) = \max_{S_i} \left(P - e^{-\frac{1}{1+k}\omega_i}\right) S_i. \tag{6}$$

We normalize the disutility by $\frac{1}{1+k}$ so that, if we interpret k as supply elasticity, then as $k \to \infty$, $e^{-\frac{1}{1+k}\omega_i}$ converges to $e^{-\zeta}$, so that prices remain finite $a.s.$ and supply is completely driven by the common supply shock. Since builders are risk-neutral, it is easy to determine the builders' optimal supply curve:

$$S_i = \begin{cases} 1 & \text{if } P \geq e^{-\frac{1}{1+k}\omega_i} \\ 0 & \text{if } P < e^{-\frac{1}{1+k}\omega_i} \end{cases}. \tag{7}$$

The parameter k measures the supply elasticity of the neighborhood. A more elastic neighborhood has a larger supply shock, i.e., the supply shock has greater mean and variance. In the housing market equilibrium, the supply shock ξ not only affects the supply side but also the demand side, as it acts as informational noise in the price signal when the households use the price to learn about the common productivity A.

1.4 Noisy Rational Expectations Cutoff Equilibrium

Our model features a noisy rational expectations cutoff equilibrium, which requires clearing of the two real estate markets that is consistent with the optimal behavior of households, homer builders and commercial developers:

- Household optimization: $\left\{\{C_i\}_{i\in\mathcal{N}}, l_i, K_i\right\}$ solves each household's maximization problem at $t = 2$ as specified in (2) and H_i solves its problem at $t = 1$ as specified in (3).

- Commercial developer optimization: K solves the representative commercial developer's maximization problem in (4).

- Builder optimization: S_i solves each builders' maximization problem in (6).

- At $t = 1$, the residential housing market clears:

$$\int_{-\infty}^{\infty} H_i\left(A_i, P, Q\right) d\Phi\left(\varepsilon_i\right) = \int_{-\infty}^{\infty} S_i\left(\omega_i, P, Q\right) d\Phi\left(e_i\right),$$

  where each household's housing demand $H_i\left(A_i, P, Q\right)$ depends on its productivity A_i and the housing price P, and each builder's housing supply $S_i\left(\omega_i, P, Q\right)$ depends on its productivity ω_i, the housing price P, and the public signal Q. The demand from households and supply from builders are integrated over the idiosyncratic components of their productivities $\{\varepsilon_i\}_{i\in[0,1]}$ and $\{e_i\}_{i\in[0,1]}$, respectively.

- At $t=2$, the market for each household's good clears:

$$\int_{\mathcal{N}} C_i(j)\,dj = e^{A_i} K_i^{\alpha} l_i^{1-\alpha}, \quad \forall\, i \in \mathcal{N},$$

and the market for commercial facilities clears:

$$\int_{\mathcal{N}} K_i di = K \int_{\mathcal{N}} di.$$

2 Equilibrium

In this section, we analyze a symmetric cutoff equilibrium, in which the choice of each household to live in the neighborhood is monotonic with respect to its own productivity A_i.

2.1 Choices of Households and Commercial Developers

We first analyze household choices. At $t=2$, households need to make their production and consumption decisions, after the strength of the neighborhood A is revealed to the public, and home builders and commercial developers have also made their choices at $t=1$. Household i has $e^{A_i} K_i^{\alpha} l_i^{1-\alpha}$ units of good i for consumption and trading with other households. It maximizes its utility function given in (2). The following proposition describes the household's consumption, labor, and commercial facility choices. Its marginal utility of goods consumption also gives the equilibrium goods price.

Proposition 1 *Households i's optimal goods consumption and labor supply at $t=2$ are:*

$$\begin{aligned}
C_i(i) &= (1-\eta_c)\, e^{A_i} K_i^{\alpha} l_i^{1-\alpha},\ C_j(i) = \frac{1}{\Phi\left(\sqrt{\tau_\theta}\,(A-A^*)\right)} \eta_c e^{A_j} K_j^{\alpha} l_j^{1-\alpha}, \\
\log l_i &= \frac{1}{1-\alpha} \frac{1+\psi}{(1-\alpha)\psi + (1+\alpha\psi)\eta_c} \frac{\eta_c}{\psi} A + \frac{1-\eta_c}{(1-\alpha)\psi + (1+\alpha\psi)\eta_c} A_i - \frac{\alpha}{1-\alpha}\frac{1}{\psi}\log R \\
&\quad + \frac{1}{1-\alpha}\frac{\eta_c}{\psi} \log \frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2} + \frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\,(A-A^*)\right)} + l_0.
\end{aligned}$$

The price of the good produced by household i is:

$$p_i = e^{\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\eta_c(A-A_i) + \frac{1}{2}\eta_c\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\right)^2 \tau_\theta^{-1}} \left(\frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2} + \frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\,(A-A^*)\right)}\right)^{\eta_c},$$

and its optimal choice of commercial facilities is:

$$
\begin{aligned}
\log K_i &= \frac{1}{1-\alpha}\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\frac{1+\psi}{\psi}\eta_c A+\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}A_i-\frac{1}{1-\alpha}\frac{\psi+\alpha}{\psi}\log R \\
&\quad +\frac{1}{1-\alpha}\frac{1+\psi}{\psi}\eta_c\log\frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\,(A-A^*)\right)}+h_0.
\end{aligned}
$$

Furthermore, the expected utility of household i at $t=1$ is given by:

$$
E\left[U\left(\{C_j(i)\}_{j\in\mathcal{N}};\mathcal{N}\right)-\frac{l_i^{1+\psi}}{1+\psi}\middle|\mathcal{I}_i\right]=\frac{\psi+\alpha}{1+\psi}E\left[p_i e^{A_i}K_i^{\alpha}l_i^{1-\alpha}\middle|\mathcal{I}_i\right],
$$

with constants l_0 and h_0 given in the Appendix.

Proposition 1 shows that each household spends a fraction $1-\eta_c$ of its wealth (excluding housing wealth) on consuming its own good $C_i(i)$ and a fraction η_c on goods produced by its neigbors $\int_{\mathcal{N}/i}C_j(i)\,dj$. When $\eta_c=1/2$, the household consumes its own good and the goods of its neighbors equally. The price of each good is determined by its output relative to that of the rest of the neighborhood. One household's good is more valuable when the rest of the neighborhood produces more, and thus each household needs to take into account the labor decisions of the other households in its neighborhood when making its own decision. The proposition demonstrates that the labor chosen by a household is determined by not only by its own productivity e^{A_i} but also the aggregate production of other households in the neighborhood. This latter component arises from the complementarity in the utility function of the household. Note that in the expressions above A^* is the equilibrium threshold for each household to enter the neighborhood.

Proposition 1 also reveals that the optimal choice of labor for each household is log-linear with the strength of the neighborhood, A, its own productivity, A_i, and the logarithm of the commercial real estate price, $\log R$. The final (nonconstant) term reflects selection, in that only households with productivity above A^* enter the neighborhood. Since A is the mean of the distribution of household productivities, it shows up in this truncation. This proposition also demonstrates that household i's optimal choice of commercial facilities has a similar functional form, with the last (nonconstant) term reflecting selection into the neighborhood. Household i's optimal labor choice and demand for commercial facilities are increasing in the strength of the neighborhood A because of consumption complementarity, since a higher A represents improved trading opportunities, while they are decreasing in the price of commercial facilities $\log R$.

We now discuss each household's decision on whether to live in the neighborhood at $t = 1$ when it still faces uncertainty about A. As a result of Cobb-Douglas utility, the household is effectively risk-neutral over its aggregate consumption, and its optimal choice reflects the difference between its expected output in the neighborhood and the cost of living there, which is the price P to buy a house. This is because the household views the transfer from its home builder Π_i and its commercial developers RK_i as exogenous to whether it lives in the neighborhood. It then follows that household i's neighborhood decision is given by:

$$H_i = \begin{cases} 1 & \text{if } \frac{\psi+\alpha}{1+\psi} E\left[p_i e^{A_i} K_i^{\alpha} l_i^{1-\alpha} \middle| \mathcal{I}_i \right] \geq P \\ 0 & \text{if } \frac{\psi+\alpha}{1+\psi} E\left[p_i e^{A_i} K_i^{\alpha} l_i^{1-\alpha} \middle| \mathcal{I}_i \right] < P \end{cases}.$$

This decision rule for neighborhood choice supports our conjecture to search for a cutoff strategy for each household, in which only households with productivities above a critical level, A^*, enter the neighborhood. This cutoff is eventually solved as a fixed point in the equilibrium.

Given each household's equilibrium cutoff A^* at $t = 1$ and optimal choices at $t = 2$, we can impose market-clearing in the market for commercial facilities to arrive at its price R at $t = 2$. Commercial developers forecast this price when choosing their optimal stock of commercial facilities to develop at $t = 1$. These observations are summarized by the following proposition.

Proposition 2 *Given K units of commercial facilities developed by commercial developers, the price of commercial facilities R at $t = 2$ takes the log-linear form:*

$$\begin{aligned} \log R \quad = \quad & \frac{1+\psi}{\psi+\alpha} A - (1-\alpha) \frac{\psi}{\psi+\alpha} \log K + \frac{1+\psi}{\psi+\alpha} \eta_c \log \frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi + (1+\alpha\psi)\eta_c} \tau_\theta^{-1/2} + \frac{A-A^*}{\tau_\theta^{-1/2}} \right)}{\Phi\left(\sqrt{\tau_\theta} \left(A - A^* \right) \right)} \\ & + (1-\alpha) \frac{\psi}{\psi+\alpha} \log \frac{\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi + (1+\alpha\psi)\eta_c} \tau_\theta^{-1/2} + \frac{A-A^*}{\tau_\theta^{-1/2}} \right)}{\Phi\left(\sqrt{\tau_\theta} \left(A - A^* \right) \right)} + r_0, \end{aligned}$$

with constant r_0 given in the Appendix. The optimal supply of commercial facilities by *commercial developers at $t = 1$ is given by:*

$$\log K = \frac{1}{\lambda - \alpha\frac{1+\psi}{\psi+\alpha}} \log E \left[e^{\frac{1+\psi}{\psi+\alpha}A} \left(\frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2} + \frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\left(A-A^*\right)\right)} \right)^{\frac{1+\psi}{\psi+\alpha}\eta_c} \cdot \left(\frac{\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2} + \frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\left(A-A^*\right)\right)} \right)^{\frac{\psi(1-\alpha)}{\psi+\alpha}} \middle| \mathcal{I}^c \right] + k_0, \tag{8}$$

with constant k_0 given in the Appendix.

Proposition 2 reveals that the commercial real estate price at $t = 2$ is increasing in the strength of the neighborhood, A, with the last two (nonconstant) terms refecting selection by households into the neighborhood, and is decreasing in the supply of commercial facilities K. It also demonstrates that the optimal supply of commercial facilities reflects expectations over not only the strength of the neighborhood, A, but also the impact of truncation from the neighborhood choice of households on the expected price of commercial facilities at $t = 2$. The expectation term captures not only the expected productivity from the terms-of-trade (relative prices of household goods) in the first ratio, but also the dispersion in labor productivity in the second ratio.

2.2 Perfect-Information Benchmark

With perfect information, all households, home builders, and commercial developers observe the strength of the neighborhood A when making their neighborhood choice decision. It is straightforward to show that the optimal choice of commercial facility K simplifies to:

$$\log K = \frac{\frac{1+\psi}{\psi+\alpha}}{\lambda - \alpha\frac{1+\psi}{\psi+\alpha}} A + \frac{\frac{1+\psi}{\psi+\alpha}}{\lambda - \alpha\frac{1+\psi}{\psi+\alpha}} \eta_c \left\{ \log \left[\frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2} + \frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\left(A-A^*\right)\right)} \right] + (1-\alpha)\frac{\psi}{1+\psi} \frac{\frac{1+\psi}{\psi+\alpha}}{\lambda - \alpha\frac{1+\psi}{\psi+\alpha}} \log \left[\frac{\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2} + \frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\left(A-A^*\right)\right)} \right] \right\} + k_0,$$

where k_0 is given in the Appendix and $\frac{\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}} > 0$ since $\lambda - \alpha\frac{1+\psi}{\psi+\alpha} > \lambda - 1 > 0$.

Similar to the labor choice of households from Proposition 1, the supply of commercial facility is log-linear with respect to the strength of the neighborhood, A, with a correction term for the truncation in the household population that occurs because of household selection into the neighborhood. This truncation term reflects two forces. The first is that a smaller population implies less demand for a given choice of commercial facility per household, while the second reflects that the price at which households charge each other for their goods, p_i, is also affected by this truncation.

We now characterize the neighborhood choice of households and the housing price. Households will sort into the neighborhood according to a cutoff equilibrium determined by the net benefit of living in the neighborhood, which trades off household income opportunities with other households in the neighborhood with the price of housing. Despite the inherent nonlinearity of our framework, we derive a tractable, unique cutoff equilibrium that is characterized by the solution to a fixed-point problem over the endogenous cutoff of entry in the neighborhood, A^*. This is summarized in the following proposition.

Proposition 3 *In the absence of informational frictions, there exists a unique cutoff equilibrium in which the following hold: 1) household i follows a cutoff strategy in its neighborhood choice such that:*

$$H_i = \begin{cases} 1 & if \;\; A_i \geq A^* \\ 0 & if \;\; A_i < A^* \end{cases},$$

where $A^*(A, \xi)$ solves equation (17) in the Appendix; 2) the housing price takes the log-linear form:

$$\log P = \frac{1}{1+k}\left(\sqrt{\frac{\tau_\theta}{\tau_e}}\left(A - A^*\right) - \xi\right);$$

3) the cutoff productivity $A^*(A, \xi)$ is monotonically decreasing in ξ and increasing in A; and 4) the population that enters the neighborhood is monotonically increasing in both A and ξ.

Proposition 3 characterizes the cutoff equilibrium in the economy in the absence of informational frictions, and confirms the optimality of a cutoff strategy for households in their neighborhood choice. Households sort based on their individual productivities into the neighborhood, with the most productive, who expect the most gains from living in the neighborhood, entering and participating in production at $t = 2$. This determines the supply of labor at $t = 2$, and, through this channel, the price of commercial facilities at $t = 2$.

Given a cutoff productivity $A^*(A, \xi)$, the housing price P positively loads on the strength of the neighborhood, A, since a higher A implies stronger demand for housing, and loads negative on the supply shock ξ, reflecting that a discount is needed to ensure that a positive shift in housing supply is absorbed by a larger household population. As one would expect, the cutoff A^* enters negatively into the price since households above the cutoff sort into the neighborhood. The higher the cutoff, the fewer the households that enter the neighborhood, and the lower the housing price that is needed to clear the market with the lower housing demand. Despite its log-linear representation, the housing price is actually a generalized linear function of $\sqrt{\frac{\tau_\theta}{\tau_e}}A - \xi$, since A^* is an implicit function of A and $\log P$.

Finally, the last part of the proposition provides comparative statics of the cutoff household that enters the neighborhood with productivity $A^*(A, \xi)$. This cutoff is decreasing in ξ, since a lower house price causes more households to enter the neighborhood for a given neighborhood strength A, and consequently a higher population enters the neighborhood as ξ increases. The cutoff, in contrast, is increasing in neighborhood strength A, since a higher A implies a higher housing price, and can also raise the price of commercial facilities, depending on the supply response of commercial developers. This dominates the countervailing force that a higher A also signals more gains from trade from complementarity in household consumption decisions, and more commercial facilities developed by commercial developers. Though the cutoff productivity increases, more households ultimately enter the neighborhood because a higher A shifts right (in the sense of FOSD) the distribution of households more than it moves the cutoff.

As a result of endogenous selection into the neighborhood, the productivity of the neighborhood is determined by which households choose to live there. The aggregate productivity of the neighborhood $A_{\mathcal{N}}$ is given by:

$$A_{\mathcal{N}} = \log \int_{A^*}^{\infty} e^{A_j} d\Phi(\varepsilon_j) = A + \frac{1}{2}\tau_\theta^{-1} + \log \Phi\left(\tau_\theta^{-1/2} + \frac{A - A^*}{\tau_\theta^{-1/2}}\right).$$

The first two terms would be what one would expect without neighborhood choice, while the third term reflects that productivity is truncated by selection. Importantly, since $A^* = A^*(A, \xi)$, it follows that A^* depends on the housing price in the neighborhood, introducing feedback from housing markets to real decisions. Similar aggregation results exist for total income $\int_{\mathcal{N}} e^{A_j} p_i K^\alpha l_j^{1-\alpha} d\Phi(\varepsilon_j)$ and labor supply $\int_{\mathcal{N}} l_j d\Phi(\varepsilon_j)$ as well.

2.3 Cutoff Equilibrium with Informational Frictions

Having characterized the perfect-information benchmark equilibrium, we now turn to the equilibrium at $t = 1$ in the presence of informational frictions. With informational frictions, households and developers must now forecast the strength of the neighborhood, A, and the realized price of commercial facilities R, when choosing whether to live in the neighborhood, and when deciding the stock of commercial facilities to develop. We again conjecture a cutoff equilibrium and, in addition, now conjecture that the housing price contains a sufficient statistic, $z(P)$ that is linear in A and in the supply shock ξ :

$$z(P) = A + \frac{1}{z_\xi}\left(\xi - \bar{\xi}\right).$$

In addition to the housing price, each household's type, A_i, also acts as a private signals about the strength of the neighborhood, A. Since types are positively correlated with this common productivity, higher types also have more optimistic expectations about A. As such, we anticipate that households will again follow a cutoff strategy when deciding whether to live in the neighborhood.

By solving for the learning of households and commercial developers, and clearing the aggregate housing demand of the households with the supply from home builders, we derive the housing market equilibrium. The following proposition summarizes the housing price from each household's housing demand, and the supply of commercial facilities in this equilibrium.

Proposition 4 *There exists a cutoff equilibrium in the presence of informational frictions in which the following hold: 1) the posterior of household i after observing housing price P, the household public signal Q, and its own productivity A_i is conditionally Gaussian with conditional mean $\hat{A}_i$ and variance $\hat{\tau}_A$ given by:*

$$\begin{aligned}
\hat{A}_i &= \hat{\tau}_A^{-1}\left(\tau_A\bar{A} + \tau_Q Q + \frac{\tau_\theta}{\tau_e}\tau_\xi\left(\sqrt{\frac{\tau_e}{\tau_\theta}}\left(\log P + \bar{\xi}\right) + A^*\right) + \tau_\theta A_i\right), \\
\hat{\tau}_A &= \tau_A + \tau_Q + \frac{\tau_\theta}{\tau_e}\tau_\xi + \tau_\theta,
\end{aligned}$$

and the posterior of commercial developers is conditionally Gaussian with conditional mean $\hat{A}^c$ and variance $\hat{\tau}_A^c$ given by:

$$\begin{aligned}
\hat{A}^c &= \hat{\tau}_A^{c-1}\left(\tau_A\bar{A} + \tau_Q Q + \frac{\tau_\theta}{\tau_e}\tau_\xi\left(\sqrt{\frac{\tau_e}{\tau_\theta}}\left(\log P + \bar{\xi}\right) + A^*\right)\right), \\
\hat{\tau}_A^c &= \tau_A + \tau_Q + \frac{\tau_\theta}{\tau_e}\tau_\xi;
\end{aligned}$$

2) household i follows a cutoff strategy in its neighborhood choice such that:

$$H_i = \begin{cases} 1 & \text{if} \quad A_i \geq A^* \\ 0 & \text{if} \quad A_i < A^* \end{cases},$$

where $A^*(A, Q, \xi)$ *solves equation (19) in the Appendix; 3) the equilibrium commercial facility supply takes the form:*

$$\log K = \frac{1}{\lambda - \alpha \frac{1+\psi}{\psi+\alpha}} \log F\left(\hat{A}^c - A^*, \hat{\tau}_A^c\right) + \frac{\frac{1+\psi}{\psi+\alpha}}{\lambda - \alpha \frac{1+\psi}{\psi+\alpha}} A^* + k_0.$$

where $F\left(\hat{A}^c - A^*, \hat{\tau}_A^c\right)$ *is given in the Appendix, and* $\log K$ *is increasing in the conditional belief of the commercial developers,* $\hat{A}^c$*; 4) the housing price takes the log-linear form:*

$$\log P = \frac{1}{1+k}\left(\sqrt{\frac{\tau_\theta}{\tau_e}}\left(A - A^*\right) - \xi\right);$$

and 5) the equilibrium converges to the perfect-information benchmark in Proposition 3 as $\tau_Q \nearrow \infty$.

Proposition 4 establishes the analogue of the cutoff equilibrium in the presence of informational frictions. Households now must form beliefs about the strength of the economy when deciding whether to live in the neighborhood. From equation (18) in the Appendix, we can express $A^*(A, Q, \xi)$ as $A^*(\log P, Q)$, which verifies that the cutoff is indeed measurable to households when making their decisions at $t = 1$. Consequently, after observing the housing price P and the public signal Q, they know the exact value of A^* and form their conditional estimate of A.

Importantly, informational frictions provide a channel for housing prices to feed into the supply decision of commercial developers through learning. With perfect information, housing prices only impact the supply of commercial facility by altering the cost that households pay to enter the neighborhood. Since the price of commercial facility depends on the imperfectly observed strength of the neighborhood, commercial developers use the housing price as a signal when forming their beliefs about this strength. Consequently, any noise in this signal will distort commercial developers' optimal choice of how much commercial facilities to develop.

Though optimal policies are qualitatively similar to those in the perfect-information benchmark, these policies are distorted by informational frictions. The cutoff household productivity A^* now reflects that households and commercial developers imperfectly observe the neighborhood's strength, and use the housing price to form their beliefs. The distortion to both beliefs and the cutoff productivity then feed back into the optimal choice of commercial facilities supplied at $t = 1$, amplifying the impact of informational frictions.

The complementarity between households is even more important for a cutoff strategy in the presence of informational frictions. Without complementarity, a stronger neighborhood, i.e., higher A, is bad news for households, since a higher A raises housing prices, and can also raise the price of commercial facilities depending on the supply response of commercial developers. Furthermore, a higher household type A_i signals a stronger neighborhood, which gives a household with a high type A_i less incentive to enter the neighborhood. With complementarity, however, a stronger neighborhood is also good news for households, since it signals that other households in the neighborhood will be more productive, and household i will get a better terms-of-trade when it trades with other households at $t = 2$. A higher A_i, consequently, can be good news since it signals a stronger neighborhood. This latter effect reinforces the cutoff equilibrium.

Informational frictions distort housing prices by shifting the productivity cutoff $A^* = A^*(\log P, Q)$. With perfect information, $\frac{dA^*(\log P,Q)}{d\log P} > 0$ captures only the direct effect that a higher price deters households from entering the neighborhood because of the cost effect, shifting up the productivity of the marginal household. In the presence of informational frictions, however, $\frac{dA^*(\log P,Q)}{d\log P}$ also reflects an information effect that prices act as a positive signal about the strength of the neighborhood, which lowers $\frac{dA^*(\log P,Q)}{d\log P}$ compared to its perfect-information benchmark value because it also encourages more households to enter the neighborhood.

The sensitivity of the price to changes in $z(P) = A - \sqrt{\frac{\tau_e}{\tau_\theta}}\xi$ (i.e., the sufficient statistic of P), by the Implicit Function Theorem, is:

$$\frac{d\log P}{dz} = \left((1+k)\sqrt{\frac{\tau_e}{\tau_\theta}} + \frac{dA^*(\log P, Q)}{d\log P}\right)^{-1}.$$

As $\frac{d\log P}{dz}$ is greater in the presence of informational frictions, this effect can make housing prices more volatile, as highlighted by Albagli, Hellwig, and Tsyvinski (2015) in their analysis of the cutoff equilibrium in an asset market. This interesting feature also differentiates our cutoff equilibrium from other type of non-linear equilibrium with asymmetric information, such as the log-linear equilibrium developed by Sockin and Xiong (2015) to analyze commodity markets. In their equilibrium, prices become less sensitive to their analogue of

z in the presence of informational frictions. This occurs because households, on aggregate, underreact to the fundamental news in the their private signals because of noise, and this is reflected in a lower weight on z in prices compared to perfect-information.

To understand how housing prices impact learning, it is instructive to consider two polar cases for supply elasticity. When $k = 0$, then housing prices are only a function of the strength of the neighborhood, A, and prices are fully revealing to households and commercial developers. Informational frictions, as a result, unravel when supply is infinitely inelastic. When, instead, $k \to \infty$, then prices converge to $\log P = -\zeta$, and the housing price is driven only by the supply shock.[7] When supply is perfectly elastic, then prices contain no information about demand, and therefore no information about the strength of the neighborhood. Consequently, the information content of prices, and the weight that households and commercial developers assign to prices in forming their conditonal estimates of the neighborhood strength, dissipates as supply elasticity increases.

3 Model Implications

We now investigate several implications of our theoretical framework for the response of the neighborhood to housing demand and supply shocks. We provide a comparative statistics analysis to illustrate how aspects of the neighborhood and its real estate markets vary across two dimensions: 1) supply elasticity k, and 2) the degree of consumption complementarity in the utility of households η_c. Supply elasticity is a natural candidate for classifying the cross-section of housing markets, as it has been emphasized in the literature, in such work as Malpezzi and Wachter (2005) and Glaeser, Gyourko, and Saiz (2008), to help explain certain features of housing cycles, such as housing price volatility. Similarly, the degree of complementarity captures the agglomeration and spillover effects that lead to coordination among firms and industries that locate in one area, such as the financial industry in New York City, the technology sector in San Francisco, the Research Triangle in North Carolina, and the oil industry in Houston. As emphasized, for instance, by Dougal, Parsons, and Titman (2015), employers and/or workers can benefit from locating in close proximity to competitors, either from knowledge spillovers or from implicit insurance in labor markets.

While we have analytical expressions for most equilibrium variables, the key equilibrium

[7] It is straightforward to see from equation (19) that A^* remains finite a.s. as $k \to \infty$, allowing us to take the limit.

cutoff A^* needs to be numerically solved from the fixed-point condition in equation (19). We therefore analyze the equilibrium properties of A^* and other variables through a series of numerical illustrations. The parameters we choose for the numerical examples are provided below in Table 1. For the share of profits of commercial facility, we treat it is as being similar to capital and select the typical estimate of about $\alpha = .33$. For the Frisch elasticity of labor supply, we choose $\psi = 2.5$, which is within the typical range found in the literature. We set τ_ζ to be four-fold larger than τ_A to ensure that, with perfect information, the log housing price variance is montonically declining in supply elasticity, as observed empirically. We set $\lambda = 1.1$ to have commercial land be in elastic supply to avoid having convexity in its production function drive our results. We choose for the neighborhood fundamentals $A = \zeta = -.5$, though the qualitative patterns we highlight hold more generically for a wide range of shock values. In addition, we set the noise in the demand signal, Q, to 0.

τ_A	0.50	τ_ζ	2.00	τ_θ	0.20
τ_θ	0.20	τ_Q	1.00	η_c	0.50
α	0.33	ψ	2.50	k	0.50
λ	1.10	$\overline{A}$	0	$\overline{\zeta}$	0

Table 1: Numerical Parameters

3.1 Equilibrium Cutoff

We first analyze how the cutoff productivity for the marginal household to enter the neighborhood varies across housing supply elasticity and the degree of consumption complementarity.

Figure 1 demonstrates that with perfect information, as housing supply becomes more elastic, more households enter the neighborhood. This shifts down the cutoff productivity A^*, above which households enter the neighborhood, because housing is cheaper. As the degree of consumption complementarity increases, the cutoff productivity falls for the same realization of fundamentals since households benefit more from participating in the neighborhood, as they derive more of their consumption from trading with other households.

Informational frictions increase the population in the neighborhood by lowering the cutoff productivity, and this holds across both supply elasticity and the degree of complementarity. Since each household's productivity now acts as a private signal about the strength of the neighborhood, A, their type plays a dual role in their housing purchase decision. With consumption complementarity, households coordinate on housing prices as a public signal

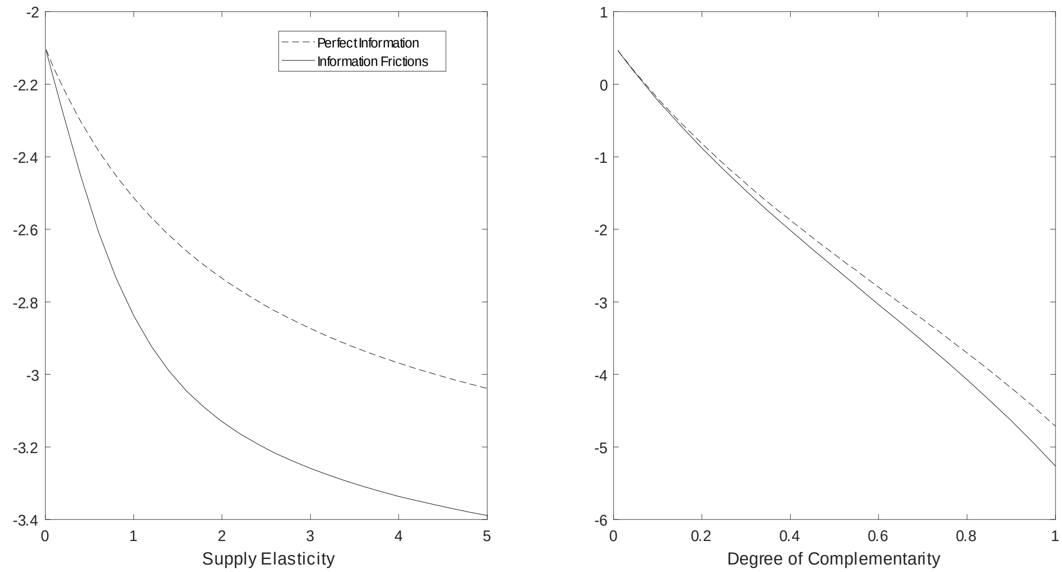

Figure 1: Equilibrium cutoff across housing supply elasticity (left) and degree of complementarity (right).

about the strength of the neighborhood, and uncertainty about A provides option value to households over their future trading opportunities. That the increased population with informational frictions relies on the complementarity between households can be seen from the limit as it goes to zero in the right panel in Figure 1.

3.2 Housing Supply Shock

We next characterize the response of the neighborhood to a supply or demand shock to understand how informational frictions could affect economic outcomes in the neighborhood. To facilitate our discussion, we also compute the price for commercial facilities at $t = 1$ as the marginal development cost at the optimal supply, $K^{\lambda-1}$. While there is no centralized trading of commercial facilities at $t = 1$, we can view the shadow price of producing another unit as its effective price. This price allows us to discuss comovement between residential and commercial real estate markets, as well as boom and bust cycles in commercial real estate.

We now consider the long-term impact of a housing supply shock to the neighborhood. One can think of this shock as a shift in the supply curve of housing because of, for instance, land speculation by builders as in Nathanson and Zwick (2017), or a non-fundamental shift in home builder expectations. To do this, we construct a non-linear impulse response or

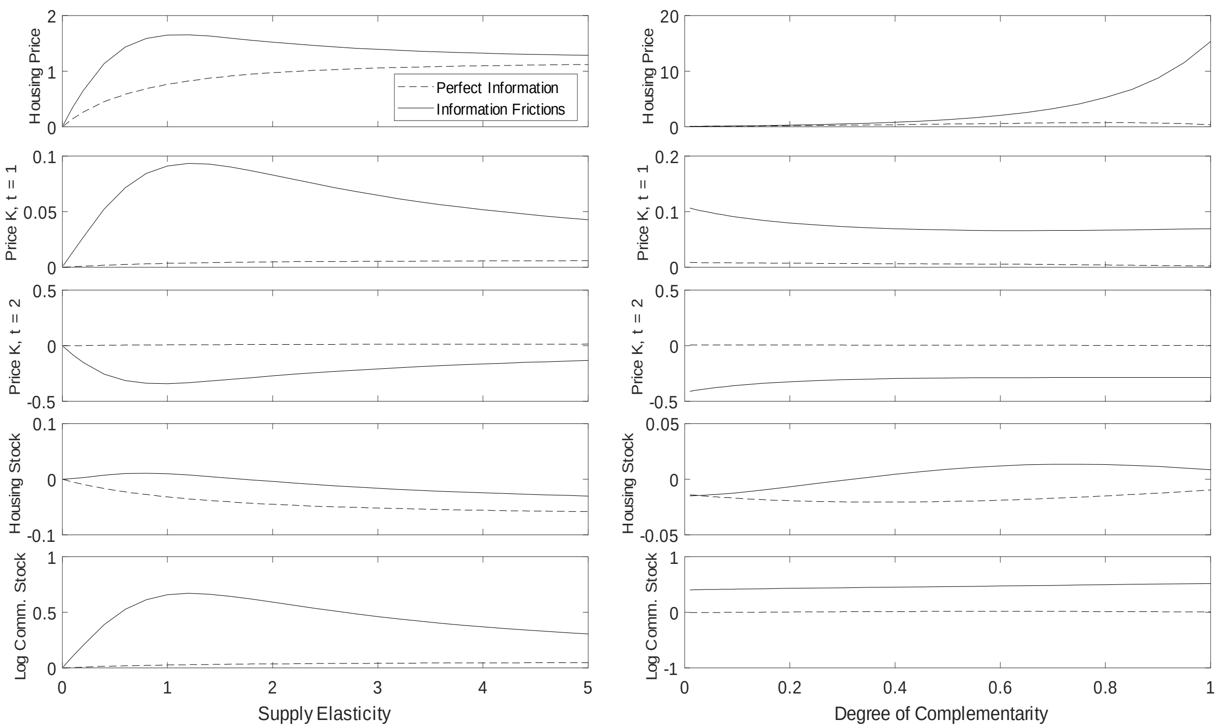

Figure 2: Neighborhood responses to a supply shock across supply elasticity (left) and degree of complementarity (right).

perturbation by taking the partial derivative of each outcome with respect to the building cost shock ζ. We measure the long-term impact of this supply shock as the partial derivative.

Figure 2 displays the response of the neighborhood to a negative supply shock across supply elasticity. The differences between the responses under perfect and imperfect information are pronounced. The housing price increases under perfect information with a negative supply shock, and the housing stock falls since the higher housing price discourages households from entering. With perfect information, the tighter supply has modest impacts on the commercial real estate prices at both $t = 1$ and $t = 2$ and on the supply of commercial facilities. The supply shock has a similar impact across complementarity as across supply elasticity, with a muted impact on the commerical real estate market.

In the presence of informational frictions, the negative supply shock is, in part, interpreted by households as a positive demand shock when they observe a higher housing price. Across supply elasticity, this overreaction is most severe at intermediate supply elasticities, since prices are fully revealing when supply is perfectly inelastic, and prices are completely uninformative about demand when supply is very elastic. As a result, the housing price and increase in housing stock is hump-shaped in supply elasticity, and peaks at an intermediate range. This increase in population and uncertainty about the strength of the neighborhood pushes up the price and supply of commercial facilities at $t = 1$, and also display hump-shaped patterns, and this price then mean-reverts with a U-shape at $t = 2$. The difference in the responses of the commercial real estate prices at $t = 1$ and $t = 2$ is that the price at $t = 1$ has a higher variance with informational frictions, which, by Jensen's Inequality, pushes up the stock of commercial land developed.

Across the degree of household consumption complementarity, the housing price increases with the supply shock monotonically. Interestingly, the housing stock panel reveals that a sufficient level of complementarity is necessary for higher housing prices as signals about demand to have a positive effect on household entry into the neighborhood. While informational frictions push up the commercial real estate price and supply at $t = 1$, and depress the commercial real estate price at $t = 2$, monotonically across complementarity, the difference across the degree of complementarity is modest. The hump-shape in the housing stock reflects the dampening effect of the escalation in the housing price on household entry at very high degrees of complementarity.

The stark differences in the responses to the supply shock in the presence of informational frictions suggests that there are long-term consequences to informational frictions. As expectations about household demand are corrected over time, any excess infrastructure in housing and commercial facilities from overbuilding will eventually have to be corrected. The differences in the responses between perfect and incomplete information in the figures above quantify the extent to which there must be a correction. Consequently, informational frictions can introduce a long-term misallocation of resources that are most severe, from Figure 2, at intermediate supply elasticities, such as in Las Vegas, and in areas with the highest degree of services complementarity, such as NYC and San Francisco.

3.3 Speculative Demand Shock

We now consider the impact of a speculative demand shock to the neighborhood. One can think of this shock as a shift in the demand curve of housing because of a nonfundamental shift in household and commercial developer expectations. This shift could arise for instance, because of optimism in the housing market, as in Ferreira and Gyourko (2011), Gao, Sockin and Xiong (2017), or Kaplan, Mitman, and Violante (2017), or noise in public information, as in Morris and Shin (2002) or Hellwig (2005). Specifically, we consider a demand shock to

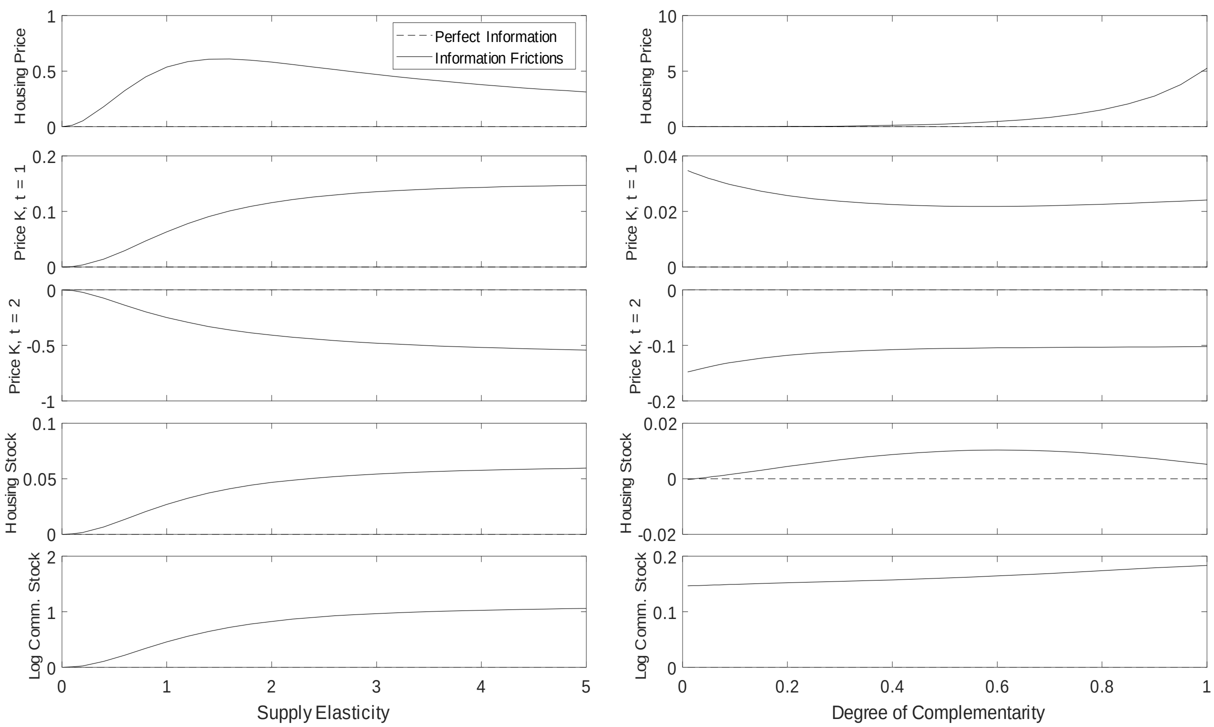

Figure 3: Neighborhood responses to a demand shock across supply elasticity (left) and degree of complementarity (right).

be a positive shock to the noise in the public signal Q about the neighborhood's common productivity A, which is a shock to expectations that is independent of the true demand fundamental, and the response to be the partial derivative of economic outcomes with respect to the shock. In the absence of informational frictions, when the common productivity is observable, then this demand shock has no impact on neighborhood outcomes.

Figure 3 illustrates the responses of the neighborhood across supply elasticity and the degree of comsumption complementarity to the speculative demand shock. Interestingly, the responses in the housing and commerical stocks and the commercial real estate price at $t = 1$ are monotonically increasing in supply elasticity, and the commercial real estate price at $t = 2$ is decreasing across supply elasticity. While supply shocks lead to the most confusion at intermediate elasticities, since this is when households and commercial developers pay the most attention to the housing price as a signal about household demand, demand shocks have the most pronounced impact in very elastic areas. This occurs because of substitution between the two sources of public information. As the housing price becomes less informative at higher elasticities, the public signal becomes relatively more informative, and households and commercial developers put more weight on it when forming their expectations. As such, a positive speculative demand shock has the most pronounced impact at high elasticities. This reflects a fundamental difference between how supply and demand shocks impact household learning across supply elasticity. The response of the housing price is humped-shaped since, though demand shocks do put upward pressure on the housing price, eventually housing prices are completely driven by supply shocks.

Similar to a supply shock, the impact of the speculative demand shock puts upward pressure on housing prices and the commercial stock, and both are monotonically increasing in the degree of complementarity. Commercial real estate prices are inflated at $t = 1$, and then reverse at $t = 2$, as a result of the overreaction of commercial developers to the noise in the public demand signal. At extremely high degrees of complementarity, the impact of a stronger neighborhood on housing prices dominates the additional benefits of trading with productive households, causing the housing stock response to peak in an intermediate range. To the extent that informational frictions distort these economic outcomes, there will be a correction in the long-term to their perfect-information analogues.

The above analysis suggests that shocks to the supply and demand sides of the housing market have different cross-sectional predictions for the joint cycles of residential and commercial real estate markets, across both supply elasticity and the degree of service complementarity. While the responses to the supply shock tends to be hump-shaped across supply elasticity, they are instead monotonic for the speculative demand shock, except in the housing price. This suggests that one must look at additional economic outcomes beyond the housing price in trying to disentangle the source of speculative shocks in housing markets. Interestingly, while the economic responses across the degree complementarity are distinct from those across supply elasticity, which provides a rich set of empirical predictions, they are similar for both speculative supply and demand shocks. This suggests that the cross-section of supply elasticity can be helpful in distinguishing between sources of speculation, whether it originates from the demand or supply side, in real estate markets. Importantly, there is not a tight link between the response of housing and commercial real estate markets to speculative shocks in the absence of informational frictions, suggesting a role for informational frictions in explaining the comovement observed between the two markets during the recent U.S. housing cycle, a role we next address.

3.4 The Recent U.S. Housing and Commercial Real Estate Cycles

We now return to one of the central motivations of our analysis, the joint dynamics of housing and commercial real estate markets. While a positive comovement is to be expected because the two markets have common fundamentals,[8] to the extent that the recent U.S. housing boom was not driven by strong economic fundamentals, it is difficult to explain the boom in commercial real estate that accompanied the housing boom across many cities before 2006. Theories linking housing and commercial real estate markets, such as the dual use of land, as in Rosen (1979) and Roback (1982), or the role of commercial land an input to household production, as in our benchmark setting with perfect information, have difficulty rationalizing why the commercial real estate market would rise with a housing bubble. The dual use theory would suggest that commercial real estate would be crowded out by a bubble in the housing market, and our model with perfect information, at best, predicts a modest comovement between the two markets in response to supply and speculative demand shocks. Gyourko (2009a) and Levitin and Wachter (2013) argue that a deteriorating composition of commercial real estate investors, and a similar relaxation of credit conditions as in the housing market, can help explain the joint boom in housing and commercial real estate markets, yet these theories are silent about why speculation occurred simultaneously in both markets.

The presence of realistic informational frictions offers an explanation as to how speculative booms, with roots in the residential real estate market, can spread to the commercial real estate market as well. Anchoring on higher signals from the housing market, such as housing prices, both households and commercial developers became optimistic about the prospects of their local economies. This shared optimism led both markets to experience not only dramatic price boom and bust cycles, but also overbuilding that would need to be absorbed once the prices burst and expectations corrected. Insofar as optimistic home buyers and commercial developers needed access to credit to act on their expectations during the boom, we view the relaxation of credit as a complementary force.

Our earlier thought experiments with supply and speculative demand shocks reveal that noise that breeds overoptimism in the housing market, a positive demand shock or a negative supply shock, can also spill over to the commercial real estate market through the learning

[8] Gyourko (2009) documents a historical correlation of approximately 0.4 between log housing values in residential and commercial real estate indices.

channel. This leads to a boom-bust in commercial real estate prices, and to an oversupply of commercial facilities. Consistent with the motivating narratives of Las Vegas and Phoenix, the remarkable housing boom also saw a pronounced expansion in hotels, apartments, and other commercial real estate properties whose construction continued even after the housing market peaked. Furthermore, from the previous two subsections, the supply of housing and commercial real estate respond similarly to speculative supply and demand shocks in the presence of informational frictions, whereas the response of commercial real estate is modest when there is perfect information.

Interestingly, Gyourko (2009a) documents that, while commercial real estate and equity REIT prices appreciated during the recent U.S. housing cycle, this appreciation appears to have been disconnected from rental rates, the cash flow fundamental for commercial real estate, which remained stable.[9] Our analysis suggests such a disconnect is rationalizable with informational frictions. Speculation in housing markets can lead to a boom-bust cycle in commercial real estate markets when commercial developers form expectations of the future demand by observing the housing market, and that this is likely to be more severe in areas with intermediate elasticities.

Consequently, our learning channel can provide a rationale in explaining the two synchronized real estate cycles. Importantly, our analysis emphasizes that the overbuilding during these cycles will have long-term consequences for neighborhoods as their economies converge to the levels that would have prevailed with perfect information.

4 Conclusion

In this paper, we introduce a model of information aggregation in housing and commercial real estate markets, and examine its implications for not only housing prices, but also economic outcomes such as neighborhood choice and the supply of commercial real estate. We provide empirical predictions for the expected response of neighborhoods to speculative demand and supply shocks across supply elasticity and the degree of service complementarity, and offer a rationale for the synchronized boom and bust cyles observed in the U.S. housing and commercial real estate markets during the 2000s. Our analysis highlights that

[9] This leads Gyourko (2009) to conclude that the appreciation in commercial real estate prices was driven by a decline in investor discount rates. Our analysis formalizes this idea by showing that, by impacting commercial investor expectations, a boom in housing markets could decrease the required rate of return in commercial real estates market compared to what would prevail under perfect information.

speculative booms in real estate markets can have long-term consequences on local economic conditions, as any overbuilding in housing and commercial real estate markets must be absorbed as the markets correct over time.

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Appendix Proofs of Propositions

A.1 Proof of Proposition 1

The first order conditions of household i's optimization problem in (2) respect to $C_i(i)$ and $C_j(i)$ at an interior point are:

$$C_i(i) \;:\; \frac{1-\eta_c}{C_i(i)} U\left(\{C_k(i)\}_{k\in\mathcal{N}};\mathcal{N}\right) = \theta_i p_i, \tag{9}$$

$$C_j(i) \;:\; \frac{\eta_c}{\int_{\mathcal{N}/i} C_j dj} U\left(\{C_k(i)\}_{k\in\mathcal{N}};\mathcal{N}\right) = \theta_i p_j, \tag{10}$$

where θ_i is the Lagrange multiplier for the budget constraint. Rewriting (10) as

$$\frac{\eta_c C_j}{\int_{\mathcal{N}/i} C_j dj} U\left(\{C_k(i)\}_{k\in\mathcal{N}};\mathcal{N}\right) = \theta_i p_j C_j$$

and integrating over $\mathcal{N}$, we arrive at

$$\eta_c U\left(\{C_k(i)\}_{k\in\mathcal{N}};\mathcal{N}\right) = \theta_i \int_{\mathcal{N}/i} p_j C_j dj.$$

Dividing equations (9) by this expression leads to $\frac{\eta_c}{1-\eta_c} = \frac{\int_{\mathcal{N}/i} p_j C_j(i) dj}{p_i C_i(i)}$, which in a symmetric equilibrium implies $p_j C_j(i) = \frac{1}{\Phi\left(\sqrt{\tau_\theta}(A-A^*)\right)} \frac{\eta_c}{1-\eta_c} p_i C_i(i)$. By substituting this equation back to the household's budget constraint in (2), we obtain:

$$C_i(i) = (1-\eta_c) e^{A_i} K_i^\alpha l_i^{1-\alpha}.$$

The market-clearing for the household's good requires that $C_i(i) + \int_{\mathcal{N}/i} C_i(j)\, dj = e^{A_i} K_i^\alpha l_i^{1-\alpha}$, which implies that $C_i(j) = \frac{1}{\Phi\left(\sqrt{\tau_\theta}(A-A^*)\right)} \eta_c e^{A_i} K_i^\alpha l_i^{1-\alpha}$.

The first order condition in equation (9) also gives the price of the good produced by household i. Since the household's budget constraint in (2) is entirely in nominal terms, the price system is only identified up to θ_i, the Lagrange multiplier. We therefore normalize θ_i to 1. It follows that:

$$p_i = \frac{1-\eta_c}{C_i(i)} U\left(\{C_j(i)\}_{j\in\mathcal{N}};\mathcal{N}\right) = \left(e^{A_i} l_i^{1-\alpha} K_i^\alpha\right)^{-\eta_c} \left(\frac{1}{\Phi\left(\sqrt{\tau_\theta}(A-A^*)\right)} \int_{\mathcal{N}/i} e^{A_j} l_j^{1-\alpha} K_j^\alpha dj\right)^{\eta_c}. \tag{11}$$

Furthermore, given equation (1), it follows since $C_i(i) = (1-\eta_c) e^{A_i} K_i^\alpha l_i^{1-\alpha}$ and $C_j(i) = \frac{1}{\Phi\left(\sqrt{\tau_\theta}(A-A^*)\right)} \eta_c e^{A_j} K_j^\alpha l_j^{1-\alpha}$ that:

$$\begin{aligned} U\left(\{C_k(i)\}_{k\in\mathcal{N}};\mathcal{N}\right) &= \left(e^{A_i} l_i^{1-\alpha} K_i^\alpha\right)^{1-\eta_c} \left(\frac{1}{\Phi\left(\sqrt{\tau_\theta}(A-A^*)\right)} \int_{\mathcal{N}/i} e^{A_j} K_j^\alpha l_j^{1-\alpha} dj\right)^{\eta_c} \\ &= p_i e^{A_i} K_i^\alpha l_i^{1-\alpha}, \end{aligned}$$

from substituting with the household's budget constraint at $t = 2$. The first-order conditions for household i's choice of l_i at an interior point is:

$$l_i^\psi = (1-\alpha)\theta_i p_i e^{A_i} \left(\frac{K_i}{l_i}\right)^\alpha = (1-\alpha) \frac{1-\eta_c}{C_i(i)} U\left(\{C_k(i)\}_{k\in\mathcal{N}};\mathcal{N}\right) e^{A_i} \left(\frac{K_i}{l_i}\right)^\alpha. \tag{12}$$

from equation (9). Imposing $C_i = (1-\eta_c) e^{A_i} K_i^\alpha l_i^{1-\alpha}$ in equation (12), and $U\left(\{C_k(i)\}_{k\in\mathcal{N}};\mathcal{N}\right) = p_i e^{A_i} K_i^\alpha l_i^{1-\alpha}$, it follows that:

$$\log l_i = \frac{1}{\psi+\alpha+(1-\alpha)\eta_c} \log(1-\alpha) + \frac{1}{\psi+\alpha+(1-\alpha)\eta_c} \log\left(\left(e^{A_i} K_i^\alpha\right)^{(1-\eta_c)} \left(\frac{\int_{\mathcal{N}/i} e^{A_j} K_j^\alpha l_j^{1-\alpha} dj}{\Phi\left(\sqrt{\tau_\theta}(A-A^*)\right)}\right)^{\eta_c}\right). \tag{13}$$

The optimal labor choice of household i, consequently, represents a fixed point problem over the optimal labor strategies of other households in the neighborhood.

Recogizing that $K_i = \left(\frac{\alpha p_i e^{A_i} l_i^{1-\alpha}}{R}\right)^{\frac{1}{1-\alpha}}$, we can substitute the price function p_i to arrive at:

$$\begin{aligned}\log K_i &= \frac{1}{1-(1-\eta_c)\alpha}\log\left(\left(e^{A_i} l_i^{1-\alpha}\right)^{1-\eta_c}\left(\frac{1}{\Phi\left(\sqrt{\tau_\theta}\left(A-A^*\right)\right)}\int_{\mathcal{N}/i} e^{A_j} l_j^{1-\alpha} K_j^{\alpha} dj\right)^{\eta_c}\right)\\ &\quad -\frac{1}{1-(1-\eta_c)\alpha}\log R+\frac{1}{1-(1-\eta_c)\alpha}\log\alpha, \qquad (14)\end{aligned}$$

which is a fixed-point problem for the optimal choice of commercial land.

Given the optimal labor supply of household i l_i and optimal demand for commercial land K_i jointly satisfy the functional fixed-point equations (13) and (14), let us conjecture for i for which $A_i \geq A^*$, so that $i \in \mathcal{N}$ is in the neighborhood, that:

$$\log l_i = l_0 + l_A A + l_s A_i + l_R \log R + l_\Phi \log \frac{\Phi\left(\left(1+\left(\alpha h_s+(1-\alpha) l_s\right)\right)\tau_\theta^{-1/2}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\left(A-A^*\right)\right)},$$

where R is the rental rate of commercial land, and that capital satisfies:

$$\log K_i = h_0 + h_A A + h_s A_i + h_R \log R + h_\Phi \log \frac{\Phi\left(\left(1+\left(\alpha h_s+(1-\alpha) l_s\right)\right)\tau_\theta^{-1/2}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\left(A-A^*\right)\right)},$$

Substituting these conjectures into the fixed-point recursion for labor, equation (13), we arrive, by the method of undetermined coefficients, at the coefficient restrictions:

$$\begin{aligned} cons &: (\psi+\alpha) l_0 = \log(1-\alpha) + \alpha h_0 + \frac{1}{2}\eta_c\left(1+\alpha h_s+(1-\alpha) l_s\right)^2 \tau_\theta^{-1},\\ A &: (\psi+\alpha) l_A = \alpha h_A + \left(1+\alpha h_s+(1-\alpha) l_s\right)\eta_c,\\ A_i &: \left(\psi+\alpha+(1-\alpha)\eta_c\right) l_s = (1-\eta_c)\left(1+\alpha h_s\right),\\ \log R &: (\psi+\alpha) l_R = \alpha h_R,\\ \Phi &: (\psi+\alpha) l_\Phi = \eta_c + \alpha h_\Phi. \end{aligned}$$

Similarly, substituting these conjectures into the fixed-point recursion for commercial land, equation (14), we arrive at the coefficient restrictions:

$$\begin{aligned} cons &: (1-\alpha) h_0 = (1-\alpha) l_0 + \frac{1}{2}\eta_c\left(1+\alpha h_s+(1-\alpha) l_s\right)^2 \tau_\theta^{-1} + \log\alpha,\\ A &: (1-\alpha) h_A = (1-\alpha) l_A + \eta_c\left(1+\alpha h_s+(1-\alpha) l_s\right),\\ A_i &: \left(1-(1-\eta)\alpha\right) h_s = (1-\eta_c)\left(1+(1-\alpha) l_s\right),\\ \log R &: (1-\alpha) h_R = (1-\alpha) l_R - 1,\\ \Phi &: (1-\alpha) h_\Phi = (1-\alpha) l_\Phi + \eta_c. \end{aligned}$$

We consequently have ten linear equations and ten coefficients, from which follows that:

$$
\begin{aligned}
l_0 &= \frac{1}{2}\frac{1}{1-\alpha}\frac{\eta_c}{\psi}\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\right)^2\tau_\theta^{-1}+\frac{\alpha}{1-\alpha}\frac{1}{\psi}\log\alpha+\frac{1}{\psi}\log(1-\alpha),\\
l_A &= \frac{1}{1-\alpha}\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\frac{\eta_c}{\psi},\\
l_s &= \frac{1-\eta_c}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c},\\
l_R &= -\frac{\alpha}{1-\alpha}\frac{1}{\psi},\\
l_\Phi &= \frac{1}{1-\alpha}\frac{\eta_c}{\psi},
\end{aligned}
$$

and

$$
\begin{aligned}
h_0 &= \frac{1}{2}\frac{1}{1-\alpha}\frac{1+\psi}{\psi}\eta_c\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\right)^2\tau_\theta^{-1}+\frac{1}{1-\alpha}\frac{\psi+\alpha}{\psi}\log\alpha+\frac{1}{\psi}\log(1-\alpha),\\
h_A &= \frac{1}{1-\alpha}\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\frac{1+\psi}{\psi}\eta_c,\\
h_s &= \frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c},\\
h_R &= -\frac{1}{1-\alpha}\frac{\psi+\alpha}{\psi},\\
h_\Phi &= \frac{1}{1-\alpha}\frac{1+\psi}{\psi}\eta_c,
\end{aligned}
$$

which confirms the conjectures.

Consequently, we find that, for $A_i \geq A^*$:

$$
\begin{aligned}
\log l_i &= \frac{1}{2}\frac{1}{1-\alpha}\frac{\eta_c}{\psi}\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\right)^2\tau_\theta^{-1}+\frac{\alpha}{1-\alpha}\frac{1}{\psi}\log\alpha+\frac{1}{\psi}\log(1-\alpha)\\
&\quad+\frac{1-\eta_c}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}A_i+\frac{1}{1-\alpha}\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\frac{\eta_c}{\psi}A\\
&\quad-\frac{\alpha}{1-\alpha}\frac{1}{\psi}\log R+\frac{1}{1-\alpha}\frac{\eta_c}{\psi}\log\frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\,(A-A^*)\right)},
\end{aligned}
$$

and:

$$
\begin{aligned}
\log K_i &= \frac{1}{2}\frac{1}{1-\alpha}\frac{1+\psi}{\psi}\eta_c\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\right)^2\tau_\theta^{-1}+\frac{1}{1-\alpha}\frac{\psi+\alpha}{\psi}\log\alpha+\frac{1}{\psi}\log(1-\alpha)\\
&\quad+\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}A_i+\frac{1}{1-\alpha}\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\frac{1+\psi}{\psi}\eta_c A\\
&\quad-\frac{1}{1-\alpha}\frac{\psi+\alpha}{\psi}\log R+\frac{1}{1-\alpha}\frac{1+\psi}{\psi}\eta_c\log\frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\,(A-A^*)\right)}.
\end{aligned}
$$

Substituting this functional form for the labor supply and commercial labor demand of household i into equation (11), the price of household $i's$ good then reduces to:

$$p_i = e^{\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\eta_c(A-A_i)+\frac{1}{2}\eta_c\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\right)^2\tau_\theta^{-1}} \left(\frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\left(A-A^*\right)\right)}\right)^{\eta_c}.$$

Finally, that $U\left(\{C_k(i)\}_{k\in\mathcal{N}};\mathcal{N}\right) = p_i e^{A_i}K_i^\alpha l_i^{1-\alpha}$, implies:

$$E\left[U\left(\{C_j(i)\}_{j\in\mathcal{N}};\mathcal{N}\right) - \frac{l_i^{1+\psi}}{1+\psi}\middle|\mathcal{I}_i\right] = \frac{\psi+\alpha}{1+\psi}E\left[p_i e^{A_i}K_i^\alpha l_i^{1-\alpha}\middle|\mathcal{I}_i\right].$$

A.2 Proof of Proposition 2

Substituting the optimal demand for commercial land K_i into the market-clearing condition for the rental market (5) reveals that the rental rate R is given by:

$$\begin{aligned}\log R &= \frac{1+\psi}{\psi+\alpha}A-(1-\alpha)\frac{\psi}{\psi+\alpha}\log K+\frac{1+\psi}{\psi+\alpha}\eta_c\log\frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\left(A-A^*\right)\right)}\\ &\quad +(1-\alpha)\frac{\psi}{\psi+\alpha}\log\frac{\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\left(A-A^*\right)\right)}+r_0,\end{aligned}$$

where K is the total amount of commercial land supplied by commercial developers at $t=2$, and:

$$r_0 = \log\alpha+\frac{1-\alpha}{\psi+\alpha}\log(1-\alpha)+\frac{1}{2}\left(\frac{1+\psi}{\psi+\alpha}\eta_c+(1-\alpha)\frac{\psi}{\psi+\alpha}(1-\eta_c)^2\right)\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\right)^2\tau_\theta^{-1}.$$

Since market-clearing in the market for commercial land imposes that $K\int_{i\in\mathcal{N}}di=\int_{i\in\mathcal{N}}K_i di$, it follows from equation (4) that the optimal choice of how much commercial land commercial developers create is given by equation (8) and constant k_0 is given by:

$$k_0 = \frac{\log\alpha+\frac{1-\alpha}{\psi+\alpha}\log(1-\alpha)+\frac{1}{2}\left(\frac{1+\psi}{\psi+\alpha}\eta_c+(1-\alpha)\frac{\psi}{\psi+\alpha}(1-\eta_c)^2\right)\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\right)^2\tau_\theta^{-1}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}$$

A.3 Proof of Proposition 3

When all households and builders observe A directly, there are no longer information frictions in the economy. Substituting for prices, the optimal labor and commercial land choices of household i, the realized rental rate R, and commercial land demand K_i from Proposition 2, the utility of household i at $t = 1$ from choosing to live in the neighborhood is then:

$$E\left[U_i|\mathcal{I}_i\right] = \frac{\psi+\alpha}{1+\psi} e^{u_0+u_A A+\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}A_i} \cdot \left(\frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\left(A-A^*\right)\right)}\right)^{u_\Phi}$$
$$\times\left(\frac{\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\left(A-A^*\right)\right)}\right)^{(1-\lambda)\frac{\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}},$$

where:

$$u_0 = \frac{1}{2}\frac{1}{1-\alpha}\frac{1+\psi}{\psi}\left(\lambda\eta_c\frac{1-\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}-\frac{\alpha\left(\lambda-1\right)}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\frac{\psi\left(1-\alpha\right)}{\psi+\alpha}\left(1-\eta_c\right)^2\right)\left(\frac{1+\psi}{\left(1-\alpha\right)\psi+\left(1+\alpha\psi\right)\eta_c}\right)^2\tau_\theta^{-1}$$
$$+\frac{1}{\psi}\frac{1-\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\left(\frac{\alpha}{1-\alpha}\left(1+\psi\right)\log\alpha+\lambda\log\left(1-\alpha\right)\right)$$
$$u_A = \frac{1}{1-\alpha}\frac{1+\psi}{\psi}\left(\frac{1+\psi}{\left(1-\alpha\right)\psi+\left(1+\alpha\psi\right)\eta_c}\eta_c-\left(\lambda-1\right)\frac{\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\right),$$
$$u_\Phi = \frac{\lambda\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\eta_c > 0.$$

Since the household with the critical productivity A^* must be indifferent to its neighborhood choice at the cutoff, it follows that $U_i - P = 0$, which implies:

$$e^{\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}A_i}\left(\frac{\Phi\left(\frac{(1+\psi)\tau_\theta^{-1/2}}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\left(A-A^*\right)\right)}\right)^{u_\Phi}\left(\frac{\Phi\left(\frac{(1+\psi)(1-\eta_c)\tau_\theta^{-1/2}}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\theta}\left(A-A^*\right)\right)}\right)^{\frac{(1-\lambda)\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}}$$
$$= \frac{1+\psi}{\psi+\alpha}e^{-u_0-u_A A}P,\ \ A_i = A^* \tag{15}$$

which implies the difference in benefit of living with more productive households must be offset by the differences in the cost of living in the neighborhood.

Fixing the critical value A^* and price P, we see that the LHS of equation (15) is increasing in monotonically in A_i, since $\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\left(1-\eta_c\right) > 0$. This confirms the optimality of the cutoff strategy that households for which $A_i \geq A^*$ enter the neighborhood, and households for which $A_i < A^*$ to live somewhere else. Since $A_i = A + \varepsilon_i$, it then follows that a fraction $\Phi\left(-\sqrt{\tau_\theta}\left(A^*-A\right)\right)$ enter the neighborhood, and a fraction $\Phi\left(\sqrt{\tau_\theta}\left(A^*-A\right)\right)$ choose to live somewhere else. As one can see, it is the integral over the idiosyncratic productivity shocks of households ε_i that determines the fraction of households in the neighborhood.

From the optimal supply of housing by builder i in the neighborhood (7), there exists a critical value ω^* :

$$\omega^* = -(1+k)\log P, \tag{16}$$

such that builders with productivity $\omega_i \geq \omega^*$ build houses. Consequently, a fraction $\Phi\left(-\sqrt{\tau_e}\left(\omega^* - \xi\right)\right)$ build households in the neighborhood.

Imposing market-clearing, it must be the case that:

$$\Phi\left(-\sqrt{\tau_\theta}\left(A^* - A\right)\right) = \Phi\left(-\sqrt{\tau_e}\left(\omega^* - \xi\right)\right).$$

Since the CDF of the normal distribution is montonically increasing, we can invert the above market-clearing conditions, and impose equation (16) to arrive at:

$$\log P = \frac{1}{1+k}\left(\sqrt{\frac{\tau_\theta}{\tau_e}}\left(A - A^*\right) - \xi\right).$$

Substituting for prices, we can express equation (15) as:

$$\begin{aligned} & e^{\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}+\frac{\sqrt{\tau_\theta/\tau_e}}{1+k}\right)A^*}\left(\frac{\Phi\left(\frac{(1+\psi)(1-\eta_c)\tau_\theta^{-1/2}}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}\right)^{\frac{(1-\lambda)\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}}\frac{\Phi\left(\frac{(1+\psi)\tau_\theta^{-1/2}}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)^{u_\Phi}}{\Phi\left(\frac{A-A^*}{\tau_\theta^{-1/2}}\right)^{u_\Phi}} \\ = \; & \frac{1+\psi}{\psi+\alpha}e^{\left(\frac{1}{1+k}\sqrt{\frac{\tau_\theta}{\tau_e}}-u_A\right)A-\frac{1}{1+k}\xi-u_0}. \end{aligned} \tag{17}$$

Taking the derivative of the log of the LHS of equation (17) with respect to A^*, which is a monotonic transformation of the LHS:

$$\begin{aligned} \frac{d\log LHS}{dA^*} = \; & u_\Phi\frac{1}{\tau_\theta^{-1/2}}\left(\frac{\phi\left(\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\frac{A-A^*}{\tau_\theta^{-1/2}}\right)} - \frac{\phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}\right) + \frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c} \\ & + \frac{1}{1+k}\sqrt{\frac{\tau_\theta}{\tau_e}} - \frac{1}{\tau_\theta^{-1/2}}\frac{(\lambda-1)\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\left(\frac{\phi\left(\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\frac{A-A^*}{\tau_\theta^{-1/2}}\right)} - \frac{\phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}\right). \end{aligned}$$

The term in parentheses are nonnegative by the properties of the normal CDF. The last term is nonpositive, since $\lambda > 1$, and attains its minimum at $A^* \to \infty$, from which follows, substituting for u_Φ, that:

$$\frac{d\log LHS}{dA^*} \to_{A^*\to\infty} \frac{1}{1+k}\sqrt{\frac{\tau_\theta}{\tau_e}} + \frac{\lambda\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}} > 0.$$

Consequently, since $\frac{d\log LHS}{dA^*} > 0$ when the last term attains its (nonpositive) minimum, it follows that $\frac{d\log LHS}{dA^*} > 0$. Therefore, $\log LHS$, and consequently LHS, since log is a monotonic transformation of the LHS, is monotonically increasing in A^*. Since the RHS of equation (17) is fixed for all A^*, it follows that the LHS and RHS of equation (17) intersect at most once. Therefore, the can be, at most, one cutoff equilibrium. Furthermore, since the LHS of equation (17) tends to 0 as $A^* \to -\infty$, and the RHS is nonnegative, it follows that a cutoff equilibrium always exist. Therefore, there exists a unique cutoff equilibrium in this economy.

Finally, it is straightforward to apply the Implicit Function Theorem to (17) to conclude that

$$\begin{aligned} \frac{dA^*}{dA} &= \frac{\frac{1}{1+k}\sqrt{\frac{\tau_\theta}{\tau_e}} - \frac{d\log LHS}{dA} - u_A}{\frac{d\log LHS}{dA^*}} \\ \frac{dA^*}{d\xi} &= -\frac{1}{1+k}\frac{1}{\frac{d\log LHS}{dA^*}} < 0, \end{aligned}$$

where:

$$\begin{aligned} \frac{d\log LHS}{dA} &= -u_\Phi \frac{1}{\tau_\theta^{-1/2}} \left(\frac{\phi\left(\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\frac{A-A^*}{\tau_\theta^{-1/2}}\right)} - \frac{\phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2} + \frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2} + \frac{A-A^*}{\tau_\theta^{-1/2}}\right)} \right) \\ &\quad + \frac{1}{\tau_\theta^{-1/2}}(\lambda-1)\frac{\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}} \left(\frac{\phi\left(\frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\frac{A-A^*}{\tau_\theta^{-1/2}}\right)} - \frac{\phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2} + \frac{A-A^*}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2} + \frac{A-A^*}{\tau_\theta^{-1/2}}\right)} \right). \end{aligned}$$

Notice that the nonpositive term in $\frac{d\log LHS}{dA}$ achieves its minimum at $A \to -\infty$, at which:

$$\frac{d\log LHS}{dA} \to_{A\to-\infty} \left((\lambda-1)\alpha(1-\eta_c) - \lambda\eta_c\right)\frac{\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}.$$

Then, as $A \to -\infty$, the numerator of $\frac{dA^*}{dA}$ converges to:

$$\begin{aligned} \frac{1}{1+k}\sqrt{\frac{\tau_\theta}{\tau_e}} - \frac{d\log LHS}{dA} - u_A \quad \to_{A\to-\infty} & -\frac{(1+\psi)\left(\frac{((\lambda-1)\alpha(1-\eta_c)-\lambda\eta_c)\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}} + \frac{1}{1-\alpha}\frac{1+\psi}{\psi}\eta_c\right)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c} \\ & + \frac{1}{1-\alpha}\frac{1+\psi}{\psi}\frac{(\lambda-1)\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}} + \frac{1}{1+k}\sqrt{\frac{\tau_\theta}{\tau_e}}, \end{aligned}$$

which is positive. Consequently:

$$\frac{dA^*}{dA} \geq 0.$$

Finally, we can rewrite equation (17) as:

$$e^{-\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}+\frac{1}{1+k}\sqrt{\frac{\tau_\theta}{\tau_e}}\right)s}\left(\frac{\Phi\left(\frac{(1+\psi)(1-\eta_c)\tau_\theta^{-1/2}}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}+\frac{s}{\tau_\theta^{-1/2}}\right)}{\Phi\left(\frac{s}{\tau_\theta^{-1/2}}\right)}\right)^{\frac{(1-\lambda)\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}}\frac{\Phi\left(\frac{(1+\psi)\tau_\theta^{-1/2}}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}+\frac{s}{\tau_\theta^{-1/2}}\right)^{u_\Phi}}{\Phi\left(\frac{s}{\tau_\theta^{-1/2}}\right)^{u_\Phi}}$$

$$= \frac{1+\psi}{\psi+\alpha}e^{-\lambda\frac{1}{1-\alpha}\frac{1+\psi}{\psi}\frac{1-\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}A-\frac{1}{1+k}\xi-u_0},$$

where $s=A-A^*$ determines the population that enter the neighborhood. It is straightforward to see, with some manipulation, that:

$$\frac{d\log LHS}{ds}=-\frac{1}{1+k}\sqrt{\frac{\tau_\theta}{\tau_e}}-\frac{\lambda}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\frac{1+\psi}{\psi+\alpha}<0,$$

and therefore:

$$\begin{aligned}\frac{ds}{d\xi} &= -\frac{\frac{1}{1+k}}{\frac{d\log LHS}{ds}}>0,\\ \frac{ds}{dA} &= -\frac{\lambda\frac{1}{1-\alpha}\frac{1+\psi}{\psi}\frac{1-\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}}{\frac{d\log LHS}{ds}}>0.\end{aligned}$$

Conequently, the population that enters, $\Phi\left(\sqrt{\tau_\theta}s\right)$, is increasing in A and ξ.

A.4 Proof of Proposition 4

We first conjecture that each household's housing purchasing, each builder's housing supply, and the housing price follow a cutoff strategy, and that learning by households and capital owners is linear as derived in the main text. Given our assumption about the sufficient statistic in prices, each household's posterior about A is Gaussian $A\,|\mathcal{I}_i\sim\mathcal{N}\left(\hat{A}_i,\hat{\tau}_A^{-1}\right)$ with conditional mean and variance

$$\begin{aligned}\hat{A}_i &= \bar{A}+\tau_A^{-1}\begin{bmatrix}1 & 1 & 1\end{bmatrix}\begin{bmatrix}\tau_A^{-1}+\tau_Q^{-1} & \tau_A^{-1} & \tau_A^{-1}\\ \tau_A^{-1} & \tau_A^{-1}+z_\xi^{-2}\tau_\xi^{-1} & \tau_A^{-1}\\ \tau_A^{-1} & \tau_A^{-1} & \tau_A^{-1}+\tau_\theta^{-1}\end{bmatrix}^{-1}\begin{bmatrix}Q-\bar{A}\\ z(P)-\bar{A}\\ A_i-\bar{A}\end{bmatrix}\\ &= \hat{\tau}_A^{-1}\left(\tau_A\bar{A}+\tau_Q Q+z_\xi^2\tau_\xi z(P)+\tau_\theta A_i\right),\\ \hat{\tau}_A &= \tau_A+\tau_Q+z_\xi^2\tau_\xi+\tau_\theta.\end{aligned}$$

We recognize that the conditional estimate of $\hat{A}_i$ of household i is increasing in its own productivity A_i. Similarly, the posterior for capital owners about A is Gaussian $A\,|\mathcal{I}^c\sim$

$\mathcal{N}\left(\hat{A}^c, \hat{\tau}_A^{c-1}\right)$, where

$$\begin{aligned}
\hat{A}^c &= \bar{A} + \tau_A^{-1}\left[\begin{array}{cc} 1 & 1 \end{array}\right]\left[\begin{array}{cc} \tau_A^{-1}+\tau_Q^{-1} & \tau_A^{-1} \\ \tau_A^{-1} & \tau_A^{-1}+z_\xi^{-2}\tau_\xi^{-1} \end{array}\right]^{-1}\left[\begin{array}{c} Q-\bar{A} \\ z\left(P\right)-\bar{A} \end{array}\right] \\
&= \hat{\tau}_A^{c-1}\left(\tau_A\bar{A}+\tau_Q Q + z_\xi^2\tau_\xi z\left(P\right)\right), \\
\hat{\tau}_A^c &= \tau_A+\tau_Q+z_\xi^2\tau_\xi.
\end{aligned}$$

This completes our characterization of learning by households and capital owners.

Let us now turn to the optimal commercial land decision of commercial developers. Since the posterior for $A-A^*$ of households is conditionally Gaussian, it follows that the expectations in the expression for K in Proposition 2 is a function of the first two conditional moments, $\hat{A}_i - A^*$ and $\hat{\tau}_A^{-1}$. Let:

$$F\left(\hat{A}^c - A^*, \hat{\tau}_A^c\right) = E\left[\left(\frac{e^{(A-A^*)}\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)^{\eta_c}}{\Phi\left(\frac{A-A^*}{\tau_\theta^{-1/2}}\right)^{\eta_c+\frac{\psi(1-\alpha)}{1+\psi}}\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)^{-\frac{\psi(1-\alpha)}{1+\psi}}}\right)^{\frac{1+\psi}{\psi+\alpha}}\middle|\,\mathcal{I}^c\right],$$

where $F\left(\hat{A}^c - A^*, \hat{\tau}_A\right) \geq 0$. Define $z = \frac{A-A^*}{\tau_\theta^{-1/2}}$, and the function $f\left(z\right)$:

$$f\left(z\right) = e^{\tau_\theta^{-1/2}z}\frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+z\right)^{\eta_c}}{\Phi\left(z\right)^{\eta_c}}\left(\frac{\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+z\right)}{\Phi\left(z\right)}\right)^{\frac{\psi}{1+\psi}(1-\alpha)}.$$

Then it follows that:

$$\begin{aligned}
\frac{1}{f\left(z\right)}\frac{df\left(z\right)}{dz} &= \tau_\theta^{-1/2}+\eta_c\left(\frac{\phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+z\right)}{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+z\right)}-\frac{\phi\left(z\right)}{\Phi\left(z\right)}\right) \\
&\quad +\frac{\psi}{1+\psi}\left(1-\alpha\right)\left(\frac{\phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+z\right)}{\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+z\right)}-\frac{\phi\left(z\right)}{\Phi\left(z\right)}\right).
\end{aligned}$$

Notice that $\frac{\phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+z\right)}{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+z\right)} - \frac{\phi(z)}{\Phi(z)}$ achieves its minimum as $z \to -\infty$. Applying

L'Hospital's Rule, it follows that the minimum of $\frac{1}{f(z)}\frac{df(z)}{dz}$ is given by:

$$\begin{aligned}
\lim_{z\to-\infty}\frac{1}{f(z)}\frac{df(z)}{dz} &= \tau_\theta^{-1/2}+\lim_{z\to-\infty}\eta_c\left(\frac{\frac{d}{dz}\phi\left(\frac{1+\psi}{\psi+\alpha+(1-\alpha)\eta_c}\tau_\theta^{-1/2}+z\right)}{\phi\left(\frac{1+\psi}{\psi+\alpha+(1-\alpha)\eta_c}\tau_\theta^{-1/2}+z\right)}-\frac{\frac{d}{dz}\phi(z)}{\phi(z)}\right)\\
&\quad+\frac{\psi}{1+\psi}(1-\alpha)\left(\frac{\frac{d}{dz}\phi\left(\frac{(1+\psi)(1-\eta_c)}{\psi+\alpha+(1-\alpha)\eta_c}\tau_\theta^{-1/2}+z\right)}{\phi\left(\frac{(1+\psi)(1-\eta_c)}{\psi+\alpha+(1-\alpha)\eta_c}\tau_\theta^{-1/2}+z\right)}-\frac{\frac{d}{dz}\phi(z)}{\phi(z)}\right)\\
&= \alpha\frac{1+\psi}{\psi+\alpha+(1-\alpha)\eta_c}(1-\eta_c)\tau_\theta^{-1/2}\\
&> 0
\end{aligned}$$

from which follows that $\frac{1}{f(z)}\frac{df(z)}{dz}\geq 0$ for all z, and therefore $\frac{df(z)}{dz}\geq 0$, since $f(z)\geq 0$. Consequently, since $f(z)^{\frac{1+\psi}{\psi+\alpha}}$ is a monotonic transformation of $f(z)$, it follows that $\frac{dF}{dx}(x,\hat{\tau}_A)$ ≥ 0 since this holds for all realizations of $A-A^*$. This establishes that the optimal choice of commercial land is increasing in the conditional estimate of commercial developers, $\hat{A}^c$, since $f(z)$ is increasing for each realization of z.

The optimal choice of commercial land then takes the form

$$\log K=\frac{1}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\log F\left(\hat{A}^c-A^*,\hat{\tau}_A^c\right)+\frac{\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}A^*+k_0.$$

Substituting this expression into the functional form for commercial land, the utility of household i is then given by:

$$\begin{aligned}
E[U_i|\mathcal{I}_i] &= \frac{\psi+\alpha}{1+\psi}e^{\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}A_i+\frac{\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\left(\log F\left(\hat{A}^c-A^*,\hat{\tau}_A^c\right)+\frac{1+\psi}{\psi+\alpha}A^*\right)+\frac{1}{1-\alpha}\frac{1+\psi}{\psi}\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\eta_c-\alpha\frac{1+\psi}{\psi+\alpha}\right)A^*+u_0}\\
&\quad\times E\left[\left(\frac{e^{\frac{1}{1-\alpha}\frac{\psi+\alpha}{\psi}\left(\frac{(1+\psi)\eta_c}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}-\alpha\frac{1+\psi}{\psi+\alpha}\right)(A-A^*)}\Phi\left(\frac{(1+\psi)\tau_\theta^{-1/2}}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)^{\eta_c}}{\Phi\left(\frac{A-A^*}{\tau_\theta^{-1/2}}\right)^{\eta_c-\alpha}\Phi\left(\frac{(1+\psi)(1-\eta_c)\tau_\theta^{-1/2}}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)^{\alpha}}\right)^{\frac{1+\psi}{\psi+\alpha}}\mid\mathcal{I}_i\right].
\end{aligned}$$

where u_0 is given in the proof of Proposition 3.

Since the posterior for $A-A^*$ of households is conditionally Gaussian, it follows that the expectations in the expressions above are functions of the first two conditional moments $\hat{A}_i-A^*$ and $\hat{\tau}_A^{-1}$. Let:

$$G\left(\hat{A}_i-A^*,\hat{\tau}_A\right)=E\left[\left(\frac{e^{\frac{1}{1-\alpha}\frac{\psi+\alpha}{\psi}\left(\frac{(1+\psi)\eta_c}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}-\alpha\frac{1+\psi}{\psi+\alpha}\right)(A-A^*)}\Phi\left(\frac{(1+\psi)\tau_\theta^{-1/2}}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)^{\eta_c}}{\Phi\left(\frac{A-A^*}{\tau_\theta^{-1/2}}\right)^{\eta_c-\alpha}\Phi\left(\frac{(1+\psi)(1-\eta_c)\tau_\theta^{-1/2}}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}+\frac{A-A^*}{\tau_\theta^{-1/2}}\right)^{\alpha}}\right)^{\frac{1+\psi}{\psi+\alpha}}\mid\mathcal{I}_i\right],$$

where $G\left(\hat{A}_i - A^*, \hat{\tau}_A\right) \geq 0$. Define $z = \frac{A-A^*}{\tau_\theta^{-1/2}}$, and the function $g(z)$:

$$g(z) = e^{\frac{1}{1-\alpha}\frac{\psi+\alpha}{\psi}\left(\frac{(1+\psi)\eta_c}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c} - \alpha\frac{1+\psi}{\psi+\alpha}\right)\tau_\theta^{-1/2} z} \frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2} + z\right)^{\eta_c}}{\Phi(z)^{\eta_c}} \frac{\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2} + z\right)^{-\alpha}}{\Phi(z)^{-\alpha}}.$$

Then it follows that:

$$\begin{aligned}
\frac{1}{g(z)}\frac{dg(z)}{dz} &= \frac{1}{1-\alpha}\frac{\psi+\alpha}{\psi}\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\eta_c - \alpha\frac{1+\psi}{\psi+\alpha}\right)\tau_\theta^{-1/2} \\
&\quad -\eta_c\left(\frac{\phi(z)}{\Phi(z)} - \frac{\phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+z\right)}{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+z\right)}\right) \\
&\quad +\alpha\left(\frac{\phi(z)}{\Phi(z)} - \frac{\phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+z\right)}{\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+z\right)}\right).
\end{aligned}$$

Notice that $\frac{\phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+z\right)}{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+z\right)} - \frac{\phi(z)}{\Phi(z)}$ achieves its minimum as $z \to -\infty$. Applying L'Hospital's Rule, it follows that the minimum of $\frac{1}{g(z)}\frac{dg(z)}{dz}$ is given by:

$$\begin{aligned}
\lim_{z\to-\infty}\frac{1}{g(z)}\frac{dg(z)}{dz} &= \frac{1}{1-\alpha}\frac{\psi+\alpha}{\psi}\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\eta_c - \alpha\frac{1+\psi}{\psi+\alpha}\right)\tau_\theta^{-1/2} \\
&\quad +\eta_c\lim_{z\to-\infty}\left(\frac{\frac{d}{dz}\phi\left(\frac{1+\psi}{\psi+\alpha+(1-\alpha)\eta_c}\tau_\theta^{-1/2}+z\right)}{\phi\left(\frac{1+\psi}{\psi+\alpha+(1-\alpha)\eta_c}\tau_\theta^{-1/2}+z\right)} - \frac{\frac{d}{dz}\phi(z)}{\phi(z)}\right) \\
&\quad +\alpha\lim_{z\to-\infty}\left(\frac{\frac{d}{dz}\phi(z)}{\phi(z)} - \frac{\frac{d}{dz}\phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+z\right)}{\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}+z\right)}\right) \\
&= \frac{1}{1-\alpha}\frac{\psi+\alpha}{\psi}\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\eta_c - \alpha\frac{1+\psi}{\psi+\alpha}\right)\tau_\theta^{-1/2} \\
&\quad +((1-\eta_c)\alpha - \eta_c)\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\theta^{-1/2}.
\end{aligned}$$

With some manipulation, the above expression collapsed to:

$$\lim_{z\to-\infty}\frac{1}{g(z)}\frac{dg(z)}{dz} = 0,$$

and it follows that $\frac{1}{g(z)}\frac{dg(z)}{dz} \geq 0$, and therefore $\frac{dg(z)}{dz} \geq 0$, since $g(z) \geq 0$. Consequently, since $g(z)^{\frac{1+\psi}{\psi+\alpha}}$ is a monotonic transformation of $g(z)$, it follows that $\frac{dG}{dx}(x, \hat{\tau}_A) \geq 0$ since this holds for all realizations of $A - A^*$.

Since the household with the critical productivity A^* must be indifferent to its neighborhood choice at the cutoff, it follows that $U_i - P = 0$, which implies

$$\begin{aligned} & e^{\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}A_i+\frac{\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\left(\log F\left(\hat{A}^c-A^*,\hat{\tau}_A^c\right)+\frac{1+\psi}{\psi+\alpha}A^*\right)+\frac{1}{1-\alpha}\frac{1+\psi}{\psi}\left(\frac{(1+\psi)\eta_c}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}-\alpha\frac{1+\psi}{\psi+\alpha}\right)A^*+u_0} G\left(\hat{A}_i - A^*, \hat{\tau}_A\right) \\ = & \frac{1+\psi}{\psi+\alpha}P, \;\; A_i = A^* \end{aligned} \tag{18}$$

which does not depend on the unobserved A or the supply shock ξ. As such, $A^* = A^*(\log P, Q)$. Furthermore, since $\hat{A}_i^*$ is increasing in A_i and $G\left(\hat{A}_i^* - A^*, \tau_A\right)$ is (weakly) increasing in $\hat{A}_i$, it follows that the LHS of equation (18) is (weakly) monotonically increasing in A_i, confirming the cutoff strategy assumed for households is optimal. Those for which the RHS is nonnegative enter the neighborhood, and those for which it is negative go choose to live elsewhere.

It then follows from market-clearing that

$$\Phi\left(-\sqrt{\tau_\theta}\left(A^* - A\right)\right) = \Phi\left(-\sqrt{\tau_e}\left(\omega^* - \xi\right)\right).$$

Since the CDF of the normal distribution is montonically increasing, we can invert the above market-clearing conditions, and impose equation (16) to arrive at

$$\log P = \frac{1}{1+k}\left(\sqrt{\frac{\tau_\theta}{\tau_e}}\left(A - A^*\right) - \xi\right)$$

from which follows that

$$z(P) = \sqrt{\frac{\tau_e}{\tau_\theta}}(1+k)\log P + A^* = A - \sqrt{\frac{\tau_e}{\tau_\theta}}\xi,$$

and therefore $z_\xi = \sqrt{\frac{\tau_\theta}{\tau_e}}$. This confirms our conjecture for the sufficient statistics in housing prices and that learning by households is indeed a linear updating rule. As a consequence, it follows we can express the conditional estimates of household i as:

$$\begin{aligned} \hat{A}_i &= \hat{\tau}_A^{-1}\left(\tau_A\bar{A} + \tau_Q Q + \frac{\tau_\theta}{\tau_e}\tau_\xi\left(\sqrt{\frac{\tau_e}{\tau_\theta}}(1+k)\log P + A^*\right) + \tau_\theta A_i\right), \\ \hat{\tau}_A &= \tau_A + \tau_Q + \frac{\tau_\theta}{\tau_e}\tau_\xi + \tau_\theta, \end{aligned}$$

and for capital owners as:

$$\begin{aligned} \hat{A}^c &= \hat{\tau}_A^{c-1}\left(\tau_A\bar{A} + \tau_Q Q + \frac{\tau_\theta}{\tau_e}\tau_\xi\left(\sqrt{\frac{\tau_e}{\tau_\theta}}(1+k)\log P + A^*\right)\right), \\ \hat{\tau}_A^c &= \tau_A + \tau_Q + \frac{\tau_\theta}{\tau_e}\tau_\xi. \end{aligned}$$

Substituting for prices, and simplifying A^* terms, we can express equation (18) as:

$$e^{\left(\frac{\lambda\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}+\frac{\sqrt{\tau_\theta/\tau_e}}{1+k}\right)A^*} G\left(\hat{A}^*-A^*,\hat{\tau}_A\right)F\left(\hat{A}^c-A^*,\hat{\tau}_A^c\right)^{\frac{\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}}=\frac{1+\psi}{\psi+\alpha}e^{\frac{1}{1+k}\sqrt{\frac{\tau_\theta}{\tau_e}}A-\frac{1}{1+k}\xi-u_0}, \tag{19}$$

where:

$$\begin{aligned}
\hat{A}_i^* &= \hat{\tau}_A^{-1}\left(\tau_A\bar{A}+\tau_Q Q+\frac{\tau_\theta}{\tau_e}\tau_\xi\left(\sqrt{\frac{\tau_e}{\tau_\theta}}(1+k)\log P+A^*\right)+\tau_\theta A^*\right),\\
\hat{A}^c &= \hat{\tau}_A^{c-1}\left(\tau_A\bar{A}+\tau_Q Q+\frac{\tau_\theta}{\tau_e}\tau_\xi\left(\sqrt{\frac{\tau_e}{\tau_\theta}}(1+k)\log P+A^*\right)\right).
\end{aligned}$$

Notice that the LHS of equation (19) is continuous in A^*. It follows, as $A^* \to -\infty$, that the LHS of equation (19) tends to:

$$\begin{aligned}
\lim_{A^*\to-\infty} LHS &= \left[e^{\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}A^*+u_0}E\left[e^{\frac{1+\psi}{\psi+\alpha}A}\middle|\mathcal{I}^c\right]^{\frac{\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}}E\left[e^{\frac{1+\psi}{\psi(1-\alpha)}\left(\frac{(1+\psi)\eta_c}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}-\alpha\frac{1+\psi}{\psi+\alpha}\right)A}\mid\mathcal{I}_i\right]\right]_{A^*\to-\infty}\\
&= 0.
\end{aligned}$$

Furthermore, by L'Hospital's Rule and the Sandwich Theorem, one also has that:

$$\lim_{A^*\to\infty} LHS = \infty.$$

Since the LHS of equation (19) is continuous in A^* and the RHS is fixed for all A^*, it follows that the LHS and RHS intersect at least once. Therefore, a cutoff equilibrium in the economy with informational frictions exists.

Finally, notice that, as $\tau_Q \nearrow \infty$, that $\hat{A}^c$ and $\hat{A}^i$ converge to A a.s., since $\hat{\tau}_A^c$, $\hat{\tau}_A^i \nearrow \infty$. Taking the limit along a sequence of τ_Q, it is straightforward to verify that equation (18) converges to equation (17), and therefore A^* converges to its perfect-information benchmark value. Taking similar limits for the expressions for capital and labor supply verify that they also converge to their perfect-information benchmark values, and therefore the noisy rational expectations cutoff equilibrium converges to the perfect-information benchmark economy as $\tau_Q \nearrow \infty$.