Forward Guidance and the Exchange Rate*

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May 2017

Abstract

I analyze the effectiveness of forward guidance policies in open economies, focusing on the role played by the exchange rate in their transmission. An open economy version of the "forward guidance puzzle" is shown to emerge. In partial equilibrium, the effect on the current exchange rate of an anticipated change in the interest rate does not decline with the horizon of implementation. In general equilibrium, the size of the effect is larger the longer is that horizon. Empirical evidence using U.S. and euro area data euro-dollar points to the presence of a *forward guidance exchange rate puzzle*: expectations of interest rate differentials in the near (distant) future have much larger (smaller) effects on the euro-dollar exchange rate than is implied by the theory.

JEL Classification: E43, E58, F41,

Keywords: forward guidance puzzle, uncovered interest rate parity, unconventional monetary policies, open economy New Keynesian model.

*I have benefited from comments by Jón Steinsson, Shogo Sakabe, and seminar participants at CREI-UPF. I thank Philippe Andrade for help with the data, and Christian Hoynck, Cristina Manea and Matthieu Soupre for excellent research assistance. I acknowledge financial support from the CERCA Programme/Generalitat de Catalunya and the Severo Ochoa Programme for Centres of Excellence.

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1 Introduction

The challenges posed by the global financial crisis to central bankers and the latter's increasing reliance on unconventional monetary policies (UMPs) has triggered an explosion of theoretical and empirical research on the effectiveness of such policies, i.e. policies that seek to substitute for changes in the short-term nominal rate –the instrument of monetary policy in normal times– when the latter attains its zero lower bound (ZLB). A prominent example of an unconventional policy adopted by several central banks in recent years is given by *forward guidance*, i.e. the attempt to influence current macroeconomic outcomes by managing expectations about the future path of the policy rate once the ZLB is no longer binding.

In the present paper I analyze the effectiveness of forward guidance policies in an *open economy*, focusing on the role played by the exchange rate in their transmission. As I discuss below, that transmission hinges to an important extent on the dependence of the exchange rate on the *undiscounted* sum of expected future interest rate differentials, as implied by the theory. Importantly, that relation relies only a (relatively) weak assumption: the existence at each point in time of some investors with access to both domestic and foreign bonds.

In the first part of the paper I analyze the effects of forward guidance on the exchange rate, under the assumption of constant prices (or, equivalently, when the induced effects of the interest rates and the exchange rate on output and prices are ignored). In that environment, the combination of uncovered interest parity with the long run neutrality of monetary policy yields a strong implication: the impact on the current exchange rate of an announcement of a future adjustment of the nominal rate is *invariant* to the timing of that adjustment.

Next I turn to the analysis of forward guidance policies in general equilibrium, i.e. allowing for feedback effects on output and prices, using a simple New Keynesian model of a small open economy. In general equilibrium, the size of the effect of forward guidance policies on the exchange rate is shown to be *larger* the *longer* is the horizon of implementation of a given adjustment in the nominal interest rate. A similar prediction applies to the effect on output and inflation. Both results are closely connected to the findings in the closed economy literature on the forward guidance puzzle, as discussed below.[1]

The same framework can be used to analyze the relation between the effectiveness of forward guidance policies and openness. I start by showing a simple condition under which the size of the effects of forward guidance policies on the exchange rate and other macro variables is invariant to the economy's openness. When that condition does not apply, the sign of that relation between openness and the size of the effects of forward looking policies can no longer be pinned down analytically. As an illustration, I show that under my baseline calibration the impact of forward guidance on some variables (output, the nominal exchange rate) increases with the degree of openness, whereas the opposite is true for some other variables (e.g., the real exchange rate).

Finally, I turn to the data, and provide some empirical evidence on the role of current and expected future interest rate differentials as a source of exchange rate fluctuations. Using data on euro-dollar exchange rate and market-based forecasts of interest rate differentials between the U.S. and the euro area, I provide evidence suggesting that expectations of interest rate differentials in the near (distant) future have much larger (smaller) effects than is implied by the theory. I refer to the apparent disconnect between theory and empirics on this issue as the *forward guidance exchange rate puzzle*, and discuss why the solutions to the forward guidance puzzle found in the closed economy literature are unlikely to apply in the presence of an exchange rate channel.

The remainder of the paper is organized a follows. Section 2 describes the related literature. Section 3 discusses the effects of forward guidance on the exchange rate in a partial equilibrium framework. Section 4 revisits that analysis in general equilibrium, using a small open economy New Keynesian model as a reference framework. Section 5 presents the empirical evidence. Section 6 summarizes and concludes.

2 Related Literature

The effectiveness of forward guidance and its role in the design of the optimal monetary policy under a binding ZLB was analyzed in Eggertsson and Woodford (2003) and Jung et al. (2005), using a standard New Keynesian model. Those papers emphasized the high effectiveness of forward guidance as a stabilizing instrument implied by the theory, at least under the maintained assumption of credible commitment.

More recently, the contributions of Del Negro et al. (2015), and McKay et al. (2016, 2017), among others, have traced the strong theoretical effectiveness of forward guidance to a "questionable" property of one of the

[1] See Del Negro et al. (2015), and McKay et al. (2016, 2017), among others,

key blocks of the New Keynesian model, the Euler equation, which in its conventional form implies that future interest rates are not "discounted" when determining current consumption. Formally, the dynamic IS equation (DIS) of the New Keynesian Model can be solved forward and written as:

$$\widehat{y}_t = -\frac{1}{\sigma}\sum_{k=0}^{\infty}\mathbb{E}_t\{\widehat{r}_{t+k}\}$$

where y_t is (log) output and $r_t \equiv i_t - \mathbb{E}_t\{r_{t+k}\}$ is the real interest rate. The $\widehat{}$ denotes deviations from steady state. Note that the predicted effect on output of a given anticipated change in the real interest rate is invariant to the horizon of implementation of that change. Furthermore, when combined with the forward-looking nature of inflation inherent to the New Keynesian Phillips curve, the previous property implies that the announcement of a future nominal rate adjustment of a given size and persistence is predicted to have a stronger effect on current output and inflation the longer is its horizon of implementation. That prediction, at odds with conventional wisdom, has been labeled the *forward guidance puzzle.*

Several potential "solutions" to the forward guidance puzzle have been proposed in the literature, in the form of modifications of the benchmark model that may generate some kind of discounting in the Euler equation, including the introduction of finite lives (Del Negro et al. (2015)), incomplete markets (McKay et al. (2016, 2017)), lack of common knowledge (Angeletos and Lian (2017)), and behavioral discounting (Gabaix (2017)).

The proposed solutions typically generate a "discounted" DIS equation of the form

$$\widehat{y}_t = \alpha\mathbb{E}_t\{\widehat{y}_{t+1}\} - \frac{1}{\sigma}\mathbb{E}_t\{\widehat{r}_t\}$$

where $\alpha \in (0,1)$, leading to the forward-looking representation

$$\widehat{y}_t = -\frac{1}{\sigma}\sum_{k=0}^{\infty}\alpha^k\mathbb{E}_t\{\widehat{r}_{t+k}\}$$

which implies that the effect of future interest rate changes on current output is more muted the longer is the horizon of their implementation.

Interestingly, and as discussed below, many of those solutions would not seem to be relevant in the presence of the exchange rate channel introduced below.

3 Forward Guidance and the Exchange Rate in Partial Equilibrium

Consider the asset pricing equations

$$1 = (1+i_t)\mathbb{E}_t\{\Lambda_{t,t+1}(P_t/P_{t+1})\} \tag{1}$$

$$1 = (1+i_t^*)\mathbb{E}_t\{\Lambda_{t,t+1}(\mathcal{E}_{t+1}/\mathcal{E}_t)(P_t/P_{t+1})\} \tag{2}$$

for all t, where i_t denotes the yield on a nominally riskless one-period bond denominated in domestic currency purchased in period t (and maturing in period $t+1$). i_t^* is the corresponding yield on an analogous bond denominated in foreign currency. $\mathcal{E}_t$ is the exchange rate, expressed as the price of foreign currency in terms of domestic currency. $\Lambda_{t,t+1}$ is the stochastic discount factor for an investor with access to the two bonds in period t.

Combining (1) and (2) we have

$$\mathbb{E}_t\{\Lambda_{t,t+1}(P_t/P_{t+1})\left[(1+i_t) - (1+i_t^*)(\mathcal{E}_{t+1}/\mathcal{E}_t)\right]\} = 0$$

In a neighborhood of a steady state, and to a first-order approximation, we can rewrite the previous equation as:

$$i_t = i_t^* + \mathbb{E}_t\{\Delta e_{t+1}\} \tag{3}$$

for all t, where $e_t \equiv \log\mathcal{E}_t$. This is the familiar uncovered interest parity condition.

Letting $q_t \equiv p_t^* + e_t - p_t$ denote the (log) real exchange rate, one can write the "real" version of (3) as:

$$q_t = r_t^* - r_t + \mathbb{E}_t\{q_{t+1}\}$$

where $r_t \equiv i_t - \mathbb{E}_t\{\pi_{t+1}\}$ is the real interest rate and $\pi_t \equiv p_t - p_{t-1}$ denotes (CPI) inflation, both referring to the home economy. r_t^* and with π_t^* are defined analogously for the foreign economy. Solving forward and taking the limit as $T \to \infty$,

$$q_t = \sum_{k=0}^{\infty} \mathbb{E}_t\{r_{t+k}^* - r_{t+k}\} + \lim_{T\to\infty} \mathbb{E}_t\{q_{t+T}\} \tag{4}$$

3.1 A Forward Guidance Experiment

Assume that at time t, the home central bank credibly announces an increase of the nominal interest rate of size δ, starting T periods from now and of duration D (i.e., from period $t+T$ to $t+T+D-1$), with no reaction expected from the foreign central bank. Furthermore, assume that the path of domestic and foreign prices remains unchanged (this assumption is relaxed below). Both the transitory nature of the intervention, as well as the assumption of long run neutrality of monetary policy, imply that $\lim_{T\to\infty} \mathbb{E}_t\{q_{t+T}\}$ should not change in response to the previous announcement. It follows from (4) that the real exchange rate will vary in response to the announcement by an amount given by

$$\widehat{q}_t = -D\delta$$

i.e. the real exchange rate appreciation at the time of the announcement is proportional to the *duration* and the *size* of the announced interest rate increase, but is *independent of its planned timing* (T). Thus, a D-period increase of the real interest rate 10 years from now is predicted to have the same effect on today's real exchange rate as an increase of equal size and duration to be implemented a year from now.

Once the interest rate increase is effectively implemented in period $t+T$, the exchange rate depreciates at a constant rate δ per period, i.e. $\Delta q_{t+T+k} = \delta$ for $k = 1, 2, ..D$ and stabilizes at its initial level once the intervention concludes, i.e. $q_{t+T+k} = q_t$ for $k = D+1, D+2, ...$

Figure 1 illustrates that prediction by displaying the implied path of the interest rate and the exchange rate when an interest rate rise of 1% (in annual terms) is announced at $t=0$, to be implemented at $T=4$ and lasting for $D=4$ periods.

It is worth noting at this point that some of the solutions to the forward guidance puzzle in the closed economy literature do not apply to the present case. More specifically, those solutions involve a "downward" adjustment in the relevant stochastic discount factor $\Lambda_{t,t+1}$, e.g. due to the risk of death (Del Negro et al. (2015) or the risk of future downward adjustment of consumption in the presence of borrowing constraints (McKay et al. (2016,2017)). The interest parity condition (3), on the other hand, holds independently of the size of the discount factor $\Lambda_{t,t+1}$. Intuitively, the reason is that (3) involves a "contemporaneous arbitrage" between two assets (whose payoffs are subject to the same discounting), as opposed to the "intertemporal arbitrage" associated with the consumer's Euler equation.

On the other hand, deviations from rational expectations that implied some discounting of subjective expectations, $\mathbb{E}_t^*\{q_{t+1}\}$, relative to rational expectations, i.e. $\mathbb{E}_t^*\{q_{t+1}\} = \alpha \mathbb{E}_t\{q_{t+1}\}$ as in the behavioral model of Gabaix (2017). In that case (4) could be rewritten

$$q_t = \sum_{k=0}^{\infty} \alpha^k \mathbb{E}_t\{r_{t+k}^* - r_{t+k}\}$$

with anticipated changes in the interest rate to be implemented in the distant future predicted to have a more muted effect on the real exchange rate than those implemented at a shorter horizon.

4 Forward Guidance and the Exchange Rate in General Equilibrium

Consider the (log-linearized) equilibrium conditions of a standard small open economy model with Calvo staggered price-setting, law of one price (producer pricing), and complete markets.[2]

$$\pi_{H,t} = \beta \mathbb{E}_t\{\pi_{H,t+1}\} + \kappa y_t - \omega q_t \tag{5}$$

[2] Detailed derivations of the equilibrium conditions can be found in Galí and Monacelli (2005) and Galí (2015, chapter 8) With little loss of generality I assume an underlying technology that is linear in labor input.

$$y_t = (1-\upsilon)c_t + \vartheta q_t \tag{6}$$

$$c_t = \mathbb{E}_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - \mathbb{E}_t\{\pi_{t+1}\}) \tag{7}$$

$$c_t = \frac{1}{\sigma} q_t \tag{8}$$

where $\pi_{H,t} \equiv p_{H,t} - p_{H,t-1}$ denotes domestic inflation, y_t is (log) output and c_t is (log) consumption. Equation (5) is a New Keynesian Phillips curve for the small open economy. Coefficient $\kappa \equiv \lambda\,(\sigma + \varphi)$ and $\omega \equiv \frac{\lambda(\sigma\eta-1)\upsilon(2-\upsilon)}{1-\upsilon}$ where $\upsilon \in [0,1]$ is an index of openness (equal the share of imported goods in domestic consumption in the steady state), $\sigma > 0$ is the (inverse) elasticity of intertemporal substitution, $\eta > 0$ is the elasticity of substitution between domestic and foreign goods, and $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} > 0$ is inversely related to the Calvo price stickiness parameter θ. (6) is the goods market clearing condition, with $\vartheta \equiv \upsilon\left(1 + \frac{1}{1-\upsilon}\right)\eta > 0$. (7) is the consumption Euler equation, with $\pi_t \equiv p_t - p_{t-1}$ denoting CPI inflation. (8) is the risk sharing condition, derived under the assumption of complete markets. The above specification of the equilibrium conditions assumes constant prices and real interest rates in the rest of the world, normalized to zero for notational ease (i.e. $r_t^* = p_t^* = 0$ all t). Also for simplicity I abstract from any non-policy shocks, with the analysis focusing instead on the effects of exogenous monetary policy changes. An extension allowing for other shocks is straightforward.

Note that (7) and (8) imply the real version uncovered interest parity analyzed in the previous section:

$$q_t = \mathbb{E}_t\{q_{t+1}\} - (i_t - \mathbb{E}_t\{\pi_{t+1}\}) \tag{9}$$

Furthermore, under the maintained assumption of full pass through, CPI inflation and domestic inflation are linked by

$$\begin{aligned} \pi_t &\equiv (1-\upsilon)\pi_{H,t} + \upsilon\Delta e_t \\ &= \pi_{H,t} + \frac{\upsilon}{1-\upsilon}\Delta q_t \end{aligned} \tag{10}$$

As emphasized in Galí and Monacelli (2005) the previous equilibrium conditions can be combined to obtain a system of two difference equations for domestic inflation $\pi_{H,t}$ and output y_t that is isomorphic to that of the closed economy, namely:

$$\pi_{H,t} = \beta\mathbb{E}_t\{\pi_{H,t+1}\} + \kappa_\upsilon y_t \tag{11}$$

$$y_t = \mathbb{E}_t\{y_{t+1}\} - \frac{1}{\sigma_\upsilon}(i_t - \mathbb{E}_t\{\pi_{H,t+1}\}) \tag{12}$$

where $\sigma_\upsilon \equiv \frac{\sigma}{1+(\sigma\eta-1)\upsilon(2-\upsilon)}$ and $\kappa_\upsilon \equiv \lambda\,(\sigma_\upsilon + \varphi)$ are now both functions of the open economy parameters (υ, η).

In addition, we have a proportional relation between the real exchange rate and output:

$$q_t = \sigma_\upsilon(1-\upsilon)y_t \tag{13}$$

In order to close the model, a description of monetary policy is required. I assume the simple rule

$$i_t = \phi_\pi \pi_{H,t} \tag{14}$$

where $\phi_\pi > 1$. It can be easily checked that in the absence of exogenous shocks the equilibrium in the above economy is (locally) unique and given by $\pi_{H,t} = y_t = q_t = i_t = 0$ for all t.

Consider next a forward guidance experiment analogous to the one analyzed in the previous section, but allowing for a response of output and inflation to changes in the interest rate and exchange rates.. More specifically, assume that at time 0, the home central bank credibly announces a *nominal* interest rate increase of size δ, starting in period T and of duration D (i.e., implemented from period T to $T+D-1$). Furthermore, the central bank commits to keeping the nominal interest rate at its initial level ($i_t = 0$) until period T, independently of the evolution of inflation. At time $T+D$, once the experiment is over, it restores the interest rate rule (14) and, with it, the initial equilibrium. I use (11), (12) and (13) to determine the response of output, domestic inflation and the real exchange rate to that forward guidance experiment. Given the response of $\pi_{H,t}$ and q_t, (10) can be used to back out the response of CPI inflation, π_t. The latter can then be used to derive the response of the (consumption) price level, which combined with the relation $e_t = q_t + p_t$ allows one to derive the response of the nominal exchange rate.

Figure 2 displays the response of interest rates, the exchange rate, output, and inflation, to the above experiment, assuming $\delta = 1$ and $D = 1$, under three alternative time horizons for implementation: $T = \{1, 2, 4\}$. The parameters of the model are calibrated as follows: $\beta = 0.99$, $\upsilon = 0.4$, $\sigma = 1$, $\eta = 2$, $\theta = 0.75$, and $\varphi = 5$. Note that a version of the "forward guidance puzzle" for the open economy emerges: the longer is the horizon of implementation, the larger is the impact of the announcement on the real exchange rate and, hence, on output and inflation. As emphasized by McKay el at. (2016), the reason for the amplification has to do with the fact that inflation depends on current and expected future output, combined with the property that the longer is the implementation of a given interest rise the more persistent the output response. It follows that the longer is the implementation horizon of a given change in the *nominal* rate the larger will be the response of the *real* rate –and hence of output and the real exchange rate– between the time of the announcement and that of policy implementation.

A similar phenomenon obtains when one varies the *duration* of the nominal rate adjustment, while keeping the time of implementation constant. This is illustrated in Figure 3, which displays the responses of a number of variables to the announcement of a nominal rate increase implemented at $T = 2$ for $D = \{1, 2, 3\}$, *with the size of the responses normalized by the duration of the policy intervention*. As the figure makes clear, and in contrast with the partial equilibrium case, the size of the response is more than proportional to the duration of the intervention (for any given starting period for the latter). This is due to the fact that longer interventions involve interest rate changes that occur further into the future and thus have stronger effects, as shown above.

4.1 Forward Guidance and Openness

Is forward guidance more effective at stimulating output in more open economies? How does openness influence the response of the real and nominal exchange rates? In order to answer that question one should first look at the impact of a change in the openness parameter υ on the coefficients of equations (11), (12), and (13). The following lemma is useful in that regard:

Lemma: $\left(\frac{\partial \sigma_\upsilon}{\partial \upsilon}\right)(\sigma\eta - 1) < 0$ if $\sigma\eta \neq 1$, and $\frac{\partial \sigma_\upsilon}{\partial \upsilon} = 0$ if $\sigma\eta = 1$.

The case of $\sigma\eta = 1$ provides a useful, albeit unlikely realistic, benchmark to answer the previous questions. In that case, $\sigma_\upsilon = \sigma$ is independent of υ (and η as well). As a result the response of output and domestic inflation to a forward guidance experiment is invariant to the degree of openness. Since $q_t = \sigma(1 - \upsilon)y_t$ in that case, the response of the real exchage rate will be more muted the more open the economy is. On the other hand, (10) and (13) jointly imply $p_t = p_{H,t} + \sigma\upsilon y_t$ in that case. Thus, the decline of CPI inflation in response to an announced future increase in the nominal rate will be larger than the fall in domestic inflation the more open is the economy, since it will be amplified by the real appreciation of the exchange rate. Accordingly, in that case $e_t = q_t + p_t = p_{H,t} + \sigma y_t$, i.e. the response of the nominal exchange rate will be invariant to the degree of openness of the economy.

How is the previous logic affected when $\sigma\eta \neq 1$? Consider, for concreteness, the case of $\sigma\eta > 1$, which is arguably the empirically relevant one. In that case, the lemma above and (12) imply that, conditional on domestic inflation, the response of output to the announcement of a future increase in the nominal rate will be larger in a more open economy. On the other hand, applying the lemma to (11) implies that, conditional of the path of output, the response of domestic inflation will be more muted in a more open economy. The impact of openness on the total effect, i.e. taking into account the feedback from inflation to output, is thus in principle ambiguous.

Figure 4 shows the responses of the key macro variables to the forward guidance experiment described above under the assumption that $T = 2$ and $D = 2$ and for $\upsilon \in [0.2, 0.4, 0.6]$. The figure illustrates that the impact of openness doesn't have a uniform sign: it amplifies the response of some variables, like output and domestic inflation, but it mutes the response of others, e.g. the real exchange rate. The proximate cause for the latter's more muted response is the smaller response of *expected* CPI inflation.

5 Interest Rate Expectations and the Exchange Rate: Does the Horizon Matter?

Consider the following decomposition of the relation between the current real exchange rate and expected future real interest rate differentials:

$$\begin{aligned} q_t &= \sum_{k=0}^{\infty} \mathbb{E}_t\{r^*_{t+k} - r_{t+k}\} + \lim_{T\to\infty} \mathbb{E}_t\{q_{t+T}\} \\ &= q_t^S(M) + q_t^L(M) + \lim_{T\to\infty} \mathbb{E}_t\{q_{t+T}\} \end{aligned} \tag{15}$$

where $q_t^S(M) \equiv \sum_{k=0}^{M-1} \mathbb{E}_t\{r^*_{t+k} - r_{t+k}\}$ and $q_t^L(M) \equiv \sum_{k=M}^{\infty} \mathbb{E}_t\{r^*_{t+k} - r_{t+k}\}$, for any $M > 0$. Note that $q_t^S(M)$ captures the predicted effect on the real exchange rate of expected interest rate differentials over the *short run* (i.e. the next M periods), while $q_t^L(M)$ captures the corresponding predicted effect of expected interest rate differentials at a longer horizon (i.e. beyond the next M periods). The absence of discounting in (15) implies that a change in $q_t^S(M)$ should have the same effect on the real exchange rate as a commensurate change in $q_t^L(M)$. Furthermore, the size of that effect should be "one-for-one" in both cases. The previous prediction lies behind some of the theoretical results uncovered above.

In the present section I seek to evaluate whether the previous prediction holds empirically, against the (natural) alternative hypothesis that changes in expected interest rate differentials further into the future have a more muted effect on the real exchange rate than those expected to occur in a less distant future.

In order to evaluate the previous hypotheses I need to construct empirical counterparts to $q_t^S(M)$ and $q_t^L(M)$. Note that under the expectations hypothesis, the annualized nominal yield on a M-period bond is given by

$$i_t(M) = \frac{J}{M} \sum_{k=0}^{M-1} \mathbb{E}_t\{i_{t+k}\}$$

where J is the number of periods per year (e.g. 12 in the case of monthly data) and where $i_t(1) = i_t$. Subtracting (annualized) expected inflation between t and $t+M$ from both sides of the previous equation we can write:

$$r_t(M) = \frac{J}{M} \sum_{k=0}^{M-1} \mathbb{E}_t\{r_{t+k}\}$$

An analogous expression holds for foreign bonds. Thus, it follows that,

$$q_t^S(M) = \frac{M}{J}[r_t^*(M) - r_t(M)] \tag{16}$$

I construct a measure of $q_t^S(M)$ for the euro-dollar real exchange rate, using *monthly* data on German and U.S. government bond (zero coupon) yields with 2, 5 and 10 year maturity (thus corresponding to $M \in \{24, 60, 120\}$, combined with monthly measures of expected inflation over the same three horizons derived from inflation swaps.

In order to obtain an empirical counterpart to $q_t^L(M)$ I assume that $\sum_{k=M_L}^{\infty} \mathbb{E}_t\{r^*_{t+k} - r_{t+k}\} \simeq 0$ beyond a sufficiently long horizon M_L. In that case, I can use the approximate expression

$$q_t^L(M) \simeq q_t^S(M_L) - q_t^S(M)$$

I construct an empirical measure for $q_t^L(M)$ by setting $M_L = 12 \times 30 = 360$ and using relation (16) to measure $q_t^S(M_L)$ and $q_t^S(M)$, together with 30-year government debt yields and market-based measures of expected inflation over the next 30 years, in addition to the data mentioned above.

Finally, I need to take some stance regarding the term $\lim_{T\to\infty} \mathbb{E}_t\{q_{t+T}\}$ in the decomposition of the real exchange rate. In the previous sections I analyzed the response of the economy to an *exogenous* monetary policy intervention in the form of forward guidance. In that context the assumption of no permanent response of the real exchange rate to that intervention could be viewed as a plausible assumption, and one consistent with a broad class of models, including those allowing for short-run monetary non-neutralities. But, more generally, there are a variety of reasons why the real exchange rate may vary in the long run, thus violating unconditional PPP. Accordingly, expectations about the long real exchange rate, $\lim_{T\to\infty}\mathbb{E}_t\{q_{t+T}\}$, may also vary over time. I follow two alternative approaches in order to deal with this issue. Under the first approach, which I refer to as *level specification*, I adopt a smooth, parsimonious model for the expected long run real exchange rate, by assuming that it can be approximated by a low order polynomial function of time. The resulting model can thus be written as:

$$q_t = q_t^S(M) + q_t^L(M) + f(t)$$

In practice, a quadratic function seems to capture well the low frequency movements of the euro-dollar real exchange rate unaccounted for by expected interest rate differentials, so below I set $f(t) = \alpha_0 + \alpha_1 t + \alpha_2 t^2$.

Under the second approach, I exploit the fact that under rational expectations $\lim_{T\to\infty}\mathbb{E}_t\{q_{t+T}\}$ is a martingale process. Thus, taking first differences on both sides of (15), we obtain the equation

$$\Delta q_t = \Delta q_t^S(M) + \Delta q_t^L(M) + \xi_t$$

where $\xi_t \equiv \lim_{T\to\infty}(\mathbb{E}_t\{q_{t+T}\} - \mathbb{E}_{t-1}\{q_{t+T}\})$. I refer to the relation above as the first-difference specification. One advantage of this second approach is its immunity to the potential "spurious regression" problem if q_t, $q_t^S(M)$ and $q_t^L(M)$ had a non-stationary component not fully captured by the deterministic function $f(t)$ (e.g. a unit root).

5.1 Findings: Level Specification

Using monthly U.S. and euro area data over the period 2004:7-2016:12, I estimate the empirical equation

$$q_t = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + \gamma_S q_t^S(M) + \gamma_L q_t^L(M) + \varepsilon_t$$

using OLS. Table 1 reports the estimated coefficients γ_S and γ_L for $M \in \{24, 60, 120\}$. With the exception of γ_L when $M = 120$, all the estimated coefficients are positive and highly significant. With the exception of γ_S for $M \in \{60, 120\}$, the estimated coefficients are significantly different from one, thus rejecting a central prediction of the model above. Most importantly, the null $\gamma_S = \gamma_L$ is easily rejected for all specifications (as reflected in reported p value for the test of that null), with the estimates of γ_S being in all cases an order of magnitude larger than those of γ_L. In words: the real exchange rate appears to respond much more strongly (weakly) than implied by the theory to variations in expected real interest rate differentials in the near (distant) future. A look at the pattern of γ_S estimates across specifications suggests that the strength of the response to expected interest rate differentials diminishes with the horizon, with the response to $q_t^L(24)$ being by far the largest one (3 times larger than implied by the theory). I refer to this apparent disconnect between theory and empirics as the *forward guidance exchange rate puzzle*.

Based on the R^2 value, reported on the last column of Table 1, the specification for $M = 24$ appears to provide the best fit. The goodness-of-fit is illustrated in Figure 5(A) which displays q_t together with its corresponding fitted value, based on the estimated regression with $M = 24$. Figure 5(B) makes clear that such good fit is not just the result of the deterministic components, by showing the estimated non-deterministic components $\widehat{q}_t \equiv q_t - (\alpha_0 + \alpha_1 t + \alpha_2 t^2)$ and $\widehat{q}_t^* \equiv \gamma_S q_t^S(M) + \gamma_L q_t^L(M)$. Figure 5(C) further decomposes $\widehat{q}_t^*$ into $\widehat{q}_t^{*,S} \equiv \gamma_S q_t^S(M)$ and $\widehat{q}_t^{*,L} \equiv \gamma_L q_t^L(M)$, and points to the dominant role of expected interest rate differentials less than two years ahead in accounting for the higher frequency fluctuations in $\widehat{q}_t$, while expected differentials beyond the two-year horizon show a visible correlation with lower frequency fluctuation in the same variable.

Motivated by the previous findings, and in order to assess the relative role played by the different horizons of expected interest rate differentials in accounting for real exchange rate fluctuations, I estimate the equation

$$q_t = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + \gamma_1 q_t^B(0,24) + \gamma_2 q_t^B(24,60) + \gamma_3 q_t^B(60,120) + \gamma_4 q_t^B(120,360) + \varepsilon_t \qquad (17)$$

where $q_t^B(M_l, M_u) \equiv q_t^S(M_l) - q_t^S(M_s)$, and $q_t^S(0) \equiv 0$. The bottom panel of Table one reports the OLS estimates of coefficients γ_1,...γ_4 using the data set described above. Note that only γ_1 and γ_4 are found to be significant, with the null $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 1$ easily rejected at conventional significance levels. Once again the coefficient on expected interest rate differentials over the next two years is more than three times larger than the unit value implied by the model above, while the point estimates of the remaining coefficients are between five and ten times smaller. Figure 6 shows the fit of the previous empirical model in a way analogous to

Figure 5(C), where now $\widehat{q}_t^{*,S} \equiv \gamma_1 q_t^B(0,24)$ and $\widehat{q}_t^{*,L} \equiv \gamma_4 q_t^B(120,360)$, which correspond to the two statistically significant components in the estimated equation above. In a way consistent with the findings above expected interest rate differentials less than two years ahead appear to account for a large fraction of the higher frequency fluctuations in $\widehat{q}_t$. In addition, expected differentials beyond the 10 horizon comove with the real exchange rate at lower frequencies and appear to explain the apparent slight upward trend in the dollar starting about 2010, coinciding with the beginning of the U.S. recovery, and with a euro area economy which has fallen behind the U.S. cycle since then.

5.2 Findings: First-Difference Specification

Table 2 reports the OLS estimates γ_S and γ_L based on the first-difference specification

$$\Delta q_t = \alpha_0 + \gamma_S \Delta q_t^S(M) + \gamma_L \Delta q_t^L(M) + \xi_t \tag{18}$$

for $M \in \{24, 60, 120\}$. Note that some of the key findings obtained using the level specification re-emerge here. Thus, the null $\gamma_S = \gamma_L = 1$ is systematically rejected, with the estimates of γ_S being significantly larger than those of γ_L, and with the latter being close to zero. Note also that the estimate of γ_S is larger than one (though not significantly so) in the case of $M = 24$, but below one for the two longer horizons, suggesting that the real exchange rate is particularly sensitive (possibly overly so) to variations in forecasts of real interest rate differentials over a relatively short horizon. The previous property is also reflected in the "declining pattern" of the estimated coefficients of the first-differenced version of the augmented equation (17), reported in the bottom panel of Table 2.

A possible concern with the use of OLS to estimate equation (18) is the potential correlation between the error term and the regressors. That correlation would arise if there was a systematic response of current and anticipated real interest rate differentials to shocks that are behind the unit root in the real exchange rate. In that case, one could in principle use any lagged variable correlated with $\Delta q_t^S(M)$ and $\Delta q_t^L(M)$ as an instrument, given that by construction $\mathbb{E}\{\xi_t Z_{t-1}\} = 0$.[3] Unfortunately, the very definition of $q_t^S(M)$ and $q_t^L(M)$ as "expectational variables" makes their first differences to be largely unpredictable, so good instruments are hard to find. That observation notwithstanding, Table 3 reports IV estimates of γ_S and γ_L in (18) (and its extension corresponding to (17) in first differences) using as instruments four lags of Δq_t and $q_t^S(24)$. The coefficient estimates display large standard errors and are, with few exceptions, insignificantly different from zero. Yet, the point estimates also display the "declining pattern" uncovered above and the $\gamma_S = \gamma_L = 1$ null is systematically rejected at negligible significance levels.

6 Concluding Comments

The present paper has analyzed the effectiveness of forward guidance policies in open economies, focusing on the role played by the exchange rate in their transmission. Uncovered interest parity implies that the current exchange rate is determined by current and expected future interest rate differentials, *undiscounted*. Accordingly, in partial equilibrium (i.e. ignoring the feedback effects on inflation) the effect on the current exchange rate of a given future change in the interest rate does not decline with the horizon of its implementation.

In general equilibrium, and using a simple New Keynesian model of a small open economy as a reference framework, I show that the size of the effect of forward guidance policies on the current exchange rate, as well as on output and inflation, is larger the longer is the horizon of implementation of the announced policies. Under my baseline calibration, the size of the effects of forward guidance policies on some variables (output, nominal exchange rate) is increasing in the degree of openness, but it is decreasing for some other variables (e.g. real exchange rate).

Finally, and using data on the euro-dollar exchange rate and market-based forecasts of interest rate differentials between the U.S. and the euro area, I provide evidence that expectations of interest rate differentials in the near (distant) future have much larger (smaller) effects than is implied by the theory, an observations which I refer to as the *forward guidance exchange rate puzzle.*

[3] This follows from the fact that $\mathbb{E}_{t-1}\{\xi_t\} = 0$.

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Table 1. Expected Interest Differentials and the Real Exchange Rate

Levels, Euro-dollar, *2004:8-2016:12*

(A)	$q_t^S(M)$	$q_t^L(M)$	p	R^2
M=24	3.11^{**}	0.16^{**}	0.00	0.88
	(0.19)	(0.02)		
M=60	1.66^{**}	0.11^{**}	0.00	0.81
	(0.15)	(0.03)		
M=120	0.87^{**}	0.06	0.00	0.74
	(0.13)	(0.04)		

(B)	$q_t^S(24)$	$q_t^B(24,60)$	$q_t^B(60,120)$	$q_t^L(120)$		R^2
	3.26^{**}	-0.29	0.36	0.15^{**}	0.00	0.88
	(0.22)	(0.32)	(0.25)	(0.03)		

Table 2. Expected Interest Differentials and the Real Exchange Rate

First-differences, Euro-dollar, *2004:9-2016:12*

(A)	$\Delta q_t^S(M)$	$\Delta q_t^L(M)$			p	R^2
M=24	1.43^{**} (0.29)	0.09^{*} (0.03)			0.00	0.16
M=60	0.83^{**} (0.19)	0.08^{*} (0.03)			0.00	0.16
M=120	0.64^{**} (0.14)	-0.005 (0.04)			0.00	0.13

(B)	$\Delta q_t^S(24)$	$\Delta q_t^B(24, 60)$	$\Delta q_t^B(60, 120)$	$\Delta q_t^L(120)$		R^2
	1.42^{**} (0.29)	0.07 (0.30)	0.63^{*} (0.27)	0.01 (0.04)	0.00	0.17

Table 3. Expected Interest Differentials and the Real Exchange Rate

First-differences + IV, Euro-dollar, *2004:9-2016:12*

(A)	$\Delta q_t^S(M)$	$\Delta q_t^L(M)$			p
M=24	1.83^{*} (0.89)	-0.25 (0.14)			0.00
M=60	1.64 (0.80)	-0.16 (0.14)			0.00
M=120	0.89 (0.59)	-0.42^{**} (0.14)			0.00

(B)	$\Delta q_t^S(24)$	$\Delta q_t^B(24,60)$	$\Delta q_t^B(60,120)$	$\Delta q_t^L(120)$	p
	1.84^{*} (0.90)	-0.15 (2.13)	0.37 (0.89)	-0.36 (0.26)	0.00

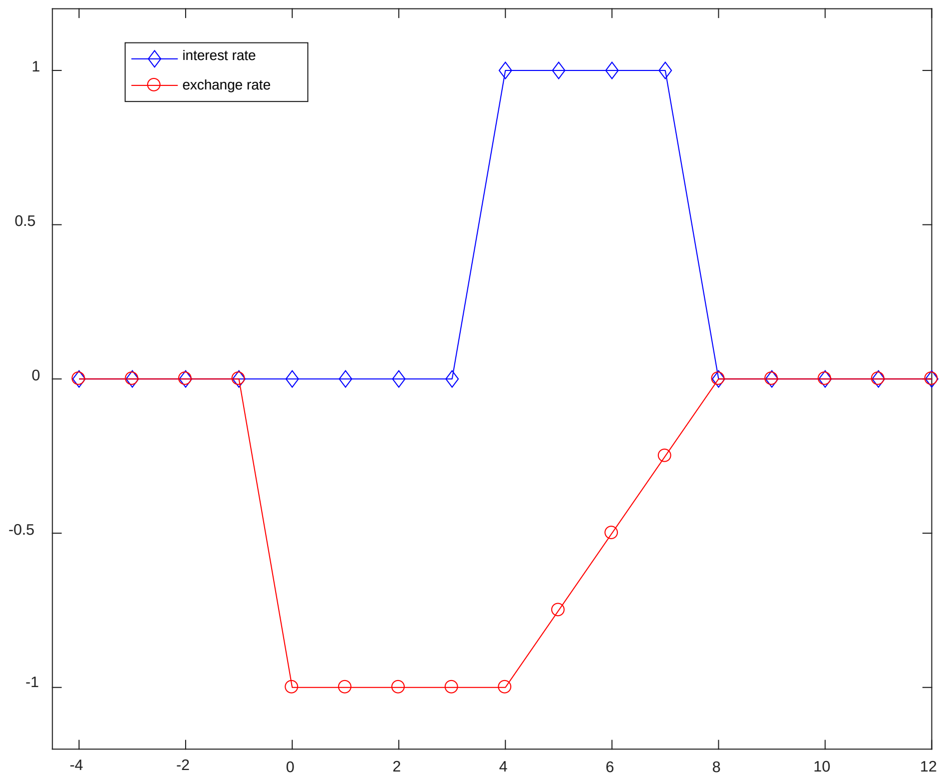

Figure 1. Forward Guidance and the Exchange Rate: Partial Equilibrium

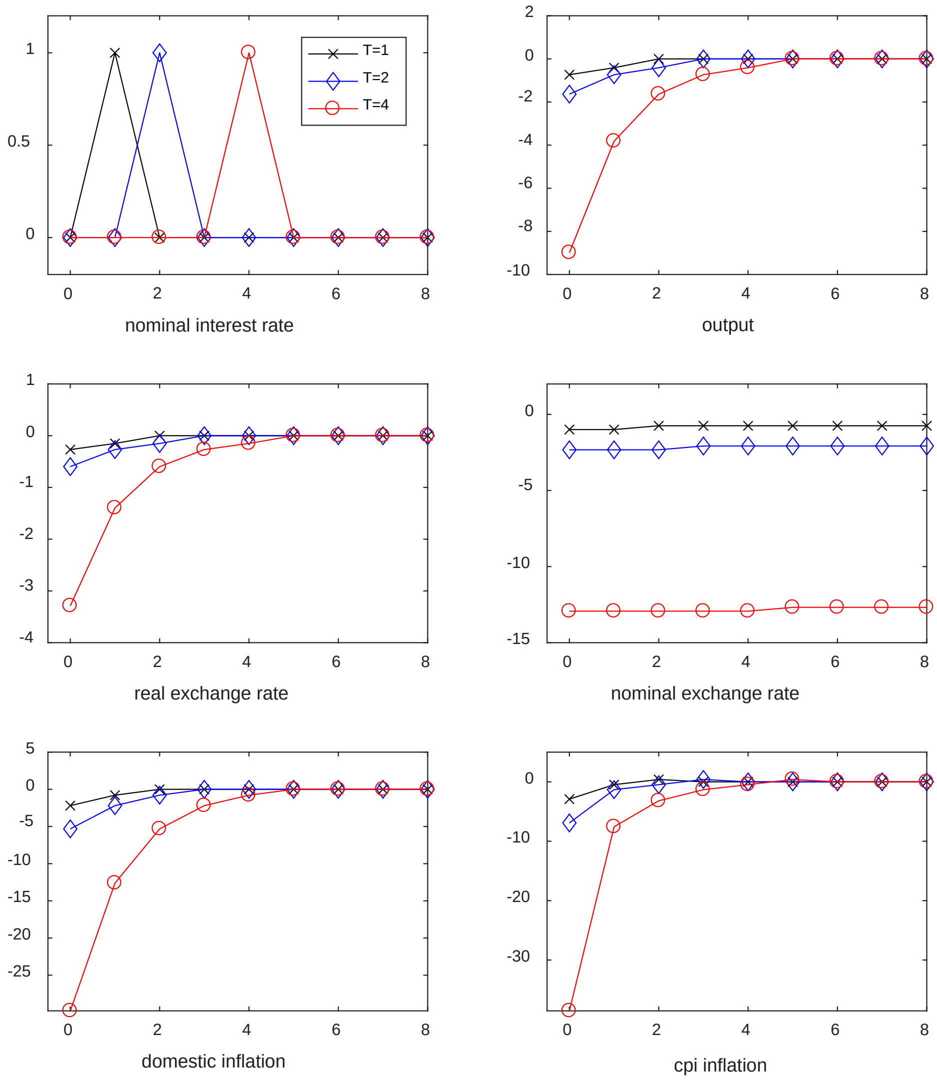

Figure 2. Forward Guidance in the Open Economy: The Role of the Horizon

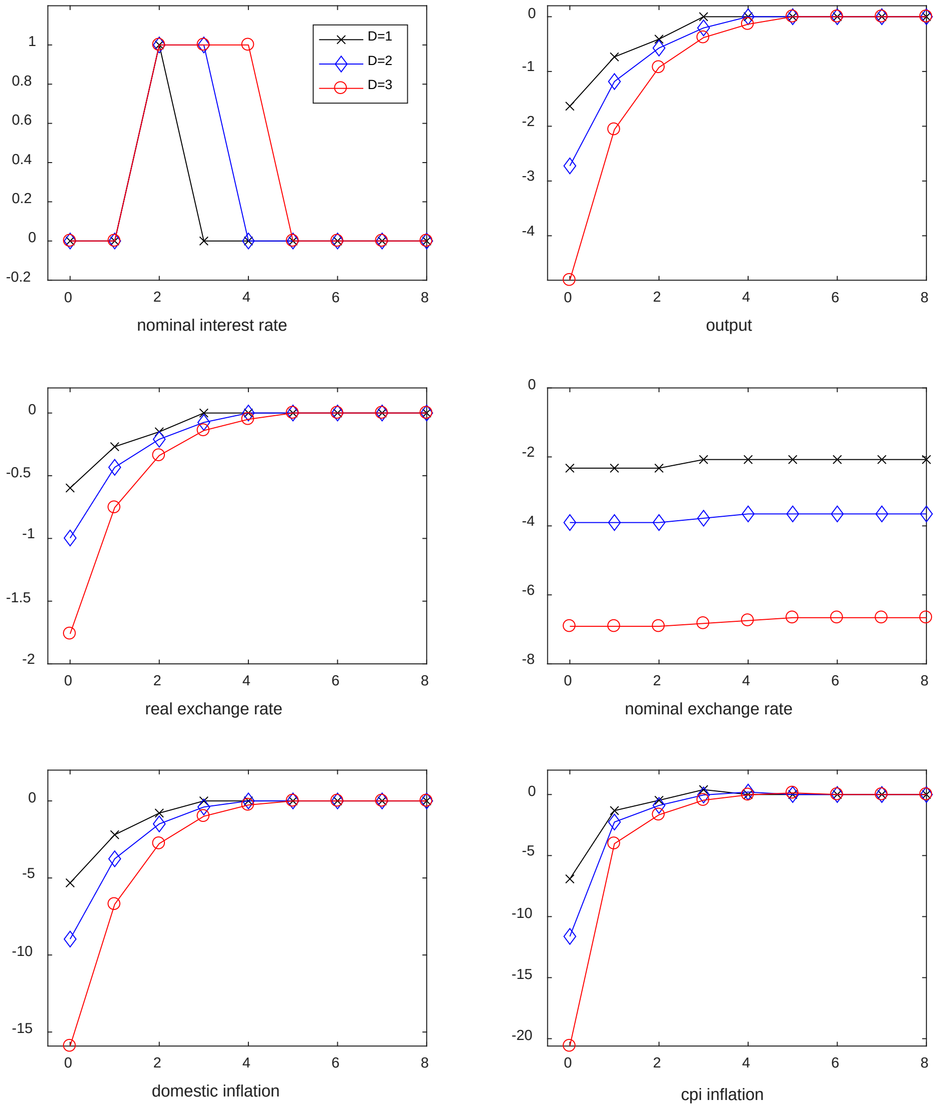

Figure 3. Forward Guidance in the Open Economy: The Role of Duration

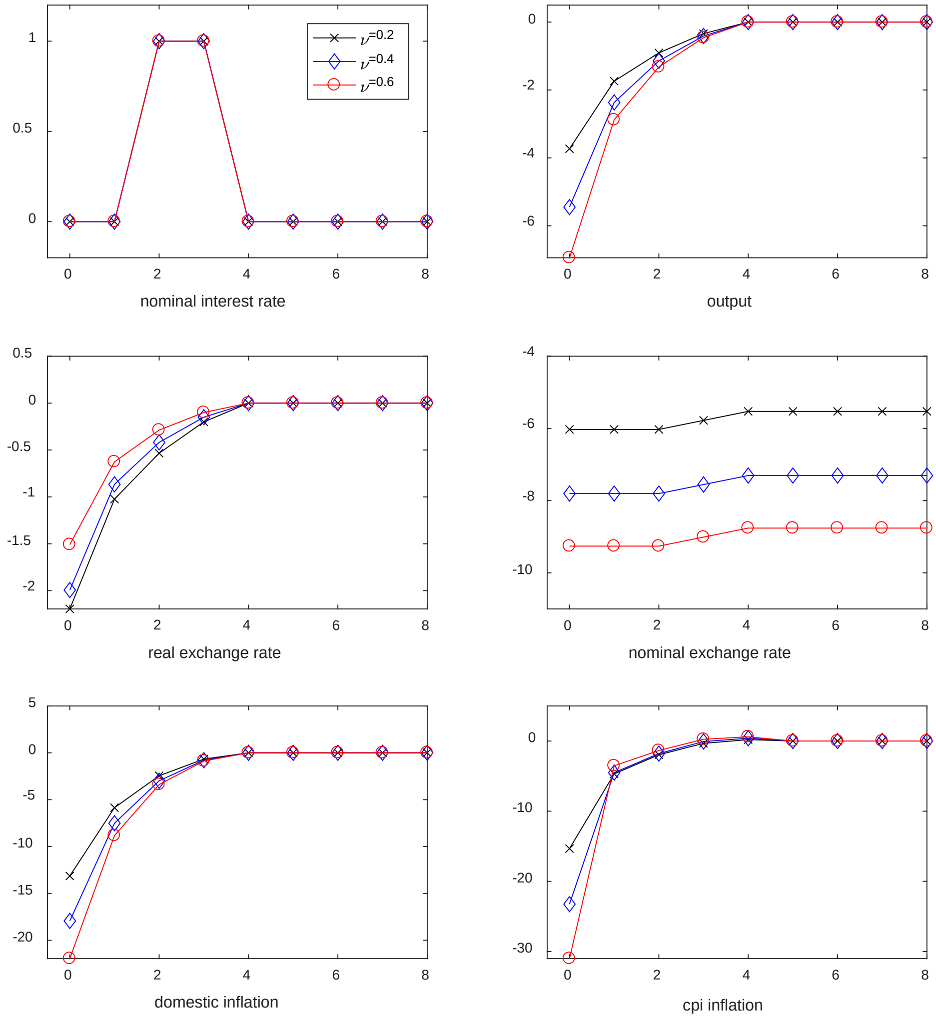

Figure 4. Forward Guidance in the Open Economy: The Role of Openness

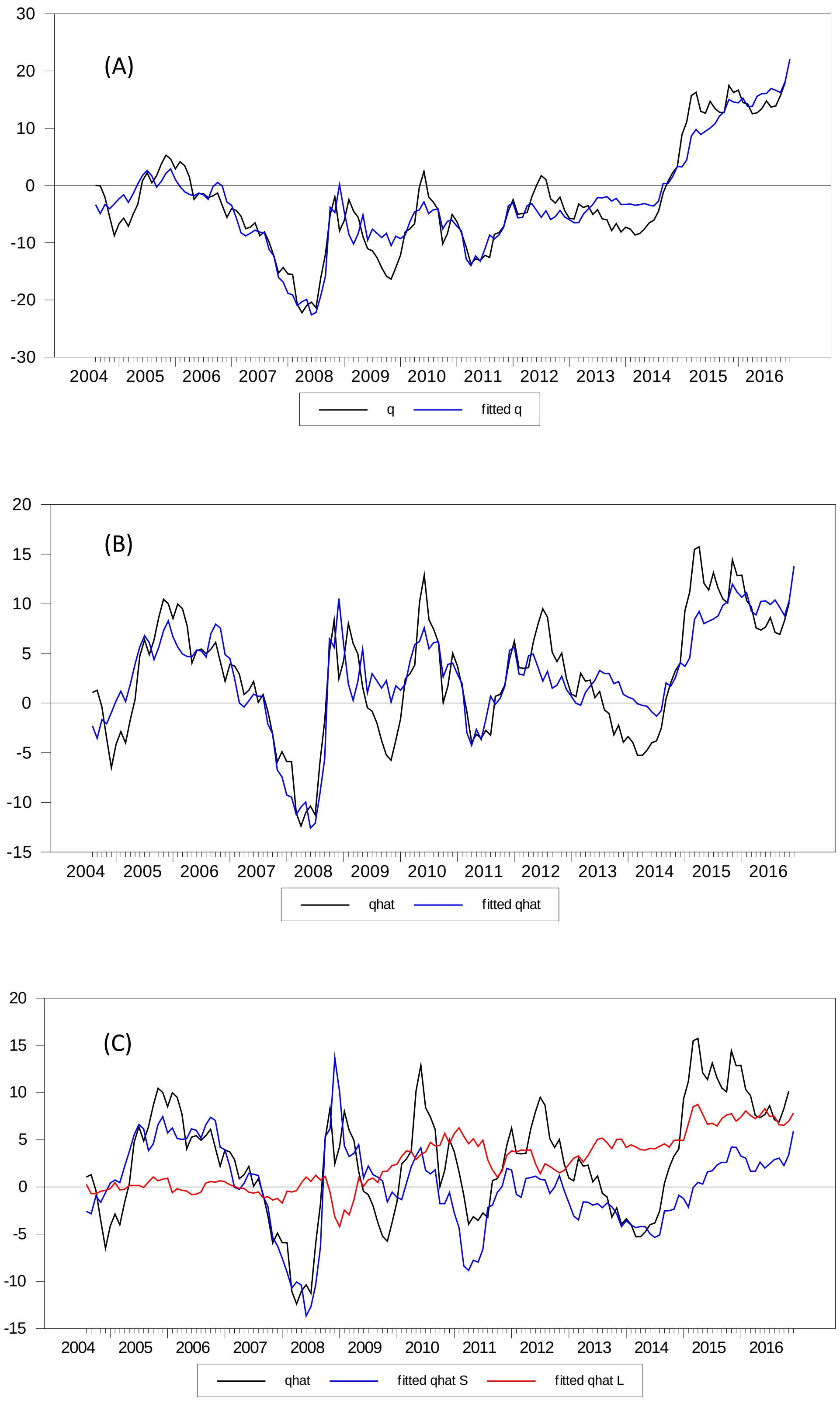

Figure 5. Expected Real Interest Rate Differentials and the Real Exchange Rate (T=24)

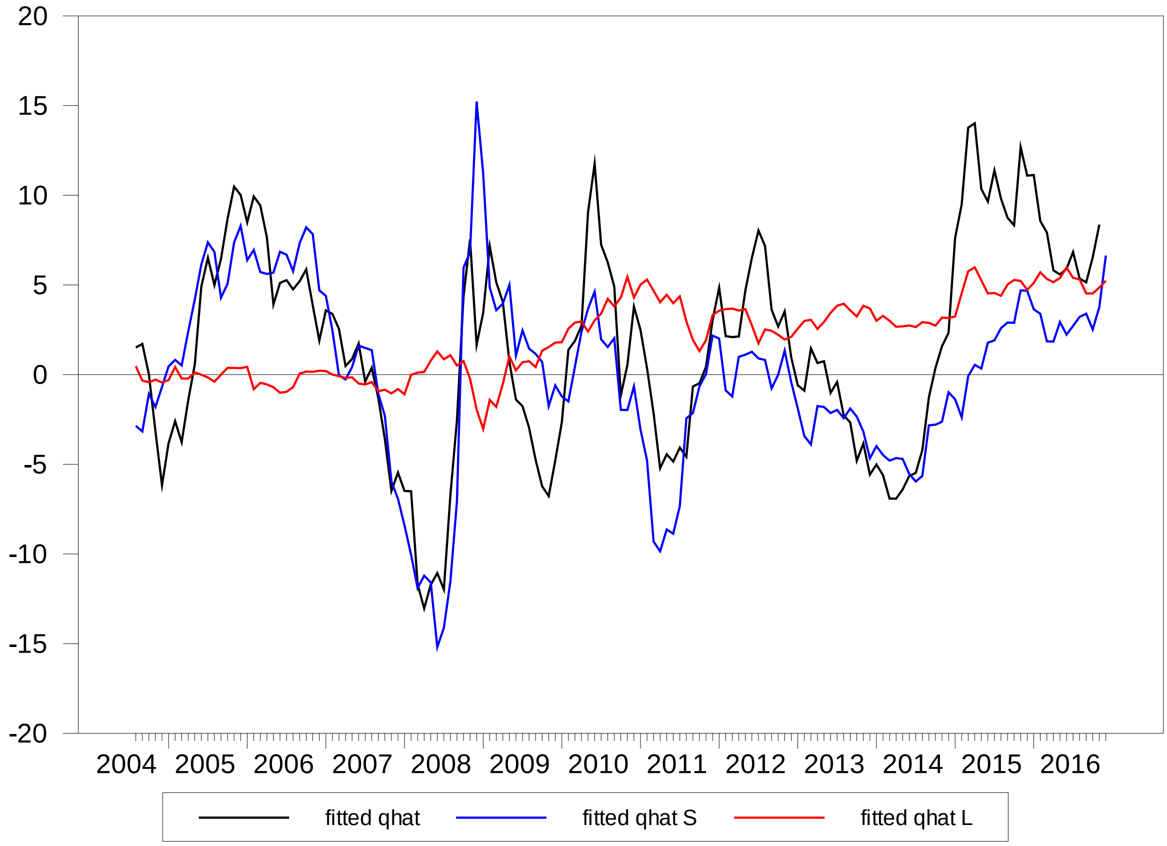

Figure 6. Expected Real Interest Rate Differentials and the Real Exchange Rate (multiple horizons)