Best Friend or Worst Enemy – Dynamics and Multiple Equilibria of Arbitrage, Production and Collateral Constraints

Ally Quan Zhang

Department of Banking and Finance - University of Zurich and Swiss Finance Institute

Universität Zürich[UZH]

Introduction

- I develop a general equilibrium model to study the economic impacts of the limits to arbitrage in the segmented markets.
- By incorporating the aggregate production and the asset mispricings in a unified framework, I provide an alternative mechanism to explain financial crises and the post-crisis recovery. I model the direct cause of crises via the breakdown of arbitrage transactions.
- I derive both the model dynamics and multiple equilibria in a closed form solution. I describe the crisis scenario as the case when the economy shifts from one equilibrium to the other.

Conclusion

- Given financial frictions, such as market segmentation and collateral constraints, certain degree of mispricings arising from insufficient arbitrage can boost the production sectors with higher capital investment and output level.
  - This is because arbitrageurs' binding collateral constraints makes their capital investment have positive shadow value serving as collateral in the financial markets, which encourages producers to invest more and produce more.
- The mispricings with limited arbitrage activities can also increase the systemic risk and render the economy vulnerable to financial crises.
- Due to the regime shifts, the economy might experience a slow and partial recovery after the financial crisis.

Model

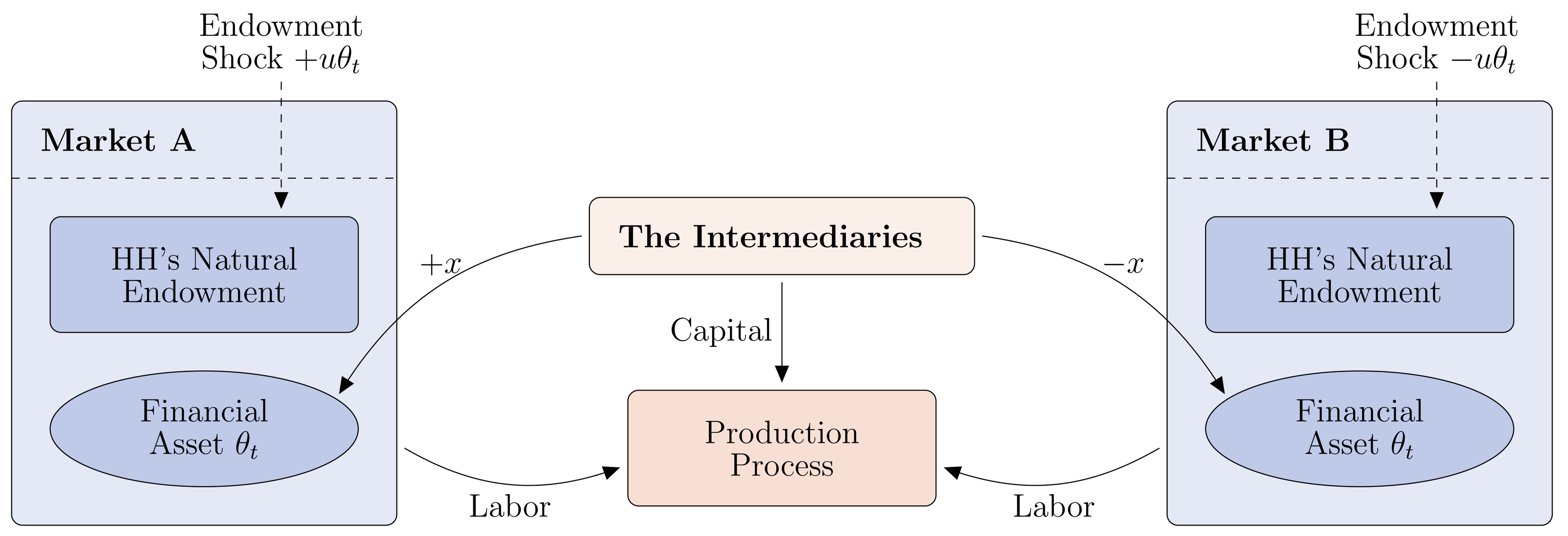


Figure 1: The structure of the economic system.

The infinite-horizon economy is populated with a continuum of competitive intermediaries (IM) and households (HH). There is only one perishable consumption good.

Households live within two separated markets

- experience equal but opposite (u) units of random endowment shock θ_t every period, i.e., $u^A = -u^B = u > 0$.

Financial Assets are identical in each market

- long-lived assets, paying out dividend equal to the endowment shock θ_t each period
- in net-zero supply
- traded each period by IM and HH with positions x_t and y_t^i, where $i \in A, B$
- perfect instrument to hedge against HH's endowment shocks
- HH from different markets have opposite hedging demand $\to$ price gaps

Intermediaries are both arbitrageurs and entrepreneurs

- can trade financial assets in both markets and exploit the price differences
- can convert consumption one-to-one into capital and vice versa
- invest capital (depreciation rate δ) and hire HH as labor with output function

$$Y_t = F(K_{t-1}) = aK_{t-1}^{\alpha}L^{\gamma} + (1-\delta)K_{t-1}$$

Collateral Constraints

- IM have to post capital input as collateral to support their arbitrage trade
- collateral has to be enough to cover HH's maximum loss if IM default or walk away from their positions in the next period
- IM's total collateral upper limit at t: $(1-\delta)K_t$.

Optimization Problems

Intermediaries' optimization problem

$$\max_{c_s^{\mathrm{IM}}, x_s, K_s} \mathbb{E}\left[\sum_{s=t}^{\infty} \rho^s \log\left(c_s^{\mathrm{IM}}\right)\right],$$

- subject to budget constraint

$$c_t^{\mathrm{IM}} + K_t = \underbrace{-x_{t-1}(P_t^B - P_t^A)}_{\text{obligation}} + \underbrace{x_t(P_t^B - P_t^A)}_{\text{arbitrage gain}} + F(K_{t-1}) + (1-\delta)K_{t-1},$$

- and collateral constraint

$$\underbrace{-x_t(P_{t+1}^B - P_{t+1}^A)}_{\text{next period obligation}} + (1-\delta)K_t \geqslant 0.$$

Households' optimization problem

$$\max_{c_s^i, y_s^i} \mathbb{E}\left[\sum_{s=t}^{\infty} \beta^s \log\left(c_s^i\right)\right], \quad i \in \{A, B\},$$

- subject to only budget constraint

$$c_t^i = \underbrace{y_{t-1}^i(P_t^i + \theta_t) - y_t^i P_t^i}_{\text{income from trading assets}} + \underbrace{a\gamma K_{t-1}^{\alpha} L^{\gamma-1}}_{\text{labor income}} + \underbrace{(b + u_i\theta_t)}_{\text{endowment}}.$$

Model Dynamics of IM's Wealth, Investment and Consumption

Under binding collateral constraints, IM's consumption and capital evolves according to

$$C_t = (1-\alpha\rho)W_t, \quad K_t = \alpha\rho W_t S_t.$$

where W_t is IM's wealth at the beginning of t,

$$W_t := F(K_{t-1}) + (1-\delta)K_{t-1} - x_{t-1}\phi_t = F(K_{t-1})$$

and the leverage ratio: $S_t = \dfrac{\phi_{t+1}}{\phi_{t+1} - (1-\delta)\phi_t} > 1$, *where*

- arbitrage gain serves as leverage to production
  - negative interest loan to IM
  - loan: immediate arbitrage gains; repayment: next period obligated settlement
- capital's collateral premium, marginal return $\uparrow$, production output $\uparrow$

Multiple Equilibria – Two Steady States

For IM the future shock intensity

- collateral premium boosts capital: $K_b^* = F'^{-1}\left(\frac{\delta}{\rho}\right) > F'^{-1}\left(\frac{1}{\rho}\right)$
- steady state capital input is higher than the one in neo-classic growth model with frictionless markets

For HH two equilibria

- binding collateral constraints $x^*\phi^* = (1-\delta)K_b^*$
- bad regime: small volume x_1^* & large price spread ϕ_1^*
- good regime: large volume x_2^* & small price spread ϕ_2^*

Crises

crises arise when shifting from good to bad after a tiny negative shock

- price spreads widen to fit the bad regime
- large initial trading positions inherited from the good one
- financial distress or insolvency

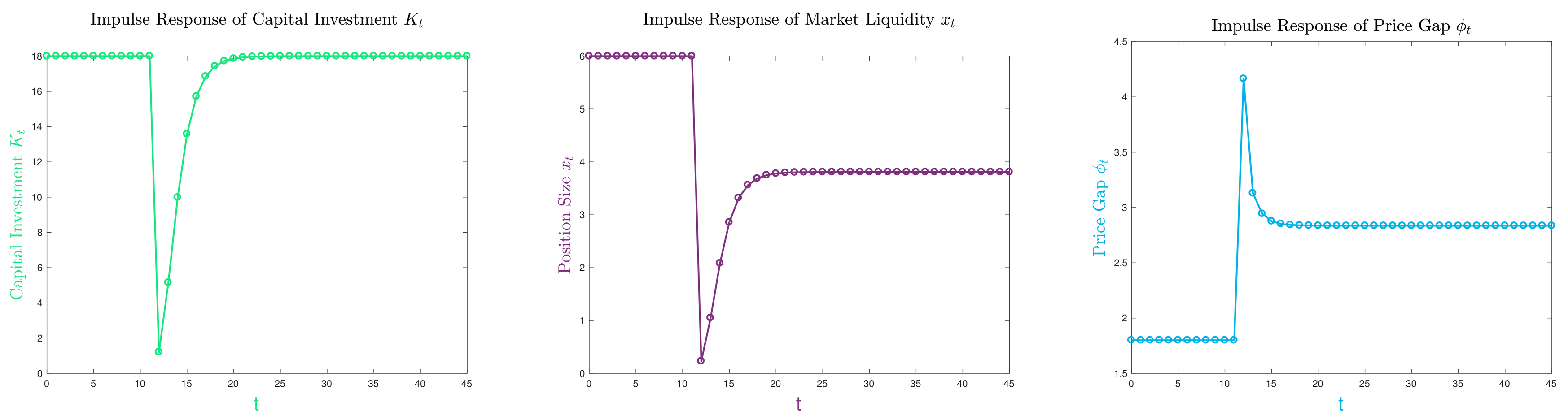
crises unavoidable even when switching to a good regime similar crises happens

- as long as the new regime features a bigger price spread

Recovery

Partial Recovery in Market Liquidity as the economy has switched to a different regime after the crisis, which features a lower trading volume

- help explain the slow and incomplete recovery of some asset markets after 2007-2009 crisis



Main References

Gromb, D. and Vayanos, D. (2002). Equilibrium and welfare in markets with financially constrained arbitrageurs. *Journal of Financial Economics*, 66(2–3):361–407.

Gromb, D. and Vayanos, D. (2017). The dynamics of financially constrained arbitrage. *Journal of Finance*, Forthcoming.