

Waiting for Affordable Housing*

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Abstract

We develop a new dynamic equilibrium model of housing markets for low- and moderate-income households, which is consistent with the key supply restrictions and search frictions that arise in rental markets for public and affordable housing. We estimate the model using data collected by the New York Housing Vacancy Survey in 2011. We find that having access to rent stabilized or affordable housing increases household welfare by up to \$55,000. Our policy simulations suggest that increasing the supply of affordable housing by ten percent significantly improves the welfare of all renters in the city. As a consequence our model provides a compelling explanation why affordable housing policies are popular at the ballot box with the vast majority of urban renters.

Keywords: Affordable Housing, Urban Housing Policies, Excess Demand, Housing Supply, Rationing, Search Frictions, Queuing, Welfare Analysis, Political Economy of City Governments.

JEL classification: C33, C83, D45, D58, H72, R31.

1 Introduction

As many urban and metropolitan areas have shifted toward a knowledge-based economy, most large cities in the U.S. have continued to attract highly skilled younger households. As a consequence real estate prices and rents have continued to soar in many metropolitan areas for the past two decades. Low- and moderate-income households have traditionally relied on a variety of affordable and public housing policies to sustain living in expensive cities.¹ Despite the importance and prevalence of affordable housing in many cities, there are few compelling dynamic models that allow us to study housing choices of low- and moderate-income households. The first objective of this paper is to fill this gap. We develop and estimate a new dynamic equilibrium model that is consistent with the observed market search frictions, the existence of long queues for public housing and the need to search for a long time to obtain access to rent stabilized housing. The second objective is to explain why affordable housing policies are increasingly popular at the ballot box and to evaluate the impact of increasing the supply of affordable housing on renters' welfare.

A compelling model of affordable housing should capture the existence of three different types of rental markets: public, regulated, and unregulated markets. It must also capture the dynamic incentives faced by households, such as income dynamics, long waiting lists for public housing, and long search times for regulated housing. We model the unregulated private housing rental market as frictionless. Households can purchase any quantity or quality of housing given the prevailing market price. All households also have access to regulated or rent stabilized housing. We assume that the rental price for rent stabilized housing is significantly lower than the equivalent market price in the unregulated market. Since the demand for rent-stabilized units typically exceeds the supply, there are significant frictions in the rent-stabilized market. Finding a rent-stabilized apartment involves significant search efforts and luck. We

¹Low- and moderate-income households play a key role in the provision of many local goods and services in the urban economy. Their presence is particularly essential with extreme-skill complementarity in the production function of large cities as discussed in detail by Eeckhout, Pinheiro, and Schmidheiny (2014).

capture these market frictions by endogenizing the probability that a household who is actively searching for rent stabilized housing will receive an offer to move into a stabilized unit. Our modeling approach is thus consistent with Glaeser and Luttmer (2003) who show that rent controls lead to misallocations in housing markets.

Low- and moderate income households are also eligible for public housing assistance in our model if income is below a threshold that depends on household composition and region. The rent charged for public housing is a fixed percentage of household income. Hence, there is no price mechanism to ensure that public housing markets clear. Demand for public housing vastly exceeds the available supply in most affluent U.S. cities. Rationing is achieved by placing households on waitlists that allocate free units to households with highest priority, i.e. households that have waited the longest.² Moreover, the housing authority does not evict households after they have lost their eligibility for housing aid. Consequently, public housing provides a consumption subsidy and also partial insurance against negative income shocks. We show that these incentives of current public housing policies give rise to a large degree of mismatch of low- and moderate-income households in housing markets.

We define and characterize a stationary equilibrium with rationing. The length of the waitlist for public housing and the probability of finding a rent-stabilized unit are all endogenously determined in equilibrium of our model. In equilibrium low-income households prefer to live in public housing due to the large rent subsidy, which implies a large increase in numeraire consumption, and the relatively high quality of these housing units. Rent stabilized housing appeals to a large range of low- and moderate-income households due to the significant rental price discount relative to the unregulated market. High-income households prefer to rent in the

²Geyer and Sieg (2013) consider a static model of public housing with myopic households and do not analyze rent-stabilization programs. Moreover, the focus on housing markets in Pittsburgh. Thakral (2015) considers a dynamic model of matching and introduces a multiple waitlist procedure. His analysis suggests that there are large potential welfare gains associated with this allocation mechanism. Halket and Nesheim (2017) also consider the problem of optimal allocation of public housing.

unregulated market. Due to the existence of rationing in public housing and search frictions in rent stabilized housing a fraction of low- and moderate-income households must also rent in the unregulated market in equilibrium. Our model can explain the existence of long wait and search times, and it is also consistent with the observed mismatch in public and rent-stabilized housing markets.³

The parameters of our model can be identified based on the observed moments in the data. Our proof of identification is constructive and can be used to define a method of moments estimator. This estimator matches the sorting of households by income and family type among housing options and the average time spent in different housing markets. The estimator also matches the average rental payment for each housing type.⁴

Our empirical analysis focuses on New York City (NYC). While many cities in the U.S. and abroad face the challenge to provide an adequate supply of affordable housing, NYC has been at the center of the debate over affordable housing. Studying the housing markets for low- and moderate-income households in NYC is promising for a variety of compelling reasons. First, NYC has the largest stock of rental apartments of all cities in the U.S. and is generally perceived to be one of the most expensive rental market in the world. Second, New York City also has the largest stock of public housing units of all cities in the U.S. Finally, NYC is the only large city in the United States that has ever declared a housing emergency and has adopted strict rent stabilization programs over an extended period of time.⁵ NYC, therefore,

³Our paper is also related to search and matching models that have been applied to study housing markets. See, for example, Wheaton (1990), Krainer (2001), Albrecht, Anderson, Smith, and Vroman (2007), Piazzesi and Schneider (2009), Diaz and Jerez (2013), Anenberg and Bayer (2013), and Anenberg and Kung (2017). Most of the papers in this literature focus on the markets for owner occupied housing which are distinctly different from affordable rental markets that are the focus of this paper.

⁴Our work is also related to the new literature on estimating dynamic models of houses and neighborhood choice, as discussed in Bayer, McMillan, Murphy, and Timmins (2016).

⁵Over one million households live in rent regulated housing units in New York City. Many other large European and Asian cities also use strict rent control laws to provide affordable housing. An early analysis of the benefits and costs of public housing in New York City is given by Olsen and Barton (1983).

serves as a laboratory to explore the effectiveness and impact of a variety of different affordable housing policies.

Our empirical analysis is based on the 2011 sample of the New York City Housing and Vacancy Survey (NYCHVS). This survey provides comprehensive data about household and housing characteristics. In particular, we observe household income and family status, the time that the household has spent in the housing unit, as well as a large number of structural characteristics of the housing unit that the household occupies. In addition, it allows us to classify households as living in public housing, rent-stabilized housing, or unregulated housing. We implement our estimator focusing on Manhattan, since waitlists for public housing in NYC are operated at the borough level. The data show that approximately 10 percent of our sample of low- and moderate-income households lived in public housing communities in 2011. 58 percent of households lived in rent stabilized units and the remaining 32 percent rented in the unregulated housing market. At the time of the survey, households spent, on average, 16 years in public housing, 9.5 years in regulated housing and only 4 years in unregulated housing. Not surprisingly, households in public housing are much poorer than households in rent stabilized and unregulated housing.

We estimate the structural parameters of the model. We find that our model fits the sorting of households by income among the three housing options. Our model is consistent with the well-known fact that public housing is particularly popular with black, female-headed households. Our model captures differences in rental prices as well as time spent in the housing units. We find that rental prices for stabilized housing are approximately 50 percent of the prices in the unregulated market in Manhattan. This significant discount explains the popularity of rent-stabilized units. The probability of finding a rent-stabilized unit is approximately 25 percent per year. The wait list for public housing is 18 years.

Policy makers and politicians have struggled to find a response to the increasing demands of voters to preserve mixed-income neighborhoods in affluent cities and to increase the supply of affordable housing. Our model provides a compelling explanation why these policies have

been so popular at the ballot box with the vast majority of urban renters. Our analysis shows that low quality units of affordable housing primarily attract households with incomes between \$20,000 and \$100,000. These households gain up to \$20,000 from having access to a stabilized units. For high quality units the results are even more striking. High quality units are attractive for households with incomes between \$30,000 and \$150,000. The welfare gains are up to \$55,000.⁶ Given these large benefits associated with having access to affordable housing, it is not surprising that rent stabilization and affordable housing policies are popular, not only with low- and moderate-income households, but with the vast majority of all urban renters in NYC.

The political popularity of these policies is in stark contrast to long term trends in the supply of affordable housing. Landlords primarily bear the burden of the rent-stabilization policies. Note that less than 8 percent of all apartments in NYC are voluntary rent-stabilized. These landlords obtain significant tax breaks as a return for making a fraction of the housing units affordable. The vast majority of rental unit in NYC – approximately 92 percent – are “involuntarily” stabilized under New York State’s Rent Stabilization Law. In NYC landlords have long been allowed to deregulate vacant apartments if the legal rent for a new renter exceeds a threshold, currently \$2,700 a month. Between 1993 and 2015 more than 139,000 apartments have been converted to market rates through vacancy decontrol which has led to a significant decline in the supply of affordable housing. Not surprisingly, this trend has not been popular with many voters in the city. As a candidate, the current mayor of NYC, Bill de Blasio, successfully ran on a platform that promised significant increases in the provision of affordable housing. Once in office, he proposed and city council recently adopted a 10-year plan to build or preserve 200,000 affordable housing units in the NYC area through various rezoning laws.

It is desirable to evaluate the impact of these new affordable housing policies on the distribution of renters’ welfare. We find that a ten percent increase in the supply of affordable improves

⁶The gains associated with public housing are of a similar magnitude.

welfare for all renters since the wait and search times decrease. The amount of mismatch in housing markets also decreases. The average welfare gains associated with a permanent 10 percent increase in the stock of affordable housing are approximately \$20,000. Of course, these findings do not imply that these policies are desirable or efficient since these policies impose large losses on land owners and housing developers. The magnitude of these losses are hard to assess.⁷ However, our findings provide a compelling explanation why affordable housing policies are increasingly popular among urban renters and populist politicians.

The rest of the paper is organized as follows. Section 2 discusses affordable housing policies in NYC and our data. Section 3 provides a new dynamic model of affordable housing markets. Section 4 discusses identification and estimation of the parameters of our model. Section 5 presents our empirical findings. Section 6 reports the findings from our welfare analysis and considers alternative housing policies. Section 7 offers our conclusions.

2 Data

Our empirical analysis focuses on the rental housing markets of NYC. Housing markets have been heavily regulated in NYC since the 1930's. As of 2011, over one million units were rent-stabilized representing roughly 47 percent of the rental housing stock in NYC.⁸ Rent stabilization generally applies to buildings of six or more units built between February 1, 1947 and December 31, 1973, and to those units that have exited from the rent-control program. Approximately 8 percent of the city's stabilized units and nearly all stabilized units in buildings constructed after 1974 were voluntarily subjected to rent stabilization by their owners in exchange for tax incentives from the city. Under the 421-a program, developers currently have

⁷The early literature on rent control suggest primarily focus on the misallocation in housing markets. See, for example, Olsen (1972), Suen (1989), and Gyourko and Linneman (1989).

⁸The stock of rent-regulated units includes a relatively small number of rent controlled units - approximately 38,000 units which are primarily older.

to set aside 20 percent of new apartments for poor and working-class tenants to receive tax abatements lasting 35 years.⁹

Involuntarily stabilized units, representing 92 percent of the stabilized stock, are regulated based on a “housing emergency” declared by the city in 1974 and renewed every three years since. Under New York State’s Rent Stabilization Law, the city may declare a housing emergency whenever the city’s rental vacancy rate drops below five percent. This law was most recently renewed in June 2015 and affects units with a maximum rent of \$2,700. Rent stabilization sets maximum rates for annual rent increases. It also entitles tenants to have their leases renewed. The rent guidelines board meets every year to determine how much the landlord can set future rents on the lease.

In addition, the low- and moderate income households may have access to public housing. Providing adequate housing and shelter for low- and moderate-income households has been a policy goal of most federal, state, and city administrations in the United States since the passage of the Public Housing Act of 1937. The New York City Housing Authority (NYCHA) provides public housing and administers Section 8 housing vouchers for low- and moderate-income residents throughout the five boroughs of New York City. Households whose incomes do not exceed 80% (50%) of median income are eligible for the public housing program (voucher program). In addition, income limits are functions of family size. For example, in 2011 the income limit for a single person household was \$45,850 (\$28,500) while it was \$65,450 (\$40,900) for a family of four.

Applications for public housing are assigned a priority code based upon information that includes employment status, income, family size, and quality of previous residence provided. Households are then placed on the housing authority’s preliminary waiting list for an eligibility interview. Households are required to update or renew their applications every two years if they have not been scheduled for an interview. Upon passing the interview and background checks, applicants are then placed on a (borough wide) waiting list.

⁹The de Blasio administration has been pushing to increase that fraction to 35 percent.

More than 403,000 New Yorkers reside in NYCHA’s 177,666 public housing apartments across the city’s five boroughs. Another 235,000 residents receive subsidized rental assistance in private homes through the NYCHA-administered Section 8 program. The NYCHA reported that 270,201 families were on the waiting list for conventional public housing and 121,356 families on the waiting list for Section 8. Little is known about the annual flows of waitlisted individuals into public housing. The NYT reported on July 23, 2013 that “the queue moves slowly. The apartments are so coveted that few leave them. Only 5,400 to 5,800 open up annually.” As of December 10, 2009 NYCHA stopped processing any new Section 8 applications due to the long waiting list. As consequence, there is almost no mobility in and out of Section 8 housing markets. We, therefore, treat Section 8 housing as a completely separate market and focus on public housing in this paper.

The empirical analysis is based on the New York City Housing Vacancy Survey (NYCHVS) in 2011. The main advantage of this data set is that it matches households with units (i.e., it contains detailed information about both household characteristics and housing characteristics).

Table 1: Descriptive Statistics

housing type	market share	rent	number of years	income	female head	kids	working family
Public	0.10	—	16.18	32,930	0.73	0.92	0.70
Regulated	0.58	1317	9.49	54,739	0.53	0.38	0.83
Unregulated	0.33	2640	3.85	71,045	0.54	0.17	0.87

Source: New York City Housing Vacancy Survey 2011

A household is defined as working if the labor income share is higher than 50 percent of total income.

Regulated units include rent-stabilized units, HUD-regulated units, and Michell-Lama rental units.

We focus on affordable housing for low- and moderate-income households which imposes three sample restrictions. First, we drop households whose average incomes exceed 200% of

median income level. This sample restriction is motivated by the fact that high-income New Yorkers are likely to own a condominium or house and, therefore, face a different choice set than low- and moderate-income households face.¹⁰ Second, we drop all low-income households that receive vouchers since that market has been closed for at least 6 years.¹¹ Finally, we drop all households not living in Manhattan since waitlists are operated at the borough level rather than city-wide. These restrictions reduce our sample size to 1,557.¹²

Table 1 provides some descriptive statistics of the Manhattan housing market for 2011. Table 1 shows that a large fraction of the rental units in Manhattan are under rent stabilization. The fraction was 58 percent in 2011. At the same time, the average rent was \$2,640 in the unregulated market and \$1,317 in the regulated market.

Households tend to stay for long periods in their apartments. On average in 2011, households had occupied their apartments 16.18 years for public housing and 9.49 years for rent stabilized housing. The turnover is much higher in the unregulated housing market. Not surprising, households in public housing are much poorer than household in rent stabilized and unregulated housing. Families in public housing tend towards single parent households, the majority headed by a female. Public housing families have more children, on average, than households in rent-stabilized or unregulated housing.

3 A Dynamic Model of Affordable Housing Markets

We consider a local housing market with three housing options: public housing (p), rent-regulated housing (r), and housing provided by the unregulated market (m). The exogenous housing supply in public and rent regulated housing are given by k_p and k_r . The assumption

¹⁰None of the key findings of this paper qualitatively or quantitatively depend on these choices.

¹¹Galiani, Murphy, and Pantano (2015) estimate a model of neighborhood choice with vouchers.

¹²Descriptive statistics for the full sample that includes renters from all five boroughs of NYC are qualitatively similar. Details available upon request from the authors.

of fixed supply of public and rent stabilized housing is appropriate for NYC. There has been limited recent construction of new housing communities in NYC.¹³ We can, therefore, treat supply as price inelastic and fixed in the short run.

Time is discrete, $t = 0, \dots, \infty$. Households are infinitely lived and forward looking. Households have a common discount factor β and maximize expected lifetime utility. In the baseline model, households only differ by income, denoted by y , which evolves according to a stochastic law of motion that can be described by a stationary Markov process with transition density $f(y'|y)$. Below we extend our model to allow for additional sources of household heterogeneity.

Household flow utility is defined over housing quality, h , and a numeraire good, b . Consider a household that rents in the unregulated market. Housing services can be purchased at price p_m .¹⁴ Flow utility is, therefore, given by:

$$u_m(y) = \max_{h,b} U(b, h) \tag{1}$$

$$s.t. \quad p_m h + b = y$$

Note that we are imposing the realistic assumption that low and moderate-income households do not save and cannot borrow against uncertain future income. They are liquidity constrained and spend their income on housing and consumption goods in each period.

There are R discrete different levels of housing quality in the stabilized market. The flow utility associated with a rent regulated unit of quality h_r and price $p_r < p_m$ is given by:

$$u_r(y) = U(y - p_r h_r, h_r) \quad r = 1, \dots, R \tag{2}$$

The next assumption captures the search frictions in that market.

Assumption 1

a) Each period, there is a positive probability q_r that a household receives an offer to move into

¹³If anything, the supply of rent stabilized housing has declined in the past decades.

¹⁴We implicitly assume that unregulated housing supply is perfectly elastic at price p_m . This assumption can be easily relaxed to endogenize the price of housing in the unregulated market by allowing for an upward sloping supply function.

a rent regulated unit of quality h_r .

b) Each household receives, at most one, offer per period.

The probabilities of receiving an offer to move into a stabilized housing unit are endogenous and depends on the supply and the voluntary outflow from regulated housing as discussed below in detail.

To simplify the notation we set $R = 1$ for the remainder of this section. But all results can be easily generalized to account for heterogeneity in the quality and supply of rent stabilized units.¹⁵ In our quantitative analysis below, we estimate a model with such heterogeneity ($R > 1$).

Public housing provides a constant level of housing consumption, h_p , and taxes individual at constant rate τ . Per period utility in public housing is, therefore, given by:

$$u_p(y) = U((1 - \tau)y, h_p) \tag{3}$$

The local housing authority that administers the public housing program manages a waitlist. The priority score of a household is a monotonic function of the time spent on the waitlist. More formally, let w denote the time that a household has been on the wait list. Let $p(w)$ denote the probability that a household that has been on the waitlist for w periods receives an offer to move into public housing. The next assumption captures the behavior of the housing authority.

Assumption 2

a) The housing authority makes take it or leave it offers, i.e if a household rejects an offer, it will go to the end of the waitlist ($w = 0$).

b) The outflow of public housing is voluntary (i.e., the housing authority does not evict households from public housing).

¹⁵An appendix that contains a detailed derivation of all key equations is available upon request from the authors.

c) Eligibility is determined by an income cut-off, denoted by \bar{y} and is checked every time period. Loss of eligibility means that the household is removed from the waitlist ($w = 0$).

These assumptions are uncontroversial and reflect common practice of housing authorities in NYC and other U.S. metropolitan areas. Note that the distribution of priority scores is endogenous and determined in equilibrium as we discuss below.

The timing of decisions is as follows:

1. Each household gets a realization of income which determines the income distributions at the beginning of the period.
2. Some households get an offer to move into public housing generated with probability $p(w)$.
3. Some households get an offer to move into rent-regulated housing generated with probability q_r .
4. Households decide to move and obtain the flow utility that depends on their decisions.
5. Wait times are updated.

Note that utility is realized after households have relocated.

The two state variables in this model are wait time, w , and income, y . Define the conditional value functions associated with the three choices:

$$\begin{aligned}
 v_p(y) &= u_p(y) + \beta \int V_p(y') f(y'|y) dy' \\
 v_m(y, w) &= u_m(y) + \beta \int V_m(y', w') f(y', w'|y, w) dy' dw' \\
 v_r(y, w) &= u_r(y) + \beta \int V_r(y', w') f(y', w'|y, w) dy' dw'
 \end{aligned} \tag{4}$$

We can derive recursive expressions for the unconditional value functions. The value function of a household with characteristics (w, y) that rents in the regulated market is given by:

$$\begin{aligned}
V_r(y, w) &= p(w) 1\{y \leq \bar{y}\} \max\{v_p(y), v_m(y, 0), v_r(y, 0)\} \\
&+ (1 - p(w)) 1\{y \leq \bar{y}\} \max\{v_m(y, w + 1), v_r(y, w + 1)\} \\
&+ 1\{y > \bar{y}\} \max\{v_m(y, 0), v_r(y, 0)\}
\end{aligned} \tag{5}$$

The value function of a household with characteristics (w, y) that rents in the unregulated market is then given by:

$$\begin{aligned}
V_m(y, w) &= q_r V_r(y, w) \\
&+ (1 - q_r) p(w) 1\{y \leq \bar{y}\} \max\{v_m(y, 0), v_p(y)\} \\
&+ (1 - q_r) (1 - p(w)) 1\{y \leq \bar{y}\} v_m(y, w + 1) \\
&+ (1 - q_r) 1\{y > \bar{y}\} v_m(y, 0)
\end{aligned} \tag{6}$$

Finally, the value function of a household living in public housing satisfies:

$$\begin{aligned}
V_p(y) &= (1 - q_r) \max\{v_p(y), v_m(y, 0)\} \\
&+ q_r \max\{v_p(y), v_m(y, 0), v_r(y, 0)\}
\end{aligned} \tag{7}$$

These value functions determine the optimal decision rules for each household.¹⁶

To illustrate the optimal decision rules we consider a simple estimated version of model with only one type of stabilized housing. Figure 1 plots the policy function for a household in public housing. The blue vertical line indicates the income eligibility threshold for public housing. Optimal decision rules can be characterized by thresholds. The blue line indicates the decision rule of a household that received an offer to move into regulated housing while the red line is a household without an offer. Low-income households prefer to live in public housing, moderate-income households prefer rent-regulated housing while higher income households prefer renting in the unregulated market.

¹⁶Note that our analysis abstracts from moving costs which are likely to be low in rental markets. It is straightforward to extend the model to allow for both monetary and psychic mobility costs.

Figure 1: Policy Function (0=public, 1=unregulated, 2=regulated)

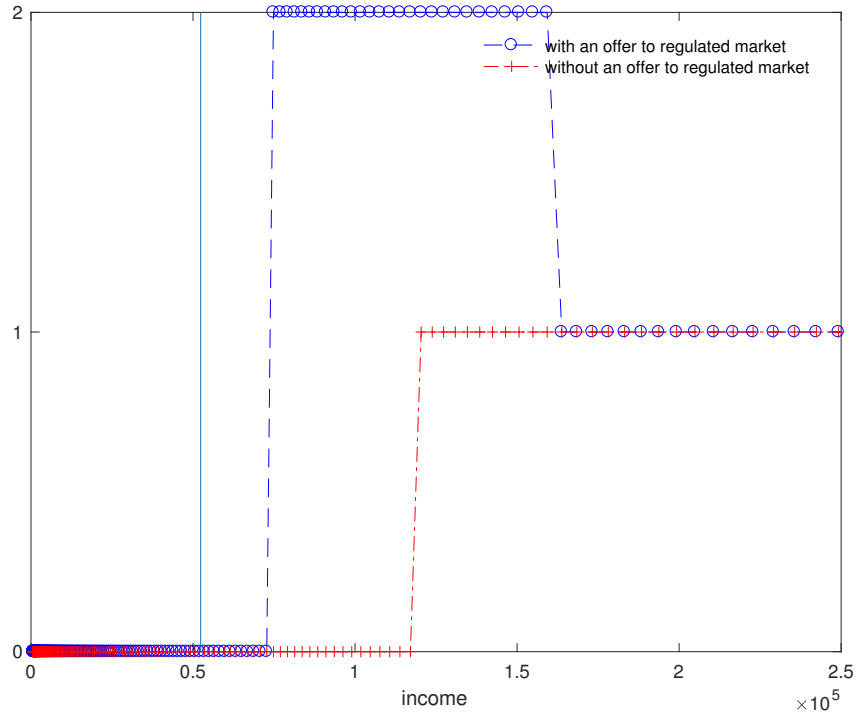
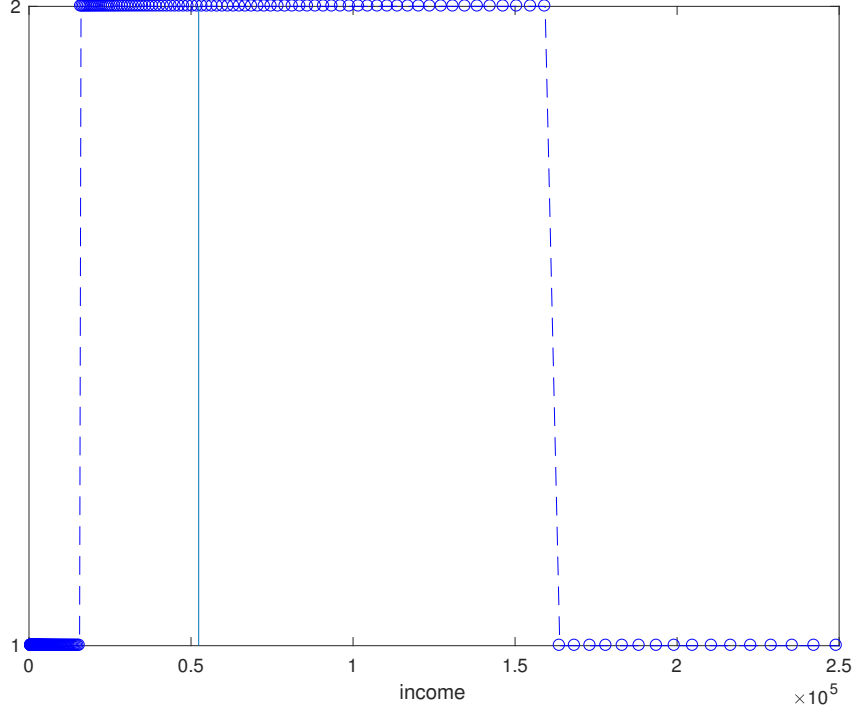


Figure 2 shows the decision rule for a household, who is currently in the regulated market, has been on the waitlist for 5 periods, and does not receive an offer to move into public housing. Here we find that low- and high-income households prefer the unregulated markets while moderate-income households prefer units in the rent-regulated market. The non-monotonicity of the decision rule is partially due to the fact that the quality of housing in the regulated market is relatively high exceeding the quality in public housing in this example.

Let $g_m(w)$ ($g_r(w)$) denote the marginal distribution of wait times for households in unregulated (rent regulated) housing in stationary equilibrium. Let $g_p(y)$ denote the density of income of households that are inside public housing at the beginning of each period (before households have moved). Similarly let $g_m(y|w)$ ($g_r(y|w)$) denote the stationary density of income conditional on wait time for households in the unregulated (regulated) market.

Figure 2: Policy Function (1=unregulated, 2=regulated, $w = 5$)



The voluntary flow of households out of public housing is given by:

$$\begin{aligned}
 OF_p &= k_p (1 - q_r) \int 1\{v_m(y, 0) > v_p(y)\} g_p(y) dy \\
 &+ k_p q_r \int 1\{v_m(y, 0) \geq \max[v_p(y), v_r(y, 0)]\} g_p(y) dy \\
 &+ k_p q_r \int 1\{v_r(y, 0) \geq \max[v_p(y), v_m(y, 0)]\} g_p(y) dy
 \end{aligned} \tag{8}$$

Note that the first two terms is the outflow to the unregulated market and the third term captures the outflow to the rent regulated market. The flow into public housing is given by:

$$\begin{aligned}
 IF_p &= k_m \sum_{j=0}^{\infty} p(w_j) g_m(w_j) IF_{mp}(w_j) \\
 &+ k_r \sum_{j=0}^{\infty} p(w_j) g_r(w_j) IF_{rp}(w_j)
 \end{aligned} \tag{9}$$

where the inflow from the unregulated market conditional on wait time is:

$$\begin{aligned}
IF_{mp}(w_j) &= (1 - q_r) \int_{y \leq \bar{y}} 1\{v_p(y) \geq v_m(y, 0)\} g_m(y|w_j) dy \\
&+ q_r \int_{y \leq \bar{y}} 1\{v_p(y) \geq \max[v_m(y, 0), v_r(y, 0)]\} g_m(y|w_j) dy
\end{aligned} \tag{10}$$

and the inflow from the rent regulated market is given by:

$$IF_{rp}(w_j) = \int_{y \leq \bar{y}} 1\{v_p(y) \geq \max[v_m(y, 0), v_r(y, 0)]\} g_r(y|w_j) dy \tag{11}$$

Similarly, the voluntary flow of households out of rent regulated housing is given by:

$$\begin{aligned}
OF_r &= k_r \sum_{j=0}^{\infty} p(w_j) g_r(w_j) \int_{y \leq \bar{y}} 1\{v_r(y, 0) \leq \max[v_p(y), v_m(y, 0)]\} g_r(y|w_j) dy \\
&+ k_r \sum_{j=0}^{\infty} (1 - p(w_j)) g_r(w_j) \int_{y \leq \bar{y}} 1\{v_m(y, w_j + 1) \geq v_r(y, w_j + 1)\} g_r(y|w_j) dy \\
&+ k_r \sum_{j=0}^{\infty} g_r(w_j) \int_{y > \bar{y}} 1\{v_m(y, 0) \geq v_r(y, 0)\} g_r(y|w_j) dy
\end{aligned} \tag{12}$$

Note that the first term is the outflow of those households that have an offer to move into public housing. The second term is the outflow of households eligible for public housing who do not have an offer to move into public housing. The last term is the outflow of households above the eligibility threshold to unregulated housing. The flow into rent regulated housing is given by:

$$\begin{aligned}
IF_r &= k_p q_r \int 1\{v_r(y, 0) \geq \max[v_m(y, 0), v_p(y)]\} g_p(y) dy \\
&+ k_m \sum_{j=0}^{\infty} p(w_j) q_r \int_{y \leq \bar{y}} 1\{v_r(y, 0) \geq \max[v_m(y, 0), v_p(y)]\} g_m(y|w_j) dy \\
&+ k_m \sum_{j=0}^{\infty} (1 - p(w_j)) q_r \int_{y \leq \bar{y}} 1\{v_r(y, w_j + 1) \geq v_m(y, w_j + 1)\} g_m(y|w_j) dy \\
&+ k_m \sum_{j=0}^{\infty} q_r \int_{y > \bar{y}} 1\{v_r(y, 0) \geq v_m(y, 0)\} g_m(y|w_j) dy
\end{aligned} \tag{13}$$

In a stationary equilibrium, the inflow has to be equal to the outflow of households for public and rent regulated housing.¹⁷

Definition 1 *A stationary equilibrium for this model consists of the following: a) offer probabilities $p(w)$ and q_r , b) distributions $g_p(y)$, $g_m(w)$, $g_r(w)$, $g_m(y|w)$, and $g_r(y|w)$, and c) value functions $V_p(y)$, $V_m(y, w)$ and $V_r(y, w)$, such that:*

1. *Households behave optimally and value functions satisfy the equations above.*
2. *The housing authority behaves according the administrative rules described above.*
3. *The densities are is consistent with the laws of motion and optimal household behavior.*
4. *$p(w)$ satisfies the market clearing condition for public housing:*

$$OF_p = IF_p \tag{14}$$

5. *q_r satisfies the market clearing condition for rent regulated housing:*

$$OF_r = IF_r \tag{15}$$

Finally note that we can endogenize the price of housing in the unregulated market by assuming that there is an upward sloping housing supply function $H_m^s(p_m)$ and by requiring that the demand for unregulated housing given by

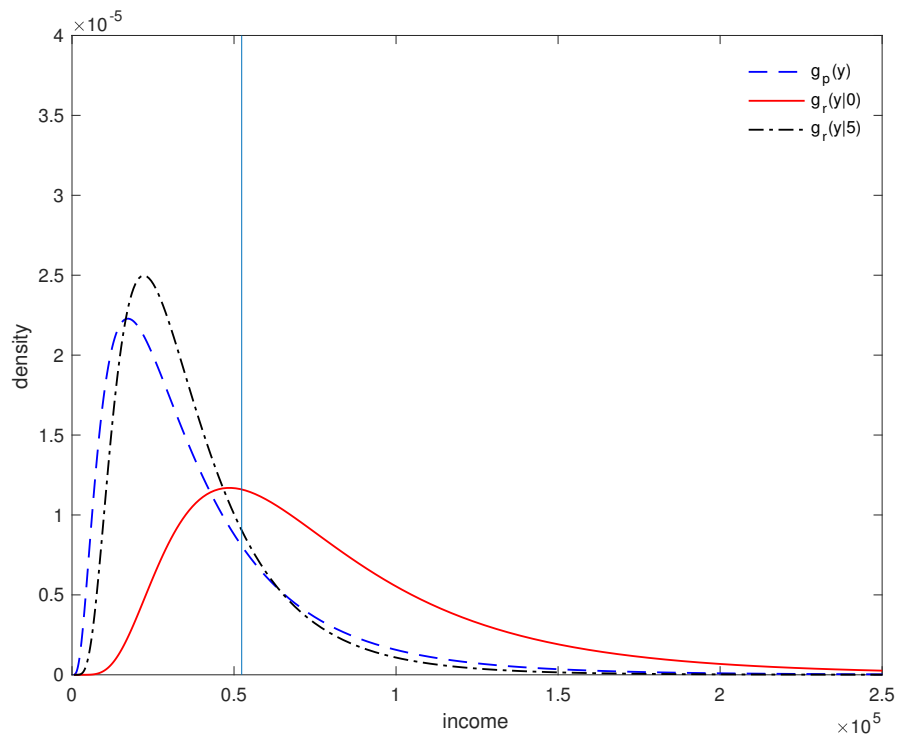
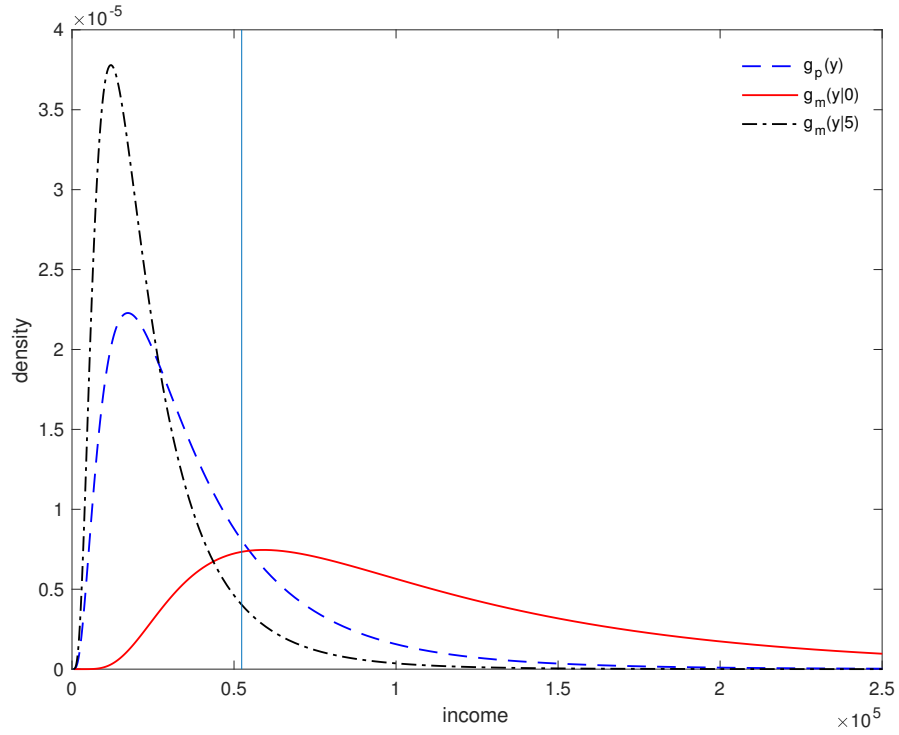
$$H_m^d = (1 - k_p - k_r) \sum_j g(w_j) \int h(p_m, y) g_m(y|w_j) dy \tag{16}$$

is equal to the supply.

Figure 3 illustrates the stationary equilibrium densities of income for a specification of our estimated model with one household type and only one type of rent stabilized housing.

¹⁷The vacancy rate in NYC has been around 2 percent during the time period of interest. Hence we ignore vacancies.

Figure 3: Stationary Distributions



The top panel compares the income distribution of households in the unregulated market with those that are in public housing. Not surprising we find that households with a priority score of zero, who are ineligible for public housing, have much higher income than those who live in public housing. More surprising is the result that households with a priority score of five years have lower income, on average. This due to the fact that households with high priority must have had income below the eligibility threshold for a number of consecutive periods to remain eligible, while this criteria does not apply for households in public housing. The lower panel compares the income distribution of households in the rent stabilized market with the distribution of households in public housing. Again we find similar qualitative patterns. Households that live in stabilized housing with high priority scores look similar to households in public housing.

Next we characterize the properties of equilibria with rationing. The main analytical result is summarized by the following proposition.

Proposition 1 *Any stationary equilibrium with sufficiently strong excess demand for public housing has the property that there exists a value $\bar{w} < \infty$ such that: $p(\bar{w}+1) = 1$, $0 \leq p(\bar{w}) < 1$, and $p(\bar{w} - j) = 0$ for all $j \geq 1$.*

Note that $p(\bar{w} + 1) = 1$ implies that there are no households with priority score greater than $\bar{w} + 1$, i.e. $g(\bar{w} + 1 + j) = 0$, for $j \geq 1$.

Proof:

We use a proof by contradiction. Suppose not, then

$$p(\bar{w} + 1) < 1 \tag{17}$$

and next period there exists some households with priority score $\bar{w} + 2$, hence $g(\bar{w} + 2) > 0$ which violates the stationarity definition and the definition of \bar{w} .

Suppose that $p(\bar{w} - j) > 0$ and $p(\bar{w}) \leq 1$. This case violates the assumption that offers to households with lower priority ranks can only be made if all households with higher ranks

receive offers.

Suppose that $p(\bar{w} - j) > 0$, $p(\bar{w}) = 1$ and $p(\bar{w} + 1) = 1$, then there will be no household in the next period which priority score $\bar{w} + 1$ which violates the stationarity assumption and that and that $g(\bar{w} + 1) > 0$.

Q.E.D.

The equilibrium has the property that everybody in the highest priority group obtains an offer to move into public housing. In addition, a fraction of the households with the second highest priority also gets an offer. The remaining households with the second highest priority score who do not get an offer this period, will obtain an offer in the next period. The intuition for this result is the following. The waitlist partitions the potential demand into $\bar{w} + 2$ cohorts. By adjusting $p(\bar{w})$, we can smooth out the fraction of individuals that obtains an offer. Note that $p(w_j)$ is not uniquely defined for $w_j > \bar{w} + 1$. Since the housing authority makers take-it-or-leave-it offers, there will be no households with wait times larger than $\bar{w} + 1$. Without loss of generality, we can, therefore, set $p(\bar{w} + j) = 1$ for all $j > 1$.

Given this equilibrium offer function, the inflow into public housing has two components and is equal to:

$$\begin{aligned} IF_p &= p(\bar{w}) [k_m g_m(\bar{w}) IF_{mp}(\bar{w}) + k_r g_r(\bar{w}) IF_{rp}(\bar{w})] \\ &+ [k_m g_m(\bar{w} + 1) IF_{mp}(\bar{w} + 1) + k_r g_r(\bar{w} + 1) IF_{rp}(\bar{w} + 1)] \end{aligned} \quad (18)$$

To finish the characterization of the equilibrium, we need to provide the laws of motion for the equilibrium densities. Equations (21) - (29) in Appendix A provide the details.

We would like to point out that households differ across many attributes besides income such as family size, race, ethnicity or gender of the household head. We extend our model to capture these differences using discrete household types, which also allows us to include differences in preferences over public housing and differences in access to rent stabilized units.¹⁸ We discuss

¹⁸This approach could also be used to capture the fact that some households may not consider public housing a desirable housing choice due to its stigma as suggested by Moffitt (1983).

in Appendix B how to extend our model to allow for heterogeneity among households. We also estimate versions of the model with heterogeneity that account for separate wait lists for households with different characteristics.

Finally, we extend the model to account the fact that housing supply for unregulated housing is price elastic. While we do not need to make an assumption when we estimate the model, we need to specify a supply function in our counterfactual policy analysis. We discuss these issue in more detail in Section 6.

4 Identification and Estimation

Since equilibria can only be computed numerically, we need to introduce a parametrization of the model and discuss identification and estimation. We can normalize the price of housing in the unregulated market to be equal to one since the units of housing services are arbitrary. To identify and estimate the price discount in the rent regulated market, we assume that market rents can be decomposed into a price and a quality index. We assume that the quality index is the same for units in the unregulated and the regulated markets, but the prices are not. We can, therefore use the techniques discussed in Sieg, Smith, Banzhaf, and Walsh (2002) to identify and estimate the price discount in the regulated market. We can also classify rent stabilized units into different types based on the quality levels predicted by the regression model. That approach allows us to discretize the underlying distribution of quality of rent regulated housing units.

We assume that the logarithm of income for each household follows an AR(1) process:

$$\ln(y_{it}) = \mu + \rho \ln(y_{it-1}) + \epsilon_{it}^y \quad (19)$$

The mean and the variance of income is identified of the observed income distributions in the data. The autocorrelation parameter is identified of the persistence of housing choices

measured by time spent in each housing type.¹⁹

We assume that the flow utility functions can be approximated by a Cobb-Douglas utility function, and hence we have:

$$\begin{aligned}
 u_p(y, h_p) &= [(1 - \tau)y]^{(1-\alpha)} h_p^\alpha \\
 u_m(y) &= \alpha^\alpha (1 - \alpha)^{1-\alpha} y p_m^{-\alpha} \\
 u_r(y, h_r) &= [y - p_r h_r]^{(1-\alpha)} h_r^\alpha
 \end{aligned} \tag{20}$$

Recall that α is the housing share parameter, which is identified from the observed joint distribution of housing and income. Public housing quality, h_p , is identified from the observed demand for public housing. The quality parameters h_r for $r = 1, \dots, R$ are identified based on our classification algorithm discussed above and the observed market rents for each type of unit type conditional on observed characteristics. The model also predicts that the time spent in the unit is an increasing function in housing quality.

All parameters of the income process and household preferences depend on household type in the extended model. As noted before, we assume that household type is observed by the econometrician. Hence, the identification argument extends to that model since all relevant moments are observed conditional on type.

Next consider the tax rate in public housing, denoted by τ . This parameter is determined by the administration of public housing programs. It is a state policy that renters in public housing pay roughly 30 percent of their income in rent.²⁰

The arguments for identification are constructive and suggest that we can estimate the parameters of our model using a methods of moments estimator. We use the following moments in estimation: the fraction of each housing type, the average time spent in unit by housing type, the average income by housing type, the variance of income by type, the autocorrelation

¹⁹Alternatively, we could use moments from a panel data set such as the SIPP to identify the autocorrelation parameter.

²⁰Also note that k_m , k_p and k_r are observed in the data.

of income by type, and the housing expenditure shares by housing type. Asymptotic standard errors can be consistently estimated using the standard formula for a parametric method of moments estimator provided, for example, in Newey and McFadden (1994).

5 Empirical Results

We first estimate the relative price of rent stabilized housing as discussed in the previous section. We find that rent stabilized apartments are offered at a 51 percent discount in Manhattan.²¹ This explains why rent stabilized units are extremely popular in Manhattan.

Table 2 reports estimated parameters and standard errors for a variety of models. First, we estimate the baseline model (Column I). We then add heterogeneity in regulated housing types (Column II), heterogeneity in preferences by household type (Column III), and finally explore a model with heterogeneity in household types and with multiple waitlists (Column IV). Overall, we find that all parameter estimates are reasonable and estimated with good precision.

First consider the baseline model in Column I. The parameter α captures the housing expenditure share for households that rent in the unregulated market. Low- and moderate-income households in Manhattan spend approximately 45 percent of their income on housing if they rent in the unregulated market. Allowing for heterogeneity among households in Columns III and IV shows that female headed households have slightly larger housing share parameters than male headed households.

The parameters of the income process, however, depend on the observed type. Comparing the estimates in Column I and II with those in Columns III and IV, we find that male headed households tend to have higher, more volatile, and less persistent incomes than female headed households. The autocorrelation coefficient ranges between 0.69 and 0.80 suggesting that

²¹See Appendix B for details of the price regression.

Table 2: Estimated Parameters

	I	II	III		IV	
	Baseline	1 Type	2 Type - 1 Queue		2 Type - 2 Queue	
	all	all	female	male	female	male
α	0.45 (0.01)	0.46 (0.01)	0.50 (0.02)	0.43 (0.01)	0.47 (0.01)	0.44 (0.01)
μ_y	10.62 (0.03)	10.64 (0.02)	10.59 (0.03)	10.69 (0.03)	10.56 (0.03)	10.70 (0.06)
σ	0.54 (0.02)	0.53 (0.02)	0.50 (0.01)	0.58 (0.03)	0.49 (0.01)	0.59 (0.06)
ρ	0.76 (0.02)	0.76 (0.03)	0.77 (0.03)	0.72 (0.04)	0.80 (0.02)	0.69 (0.02)
h_p	26,552 (515)	25,902 (866)	25,985 (670)		24,189 (2296)	29,841 (1278)
h_1	32,240 (673)	26,795 (604)	27,110 (620)		26,527 (618)	
h_2		37,980 (1087)	37,605 (918)		37,072 (440)	

Standard errors are in parenthesis.

income shocks are fairly persistent.

Housing quality is measured as equivalent expenditures in the unregulated market. Column I shows that an average public housing unit in Manhattan provides the same quality as a unit that rents for approximately \$26,000 dollars in the unregulated market. The average quality of rent controlled housing in the baseline model is approximately \$32,000. Allowing for heterogeneity in rent stabilized housing in Columns II-IV indicates that the quality for a low (high) quality rent stabilized apartment is approximately \$27,000 (\$38,000). Low quality stabilized units are, therefore, similar to public housing units while high quality units are significantly nicer than units in public housing.

Turning our attention to the model with multiple wait lists in Column IV, we find that the main empirical results are qualitatively and quantitatively similar to the one wait list (Columns I-III). The main difference is that male headed households tend to value public housing higher

than female headed households.

Table 3: Properties of Equilibrium

		Baseline	1 Type	2 Type - 1 Queue	2 Type - 2 Queue
wait	\bar{w}	18	17	17	19
times	$p(\bar{w})$	0.82	0.53	0.96	0.75
search	q_1	0.25	0.13	0.14	0.14
frictions	q_2		0.10	0.11	0.11

Table 3 summarizes some properties that correspond to the equilibria that are implied by the parameter estimates. In equilibrium, our model generates a wait times of approximately 18 years. The probability of finding a rent stabilized unit is approximately 25 percent, 11 percent for high quality units and 14 percent for low quality units. Male headed households tend to have slightly shorter wait times than female headed households, but the predicted difference is only one year or approximately 5 percent of the wait time.

Tables 4 reports a variety of goodness of fit statistics. We report the key statistics for both models. Overall, we find that our model fits the key moments used in estimation well.

Our estimates imply that affordable housing is an attractive option for low- and moderate-income households in Manhattan. To illustrate the magnitude of these effects, we compare differences in welfare between households in unregulated housing and households in rent stabilized housing. Figure 4 plots the differences in welfare by income for the two quality levels of rent stabilized housing.

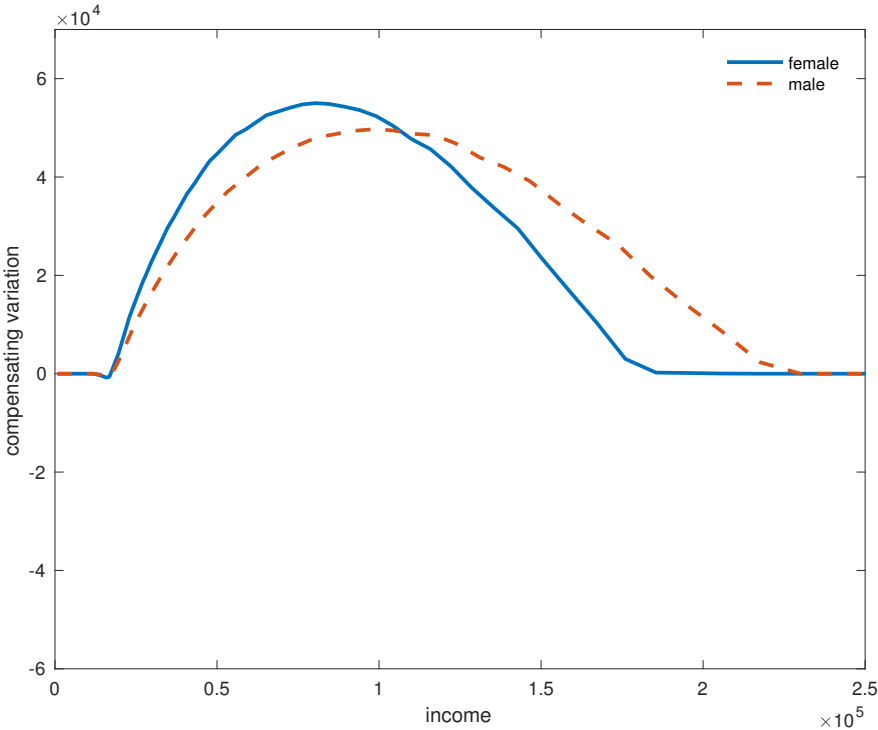
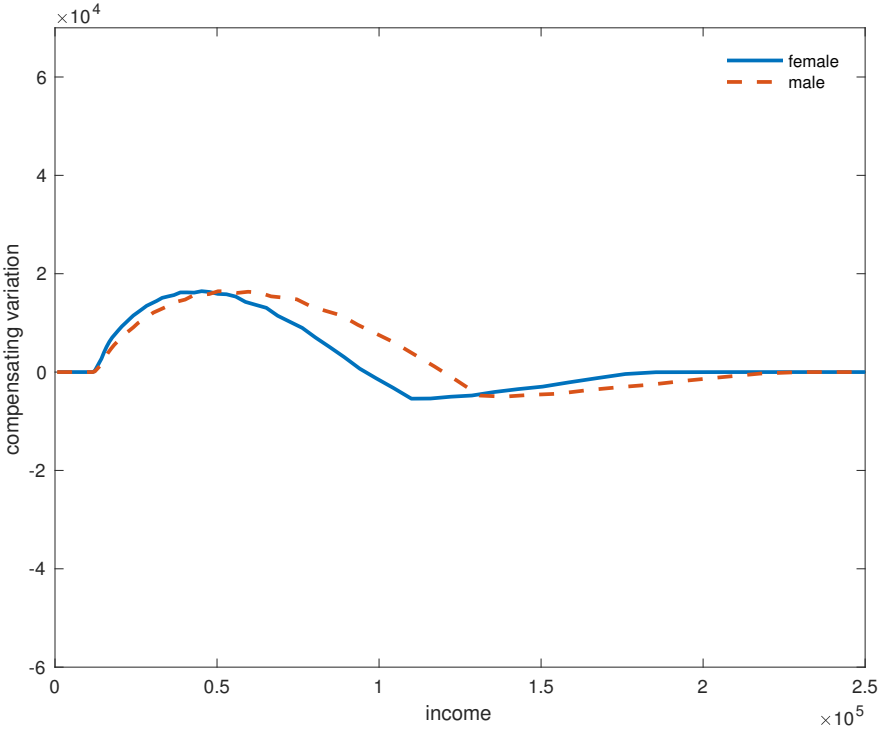
Figure 4 shows that there is an inverted-u shaped relationship between income and welfare gains. As a consequence, rent-stabilized policies create a fair amount of mismatch in affordable housing markets.²² Our analysis suggests that low quality units of affordable housing primarily

²²Glaeser and Luttmer (2003) find that 21 percent of New York apartment renters live in units with more or fewer rooms than they would if they rented in the unregulated market in 1990.

Table 4: Model Fit

	housing	percent		years		income	market rent		
Baseline									
	Public	9.90	9.90	16.18	16.37	32930	33914	—	—
	Regulated	57.20	57.20	9.49	9.20	54739	55615	1317	1309
	Market	32.90	32.90	3.85	4.22	71045	70262	2640	2642
2 Type - 1 Queue									
	Public	6.55	6.55	15.39	16.75	28796	33732	—	—
female	Regulated1	12.55	13.15	10.03	8.90	45516	43625	1048	1101
	Regulated2	14.90	14.79	10.41	10.41	55184	59342	1484	1527
	Market	16.00	15.51	3.70	4.19	69970	65844	2555	2729
	Public	2.95	2.95	18.34	13.41	44298	36075	—	—
male	Regulated1	16.55	15.95	8.37	8.54	53550	50321	1093	1101
	Regulated2	13.45	13.56	8.99	8.59	66288	66296	1695	1527
	Market	17.05	17.55	4.04	4.22	72300	74908	2743	2673
2 Type - 2 Queue									
	Public	6.55	6.55	15.39	16.02	28796	30942	—	—
female	Regulated1	12.55	13.20	10.03	8.67	45516	44186	1048	1077
	Regulated2	14.90	14.38	10.41	10.29	55184	59558	1484	1506
	Market	16.00	15.87	3.70	4.21	69970	67341	2555	2654
	Public	2.95	2.95	18.34	18.99	44298	42304	—	—
male	Regulated1	16.55	15.90	8.37	7.90	53550	48874	1093	1077
	Regulated2	13.45	13.97	8.99	8.71	66288	64866	1695	1506
	Market	17.05	17.18	4.04	3.95	72300	74827	2743	2743

Figure 4: Difference in Welfare between Rent Stabilized and Unregulated Housing



attract households with incomes between \$20,000 and \$100,000. These households gain up to \$20,000 from having access to a stabilized units. For high quality units the results are even more striking. High quality units are attractive for households with incomes between \$30,000 and \$150,000. The welfare gains are up to \$55,000.²³

Given these large benefits associated with having access to affordable housing, it is not surprisingly that rent control and affordable housing policies are popular, not only with low- and moderate-income households, but with the vast majority of all urban renters in NYC at the ballot box. As a consequence our model explains the the prevalence of the affordable housing policies in places such as NYC.

We focused in this section on the Manhattan subsample. We also estimated the model using the full sample that includes renters of all five boroughs are available upon request from the authors. The results are qualitatively similar to the results reported above. The main difference is that the price discount for affordable housing, wait and search times are lower in equilibrium.²⁴

6 Policy Analysis

The popularity of affordable policies is in stark contrast to long term trends in the supply of affordable housing in NYC. As we discussed above, landlords have long been allowed to deregulate vacant apartments if the legal rent for a new renter exceeds a threshold, currently \$2,700 a month. As a consequence, vacant apartments have been upgraded to take them into the unregulated market. Between 1993 and 2015 more than 139,000 apartments have been converted to market rates through vacancy decontrol which has led to a significant decline in

²³The welfare gains associated with public housing are up to \$60,000.

²⁴More details are available upon request from the authors.

the supply of affordable housing (WSJ, 2015).²⁵

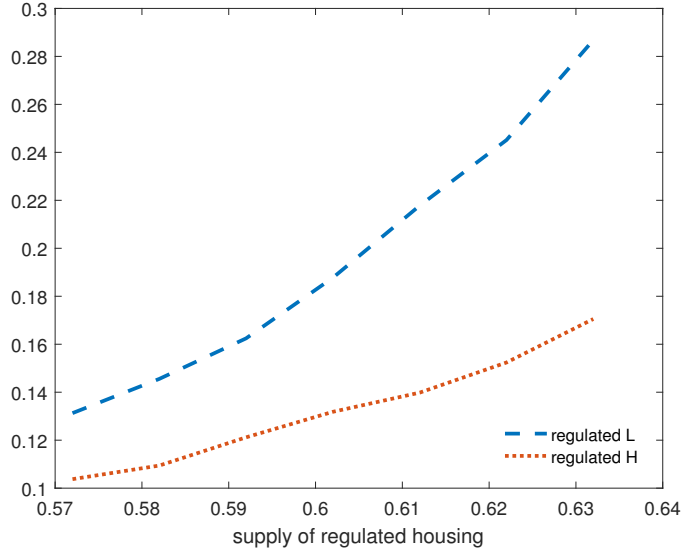
Not surprisingly, this long term trend has not gone unnoticed. Local politicians and policy makers have struggled with the voters' demands to reverse this trend. As a candidate, the current mayor of NYC, Bill de Blasio, successfully ran on a platform that promised significant increases in the provision of affordable housing. Once in office, he proposed and city council recently adopted a 10-year plan to build and retain 200,000 affordable housing units in the NYC area through various rezoning laws. We can use our model to simulate the effects of these types of policy changes on renters' welfare. Using our model with two affordable housing types and one public housing queue, we increase the supply of affordable housing by up to 10 percent. We consider the impact of this policy change on the probability of finding an affordable housing unit, the wait time for public housing, as well as the distribution of renters' welfare in the economy.

Here we consider the impact of increasing the supply of regulated housing from 57.2% to 63.2% of total rental units in Manhattan. We increase the supply of low quality and high quality regulated housing equally. Figure 5 shows that a ten percent increase in the supply of affordable housing substantially increases the probability of receiving an offer to move into a regulated housing unit in equilibrium. The probability of finding a low-quality unit increases from 14 to 28 percent, while the probability of finding a high-quality unit increases from 10 to 16 percent. Wait times for public housing also decrease by up to 1.5 years.

The reduced waiting and search times are associated with a more efficient allocation of public and rent stabilized housing in equilibrium. Households are more likely to move out of affordable units when they receive positive income shocks. Hence, those units can be reallocated faster to more needy households. As a consequence, the time spent in public or regulated housing decreases significantly. Similarly, household in the unregulated market also spend less

²⁵The NYCHVS suggests that more than 70 percent of all renters in Manhattan with incomes less than \$200,000 live in a rent-stabilized unit in 2002.

Figure 5: Offer Probability (p_r)



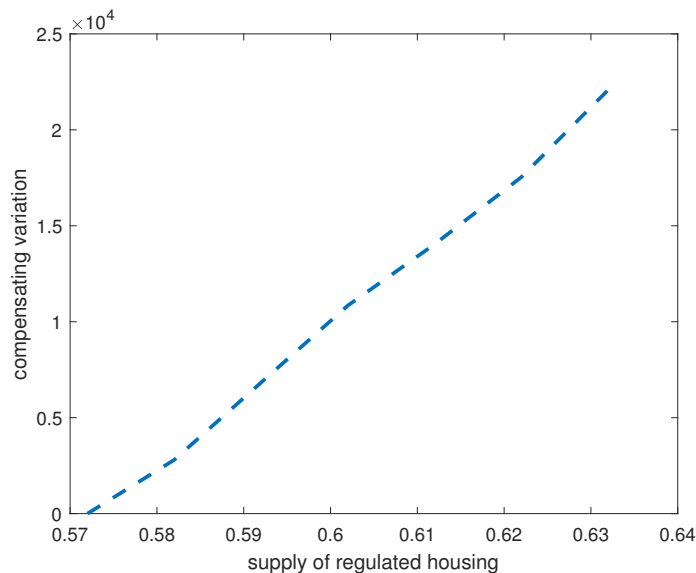
time in the unit because of the reduced wait and search times for affordable housing.²⁶

In a static framework subsidized housing is primarily occupied by low-income households. An increase in the supply of subsidized housing implies that the average income of households in those subsidized units increases as more higher income households move into those units. In our dynamic framework, there is another countervailing effect. As the allocation of affordable housing becomes more efficient, those units are occupied by households whose realized income is low due to a negative income shocks. As we increase the supply of regulated housing we find that the average income of households in public housing and low-quality regulated housing decreases.

We can measure the welfare gains for households using compensating variations. Figure 6 plots the average welfare gain as a function of the increase in the regulated housing stock. We find that the average welfare gain of a 10% increase in affordable housing is approximately \$20,000. Note that all households in the income distribution benefit from a permanent increase

²⁶Another measure of inefficiency of the equilibrium allocation is fraction of ineligible household in the public housing. We find that increasing the supply of regulated housing can also mitigate this problem to some degree.

Figure 6: Change in Welfare



in rent stabilized housing. The lower wait and search times imply that all households are better insulated against negative future income shocks. Overall, the welfare gains from this insurance is larger for high-income households. Of course, the biggest gains are for those who prefer rent stabilized housing and now are more likely to obtain access to it.

Finally, we evaluate how sensitive the results of our policy analysis are to changes in the supply of unregulated housing. In the baseline model we assume that the housing supply for unregulated housing is perfectly elastic. Hence, prices in unregulated housing are not affected by the change in the supply of affordable housing. Alternatively, we can use an aggregate housing supply function given by $H_m^s(p_m) = l [p_m]^\epsilon$. We set the constant l such that demand and supply are equal when $p_m = 1$ in our baseline year 2011. To evaluate the robustness of our findings we repeated the exercise above assuming a variety of different values for the supply elasticity of unregulated housing. Here we focus on the case when $\epsilon = 0.5$. A ten percent increase in regulated housing reduces the demand for unregulated housing. As a consequence, the rental price for unregulated housing drops by 6 percent. As private housing gets cheaper, it becomes more attractive. As a consequence, the reduction in waiting and search times are

even steeper than in the baseline model. But overall, we obtain the same qualitative and quantitative results.²⁷

7 Conclusions

We have developed a new dynamic model that captures search and queuing frictions in the rental markets for affordable housing. We have characterized the stationary equilibrium with rationing that arises in the model. We have shown how to identify and estimate the structural parameters of the model. Our application focuses on the housing markets of Manhattan in 2011. Overall, our model fits the observed sorting of households well. We have characterized the distribution of welfare that arises in our model and shown that access to low (high) quality of affordable housing can increase welfare by as much as \$20,000 (\$55,000). As consequence, our model provides a compelling explanation why affordable housing policies have been popular with the vast majority of urban renters in NYC. Finally, we study the effects of expanding the supply of affordable housing. We find that a ten percent increase in the supply of affordable improves welfare for all renters as the wait and search times decrease.

We should point out that we cannot conclude from this analysis that affordable housing policies such as those in NYC are desirable. First, our analysis does not allow us to measure the costs that are imposed on landlords. Clearly, these policies primarily redistribute wealth and income from landlords to renters. The magnitude of the welfare losses imposed on landlords is largely unknown. Second, rent stabilization policies weaken the incentive to invest in housing. As a consequence these policies have a significant negative impact on long-term housing supply.

The main focus of this paper is on positive analysis. We provide a clean measure of the benefits that renters obtain from living in an affordable unit. Our analysis provides a compelling explanation of why affordable housing policies are popular with the vast majority of

²⁷Details are available upon request from the authors.

urban renters and populist politicians. From the perspective of land owner and housing developers affordable housing policies are undoubtedly very costly. The main political advantage of affordable housing policies is that they allow local politicians to finance redistribution by implicitly taxing landlords and housing developers. Since housing developers and land owners tend to benefit the most from improvements in urban quality via capitalization effects, affordable housing policies effectively redistribute part of these gains of urban redevelopment and improvement to low- and moderate income renters. These renters tend to be the majority of voters in city elections.

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A Law of Motions for the Income Distributions

The equilibrium rationing rule then implies the following law of motion for the stationary income distributions:

$$\begin{aligned}
g_p(y) &= k_p (1 - q_r) \int 1\{v_p(x) \geq v_m(x, 0)\} f(y|x) g_p(x) dx \\
&+ k_p q_r \int 1\{v_p(x) \geq \max[v_m(x, 0), v_r(x, 0)]\} f(y|x) g_p(x) dx \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j) p(w_j) (1 - q_r) \int_{x \leq \bar{y}} 1\{v_p(x) \geq v_m(y, 0)\} f(y|x) g_m(x|w_j) dx \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j) p(w_j) q_r \int_{x \leq \bar{y}} 1\{v_p(x) \geq \max[v_m(x, 0), v_r(x, 0)]\} f(y|x) g_m(x|w_j) dx \\
&+ k_r \sum_{j=0}^{\infty} g_r(w_j) p(w_j) \int_{x \leq \bar{y}} 1\{v_p(x) \geq \max[v_m(x, 0), v_r(x, 0)]\} f(y|x) g_r(x|w_j) dx
\end{aligned} \tag{21}$$

and

$$\begin{aligned}
g_m(y|0) &= k_p (1 - q_r) \int 1\{v_m(x, 0) \geq v_p(x)\} f(y|x) g_p(x) dx \\
&+ k_p q_r \int 1\{v_m(x, 0) \geq \max[v_p(x), v_r(x, 0)]\} f(y|x) g_p(x) dx \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j) p(w_j) (1 - q_r) \int_{x \leq \bar{y}} 1\{v_m(x, 0) \geq v_p(x)\} f(y|x) g_m(x|w_j) dx \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j) p(w_j) q_r \int_{x \leq \bar{y}} 1\{v_m(x, 0) \geq \max[v_p(x), v_r(x, 0)]\} f(y|x) g_m(x|w_j) dx \\
&+ k_r \sum_{j=0}^{\infty} g_r(w_j) p(w_j) \int_{x \leq \bar{y}} 1\{v_m(x, 0) \geq \max[v_p(x), v_r(x, 0)]\} f(y|x) g_r(x|w_j) dx \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j) (1 - q_r) \int_{x > \bar{y}} f(y|x) g_m(x|w_j) dx \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j) q_r \int_{x > \bar{y}} 1\{v_m(x, 0) \geq v_r(x, 0)\} f(y|x) g_m(x|w_j) dx \\
&+ k_r \sum_{j=0}^{\infty} g_r(w_j) \int_{x > \bar{y}} 1\{v_m(x, 0) \geq v_r(x, 0)\} f(y|x) g_r(x|w_j) dx
\end{aligned} \tag{22}$$

and

$$\begin{aligned}
k_m g_m(0) &= k_p (1 - q_r) \int 1\{v_m(x, 0) \geq v_p(x)\} g_p(x) dx & (23) \\
&+ k_p q_r \int 1\{v_m(x, 0) \geq \max[v_p(x), v_r(x, 0)]\} g_p(x) dx \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j) p(w_j) (1 - q_r) \int_{x \leq \bar{y}} 1\{v_m(x, 0) \geq v_p(x)\} g_m(x|w_j) dx \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j) p(w_j) q_r \int_{x \leq \bar{y}} 1\{v_m(x, 0) \geq \max[v_p(x), v_r(x, 0)]\} g_m(x|w_j) dx \\
&+ k_r \sum_{j=0}^{\infty} g_r(w_j) p(w_j) \int_{x \leq \bar{y}} 1\{v_m(x, 0) \geq \max[v_p(x), v_r(x, 0)]\} g_r(x|w_j) dx \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j) (1 - q_r) \int_{x > \bar{y}} g_m(x|w_j) dx \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j) q_r \int_{x > \bar{y}} 1\{v_m(x, 0) \geq v_r(x, 0)\} g_m(x|w_j) dx \\
&+ k_r \sum_{j=0}^{\infty} g_r(w_j) \int_{x > \bar{y}} 1\{v_m(x, 0) \geq v_r(x, 0)\} g_r(x|w_j) dx
\end{aligned}$$

Moreover,

$$\begin{aligned}
g_r(y|0) &= k_p q_r \int 1\{v_r(x, 0) \geq \max[v_p(x), v_m(x, 0)]\} f(y|x) g_p(x) dx & (24) \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j) p(w_j) q_r \int_{x \leq \bar{y}} 1\{v_r(x, 0) \geq \max[v_p(x), v_m(x, 0)]\} f(y|x) g_m(x|w_j) dx \\
&+ k_r \sum_{j=0}^{\infty} g_r(w_j) p(w_j) \int_{x \leq \bar{y}} 1\{v_r(x, 0) \geq \max[v_p(x), v_m(x, 0)]\} f(y|x) g_r(x|w_j) dx \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j) q_r \int_{x > \bar{y}} 1\{v_r(x, 0) \geq v_m(x, 0)\} f(y|x) g_m(x|w_j) dx \\
&+ k_r \sum_{j=0}^{\infty} g_r(w_j) \int_{x > \bar{y}} 1\{v_r(x, 0) \geq v_m(x, 0)\} f(y|x) g_r(x|w_j) dx
\end{aligned}$$

and

$$\begin{aligned}
k_r g_r(0) &= k_p q_r \int 1\{v_r(x, 0) \geq \max[v_p(x), v_m(x, 0)]\} g_p(x) dx & (25) \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j) p(w_j) q_r \int_{x \leq \bar{y}} 1\{v_r(x, 0) \geq \max[v_p(x), v_m(x, 0)]\} g_m(x|w_j) dx \\
&+ k_r \sum_{j=0}^{\infty} g_r(w_j) p(w_j) \int_{x \leq \bar{y}} 1\{v_r(x, 0) \geq \max[v_p(x), v_m(x, 0)]\} g_r(x|w_j) dx \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j) q_r \int_{x > \bar{y}} 1\{v_r(x, 0) \geq v_m(x, 0)\} g_m(x|w_j) dx \\
&+ k_r \sum_{j=0}^{\infty} g_r(w_j) \int_{x > \bar{y}} 1\{v_r(x, 0) \geq v_m(x, 0)\} g_r(x|w_j) dx
\end{aligned}$$

$$\begin{aligned}
g_m(y|w_j) &= k_m g_m(w_j - 1) (1 - q_r) \int_{x \leq \bar{y}} f(y|x) g_m(x|w_j - 1) dx & (26) \\
&+ k_m g_m(w_j - 1) q_r \int_{x \leq \bar{y}} 1\{v_m(x, w_j) \geq v_r(x, w_j)\} f(y|x) g_m(x|w_j - 1) dx \\
&+ k_r g_r(w_j - 1) \int_{x \leq \bar{y}} 1\{v_m(y, w_j) \geq v_r(y, w_j)\} f(y|x) g_r(x|w_j - 1) dx
\end{aligned}$$

and

$$\begin{aligned}
g_m(w_j) &= g_m(w_j - 1) (1 - p(w_j - 1)) (1 - q_r) \int_{x \leq \bar{y}} g_m(x|w_j - 1) dx & (27) \\
&+ g_m(w_j - 1) (1 - p(w_j - 1)) q_r \int_{x \leq \bar{y}} 1\{v_m(x, w_j) \geq v_r(x, w_j)\} g_m(x|w_j - 1) dx \\
&+ \frac{k_r}{k_m} g_r(w_j - 1) (1 - p(w_j - 1)) \int_{x \leq \bar{y}} 1\{v_m(x, w_j) \geq v_r(x, w_j)\} g_r(x|w_j - 1) dx
\end{aligned}$$

$$\begin{aligned}
g_r(y|w_j) &= k_r g_r(w_j - 1) \int_{x \leq \bar{y}} 1\{v_r(x, w_j) \geq v_m(x, w_j)\} f(y|x) g_r(x|w_j - 1) dx & (28) \\
&+ k_m g_m(w_j - 1) q_r \int_{x \leq \bar{y}} 1\{v_r(x, w_j) \geq v_m(x, w_j)\} f(y|x) g_m(x|w_j - 1) dx
\end{aligned}$$

and

$$\begin{aligned}
g_r(w_j) &= g_r(w_j - 1) (1 - p(w_j - 1)) \int_{x \leq \bar{y}} 1\{v_r(x, w_j) \geq v_m(x, w_j)\} g_r(x|w_j - 1) dx \\
&+ \frac{k_m}{k_r} g_m(w_j - 1) (1 - p(w_j - 1)) q_r \int_{x \leq \bar{y}} 1\{v_r(x, w_j) \geq v_m(x, w_j)\} g_m(x|w_j - 1) dx
\end{aligned} \tag{29}$$

B Extending the Model to Allow for Multiple Household Types

We can allow for different discrete types allowing for differences in family structure. Assume that there are I types of households. Household types are defined by family structure (number of kids, number of adults etc.) Each household has a fixed share denoted by s_i , where $\sum_{i=1}^I s_i = 1$.²⁸ We make the following simplifying assumption which can be easily relaxed.

Assumption 3 *The housing authority operates one waitlist for all types and all types compete for the same housing units in the unregulated and regulated markets.*

Let (k_{ip}, k_{ir}, k_{im}) denote the relevant type specific market shares. Let $g_{im}(w)$ ($g_{ir}(w)$) denote the marginal distribution of wait times for households of type i in unregulated (rent regulated) housing in stationary equilibrium. Let $g_{ip}(y)$ denote the density of income of households of type i that are inside public housing at the beginning of each period. Similarly let $g_{im}(y|w)$ ($g_{ir}(y|w)$) denote the stationary density of income conditional on wait time for households in the unregulated (regulated) market.

²⁸This approach is in the spirit of Heckman and Singer (1984) although we will treat the household type as observed.

The voluntary flow of type i households out of public housing is given by:

$$\begin{aligned}
OF_{ip} &= k_{ip} (1 - q_r) \int 1\{v_{im}(y, 0) > v_{ip}(y)\} g_{ip}(y) dy \\
&+ k_{ip} q_r \int 1\{v_{im}(y, 0) \geq \max[v_{ip}(y), v_{ir}(y, 0)]\} g_{ip}(y) dy \\
&+ k_{ip} q_r \int 1\{v_{ir}(y, 0) \geq \max[v_{ip}(y), v_{im}(y, 0)]\} g_{ip}(y) dy
\end{aligned} \tag{30}$$

Note that the first two terms is the outflow to the unregulated market and the third term captures the outflow to the rent regulated market.

The flow into public housing of type i households is given by:

$$\begin{aligned}
IF_{ip} &= p(\bar{w}) [k_{im} g_{im}(\bar{w}) IF_{imp}(\bar{w}) + k_{ir} g_{ir}(\bar{w}) IF_{irp}(\bar{w})] \\
&+ [k_{im} g_{im}(\bar{w} + 1) IF_{imp}^i(\bar{w} + 1) + k_{ir} g_{ir}(\bar{w} + 1) IF_{irp}(\bar{w} + 1)]
\end{aligned} \tag{31}$$

where the inflow from the unregulated market conditional on wait time is:

$$\begin{aligned}
IF_{imp}(w) &= (1 - q_r) \int_{y \leq \bar{y}} 1\{v_{ip}(y) \geq v_{im}(y, 0)\} g_{im}(y|w) dy \\
&+ q_r \int_{y \leq \bar{y}} 1\{v_{ip}(y) \geq \max[v_{im}(y, 0), v_{ir}(y, 0)]\} g_{im}(y|w) dy
\end{aligned} \tag{32}$$

and the inflow from the rent regulated market is given by:

$$IF_{irp}(w) = \int_{y \leq \bar{y}} 1\{v_{ip}(y) \geq \max[v_{im}(y, 0), v_{ir}(y, 0)]\} g_{ir}(y|w) dy \tag{33}$$

Equilibrium in public housing requires that for each housing type i , we have

$$IF_p = \sum_{i=1}^I IF_{ip} = \sum_{i=1}^I OF_{ip} = OF_p \tag{34}$$

Next consider the market for regulated housing. The voluntary flow of type i households

out of rent regulated housing is given by:

$$\begin{aligned}
OF_{ir} &= k_{ir} \sum_{j=0}^{\infty} p(w_j) g_{ir}(w_j) \int_{y \leq \bar{y}} 1\{v_{ip}(y) \geq \max[v_{im}(y, 0), v_{ir}(y, 0)]\} g_{ir}(y|w_j) dy \\
&+ k_{ir} \sum_{j=0}^{\infty} p(w_j) g_{ir}(w_j) \int_{y \leq \bar{y}} 1\{v_{im}(y, 0) \geq \max[v_{ip}(y), v_{ir}(y, 0)]\} g_{ir}(y|w_j) dy \\
&+ k_{ir} \sum_{j=0}^{\infty} (1 - p(w_j)) g_{ir}(w_j) \int_{y \leq \bar{y}} 1\{v_{im}(y, w_j + 1) \geq \max[v_{ir}(y, w_j + 1)]\} g_{ir}(y|w_j) dy \\
&+ k_{ir} \sum_{j=0}^{\infty} g_{ir}(w_j) \int_{y > \bar{y}} 1\{v_{im}(y, 0) \geq \max[v_{ir}(y, 0)]\} g_{ir}(y|w_j) dy
\end{aligned} \tag{35}$$

Note that the first term is the outflow to public housing. The second term is the outflow to unregulated housing if you have an offer to move into public housing. The last two terms are the outflow to unregulated housing if you do not have an offer to move into public housing.

The flow into rent regulated housing is given by:

$$IF_{ir} = k_{im} \sum_{j=0}^{\infty} g_{im}(w_j) IF_{imr}(w_j) + k_{ip} IF_{ipr} \tag{36}$$

where the inflow from the unregulated market conditional on wait time is:

$$\begin{aligned}
IF_{imr}(w_j) &= q_r p(w_j) \int_{y \leq \bar{y}} 1\{v_{ir}(y, 0) \geq \max[v_{im}(y, 0), v_{ip}(y)]\} g_{im}(y|w_j) dy \\
&+ q_r (1 - p(w_j)) \int_{y \leq \bar{y}} 1\{v_{ir}(y, w_j + 1) \geq v_{im}(y, w_j + 1)\} g_{im}(y|w_j) dy \\
&+ q_r \int_{y > \bar{y}} 1\{v_{ir}(y, 0) \geq v_{im}(y, 0)\} g_{im}(y|w_j) dy
\end{aligned} \tag{37}$$

and the flow from public housing market to rent regulated housing is given by:

$$IF_{ipr} = q_r \int 1\{v_{ir}(y, 0) \geq \max[v_{im}(y, 0), v_{ip}(y)]\} g_{ip}(y) dy \tag{38}$$

Equilibrium requires that the aggregate outflow equal the aggregate inflow

$$IF_r = \sum_{i=1}^I IF_{ir} = \sum_{i=1}^I OF_{ir} = OF_r \tag{39}$$

As before, we can define a stationary equilibria with rationing as follows:

Definition 2 *A stationary equilibrium with rationing for the extended model consists of the following: a) market shares (k_{ip}, k_{ir}, k_{im}) $i = 1, \dots, I$, b) offer probability $p(w)$ and q_r , c) distributions $g_{ip}(y)$, $g_{im}(w)$, $g_{ir}(w)$, $g_{im}(y|w)$, and $g_{ir}(y|w)$, and d) value functions $V_{ip}(y)$, $V_{im}(y, w)$ and $V_{ir}(y, w)$, such that:*

1. *Households behave optimally and value functions satisfy the equations above.*
2. *The housing authority behaves according the administrative rules described above.*
3. *The densities are is consistent with the laws of motion and optimal household behavior.*
4. *$p(w)$ satisfies the market clearing condition for public housing:*

$$OF_p = IF_p \quad (40)$$

5. *q_r satisfies the market clearing condition for rent regulated housing:*

$$OF_r = IF_r \quad (41)$$

6. *The following identities hold for the market shares:*

$$\begin{aligned} \sum_{i=1}^I k_{ir} &= k_r \\ \sum_{i=1}^I k_{im} &= k_m \\ k_{ip} + k_{ir} + k_{im} &= s_i \quad i = 1, \dots, I \end{aligned} \quad (42)$$

It is fairly straightforward to extend the law of motions for the equilibrium densities.²⁹

²⁹An appendix is available upon request from the authors that provides the relevant equations.

C Measuring the Discount in Rent Stabilized Housing

To measure the relative price between unregulated and regulated housing, we estimate a hedonic regression using data on housing units in both market. As discussed in Section 4 we assume that the quantity index that relates structural and neighborhood characteristics to housing service flows is constant among the two markets. We can, therefore, use these regressions to measure price differences between regulated and unregulated housing markets.

Table 5: Log Rent Regression

regulated	-0.513***
# of bed rooms	0.124***
# of other rooms	-0.00249
complete kitchen	0.370**
complete plumbing	0.622**
Constant	7.188***
Observations	1416

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The regression also includes dummy variables that indicate whether the building has an elevator, the building age, the building size, a dummy for the fuel type, a dummy for condo/coop, a dummy for bad walls, a unit floor control and household characteristic controls, as well as sub-borough controls. Table 5 summarizes our findings. We find that rent regulated units are, on average, 51 percent cheaper in Manhattan compared to the market rated units.