Coordinating Monetary and Financial Regulatory Policies

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Abstract

How to conduct macro-prudential regulation? How to coordinate monetary policy and macro-prudential policy? To address these questions, I develop a continuous-time New Keynesian economy in which a financial intermediary sector is subject to a leverage constraint. Coordination between monetary and macro-prudential policies helps to reduce the risk of entering into a financial crisis and speeds up exit from the crisis. The downside of coordination is variability in inflation and in the employment gap.

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1 Introduction

The Global Financial Crisis of 2008 has called into question the conduct of monetary policy. Prior to the crisis, traditionally, monetary policy adjusted the short-term nominal interest rate to maintain price stability and sustain full employment. After the crisis, a debate began in academic and policy circles (BIS 2014, 2016; Bernanke 2015; Svensson 2016) concerning whether monetary policy should also respond to financial stability concerns. The crisis also has fostered the development of new policy instruments whose primary objective has been to safeguard financial stability. Those policy instruments are usually referred to as macro-prudential policies, and usually consist of quantity restrictions that target the sector level, such as payment-to-income ratios (PTI) and loan-to-value ratios (LTV) on households, and liquidity coverage ratios (LCR) and capital requirements (CR) on financial institutions.

Should monetary policy and mac ro-prudential policy coordinate to jointly respond to macroeconomic and to financial stability concerns? And if so, should they coordinate all the times, only during times of financial turmoil and deep contraction in economic activity, or only during times of financial booms and rapid economic expansions? What are the costs and benefits of coordinating monetary policy and macro-prudential policy optimally throughout the economic cycle? This paper addresses these questions.

The paper's first contribution is to develop a tractable model economy that is suitable for studying coordination between monetary policy and macro-prudential policy over the multiple phases of the economic cycle. The model is a continuous-time New Keynesian economy in which a financial intermediary sector is subject to an incentive-compatible (IC) leverage constraint. The leverage constraint occasionally binds in equilibrium, giving rise to an endogenous economic cycle that: (*i*) fluctuates continuously in accord with the continuous fluctuations in the intermediaries' aggregate capitalization, and in the gap between potential and actual aggregate output; (*ii*) recurrently transitions along the entire continuum, from good phases of sound financial conditions and high economic activity, i.e. "normal times", to extremely bad phases of severe financial distress and deep economic recession, i.e. "crisis times"; and (*iii*) responds to changes on the phase-contingent rules for monetary policy and for macro-prudential policy. The continuous-time framework adopted for the model economy is useful for capturing the highly nonlinear dynamics in the economic cycle associated with financially constrained agents (Brunnermeier and Sannikov 2014 and Moll 2014).

To study coordination between monetary policy and macro-prudential policy, I restrict attention to two specific policy instruments and two specific policy mandates. Specifically, monetary policy sets the benchmark short-term nominal interest rate, while macro-prudential policy sets a state-contingent capital requirement on financial intermediaries. In a traditional mandate, monetary policy and macro-prudential policy do not cooperate, but instead interact strategically while taking each other's policy rules as given. The objective of monetary policy is to keep inflation and the employment gap stable at their structural levels (i.e., macroeconomic stability); while the objective of macro-prudential policy is to curb the fluctuations in asset prices and in the intermediary aggregates that result from the IC leverage constraint and financial frictions in the economy (i.e., financial stability).[1] In a coordinated mandate, monetary policy and macro-prudential policy share a joint objective, which consists of maximizing social welfare and is consistent with the conjunction of individual objectives under the traditional mandate.

The paper's second contribution is to derive the optimal monetary policy and the optimal macro-prudential policy under each mandate, and to quantitatively assess the costs and benefits of the coordinated mandate relative to the traditional mandate.

In the traditional mandate, I obtain that the optimal monetary policy mimics the natural rate, and the optimal macro-prudential policy replicates the constrained-efficient capital requirement of the counterfactual economy, in which nominal prices are flexible. The natural rate is the short-term real interest rate that accommodates aggregate demand, in the manner required to keep inflation and the employment gap stable at their structural levels of zero. The constrained-efficient capital requirement of the counterfactual flexible price economy restricts intermediary leverage occasionally, only when financial intermediaries, on aggregate, are average capitalized relative to the total wealth in the economy. Restricting intermediary leverage in the manner described above forces financial intermediaries to internalize the pecuniary externalities related to the IC leverage constraint, when the intermediaries, on aggregate, are large enough to have sizeable price effects, but not large enough to have sufficient borrowing capacity to absorb all of the aggregate stock of capital investments locally.

In the coordinated mandate, I obtain that the optimal monetary policy deviates from

[1] Dávila and Korinek (2017) show that in financially constrained economies in which agents are subject to ocassionally binding financing constraints and/or financial contracts are incomplete, pecuniary externalities that operate through asset prices generate excessive fluctuations in macroeconomic aggregates relative to the constrained efficient allocation.

the natural rate, and the optimal macro-prudential policy relaxes the capital requirement relative to the traditional mandate. I also obtain that the optimal monetary policy deviates in accord with the prescriptions of the Greenspan put and of leaning against the wind, but that it relies more heavily on the prescriptions of the latter. The Greenspan put prescribes over stimulating economic activity beyond the flexible price economy benchmark during times of financial distress, while leaning against the wind prescribes slowing economic activity down beyond the same benchmark, but during times of (seemingly) sound financial conditions. Through the lens of the model economy, times of financial distress occur when financial intermediaries, on aggregate, are poorly capitalized — and the aggregate share of intermediated capital is way below its first-best level — while times of sound financial conditions occur when financial intermediaries on aggregate are average to richly capitalized.

Relative to the traditional mandate, and to mimicking the natural rate, deviating from the natural rate in the manner described above helps to improve financial stability. This is because it temporarily boosts economic activity and the intermediation margin precisely when financial intermediaries, on aggregate, are poorly capitalized and need the temporary stimulus the most. Leaning against the wind is particularly useful for further boosting the intermediation margin beyond the stimulus provided by the Greenspan put: Because the price of capital investments is forward-looking, slowing economy activity down in times of sound financial conditions puts downward pressure on the price of capital investment in times of financial distress, which in turn puts upward pressure on the rate of return of capital investments and on the intermediation margin when financial intermediaries are poorly capitalized. Leaning against the wind is not particularly useful for restricting intermediary leverage during times of sound financial conditions, because for that reason there is a capital requirement. The capital requirement softens relative to the traditional mandate, because a binding capital requirement places intermediary leverage and the aggregate share of intermediated capital below their potential capacities in the short term. Softening the capital requirement is evidence that in the model economy, monetary policy and macro-prudential policy are substitutes as far as financial stability is concerned.

Relative to the traditional mandate, deviating from the natural rate nonetheless also generates macroeconomic instability. If deviations from the natural rate are sufficiently small, losses in macroeconomic instability are of second-order importance relative to the gains in financial stability. The reason is that in the traditional mandate, inflation and the employment gap remain stable at their structural levels, while the aggregate share of intermediated capital, in general, does not remain stable at its first-best level.

I then calibrate the model economy to quantify the costs and benefits of the coordinated mandate relative to the traditional mandate. In the baseline calibration, I obtain that social welfare gains from the coordinated mandate over the traditional mandate amount to 0.07% in terms of annual consumption equivalent. I obtain also that gains from improving on financial stability amount to 0.11%, while losses from worsening on macroeconomic stability amount to 0.04%.

Related Literature This paper relates to a body of literature that studies the interaction of and coordination between monetary policy and macro-prudential policy. A group of papers in this literature — for instance, Angelini et al. (2012) and Gelain and Ilbas (2017), among others — specifies policy mandates that are grounded in macroeconomic aggregates (such as inflation, output gap, credit growth, and so on), but not necessarily grounded in social welfare. Another group of papers in this literature — e.g., Bailliu et al. (2015) and Carrillo et al. (2017), among others — restricts attention to simple policy rules such as Taylor rules. This paper differentiates from the papers in these two groups by considering policy mandates that are grounded in social welfare, and general policy rules whose only restriction is to be polynomial functions of the aggregate state.

De Paoli and Paustian (2017), Collard et al. (2017), and Farhi and Werning (2016) follow a similar approach to this paper concerning the specification of policy mandates and policy rules.[2] The main difference with respect to De Paoli and Paustian (2017) is that in their model economy the financing constraint always binds. Occasionally binding financing constraints, in general, are useful for analyzing the effects of policies that are truly prudential in nature. The main difference with respect to Collard et al. (2017) is that in my model, economic cycles are fueled by feedback loops and pecuniary externalities whose strength depend on policy. Brunnermeier and Sannikov (2014, 2016) underscore the importance of endogenous feedback loops in driving nonlinear dynamics in financial and macroeconomic variables. The main difference with respect to Farhi and Werning (2016) is that I consider pecuniary externalities in financial markets that operate through asset prices and/or asset returns, while they consider market failures that result from aggregate demand externalities.

This paper also relates to a body of literature that studies whether monetary policy

[2] To be more precise concerning the specification of the policy rules, none of those papers place any restrictions on their domain.

should lean against the wind of credit booms and financial imbalances. Most of the papers in this literature — for instance, Svensson (2016), Ajello et al. (2016), and Gourio et al. (2017), among others — consider an economic cycle that has only two stages: "normal times" and "crisis times." A notable exception is Filardo and Rungcharoenkitkul (2016), who introduce an endogenous economic cycle with an arbitrarily large number of stages, into an otherwise standard quadratic-function-loss model for the stabilization problem of monetary policy. The main difference with respect to the papers in this literature is that, in this paper, the economic cycle is microfounded, being the microfoundation based on the leverage behavior of financial intermediaries; in constrast, those papers model the economic cycle in reduced-form. The microfoundation of the economic cycle in this paper is critical for assessing the benefits of leaning against the wind.

This paper relates also to a body of literature that studies optimal macro-prudential policy in the context of a flexible price economy. The theoretical foundation for macro-prudential policy is to correct pecuniary externalities and general failures in financial markets, that may pose threats to the stability of the financial system as a whole (Hanson et al. 2011). This paper contributes to the literature by identifying a new type of pecuniary externality, which differs from existing distributive and collateral externalities identified by Dávila and Korinek (2017). This new type of pecuniary externality, which I refer to as the *dynamic pecuniary externality*, arises when individual agents can also affect the dynamic behavior of asset prices and/or asset returns. The dynamic pecuniary externality is not considered by Dávila and Korinek (2017), because in their model economy asset prices and macroeconomic aggregates display no dynamic behavior.

On methodological grounds, my model economy builds on the works of Calvo (1983), Brunnermeier and Sannikov (2014, 2016), Gertler and Karadi (2011), Gertler and Kiyotaki (2010), and Maggiori (2017). The main difference with respect to Brunnermeier and Sannikov (2016) is that in my model, money serves the role of a unit of account, whereas in their model money serves the role of a store of value. As in Drechsler et al. (2017), my model economy is a continuous-time production economy with nominal rigidities and financial frictions; in constrast to their paper, however, in my model nominal rigidities are grounded on the sluggish nominal price adjustments of firms, as in the New Keynesian framework.

Layout Section 2 develops the model economy. Section 3 solves for the competitive equilibrium of the model economy. Section 4 defines the policy mandates. Section 5 derives the optimal monetary policy and the optimal macro-prudential policy under the traditional mandate. Section 6 repeats the same exercise, but for the coordinated mandate. Section 7 quantitatively assesses the costs and benefits of the coordinated mandate relative to the traditional mandate. Section 8 concludes.

2 The Model

The model is a continuous-time New Keynesian economy in which a financial intermediary sector is subject to a leverage constraint. The specification for the sluggish nominal price adjustments of firms, which is the key feature of the New Keynesian framework, follows the work of Calvo (1983). The setup of financial intermediation builds on the works of Brunnermeier and Sannikov (2014), Gertler and Karadi (2011), Gertler and Kiyotaki (2010), and Maggiori (2017).

2.1 Production in Goods Markets and Price-setting Behavior

In the model economy, there is a continuum of firms that produce a continuum of intermediate good varieties $y_{j,t}$, with $j \in [0,1]$, using labor $l_{j,t}$ and capital services $k_{j,t}$ as inputs. Each firm produces a single intermediate good variety using a Cobb-Douglas production technology:

$$y_{j,t} = A_t l_{j,t}^{\alpha} k_{j,t}^{1-\alpha}, \tag{1}$$

that has a common labor share of output α and a common productivity factor A_t across $j \in [0,1]$. The productivity factor A_t is exogenous and evolves stochastically according to the Ito process:

$$dA_t/A_t = \mu_A dt + \sigma_A dZ_t, \tag{2}$$

with drift process μ_A and diffusion process $\sigma_A > 0$, being $\{Z_t \in \mathbb{R} : t \geq 0\}$ a standard Brownian process defined on a filtered probability space $(\Omega, \mathcal{F}, P)$. Intuitively, the Brownian shock dZ_t is an i.i.d. shock to the growth rate of aggregate productivity that is normally $(0,1)$ distributed. The shock dZ_t is the only source of risk in the model economy.

To produce their intermediate good variety, firms hire labor and rent capital services in competitive markets at the real wage rate of w_t and at the real rental rate of $r_{k,t}$, respectively. Firms combine labor and capital services optimally to minimize their production costs $x_t\left(y_{j,t}\right)$, which end up being:

$$x_t\left(y_{j,t}\right) = \frac{1}{A_t}\left(\frac{w_t}{\alpha}\right)^{\alpha}\left(\frac{r_{k,t}}{1-\alpha}\right)^{1-\alpha} y_{j,t}. \tag{3}$$

In the intermediate goods markets, firms compete monopolistically with each other, resetting their nominal price $p_{j,t}$ sluggishly according to Calvo (1983) pricing. Each firm faces an indirect demand function $y_{d,t}\left(p_{j,t}\right) \equiv \left(p_{j,t}/p_t\right)^{-\varepsilon} y_t$, which follows from a constant-elasticity-of-substitution (CES) aggregator,

$$y_t = \left[\int_0^1 y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{4}$$

that aggregates $\{y_{j,t}\}_{j\in[0,1]}$ into a final consumption good y_t optimally given $\{p_{j,t}\}_{j\in[0,1]}$, being $\varepsilon > 1$ the elasticity of substitution across the intermediate goods in the CES aggregator. The nominal price p_t measures the minimum cost required to produce one unit of the final consumption good, equals the consumer price index:

$$p_t = \left[\int_0^1 p_{j,t}^{1-\varepsilon} dj\right]^{\frac{1}{1-\varepsilon}}; \tag{5}$$

and it can therefore be interpreted as the aggregate price level.

Price-setting Problem In the Calvo (1983) pricing specification, firms have the opportunity to reset their nominal price only when they are hit by an idiosyncratic Poisson shock that has a common arrival rate θ across firms.[3] When they have the opportunity to reset their nominal price, firms maximize the present discounted value of the profits flows, that follow from fixing their nominal price at $p_{j,t}$:

$$\max_{p_{j,t}>0} E_t \int_t^{\infty} \theta e^{-\theta(s-t)} \frac{\Lambda_s}{\Lambda_t}\left[\left(1-\tau\right)\frac{p_{j,t} y_{d,s}\left(p_{j,t}\right)}{p_s} - x_s\left[y_{d,s}\left(p_{j,t}\right)\right]\right] ds\ . \tag{6}$$

I assume that firms discount future profit flows with the Stochastic Discount Factor (SDF) of households Λ_t — weighted, of course, by the survival density function $\theta e^{-\theta(s-t)}$ of their fixed nominal price. The SDF Λ_t is an endogenous object to be determined in equilibrium.

[3] Additionally, in the Calvo (1983) pricing specification, firms pay no "menu" cost for resetting their nominal price, and the firms that cannot reset their price must accommodate their indirect demand at the prevailing market price.

The coefficient τ is an advalorem sales subsidy on firms.

Optimal Prices in Goods Markets Firms that have the opportunity to reset their nominal price set the same optimal price $p_{*,t}$, because their price-setting problems (6) are identical. The optimal real price $p_{*,t}/p_t$ is the product of two factors:

$$p_{*,t}/p_t = \underbrace{\frac{\varepsilon}{(1-\tau)(\varepsilon-1)}}_{=1} \frac{E_t \int_t^{\infty} \theta e^{-\theta(s-t)} \frac{\Lambda_s}{\Lambda_t} x_s \left[y_{d,s}\left(p_t\right)\right] ds}{E_t \int_t^{\infty} \theta e^{-\theta(s-t)} \frac{\Lambda_s}{\Lambda_t} \frac{p_t y_{d,s}(p_t)}{p_s} ds} . \tag{7}$$

The first factor is the product of a sales subsidy multiplier $1/(1-\tau)$ and a distortion coefficient from monopoly pricing $\varepsilon/(\varepsilon-1)$. I impose that $\tau = -1/(\varepsilon-1)$ to eliminate the distortions from monopoly pricing. This implies that firms set competitive prices. The second factor is the ratio of the present discounted value of production costs to that of sales revenues (gross of sales subsidies) of a hypothetical firm that charges a nominal price equal to the aggregate price level p_t. The second factor is forward-looking, because firms can reset their nominal price only sporadically: If they could instead reset their price continuously, i.e., $1/\theta \to 0$, the second factor would reduce to the spot marginal production costs $x_t(y_j)/y_j$.

2.2 Investment Portfolios and Financial Intermediation

In the model economy, there is also a continuum of financial intermediaries and a continuum of households. Households are the residual claimants of the profits flows that firms make and of the dividends flows that financial intermediaries pay out.

To create a meaningful role for financial intermediation, I assume that financial intermediaries have a comparative advantage relative to households for providing capital services to firms. The capital services that firms use in production are made out of physical capital, which is a real asset in positive fixed supply. Financial intermediaries transform physical capital into capital services at a one-to-one rate, whereas households do it at a rate $a_h < 1$. The productivity gap $1 - a_h$ can be rationalized as a productivity difference that originates from a moral hazard problem in equity markets, in which: (*i*) neither financial intermediaries nor households directly hold physical capital; (*ii*) the direct holders of physical capital issue equity shares against the present discounted value of the profit flows made from renting the capital services to firms; and (*iii*) the shareholders can monitor the activities of the equity issuers to induce the latter to provide net present value, having financial intermediaries a comparative advantage at monitoring relative to households.[4] The productivity gap $1 - a_h$ is the only reason financial intermediaries provide value in the model economy.

Physical capital is tradable, being all of the aggregate capital stock $\bar{k}$ traded in fully liquid markets at the spot real price of $q_t\bar{k}$. By raising deposits b_t from households, financial intermediaries can take positions $q_t\bar{k}_{f,t} = b_t + n_{f,t}$ on physical capital, beyond the limits given by their own net worth $n_{f,t}$. To create a meaningful link between aggregate intermediary net worth and the real economy, I assume that financial intermediaries are subject to a limited enforcement problem, that limits deposits b_t and levered capital positions $q_t\bar{k}_{f,t}$ according to:

$$q_t\bar{k}_{f,t} = b_t + n_{f,t} \leq \lambda V_t, \tag{8}$$

being $\lambda > 1$ a real number, and V_t the franchise value of the financial intermediary company. The limited enforcement problem is such that financial intermediaries can divert a share $1/\lambda$ of their assets, at the expense of losing access to their intermediary company. For this problem to be relevant, I assume that each financial intermediary is owned by a single household, and that each household deposits funds with financial intermediaries other than the one they own. In the incentive-compatible (IC) constraint (8), deposits b_t are also bounded from above, because financial intermediaries cannot issue equity, which ensures that $n_{f,t} \geq 0$. Later in the paper, I show that $V_t \equiv v_t n_{f,t}$ is proportional to net worth $n_{f,t}$, with $v_t \geq 1$, which delivers the linear incentive-compatible (IC) constraints $b_t \leq (\lambda v_t - 1)\, n_{f,t}$ and $0 \leq q_t\bar{k}_{f,t} \leq \lambda v_t n_{f,t}$, and the corresponding linear upper bounds on b_t and $q_t\bar{k}_{f,t}$.

Let $dR_{e,t}$, with $e = \{f, h\}$, denote the rates on return on physical capital that financial intermediaries (f) and households (h) earn. The rates $dR_{e,t}$ are the sum of the specific dividend yields that agents obtain and the common capital gain/loss rate dq_t/q_t :

$$dR_{e,t} \equiv \left[a_h \mathbf{1}_{e=h} + 1 - \mathbf{1}_{e=h}\right] \frac{r_{k,t}}{q_t} dt + \frac{dq_t}{q_t} \text{ , with } e = \{f, h\}\,.$$

Because financial intermediaries earn higher rates of return on physical capital relative to households, i.e., $dR_{f,t} > dR_{h,t}$, financial intermediaries would eventually accumulate enough net worth to grow out of the IC constraint if they were to not pay out dividends

[4] See Appendix A for details.

sufficiently often. To avoid that scenario, I assume that financial intermediaries pay out dividends according to an idiosyncratic Poisson process that has a common arrival rate of γ across them. I also assume that when financial intermediaries pay out dividends, they transfer all of their net worth to the households, and that after the dividend payout, financial intermediaries automatically receive a share κ/γ of the aggregate capital stock as a start-up endowment from the households. Financial intermediaries must receive a positive endowment after paying out dividends, because without net worth they cannot issue deposits or operate.

I postulate that the capital gain/loss rate dq_t/q_t, and therefore the rates of return $dR_{e,t}$, are locally risky:

$$\frac{dq_t}{q_t} = \mu_{q,t}dt + \sigma_{q,t}dZ_t,$$

being the drift and the diffusion processes $\mu_{q,t}$ and $\sigma_{q,t}$ endogenous objects to be determined in equilibrium. I also postulate that the inflation rate dp_t/p_t, and hence the real deposit rate $i_t dt - dp_t/p_t$, are locally risk-free:

$$\frac{dp_t}{p_t} = \pi_t dt + 0dZ_t,$$

being the expected inflation rate $\pi_t dt \equiv E_t\left[dp_t/p_t\right]$ an endogenous object to be determined in equilibrium. Deposits by design are short-term nominal debt contracts that pay out a locally risk-free nominal rate of return of $i_t dt$. These two postulates ensure that financial intermediaries are subject to a liquidity mismatch problem in their balance sheets, and hence that financial intermediaries concentrate aggregate risk when they take on leverage. These two postulates will be consistent with the conditions that characterize the competitive equilibrium.

2.3 Portfolio Problems

Intermediaries' Portfolio Problem The objective of financial intermediaries is to maximize the present discounted value of their dividend payouts. I assume that financial intermediaries discount future dividend payouts with the SDF of the household Λ_t, weighted by the probability density function $\gamma e^{-\gamma(s-t)}$ of paying out dividends.

To incorporate macro-prudential policy in the analysis, I assume that financial intermediares are subject to an additional leverage constraint, that restricts $q_t\bar{k}_{f,t}$ according to:

$$q_t \bar{k}_{f,t} \leq \Phi_t n_{f,t}, \tag{9}$$

being $\Phi_t \geq 1$ a common capital requirement across financial intermediaries. The capital requirement Φ_t is contingent on the aggregate state and indicates the stance of macro-prudential policy. Financial intermediaries take Φ_t as given.

Financial intermediaries solve the portfolio problem:

$$\begin{aligned} V_t \quad \equiv \quad & \max_{\bar{k}_{f,t} \geq 0} E_t \int_t^\infty \gamma e^{-\gamma(s-t)} \frac{\Lambda_s}{\Lambda_t} n_{f,s} ds \\ & \textit{subject to}: n_{f,t} \geq 0,\ (8),\ (9),\ (11)\ , \end{aligned} \tag{10}$$

being (11) the condition that describes the evolution of the intermediary net worth,

$$dn_{f,t} = dR_{f,t} q_t \bar{k}_{f,t} - (i_t - \pi_t) \left(q_t \bar{k}_{f,t} - n_{f,t} \right) dt. \tag{11}$$

The value $V_t \equiv v_t n_{f,t}$ is proportional to net worth $n_{f,t}$ because the portfolio problem (10) is linear. The marginal value of wealth v_t is common to all financial intermediaries, and therefore can be interpreted as Tobin's Q.

Leverage Multiple and Tobin's Q In Appendix B, I show that the value $\Lambda_t V_t = \Lambda_t v_t n_{f,t}$ satisfies a standard Hamilton-Jacobi-Bellman (HJB) equation, which delivers two optimality conditions.[5]

The first optimality condition is an asset pricing condition for physical capital that can be represented accordingly:

$$E_t \left[dR_{f,t} \right] - (i_t - \pi_t) \, dt + Cov_t \left[d\Lambda_t / \Lambda_t + dv_t / v_t, dR_{f,t} \right] \geq 0\ , \tag{12}$$

with equality if the leverage constraint $\phi_t \equiv q_t \bar{k}_{f,t} / n_{f,t} \leq \min \left\{ \lambda v_t, \Phi_t \right\}$ is slack.

The LHS in (12) is the (expected) risk-adjusted excess return on capital over deposits that financial intermediaries earn. When they earn a positive risk-adjusted excess return, financial intermediaries strictly prefer physical capital to deposits, and take levered positions on physical capital until they hit their leverage constraint. When they earn a null risk-adjusted excess return, financial intermediaries are indifferent between physical capital

[5] To derive the HJB equation, I conjecture that processes v_t and Λ_t evolve stochastically according to Ito processes.

and deposits, and are willing to take any leverage multiple ϕ_t.[6] Financial intermediaries are concerned with comovement between the percentage change in their marginal value of wealth dv_t/v_t and the rate of return $dR_{f,t}$ (and therefore demand compensation for holding capital risk that differs from the usual compensation a representative household with an SDF of Λ_t would demand), because they are subject to a leverage constraint.

The second optimality condition is an asset pricing condition for v_t that can be represented accordingly:

$$\tilde{E}_t\left[dR_{n_f,t}\right] + \frac{\gamma}{v_t}dt + E_t\left[dv_t/v_t\right] - \gamma dt + Cov_t\left[d\Lambda_t/\Lambda_t, dv_t/v_t\right] = 0\ , \tag{13}$$

with[7]

$$\tilde{E}_t\left[dR_{n_f,t}\right] \equiv E_t\left[dn_{f,t}/n_{f,t}\right] - \left(i_t - \pi_t\right)dt + Cov_t\left[d\Lambda_t/\Lambda_t + dv_t/v_t, dn_{f,t}/n_{f,t}\right].$$

The conditional expectation $\tilde{E}_t\left[dR_{n_f,t}\right]$ is the (expected) risk-adjusted excess return on net worth over deposits that financial intermediaries earn. It equals the product of the leverage multiple ϕ_t and the (expected) risk-adjusted excess return on capital in (12). The conditional expectation $\tilde{E}_t\left[dR_{n_f,t}\right]$ enters as a dividend yield component in the asset pricing condition for v_t. The value v_t can therefore be interpreted as a present discounted value of the marginal profit flows that financial intermediaries make.

Households' Portfolio Problem To close the model economy, I specify the portfolio problem of households. Households choose their consumption c_t, labor supply l_t, and investment portfolio. Households are subject to no leverage constraints; their objective is to maximize the present discounted value of their utility flows:

$$E_t \int_t^\infty e^{-\rho(s-t)}\left[\ln c_s - \chi\frac{l_s^{1+\psi}}{1+\psi}\right]ds, \tag{14}$$

being ρ the time discount rate; χ the weight assigned to the disutility from labor; and ψ the inverse of the Frish elasticity of the labor supply. Households have logarithmic preferences for consumption, which implies that their SDF is $\Lambda_t \equiv e^{-\rho t}/c_t$.

[6] Financial intermediaries cannot earn a negative risk-adjusted excess return; otherwise, they would not be willing to take levered positions on physical capital.

[7] The expression in (13) assumes that $\left(i_t - \pi_t\right)dt = -E_t\left[d\Lambda_t/\Lambda_t\right]$. This latter condition follows from the optimality conditions in the households' portfolio problem.

Households solve a standard portfolio problem, which consists of maximizing (14) subject to $c_t, l_t, \bar{k}_{h,t} \geq 0$ and to the evolution of their net worth,

$$dn_{h,t} = dR_{h,t}q_t\bar{k}_{h,t} + (i_t - \pi_t)\left(n_{h,t} - q_t\bar{k}_{h,t}\right)dt + w_t l_t dt + Tr_t dt - c_t dt, \tag{15}$$

being $n_{h,t}$ the net worth of households; $\bar{k}_{h,t}$ the position households take on physical capital; and Tr_t the net transfers households receive from firms and financial intermediaries. The position $n_{h,t} - q_t\bar{k}_{h,t}$ is the funds households deposit with financial intermediaries.

Consumption, Labor, and Savings In Appendix B, I show that the value of households $U_t \equiv \max\left\{(14) : c_t, l_t, \bar{k}_{h,t} \geq 0 \wedge (15)\right\}$ satisfies a standard HJB equation, which delivers three optimality conditions.

The first optimality condition is an intra-temporal condition between consumption and labor:

$$\frac{1}{c_t}w_t = \chi l_t^{\psi}. \tag{16}$$

The second optimality condition is an asset pricing condition for deposits that can be represented accordingly:

$$(i_t - \pi_t)\,dt = -E_t\left[d\Lambda_t/\Lambda_t\right] \equiv \rho dt + E_t\left[dc_t/c_t\right] - Var_t\left[dc_t/c_t\right]. \tag{17}$$

This condition implies that households match their expected utility return from consumption to the real deposit rate, and that households are therefore indifferent on the margin between consumption and deposits.

The third optimality condition is an asset pricing condition for physical capital that can be represented accordingly:

$$E_t\left[dR_{h,t}\right] - (i_t - \pi_t)\,dt + Cov_t\left[d\Lambda_t/\Lambda_t, dR_{h,t}\right] \leq 0\ , \tag{18}$$

with equality if $\bar{k}_{h,t} > 0$.

The LHS in (18) is the (expected) risk-adjusted excess return on capital over deposits that households earn. When they earn a null risk-adjusted excess return, households are indifferent on the margin between capital and deposits, and therefore they are willing to take a capital position $\bar{k}_{h,t} \geq 0$. When they earn a negative risk-adjusted excess return, households strictly prefer on the margin deposits to capital, and therefore $\bar{k}_{h,t} = 0$.[8] Because they are subject to no leverage constraint, households demand compensation for holding capital risk, which is based only on consumption risk.

2.4 Competitive Equilibrium

The definition of the competitive equilibrium takes monetary policy i_t and macro-prudential policy Φ_t as given. Monetary policy sets the benchmark short-term nominal interest rate, which in equilibrium is perfectly arbitraged with nominal deposit rate i_t, because the implementation mechanism of monetary policy is the same as in the New Keynesian framework.[9] The definition of the competitive equilibrium is based on: the existence of a representative financial intermediary, the existence of a representative household, and an indexation of firms that labels firms according to the last time they had the opportunity to reset their nominal price.[10] To economize in notation, in what follows I make no distinction between individual and aggregate variables. I refer to firms that had the opportunity to reset their nominal price for the last time at a time $s \leq t$ as the firms (s, t).

Definition 1 A competitive equilibrium is a set of stochastic processes adapted to the filtration generated by Z : the real wage rate $\{w_t\}$; the real rental rate of capital services $\{r_{k,t}\}$; the aggregate price level $\{p_t\}$; the inflation rate $\{\pi_t\}$; the real price of capital $\{q_t\}$; the optimal nominal price $\{p_{*,t}\}$; the intermediate good that each firm (s, t) produces $\{y_{s,t}\}$; the quantity of labor that each firm (s, t) employs $\{l_{s,t}\}$; the units of capital services that each firm (s, t) employs $\{k_{s,t}\}$; the final consumption good $\{y_t\}$; labor $\{l_t\}$; the capital position of households $\left\{\bar{k}_{h,t}\right\}$; the capital position of financial intermediaries $\left\{\bar{k}_{f,t}\right\}$; the leverage multiple $\{\phi_t\}$; the marginal value of wealth $\{v_t\}$; productivity $\{A_t\}$; the policy rate $\{i_t\}$; and the macro-prudential capital requirement $\{\Phi_t\}$, such that:

1. $\{l_{s,t}, k_{s,t}\}_{s \leq t}$ are consistent with the labor and capital services demand functions related to the cost function (3) ;

[8] Households cannot earn a positive risk-adjusted excess return, because they are not subject to portfolio constraints. If they were to obtain a positive risk-adjusted excess return, they would take unbounded levered positions on capital, and $\bar{k}_{h,t} = +\infty$.

[9] See Clarida, Galí, and Gertler (1999) for a reference.

[10] A representative financial intermediary exists because the leverage multiple ϕ_t and marginal value of wealth v_t do not depend on individual net worth $n_{f,t}$. A representative household exists because households are identical.

2. $\left\{ \{l_{s,t}, k_{s,t}, y_{s,t}\}_{s\leq t}, y_t \right\}$ are consistent with production functions (1) and (4);

3. $\left\{ \{p_{*,s}\}_{s\leq t}, p_t \right\}$ are consistent with the consumer price index (5);

4. $\{p_{*,t}\}$ satisfies the optimality condition (7) in the price-setting problem of firms;

5. $\{\phi_t, v_t\}$ satisfy optimality conditions (12) and (13) in the intermediaries' portfolio problem;

6. $\{y_t, l_t, \bar{k}_{h,t}\}$ satisfy optimality conditions (16), (17), and (18) in the households' portfolio problem;

7. The labor market, the rental market for capital services, and the market for physical capital, clear:

$$\int_{-\infty}^{t} \theta e^{-\theta(t-s)} l_{s,t} ds = l_t \ ; \quad \int_{-\infty}^{t} \theta e^{-\theta(t-s)} k_{s,t} ds = a_h \bar{k}_{h,t} + \bar{k}_{f,t} \ ; \text{ and } \ \bar{k}_{h,t} + \bar{k}_{f,t} = \bar{k} \ .$$

In equilibrium, because a law of large numbers applies, the aggregate share of firms (s,t) equals the survival density function $\theta e^{-\theta(t-s)}$ of the optimal nominal price $p_{*,s}$. Aggregate consumption c_t equals aggregate output y_t because there is no investment technology or fiscal policy. The market for deposits automatically clears because of Walras Law.

3 Equilibrium Results

I summarize the key features of the competitive equilibrium with the following four results.

3.1 The Leverage Multiple and Equilibrium Regions

Result 1 The leverage constraint occasionally binds in equilibrium. It binds when financial intermediaries on aggregate lack enough borrowing capacity to absorb all of the aggregate capital stock. It is slack otherwise.

Let $\eta_t \equiv n_{f,t}/q_t\bar{k} \in [0,1]$ denote the wealth share of financial intermediaries. The total wealth in the economy, i.e., $n_{f,t} + n_{h,t}$, equals $q_t\bar{k}$ because physical capital is the only real asset. Financial intermediaries lack enough borrowing capacity to absorb all of the aggregate capital stock when $\min\{\lambda v_t, \Phi_t\}\eta_t < 1$; they have enough borrowing capacity to absorb all of the aggregate capital stock when $\min\{\lambda v_t, \Phi_t\}\eta_t \geq 1$.

When $\min\{\lambda v_t, \Phi_t\}\eta_t < 1$, households hold a positive amount of physical capital in equilibrium, and are therefore indifferent on the margin between physical capital and deposits. Financial intermediaries strictly prefer physical capital to deposits,[11] hit their leverage constraint, and $\phi_t = \min\{\lambda v_t, \Phi_t\}$. When $\min\{\lambda v_t, \Phi_t\}\eta_t \geq 1$, financial intermediaries are indifferent between deposits and physical capital, and households therefore strictly prefer deposits to physical capital on the margin. Households hold no physical capital in equilibrium, financial intermediaries hold all of the aggregate capital stock, and $\phi_t = 1/\eta_t \leq \min\{\lambda v_t, \Phi_t\}$.

Both regions $\min\{\lambda v_t, \Phi_t\}\eta_t < 1$ and $\min\{\lambda v_t, \Phi_t\}\eta_t \geq 1$ are always feasible in equilibrium because $v_t \geq 1$. Tobin's Q v_t is never below 1, because the Tobin's Q of a hypothetical financial intermediary that can invest only in deposits is always 1. This is because households are always indifferent on the margin between consumption and deposits.

3.2 Pecuniary Externalities and Their Implications for Leverage

Result 2 The competitive equilibrium has three distinct types of pecuniary externalities: distributive, binding constraint, and dynamic externality. The existence of pecuniary externalities implies that the competitive equilibrium, if left unregulated, is constrained inefficient.

All three types of pecuniary externalities follow from the combination of deposit contracts and occasionally binding leverage constraints. The rationale behind the pecuniary externalities are as follows.

Distributive Pecuniary Externality Deposit contracts prevent financial intermediaries from off-loading the aggregate risk in their levered capital positions to households. Because financial intermediaries concentrate aggregate risk in their balance sheets, the net worth gain/loss rate $dn_{f,t}/n_{f,t}$ responds more than the rate of return $dR_{f,t}$ and than the capital gain/loss rate dq_t/q_t to the shock dZ_t.[12] The leverage constraint, when it binds,

[11] Financial intermediaries have to strictly prefer physical capital to deposits in this region, because otherwise more asset pricing conditions would be holding with equality than endogenous processes to be determined in equilibrium.

[12] Specifically, $dn_{f,t}/n_{f,t}$ responds to dZ_t according to $\sigma_{n_f,t} = \phi_t \sigma_{q,t}$ while dR_f and dq_t/q_t respond to dZ_t according to $\sigma_{q,t}$. In equilibrium, $\sigma_{q,t} > 0$ because q_t is a present discounted value of the future rental

forces financial intermediaries to maintain capital positions proportional to net worth. The constrained capital positions $q_t\bar{k}_{f,t} = \min\{\lambda v_t, \Phi_t\}\, n_{f,t}$, together with the relatively larger response of $dn_{f,t}/n_{f,t}$ to dZ_t, force a positive response of the intermediary capital rate $d\bar{k}_{f,t}/\bar{k}_{f,t}$ to dZ_t, which triggers a positive feedback loop and a distributive pecuniary externality:[13]

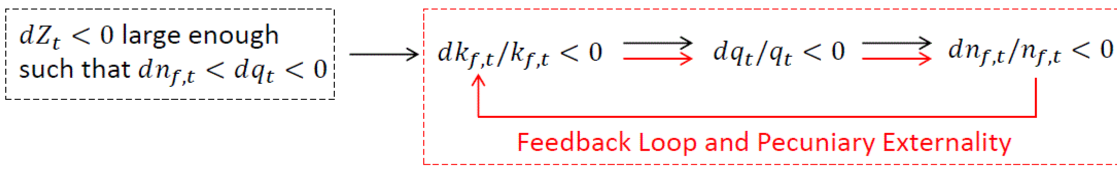

FIGURE 1: SCHEMATIC REPRESENTATION OF THE DISTRIBUTIVE PECUNIARY EXTERNALITY

The feedback loop has a built-in distributive pecuniary externality, because financial intermediaries do not internalize the effects of their leverage decision on the capital gain/loss rate dq_t/q_t or on the net worth gain/loss rate $dn_{f,t}/n_{f,t}$, of the other financial intermediaries in the economy. Notice that financial intermediaries take the rates $dR_{f,t}$ and dq_t/q_t as given in their portfolio problem (10).

Binding-constraint Pecuniary Externality The intermediary borrowing capacity is $\min\{\lambda v_t, \Phi_t\}$. I restrict attention to competitive equilibria in which the IC borrowing capacity λv_t occasionally binds. In those competitive equilibria, there is also a binding-constraint pecuniary externality, because the Tobin's Q is inversely related to the price of capital, and because individual financial intermediaries take the price of capital q_t as given in their portfolio problem (10). The Tobin's Q is inversely related to q_t, because the dividend yield $r_{k,t}/q_t$, the rate of return $dR_{f,t}$, and the expected risk-adjusted excess return

rates $\{r_{k,s}\}_{s>t}$ and therefore also of the future productivity levels $\{A_s\}_{s>t}$.

[13] In the feedback loop (i.e., the box on the RHS in Figure 1), dq_t/q_t positively responds to changes in $d\bar{k}_{f,t}/\bar{k}_{f,t}$, because financial intermediaries have a larger valuation for physical capital than households in equilibrium. The rate $dn_{f,t}/n_{f,t}$ positively depends on $dR_{f,t}$ and therefore also on dq_t/q_t. The rate $d\bar{k}_{f,t}/\bar{k}_{f,t}$ positively responds to changes in $dn_{f,t}/n_{f,t}$ because $q_t\bar{k}_{f,t} = \min\{\lambda v_t, \Phi_t\}\, n_{f,t}$ and because $dn_{f,t}/n_{f,t}$ is linear on $\phi_t dq_t/q_t$.

The borrowing capacity $\min\{\lambda v_t, \Phi_t\}$ responds to changes in dZ_t, but its response is of second-order importance.

$\tilde{E}_t\left[dR_{n_f,t}\right]$ are inversely related to q_t.[14]

Dynamic Pecuniary Externality The dynamic pecuniary externality is the dynamic counterpart of the distributive and binding-constraint externalities. Specifically, because financial intermediaries do not internalize the effect of their leverage decision on the dynamic behavior of aggregate variables such as η_t and v_t, neither do financial intermediaries internalize the effects of their leverage decisions on the share of time the economy spends in each equilibrium region, and on the strength of the aforementioned two externalities.

3.3 The Aggregate Production Function

Result 3 The competitive equilibrium admits an aggregate production function. The endogenous total factor productivity (TFP) in the aggregate production function determines the gap between potential and actual aggregate output, as well as the phase of the economic cycle.

In equilibrium, the aggregate production function is Cobb-Douglas:

$$y_t = \zeta_t l_t^{\alpha} \bar{k}^{1-\alpha}, \text{ with } \zeta_t \equiv A_t a_t^{1-\alpha}/\omega_t.^{15}$$

The inputs in the aggregate production function are aggregate labor l_t and the aggregate stock of physical capital $\bar{k}$. The labor share of output α and the exogenous productivity factor A_t are the same as in the individual production function of firms. The endogenous TFP is $\zeta_t/A_t \leq 1$. The endogenous productivity factor a_t is:

$$a_t \equiv a_h \bar{k}_{h,t}/\bar{k} + \bar{k}_{f,t}/\bar{k} = a_h\left(1 - \phi_t \eta_t\right) + \phi_t \eta_t.$$

The factor $a_t^{1-\alpha}$ measures the extent to which allocative efficiency problems in financial markets hinder economic activity. It brings the excessive fluctuations that result from pecuniary externalities into aggregate output. The endogenous productivity factor ω_t is

[14] Intuitively, a higher price of capital depresses the rate of return on physical capital, and therefore also intermediaries' excess returns and profit flows.

[15] The aggregate production function follows from replacing the inputs demand functions of firms into the market clearing conditions for inputs. The inputs demand functions of firms are consistent with the cost function in (3).

the consumption-based measure of quantity dispersion on intermediate goods:

$$\omega_t \equiv \int_{-\infty}^{t} \theta e^{-\theta(t-s)} \frac{y_{s,t}}{y_t} ds = \int_{-\infty}^{t} \theta e^{-\theta(t-s)} \left(\frac{p_{*,s}}{p_t}\right)^{-\varepsilon} ds. \tag{19}$$

The factor ω_t measures the quantity of the final consumption good that could have been produced relative to the actual quantity y_t if the aggregate quantity of intermediate goods $\omega_t y_t$ had been evenly allocated across intermediate-goods varieties. Jensen's inequality implies that $\omega_t \geq 1$, and hence that quantity dispersion across intermediate goods is inefficient. The indirect demand function $y_{d,t}(p_{*,s})$ implies that ω_t can be interpreted as the consumption-based measure of price dispersion.

3.4 The Labor Wedge, Optimal Prices, and Inflation Rate

Result 4 In equilibrium, a labor wedge exists only if, and when, real optimal prices $\{p_{*,s}/p_t\}_{s\leq t}$ deviate from spot marginal production costs $x_t(y_j)/y_j$. Optimal real prices $\{p_{*,s}/p_t\}_{s\leq t}$ may deviate from $x_t(y_j)/y_j$, depending on the behavior of the cost-revenue ratio b_t/m_t.

Let B_t denote the numerator on the RHS of $p_{*,t}/p_t$ in (7). Let M_t denote the corresponding denominator. Processes B_t and M_t satisfy in equilibrium that $B_t/\theta y_t = b_t$ and $M_t/\theta y_t = m_t$, with:[16]

$$b_t \equiv E_t \int_t^{\infty} e^{-(\theta+\rho)(s-t)} \frac{x_s(y_j)}{y_j} \left(\frac{p_t}{p_s}\right)^{-\varepsilon} ds\ ,$$

$$m_t \equiv E_t \int_t^{\infty} e^{-(\theta+\rho)(s-t)} \frac{p_t}{p_s} \left(\frac{p_t}{p_s}\right)^{-\varepsilon} ds\ .$$

Marginal production costs $x_t(y_j)/y_j$ satisfy that:

$$\frac{x_t(y_j)}{y_j} = \left(\frac{l_t}{l_*}\right)^{1+\psi} \frac{1}{\omega_t},$$

with $(l_*/l_t)^{1+\psi}$ being a labor wedge, and $l_* \equiv (\alpha/\chi)^{\frac{1}{1+\psi}}$ the equilibrium quantity of aggre-

[16]Processes b_t and m_t follow from replacing the SDF $\Lambda_t = e^{-\rho t}/y_t$ into the numerator and denominator on the RHS of $p_{*,t}/p_t$ in (7). Processes b_t and m_t do not depend on future aggregate output because the intertemporal elasticity of substitution (IES) for consumption is 1.

gate labor in the flexible price economy in which $1/\theta \to 0$.[17]

The Labor Wedge A labor wedge may exist only in the sticky price economy in which $1/\theta \not\to 0$. In the flexible price economy, no labor wedge can exist because prices are flexible as well as competitive. In the sticky price economy, a labor wedge exists only if, and when, real optimal prices $\{p_{*,s}/p_t\}_{s\leq t}$ deviate from spot marginal production costs $x_t(y_j)/y_j$. Deviations of $\{p_{*,s}/p_t\}_{s\leq t}$ from $x_t(y_j)/y_j$ are equivalent in equilibrium to deviations of real prices from marginal production costs in intermediate goods markets. The reason is that the optimal nominal prices $p_{*,s}$, along with their aggregate shares $\theta e^{-\theta(t-s)}$, comprise the cross-section distribution of intermediate goods prices. The aforementioned deviations create distortions in the quantities demanded of intermediate goods and the quantities demanded of inputs, which in turn create wedges between input prices w_t and $r_{k,t}$ and their corresponding marginal productivities:

$$w_t = \left(\frac{l_t}{l_*}\right)^{1+\psi} \alpha \frac{y_t}{l_t}; \quad r_{k,t} = \left(\frac{l_t}{l_*}\right)^{1+\psi} (1-\alpha) \frac{y_t}{a_t \bar{k}}.$$

The Optimal Prices Optimal real prices $p_{*,t}/p_t = b_t/m_t$ may deviate from $x_t(y_j)/y_j$, because the cost-revenue ratio b_t/m_t depends also on the future factors, $\{x_s(y_j)/y_j, \pi_s\}_{s>t}$. The cost-revenue ratio b_t/m_t depends on future inflation $\{\pi_s\}_{s>t}$, because future inflation rates affect the real price $p_t/p_s = \exp\left\{-\int_t^s \pi_{\tilde{s}} d\tilde{s}\right\}$ and the indirect quantity demanded share $(p_t/p_s)^{-\varepsilon} = \exp\left\{\varepsilon \int_t^s \pi_{\tilde{s}} d\tilde{s}\right\}$ related to fixed nominal price p_t. Future inflation rates indeed have an asymmetric effect on b_t/m_t at the null rate $\pi = 0$. The reason is that the coefficient associated with inflation in b_t is larger than that in m_t. The present discounted cost b_t is more sensitive to future inflation than the present discounted revenue m_t, because inputs prices are flexible in nominal terms (and therefore adjust one-to-one to spot inflation), whereas intermediate goods prices are rigid in nominal terms (and therefore do not adjust to inflation at all). The relatively higher sensitivity of b_t to future inflation implies that for any given absolute rate of future inflation, positive inflation rates have a larger impact on the price-setting behavior of firms than their negative counterparts.

[17] The labor wedge is the ratio of the marginal product of labor $\alpha y_t/l_t$ to the households' marginal rate of substitution of labor for consumption $\chi l_t^{\psi} y_t$. To derive the quantity $l_* \equiv (\alpha/\chi)^{\frac{1}{1+\psi}}$, replace the conditions $w_t = \chi l_t^{\psi} y_t$ and $r_{k,t} = (1-\alpha) y_t/a_t \bar{k}$ into the optimality condition $p_{*,t}/p_t = x_t(y_j)/y_j = 1$. In the flexible price economy, $p_{*,t}/p_t = 1$ because all of the firms reset their nominal price at every instant.

The Inflation Rate The discussion concerning optimal real prices presupposes a locally risk-free inflation rate. The inflation rate is locally risk-free because the aggregate price level p_t is time-differentiable:

$$p_t = \left[\int_{-\infty}^{t} \theta e^{-\theta(t-s)} p_{*,s}^{1-\varepsilon} ds \right]^{\frac{1}{1-\varepsilon}} .$$

Intuitively, the inflation rate is locally risk-free because firms that can reset their nominal price during time interval $[t, t+dt]$ set the same nominal price of $p_{*,t}$. They set the same nominal price because the Brownian shock dZ_t is a cumulative shock that realizes just before time $t+dt$ arrives. A locally risk-free inflation rate is consistent, in particular, with a sluggish response of aggregate price level p_t to shock dZ_t.

The expression for the expected inflation rate π_t is:

$$\pi_t = \frac{\theta}{\varepsilon - 1} \left[1 - \left(\frac{p_{*,t}}{p_t} \right)^{-(\varepsilon-1)} \right] . \tag{20}$$

4 Policy Mandates and Markov Equilibrium

4.1 Policy Mandates

To analyze the behavior of policy, I restrict attention to two specific policy mandates, which I refer to as the traditional mandate and the coordinated mandate.

Decomposition of Utility Losses The traditional and coordinated mandates are based on the following partition of the utility flows of households:

$$\ln \frac{1}{\omega_t} + \alpha \ln l_t - \chi \frac{l_t^{1+\psi}}{1+\psi} + (1-\alpha) \ln a_t + \ln A_t + (1-\alpha) \ln \bar{k} . \tag{21}$$

The first term in (21) accounts for the utility losses from price dispersion, the difference between the second and third terms accounts for the utility losses from the labor wedge, and the fourth term accounts for the utility losses from financial disintermediation. The last two terms in (21) are exogenous and therefore uninteresting.

Traditional and Coordinated Mandates In the traditional mandate, monetary policy and macro-prudential policy have separate objectives and take each other's policy rules as given. The objective of monetary policy is to maximize the present discounted value of the first three terms in (21). The objective of macro-prudential policy is to maximize the present discounted value of the corresponding fourth term. In the coordinated mandate, monetary policy and macro-prudential policy are set together and share a joint objective, which consists of maximizing the present discounted value of the utility flows in (21). Later in the paper, I show that the traditional mandate is consistent with a policy mandate in which the objective of monetary policy is to minimize social welfare losses from nominal rigidities, and the objective of macro-prudential policy is to minimize social welfare losses from financial frictions.

4.2 The Markov Competitive Equilibrium

For simplicity, I derive optimal policy only in the context of a Markov competitive equilibrium.

Definition 2 A Markov competitive equilibrium is a set of state variables Γ and a set of mappings $x : \Gamma \rightarrow \Gamma^c$ such that (i) mappings $x : \Gamma \rightarrow \Gamma^c$ are consistent with the conditions of the competitive equilibrium, and (ii) endogenous state variables in Γ evolve in accord with the conditions of the competitive equilibrium.

State Variables and Their Evolution I conjecture that the set of state variables is $\Gamma = \{A, \omega, \eta\}$. This conjecture requires that the policy rules i and Φ depend only on $\{A, \omega, \eta\}$.

Further Restrictions on Policy Rules To simplify the analysis, I restrict policy rules i and Φ to not depend on A. This restriction, together with the law of motion dA_t/A_t, implies that the Markov equilibrium is scale invariant with respect to A. I also restrict capital requirement Φ to be strictly decreasing in η. This additional restriction ensures that financial intermediaries are financially constrained, i.e., $\phi = \min\{\lambda v, \Phi\}$, when intermediary wealth share η is sufficiently low.[18] Lastly, I restrict monetary policy and macro-prudential policy to have commitment. Commitment allows policy rules $\{i, \Phi\}$ to be set contigent upon the aggregate state. Commitment nonetheless restricts $\{i, \Phi\}$ to be designed just before

[18]Tobin's Q v is also strictly decreasing in η, because dividend returns r_k, and therefore expected risk-adjusted excess returns $\tilde{E}\left[dR_{n_f}|\omega, \eta\right]$, are strictly increasing in aggregate supply of capital services to firms $a\bar{k}$.

the economy unravels and to remain unchanged forever after. Commitment implies that policy uses the unconditional invariant distribution $G(\omega, \eta)$ over aggregate states (ω, η) to compute present discounted values. Intuitively, $dG(\omega, \eta)$ indicates the share of time the economy spends in states (ω, η) on average.

5 Traditional Mandate

In this section, I derive the optimal monetary policy and the optimal macro-prudential policy under the traditional mandate. In the traditional mandate, monetary policy and macro-prudential policy interact strategically in accord with a static game. The outcome of their strategic interaction is consistent with the Nash equilibrium.

5.1 Monetary Policy

Problem Monetary policy minimizes the unconditional present discounted value of utility losses from price dispersion and the labor wedge, subject to the conditions of the Markov competitive equilibrium and the behavior of macro-prudential policy. Specifically:

$$\begin{aligned} &\max_i \int \hat{U}(\omega, \eta)\, dG(\omega, \eta) \\ &\text{subject to the conditions in Definition 2 ,} \\ &\text{taking } \Phi \text{ as given .} \end{aligned} \tag{22}$$

The function $\hat{U}(\omega, \eta)$ is the present discounted value of the first three terms in (21) conditional on states (ω, η). It solves the HJB equation:

$$\rho \hat{U} = \ln \frac{1}{\omega} + \alpha \ln l - \chi \frac{l^{1+\psi}}{1+\psi} + \frac{\partial \hat{U}}{\partial \omega} \mu_\omega \omega + \frac{\partial \hat{U}}{\partial \eta} \mu_\eta \eta + \frac{1}{2} \frac{\partial^2 \hat{U}}{(\partial \eta)^2} (\sigma_\eta \eta)^2 , \tag{23}$$

with μ_ω being the diffusion process of price dispersion, and μ_η and σ_η the drift and the diffusion processes of the wealth share η. The diffusion process of price dispersion is null, because ω is a Riemann Integral. The drift process μ_ω depends on the optimal price p_*/p and on inflation π according to:

$$\mu_\omega = \left[\left(\frac{p_*}{p} \right)^{-\varepsilon} \frac{1}{\omega} - 1 \right] \theta + \varepsilon \pi.$$

The drift and diffusion processes μ_η and σ_η reflect the realized excess returns on internal financing and on external financing over the total wealth in the economy that financial intermediaries earn:[19]

$$\mu_\eta = \frac{r_k}{q} + \left[\frac{r_k}{q} + \mu_q - (i - \pi) - \sigma_q^2\right](\phi - 1) - \left(\gamma - \frac{\kappa}{\eta}\right), \tag{24}$$

$$\sigma_\eta = \sigma_q (\phi - 1). \tag{25}$$

Invariant distribution $G(\omega, \eta)$ is endogenously determined by the joint evolution of ω and η in accord with a Kolmogorov forward equation.[20]

Solution In the traditional mandate, monetary policy has a dominant strategy that consists of mimicking the natural rate with policy rate i. Natural rate $\tilde{r}$ is the real interest rate in the flexible price economy:

$$\tilde{r}dt \equiv \rho dt + E[d\tilde{y}/\tilde{y}|\eta] - Var[d\tilde{y}/\tilde{y}|\eta],$$

with $\tilde{y} \equiv A\tilde{a}^{1-\alpha} l_*^\alpha \bar{k}^{1-\alpha}$ being the aggregate output level in the flexible price economy, and $\tilde{a}^{1-\alpha}$ the endogenous TFP, also in the flexible price economy. In the flexible price economy, there is no price dispersion because all of the firms can reset their nominal price at every instant. Therefore, $\omega = 1$.

Mimicking the natural rate is a dominant strategy for monetary policy, because $i = \tilde{r}$ implements the efficient mappings:

$$l = l_* \text{ and } \pi = \pi_* \equiv \frac{\theta}{\varepsilon - 1}\left(1 - \omega^{\varepsilon - 1}\right),$$

independent of macro-prudential policy Φ. The efficient inflation rate π_* maximizes the rate at which price dispersion decays:

$$\pi_* \equiv \arg\min_\pi \mu_\omega = \min_\pi \mu_\omega .$$

The efficient inflation rate π_* is such that appreciation in the aggregate price level fully

[19] The law of motion of η results from applying the Ito quotient rule to the ratio $n_f/q\bar{k}$ and from subsequently deducting from $d\left(n_f/q\bar{k}\right)/\left(n_f/q\bar{k}\right)$ the net transfers that financial intermediaries pay out to households.

[20] See Appendix B for details.

reflects productivity gains from reducing quantity dispersion across intermediate goods. The efficient inflation rate requires that firms set nominal prices according to $p_*/p = 1/\omega$. Over the efficient inflation rate, the aggregate price level and price dispersion evolve in tandem, and therefore $dp/p = d\omega/\omega$. Price dispersion converges uniformly to $\omega = 1$, and there is neither price dispersion nor inflation at the invariant distribution.

Mimicking the natural rate implements the efficient mappings $l = l_*$ and $\pi = \pi_*$, because those mappings, along with $i = \tilde{r}$, are consistent with the conditions of the Markov competitive equilibrium. Specifically, firms are willing to set nominal prices according to $p_*/p = 1/\omega$, because their marginal production costs and their costs-to-revenues ratio b/m become $1/\omega$. Firms break even when they price at $x(y_j)/y_j = 1/\omega$, because their average costs and the real value of their fixed nominal price appreciate at the same rate of $-\pi_*$. Households are willing to supply labor according to $l = l_*$, because the mapping $l = l_*$ satisfies their intra-temporal condition between consumption and labor, given that $y = \tilde{y}/\omega$. Households are willing to consume according to $\tilde{y}/\omega$, because $c = \tilde{y}/\omega$ matches their expected marginal utility return from consumption to the real interest rate. The real interest rate is $\tilde{r}dt - d\omega/\omega$, since $i = \tilde{r}$ and $\pi_* = \mu_\omega$. Households and financial intermediaries are willing to take portfolio positions consistent with $a = \tilde{a}$, because their risk-adjusted excess returns remain the same as in the flexible price economy. Dividend yield r_k/qdt and excess returns $dR_e - (i - \pi_*)\,dt$ remain the same, because variables r_k and q become $\tilde{r}_k/\omega$ and $\tilde{q}/\omega$, and rate dq/q becomes $d\tilde{q}/\tilde{q} - d\omega/\omega$, with being $\tilde{r}_k$ and $\tilde{q}$ the corresponding variables in the flexible price economy. Compensations for holding capital risk also remain the same as in the flexible price economy, but because price dispersion evolves in accord with a time-differentiable process.[21]

Mimicking the natural rate can implement efficient mappings $l = l_*$ and $\pi = \pi_*$ independent of Φ, because there is no binding zero-lower-bound (ZLB) constraint on the nominal rate. A slack ZLB constraint allows monetary policy to always mimic the natural rate with the policy rate.

Discussion of Commitment Assumption Monetary policy does not require commitment under the traditional mandate. The reason is that efficient mappings $l = l_*$ and $\pi = \pi_*$ also maximize the RHS in the HJB (23). Notice that value $\hat{U}$ is such that

[21] Intuitively, neither price dispersion nor its dynamics affect risk-adjusted excess returns, because price dispersion evolves locally deterministically. Local deterministic law of motions cannot affect the riskiness in the rate of return of physical capital or the riskiness in the aggregate economy.

$\partial \hat{U}/\partial \omega < 0$ and $\partial \hat{U}/\partial \eta = 0$.

5.2 Macro-prudential Policy

Problem Macro-prudential policy faces the same problem it would face in a flexible price economy, in which monetary policy has no real effects. The reason is that at the invariant distribution, the sticky price economy behaves like the flexible price economy if $i = \tilde{r}$.

Macro-prudential policy solves the same problem as (22), but with a control variable of Φ, with an objective function of:

$$(1-\alpha) \int_0^1 \tilde{U}(1,\eta)\, dG(1,\eta),$$

and with the behavioral constraint for monetary policy of $i = \tilde{r}$. The value function $\tilde{U}$ satisfies the HJB equation:

$$\rho \tilde{U} = \ln a + \frac{\partial \tilde{U}}{\partial \eta} \mu_\eta \eta + \frac{1}{2} \frac{\partial^2 \tilde{U}}{(\partial \eta)^2} (\sigma_\eta \eta)^2 .$$

I set $\omega = 1$ in the problem of macro-prudential policy, because $\int_0^1 dG(1,\eta) = 1$.

Solution Let Φ_e denote the solution to the problem of macro-prudential policy. The macro-prudential capital requirement Φ_e is equivalent to the constrained efficient capital requirement of the flexible price economy. The best response of macro-prudential policy to mimicking the natural rate is to replicate Φ_e.

5.2.1 Macro-prudential Policy in the Flexible Price Economy

In what follows, I restrict the analysis to the flexible price economy. I restrict the functional form of capital requirement Φ to a polynomial function of state η. This is done for simplicity.[22] This restriction captures the notion that in general, capital requirements cannot be freely adjusted in response to fluctuations in the aggregate state. I solve for the competitive equilibrium and for the constrained efficient capital requirement Φ_e numerically, using spectral methods. See Appendix C for a description of the numerical solution method.

[22] See Appendix C for further details on the set of admissible capital requirements.

Figures 2 and 3 contrast the Markov competitive equilibria corresponding to the macro-prudential policies $\Phi = \Phi_e$ and $\Phi = \Phi_L$ with $\Phi_L > \min\{\lambda v, 1/\eta\}$. The second macro-prudential policy does not restrict leverage, and can therefore be interpreted as a laissez-faire policy.

Contrast of State Functions The constrained-efficient macro-prudential policy $\Phi = \Phi_e$, restricts ϕ below its natural upper bound of $\min\{\lambda v, 1/\eta\}$ only when financial intermediaries on aggregate are average capitalized, and η attains intermediate values (Figure 2a).[23] The relative benefits of $\Phi = \Phi_e$ over $\Phi = \Phi_L$ come from three different sources.

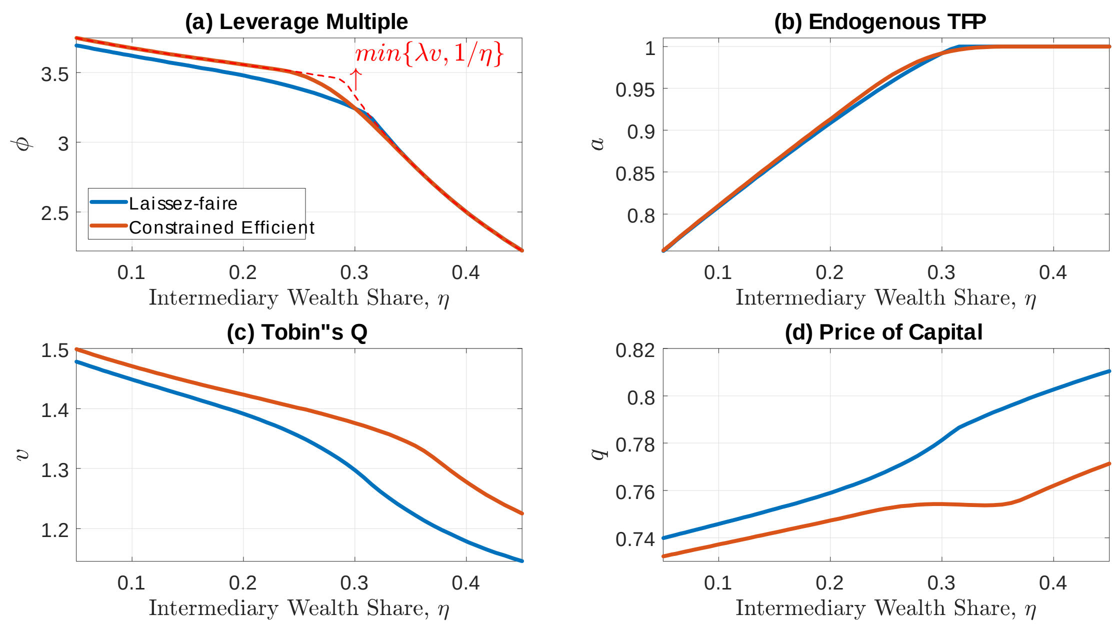

Figure 2: Laissez-Faire vs. Constrained-efficient Macro-prudential Policies in the Flexible Price Economy

First, $\Phi = \Phi_e$ flattens the slope of the price of capital q with respect to wealth share η in Figure 2d when η attains intermediate values. The slope of q gets flattened in that

[23]When financial intermediaries are poorly capitalized, and η is low, the leverage multiple hits its incentive-compatible borrowing limit, i.e., $\phi = \lambda v < \min\{\Phi_e, 1/\eta\}$, and Φ_e does not restrict intermediary leverage. When financial intermediaries are richly capitalized, and η is high, the leverage multiple hits its efficient quantity, i.e., $\phi = 1/\eta < \min\{\Phi_e, \lambda v\}$, and Φ_e also does not restrict intermediary leverage.

intermediate region, because a binding capital requirement keeps households as marginal investors, and therefore eliminates the large swings in q associated with changes in the identity of the marginal investor. A lower sensitivity of q with respect to η mitigates the feedback loop and the distributive pecuniary externality, and therefore helps to keep the fluctuations in dn_f/n_f; dq/q; $d\bar{k}_f/\bar{k}_f$ in check.

Second, $\Phi = \Phi_e$ boosts dividend yields r_k/q and the Tobin's Q. Dividend yields r_k/q increase mainly because the price of capital q falls (Figure 2d). When η attains intermediate values, the price of capital falls, because a binding capital requirement extends the region in which households are the marginal investors; in the other regions of the state space, the price of capital also falls, because q is a forward-looking variable that also takes into account the identity of the marginal investor in the future. The Tobin's Q and IC borrowing capacity λv increase (Figure 2c), as a consequence of higher intermediary profitability: Notice that higher dividend yields r_k/q boost the return on capital dR_f; excess returns on internal financing and on external financing; and rate the $\mu_\eta \eta$ at which the intermediary wealth share recovers in expectation (Figure 3a). Concerning pecuniary externalities, the positive effect on λv helps to improve on the binding-constraint pecuniary externality. The positive effect on $\mu_\eta \eta$ helps to improve on the dynamic pecuniary externality.

Third, and related to the second benefit, $\Phi = \Phi_e$ redistributes the leverage multiple progressively across the wealth share η. Specifically, the leverage multiple increases when η is low and Φ_e is slack, while it decreases when η attains intermediate values and Φ_e binds (Figure 2a) — the leverage multiple remains the same as in the laissez-faire economy when η is high, because in that region financial intermediaries clear the market for physical capital, and $\phi = 1/\eta$. Progressive redistributions of leverage across η are beneficial, because the preferences for consumption are strictly concave, and because the endogenous TFP is strictly increasing in η. This third benefit helps to improve on the dynamic pecuniary externality.

The constrained-efficient policy $\Phi = \Phi_e$ nonetheless also brings costs relative to $\Phi = \Phi_L$. Specifically, restricting intermediary leverage according to $\Phi = \Phi_e$ places endogenous TFP below its potential level when the capital requirement binds (Figure 2b). These costs, together with the strict concavity the preferences for consumption, imply that restricting intermediary leverage below $\min\{\lambda v, 1/\eta\}$ when η is low is not constrained efficient. They also imply that only moderate restrictions on intermediary leverage when η attains intermediate values are constrained efficient.

Contrast of Invariant Distributions The constrained efficient macro-prudential policy not only affects the Markov equilibrium state-by-state, but also at the invariant distribution (Figures 3c and 3d). In Appendix B, I show that invariant density function $dG(1,\eta)$ satisfies:

$$dG(1,\eta) \propto \frac{1}{(\sigma_\eta \eta)^2} \exp\left\{2\int_0^\eta \frac{\mu_{\tilde{\eta}}\tilde{\eta}}{(\sigma_{\tilde{\eta}}\tilde{\eta})^2} d\tilde{\eta}\right\},$$

with $\int_0^1 dG(1,\eta)\, d\eta = 1$.

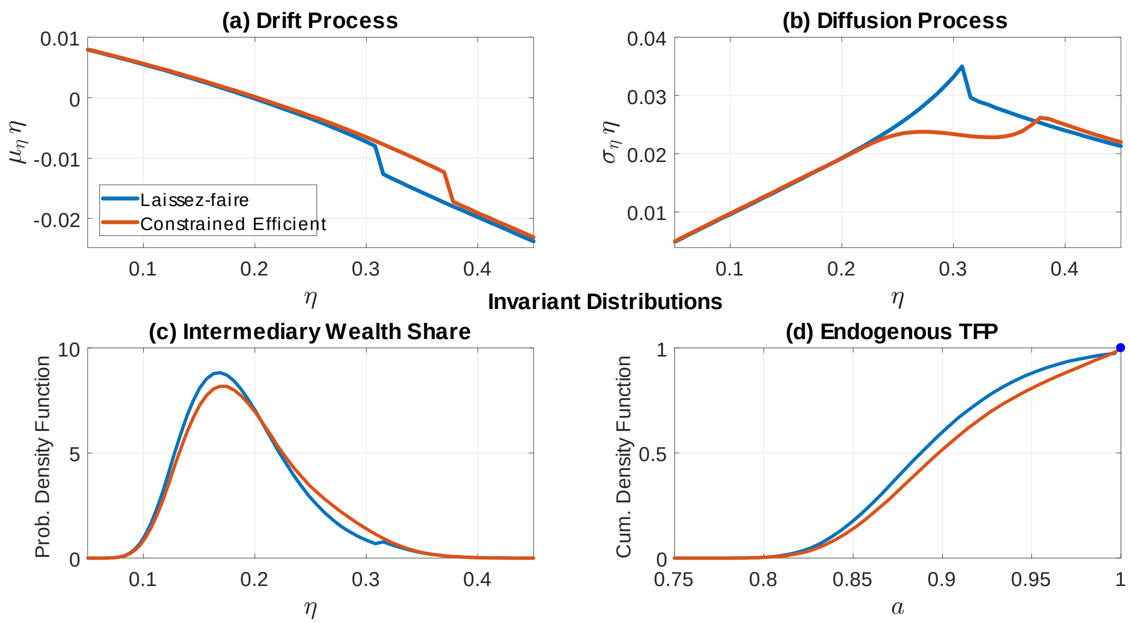

Figure 3: Laissez-Faire vs. Constrained-efficient Macro-prudential Policies in the Flexible Price Economy: Dynamics

A larger expected recovery rate $\mu_\eta \eta$ implies that the economy spends more time in states in which financial intermediaries are better capitalized. It therefore helps to shift $dG(1,\eta)$ rightward in the η domain (Figure 3c). A lower volatility $\sigma_\eta \eta$ when η attains intermediate values implies that the economy spends more time in states in which financial intermediaries are average-capitalized. It therefore helps to shift $dG(1,\eta)$ upward in that same region. The downward shift in the invariant cumulative probability function of endogenous TFP verifies that Φ_e improves social welfare relative to Φ_L at the invariant distribution (Figure

3d). The effects of Φ_e on the invariant distribution are related to the dynamic pecuniary externality.

Discussion of Commitment Assumption In the traditional mandate, macro-prudential policy does require commitment. Intuitively, the reason is that the costs from $\Phi = \Phi_e$ materialize on impact, while its benefits materialize in the medium and long terms. The first benefit materializes mainly around the region in which Φ_e binds. The second and third benefits materialize only when wealth share η is low and Φ_e is slack. Commitment is indeed critical for the second and third benefits to materialize: If macro-prudential policy were to not have commitment, financial intermediaries and households would not believe that Φ_e would restrict intermediary leverage eventually when η recovers and, as a consequence, the price of capital would not fall when η is low.

6 Coordinated Mandate

In this section, I derive the optimal monetary policy and the optimal macro-prudential policy under the coordinated mandate.

Problem In the coordinated mandate, monetary policy and macro-prudential policy face the same problem as (22), but with control variables i and Φ, and an objective function of:

$$\int \left[\hat{U}(\omega, \eta) + (1-\alpha)\tilde{U}(\omega, \eta)\right] dG(\omega, \eta).$$

I make a change of variable in the optimization problem above to simplify the analysis. Specifically, I replace the policy rate i with the employment gap $\ln(l/l_*)$. This change of variable is admissible, because in any competitive equilibrium, the policy rate can be derived as a residual using the asset pricing condition for deposits. It is convenient, because the employment gap can be interpreted as the monetary policy stance. For instance, a positive employment gap can be interpreted as an expansionary monetary policy, while a negative employment gap can be interpreted as a contractionary monetary policy. A positive employment gap precisely when financial intermediaries on aggregate are poorly capitalized can be interpreted as a Greenspan put, while a negative employment gap when financial intermediaries are average- to richly capitalized can be interpreted as leaning against the wind. A permanently null employment gap can be interpreted as macroeconomic stabilization. The employment gap can be interpreted as the monetary policy stance, because a slack ZLB constraint implies that monetary policy can implement any employment gap independent of macro-prudential policy.

I restrict attention to employment gaps and capital requirements that are contingent only on wealth share η. Furthermore, I restrict attention to employment gaps that are a linear function of η and capital requirements that are a polynomial function of η. This is done for simplicity.[24]

Numerical Solution Figure 4 contrasts the Markov competitive equilibria between the traditional mandate and the coordinated mandate. In the coordinated mandate, monetary policy deviates from its traditional objective of macroeconomic stabilization (Figure 4a). Monetary policy deviates in accord with the prescriptions of the Greenspan put and of leaning against the wind, but relies more heavily on the prescriptions of the latter. Macro-prudential policy softens the capital requirement relative to the traditional mandate, though the adjustment state-by-state is small (Figure 4b).

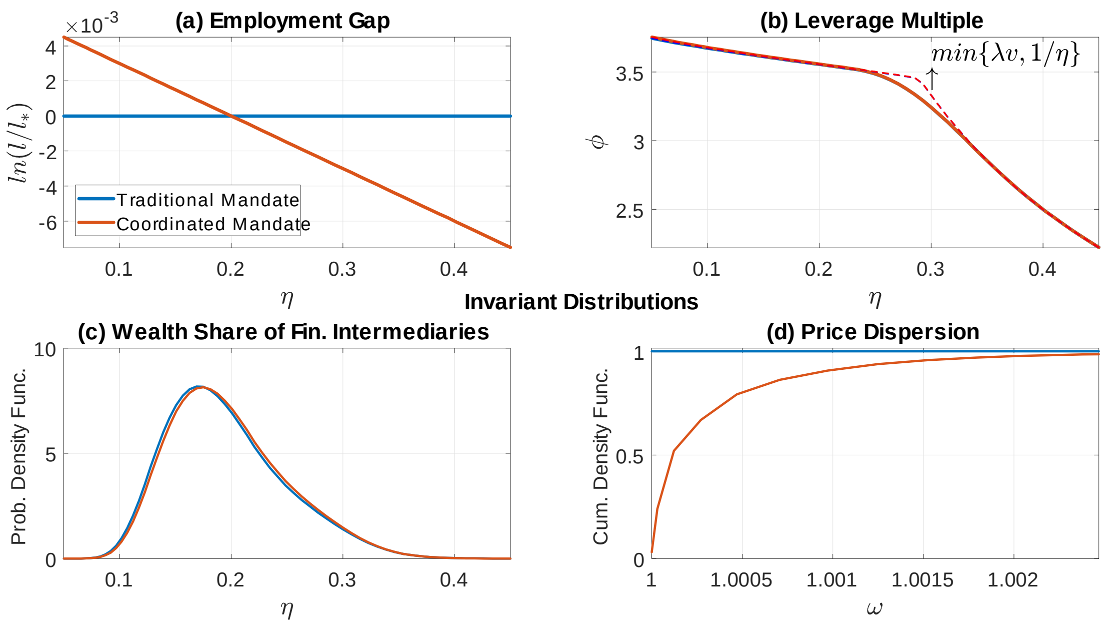

FIGURE 4: TRADITIONAL MANDATE VS. COORDINATED MANDATE

[24] See Appendix C for further details on the set of admissible capital requirements.

To explain the rationale behind the behavior of monetary policy in the coordinated mandate, I first analyze three candidate monetary policies. The first is a non-contingent employment gap that is constant over state η. I analyze only a positive employment gap; the analysis for a negative non-contingent employment gap is equivalent.

A positive non-contingent employment gap increases inputs prices w and r_k relative to the flexible economy. The reason is that real wages must increase in equilibrium to induce households to supply more labor. Higher inputs prices boost marginal production costs, induce firms to target higher real prices, and generate positive inflation rates. In equilibrium, inflation rate $\pi > 0$ and price dispersion $\omega > 1$ are constant, and in particular, satisfy that:

$$1 = \frac{\varepsilon - 1}{\theta}\pi + \left[\frac{\theta - (\varepsilon - 1)\pi}{\theta}\right]^{\varepsilon} \left[\left(\frac{l_*}{l}\right)^{(1+\psi)} \frac{\theta + \rho - \varepsilon\pi}{\theta + \rho - (\varepsilon - 1)\pi} \frac{\theta}{\theta - \varepsilon\pi}\right]^{\varepsilon - 1},$$

$$\omega = \frac{\theta}{\theta - \varepsilon\pi} \left[\frac{\theta - (\varepsilon - 1)\pi}{\theta}\right]^{\frac{\varepsilon}{\varepsilon - 1}}.$$

A positive non-contingent employment gap nonetheless does not affect the productivity factor a or the utility losses $(1 - \alpha)\ln a$ from financial disintermediation. Those variables remain the same as in the flexible price economy, because dividend yields r_k/qdt, excess returns $dR_e - (i - \pi)\,dt$, and compensations for holding capital risk remain the same. Dividend yields remain the same, because the price of capital q is a present discounted value of dividend returns — and hence q increase in tandem with the permanent increase in r_k. Excess returns and compensations for holding capital risk remain the same, because non-contingent employment gaps bring no additional risk into the economy.

The first candidate monetary policy delivers two takeaways. The first is that non-contingent employment gaps do not help to improve on financial stability relative to macroeconomic stabilization. The key problem with non-contingent employment gaps is that they generate no transitory effects on dividend returns $r_k dt$. Only transitory effects prevent the price of capital from adjusting one-to-one to changes in $r_k dt$. The second takeaway is that non-contingent employment gaps are actually worse in terms of social welfare than macroeconomic stabilization. Positive non-contingent employment gaps are even worse than their negative counterparts, because of the asymmetric responses of optimal real price p_*/p and price dispersion ω to inflation at $\pi = 0$.

The second and third candidate monetary policies are a Greenspan put and leaning against the wind. In contrast to non-contingent employment gaps, the Greenspan put and leaning against the wind generate transitory effects on $r_k dt$. This is because they target employment gaps that are contingent upon the wealth share η. The Greenspan put and the leaning against the wind generate opposite transitory effects on dividend returns $r_k dt$, but similar transitory effects on dividend yields r_k/qdt and productivity factor a.

Specifically, the Greenspan put, by design, targets positive employment gaps when η is low, while it stabilizes the employment gap at zero when η is average to high. Relative to the flexible price economy, dividend returns $r_k dt$ increase when η is low, while they remain fairly constant when η is average to high. Dividend yields r_k/qdt also increase when η is low, but decrease when η is average to high, because the price of capital is forward-looking. The shift in dividend yields boosts the profitability of financial intermediaries, relaxes moral hazard problems in credit markets, and speeds up the recapitalization of financial intermediaries in expectation only when η is low. It has the opposite effects when η is average to high. The resulting progressive redistributions across η of the intermediary profitability and of IC borrowing capacity help to reduce the present discounted value of $(1-\alpha)\ln a$.

Leaning against the wind stabilizes the employment gap at zero when η is low, while it targets negative employment gaps when η is average to high. Relative to the flexible price economy, dividend returns $r_k dt$ then remain fairly constant when η is low, and they decrease when η is average to high. Dividend yields r_k/qdt nonetheless increase when η is low and decrease when η is average to high, because the price of capital falls over the entire domain of η. The shift in dividend yields is therefore similar to the shift in the Greenspan put. Intermediary profitability, IC borrowing capacity, and productivity factor a, also shift similar to the Greenspan put.

Overall, when compared to macroeconomic stabilization, the Greenspan put and leaning against the wind always perform better in terms of financial stability, but worse in terms of macroeconomic stability. Gains in financial stability outweigh losses in macroeconomic stability only if deviations in the employment gap are sufficiently small. The reason is that in the traditional mandate, $l = l_*$ and $\omega = 1$ are efficient, but $\phi \neq 1/\eta$ is not efficient. For any given absolute deviation in the employment gap, losses in macroeconomic stability are larger for the Greenspan put than for leaning against the wind, because of the asymmetric responses of p_*/p and ω to inflation at $\pi = 0$.

In the coordinated mandate, monetary policy leverages on the takeaways provided by the three candidate monetary policies. Specifically, monetary policy combines the Greenspan put and leaning against the wind to help macro-prudential policy increase the present discounted value of $(1-\alpha)\ln a$ relative to the traditional mandate. Combining the Greenspan put and leaning against the wind strengthens the temporary effects on dividend returns r_k and dividend yields r_k/qdt. Furthermore, it smooths utility losses from price dispersion $\ln\omega$ and employment gap instability $\alpha \ln l_*/l + \frac{1}{1+\psi}\chi l^{1+\psi}\left[1-(l_*/l)^{1+\psi}\right]$ across η.

Macro-prudential policy softens the capital requirement relative to the traditional mandate, because the capital requirement becomes less valuable once monetary policy also responds to financial stability concerns. Specifically, the second and third benefits from Φ_e become less valuable, because monetary policy also redistributes intermediary profitability and effective borrowing capacity $\min\{\lambda v, \Phi\}$ progressively across wealth share η. The first benefit from Φ_e becomes less valuable as well, but because monetary policy reduces the IC borrowing capacity λv when η is average to high.

Discussion of Commitment Assumption In the coordinated mandate, monetary policy and macro-prudential policy require commitment. The reason is that the costs from leaning against the wind and from the capital requirement materialize on impact while their benefits materialize in the medium and long terms. Commitment is critical for those benefits to materialize in the first place.

7 Social Welfare Gains from Coordination

I calibrate the model economy to quantify the costs and benefits from coordinating monetary policy and macro-prudential policy.

Calibration Table 1 reports parameter values in the baseline calibration. The time frequency is annual.

Table 1: Parameter Values

Parameter	a_h	λ	κ	γ	μ_A	σ_A	α	$\bar{k}$	ε	θ	ρ	ψ	χ
Value	70%	2.5	1%	10%	1.5%	3.5%	65%	1	2	$\frac{6}{5}\ln 2$	2%	3	2.8

The first three parameters in Table 1 target unconditional averages in the laissez-faire flexible price economy, in which there is no macro-prudential policy. The productivity coefficient of households a_h targets an unconditional average Sharpe ratio of 30%, which is standard. The value of a_h is 70%. The fraction of divertable assets λ targets an unconditional average leverage multiple of 3.5. The value of λ is 2.5. The initial capital endowment κ of starting financial intermediaries targets the unconditional average wealth-to-capital ratio in the financial intermediary sector. I use a target of 20%, which is consistent with the estimates of Hirakata, Sudo and Ueda (2013). The value of κ is 1%. The cycle for intermediary dividend payouts can be interpreted as the life cycle of individual financial intermediary companies (Gertler and Karadi 2011; Gertler and Kiyotaki 2010; Maggiori 2017) I set arrival rate γ to target an unconditional average survival frequency of 10 years, which is consistent with Gertler and Kiyotaki (2010).

The drift and diffusion processes μ_A and σ_A match the unconditional mean and tunconditional standard deviation of the Utilization-Adjusted Series on Total Factor Productivity (see Fernald 2014). The value of μ_A is 1.5%. The value of σ_A is 3.5%. The labor share of output α is 65%, which is consistent with the empirical findings of Karabarbounis and Nieman (2014). The aggregate stock of physical capital $\bar{k}$ is normalized to 1.

The elasticity of substitution between intermediate goods ε is 2. This value is below the regular values, ranging from 4 to 6, that are usually set in sticky price economies in which firms reset their nominal price sluggishly according to Calvo (1983) pricing. I set a relatively low value for ε to accommodate the recent empirical findings of Nakamura and Steinsson (2017), who show that Calvo (1983) pricing over estimates social welfare costs from price dispersion relative to those measured in data, even for low inflation rates. Nakamura and Steinsson (2017) also show that the resulting over estimation critically depends on, and is positively related to, the value of ε. I use the expression for the inflation rate (20) to set the value of ε. The value $\varepsilon = 2$ is consistent with an annual inflation rate of 3% and with a price percentage change of $p_*/p = 1.075$, given a constant inflation rate $\pi_t = \pi$. Nakamura and Steinsson (2017) argue that the absolute size of price changes is an acceptable proxy indicator for inefficient price dispersion; they report an unconditional average for the absolute size of price changes of 7.5% in the U.S. from 1988-2014.

The arrival rate of the Poisson process that allows firms to reset their nominal price θ is $\frac{6}{5}\ln 2$. This value yields a median frequency of price change of 10% per month, which is consistent with Nakamura and Steinsson (2008, 2017).

The time discount rate ρ is 2%. The Frisch elasticity of labor supply ψ is 0.5, which is consistent with the empirical findings of Chetty, Guren, Manoli and Weber (2011). The relative utility weight of labor χ matches an unconditional average share of labor hours of 1/3 per unit of time.

Quantitative Gains Table 2 reports social welfare gains from coordinating monetary policy and macro-prudential policy. Social welfare gains are computed relative to the traditional mandate; they are expressed in terms of annual consumption equivalent.

Table 2: Social Welfare Gains from Coordination

	Present Discounted Value of			
	$\ln 1/\omega$	$\alpha \ln l - \chi \frac{l^{1+\psi}}{1+\psi}$	$(1-\alpha) \ln a$	Utility Flows
Baseline Calibration	-0.04%	-0.00%	$+0.11\%$	$+0.07\%$
Baseline but with $a_h = 60\%$	-0.05%	-0.01%	$+0.15\%$	$+0.09\%$
Baseline but with $\theta = \frac{4}{5} \ln 2$	-0.06%	-0.01%	$+0.20\%$	$+0.13\%$
Baseline but with $\varepsilon = 4$	-0.05%	-0.00%	$+0.07\%$	$+0.02\%$

Table 2 shows that social welfare gains from coordinating monetary policy and macro-prudential policy amount to 0.07% in the baseline calibration. Table 2 also shows that social welfare gains are larger if productivity gap $1 - a_h$ is larger and/or if the frequency at which firms can reset their nominal price θ is lower. Social welfare gains are strictly increasing in $1-a_h$, because the price of capital and the intermediary wealth share fluctuate more if valuation differences concerning risky assets are larger. They are strictly decreasing in θ, because a lower share of firms sets a nominal price away from the aggregate price level if nominal prices are more rigid.

8 Conclusion

In this paper I develop a model economy to study coordination between monetary policy and macro-prudential policy. I restrict attention to two specific policy mandates: a traditional mandate and a coordinated mandate. In the traditional mandate, I obtain that monetary policy mimics the natural rate, and that macro-prudential policy implements the constrained-efficient capital requirement of the flexible price economy. In the coordinated mandate, I obtain that monetary policy deviates from the natural rate in accord with the prescriptions of the Greenspan put and leaning against the wind, and that macro-prudential policy softens the capital requirement relative to the traditional mandate. In the baseline calibration, I obtain that social welfare gains from coordinating monetary policy and macro-prudential policy amount to 0.07% in terms of annual consumption equivalent.

The main results in this paper are robust to the source of fundamental shocks that hit the economy. The main mechanisms in play are robust to the micro-foundations concerning the price-setting behavior of firms. The main results depend, nonetheless, on the binding status of the zero-lower-bound constraint on the nominal interest rate. This is because if the zero-lower-bound constraint binds (or occasionally binds), aggregate labor and inflation do not remain stable at their structural levels. A detailed analysis concerning coordination between monetary policy and macro-prudential policy when the zero-lower-bound constraint occasionally binds remains for future research.

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Appendices

Appendix A lays out the moral hazard problem in equity markets. Appendix B derives the analytical solution of the model economy. Appendix C describes the numerical solution method.

Appendix A

The Moral Hazard Problem in Equity Markets

The moral hazard problem between the direct holders of physical capital (hereafter capital lessors) and the holders of their equity shares (hereafter shareholders) is based on the textbook moral hazard problems in Tirole (1998).

The Moral Hazard Problem Capital lessors own all of the aggregate capital stock in the economy. By exerting costly effort, capital lessors can increase in probability the productivity rate a at which they transform physical capital into capital services.

Productivity rate a is stochastic at the individual level, and either can be high or low. If the rate is high, the firms involved in the rental transaction receive $a_S > 1$ units of capital services per unit of physical capital rented out. If the rate is low, those firms receive no units of capital services. Conditional on a same-effort decision, productivity rates are i.i.d. across capital lessors. For simplicity, and to ensure that the quantity of capital services that each firm receives is deterministic, I assume that each firm rents physical capital from a continuum of different capital lessors that take the same effort decision. Exerting effort improves the probability of a high rate from $P_n > 0$ to $P_e > P_n$, with $P_e < 1$, being P_n the probability of a low rate conditional on not exerting effort. Exerting effort nonetheless entails the loss of a positive private benefit for capital lessors. Such private benefit is proportional to the stock of physical capital rented out to firms and, for simplicity, is expressed in terms of units of capital services.

Let $\beta > 0$ denote the private benefit of capital lessors per unit of physical capital rented out. The net present value (NPV) condition $P_e a_S > P_n a_S + \beta$, together with the private benefit $\beta > 0$, ensures a moral hazard problem between capital lessors and their shareholders.

Solution to the Moral Hazard Problem Shareholders can solve the moral hazard problem by implementing one of the following two strategies. The first is to monitor effort decisions. Monitoring eliminates the possibility of not exerting effort. When conducted by financial intermediaries, monitoring is costless, but when conducted by households, monitoring scales down the high productivity rate by $a_h < 1$.[25] The second strategy is to write a contract contingent on the realization of the outcome of the productivity rate, to incentivize capital lessors to exert effort. From the point of view of shareholders, who are the agents who write the contract, the optimal incentive-compatible contract (i.e., that incentivizes capital lessors to exert effort and minimizes the expected payment to lessors) promises a unitary payment of $\frac{1}{P_e - P_n}\beta > 0$ in terms of capital services contingent on a high productivity rate. The optimal incentive-compatible contract cannot promise negative payments because capital lessors are protected from limited liability. The cost to the shareholders of the optimal incentive-compatible contract is $\frac{P_e}{P_e - P_n}\beta > 0$.

Financial intermediaries prefer monitoring to the optimal incentive-compatible contract, because to them monitoring is costless. I impose that $P_e a_S a_h > P_e a_S - \frac{P_e}{P_e - P_n}\beta$ to ensure that household also prefer monitoring to the optimal incentive-compatible contract.

Interpretation I normalize $P_e a_S$ to 1, and interpret $P_e a_S = 1$ as the quantity of capital services, per unit of physical capital, that financial intermediaries can rent out to firms in a reduced-form economy in which there are no capital lessors, and financial intermediaries and households own all of the aggregate capital stock. I interpret $P_e a_S a_h = a_h < 1$ similarly, but for households.

Appendix B

Solving the Portfolio Problem of Financial Intermediaries

Let $G_{v,t}$ denote the gain process:

$$G_{v,t} \equiv E_t \int_0^\infty \gamma e^{-\gamma s} \Lambda_s n_{f,s} ds = \int_0^t \gamma e^{-\gamma s} \Lambda_s n_{f,s} ds + e^{-\gamma t} \Lambda_t v_t n_{f,t}.$$

The equality on the RHS follows from the definition of V_t and from the result that $V_t = v_t n_{f,t}$. The drift process of $G_{v,t}$ is null, because $G_{v,t}$ is the conditional expectation of a random variable. Applying Ito's Lemma to the RHS in $G_{v,t}$, and then equalizing the

[25] For monitoring to play a role, I assume that financial intermediaries cannot monitor on behalf of shareholders who are households. To that end, I assume that capital lessors can issue a single share or, alternatively, that shareholders must monitor individual units of physical capital, to ensure that capital lessors exert effort on each unit.

resulting drift process to zero, delivers the following HJB equation:

$$
\begin{aligned}
\gamma v_t &= \max_{\phi_t} \left\{ \gamma + \left[\mu_{\Lambda,t} + \mu_{v,t} + \mu_{n_f,t} + \sigma_{\Lambda,t}\sigma_{v,t} + \sigma_{\Lambda,t}\sigma_{n_f,t} + \sigma_{v,t}\sigma_{n_f,t} \right] v_t \right\} \\
& \quad s.t. : \phi_t \leq \min\{\lambda v_t, \Phi_t\} ,
\end{aligned} \tag{26}
$$

with $\mu_{x,t}$ and $\sigma_{x,t}$ being the drift and diffusion processes of the generic process x_t, with $x_t = \{\Lambda_t, v_t, n_{f,t}\}$. Processes $\mu_{n_f,t}$ and $\sigma_{n_f,t}$ depend on leverage multiple ϕ_t, in accord with (11). Processes $\mu_{\Lambda,t}$ and $\sigma_{\Lambda,t}$ do not depend on ϕ_t, because the SDF Λ_t depends only on aggregate consumption. Neither do the value v_t nor its drift and diffusion processes $\mu_{v,t}$ and $\sigma_{v,t}$ depend on ϕ_t, because the value $V_t = v_t n_{f,t}$ is the value function of the optimization problem in (10). Leverage constraint $\phi_t \leq \min\{\lambda v_t, \Phi_t\}$ follows from the financing constraint.

The asset pricing condition for physical capital (12) follows from the F.O.C. in the optimization problem above. The asset pricing condition for v_t follows from replacing (12) into the HJB equation (26).

Solving the Portfolio Problem of Households

I conjecture that the value of households U_t satisfies:

$$
U_t = U(n_{h,t}, J_t),
$$

with $U : \mathbb{R}^2 \to \mathbb{R}$ being a twice continuously differentiable function, and J_t a sufficient statistic of the aggregate state variables in the households' problem. The process J_t is a scalar. I further conjecture that J_t follows an Ito process with drift process $\mu_{J,t}$ and diffusion process $\sigma_{J,t}$.

The value U_t is the solution to the HJB equation:

$$
\rho U_t = \max_{c_t, l_t, \bar{k}_{h,t} \geq 0} \left\{ \begin{array}{l} \ln c_t - \chi \frac{l_t^{1+\psi}}{1+\psi} + \frac{\partial U_t}{\partial n_{h,t}} \mu_{n_h,t} n_{h,t} + \frac{\partial U_t}{\partial J_t} \mu_{J,t} J_t + \\ \frac{1}{2} \frac{\partial^2 U_t}{(\partial n_{h,t})^2} (\sigma_{n_h,t} n_{h,t})^2 + \frac{\partial^2 U_t}{\partial J_t \partial n_{h,t}} \sigma_{J,t} J_t \sigma_{n_h,t} n_{h,t} + \frac{1}{2} \frac{\partial^2 U_t}{(\partial J_t)^2} (\sigma_{J,t} J_t)^2 \end{array} \right\},
$$

with $\mu_{n_h,t}$ and $\sigma_{n_h,t}$ being the drift and diffusion processes of $n_{h,t}$. Processes $\mu_{n_h,t}$ and $\sigma_{n_h,t}$ depend on the controls $c_t, l_t, \bar{k}_{h,t}$, in accord with (15). Neither process J_t nor its drift and diffusion processes $\mu_{J,t}$ and $\sigma_{J,t}$ depend on individual controls $c_t, l_t, \bar{k}_{h,t}$.

The HJB equation above and its optimization problem on the RHS deliver the three optimality conditions in the portfolio problem of households. The first-order condition with respect to consumption c_t is:

$$\frac{1}{c_t} = \frac{\partial U_t}{\partial n_{h,t}}.$$

The first-order condition with respect to labor l_t is:

$$\chi l_t^{\psi} = w_t \frac{\partial U_t}{\partial n_{h,t}}.$$

The first-order condition with respect to physical capital $\bar{k}_{h,t}$ is:

$$\left[\mu_{n_h,t}\frac{r_{k,t}}{q_t} + \mu_{q,t} - (i_t - \pi_t)\right]\frac{\partial U_t}{\partial n_{h,t}} + \sigma_{q,t}\frac{\partial^2 U_t}{(\partial n_{h,t})^2}\sigma_{n_h,t} n_{h,t} + \sigma_{q,t}\frac{\partial^2 U_t}{\partial J_t \partial n_{h,t}}\sigma_{J,t} J_t \leq 0.$$

with equality if $\bar{k}_{h,t} > 0$.

The intra-temporal condition between consumption and labor follows from combining the first two first-order conditions. The asset pricing conditions for deposits and the price of capital follow from applying the same methodology as in Cox, Ingersoll and Ross (1985). Specifically, first, replace the first-order conditions in the HJB equation above; second, take the first-order condition with respect to $n_{h,t}$ in the expression obtained in the first step; and third, re arrange the expression obtained in the second step accordingly.

Markov Competitive Equilibrium

Asset Pricing Conditions in the Competitive Equilibrium

Asset pricing conditions in the competitive equilibrium are useful for characterizing the Markov equilibrium.

In equilibrium, the asset pricing conditions for deposits is:

$$i_t - \pi_t = \rho + \mu_{y,t} - \sigma_{y,t}^2.$$

The asset pricing condition for physical capital depends on whether financial intermediaries are financially constrained. When financial intermediaries are financially constrained, the asset pricing condition is (18) , holding with equality. When they are financially unconstrained, the asset pricing condition is instead (12) , also holding with equality. I conjecture that the price of capital q_t is proportional to aggregate output y_t. Let $\hat{q}_t = q_t/y_t$ denote the price of capital per unit of aggregate output. In equilibrium, the asset pricing condition for physical capital is:

$$\frac{a_h}{\hat{q}_t}\frac{1-\alpha}{a_t\bar{k}}\left(\frac{l_t}{l_*}\right)^{1+\psi} + \mu_{\hat{q},t} - \rho = 0 \ ,$$

with $\phi_t = \min\{\lambda v_t, \Phi_t\}$ if financial intermediaries are financially constrained; the asset pricing condition for physical capital is

$$\frac{1}{\hat{q}_t}\frac{1-\alpha}{\bar{k}}\left(\frac{l_t}{l_*}\right)^{1+\psi} + \mu_{\hat{q},t} + \sigma_{v,t}\left(\sigma_{\hat{q},t} + \sigma_{y,t}\right) - \rho = 0 \ ,$$

with $\phi_t = 1/\eta_t < \min\{\lambda v_t, \Phi_t\}$ if financial intermediaries are financially unconstrained.[26]

The asset pricing condition for Tobin's Q is:

$$\left[\frac{1-a_h}{\hat{q}_t}\frac{1-\alpha}{a_t\bar{k}}\left(\frac{l_t}{l_*}\right)^{1+\psi} + \sigma_{v,t}\left(\sigma_{\hat{q},t} + \sigma_{y,t}\right)\right]\phi_t \mathbf{1}_{\phi_t<1/\eta_t} + \frac{\gamma}{v_t} + \mu_{v,t} - \sigma_{y,t}\sigma_{v,t} - \gamma = 0 \ .$$

Notice that $\frac{1}{dt}\tilde{E}_t\left[dR_{n_f,t}\right]$ equals the first term on the LHS when financial intermediaries are financially constrained. Notice also that $\tilde{E}_t\left[dR_{n_f,t}\right] = 0$ when financial intermediaries are financially unconstrained.

Characterization of the Markov Equilibrium

The mappings that characterize the Markov equilibrium are $\{\hat{q}, v, \pi; l, \Phi\}$. I restrict attention to Markov equilibria in which employment gap $\ln l/l_*$ and capital requirement Φ are contingent only on wealth share η. I conjecture that price $\hat{q}$ and value v depend only on wealth share η.

Law of Motion Revisited

Inflation equation (20) implies that process μ_ω satisfies that:

$$\mu_\omega = \theta\left[\frac{1}{\omega}\left[1 - \frac{\varepsilon-1}{\theta}\pi\right]^{\frac{\varepsilon}{\varepsilon-1}} - 1\right] + \varepsilon\pi \qquad (27)$$

Let $\varepsilon_{x,\eta}$ denote the elasticity of a given mapping x with respect to state η. Let x_η denote

[26] The conjecture $q_t = \hat{q}_t y_t$ implies that $\mu_{q,t} = \mu_{\hat{q},t} + \mu_{y,t} + \sigma_{\hat{q},t}\sigma_{y,t}$ and that $\sigma_{q,t} = \sigma_{\hat{q},t} + \sigma_{y,t}$. Notice that $a_t = 1$ when $\phi_t = 1/\eta_t$.

the partial derivative of mapping x with respect to state η. Ito's Lemma implies that drift and the diffusion processes μ_x and σ_x of the process x satisfy that:

$$\begin{aligned} \mu_x &= \varepsilon_{x,\eta}\mu_\eta + \frac{1}{2}\varepsilon_{x_\eta,\eta}\varepsilon_{x,\eta}\sigma_\eta^2 \\ \sigma_x &= \varepsilon_{x,\eta}\sigma_\eta, \end{aligned}$$

Diffusion processes $\sigma_{\hat{q}}$ and σ_y satisfy, in particular, that:

$$\begin{aligned} \sigma_{\hat{q}} &= \varepsilon_{\hat{q},\eta}\sigma_\eta \\ \sigma_y &= \sigma_A + \alpha\varepsilon_{l,\eta}\sigma_\eta + (1-\alpha)\,\varepsilon_{a,\eta}\sigma_\eta. \end{aligned}$$

From the law of motion (**??**) , it follows then that diffusion process σ_η satisfies that:

$$\sigma_\eta = \frac{\phi - 1}{1 - \left[\varepsilon_{\hat{q},\eta} + \alpha\varepsilon_{l,\eta} + (1-\alpha)\,\varepsilon_{a,\eta}\right](\phi - 1)}\sigma_A, \tag{28}$$

and that drift process μ_η satisfies that:

$$\mu_\eta = \frac{1}{1 - \varepsilon_{\hat{q},\eta}(\phi - 1)}\left[\phi\frac{1}{\hat{q}}\frac{1-\alpha}{a\bar{k}}\left(\frac{l}{l_*}\right)^{1+\psi} + \left[(\phi-1)\,\frac{1}{2}\varepsilon_{\hat{q}_\eta,\eta} - 1\right]\varepsilon_{\hat{q},\eta}\sigma_\eta^2 - (\phi-1)\,\rho - \left(\gamma - \frac{\kappa}{\eta}\right)\right]. \tag{29}$$

Asset Pricing Conditions Revisited

The asset pricing condition for physical capital is:

$$\frac{a_h}{\hat{q}}\frac{1-\alpha}{a\bar{k}}\left(\frac{l}{l_*}\right)^{1+\psi} + \varepsilon_{\hat{q},\eta}\mu_\eta + \frac{1}{2}\varepsilon_{\hat{q}_\eta,\eta}\varepsilon_{\hat{q},\eta}\sigma_\eta^2 - \rho = 0, \text{ if } \phi = \min\{\lambda v, \Phi\}; \tag{30}$$

$$\frac{1}{\hat{q}}\frac{1-\alpha}{\bar{k}}\left(\frac{l}{l_*}\right)^{1+\psi} + \varepsilon_{\hat{q},\eta}\mu_\eta + \frac{1}{2}\varepsilon_{\hat{q}_\eta,\eta}\varepsilon_{\hat{q},\eta}\sigma_\eta^2 + \sigma_v\left(\sigma_{\hat{q}} + \sigma_y\right) - \rho = 0, \text{ if } \phi = 1/\eta < \min\{\lambda v, \Phi\} \tag{31}$$

The asset pricing condition for Tobin's Q is:

$$\left[\frac{1-a_h}{\hat{q}}\frac{1-\alpha}{a\bar{k}}\left(\frac{l}{l_*}\right)^{1+\psi}+\sigma_v\left(\sigma_{\hat{q}}+\sigma_y\right)\right]\phi\mathbf{1}_{\phi<1/\eta}+\frac{\gamma}{v}+\varepsilon_{v,\eta}\mu_\eta+\frac{1}{2}\varepsilon_{v_\eta,\eta}\varepsilon_{v,\eta}\sigma_\eta^2-\sigma_y\sigma_v-\gamma=0 \tag{32}$$

Notice in particular that:

$$\sigma_v = \varepsilon_{v,\eta}\sigma_\eta \tag{33}$$

$$\sigma_{\hat{q}} = \varepsilon_{\hat{q},\eta}\sigma_\eta \tag{34}$$

$$\sigma_y = \frac{1}{1-\left[\varepsilon_{\hat{q},\eta}+\alpha\varepsilon_{l,\eta}+(1-\alpha)\,\varepsilon_{a,\eta}\right](\phi-1)}\sigma_A-\sigma_{\hat{q}} \tag{35}$$

ODEs

Asset pricing conditions (30)-(35) deliver an ordinary differential equation system (ODEs) of second order. The independent variable in the ODEs is η. The dependent variables are $\hat{q}$ and v.[27] The boundary conditions for the ODEs are similar to those in the autarky banking economy of Maggiori (2017). Specifically, I impose that:

$$\lim_{\eta\to1}\sigma_{\hat{q}}+\sigma_y=1 \quad \text{and} \quad \lim_{\eta\to1}\frac{d}{d\eta}\left(\sigma_{\hat{q}}+\sigma_y\right)=0, \tag{36}$$

and that:

$$\lim_{\eta\to1}\sigma_v=0 \quad \text{and} \quad \lim_{\eta\to1}\frac{d}{d\eta}\sigma_v=0. \tag{37}$$

Intuitively, boundary conditions (36) and (37) imply that endogenous risk vanishes smoothly, as financial intermediaries own all of the wealth in the economy.

Consistency Condition for Inflation

[27] The quantity of aggregate labor l and capital requirement Φ are taken as given in the Markov equilibrium. Notice that $a=a_h(1-\phi\eta)+\phi\eta$ and that $\phi=\min\{\lambda v,\Phi,1/\eta\}$.

The inflation rate is the solution to the equation:

$$\left[1-\frac{\varepsilon-1}{\theta}\pi\right]^{-\frac{1}{\varepsilon-1}}=\frac{\frac{1}{\omega}E\left[\int_t^\infty \exp\left\{\int_t^s\left[\theta\frac{1}{\omega_{\tilde{s}}}\left(1-\frac{\varepsilon-1}{\theta}\pi_{\tilde{s}}\right)^{\frac{\varepsilon}{\varepsilon-1}}-\rho\right]d\tilde{s}\right\}\left(\frac{l_s}{l_*}\right)^{1+\psi}ds|\omega,\eta\right]}{E\left[\int_t^\infty \exp\left\{\int_t^s\left[(\varepsilon-1)\pi_{\tilde{s}}-(\theta+\rho)\right]d\tilde{s}\right\}ds|\omega,\eta\right]} \tag{38}$$

The LHS equals p_*/p. The numerator on the RHS is expected production costs b; the denominator is expected sales revenues m. In this notation, $\pi_s = \pi(\omega_s, \eta_s)$ and $l_s = l(\omega_s, \eta_s)$.

Conditions that Characterize the Markov Equilibrium

The Markov equilibrium is characterized by the following conditions: $a = a_h(1-\phi\eta) + \phi\eta$; $\phi = \min\{\lambda v, \Phi, 1/\eta\}$; (27)-(38).

Invariant Distribution in the Flexible Price Economy

Invariant density function $g(1,\eta)$ solves the Kolmogorov forward equation:

$$-\frac{\partial}{\partial\eta}\left[\mu_\eta\eta * g(1,\eta)\right] + \frac{\partial^2}{\partial\eta^2}\left[(\sigma_\eta\eta)^2 * g(1,\eta)\right] = 0.$$

Invariant density function $g(1,\eta)$ therefore satisfies that:

$$g(1,\eta) \propto \frac{1}{(\sigma_\eta\eta)^2}\exp\left\{2\int_0^\eta \frac{\mu_{\tilde{\eta}}\tilde{\eta}}{(\sigma_{\tilde{\eta}}\tilde{\eta})^2}d\tilde{\eta}\right\} \tag{39}$$

with $\int_0^1 g(1,\eta)\,d\eta = 1$.

Appendix C

The Numerical Method

The numerical method has two steps. The first is similar for both policy mandates, but the second differs.

The first step solves the ODEs taking policy rules $\{\ln l/l_*, \Phi\}$ as given. To solve for the ODEs, I use spectral methods. Specifically, I interpolate mappings $\hat{q}$ and v with linear combinations of Chebyshev Polynomials of the First Kind. I evaluate the interpolation at the Chebyshev nodes using a grid with 190 points. I use a nonlinear solver to find the coefficients associated with the Chebyshev Polynomials in the linear combination. I use as my initial guess the Markov equilibrium in the frictionless economy. That is, $l = l_*$; $\phi = 1/\eta$; $q/r_k = 1/\rho$; $v = 1$; $\omega = 1$. In the traditional mandate $l = l_*$ always, whereas in the coordinated mandate $l = l_*$ is not necessarily the case.

The second step proceeds differently, depending on the policy mandate. In the traditional mandate, the second step derives the constrained efficient capital requirement Φ_e. To this end, first, I compute the invariant density function $g(1, \eta)$ using drift and the diffusion processes μ_η and σ_η, in accord with (39). Second, I compute the present discounted value of $\ln a$ and the value the given capital requirement Φ in the first step achieves. I repeat the first and second steps for different capital requirements until I find the capital requirement Φ_e that achieves the maximum possible value. Below, I specify the capital requirements among which I searched.

In the coordinated mandate, the second step derives the policy rules that maximize social welfare. To this end, first, I derive the inflation π that satisfies the consistency condition for inflation, given the policy rules $\{\ln l/l_*, \Phi\}$ and the solution to the ODEs. Below, I explain the process I follow to solve for the consistency condition for inflation. Second, I use π to derive drift process μ_ω, and then use μ_ω, together with drift and diffusion processes μ_η and σ_η, to simulate the invariant density function $g(\omega, \eta)$. With the invariant density function $g(\omega, \eta)$, I compute social welfare and the value the given policy rule $\{\ln l/l_*, \Phi\}$ in the first step achieves. I repeat the first and second steps for different policy rules until I find the policy rules that maximize social welfare. Below, I also specify the policy rules among which I searched.

Restrictions on Policy Rules

I impose a polynomial functional form for the capital requirement. Specifically:

$$\Phi(\eta) = \sum_{d=0}^{D} \frac{a_d}{(\eta_2 - \eta_1)^d} (\eta - \eta_1)^d .$$

The constants η_1 and η_2 are the values of state η at which capital requirement Φ intersects with mappings λv and $1/\eta$, respectively. The constant η_2 is always greater than η_1. The natural number D denotes the degree of the polynomial. The real constants $\{a_d\}$ are such that: (*i*) Φ and its first $\frac{1}{2}(D-1)$ derivatives match λv and its corresponding derivatives at $\eta = \eta_1$; and (*ii*) Φ and its first $\frac{1}{2}(D-1)$ derivatives match $1/\eta$ and its corresponding derivatives at $\eta = \eta_2$. The natural number D is always odd. The restriction on the real constants $\{a_d\}$ is imposed to reduce the dimensionality of the search problem. Notice that the constants η_1 and η_2 are the only free parameters in $\Phi(\eta)$ independent of the value of D. In the numerical solution, a value of D beyond 7 does not improve social welfare.

I impose a linear functional form for the employment gap. Specifically:

$$\ln\left[l(\eta)/l_*\right] = a_l * (\eta - \eta_l).$$

The constant a_l is the semi-elasticity of aggregate labor with respect to wealth share η. The constant η_l indicates the state η at which the sign of the employment gap switches.

Consistency Condition for Inflation

In the competitive equilibrium, inflation rate π_t is related to expected marginal sales revenues m_t and to expected marginal production costs b_t accordingly:

$$\pi_t = \frac{\theta}{\varepsilon - 1}\left[1 - \left(\frac{b_t}{m_t}\right)^{-(\varepsilon-1)}\right].$$

I characterize m_t and b_t as the solution to a system of partial differential equations (PDEs).

Asset Pricing Conditions

The expected marginal sales revenues m_t satisfies that:

$$m_t = E_t \int_t^\infty \exp\left\{\int_t^s \left[(\varepsilon-1)\pi_{\tilde{s}} - (\theta+\rho)\right] d\tilde{s}\right\} ds .$$

Let $G_{m,t}$ denote the gain process:

$$\begin{aligned} G_{m,t} &\equiv E_t \int_0^\infty \exp\left\{\int_0^s \left[(\varepsilon-1)\pi_{\tilde{s}} - (\theta+\rho)\right] d\tilde{s}\right\} ds \\ &= \int_0^t \exp\left\{\int_0^s \left[(\varepsilon-1)\pi_{\tilde{s}} - (\theta+\rho)\right] d\tilde{s}\right\} ds + \exp\left\{\int_0^t \left[(\varepsilon-1)\pi_{\tilde{s}} - (\theta+\rho)\right] d\tilde{s}\right\} m_t. \end{aligned}$$

The equality in the second line follows from the definition of m_t. An asset pricing condition for m_t follows from applying Ito's Lemma to the RHS and from equalizing the resulting drift process to zero.[28] The asset pricing condition for m_t is:

$$\frac{1}{m_t} + (\varepsilon - 1)\,\pi_t - (\rho + \theta) + \mu_{m,t} = 0\ .$$

The expected marginal production costs b_t satisfies that:

$$b_t = E_t \int_t^{\infty} \exp\left\{ \int_t^s \left[\varepsilon \pi_{\tilde{s}} - (\theta + \rho)\right] d\tilde{s} \right\} \left(\frac{l_s}{l_*}\right)^{1+\psi} \frac{1}{\omega_s} ds\ .$$

Let $G_{b,t}$ denote the gain process:

$$\begin{aligned} G_{b,t} &\equiv E_t \int_0^{\infty} \exp\left\{ \int_0^s \left[\varepsilon \pi_{\tilde{s}} - (\theta + \rho)\right] d\tilde{s} \right\} \left(\frac{l_s}{l_*}\right)^{1+\psi} \frac{1}{\omega_s} ds \\ &= \int_0^t \exp\left\{ \int_0^s \left[\varepsilon \pi_{\tilde{s}} - (\theta + \rho)\right] d\tilde{s} \right\} \left(\frac{l_s}{l_*}\right)^{1+\psi} \frac{1}{\omega_s} ds + \exp\left\{ \int_0^t \left[\varepsilon \pi_{\tilde{s}} - (\theta + \rho)\right] d\tilde{s} \right\} b_t. \end{aligned}$$

The equality in the second line follows from the definition of b_t. The asset pricing condition for b_t is:

$$\left(\frac{l_t}{l_*}\right)^{1+\psi} \frac{1}{\omega_t}\frac{1}{b_t} + \varepsilon \pi_t - (\rho + \theta) + \mu_{b,t} = 0\ .$$

PDEs and The Numerical Method

The PDEs follows from the asset pricing conditions for m_t and b_t. The PDEs is:

$$\frac{1}{m} + (\varepsilon - 1)\,\pi - (\rho + \theta) + \varepsilon_{m,\eta}\mu_\eta + \varepsilon_{m,\omega}\mu_\omega + \frac{1}{2}\varepsilon_{m_\eta,\eta}\varepsilon_{m,\eta}\sigma_\eta^2 = 0$$

$$\left(\frac{l}{l_*}\right)^{1+\psi} \frac{1}{\omega}\frac{1}{b} + \varepsilon \pi - (\rho + \theta) + \varepsilon_{b,\eta}\mu_\eta + \varepsilon_{b,\omega}\mu_\omega + \frac{1}{2}\varepsilon_{b_\eta,\eta}\varepsilon_{b,\eta}\sigma_\eta^2 = 0\ .$$

The independent variables in the PDEs are ω and η. The dependent variables are m and b.

To solve for the PDEs, I use spectral methods. Specifically, I interpolate mappings m and b with a linear combination of Chebyshev Polynomials of the First Kind. I evaluate the interpolation at the Tensor basis (i.e., Tensor product plus Cartesian product of Chebyshev

[28] The drift process of the gain process $G_{m,t}$ is null, because $G_{m,t}$ is the conditional expectation of a random variable.

nodes) using a grid with 15×15 points. I use a nonlinear solver to find the coefficients associated with the Chebyshev Polynomials in the linear combination. I use as initial guess the mappings m_0 and b_0 corresponding to the traditional mandate. That is,

$$
\begin{aligned}
m_0(\omega, \eta) &= \int_t^{\infty} \exp\left\{\int_t^{s} \left[(\varepsilon - 1)\pi_{\tilde{s}} - (\theta + \rho)\right] d\tilde{s}\right\} ds \\
b_0(\omega, \eta) &= \int_t^{\infty} \exp\left\{\int_t^{s} \left[\varepsilon \pi_{\tilde{s}} - (\theta + \rho)\right] d\tilde{s}\right\} \frac{1}{\omega_s} ds,
\end{aligned}
$$

with initial state $\omega_t = \omega$.[29] I compute the integrals in the RHS numerically.

[29] Notice that m_0 and b_0 do not depend on the state η.