Uninsured Unemployment Risk and Optimal Monetary Policy*

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ABSTRACT. I study optimal monetary policy in a New Keynesian economy wherein households precautionary-save against uninsured, endogenous unemployment risk. In this economy greater unemployment risk raises desired savings, causing aggregate demand to fall and feed back to greater unemployment risk. I show this deflationary feedback loop to be constrained-inefficient and to call for an accommodative monetary policy response: after a contractionary aggregate shock the policy rate should be kept significantly lower and for longer than in the perfect-insurance benchmark. For example, the usual prescription obtained under perfect insurance of a hike in the policy rate in the face of a bad supply (i.e., productivity or cost-push) shock is easily overturned. If implemented, the optimal policy effectively breaks the deflationary feedback loop and takes the dynamics of the imperfect-insurance economy close to that of the perfect-insurance benchmark.

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1. INTRODUCTION

Households' precautionary-saving response to uninsured unemployment risk may generate substantial aggregate volatility, relative to a hypothetical situation of perfect insurance. The reason for this is that greater unemployment risk strengthens the precautionary motive for saving, causing aggregate demand, output and employment to fall, which ultimately feeds back to greater unemployment risk.[1] In this paper I ask how *should* the central bank respond to aggregate shocks when faced with this feedback loop, *by how much* does this response differ from that under perfect insurance, and *how effective* is it at stabilising welfare-relevant aggregates. To this purpose, I construct a New Keynesian model with imperfect unemployment insurance and a frictional labour market and then derive the optimal monetary policy response to two prominent aggregate shocks,

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[1] See, e.g., Auclert and Rognlie (2016), Beaudry et al. (2017), Challe et al. (2017), Chamley (2014), Den Haan et al. (2017), Heathcote and Perri (2017), Kekre (2017), McKay and Reis (2017), Ravn and Sterk (2017a, 2017b) and Werning (2015) for alternative formulations of this feedback loop. Challe et al. (2017), Den Haan et al (2017) and Ravn and Sterk (2017a) provide quantifications of this feedback loop for the U.S. and the euro area.

namely transitory (but persistent) productivity and "cost-push" shocks.[2] The optimal policy is the one that best tracks a well-defined constrained-efficient allocation derived from a social welfare function aggregating the intertemporal utilities of heterogenous households and capturing all the frictions that they are facing. In find the feedback loop between unemployment risk and aggregate demand to be constrained-inefficient and to critically affect optimal policy. Essentially, monetary policy should be (much) more accommodative during recessions so as to *counter the inefficient rise in desired precautionary savings and associated fall in aggregate demand.* And conversely, it should be less accommodative in expansion, as consumption demand is boosted by the fall in desired precautionary savings.

Consider first the response to a contractionary cost-push shock, i.e., an exogenous increase in unit production costs that is passed through to final goods prices. With uninsured, endogenous unemployment risk the optimal response of the policy rate is in general ambiguous. On the one hand, the central bank should act to mitigate the direct inflationary impact of the shock, which typically commands an increase in the policy rate; such is the optimal policy in the Representative-Agent New Keynesian model ("RANK model" henceforth), and I recover this policy in the perfect-insurance limit of my imperfect-insurance model. On the other hand, the shock harms job creation and sets in motion a *deflationary* feedback loop between unemployment risk and aggregate demand; this calls for a muted, or even reverted, response of the policy rate. Under a parametric restriction that gives the optimal response of the policy rate in closed form, these two effects can be additively decomposed into a *perfect-insurance response* and an *imperfect-insurance correction.* The perfect-insurance response is the same as in the RANK model, but the imperfect-insurance correction pushes the policy rate in the opposite direction and is greater the larger workers' mean consumption drop upon unemployment (a summary measure of the lack of consumption insurance). Away from this parametric restriction the contribution of imperfect insurance can be recovered numerically by comparing the optimal responses of the policy rate in the imperfect-insurance economy and in the perfect-insurance benchmark. In the calibrated imperfect-insurance model the central bank adopts a much more accommodative stance after a contractionary cost-push shock in order to offset its inefficient impact on aggregate demand; in several specifications the policy rate should be persistently *lowered*, not raised, after the shock. Moreover, implementation of the optimal policy is effective in that it breaks the deflationary spiral and takes the aggregate dynamics of the imperfect-insurance economy close to that of the perfect-insurance benchmark.

Uninsured unemployment risk also crucially affects the optimal response of the policy rate to productivity shocks. Indeed, a persistent productivity-driven contraction (for example) generates an increase in unemployment risk and elicits a precautionary response on the part of the households. The resulting fall in aggregate demand exerts an inefficient downward pressure on inflation and employment that the central bank must stabilise, which usually requires *lowering* the policy rate. This optimal response is the opposite of that in the RANK model, which typically prescribes a *rise*

[2] In New Keynesian models, persistent productivity shocks move the IS curve that determines the dynamics of the output gap, while cost-push shocks move the Phillips curve that determines the dynamics of inflation. Clarida et al. (1999), Woodford (2003) and Gali (2008) have extensively analysed the optimal policy response to those shocks within the standard New Keynesian model with a Representative Agent.

in the policy rate in order to counter the excess aggregate demand generated by the expected recovery. To be more specific, I show that under imperfect insurance the required degree of policy accommodation after a contractionary productivity shock depends on the two forces that ultimately determine workers' consumption response, namely the *precautionary motive* (against unemployment risk) and *aversion to intertemporal substitution* (as determined by the expected path of the real wage, conditional on remaining employed). The optimal policy is to cut in the policy rate whenever the precautionary motive dominates aversion to intertemporal substitution. This happens to be the case under my baseline calibration, but even away from it any plausible alternative calibration implies that substantially more accommodation than under perfect insurance is needed. Finally, just as in the case of cost-push shocks, implementation of the optimal policy after a productivity shock successfully undoes much of the propagating effect of imperfect insurance on aggregate dynamics.

I reach these conclusions by first focusing on a baseline specification of the model and then exploring several departures from this baseline. For example, in the baseline imperfect-insurance model I assume that the real wage that splits the match surplus between a firm and a worker is constrained-efficient; this ensures that the optimal policy responses that I derive are not an artefact of an inefficient wage-setting mechanism. But I also consider a model variant with generalised Nash bargaining and show that my results continue to hold.[3] Another feature of the baseline specification is that there is a set of (constant) taxes and subsides that align the steady state of the decentralised equilibrium to its constrained-efficient counterpart. This ensures that the optimal policy I obtain is not unduly driven by steady state distortions, but this requires introducing firm subsidies that we do not observe in practice. I therefore check that my results continue to hold without these subsidies. Finally, I systematically compare my baseline results not only to the perfect-insurance benchmark – wherein the precautionary motive for saving is shut down – but also to a *constant-wage model* – wherein the precautionary motive is maintained but it is aversion to intertemporal substitution that is shut down instead.

The present paper integrates two strands of the existing literature: one that examines the propagation of aggregate shocks within the extended New Keynesian model with uninsured unemployment risk; and one that derives the optimal monetary policy response to aggregate shocks under the simplifying assumption of perfect insurance. The feedback loop that arises under imperfect insurance, labour market frictions and nominal price stickiness was identified and quantitatively evaluated by Challe et al. (2017) and Ravn and Sterk (2017a).[4] Den Haan et al. (2017) present and quantify a related feedback loop working through nominal *wage* stickiness. Gornemann et al. (2016) also construct a model with a similar set of frictions, but their focus is on the redistributive effect of monetary policy rather than the propagation of aggregate shocks. Werning (2015, Section 3.4) examines the sensitivity of aggregate demand to the nominal interest rate under the same frictions, focusing on the aggregated Euler condition and bypassing an explicit modelling of firm

[3] The Nash wage is generically inefficient in my model because the real wage not only affects firms' surplus from hiring but also redistributes income across heterogenous households.

[4] Ravn and Sterk (2017b) analyse the implications of this feedback loop for equilibrium uniqueness under a Taylor rule, for the propagation of productivity shocks, and for the behaviour of risk premia.

behaviour.[5]

A common feature of the above-mentioned papers is to specify the way monetary policy is conducted by means of an exogenous nominal interest rate or a simple nominal interest rate rule. By contrast, in the present paper the central bank sets the policy rate with the aim of tracking the constrained-efficient allocation. This generalises the analysis of optimal monetary policy traditionally undertaken within the RANK benchmark, be it without labour-market frictions (e.g., Clarida et al., 1999; Woodford, 2003; Gali, 2008) or with such frictions (Thomas, 2008; Faia, 2009; Blanchard and Gali, 2010; Ravenna and Walsh, 2011). Braun and Nakajima (2012) studied optimal policy within a New Keynesian model with *exogenous* uninsured idiosyncratic shocks; there is no feedback loop between unemployment risk and aggregate demand under this assumption and, as a consequence, the optimal policy does not significantly differ from that in the RANK model. Similarly, this feedback loop and implied monetary policy response is absent from Bilbiie and Ragot (2016), who compute the optimal policy under imperfect insurance and endogenous liquidity, from Nuño and Thomas (2017), who undertake a similar exercise in an economy with imperfect insurance and long-term nominal bonds, and from Debortoli and Gali (2017, Section 5), who look at optimal policy within a Two-Agent New Keynesian ("TANK") model. My analysis thus shows that it is the *interaction* between the endogeneity of unemployment risk and the fact that it is imperfectly insured that is key in overturning some of the policy prescriptions of the RANK model.

Finally, two papers examine optimal *unemployment insurance* (UI) policies under the same frictions as those I consider: McKay and Reis (2017), who show that they raise the optimal ex ante level of UI (due to its role as an automatic stabiliser), and Kekre (2017), who show that they rationalise state-contingent UI duration.[6] One key advantage of monetary policy over state-contingent UI is that a change in the policy rate can be implemented readily and at virtually no cost to the public authority (aside from the potential loss in seigniorage revenues). But UI policies can usefully complement monetary policy in situations where the policy rate is constrained (e.g., by an effective lower bound).

Section 2 presents the model and its equilibrium. Section 3 derives the constrained-efficient allocation and associated steady state. Section 5 formulates and solves a linear-quadratic approximation of the optimal policy problem under a particular parametric restriction; this allows deriving analytical expressions for the optimal nominal interest rate that make the specific role played by imperfect insurance and the precautionary motive fully transparent. Section 4 calibrates and numerically solves the general model. In that section alternative wage-setting mechanisms and the implications of steady-state distortions are also explored.

[5] Other papers quantitatively examine the effect of monetary policy under imperfect insurance but exogenous unemployment risk – so that there is no feedback from aggregate demand to unemployment risk. This includes Kaplan et al. (2017), who study the impact of conventional interest-rate changes, and McKay et al. (2016), who examine the effect of forward guidance.

[6] Den Haan et al. (2017) study quantitatively the impact of the level of UI on aggregate volatility.

2. The model

2.1. Households. Time is discrete: $t \in \{0, 1, ...\}$. Households are of two types: there is a unit measure of "workers", who can be employed or unemployed, and a measure $\nu > 0$ of "firm owners" who manage the firms and collect dividends. All households are infinitely-lived and discount the future at the factor $\beta \in [0, 1)$, and none of them can borrow against future income.

Workers. A worker $i \in [0, 1]$, who can be employed or unemployed, chooses the consumption sequence $\{c_{i,t+k}\}_{k=0}^{\infty}$ that maximises $V_t^i = \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k u(c_{i,t+k})$, where $c_{i,t} \geq 0$ is consumption and $e_{i,t} \in \{0, 1\}$ worker's i status in the labour market – with $e_{i,t} = 1$ if the worker is employed and 0 otherwise. $\mathbb{E}_t$ is the expectations operator – over both aggregate and idiosyncratic uncertainty – and $u(\cdot)$ is a period utility function such that $u' > 0$ and $u'' < 0$ for all $c \geq 0$. Employed workers earn the real wage w_t, while unemployed workers earn the home production income $\delta < w_t$. Workers transit randomly between labour market statuses and the associated income risk is uninsured. The budget and borrowing constraints of worker $i \in [0, 1]$ at date t are given by, respectively:

$$a_{i,t} + c_{i,t} = e_{i,t} w_t + (1 - e_{i,t}) \delta + R_t a_{i,t-1} \quad \text{and} \quad a_{i,t} \geq 0, \tag{1}$$

where $a_{i,t}$ is the real value of worker's bond wealth at the end of date t and R_t the gross real return on assets. Workers' optimal consumption-saving choices must satisfy the Euler condition $\mathbb{E}_t \beta u'(c_{i,t+1}) R_{t+1}/u'(c_{i,t}) \leq 1$, with an equality if the borrowing constraint is slack and a strict inequality if it is binding.

Firm owners. Firm owners share the period utility function $\tilde{u}(c)$, with $\tilde{u}' > 0$ and $\tilde{u}'' \leq 0$, which may differ from $u(c)$.[7] They do not face any idiosyncratic income risk, and they all hold the same asset wealth a_{-1}^F at the beginning of time; they thus stay symmetric at all times and I denote their common individual consumption and end-of-period asset wealth by c_t^F and a_t^F, respectively. In every period they get an equal share of the aggregate dividend D_t that results from firms' rents (see below), as well as a home production income, of amount $\varpi \geq 0$ in the aggregate, and a lump sum transfer, of amount τ_t in the aggregate. A firm owner thus maximises $V_t^F = \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \tilde{u}\left(c_{t+k}^F\right)$, subject to:

$$a_t^F + c_t^F = (D_t + \varpi + \tau_t)/\nu + R_t a_{t-1}^F \quad \text{and} \quad a_t^F \geq 0. \tag{2}$$

Given their preferences and constraints, the optimal consumption plan of a firm owner must satisfy $\mathbb{E}_t M_{t+1}^F R_{t+1} \leq 1$, where M_{t+1}^F denotes firm owners' common marginal rate of intertemporal substitution ("MRIS" henceforth):

$$M_{t+1}^F = \beta \tilde{u}(c_{t+1}^F)/\tilde{u}(c_t^F). \tag{3}$$

2.2. Firms. The production structure has three layers: intermediate goods firms produce out of workers' labour units, which they hire in a frictional labour market with search costs. Those

[7] As shown in Section 3, the preferences of workers and firm owners will affect the efficient sharing of aggregate risk between the two groups and thereby the extent of wage fluctuations.

goods are sold to wholesale firms, each of whom turn them into a differentiated good. Finally, wholesale goods are purchased and reassembled by final goods firms, the output of which is used for consumption and search costs.

Final goods sector. There is a representative, competitive firm that produces the final good by combining wholesale inputs according to the function:

$$y_t = \left(\int_0^1 y_{h,t}^{\frac{\theta-1}{\theta}} \mathrm{d}h \right)^{\frac{\theta}{\theta-1}}, \tag{4}$$

where $y_{h,t}$ is the quantity of wholesale good h used in production and $\theta > 1$ the cross-partial elasticity of substitution between wholesale inputs. Denoting $p_{h,t}$ as the price of wholesale good h in terms of the final good, the optimal combination of inputs gives the following demands:

$$y_{h,t} = y_t p_{h,t}^{-\theta}, \;\; h \in [0,1], \tag{5}$$

while the zero-profit condition in the final goods sector implies that $\int_0^1 p_{h,t}^{1-\theta} \mathrm{d}j = 1$.

Wholesale sector. Wholesale firm $h \in [0,1]$ turns every intermediate good into a specialised good that is monopolistically supplied to the final goods sector. The profit of wholesale firm h is

$$\Pi_{h,t}^W = y_{h,t}[p_{h,t} - \varphi_t(1 - \tau^W)], \tag{6}$$

where φ_t is the price of intermediate goods in terms of the final goods and τ^W a production subsidy to the wholesale sector, financed through a lump sum tax on firm owners.[8]

Wholesale firms face nominal pricing frictions a la Calvo: in every period a fraction $1 - \omega \in [0,1]$ of the firms are able to reset their price optimally, while the other firms keep it unchanged. The resulting time-varying distribution of wholesale prices can be summarised by three moments, namely the optimal reset price common to all price-resetting firms $\tilde{p}_t$, final goods' inflation π_t, and the price dispersion index $\Delta_t \equiv \int_0^1 p_{h,t}^{-\theta} \mathrm{d}h \geq 1$ (see Woodford, 2003, for details). These moments evolve as follows. First, the optimal reset price is given by:

$$\tilde{p}_t = \frac{\theta(1 - \tau^W)\Xi_t}{(\theta - 1)\,\Sigma_t}, \tag{7}$$

where Ξ_t and Σ_t obey the following forward recursions:

$$\Xi_t = \varphi_t y_t + \omega\,(1 + \pi_{t+1})^{\theta}\,\mathbb{E}_t M_{t+1}^F \Xi_{t+1} \;\; \text{and} \;\; \Sigma_t = y_t + \omega\,(1 + \pi_{t+1})^{\theta-1}\,\mathbb{E}_t M_{t+1}^F \Sigma_{t+1},$$

where M_{t+1}^F is given by equation (3).

Second, current inflation depends on the optimal reset price according to:

$$\pi_t = [\omega^{-1} - \left(\omega^{-1} - 1\right)(\tilde{p}_t)^{1-\theta}]^{\frac{1}{\theta-1}} - 1. \tag{8}$$

[8]This subsidy will serve in Section 3 to correct the steady-state distortion due to monopolistic competition.

Third, the dynamics of the price dispersion index as a function of $(\tilde{p}_t, \pi_t)$ is given by:

$$\Delta_t = (1-\omega)\left(\tilde{p}_t\right)^{-\theta} + \omega\left(1+\pi_t\right)^{\theta}\Delta_{t-1}, \tag{9}$$

and I assume that prices are symmetric at the beginning of time (i.e., $\Delta_{-1} = 1$).

From equations (5)–(6) and the definition of Δ_t, the total rent generated by the wholesale sector, which will contribute to the aggregate dividend paid out to firm owners, is given by:

$$\Pi_t^W = \int_0^1 \Pi_{h,t}^W \mathrm{d}h = y_t[1-\varphi_t\left(1-\tau^W\right)\Delta_t]. \tag{10}$$

Intermediate goods sector and labour market flows. Intermediate goods firms produce z_t units of good out of one unit of labour, and labour productivity evolves as follows:

$$z_t = 1 + \mu_z\left(z_{t-1} - 1\right) + \epsilon_{z,t},$$

where $\mu_z \in [0,1)$ and $\epsilon_{z,t}$ is a white noise process with mean zero and small bounded support.

These firms hire labour in a frictional market with search costs. At the beginning of date t a constant fraction $\rho \in (0,1]$ of existing employment relationships are destroyed, at which point the size of the unemployment pool goes from $1-n_{t-1}$ to $1-(1-\rho)\,n_{t-1}$. At that time intermediate goods firms post v_t vacancies, at a unit cost $c>0$, a random matching market opens and $m\left(1-(1-\rho)\,n_{t-1}\right)^{\gamma} v_t^{1-\gamma}$ (with $m>0$ and $\gamma \in (0,1)$) new employment relationships are formed.[9] It follows that the job-finding and vacancy-filling rates are, respectively:

$$f_t = m\left[\frac{v_t}{1-(1-\rho)\,n_{t-1}}\right]^{1-\gamma} \text{ and } \lambda_t = m\left[\frac{v_t}{1-(1-\rho)\,n_{t-1}}\right]^{-\gamma}. \tag{11}$$

The value to firm owners of an employment relationship, denoted J_t, is the sum of a flow payoff – the after-tax rent generated by the match – and a continuation value that depends on the survival rate of the match and firm owners' MRIS:

$$J_t = (1-\tau^I)(z_t\varphi_t - w_t + T - \zeta_t) + (1-\rho)\,\mathbb{E}_t M_{t+1}^F J_{t+1}, \tag{12}$$

where $\tau^I \in [0,1]$ is the corporate tax rate and T a wage subsidy. ζ_t is a random wage tax evolving as follows:

$$\zeta_t = \mu_\zeta \zeta_{t-1} + \epsilon_{\zeta,t},$$

where $\mu_\zeta \in [0,1)$ and $\epsilon_{\zeta,t}$ is a white noise process with mean zero and small bounded support.

The taxes and subsidy τ^I and T will serve the same purpose as the production subsidy τ^W in the wholesale sector: they will be set in such a way that the steady state of the decentralised equilibrium be constrained-efficient. Unlike in the basic RANK model the production subsidy τ^W does not suffice for this here because the economy has two distortions in addition to monopolistic

[9] This standard timing assumption implies that firms may fill vacancies within the period in which they are opened, while workers may change job without going through a period of unemployment.

competition in the wholesale sector: congestion externalities in the intermediate-good sector (due to labour-market frictions) and imperfect insurance against unemployment risk; we will see in Section 3 below how τ^I and T eliminate these additional distortions in steady state. The random tax ζ_t perturbs the real marginal cost of intermediate goods firms and is partly pass-through to final-good prices. It will manifest itself as a pure cost-push shock and make the decentralised equilibrium of the stochastic economy generically constrained-inefficient. The net proceeds of all taxes and subsidies to the intermediate goods sector are rebated lump-sum to firm owners (they enter the transfer τ_t in equation (2)).

Under free entry, the cost of a vacant job (c) must equate its expected payoff ($\lambda_t J_t$, since vacancies can be filled immediately). Then, using equations (11)–(12) and the fact that $\lambda_t^{-1} = f_t^{\frac{\gamma}{1-\gamma}}/m^{\frac{1}{1-\gamma}}$, I get the following forward recursion for the job-finding rate:

$$f_t^{\frac{\gamma}{1-\gamma}} = \frac{(1-\tau^I)m^{\frac{1}{1-\gamma}}}{c}(z_t\varphi_t - w_t + T - \zeta_t) + (1-\rho)\,\mathbb{E}_t M_{t+1}^F f_{t+1}^{\frac{\gamma}{1-\gamma}}. \tag{13}$$

Since employed workers are separated from their firm with probability ρ at the very beginning of the period, but can immediately find a job with probability f_t, the period-to-period transition rate from employment to unemployment is given by:

$$s_t = \rho\,(1 - f_t)\,. \tag{14}$$

Note that it is the transition probability s_t, and not the beginning-of-period match destruction rate ρ, that measures the extent of unemployment risk faced by employed workers; consequently, it is this variable that will determine their desired precautionary savings.

From (f_t, s_t) in equations (13)–(14), we obtain the law of motion for total employment:

$$n_t = f_t\,(1 - n_{t-1}) + (1 - s_t)\,n_{t-1}. \tag{15}$$

Finally, from the flow payoff in equation (12), the aggregate rent generated by intermediate goods firms at time t is:

$$\Pi_t^I = n_t(1-\tau^I)(z_t\varphi_t - w_t + T - \zeta_t) - cv_t, \tag{16}$$

so the aggregate dividend D_t paid out to firm owners in equation (2) is $D_t = \Pi_t^W + \Pi_t^I$.

Firms' vacancy-posting decisions depend on the real wage w_t, which under random matching is indeterminate within the bargaining set (see Hall, 2005, for an extensive discussion). The baseline specification throughout the paper that w_t is equal to its socially efficient level w_t^*, which is derived in Section 3 below. I will also consider alternative – hence in general inefficient – wage-setting mechanisms in Section 5.3.

2.3. Policymakers. There are two policymakers, the government and the central bank. The government sets the (constant) taxes and subsidies τ^W, τ^I and T and rebates the (possibly negative) net revenue to firm owners in a lump sum manner. From equations (10) and (16), the net transfer to firm owners is:

$$\tau_t = \underbrace{\tau^I n_t \left(z_t \varphi_t - w_t\right)}_{\text{corporate taxes}} - \underbrace{\tau^W \varphi_t \Delta_t y_t}_{\text{production subsidies}} - \underbrace{n_t (1 - \tau^I) \left(T - \zeta_t\right)}_{\text{wage subsidies}}. \tag{17}$$

In most of my analysis I assume that the taxes and transfers are set in a way that decentralises the constrained-efficient allocation in the absence of aggregate shocks. However, in Section 5.4 I also explore a model variant wherein the government has a more restricted set of instruments, which results in a distorted steady state.

The central bank controls the nominal interest rate on bonds i_t (the "policy rate"). The gross real ex post return that results from the policy rate and the dynamics of inflation is:

$$R_t = \left(1 + i_{t-1}\right) / \left(1 + \pi_t\right). \tag{18}$$

2.4. Market clearing. Given the measures of workers and firm owners (1 and ν, respectively) and the market and home production of final goods, the market-clearing conditions for bonds and final goods are given by $\int_{[0,1]} a_{i,t} \mathrm{d}i + \nu a_t^F = 0$ and $\int_{[0,1]} c_{i,t} \mathrm{d}i + \nu c_t^F + c v_t = y_t + \delta \left(1 - n_t\right) + \varpi$, respectively. The supply of intermediate goods is $z_t n_t$, while from (5) the demand for intermediate goods is $\int_{[0,1]} y_{h,t} \mathrm{d}h = \Delta_t y_t$. Hence, clearing of the market for intermediate goods requires:

$$\Delta_t y_t = z_t n_t \tag{19}$$

2.5. Equilibrium: definition and characterisation. An equilibrium is a set of sequences of (i) households' $(\{c_t^F, a_t^F, c_t^i, a_t^F, a_t^i\}_{t=0}^{\infty},\ i \in [0,1])$, firms' $(\{y_t, y_{h,t}, , p_t^*\}_{t=0}^{\infty},\ h \in [0,1])$ and central bank's $(\{i_t\}_{t=0}^{\infty})$ decisions that are individually optimal given prices; and (ii) aggregate variables $\{v_t, J_t, \lambda_t, f_t, s_t, n_t, \Delta_t, \varphi_t, \pi_t, \Pi_t^W, \Pi_t^I, R_t\}_{t=0}^{\infty}$ that solve equations (8) to (19) together with the free entry condition $c = \lambda_t J_t$.

Under the assumptions made so far, the model does not generate a distribution of wealth across workers, despite imperfect unemployment insurance. The reason for this is that with a zero debt limit no one is issuing the assets that the precautionary savers would be willing to purchase for self-insurance – see Krusell et al. (2011), McKay and Reis (2016) and Ravn and Sterk (2017a, 2017b). Intuitively, employed workers' precautionary-saving behaviour pushes down the real interest rate below households' common rate of time preference. And at that interest rate, both unemployed workers (who face a rising expected income profile) and firm owners (who face no idiosyncratic risk) would like to borrow against future income, but they cannot due to a binding debt limit (this is established formally farther down). Hence the supply of assets is zero in equilibrium and no asset trade ever takes place when workers change employment statuses. This feature of the equilibrium allows the precautionary motive to be operative – as shows up in the fact that the interest rate fluctuates *below* households' rate of time preference – without the need of tracking a time-varying wealth distribution, thereby maintaining the high level of tractability that is needed for the analysis of optimal monetary policy.

Two remarks about the quantitative implications of this no-trade property are in order here. First, in the numerical analysis of Section 5 I calibrate the δ/w ratio to 90%. In an equilibrium without asset trades this implies that workers' consumption loss upon unemployment is of 10%. This values lies in the lower range of available estimates for the U.S. and the euro area.[10] Therefore, that employed workers do not hold assets in equilibrium will *not* translate into an unrealistically low level of consumption insurance that could artificially overestimate the precautionary motive. Second, one may argue that it is *liquid wealth*, rather than the entire net worth, that households can use to insulate nondurables consumption from income fluctuations, and liquid wealth is very low for many households in the U.S. (see, e.g., Challe et al., 2017). For both reasons, it is unlikely that the focus on an equilibrium without asset trades significantly distorts the response of desired savings to aggregate shocks and the implied optimal policy response.

The existence of the no-trade equilibrium can be established formally by spelling out the corresponding equilibrium conditions and showing that they hold in steady state. Provided that aggregate shocks have small bounded support (my maintained assumption), then these conditions will also hold in stochastic equilibrium. The first property of the equilibrium is that employed workers do not face a binding debt limit (because they wish to precautionary-save). Hence their Euler condition holds with equality:

$$\mathbb{E}_t M_{t+1}^e R_{t+1} = 1, \tag{20}$$

where their MRIS, incorporating both aggregate and idiosyncratic risk, and taking account of the fact that all workers consume their current income (δ or w_t), is given by:

$$M_{t+1}^e = \beta \frac{(1 - s_{t+1})\, u'(w_{t+1}) + s_{t+1} u'(\delta)}{u'(w_t)}. \tag{21}$$

The MRIS in equation (21) summarises an employed workers' desire to save and it is driven by two forces here: *aversion to intertemporal substitution* and the *precautionary motive*. Aversion to intertemporal substitution shows up in the fact that transitory wage fluctuations affect M_{t+1}^e: employed workers wish to save when the current wage is unusually high and borrow when it is unusually low. The precautionary motive shows up in the fact that changes in unemployment risk (s_{t+1}) also affect M_{t+1}^e: the greater this risk, the stronger the desire to save (since by assumption $\delta < w_t \; \forall t$, hence $u'(w_{t+1}) > u'(\delta)$). Hence, by equation (20), a declining wage profile or an increase in unemployment risk both exert a downward pressure on the equilibrium real interest rate R_{t+1}. Holding the policy rate i_t constant, a fall in R_{t+1} is brought about by deflationary pressures in the current period associated with a rise in expected inflation.

The second feature of the equilibrium is that unemployed workers face a binding debt limit,

[10] See Den Haan et al. (2017, Appendix A) for an extensive discussion of the available evidence on this parameter.

i.e., their Euler condition holds with strict inequality:

$$\mathbb{E}_t M_{t+1}^u R_{t+1} < 1, \tag{22}$$

where

$$M_{t+1}^u = \beta \frac{(1 - f_{t+1}) u'(\delta) + f_{t+1} u'(w_{t+1})}{u'(\delta)}. \tag{23}$$

The conditions (20) and (22) can jointly hold because employed workers face a decreasing expected consumption profile – due to the risk of losing one's job – while unemployed workers face a rising expected consumption profile – due to the possibility of finding one. Hence current marginal utility is low relative to expected marginal utility for the former, while the opposite is true for the latter.

The third feature of the equilibrium is that firm owners also face a binding debt limit, i.e.,

$$\mathbb{E}_t M_{t+1}^F R_{t+1} < 1. \tag{24}$$

Conditions (20) and (24) are mutually consistent because employed workers' precautionary motive take the gross real interest rate down below $1/\beta$, while firm owners face no idiosyncratic income shocks and hence have no reason to self-insure. Thus, instead of accepting a low return on their savings, they turn (frustrated) borrowers and consume their current income in every period. From equations (10), (16), (17) and (19), the consumption of a firm owner, after all taxes and subsidies have been rebated lump-sum, is given by:

$$c_t^F = \nu^{-1}(\Pi_t^W + \Pi_t^I + \tau_t) = \nu^{-1}(n_t(z_t/\Delta_t - w_t) - cv_t + \varpi). \tag{25}$$

Equation (25) shows that, holding labour market conditions (n_t, v_t, w_t) (hence workers' welfare) fixed, price dispersion Δ_t creates a productive inefficiency that is directly borne by firm owners. Whether and by how much this inefficiency is passed through to workers through lower wages depends on the wage-setting mechanism, and we explore several possibilities farther down.[11]

Let us now verify that equations (20), (22) and (24) hold simultaneously in steady state. From equations (20)–(21), in the absence of aggregate shocks R is given by:

$$R = 1 + i = \frac{1}{\beta \left[1 - s + s u'(\delta)/u'(w)\right]} < \frac{1}{\beta}. \tag{26}$$

For $f \in (0,1)$ we have $s = \rho(1 - f) > 0$ and hence (since $\delta < w$), $M^u < M^e$ and $M^F = \beta < M^e$. Thus, with $M^e R = 1$ – i.e., employed workers are not borrowing-constrained – we have $M^u, M^F < M^e$ – so that both unemployed workers and firm owners are. The same is true in stochastic equilibrium provided that aggregate shocks are sufficiently small. Finally, I assume for simplicity that households' initial bond holdings are at their steady state value, i.e., $a_{-1}^F = a_{i,-1} = 0$ $\forall i \in [0,1]$.

[11] In a nutshell, the constrained-efficient wage derived in Section 3 implies no pass-through, but the inefficient Nash bargaining variants examined in Section 5.3 generate some pass-through.

3. Constrained efficiency

The economy is potentially plagued by four distortions: monopolistic competition in the wholesale sector, asymmetric wholesale prices due to nominal rigidities, congestion externalities in the labour market, and imperfect insurance against unemployment risk. In what follows I characterise the constrained-efficient allocation and derive the values of steady-state inflation (π) and the tax instruments (τ^W, T, τ^I) that decentralise this allocation in the absence of aggregate shocks.

3.1. Social welfare function. Since in equilibrium all households consume their current income in every period, the ex ante intertemporal utilities of employed workers, unemployed workers and firm owners are given by, respectively:

$$V_t^e = u(w_t) + \beta \mathbb{E}_t[(1 - s_{t+1})\, V_{t+1}^e + s_{t+1} V_{t+1}^u], \tag{27}$$

$$V_t^u = u\,(\delta) + \beta \mathbb{E}_t[f_{t+1} V_{t+1}^e + (1 - f_{t+1})\, V_{t+1}^u], \tag{28}$$

and

$$V_t^F = \tilde{u}(c_t^F) + \beta \mathbb{E}_t V_{t+1}^F. \tag{29}$$

The social welfare function W_t aggregates the intertemporal utilities of all the households, assigning a relative welfare weight $\Lambda \geq 0$ to firm owners: $W_t = n_t V_t^e + (1 - n_t)\, V_t^u + \Lambda \nu V_t^F$. Using equations (15) and (25) to (29) and rearranging, the social welfare function can be written recursively as follows:

$$W_t = U_t + \beta \mathbb{E}_t W_{t+1}, \tag{30}$$

where the flow payoff U_t is given by:

$$U_t = \underbrace{n_t u\,(w_t) + (1 - n_t)\, u\,(\delta)}_{\text{workers}} + \underbrace{\Lambda \tilde{u}\,([\varpi + n_t\,(z_t/\Delta_t - w_t) - c v_t]\,/\nu)}_{\text{firm owners}}. \tag{31}$$

3.2. Constrained-efficient allocation. The constrained-efficient allocation is the sequence $\{\Delta_t, w_t, n_t, v_t\}_{t=0}^{+\infty}$ that maximises W_t in (30)–(31), taking as given the initial conditions (n_{-1}, Δ_{-1}), the law of motion of Δ_t (equation (9)) and the economy-wide relationship between employment and vacancies:

$$n_t = (1 - \rho)\, n_{t-1} + (1 - (1 - \rho)\, n_{t-1})^{\gamma}\, v_t^{1-\gamma}.$$

Solving the latter equation for v_t gives:

$$v_t = \left[\frac{n_t - (1 - \rho)\, n_{t-1}}{(1 - (1 - \rho)\, n_{t-1})^{\gamma}} \right]^{\frac{1}{1-\gamma}}, \tag{32}$$

which can be substituted into (31). Equation (32) makes clear that, at any level of employment inherited from the previous period (i.e., $(1 - \rho)\, n_{t-1}$), raising current employment n_t can only be achieved by raising vacancies and hence the total hiring cost borne by firm owners. On the other hand, inherited employment $(1 - \rho)\, n_{t-1}$ affects the amount of vacancies needed to reach a particular value of n_t in two ways. First, high past employment reduces the need for new vacancies (the numerator); and second, it reduces the size of the unemployment pool, which makes hiring more difficult and raises the need for new vacancies.

Formally, the constrained-efficient allocation is the solution to

$$W_t\left(n_{t-1}, \Delta_{t-1}, z_t\right) = \max_{\tilde{p}_t, w_t, n_t \geq 0} \left\{U_t + \beta \mathbb{E}_t W_{t+1}\left(n_t, \Delta_t, z_{t+1}\right)\right\}, \tag{33}$$

subject to (8), (9) and (32).

From equations (8)–(9), it is clear that $\tilde{p}_t = 1$ for all t is optimal: starting from $\Delta_{-1} = 1$, this sequence ensures that $(\pi_t, \Delta_t) = (0, 1)$ for all t, which maximises U_t in (31) in every period. Hence the constrained-efficient allocation has zero inflation and symmetric wholesale prices at all times.

Given this and equation (31), the value of w_t that maximises W_t satisfies:

$$u'\left(w_t^*\right) = \Lambda \tilde{u}'\left(\nu^{-1}\left[n_t^*\left(z_t - w_t^*\right) - c v_t^* + \varpi\right]\right), \tag{34}$$

where starred variables denote their values in the constrained-efficient allocation.

The latter condition states that the efficient real wage is that which equates the (weighted) marginal utilities of employed workers and firm owners. This condition determines how the burden of aggregate shocks is shared between workers and firm owners over the business cycle. In the extreme case where firm owners are risk neutral, the condition results in the constant wage $w_t = u'^{-1}\left(\Lambda\right)$ because firm owners are happy to fully insure risk-averse workers against wage fluctuations. Away from this limiting case efficiency requires employed workers to bear some of the burden of aggregate fluctuations through time-variations in their wage income (for example, the real wage covaries with productivity, as Section 5 below illustrates).

Finally, the first-order and envelope conditions with respect to n_t give, respectively:

$$u\left(w_t^*\right) - u\left(\delta\right) + \Lambda \tilde{u}'\left(c_t^{F*}\right)\left[\left(1-\alpha\right) z_t - w_t^* - \frac{c}{\left(1-\gamma\right)\lambda_t^*}\right] + \beta \mathbb{E}_t \frac{\partial W_{t+1}\left(n_t, 1, z_{t+1}\right)}{\partial n_t} = 0,$$

and

$$\frac{\partial W_t\left(n_{t-1}, 1, z_t\right)}{\partial n_{t-1}} = \Lambda \tilde{u}'\left(c_t^{F*}\right) c \frac{\partial v_t}{\partial n_{t-1}} = \frac{\Lambda \tilde{u}'\left(c_t^{F*}\right) c\left(1-\rho\right)\left(1-\gamma f_t^*\right)}{\lambda_t^*\left(1-\gamma\right)}.$$

Combining those two expressions, and using equations (14)–(15) and the fact that $\lambda_t^{*-1} = f_t^{*\frac{\gamma}{1-\gamma}} / m^{\frac{1}{1-\gamma}}$, gives the following forward recursion for the constrained-efficient job-finding rate:

$$f_t^{*\frac{\gamma}{1-\gamma}} = \frac{\left(1-\gamma\right) m^{\frac{1}{1-\gamma}}}{c}\left[z_t - w_t^* + \frac{u\left(w_t^*\right) - u\left(\delta\right)}{u'\left(w_t^*\right)}\right] + \left(1-\rho\right) \mathbb{E}_t M_{t+1}^{F*} f_{t+1}^{*\frac{\gamma}{1-\gamma}}\left(1 - \gamma f_{t+1}^*\right), \tag{35}$$

from which I recover the constrained-efficient employment level n_t^* using (14)–(15).

It is instructive to compare the constrained-efficient employment dynamics, as determined by equations (15) and (35), with its dynamics in the decentralised equilibrium, as given by equations (13) and (15). Since the law of motion (15) is common to both dynamics, this amount to comparing the job-finding recursions (13) and (35).

First, in the actual sticky-price dynamics the flow payoff to intermediate goods firms, and hence the job-finding rate, are affected by variations in intermediate goods prices φ_t, while they are not in the constrained-efficient outcome (where the corresponding price is equal to 1 at all times).

Second, even in the flex-price limit the decentralised equilibrium is generically not constrained-efficient in the absence of appropriate taxes and transfers. On the one hand, imperfect insurance tends to make the decentralised job-finding rate excessively low, since firm owners do not internalise the impact of their hiring intensity on workers' idiosyncratic income risk. Formally, this shows up in the fact that $[u(w_t^*) - u(\delta)]/u'(w_t^*) > 0$ in equation (35), which calls for a positive wage subsidy T in equation (13). On the other hand, congestion externalities cause intermediate goods firms to crowd out each other in the labour market, which tends to generate excessive hiring. There are two sides to this crowding out: first, a static one operating in the current period, which shows up in the fact that $1 - \gamma < 1$ in (35); and second, an intertemporal one coming from the fact that current hiring persists over time (whenever $\rho < 1$) and hence crowds out hiring *in the next period* – which shows up in the term $1 - \gamma f_{t+1}^*$ in (35). Both types of crowding out call for setting $\tau^I > 0$ in equation (13).

3.3. Constrained-efficient steady state. The restriction that taxes and subsidies (τ^W, τ^I, T) are constant implies that they cannot, in general, decentralise the constrained-efficient allocation in the presence of aggregate shocks.[12] However, the government can at least set the tax instruments, and the central bank trend inflation, in such a way that (π, τ^W, τ^I, T) decentralise the constrained-efficient allocation *in steady state*. First, as shown above the constrained-efficient allocation has $(\tilde{p}_t, \pi_t, \Delta_t) = (1, 0, 1)\ \forall t$, while from equation (7) we have $\varphi_t = (\theta - 1)/\theta(1 - \tau^W)\ \forall t$ in any zero-inflation steady state. Then, comparing equations (13) and (35), we get that the steady state of the decentralised equilibrium is constrained-efficient provided that:

$$\pi = 0, \quad \tau^W = \frac{1}{\theta}, \quad T = \frac{u(w^*) - u(\delta)}{u'(w^*)} \quad \text{and} \quad \tau^I = 1 - \frac{(1-\gamma)[1 - \beta(1-\rho)]}{1 - \beta(1-\rho)(1-\gamma f^*)}, \tag{36}$$

where f^* satisfies

$$f^{*\frac{\gamma}{1-\gamma}} = \frac{(1-\tau^I) m^{\frac{1}{1-\gamma}}}{c[1 - \beta(1-\rho)]} \left[1 - w^* + \frac{u(w^*) - u(\delta)}{u'(w^*)}\right], \tag{37}$$

and w^* solves the steady state counterpart of equation (34). Intuitively, inflation creates relative price dispersion in wholesale prices and having $\pi = 0$ eliminates this distortion; the production subsidy τ^W corrects for monopolistic competition in the wholesale sector and is greater when wholesale goods are less substitutable (i.e., when wholesale firms have more market power); the hiring subsidy T corrects for the lack of insurance and is greater when the utility cost of falling into unemployment $(u(w^*) - u(\delta))$ is high; and the corporate tax rate τ^I corrects for congestion externalities in the labour market and is greater when the elasticity of total matches with respect to vacancies $(1-\gamma)$ is low. In what follows I assume that (36) always holds, except in Section 5.4 where I investigate the robustness of my results to the introduction of steady-state distortions.

[12] For example, equation (13) makes it clear that a suitably time-varying wage subsidy T_t would undo the impact of inefficient cost-push shocks, while a constant subsidy cannot.

4. Optimal policy with full worker reallocation

In this section I impose two parametric restrictions that allow approximating the true optimal policy problem by an linear-quadratic problem, and thereby deriving an explicit formula for the optimal nominal interest rate. This formula is meant to develop intuition about the role of imperfect insurance in affecting optimal monetary policy and to pave the way for the numerical analysis of Section 5.

The first assumption is that ρ is equal to 1, so that all employed workers are reallocated (either towards other firms or towards unemployment) in every period and hence employment ceases to be a state variable. In Section 5 the parameter ρ is instead calibrated to match the size and cyclicality of empirical worker flows in the U.S. economy. The second assumption made here is that firm owners are risk neutral (i.e., $\tilde{u}(c) = c$) and thus willing to insulate the real wage from aggregate shocks – see equation (34) and the discussion that follows. Hence, workers' desired savings are exclusively driven by the precautionary motive against unemployment risk. In Section 5 I instead calibrate $\tilde{u}(c)$ to match the observed cyclicality of the real wage and I examine how aversion to intertemporal substitution and the precautionary motive jointly affect savings and the optimal policy response. Finally I also assume in this section that $m = 1$ (this is purely for expositional clarity).

4.1. Constrained-efficient, natural, and actual employment levels.

With $\tilde{u}(c) = c$ and $\rho = m = 1$ we have, from equations (11), (15), (34) and (36):

$$w_t^* = w^* = u'^{-1}(\Lambda), \quad f_t = n_t = \lambda_t v_t = v_t^{1-\gamma} \quad \text{and} \quad \tau^I = \gamma. \tag{38}$$

Equation (35) then gives the following expression for the constrained-efficient level of employment:

$$n_t^* = \left[\frac{1-\gamma}{c}\left(z_t - w^* + \frac{u(w^*) - u(\delta)}{u'(w^*)}\right)\right]^{\frac{1-\gamma}{\gamma}}. \tag{39}$$

On the other hand, from equations (13) and (36) the *actual* level of employment is given by:

$$n_t = \left[\frac{1-\gamma}{c}\left(\varphi_t z_t - \zeta_t - w^* + \frac{u(w^*) - u(\delta)}{u'(w^*)}\right)\right]^{\frac{1-\gamma}{\gamma}}. \tag{40}$$

Finally, the *natural* level of employment – i.e., that which would prevail under flexible prices – is the same as n_t in equation (40) but with $\varphi_t = 1\ \forall t$.

In the remainder of this section I will use the linearised versions of equations (39) and (40). Using hatted variables to denote first-order level-deviations from the steady state, we have:

$$\hat{n}_t^* = \Phi \hat{z}_t \tag{41}$$

and

$$\hat{n}_t = \hat{n}_t^* + \Phi(\hat{\varphi}_t - \hat{\zeta}_t) \tag{42}$$

where

$$\Phi = \frac{(1-\gamma)^2}{\gamma c} n^{\frac{1-2\gamma}{1-\gamma}} > 0 \quad \text{and} \quad n = \frac{f^*}{f^* + \rho(1-f^*)}.$$

Looking at (42) makes it clear that the central bank cannot replicate the constrained-efficient allocation after a cost-push shock, because it cannot simultaneously close the employment gap $\hat{n}_t - \hat{n}_t^* = \Phi(\hat{\varphi}_t - \hat{\zeta}_t)$ and stabilise intermediate goods prices $\hat{\varphi}_t$.

4.2. Linear-quadratic problem. One may now derive the linear-quadratic approximation to the optimal policy problem. Appendix A shows that, to second order, maximising W_t in equation (30) is equivalent to minimising

$$L_t = \frac{1}{2}\mathbb{E}_t \sum_{k=0}^{\infty} \beta^k (\tilde{n}_{t+k}^2 + \Omega \pi_{t+k}^2), \tag{43}$$

where $\tilde{n}_t = \hat{n}_t - \hat{n}_t^*$ denotes the employment gap and

$$\Omega = \frac{\theta n \Phi}{\kappa} > 0 \quad \text{and} \quad \kappa = \frac{(1-\omega)(1-\beta\omega)}{\omega} \geq 0. \tag{44}$$

The constraints faced by the central bank are the bond Euler equation for employed workers (equations (20)–(21)) and the optimality conditions for firms in the wholesale (equations (7)–(9)) and intermediate goods (equation (13)) sectors. Linearising equation (14) with $\rho = 1$ gives $\hat{s}_t = -\hat{f}_t = -\hat{n}_t$. Linearising the Euler condition for employed workers (equations (20)–(21)) around the zero-inflation steady state gives:

$$\Psi \mathbb{E}_t \hat{n}_{t+1} = \hat{\imath}_t - \mathbb{E}_t \pi_{t+1}, \tag{45}$$

where

$$\Psi = \left[1 - n + \frac{1}{u'(\delta)/u'(w^*) - 1}\right]^{-1} \geq 0.$$

Equation (45) determines the path of the policy rate that implements a given target path of inflation and employment, given workers' precautionary response to the employment risk that they are facing. The strength of this precautionary response is measured by the composite parameter Ψ, which in turn depends on workers' consumption loss upon unemployment (through its impact on the marginal utility ratio $u'(\delta)/u'(w^*)$). In the perfect-insurance limit ($\delta/w^* \to 1$) we have $\Psi \to 0$, so the precautionary motive vanishes and labour-market risk no longer affects the equilibrium real interest rate. As δ/w^* falls and Ψ increases, the precautionary motive gains strength has a larger impact on the equilibrium real interest rate $\hat{\imath}_t - \mathbb{E}_t\pi_{t+1}$; consequently, it has a larger impact on the policy rate $\hat{\imath}_t$ that the central bank must set in order to reach a given targeted outcome.

Linearising equations (7)–(8) gives the New Keynesian Phillips curve:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \hat{\varphi}_t. \tag{46}$$

Recall that in the present framework there are two potential sources of procyclical variations in wholesale firms' real marginal cost ($\hat{\varphi}_t$): fluctuations in *wage* and in *search* costs (search costs are convex in the aggregate because it is relatively harder for everyone to hire in an expansion, and relatively easier in a recession). In the present section the first source has been assumed away, so the cyclicality of the marginal cost is entirely driven by search costs. In Section 4 I consider both source jointly.

One may now use equations (19) and (42) to express (45) and (46) in terms of the employment gap $\tilde{n}_t$ that enters the loss function (43). This gives the two constraints, imposed by households' and firms' optimal behaviour, that the central bank faces when attempting to minimise its loss:

$$\Psi \mathbb{E}_t \tilde{n}_{t+1} = \hat{\imath}_t - \mathbb{E}_t \pi_{t+1} - r_t^*, \tag{47}$$

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \frac{\kappa}{\Phi} \tilde{n}_t + \kappa \hat{\zeta}_t. \tag{48}$$

In equation (47) r_t^* is the efficient interest rate (in terms of deviation from its steady state value $R-1$), i.e., the real interest rate which would equate actual employment $\hat{n}_t$ with its efficient level $\hat{n}_t^*$. From equations (41) and (45), r_t^* is given by:

$$r_t^* = \Psi \Phi \mu_z \hat{z}_t. \tag{49}$$

The efficient interest rate covaries with productivity because of the precautionary motive: a persistent productivity slump worsens future labour market conditions and urges workers to save more (and all the more so that Ψ is large). To close the employment gap the central bank should close the interest rate gap, i.e., the difference between the actual and efficient interest rates (the right hand-side of (47)). However, because the inefficiency of the employment level due to cost-push shocks persists even under flexible prices, the efficient interest rate differs from the *natural* interest rate, which (from equations (42) and (45)) is given by:

$$r_t^n = r_t^* - \Psi \Phi \mu_\zeta \hat{\zeta}_t. \tag{50}$$

Just like negative productivity shocks, persistent cost-push shocks reduce future hiring, which raises unemployment risk and employed workers' precautionary response; thus the impact of the cost-push shock ($\hat{\zeta}_t$) on the natural interest rate (r_t^n) adds up to the effect of labour productivity ($\hat{z}_t$) working through the efficient interest rate (r_t^*).

4.3. Optimal Ramsey policy. The optimal Ramsey policy is the sequence of policy rates $\{i_{t+k}\}_{k=0}^{\infty}$ that minimises L_t in equation (43) subject to (47)–(48). Formally, I first minimise (43) subject to (48) to solve for the optimal target sequences $\{\tilde{n}_t, \pi_t\}_{t=0}^{\infty}$ after one-off productivity and cost push innovations $\hat{z}_0$ and $\hat{\zeta}_0$ occurring at $t=0$; then, I use equation (47) to infer the sequence of policy rates $\{i_t\}_{t=0}^{\infty}$ that implements those target sequences.

Table 1, whose content is derived in Appendix B, shows the optimal targeted paths of inflation and the employment gap. Following a cost-push shock, the central bank promises, and then implements, a durable recession so as to mitigate the impact of the shock on current inflation.

The shapes of the optimal paths for inflation and the employment gap after this shock mirror those obtained in the baseline RANK model (see Woodford, 2003; Gali, 2008); for example, when $\alpha + \mu_\zeta > 1$ the responses of inflation and the employment gap to the shock are both U-shaped, hence the response of the output gap also is. In contrast, productivity shocks do not generate a policy tradeoff, thereby making it possible for the central bank to simultaneously close both gaps; this implies that under the optimal policy neither inflation nor the employment gap respond to $\hat{z}_0$.

Table 1. Optimal targets for inflation and the employment gap (see Appendix B).

	$\tilde{n}_t$	π_t
$t = 0$	$-\Upsilon\theta n\hat{\zeta}_0$	$\Upsilon\hat{\zeta}_0 > 0$
$t = 1$	$-\Upsilon\theta n(\alpha + \mu_\zeta)\hat{\zeta}_0$	$\Upsilon(\alpha + \mu_\zeta - 1)\hat{\zeta}_0$
$t \geq 2$	$-\Upsilon\theta n(\sum_{k=0}^{t} \alpha^k \mu_\zeta^{t-k})\zeta_0$	$\Upsilon[\mu_\zeta^t - (1-\alpha)\sum_{k=0}^{t-1} \alpha^k \mu_\zeta^{t-k}]\zeta_0$

Note: $\Upsilon = \frac{\alpha\kappa}{1-\alpha\beta\mu_\zeta} > 0$ and $\alpha = \frac{1+\beta+\kappa\theta n/\Phi}{2\beta}[1 - (1 - 4\beta(1+\beta+\frac{\kappa\theta n}{\Phi})^{-2})^{1/2}] \in (0,1)$.

From equations (47) and (49), the path of the policy rate that implements a given (perfect-foresight) sequence $\{\tilde{n}_t, \pi_t\}_{t=0}^{\infty}$ is given by:

$$\hat{\imath}_t = \Psi\Phi\mu_z\hat{z}_t + \Psi\tilde{n}_{t+1} + \pi_{t+1}. \tag{51}$$

Using the values of $\tilde{n}_{t+1}$ and π_{t+1} in Table 1 gives the optimal sequence of policy rates:

$$\text{For } t = 0: \quad \hat{\imath}_0(\hat{z}_0, \hat{\zeta}_0) = \underbrace{\Upsilon(\alpha + \mu_\zeta - 1)\hat{\zeta}_0}_{\text{perfect-insurance response}} \underbrace{- \Psi\Upsilon\theta n(\alpha + \mu_\zeta)\hat{\zeta}_0 + \Psi\Phi\mu_z\hat{z}_0}_{\text{imperfect-insurance correction}},$$

and, for $t \geq 1$:

$$i_t(\hat{z}_0, \hat{\zeta}_0) = \underbrace{\Upsilon[\mu_\zeta^t - (1-\alpha)\textstyle\sum_{k=0}^{t} \alpha^k \mu_\zeta^{t-k}]\hat{\zeta}_0}_{\text{perfect-insurance response}} \underbrace{- \Psi\Upsilon\theta n[\textstyle\sum_{k=0}^{t} \alpha^k \mu_\zeta^{t-k}]\hat{\zeta}_0 + \Psi\Phi\mu_z^{t+1}\hat{z}_0}_{\text{imperfect-insurance correction}}.$$

The optimal policy responses to productivity and cost-push shocks can be explained as follows. First, the policy rate i_t should perfectly track movements in the efficient interest rate r_t^* that are driven by productivity shocks; for example, a (persistent) productivity-driven contraction ($\hat{z}_0 < 0$) should lead to a persistent cut in the nominal interest rate – and hence an equal fall in the *real* interest rate (since inflation remains at zero all along the optimal path). This response is due to the fact that, under imperfect insurance, a persistent productivity-driven contraction raises unemployment risk and hence strengthens the precautionary motive for saving. In the absence of a policy response employment and inflation would deviate from target *downwards*, while a suitably sized cut in the policy rate can simultaneously close the employment and inflation gaps. Crucially, the size of the cut depends on the extent of imperfect insurance (as encoded in Ψ), because the latter determines the strength of the precautionary motive and hence the size of the fall in aggregate demand that would prevail without the offsetting action of the central bank. This monetary policy response to productivity shocks is in contrast with that under standard calibrations of the RANK model, wherein desired savings are entirely governed by aversion to intertemporal substitution; then, a persistent productivity-driven contraction forecasts high future income growth (on the way to the recovery) against which households seek to borrow, and this causes a rise in the efficient interest rate that is adequately tracked by an *increase* in the policy rate (Clarida et al., 1999; Woodford, 2003). In Section 5 I allow for time-variations in the real wage, so that both aversion to intertemporal subsitution and the precautionary motive are operative and compete in determining the response of desired savings and the optimal policy rate to aggregate shocks.

Second, the strength of the precautionary motive in general affects both the size and sign of the optimal interest-rate response to cost-push shocks. Just as in the RANK model, the optimal policy response is such that both inflation and the employment gap persistently deviate from target (inflation upwards and the employment gap downwards). However, the fall in employment strengthens the precautionary motive and generates deflationary pressures in the current period; this mutes down the optimal response of the policy rate and even reverts it if the precautionary motive is sufficiently strong (i.e., if insurance is sufficiently poor).

Third, the optimal policy brings about a path of $\{\tilde{n}_t, \pi_t\}_{t=0}^{\infty}$ that is independent of the degree of insurance – see Table 1, wherein none of the coefficients depend on Ψ – and is thus the same as in the perfect insurance limit (i.e., when $\delta/w^* \to 1$). This means that, under the parameter restriction of this section, implementation of the optimal policy *fully undoes the effect of imperfect insurance on the propagation of aggregate shocks.*

4.4. Optimal discretionary policy. I conclude this section by computing the optimal response of the policy rate under an additional source of inefficiency, namely, the inability of the central bank to commit to future policies. This serves to show that imperfect insurance has the same effect as under commitment of muting (and possibly reverting) the response of the policy rate relative to the perfect-insurance case. Under discretion the central bank chooses period by period the value of $\hat{\imath}_t$ that minimises $\tilde{n}_t^2 + \Omega\pi_t^2$ subject to (47)–(48). I first solve for $(\tilde{n}_t, \pi_t)$ by minimising this loss subject to (48) and then infer i_t from equation (47). The first step gives:

$$\tilde{n}_t + (\theta n)\,\pi_t = 0. \tag{52}$$

The optimal target sequence $\{\tilde{n}_t, \pi_t\}_{t=0}^{\infty}$ jointly solves equations (48) and (52) and is given by:

$$\tilde{n}_t = -\left(\frac{1-\beta\mu_\zeta}{\theta n\kappa} + \frac{1}{\Phi}\right)^{-1}\hat{\zeta}_t \;\text{ and }\; \pi_t = \left(\frac{1-\beta\mu_\zeta}{\kappa} + \frac{\theta n}{\Phi}\right)^{-1}\hat{\zeta}_t. \tag{53}$$

In as much as the cost-push shock raises inflation, a central bank operating under discretion mitigates the impact of the shock on inflation by lowering the current value of the employment gap. Using equations (51) and (53) gives the response of the policy rate to one-off productivity and cost-push shocks:

$$\hat{\imath}_t(\hat{z}_0,\hat{\zeta}_0)=\underbrace{\left(\frac{\kappa\Phi\mu_\zeta^{t+1}}{\left(1-\beta\mu_\zeta\right)\Phi+\kappa\theta n}\right)\hat{\zeta}_0}_{\text{perfect-insurance response}}\ \underbrace{-\ \Psi\left(\frac{\kappa\Phi\theta n\mu_\zeta^{t+1}}{\left(1-\beta\mu_\zeta\right)\Phi+\theta n\kappa}\right)\hat{\zeta}_0}_{\text{imperfect-insurance correction}}\ +\ \Psi\Phi\mu_z^{t+1}z_0.$$

The response to the productivity shock is the same as under commitment since this shock generates no policy tradeoff here. Regarding the cost-push shock, we first observe that in the perfect-insurance limit we recover the standard result that a cost-push shock should be fought by raising the policy rate. As unemployment insurance is reduced (i.e., Ψ rises) this response is dampened or even reverted by households' own precautionary reaction to the shock.

5. Optimal policy with partial worker reallocation

Having analytically identified how the precautionary motive affects optimal policy in the special case of full worker reallocation – and a constant wage –, I now study numerically the optimal interest-rate response to aggregate shocks under partial worker reallocation (i.e., $\rho < 1$) and a time-varying wage (which occurs when $\tilde{u}(c) \neq c$). The first feature makes hiring decisions intertemporal: in decentralised equilibrium firms take into account the future rents they will earn on newly hired employees (in addition to the current rent), while the constrained-efficient allocation incorporates the impact of current employment on future aggregate hiring costs (in addition to the current aggregate hiring costs). The second feature implies that workers' saving behaviour will not only be governed by the precautionary motive but also by aversion to intertemporal substitution. Since those generalisations preclude the derivation of an analytical formula for the policy rate, I solve numerically the Ramsey problem of finding the sequence $\{i_t\}_{t=0}^{\infty}$ that maximises W_t subject to (7)–(9), (13)–(15), (19), (20)–(21), and (36), after one-off productivity and cost-push innovations occurring at $t = 0$. In the baseline specification the real wage is assumed to be equal to the efficient wage w_t^* computed in Section 3. This is meant to ensure that my results about optimal monetary policy are not artificially driven by inefficiencies in the way the wage is set. I also consider alternative (hence inefficient) wage-setting mechanisms in Section 5.3 and show that my results continue to hold.

5.1. Calibration. I interpret the period as a quarter and I calibrate the model so as to match a certain number of standard targets – see Table 2 for a summary. The cross-partial elasticity of substitution θ is set to 6, which generates a mean markup rate of 20% for wholesale firms. The fraction of unchanged wholesale goods prices ω is set to 0.75, so that the mean duration of wholesale prices is a year. Regarding labour market variables, I first set γ to 2/3, very close to the values estimated by Shimer (2005) and Monacelli et al. (2015). I then have four parameters (c, w^*, m and ρ) for four targets (f, s, λ and c/w^*). Quarterly series for f_t and s_t where computed in Challe et al. (2017) by time-aggregating monthly series constructed as in Shimer (2005); their averages are very close to 80% and 5%, respectively. The targets for λ and c/w^* are, respectively, 70% (see, e.g., Den Haan et al., 2000; Walsh, 2005; Monacelli et al. 2015) and 4.5% (Hagedorn and Manovskii, 2008).

A key parameter in the model is workers' home production δ, which determines the extent of consumption insurance and hence the strength of the precautionary motive. One possibility would be to parameterise δ to match the UI replacement ratio. However, this would most likely underestimate the amount of consumption insurance that households effectively enjoy, by ignoring factors that are not included in the model such as (i) self-insurance using liquid assets, (ii) other forms of direct insurance, and (iii) the subjective valuation of not working. Following this concern I broadly interpret $(w^* - \delta)/w^*$ as the proportional consumption loss upon unemployment and give it the conservative value of 10% (see den Haan et al., 2017, Appendix A, for an discussion of this parameter). I interpret $vc_t^F = D_t - \tau_t + \varpi$ as aggregate capital income and accordingly set ϖ to 1/2, which generates a labour share of 65%.

Preferences are specified as follows. First, I restrict my attention to the following utility functions:

$$u(c) = \ln c \quad \text{and} \quad \tilde{u}(c) = \frac{c^{1-\tilde{\sigma}} - 1}{1 - \tilde{\sigma}}, \quad \text{with } \tilde{\sigma} \geq 0. \tag{54}$$

Given those preferences, the replacement ratio δ/w^* and the transition rates in the labour market (f, s), equation (21) determines the value of the subjective discount factor β consistent with a given interest rate. Following McKay et al. (2016), β is set such that the annualised real interest rate $(1+i)^4 - 1 \simeq 4i$ be equal to 2%.

The curvature of firm owners' utility function $(\tilde{\sigma})$ does not play a role in the steady state of the model but plays a key role in the dynamic response to aggregate shocks. Indeed, according to the efficiency condition (34), the extent to which productivity shocks are passed through to real wages depends on the willingness of firm owners to bear aggregate risk, as captured by their relative risk aversion coefficient $\tilde{\sigma}$: when $\tilde{\sigma}$ is small then firm owners are willing to bear much of the burden of aggregate fluctuations so the wage varies little over the business cycle, and the other way around. I thus pin down $\tilde{\sigma}$ using the impact elasticity of the wage with respect to productivity, $\text{dlog}\, w_0/\text{dlog}\, z_0$. Setting $\tilde{\sigma} = 0.38$ generates a wage elasticity of 1/3, in the ballpark of available estimates – see e.g. Blanchard and Gali (2010), Hagedorn and Manovskii (2008), and Den Haan et al. (2017). This value implies that firm owners provide workers with substantial insurance against wage fluctuations.

Given the assumed functional forms, the steady state of the model is constructed as follows. Given the targets for f, c/w^* and δ/w^*, w^* is recovered as the unique solution to equation (37). Then, given w^* and the targets for f and s (which give $n = f/(f+s)$ and v by equation (32)), I recover the value of the transformed welfare weight $\tilde{\Lambda} \equiv \Lambda\nu^{\tilde{\sigma}}$. Since it is this transformed weight that enters the flow payoff U_t in equation (31), the measure of firm owners ν is irrelevant.

In what follows I compare the optimal policy – and implied outcomes – under the baseline precautionary-saving model with those under a counterfactual perfect-insurance benchmark, which I recovered in the limit as $\delta/w^* \to 1$. In constructing this benchmark I adjust the deep parameters of the model whenever this is needed in order to keep matching all the steady state targets in Table 2; in other words I interpret the same observed moments in the right-hand side of Table

2 as having been generated by a perfect-insurance, rather than imperfect-insurance, model. This requires adjusting the parameters β, c and w^* – see the fourth column of Table 2.

Table 2. Calibration.

Parameters				*Targets*		
Symb	Description	Baseline	Full ins.	Symb	Description	Value
β	Discount factor	0.989	0.995	$4i$	Annual interest rate	2%
θ	Elasticity of subst.	6.000	(same)	$\frac{1}{\theta-1}$	Markup rate	20%
ω	Share of unchanged price	0.750	(same)	$\frac{1}{1-\omega}$	Mean price duration	1 y.
c	Vacancy cost	0.044	0.040	c/w^*	Vacancy cost (% of wage)	4.5%
w^*	Real wage	0.979	0.888	f	Job-finding rate	80%
m	matching efficiency	0.765	(same)	λ	Vacancy-filling rate	70%
ρ	Job-destruction rate	0.250	(same)	s	Job-loss rate	5%
δ	Workers' home prod.	0.882	$(= w^*)$	δ/w^*	Opportunity cost of empl.	90%
ϖ	Firm owners' home prod.	1/2	(same)	$\frac{nw}{\nu c^F+nw}$	Labor share	65%
$\tilde{\sigma}$	CRRA of firm owners	0.38	(same)	$\frac{\mathrm{d}\log w}{\mathrm{d}\log z}$	Wage cyclicality	1/3

5.2. Optimal policy and macroeconomic outcomes. Figures 1 and 2 show the optimal responses of the policy rate, and implied macroeconomic aggregates, after contractionary productivity and cost-push shocks. To understand those responses, recall the two determinants employed workers' consumption demand already discussed in Section 2.5. The first determinant is *aversion to intertemporal substitution*: an employed worker contemplating a rising wage profile (conditional on remaining employed) is willing to save less in order to consume more in the present – and the other way around for a worker contemplating a falling wage profile. The second determinant of employed workers' consumption is the *precautionary motive* isolated in Section 4: an employed worker who expects to lose his or her job with greater probability in the near future tends to consume less in the present. The competition between the two effects is summarised in employed workers' MRIS. Linearising equation (21) gives (with log preferences):

$$\hat{M}^e_{t+1} = \underbrace{\tilde{\Psi} s_{t+1}}_{\text{precautionary motive}} + \underbrace{\hat{w}_t - \Xi \hat{w}_{t+1}}_{\text{aversion to intertemporal substitution}},$$

where hats denote proportional deviations from steady state and

$$\tilde{\Psi} = \left[s + ((\delta/w^*)^{-1} - 1)^{-1}\right]^{-1} > 0 \quad \text{and} \quad \Xi = \left[1 + \left(\frac{s}{1-s}(\delta/w^*)^{-1}\right)\right]^{-1} \in (0,1)\,.$$

We observe that $\partial\tilde{\Psi}/\partial(\delta/w^*) < 0$ while $\lim_{\delta/w^* \to 1} \tilde{\Psi} = 0$: as the mean level of consumption insurance rises, desired (precautionary) savings become less and less sensitive to changes in unemployment risk, up to the point of becoming fully insensitive in the perfect-insurance limit. Put differently, in the perfect-insurance benchmark desired savings – and the implied optimal policy response – are exclusively driven by workers' aversion to intertemporal substitution (just as in the RANK model). At the extreme opposite, if the real wage is constant but employed workers are imperfectly insured against unemployment risk, then desired savings are entirely driven by the precautionary motive.

This explains the optimal policy responses to productivity shocks displayed in Figure 1. The perfect-insurance response is essentially the same as that of the typical RANK model: after a transitory contraction in productivity the central bank should contract demand to align it to supply, otherwise there would be excess consumption and inflation (based on workers' high *future* wages). The constant-wage response is obtained when $\tilde{\sigma} = 0$, so that $w_t = w^*$ for all t (see equation (34)). In this case the central bank should stimulate demand, else workers' consumption would *fall* too much (due to their fear of unemployment), which would be deflationary. The optimal policy in the baseline imperfect-insurance model lies between these two extremes, and under the calibration of Table 2 it implies a mild but persistent cut in the policy rate. Put differently, without a suitable monetary policy response the precautionary motive would dominate aversion to intertemporal substitution and aggregate demand would be too *low*, not too high. Importantly, the optimal policy almost aligns the dynamics of inflation, employment and output under imperfect insurance to those under perfect insurance; that is, the optimal policy successfully undoes much of the potentially destabilising impact of the precautionary motive.[13]

The same pattern emerges from the optimal response to cost-push shocks: more accommodation is required, and the optimal policy almost aligns the dynamics of the imperfect-insurance baseline with that of the perfect-insurance benchmark. Unlike in the case of productivity shocks, the efficient wage does not respond to cost-push shock; therefore, only the precautionary motive governs the optimal policy.

5.3. Nash bargaining. In the baseline imperfect-insurance specification the real wage w_t is given by the efficient wage w_t^* of Section 3. This ensures that my results about optimal monetary policy are not an artefact of an inefficient wage-setting mechanism. Here I depart from this assumption and consider a popular alternative, namely (generalised) Nash bargaining between the parties over the wage upon a match. The Nash-bargained wage is generically inefficient since it differs from w_t^*.[14] However, in the same spirit as above I will set the relative bargaining powers of workers and firm owners in such a way that the bargained wage correspond to the efficient wage *in steady state* – hence the steady state will remain undistorted. The focus here is thus on how the inefficient *response* of the wage to aggregate shocks under Nash bargaining affects business cycles and optimal policy.

[13]Note that, unlike in Section 4, under the optimal policy inflation does not stay at zero at all times after a productivity shock. This is because when $\rho < 1$ the taxes and subsidy (τ^I, T) no longer decentralise the constrained-efficient outcome in the absence of cost-push shocks – they only decentralise it in the absence of *both* aggregate shocks. Consequently, monetary policy cannot exactly replicate the constrained-efficient outcome after a productivity shock.

[14]Since the real wage has a redistributive effect here (between employed workers and firm owners), generalized Nash barganing can never decentralise the constrained-efficient wage. This is in contrast with the Representative-Agent model, where the Hosios condition ensures the (constrained-) efficiency of the bargained wage.

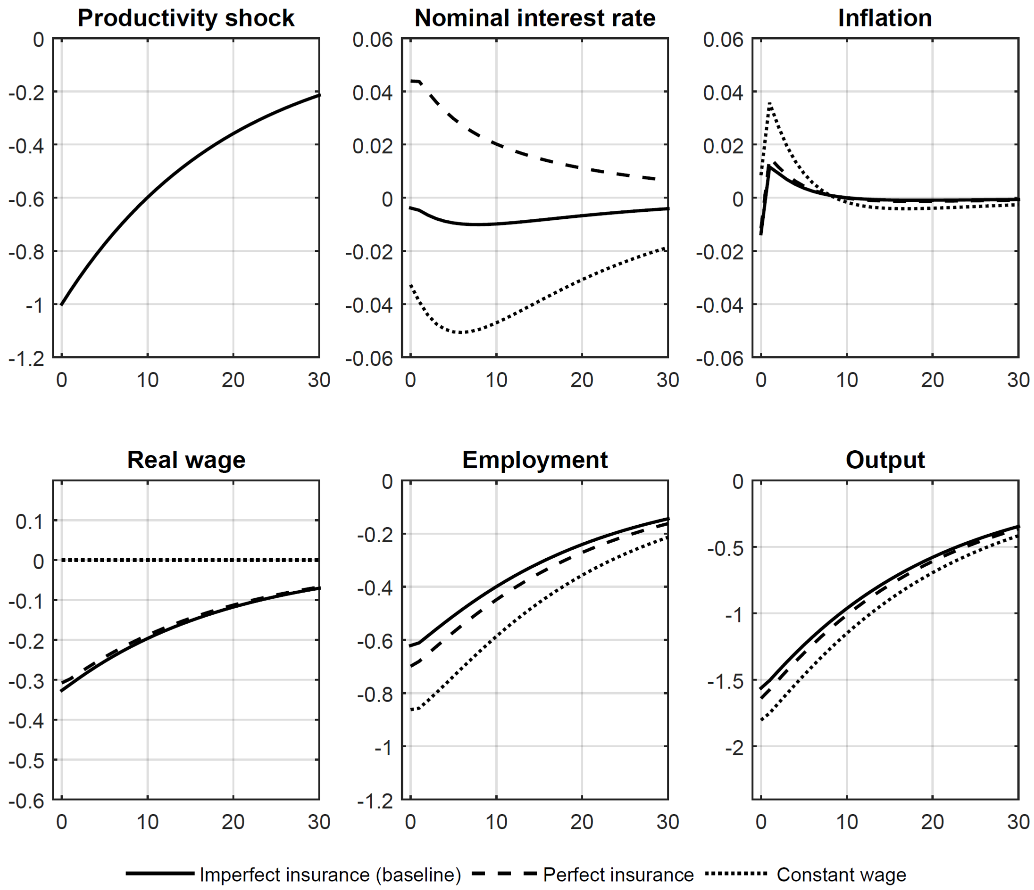

Figure 1: Responses to a contractionary productivity shock (imperfect versus perfect insurance).

Under Nash bargaining we have $w_t = \arg\max(S_t^W)^{1-\alpha} J_t^{\alpha}$, $\alpha \in (0,1)$ $\forall t$, where S_t^W and J_t are the values of the match to the worker and the hiring firm, respectively. J_t is given by equation (12), while $S_t^W = V_t^e - V_t^u$, where V_t^e and V_t^u are given by equations (27)–(28). Note that S_t^W can be written recursively as:

$$S_t^W = u(w_t) - u(\delta) + \beta \mathbb{E}_t (1 - f_{t+1} - s_{t+1}) S_{t+1}^W. \tag{55}$$

The first-order condition associated with the bargaining problem gives:

$$(1-\alpha) J_t = \alpha S_t^W / u'(w_t). \tag{56}$$

The requirement that the bargaining process decentralise the efficient wage in steady state will pin down α. First, the steady state values of J_t and S_t^W are given by, respectively:

$$J = \left[\frac{1-\tau^I}{1-(1-\rho)\beta}\right]\left[1 - w + \frac{u(w)-u(\delta)}{u'(w)}\right] \quad \text{and} \quad S^W = \frac{u(w)-u(\delta)}{1-\beta(1-s-f)}.$$

Then, setting $w = w^*$ in those expressions gives the values of J and S^W in the constrained-efficient allocation without aggregate shocks, from which we infer $\alpha = [1 + S^W / J u'(w^*)]^{-1}$ in

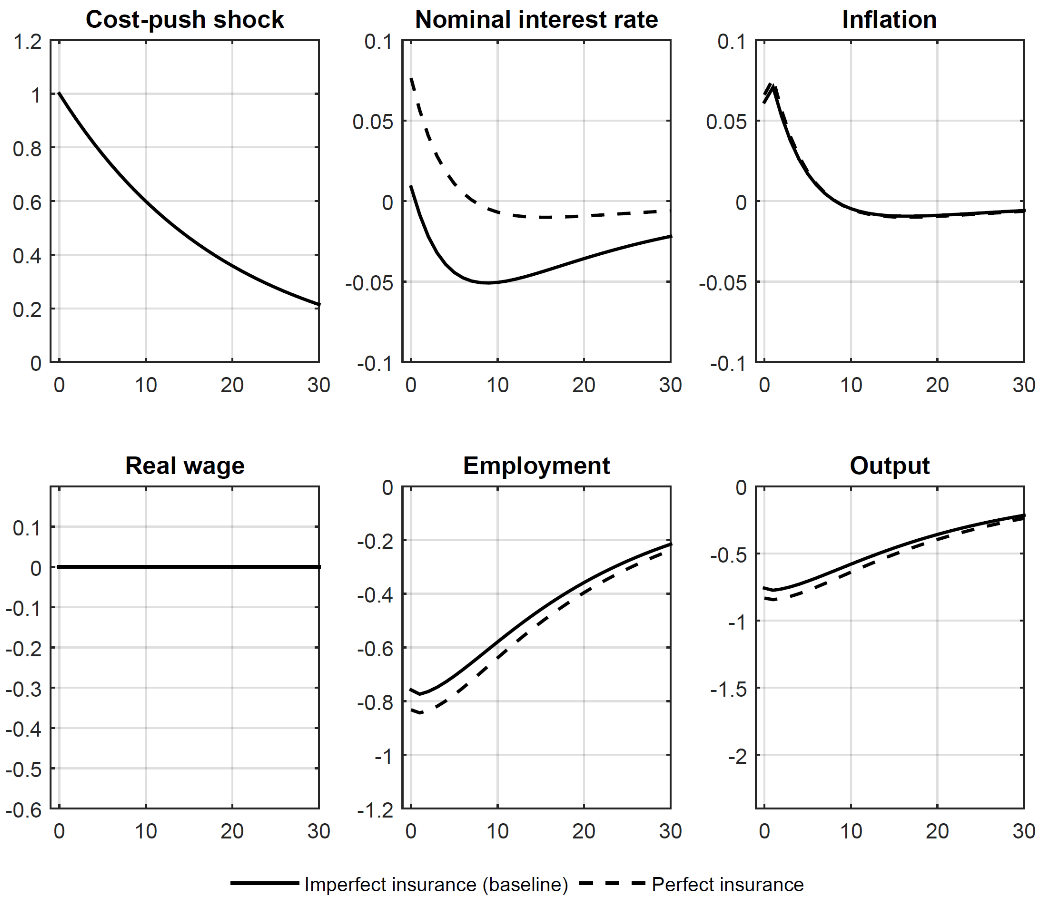

Figure 2: Responses to a contractionary cost-push shock (imperfect versus perfect insurance).

equation (56).

Figures 3 and 4 display the optimal interest-rate policy and aggregate dynamics under alternative wage-setting mechanisms. As is known since Shimer (2005) and is also apparent from Figure 3, an unattractive feature of the basic Nash bargaining process is to generate too strong a response of the real wage to productivity shocks (the impact elasticity $\mathrm{dlog}\, w_0/\mathrm{dlog}\, z_0$ lies above 1/2, instead of the targeted 1/3). This overshooting of the real wage magnifies the response of savings due to aversion to intertemporal substitution and pushes towards less policy accommodation than with a smoother wage response.

To generate the degree of wage inertia that is observed in the data in the simplest possible way, I consider a slightly more general specification of the Nash bargaining process, in the spirit of Hall (2005) and Krause and Lubik (2007): I assume that the basic Nash wage is a notional wage (w_t^N) that must be weighted against the long-run wage (w^*) in determining the actual wage:

$$w_t = (w_t^N)^{1-\mu} (w^*)^{\mu}, \quad 0 \leq \mu \leq 1.$$

This specification nests the basic Nash bargaining process ($\mu = 0$), as well as the constant-wage specification already examined in Figures 1 and 2 ($\mu = 1$). Here I adjust μ to match the

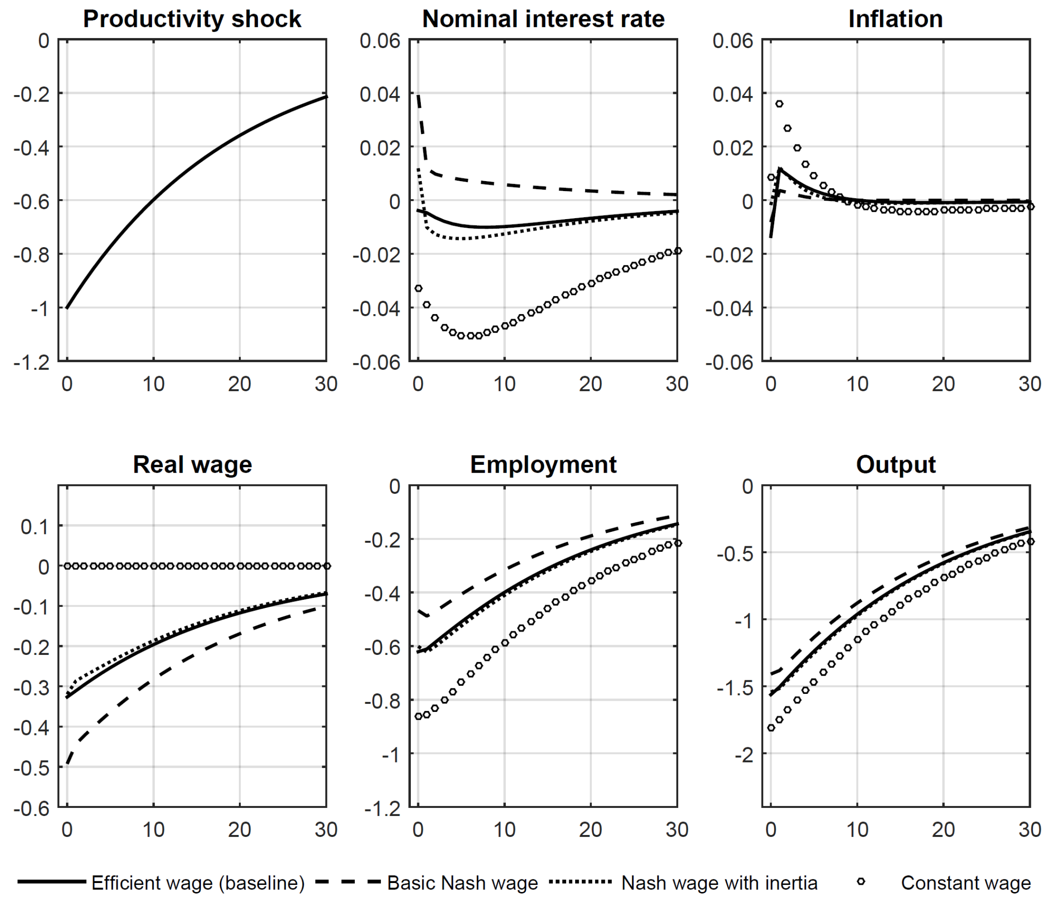

Figure 3: Responses to a contractionary productivity shock (alternative wage-setting mechanisms).

target $\mathrm{dlog}\, w_0/\mathrm{dlog}\, z_0 = 1/3$ in Table 2, which gives $\mu = 0.91$. Under this specification, the optimal policy response to a negative productivity shock closely tracks that under the efficient wage, and the implied responses of inflation, the real wage, employment and output are almost indistinguishable – see Figure 3. The biggest difference between Nash bargaining and the efficient wage pertains to the policy and aggregate responses to cost-push shocks in Figure 4. This is because the efficient wage does not respond to such shocks while the Nash wage always does whenever $\mu \neq 1$. As a result expected wage growth rises after the shock and desired savings fall, which shifts the path of the optimal interest rate upwards relative to the baseline (efficient-wage) model. Nevertheless, the optimal policy still provides substantial accommodation under the Nash wage; for example, the policy rate is almost always negative when the real wage has the targeted level of inertia. Overall, then, the optimal policy and aggregate dynamics obtained under the (inefficient) Nash wage confirm those obtained under the baseline efficient wage.[15]

5.4. Steady state distortions. I have so far been working under the assumption that taxes and subsidies were set in a way that aligned the steady state of the decentralised equilibrium with

[15] This is also true of simpler wage rules that match the cyclicality of the wage. For example, a wage process of the type $w_t = w^* z_t^{1/3}$ (see, e.g., Blanchard and Gali, 2010) generates similar responses of the optimal interest rate and macroeconomic aggregates to productivity and cost-push shocks as in the baseline model with an efficient wage.

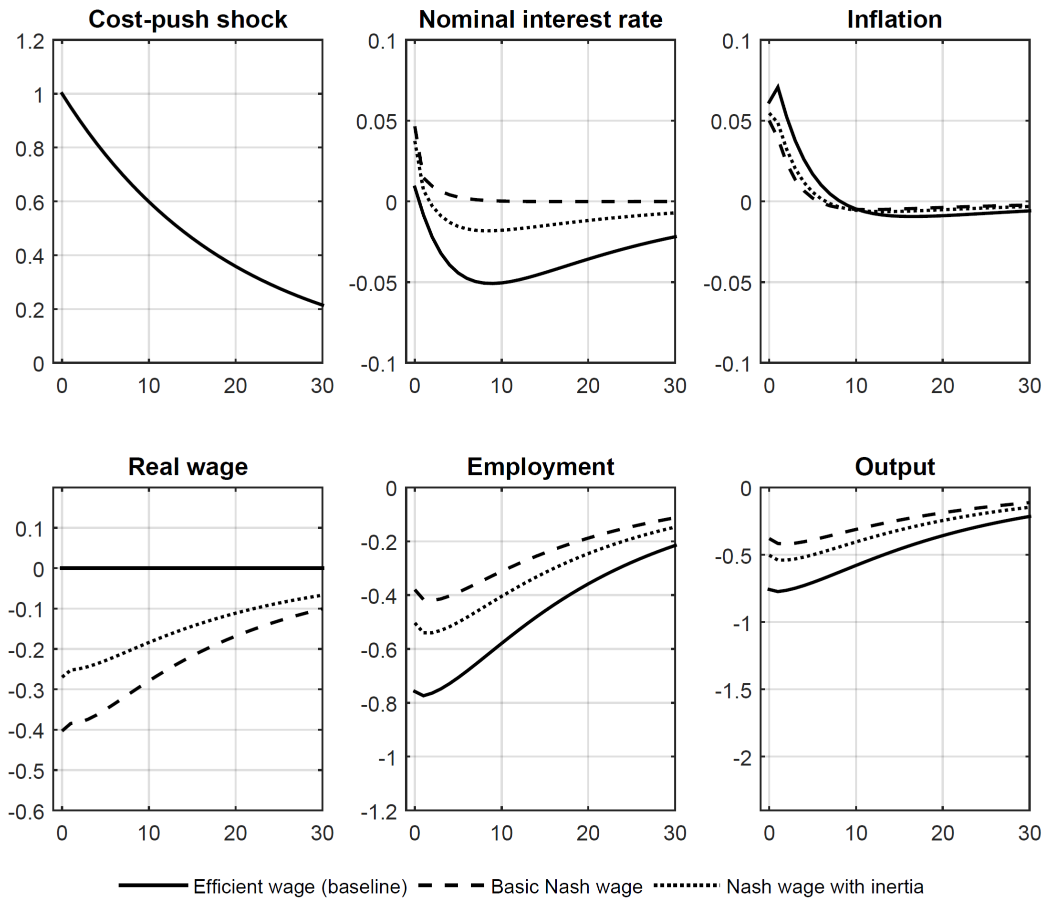

Figure 4: Responses to a contractionary cost-push shock (alternative wage-setting mechanisms).

that of the constrained-efficient allocation. This is a natural assumption to start with, for it ensures that observed differences in optimal policies according to the degree of consumption insurance are not unduly driven by differences in steady-state distortions. However, this assumption is unrealistic in the sense that one does not observe, in practice, significant wage or production subsidies (i.e., T and τ^W) of the type that I considered. I therefore explore the optimal response of the policy rate to aggregate shocks under alternative assumptions about those subsidies. In so doing I am still careful to adjust the deep parameters of the model so as to keep matching all the steady state targets in Table 2 – so that the same observed steady state as in the baseline scenario is now considered as distorted rather than undistorted. To operate the required adjustments, write the steady state counterpart of equation (13) as follows:

$$f^{\frac{\gamma}{1-\gamma}}\left[1-\beta\left(1-\rho\right)\right]c=(1-\tau^{I})m^{\frac{1}{1-\gamma}}\left[\varphi-w+T\right].$$

Noticing that c/w is among the targets in Table 2 and that $\varphi=(\theta-1)/\theta(1-\tau^W)$, one can express the steady state real wage as follows:

$$w=\left[\frac{\theta-1}{\theta(1-\tau^{W})}+T\right]\left\{1+\frac{(c/w)\times f^{*\frac{\gamma}{1-\gamma}}\left[1-\beta\left(1-\rho\right)\right]}{(1-\tau^{I})m^{\frac{1}{1-\gamma}}}\right\}^{-1}, \tag{57}$$

which can always be made consistent with the efficiency condition (34) by appropriately adjusting $\Lambda = (c^F)^{\tilde{\sigma}}/w$.

Equation (57) is informative about the impact of the subsidies T and τ^W on the labour market. If the wage subsidy T were to be lowered but everything else were kept unchanged, job creation would fall; the job-finding and unemployment rates f and n would fall and the job-loss rate s would rise. Those labour-market targets can however be maintained if w is lowered by the appropriate amount. Similarly, the impact of a fall in the production subsidy τ^W on the labour market can be offset by an appropriate reduction in w.

Figures 5 to 7 show the optimal-policy responses to aggregate shocks when the subsidies τ^W and T are alternatively, and then jointly, set to zero. The optimal responses to the shocks are affected by steady state distortions, a reflection of the fact that the distortions under considerations are large.[16] However, the general lessons that imperfect insurance calls for more policy accommodation following contractionary aggregate shocks, and that such an accommodation almost eliminates the destabilising impact of imperfect insurance, unambiguously survive.

6. Conclusion

In this paper, I have computed the optimal interest-rate response to aggregate shocks in a model economy wherein workers have a precautionary motive against uninsured, endogenous unemployment risk. In this economy aggregate "supply" shocks such as productivity or cost-push shocks may have powerful aggregate demand effects, due to the feedback loop between unemployment risk, desired savings and aggregate demand. The general lesson from this analysis is that the destabilising role of the precautionary motive calls from a policy response to contractionary aggregate shocks that is much more accommodative under imperfect insurance than under perfect insurance. Using a calibrated version of the model whose perfect-insurance limit replicates the policy prescriptions of the Representative-Agent New Keynesian model, I found those prescriptions to be significantly altered, if not overturned, under imperfect insurance. Interestingly, in all the specifications that I considered the optimal policy successfully broke the feedback loop between unemployment risk and aggregate demand and almost aligned the economy's dynamics to that under perfect insurance.

Of course, this form of policy accommodation requires the policy rate to be unconstrained by the zero lower bound on the nominal interest rate. This is always true in my calibration with a positive steady-state interest rate and the maintained assumptions that aggregate shocks are small. Extrapolating on this local analysis, the model suggests that under imperfect insurance contractionary supply shocks may also put the economy at risk of entering a liquidity trap – inasmuch as the optimal *unconstrained* policy calls for substantial interest rate cuts – and not only the contractionary demand shocks that have more commonly been considered in the literature on the liquidity trap.

[16] Here "large" means: an order of magnitude greater that the size of aggregate shocks. As discussed in Woodford (2016), small steady state distortions do not alter the optimal Ramsey response to aggregate shocks, but large steady state distortions generally do.

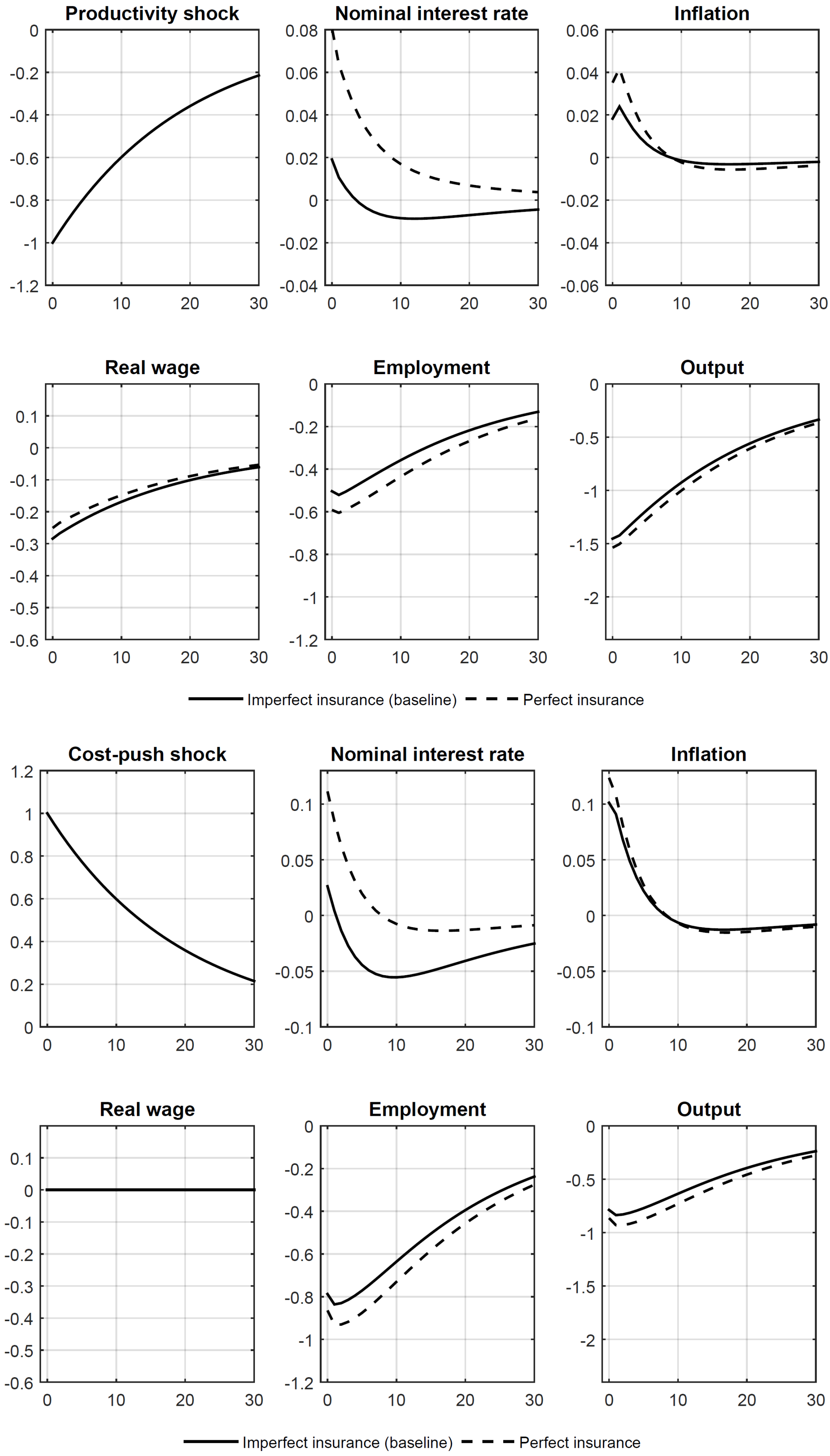

Figure 5: Responses to contractionary aggregate shocks without the production subsidy.

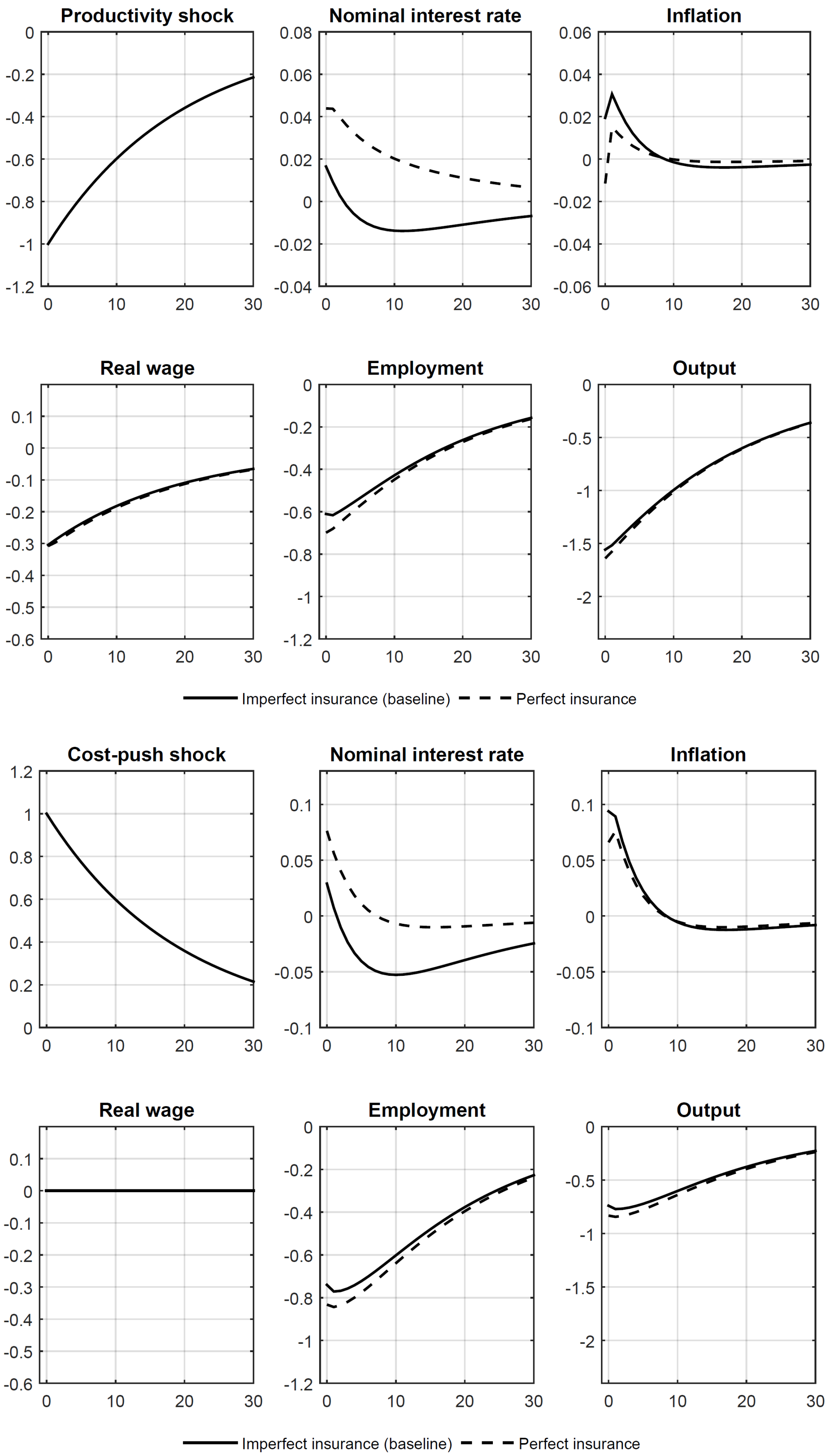

Figure 6: Responses to contractionary aggregate shocks without the wage subsidy.

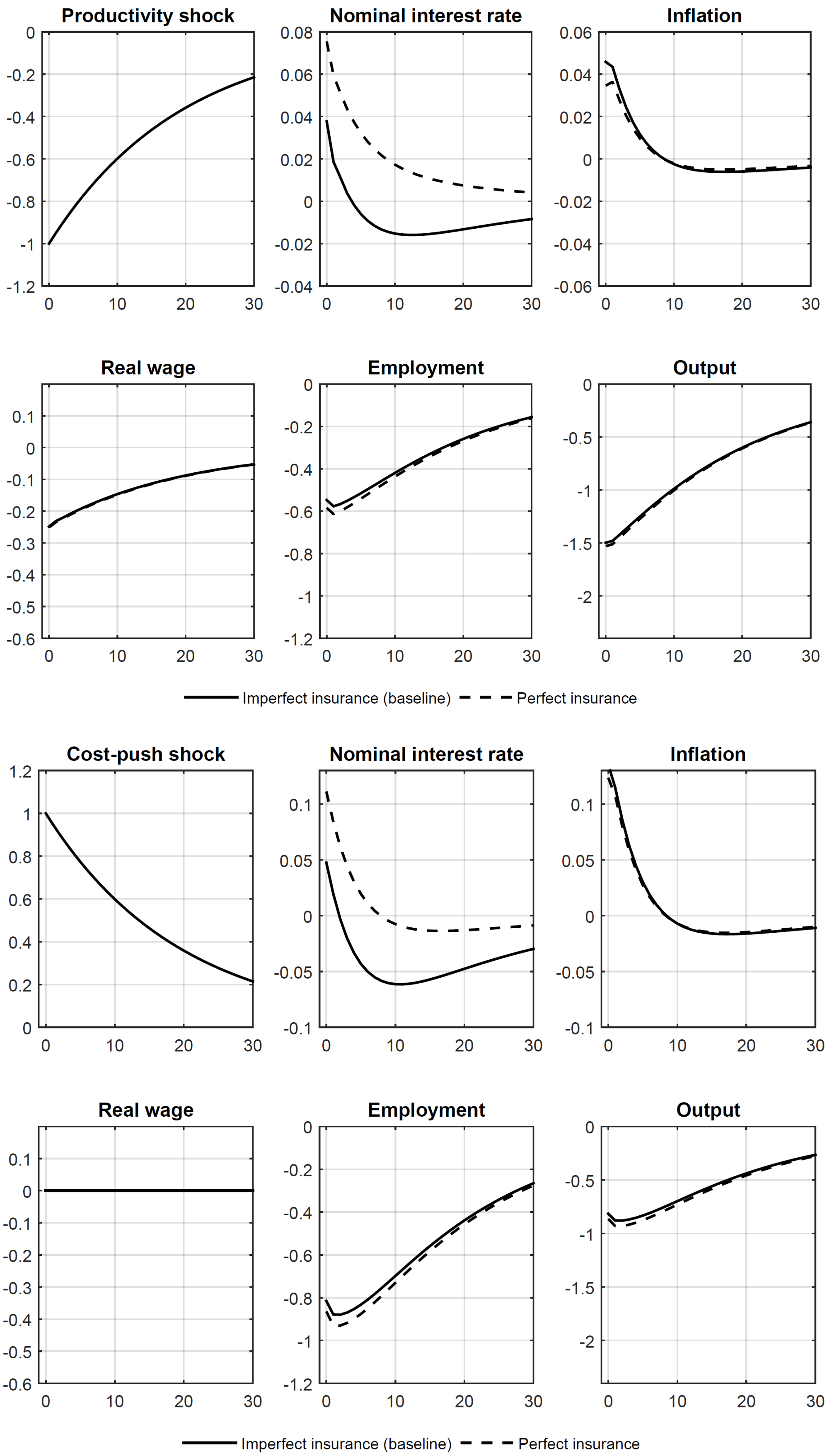

Figure 7: Responses to contractionary aggregate shocks without any subsidy.

Appendix to Section 4

A. Derivation of the quadratic loss function With $\rho = m = 1$ and $w_t = w_t^*$ we have:

$$U_t = u(\delta) + n_t [u(w_t^*) - u(\delta)] + \Lambda[\varpi + n_t (z_t/\Delta_t - w_t^*) - c n_t^{\frac{1}{1-\gamma}}].$$

We will use the facts that $\hat{n}_t^* = \Phi \hat{z}_t$ and that

$$\frac{\partial U_t}{\partial n_t} = u(w^*) - u(\delta) + \Lambda \left[z_t/\Delta_t - w^* - \frac{c}{1-\gamma} n_t^{\frac{\gamma}{1-\gamma}} \right] = 0.$$

We then get the following quadratic flow utility:

$$\begin{aligned}
U_t &= \{u(w^*) - u(\delta) + \Lambda[1 - w^* - c n^{\frac{\gamma}{1-\gamma}} / (1-\gamma)]\} \hat{n}_t - \frac{\Lambda \gamma c n^{\frac{2\gamma-1}{1-\gamma}}}{2(1-\gamma)^2} \hat{n}_t^2 \\
&\quad + \Lambda \hat{z}_t \hat{n}_t - \Lambda n (\Delta_t - 1) + \text{terms independent of policy (t.i.p.)} + \mathcal{O}(||\zeta||^3) \\
&\simeq -\frac{\Lambda}{2\Phi} \hat{n}_t^2 + \frac{\Lambda}{\Phi} \hat{n}_t^* \hat{n}_t - \Lambda n (\Delta_t - 1) + \text{t.i.p.} \\
&= -\frac{\Lambda}{2\Phi} \tilde{n}_t^2 - \Lambda n (\Delta_t - 1) + \text{t.i.p., where } \tilde{n}_t = \hat{n}_t - \hat{n}_t^*.
\end{aligned}$$

We now use the facts that (see Woodford, 2003, chapter 6):

$$\Delta_t \simeq 1 + \frac{\theta}{2} \operatorname{Var}(p_t(i)) \quad \text{and} \quad \sum_{t=0}^{\infty} \beta^t \operatorname{Var}(p_t(i)) = \frac{1}{\kappa} \sum_{t=0}^{\infty} \beta^t \pi_t^2, \quad \text{with } \kappa = \frac{(1-\omega)(1-\beta\omega)}{\omega}.$$

This allows us to write the social welfare function as follows:

$$\begin{aligned}
W_t &= \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k U_{t+k} \\
&\simeq \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \left[-\frac{\Lambda}{2\Phi} \tilde{n}_{t+k}^2 - \frac{\Lambda n \theta}{2} \operatorname{Var}(p_{t+k}(i)) \right] + \text{t.i.p.} \\
&= -\frac{\Lambda}{2\Phi} \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \tilde{n}_{t+k}^2 - \frac{\Lambda n \theta}{2} \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \operatorname{Var}(p_{t+k}(i)) + \text{t.i.p.} \\
&= -\frac{\Lambda}{2\Phi} \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \left(\tilde{n}_{t+k}^2 + \Omega \pi_{t+k}^2 \right) + \text{t.i.p.,} \quad \text{with} \quad \Omega = \frac{\theta \Phi n}{\kappa}.
\end{aligned}$$

Maximising W_t is thus equivalent to minimising $L_t = \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k (\tilde{n}_{t+k}^2 + \Omega \pi_{t+k}^2)/2$.

B. Optimal Ramsey policy This adapts Gali (2008, Section 5.1.2) to the present model. The Lagrangian associated with the central bank's problem is:

$$\mathcal{L}_t = \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \left[\frac{\tilde{n}_{t+k}^2 + \Omega \pi_{t+k}^2}{2} + \Gamma_{t+k} \left(\pi_{t+k} - \beta \mathbb{E}_t \pi_{t+1+k} - \frac{\kappa}{\Phi} \tilde{n}_{t+k} \right) \right].$$

The first-order conditions with respect to the $\tilde{n}_{t+k}$s and π_{t+k}s are:

$$\mathbb{E}_t \tilde{n}_{t+k} - (\kappa/\Phi)\, \mathbb{E}_t \Gamma_{t+k} = 0 \text{ for all } k \geq 0,$$
$$\Omega \pi_t + \Gamma_t = 0,$$
$$\text{and} \quad -\mathbb{E}_t \Gamma_{t+k} + \Omega \mathbb{E}_t \pi_{t+1+k} + \mathbb{E}_t \Gamma_{t+k+1} = 0 \text{ for all } k \geq 1.$$

Using those conditions, dropping the $\mathbb{E}_t$-operator – since I am looking at the response to a one-time shock – and using (44), I find that $\{\tilde{n}_{t+k}, \pi_{t+k}\}_{k=0}^{\infty}$ must satisfy:

$$\text{for } k = 0 : \tilde{n}_t + (\theta n)\, \pi_t = 0; \tag{58}$$
$$\text{for } k \geq 1 : \tilde{n}_{t+k} - \tilde{n}_{t+k-1} + (\theta n)\, \pi_{t+k} = 0. \tag{59}$$

Equations (58) and (59) can be more compactly written as, for all $k \geq 0$:

$$\tilde{n}_{t+k} = -\,(\theta n)\, \hat{p}_{t+k}, \text{ with } \hat{p}_{t+k} \equiv p_{t+k} - p_{t-1}, \tag{60}$$

and where p_{t-1} was the price level before the shock hit. Substituting this expression into (48) and rearranging, we obtain the following difference equation for $\hat{p}_t$:

$$(1 + \beta + \kappa\theta n/\Phi)\, \hat{p}_{t+k} = \hat{p}_{t+k-1} + \beta \hat{p}_{t+k+1} + \kappa \zeta_{t+k}.$$

The stationary solution to this equation is $\hat{p}_{t+k} = \alpha \hat{p}_{t+k-1} + \Upsilon \zeta_{t+k}$, with

$$\Upsilon = \frac{\alpha\kappa}{1 - \alpha\beta\mu_\xi} \quad \text{and} \quad \alpha = \frac{1 + \beta + \frac{\kappa\theta n}{\Phi}}{2\beta} \left[1 - \sqrt{1 - 4\beta \left(1 + \beta + \frac{\kappa\theta n}{\Phi}\right)^{-2}} \right] \in (0,1)\,.$$

This solution can be used to recover $\{\tilde{n}_{t+k}, \pi_{t+k}\}_{k=0}^{\infty}$ using (58)–(60). For $k = 0$ we get:

$$\tilde{n}_t = -\,(\theta n)\, \hat{p}_t = -\Upsilon \theta n \zeta_t,$$

where I have used the fact that $\Omega = \theta \Phi n/\kappa$ (see Appendix A). For $k \geq 1$ we have:

$$\tilde{n}_{t+k} = \alpha \tilde{n}_{t+k-1} + \Upsilon \theta n \zeta_{t+k} = -\Upsilon \theta n (\textstyle\sum_{\iota=0}^{k} \alpha^{\iota} \mu_\zeta^{k-\iota}) \zeta_t.$$

Then, we recover the path of inflation using (58)–(59). We obtain:

$$\text{for } k = 0 : \pi_t = -\frac{\tilde{n}_t}{\theta n} = \Upsilon \zeta_t;$$
$$\text{for } k = 1 : \pi_{t+1} = \frac{\tilde{n}_t - \tilde{n}_{t+1}}{\theta n} = \Upsilon \left(\alpha + \mu_\zeta - 1\right) \zeta_t$$
$$\text{for } k \geq 2 : \pi_{t+k} = \frac{\tilde{n}_{t+k-1} - \tilde{n}_{t+k}}{\theta n} = \Upsilon [\mu_\zeta^k - (1-\alpha) \textstyle\sum_{\iota=0}^{k-1} \alpha^{\iota} \mu_\zeta^{k-\iota}] \zeta_t$$

Table 1 summarises the effect of a shock occurring at $t = 0$.

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