Trade and Minimum Wages in General Equilibrium: Theory and Evidence

Xue Bai
Brock University
Arpita Chatterjee
University of New South Wales
Kala Krishna
The Pennsylvania State University, CES-IFO, IGC and NBER
Hong Ma
Tsinghua University and UC Davis

January 6, 2018

Motivation

- Impact of minimum wage on the whole economy. Open and policy relevant question.
- Increasing importance for U.S.: cities setting own minimum wage: D.C. \$12.50 *v.s.* Federal $\simeq$ \$8.00.
  - Most work is on employment effects.
  - Mixed results, possibly because U.S. setting and fast foods may be wrong place to look.
  - Too low. Binding only in industries like fast food with limited factor substitution possibilities
- First paper to connect the effects of minimum wage in GE setting to production and export data in China in a more natural setting where cities set own minimum wage which is binding across a whole range of industries
- Key insight: look for entire set of predictions, not just on employment.

What we do

- Build a GE model with two sectors, two factors, firm heterogeniety and rural urban migration to study impact of rise in minimum wage
- Derive theoretical predictions on price, output, input mix, selection into production and exporting for different sectors with different capital/ skill intensity in response to a change in minimum wage
- Test our theoretical conclusions using comprehensive customs data matched with firm level survey data using city level variation in minimum wage across China

Literature Review

- Labor literature: mostly partial equilibrium and reduced form.
  - Surveys: Neumark and Wascher (2008), Neumark, Salas, and Wascher (2013), Cahuc and Zylberberg (2004).
  - Some GE ideas in search settings: Flinn (2006), Engbom and Moser (2017) .
- Trade literature: HO setting without firm heterogeniety and empirical component
  - Brecher (1974)
  - Davis (1998)): Trade between an economy with binding minimum wages and one without will raise wages in the latter and increase unemployment in the former.
- Insights from Chinese Data
  - Wang and Gunderson (2012), Fang and Lin (2013)
  - Huang, Lougani and Wang (2014) find firm employment falls, especially for low wage firms. Hau, Huang and Wang (2016): Minimum wages change input mix in low wage firms, reduce employment growth, management differences?
  - Gan, Hernandez and Ma (2016) study impact on firms' export behavior in China using survey data.

Model Setting

- Perfect competition.
- Three aggregate goods, X and Y, A.
- Consumers have a (homothetic) utility function

$$U = U(X, Y, A) = S(X, Y)^{1-\alpha} A^{\alpha}.$$

- $S(X, Y)$ is a CES aggregator of X, Y with elasicity σ. Services made by X and Y.
- X and Y are CES aggregators of varieties of x and y with elasicity σ_x, σ_y.
- x is labor intensive, y is capital intensive.

Model Setting

- $j \in J$ cities, each with its agricultural hinterland, and endowment $\left[K^j, L^j\right]$.
- Each city makes A in hinterland, and one variety of x or y.
- Labor of effeciency γ gets $\gamma \backsim G(\gamma)$ in agriculture and w in manufacturing.
  - high γ workers stay in agriculture.
  - low γ workers migrate to the city to work in manufacturing.
- Heterogeneous firms and free entry:
  - Fixed entry costs, $f_e c^e(w, r)$.
  - Firms do not know their costs ex-ante, but discover them ex-post to be $c(w, r)\theta$, with $\theta \backsim F(\theta)$.
  - Capacity constraints: each firm can make one unit.

Demand

The demand for variety $j \in J$ of good x is a derived demand and comes from all cities $k = 1...J$

$$D^{jx}(.) = \sum_{k=1,..J} T^{jk} \left(\frac{p^{jx} T^{jk}}{P_k^X}\right)^{-\sigma_x} \left(\frac{P_k^X}{P_k}\right)^{-\sigma} \frac{(1-\alpha) I_k}{P_k}$$

In city k.

- p^{jx} is the factory price of the variety made in city j.
- T^{jk} is the iceberg transport cost of shipping variety j to city k.
- σ_x (σ_y) is the substitution between varieties of x (y).
- σ is the substitution between X and Y.
- P_k^X (P_k^Y) is the aggregate price index of X (Y) in city k;
- P_k is the price index of the overall aggregate good S.
- $\frac{(1-\alpha) I_k}{P_k}$ is the demand for S.

With price indices fixed, demand depends only on own price.

Supply

- Only those suppliers with cost below p produce

$$\theta \leq \tilde{\theta}(.) = \frac{p}{c(w,r)}.$$

- Supply is

$$\begin{aligned} s(p,N,c(w,r)) &= N \int_0^{\tilde{\theta}(p,w,rP^{kX})} f(\theta)d\theta \\ &= N\left[F\left(\tilde{\theta}\right)\right]. \end{aligned}$$

- N (mass of firms) and $\tilde{\theta}$ (cost cutoff) are endogenously determined.

Solving the model: selection and factor prices

- Given (w, r), free entry conditions give cutoffs $(\tilde{\theta}^x, \tilde{\theta}^y)$

$$\left[\int_0^{\tilde{\theta}^x} F^x(\theta)d\theta\right] = \frac{c^{e,x}(w,r)f_e^x}{c^x(w,r)}$$

$$\left[\int_0^{\tilde{\theta}^y} F^y(\theta)d\theta\right] = \frac{c^{e,y}(w,r)f_e^y}{c^y(w,r)}$$

- Marginal firms just cover their operating costs

$$\begin{aligned} p^x &= \tilde{\theta}^x(w,r)c^x(w,r) \\ p^y &= \tilde{\theta}^y(w,r)c^y(w,r) \\ 1 &= w^e = p^A \end{aligned}$$

Solving the model: Outputs and Factors

- Migration and Income: Given prices, we have factor prices, and so income:

$$\begin{aligned} I &= \bar{\gamma}(w)L + wG(w)L + rK \\ &= \bar{\gamma}(w)L + p^x x + p^y y \end{aligned}$$

where $\bar{\gamma}(w) = \int_w \gamma g(\gamma) d\gamma$ and is increasing in w.

- Factor market clearing (FMC) gives:

$$\begin{aligned} N^x A_{Lx}(w,r) + N^y A_{Ly}(w,r) &= G(w)L \\ N^x A_{Kx}(w,r) + N^y A_{Ky}(w,r) &= K \end{aligned}$$

where $A_{Lx}(w,r) = c_w^x(.)\bar{\theta}^x(.) + f_e c_w^{ex}(.)$ etc and $\bar{\theta}^x(\tilde{\theta}^x) = \int_0^{\tilde{\theta}} \theta f(\theta) d\theta$.

Solving the model: product prices

- Prices come from setting supply equal to demand.
- In general:

$$\begin{aligned} N^x(\overset{+}{p^x}, \overset{-}{p^y}; \overset{+}{L}, \overset{-}{K}) F^x(\tilde{\theta}^x(\overset{-}{p^x}, \overset{+}{p^y})) &= D^x(.) \\ N^y(\overset{-}{p^x}, \overset{+}{p^y}; \overset{-}{L}, \overset{+}{K}) F^y(\tilde{\theta}^y(\overset{+}{p^x}, \overset{-}{p^y})) &= D^y(.). \end{aligned}$$

and with limited selection $S^x(\overset{+}{p^x}, \overset{-}{p^y}; \overset{+}{L}, \overset{-}{K}), S^y(\overset{-}{p^x}, \overset{+}{p^y}; \overset{-}{L}, \overset{+}{K})$, where demand is as before.

Equilibrium with Minimum Wages

- With minimum wage, labor markets need not clear.
- The supply of labor can exceed the demand giving unemployment.
- Capital markets clear.
- Firms face $\bar{w}$, the minimum wage.
- In terms of solution we solve for equilibrium unemplyment instead of equilibrium wage.
- $\hat{w}(\bar{w})$ is the expected wage that drives migration. Migration can rise or fall with the minimum wage.

Equilibrium with Minimum Wages

- If $\bar{w}$ is binding, labor markets will not clear.
- In Factor Market

$$\begin{aligned} N^x A_{Lx}(\bar{w}, r) + N^y A_{Ly}(\bar{w}, r) &= L^D \leq G(\hat{w}(\bar{w}))L = L^s \\ N^x A_{Kx}(\bar{w}, r) + N^y A_{Ky}(\bar{w}, r) &= K \end{aligned}$$

- Where

$$\hat{w}(\bar{w}) = \left(\frac{L^D}{G(\hat{w}(\bar{w}))L} \right) \bar{w}.$$

  is the expected wage which drives migration. $\hat{w}(\bar{w})G(\hat{w}(\bar{w})$ is increasing in $\hat{w}(\bar{w})$.
- If labor demand is elastic, $\bar{w}L^D$ falls as minimum wage rises, and so must $G(\hat{w}(\bar{w}))L\hat{w}(\bar{w})$, and hence $\hat{w}(\bar{w})$ and migration.

- There is unemployment, but capital is fully utilized.

Implications of Minimum Wages

- **Production and Export Pattern**: The output of labor intensive goods should fall and price rise with a binding minimum wage. The effects should be more pronounced for more labor intensive sectors and for higher minimum wages. Similar implication for export. Whether the *value* rise or fall depends on elasticity.
- **Factor Intensity**: A higher minimum wage should raise capital intensity. This increase will be lower the less relevant is the minimum wage. Labor use should fall but less so where the minimum wage is less relevant (as with less labor intensive sectors and with higher wages paid).

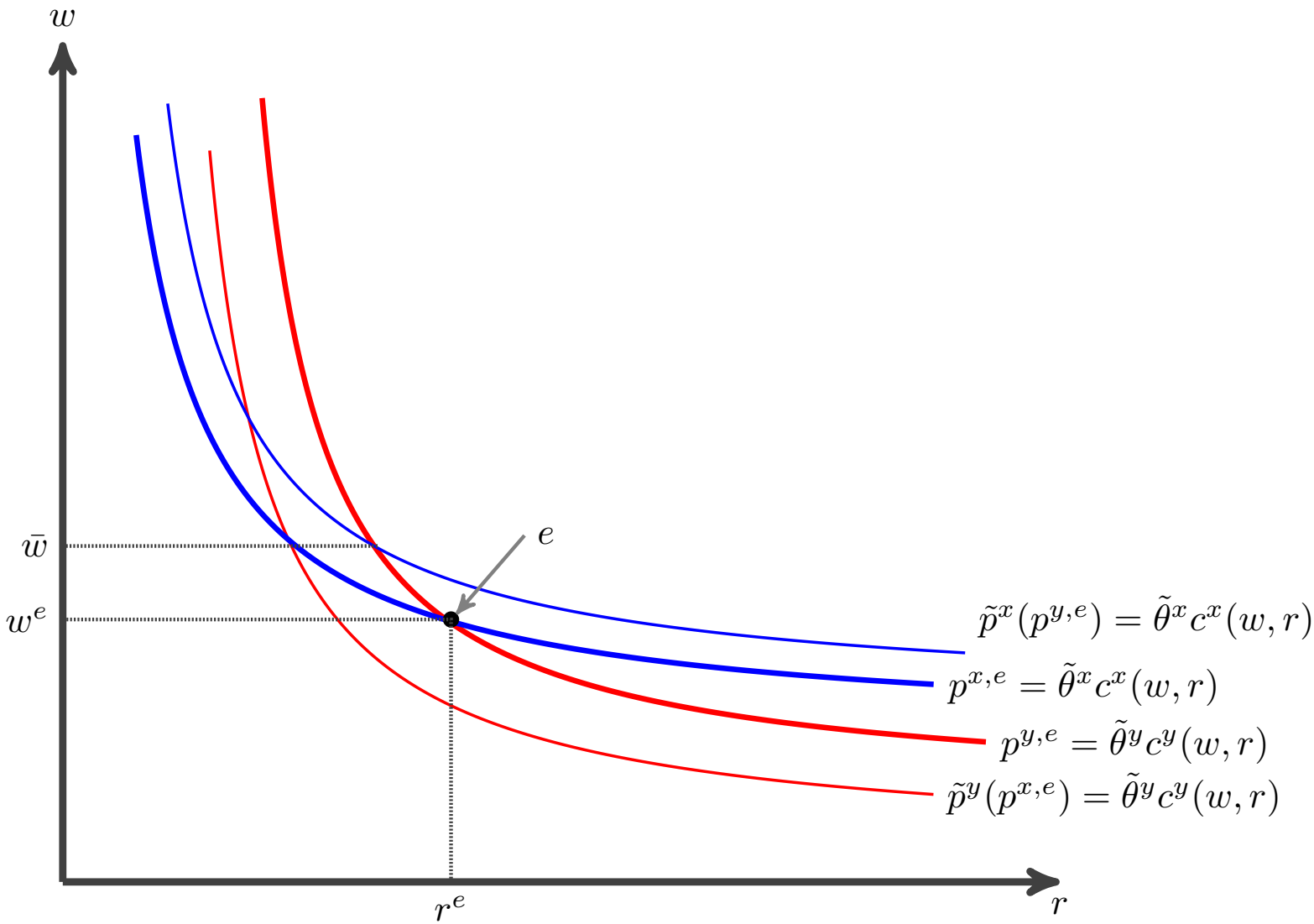
Implications of Minimum Wages

- **Selection of Firms**: Exit of existing firms should rise in labor intensive sectors with a binding minimum wage, and more so in more labor intensive sectors, other things constant. In the capital intensive sector, selection becomes weaker.
- **TFP Distribution**: Cities with high minimum wages should have a distribution of productivity that has a higher mean than that of low minimum wage cities, and this should be more pronounced in more labor intensive sectors.

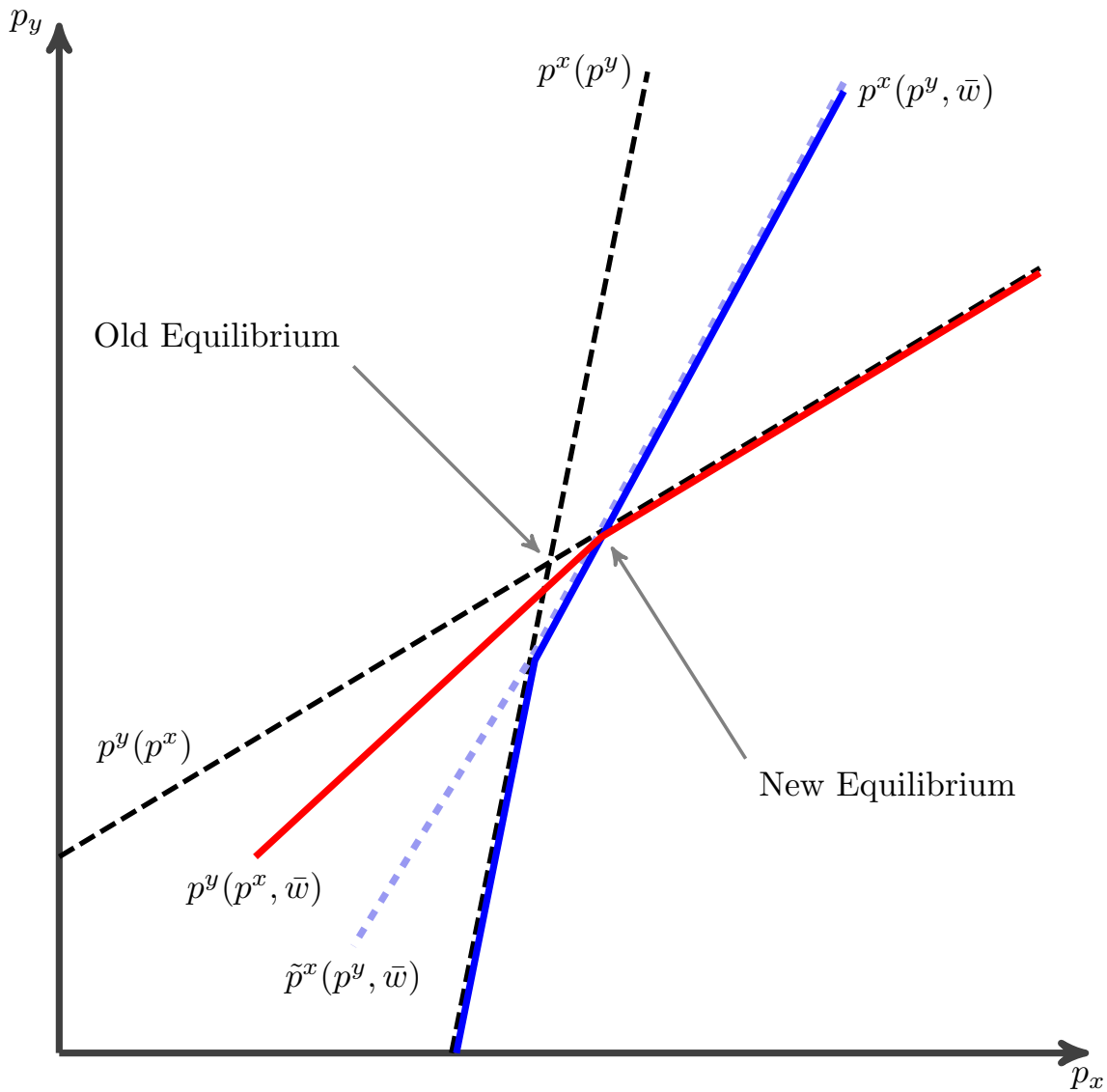
Market to Firm Level

- All predictions are at the city level.
- What about firm level predictions? Think of a firm as arising out of randomness as in Armenter and Koren (2014).
- A bin is a firm. Balls (draws of a unit of capacity at a particular cost) are randomy assigned to bins.
- The balls in a bin define that firm's supply function and aggregating over all firms gives industry supply.
- Some firms have lots of good cost draws so they are large low cost firms,...
- The cutoff, $\tilde{\theta}$, determines which of its capacity the firm uses.
- With this interpretation, predictions carry over to the firm level.

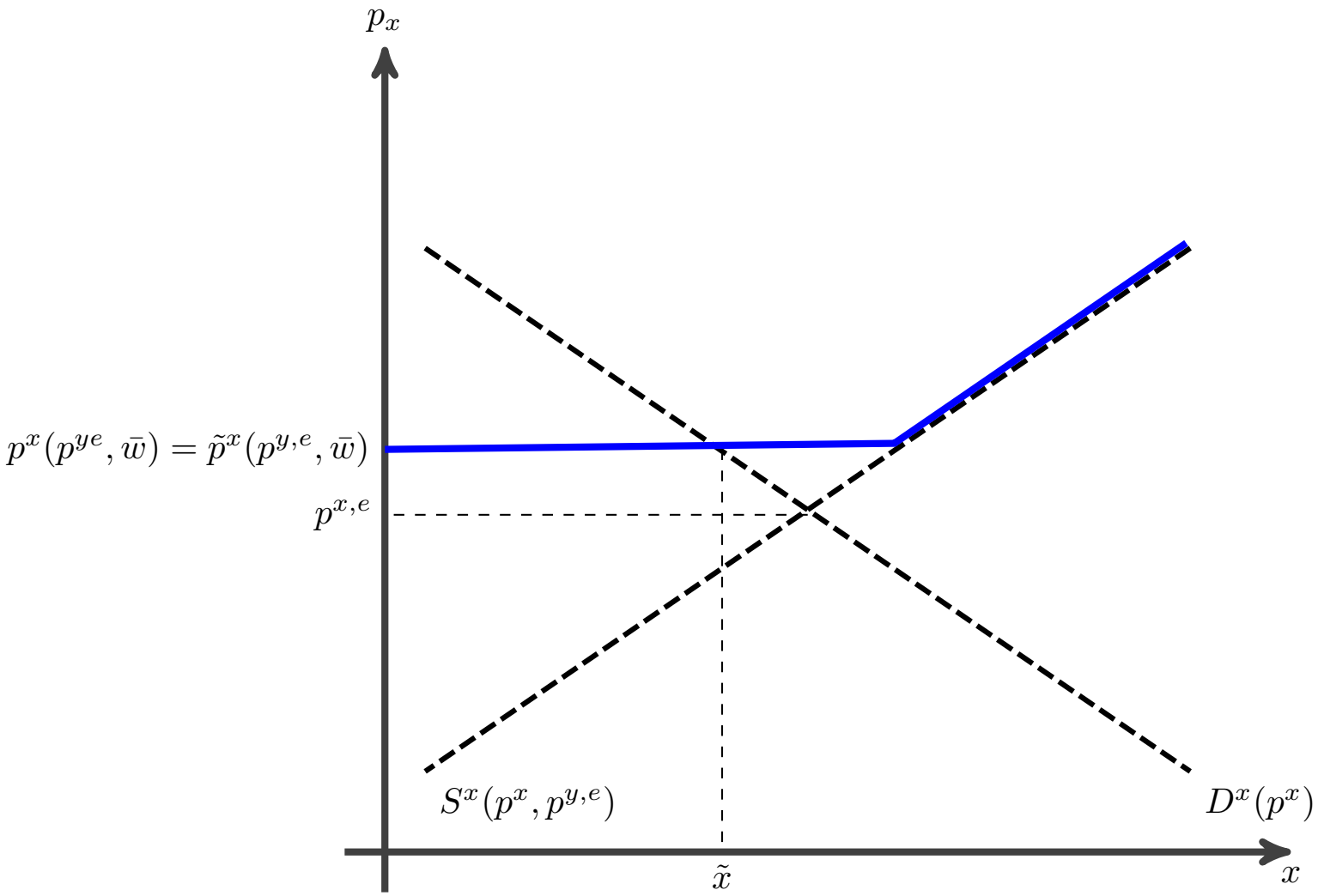
Graphical Illustration: Product Prices and Factor Prices with a Minimum Wage



Graphical Illustration: Equilibrium Prices with and without a Minimum Wage



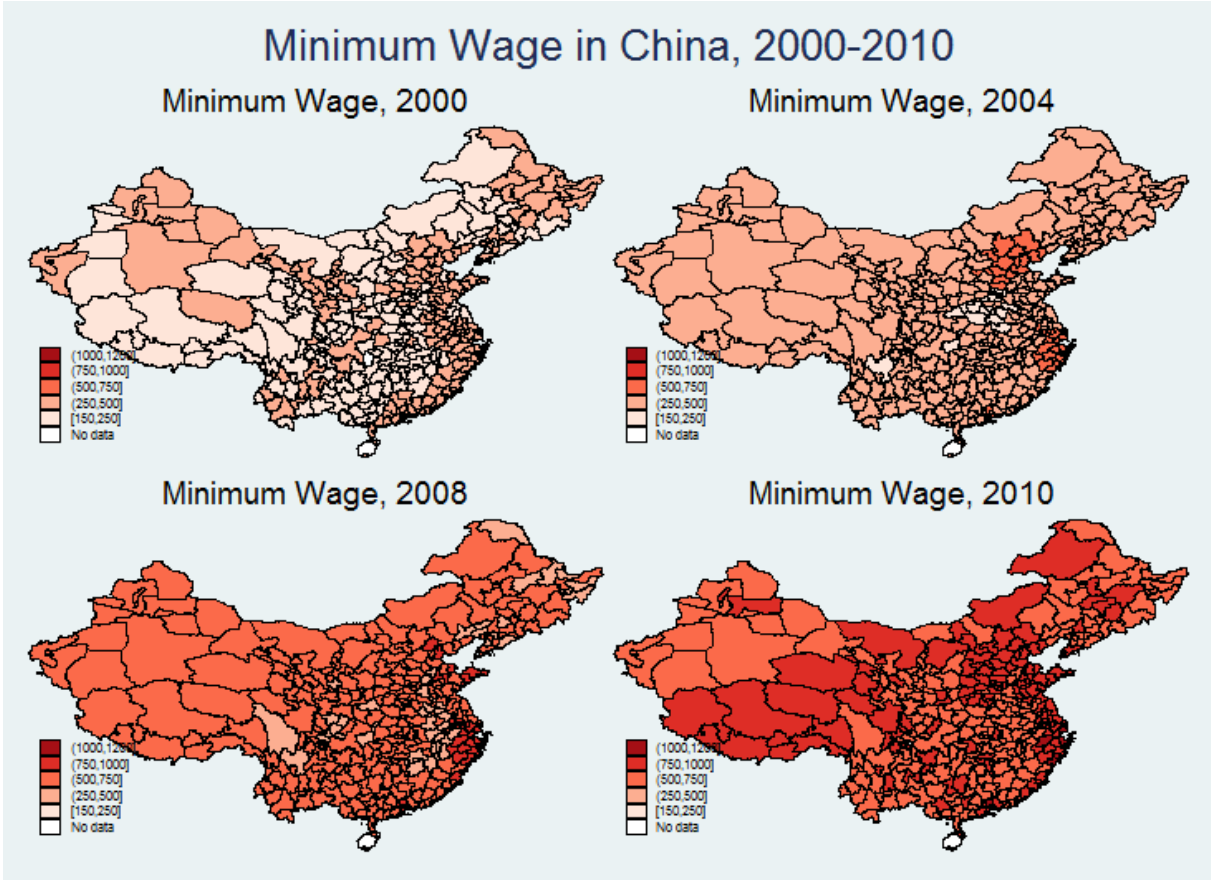
Graphical Illustration: Supply, Demand and Minimum Wages in Good X



Data

- Monthly minimum wage data is manually collected from local government websites and statistical bulletins.
  - Covers all 31 mainland provinces and provincial municipalities, 345 prefectures (similar to US county).
  - Minimum wage set at prefecture level using two methods:
    - The proportion method - minimum income necessary to cover the standard living costs
    - Engel Coefficient method - minimum food expenditure divided by the Engel coefficient in a minimum living cost.

Spatial Distribution of Minimum Wage in China



Data

- Survey data:
  - Firm-level data - ASIP - from 1998 through 2007, National Bureau of Statistics and Census of manufacturing for 2004 - information on skilled and unskilled labor employed and wage bill.
  - Includes all SOEs and non-SOEs with sales over 5 million Chinese Yuan.
  - Firms' industry of production, ownership type, age, employment, capital stocks, revenues, export values.
- Customs data
  - Transaction-level data. Collected and made available by the Chinese Customs Office.
  - Universe of transactions by Chinese firms over the 2000-2006.
  - Includes basic firm information, the value of each transaction (in US dollars) by product and trade partner for 243 destination/origin countries and $7,526$ different products at the 8-digit Harmonized System.
- Merge of the survey data and customs data matched on firm name, region code, address, legal person, and so on

Pluses and Minuses

- Survey data:
  - information on wages paid as well as estimated TFP.
  - lacks complete coverage - minimum size.
  - no information on destination of export or unit values or quantities. Only value data reported.
- Customs data:
  - unless matched with the survey data, many firm level variables are not available.
  - census so there is no lack of coverage.
  - product level information is very detailed as is destination and source of exports.
  - both quantity and value are reported, we can get unit value (price) information as well.
- Merged data strengths of both, but limited by coverage and ability to match.

Empirical Strategy

- Endogeneity of minimum wage, especially for average city-level TFP
- Possible instruments.
  - Within-province average wage for all other prefectures. One might wish to put more weight on the minimum wages of prefectures "similar" to the city than on those that differ a lot in terms of size or income per capita.
  - Initial minimum wage. Synchronization of across-prefecture minimum wage seems to be a clear policy initiative. However because this instrument is not time varying, using it would require changing our baseline empirical specification to a first-difference setup.
- We present next OLS and IV results for the first instrument

Export Pattern: Price, Quantity and Value

- The baseline regression is.

$$ln(V_{ihdt}) = \alpha_1 \cdot ln(mw_{ct}) + \beta_1 \cdot ln(mw)_{ct} \cdot (S/L)_h$$

$$+ \beta_1 \cdot ln(mw)_{ct} \cdot ln(K/L)_h + \mu_1 X_{ct} + \gamma_1 Y_{dt} + \lambda_{ihd} + \lambda_t + \varepsilon_{ihdt}(1)$$

- We expect: as the minimum wage rises, export quantity falls, and price rises, but less so for skill or capital intensive goods. If export demand is elastic, value acts in line with quantity of exports.
- Control for city size and and GDP per capita as more productive, richer city may have lower costs and prices.
- We see this in Table 1.

Table 1: Minimum Wage & Export Value, Quantity, and Price

VARIABLES	(1) ln(export value)	(2) ln(unit value)	(3) ln(export quantity)	(4) ln(export value)	(5) ln(unit value)	(6) ln(export quantity)
ln(min wage)	-0.494***	0.178***	-0.672***	-0.565***	0.199**	-0.764***
	[0.0565]	[0.0338]	[0.0600]	[0.146]	[0.0880]	[0.151]
ln(min wage)× Industry-City (S/L)	1.157***	-0.265***	1.422***	1.323***	-0.306***	1.629***
	[0.120]	[0.0750]	[0.129]	[0.108]	[0.0672]	[0.115]
ln(min wage)× Industry-City ln(K/L)	0.122***	-0.0366***	0.159***	0.139***	-0.0384***	0.178***
	[0.0151]	[0.0101]	[0.0160]	[0.0139]	[0.00928]	[0.0148]
city ln(GDP/population)	0.0462***	-0.0259**	0.0721***	0.0438***	-0.0251***	0.0689***
	[0.0157]	[0.0115]	[0.0160]	[0.0133]	[0.00905]	[0.0134]
city ln(population)	0.00512	0.0495	-0.0444	0.00560	0.0517	-0.0461
	[0.0407]	[0.0511]	[0.0617]	[0.0379]	[0.0411]	[0.0529]
destination ln(GDP/population)	0.0643*	0.0104	0.0540	0.0581*	0.0121	0.0460
	[0.0361]	[0.0175]	[0.0396]	[0.0307]	[0.0149]	[0.0335]
Observations	12,885,836	12,885,836	12,885,836	12,881,448	12,881,448	12,881,448
R-squared	0.793	0.949	0.847	0.016	0.048	0.002

Note: Robust standard errors in parentheses, clustered on city-product.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Factor Intensity

- We expect: an increase in minimum wage will (i) increase firms' capital intensity; (ii) increase firms' capital stock; (iii) decrease firms' labor employment and share in costs (with labor costs evaluated at a base wage);
- We see:
  - K/L rises, more so for skill intensive firms - capital skilled labor complementarities; less so for capital intensive sectors - minimum wage is less relevant.
    - L falls, less so for firms with higher skill intensity - substitute towards skill labor and minimum wage will matter less for them.
  - K rises, more so for skill intensive firms, and less so for higher K/L.
  - Labor cost share (at fixed wages) falls with the rise in minimum wage, less so for higher K/L, more so for skill intensive firms.

Table 2: Minimum Wage and Factor Intensity

VARIABLES	(1) ln(K/L)	(2) ln(L)	(3) ln(K)	(4) Labor cost share	(5) ln(K/L)	(6) ln(L)	(7) ln(K)	(8) Labor cost share
ln(min wage)	0.823***	-0.119***	0.704***	-0.0236***	1.370***	-0.0281	1.342***	-0.0453***
	[0.0586]	[0.0451]	[0.0628]	[0.00172]	[0.158]	[0.114]	[0.144]	[0.00542]
ln(min wage) × Industry-City (S/L)	0.193**	0.123**	0.316***	-0.00970***	0.176*	0.181***	0.358***	-0.0109***
	[0.0941]	[0.0626]	[0.0942]	[0.00306]	[0.0912]	[0.0598]	[0.0940]	[0.00297]
ln(min wage) × Industry-City ln(K/L)	-0.198***	0.0419***	-0.156***	0.00546***	-0.205***	0.0423***	-0.162***	0.00616***
	[0.0135]	[0.0111]	[0.0142]	[0.000419]	[0.0135]	[0.0107]	[0.0142]	[0.000419]
city ln(GDP/population)	0.295***	-0.0770***	0.218***	-0.00560***	0.285***	-0.0791***	0.206***	-0.00522***
	[0.0187]	[0.0157]	[0.0208]	[0.000699]	[0.0181]	[0.0136]	[0.0207]	[0.000557]
city ln(population)	-0.247***	0.0469	-0.200***	0.000260	-0.192***	0.0545	-0.137**	-0.00177
	[0.0566]	[0.0401]	[0.0591]	[0.00108]	[0.0558]	[0.0361]	[0.0601]	[0.00129]
Observations	810,177	810,177	810,177	760,269	784,870	784,870	784,870	735,816
R-squared	0.841	0.921	0.910	0.724	0.048	0.017	0.084	0.059

Note: Robust standard errors in parentheses, clustered on industry-city pair.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 3: Minimum Wage and Productivity Distribution

VARIABLES	(1) Avg TFP	(2) Avg TFP	(3) Avg TFP	(4) Avg TFP
ln(min wage)	0.161***	0.150***	0.590***	0.576***
	[0.0247]	[0.0288]	[0.0880]	[0.0912]
ln(min wage) ×Industry-City S/L	-0.0443	-0.0604	-0.0318	-0.0499
	[0.0410]	[0.0479]	[0.0464]	[0.0536]
ln(min wage) ×Industry-City ln(K/L)	-0.0141**	-0.0163**	-0.0200***	-0.0167**
	[0.00554]	[0.00639]	[0.00624]	[0.00712]
ln(min wage) ×(shr of firms < min wage in 2001)		0.208***		0.268***
		[0.0269]		[0.0307]
city ln(avg wage)	0.0918***	0.0998***	0.0862***	0.0914***
	[0.00984]	[0.0112]	[0.00994]	[0.0113]
city ln(population)	-0.176***	-0.182***	-0.144***	-0.147***
	[0.0364]	[0.0383]	[0.0372]	[0.0392]
city population density	0.0351***	0.0412***	0.0408***	0.0470***
	[0.0103]	[0.0108]	[0.0104]	[0.0109]
Observations	160,711	117,231	159,752	117,051
R-squared	0.973	0.975	0.042	0.043

Note: Robust standard errors in parentheses, clustered on industry-city pair.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 4: Minimum Wage and Exit From Export

VARIABLES	(1) Exit Share	(2) Exit Share	(3) Exit Share	(4) Exit Share
ln(min wage)	0.0362***	0.0377***	0.0528**	0.0478*
	[0.00689]	[0.00774]	[0.0245]	[0.0245]
ln(min wage) ×Industry-City S/L	-0.0385***	-0.0249*	-0.0264**	-0.0158
	[0.0114]	[0.0129]	[0.0129]	[0.0143]
ln(min wage) ×Industry-City ln(K/L)	-0.00271*	-0.00486***	-0.00353**	-0.00604***
	[0.00154]	[0.00172]	[0.00173]	[0.00191]
ln(min wage) ×(shr of firms < min wage in 2001)		0.0357***		0.0339***
		[0.00723]		[0.00823]
Avg TFP	0.00535***	0.00636***	0.00528***	0.00633***
	[0.000776]	[0.000868]	[0.000783]	[0.000875]
city ln(avg wage)	0.0206***	0.0185***	0.0203***	0.0185***
	[0.00274]	[0.00301]	[0.00276]	[0.00303]
city ln(population)	-0.0118	-0.0126	-0.0113	-0.0126
	[0.0101]	[0.0103]	[0.0103]	[0.0105]
city population density	0.000421	0.000264	0.000615	0.000307
	[0.00286]	[0.00291]	[0.00288]	[0.00292]
Observations	160,711	117,231	159,752	117,051
R-squared	0.225	0.205	0.020	0.022

Note: Robust standard errors in parentheses, clustered on firm.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Robustness check: firm- or product- specific trends

VARIABLES	(1) Δ ln(export value)	(2) Δ ln(unit value)	(3) Δ ln(export quantity)
Δ ln(min wage)	-0.574***	0.388***	-0.962***
	[0.114]	[0.0410]	[0.107]
Δ ln(min wage) × Industry-City (S/L)	1.227***	-0.478***	1.705***
	[0.265]	[0.106]	[0.275]
Δ ln(min wage) × Industry-City ln(K/L)	0.155***	-0.0340***	0.189***
	[0.0320]	[0.00980]	[0.0284]
Δ TFP	0.220***	0.0266***	0.193***
	[0.0154]	[0.00445]	[0.0161]
Δ city ln(GDP/population)	0.0781***	0.0472	0.0310
	[0.0190]	[0.0294]	[0.0283]
Δ city ln(population)	0.0107	-0.000862	0.0116
	[0.0415]	[0.0593]	[0.0468]
Δ destination ln(GDP/population)	0.213***	0.236***	-0.0231
	[0.0786]	[0.0253]	[0.0808]
young	0.126***	-0.00545**	0.131***
	[0.00754]	[0.00223]	[0.00758]
old	0.0128	-0.00483	0.0176*
	[0.00859]	[0.00511]	[0.0100]
1 if firm is in Special Economic Zone	-0.0340***	0.0103	-0.0443***
	[0.0123]	[0.00811]	[0.0128]
1 if firm is a high-tech firm	0.0230*	-0.0202***	0.0432***
	[0.0129]	[0.00492]	[0.0124]
least traded goods	0.0695***	-0.0163***	0.0858***
	[0.0110]	[0.00500]	[0.0132]
Observations	1,663,673	1,663,673	1,663,249
R-squared	0.004	0.004	
0.004			

Note: Robust standard errors in parentheses, clustered on firm.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

More Patterns on Exports

- Share to OECD rises with minimum wage and less so for skill/capital intensive goods. Suggests upgrading?
- Processing share falls with higher minimum wages and this is less so for skill or capital intensive goods. Upgrading?

VARIABLES	(1) share to OECD	(2) processing share	(3) share to OECD	(4) processing share
ln(min wage)	0.0901***	-0.0229***	0.0840***	-0.0200***
	[0.0168]	[0.00666]	[0.0156]	[0.00676]
ln(min wage)× Industry-City (S/L)	-0.254***	0.0138	-0.236***	0.0163
	[0.0258]	[0.00983]	[0.0255]	[0.0101]
ln(min wage)× Industry-City ln(K/L)	-0.0170***	0.00738***	-0.0173***	0.00701***
	[0.00465]	[0.00175]	[0.00433]	[0.00178]
lag ln(firm total export value)			0.00163***	0.00235***
			[0.000312]	[0.000135]
city ln(GDP/population)	-0.0197***	0.0155***	-0.0241***	0.0124***
	[0.00613]	[0.00233]	[0.00575]	[0.00221]
city ln(population)	0.118***	0.0158***	0.0628***	0.0108***
	[0.0138]	[0.00270]	[0.0101]	[0.00266]
Observations	10,191,176	10,191,176	8,191,565	8,191,565
R-squared	0.868	0.935	0.873	0.937

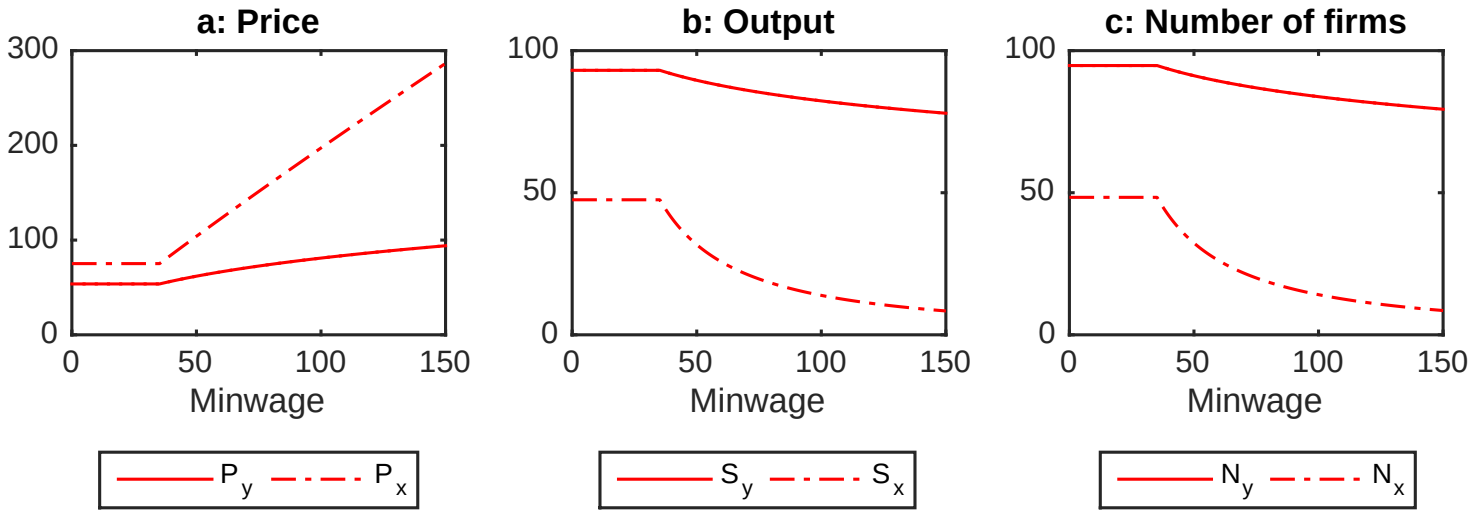
Note: Robust standard errors in parentheses, clustered on prefecture-product.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

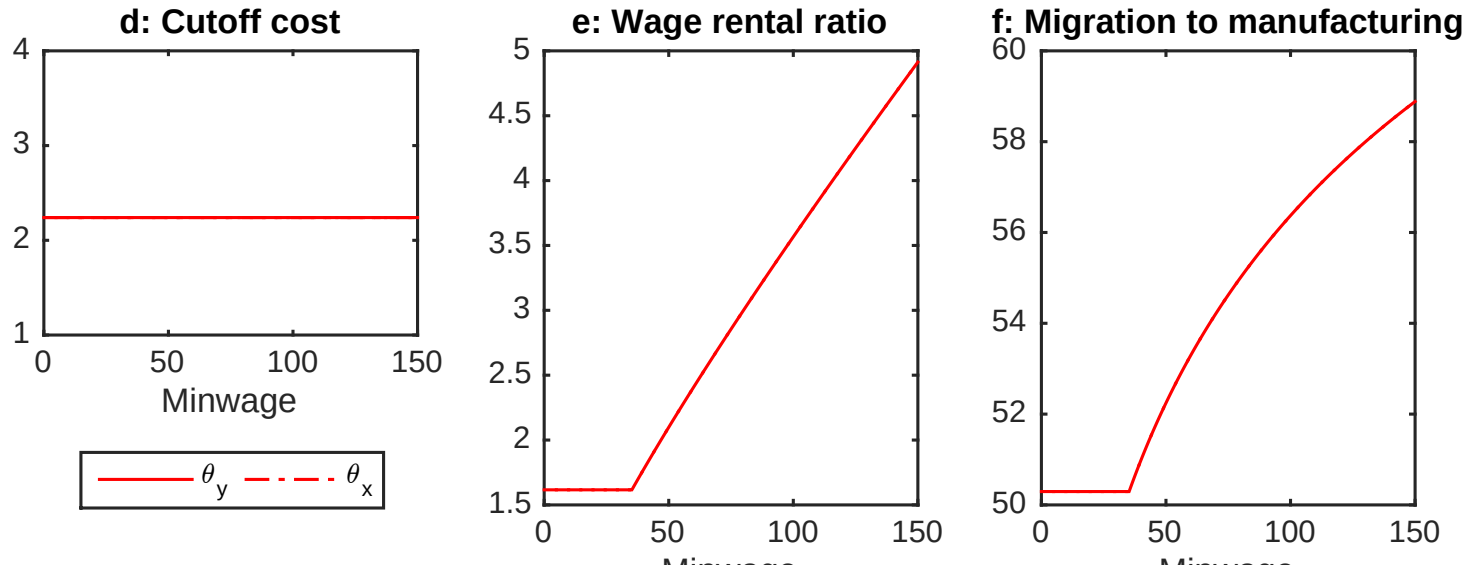
Conclusion

- Minimum wages do seem to have significant effects in a general equilibrium setting on firms' production and export pattern, choice of inputs and survival
- Future work: extensions to structural model (new urban economics) incorporating bilateral migration decision

Simulations: No Selection

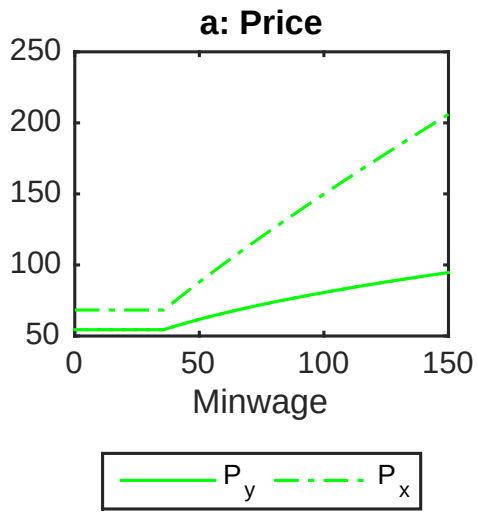
Back to Prediction

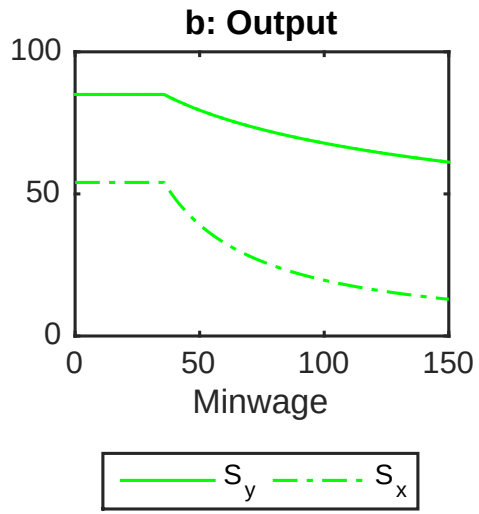


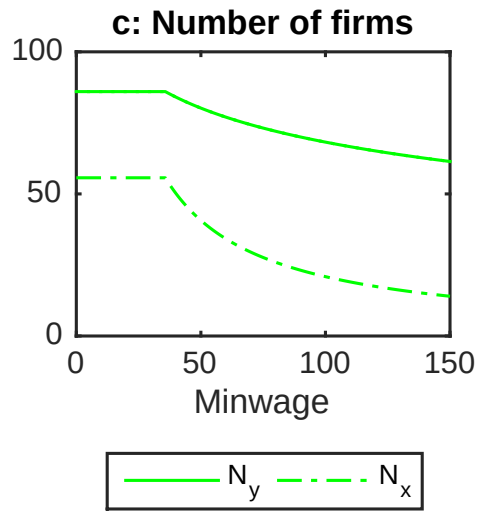


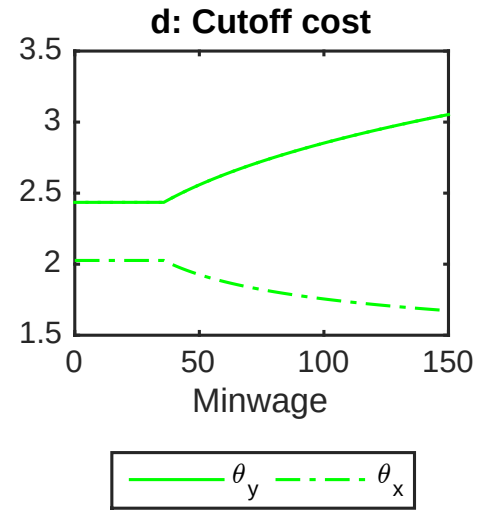
Simulations: With Selection

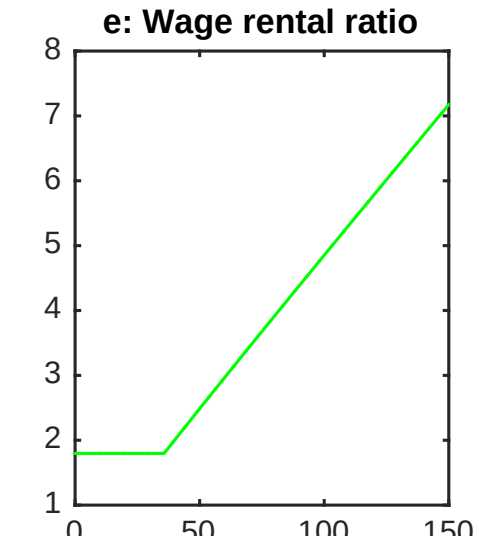
Back to Prediction

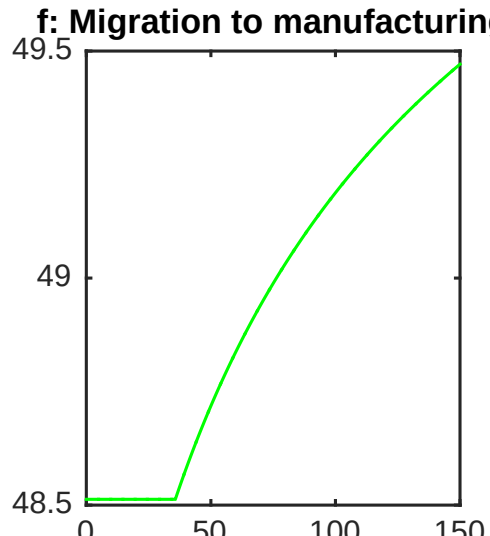












Proof of Lemma 1

Back to Model

Integrating by parts over area in which the firm chooses to produce gives:

$$\begin{aligned}
\int_0^{\frac{p}{c(w.r)}} (p-\theta c(.))\, f(\theta)d\theta &= (p-\theta c(w,r))\, F(\theta)\big|_0^{\tilde{\theta}} + \int_0^{\tilde{\theta}} c(w,r)F(\theta)d\theta \\
&= \left[c(w,r)\int_0^{\tilde{\theta}} F(\theta)d\theta\right] \\
&= c^e(w,r)f_e
\end{aligned}$$

- In Figure 1, area OAB $= \frac{c^e(w,r)f_e}{c(w,r)}$.

Back to Model