Empirical Tests of Asset Pricing Models with Individual Stocks

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AFA Annual Meeting
Jan. 6th, 2018

What Explain the Cross-section of Asset Returns?

- Many asset pricing models have been proposed to explain the cross-section of stock returns.
  - Standard CAPM
  - Fama-French three-factor and five-factor models
    - MKT, SMB, HML / RMW, CMA factors
  - q-factor asset pricing model
    - MKT, size, investment, and profitability factors
    - Hou, Xue, and Zhang (2015)
  - Liquidity-adjusted CAPM
    - Liquidity-adjusted market beta and three illiquidity betas
    - Acharya and Pedersen (2005)
- Except the standard CAPM, these asset pricing models are known successful empirically.
- Asset pricing models have been typically tested with portfolios sorted on chosen characteristics.
  - These characteristics have been known to affect the average returns.

Test Assets: Portfolios vs Individual Stocks

- Portfolios
  - Factor loadings can be estimated more accurately, i.e., with smaller measurement errors in betas.
  - Suffer from low dimensionality problem
  - Testing results can be sensitive to sorting variables.
  - Small correlation between the factor and sorted characteristics can lead to a large pricing effect.
    - Lewellen et al. (2010)
- Individual stocks
  - More severe error-in-variable (EIV) problem
  - Not suffer from low dimensionality problem
  - Not suffer from subjective choice of sorting variables
  - Can provide more powerful tests than portfolios.
- Other researches using individual stocks as test assets:
  - Litzenberger and Ramaswamy (1979), Kim (1995), Brennan et al. (1998), Chen and Kan (2004), Chordia et al. (2015), Kim and Skoulakis (2015), Gagliardini et al. (2016), Raponi et al (2016)

Main Contributions and Findings:

- We develop a consistent estimator of ex-post risk premiums and a test of asset pricing models with individual stocks.
    - Derive the asymptotic properties of the proposed risk premium estimator for large N (number of stocks).
        - Consistency and asymptotic distribution of the estimator
    - Investigate the small sample properties of the proposed IV estimator and associated t-test.
    - Very easy to implement
- Use the developed testing procedure to test several asset pricing models recently proposed in the literature.
    - Contribute to the debate on betas vs characteristics
    - Characteristics seem to play more important roles than betas.
        - Pricing evidence of SMB and HML risk
        - No pricing evidence of factor risks w/ characteristics
        - Characteristics have significant slope coefficients.

EIV Biases

- For simplicity, consider the single factor model.
- Factor loadings (=betas) are unobservable and typically estimated by running time-series regressions.
  - The standard Fama-MacBeth (FM) produces

$$\gamma_{FM} = \frac{\text{Cov}(r, \widehat{\beta})}{\text{Var}(\widehat{\beta})} = \frac{\text{Cov}(r, \beta_{true})}{\text{Var}(\beta_{true}) + \text{Var}(u)} < \frac{\text{Cov}(r, \beta_{true})}{\text{Var}(\beta_{true})}$$

where

$$\widehat{\beta} = \beta_{true} + u.$$

  - Thus the EIV bias can be fixed by

$$\gamma_{CGS} = \frac{\text{Cov}(r, \widehat{\beta})}{\text{Var}(\widehat{\beta}) - \widehat{\text{Var}}(u)}$$

    - LR (1979), Kim (1995), Kim and Skoulakis (2015), Chordia et al. (2015) and Raponi et al (2016)

Instrumental Variables (IV) Estimator and Test Statistics

1. **First-pass**: run separate time-series regressions to estimate even- and odd-month betas

$$r_{i,t} = \alpha_i + \sum_{k=1}^{K} \beta_{i,k} f_{k,t} + \varepsilon_{i,t},$$

2. **Second-pass**: run cross-sectional regressions with instrumental variables

$$\widehat{\Gamma}_{IV,t_{even}} = \left(\widehat{B}'_{odd}\widehat{B}_{even}\right)^{-1}\widehat{B}'_{odd}R_{t_{even}},$$

$$\widehat{\Gamma}_{IV,t_{odd}} = \left(\widehat{B}'_{even}\widehat{B}_{odd}\right)^{-1}\widehat{B}'_{even}R_{t_{odd}},$$

3. Risk premium estimate: average of $\widehat{\Gamma}_{IV,t_{even}}$ and $\widehat{\Gamma}_{IV,t_{odd}}$.
4. Standard error (SE): compute the Fama-MacBeth SE.
5. Construct the t-statistic:

$$t = \frac{\widehat{\gamma}_{IV}}{SE\left(\widehat{\gamma}_{IV}\right)}$$

Intuitions behind the IV Estimator

- For simplicity, consider the single factor model.
  - The IV estimator produces

$$\gamma_{IV} = \frac{\text{Cov}(r, \widehat{\beta}_{odd})}{\text{Cov}(\widehat{\beta}_{odd}, \widehat{\beta}_{even})} = \frac{\text{Cov}(r, \beta_{true})}{\text{Var}(\beta_{true})}$$

    where

$$\widehat{\beta}_{odd} = \beta_{true} + u_{odd}, \qquad \widehat{\beta}_{even} = \beta_{true} + u_{even}.$$

  - Thus no EIV bias arises in the IV estimator.

Simulation Study: Small Sample Properties

- Our asymptotic theory provides the large sample properties of the IV estimator.
  - N-consistency is proven when N grows.
  - Asymptotic normal distributions.
    - Asymptotic variance-covariance matrix

- For practical purposes, we examine the small sample properties. We investigate
  - Bias and RMSE of the IV estimator.
  - Size and power of the associated t-test.
  - Simulation parameters are based on real data.
  - Focus on the cases with fixed T ($<< N$).

Simulation Results (Constant Betas)

- Biases and RMSEs of the IV Estimator
  - Under the FF3M

Risk Factor	Estimator	Ex-ante Bias (%)	Ex-post Bias (%)	Ex-ante RMSE	Ex-post RMSE
MKT	OLS	-28.7	-29.4	0.199	0.158
	IV	1.2	0.5	0.189	0.084
SMB	OLS	-54.4	-55.2	0.136	0.135
	IV	-1.4	-2.1	0.126	0.096
HML	OLS	-50.6	-51.2	0.194	0.193
	IV	1.6	1.0	0.124	0.092

  - Similar results are obtained with time-varying betas.

Simulation Results (Constant Betas)

- Size and Power of the IV Test under the FF3M
  - Test Size:

Risk Factor	Theoretical Percentiles 1%	2.5%	5%	7.5%	10%
MKT	1.3%	2.4%	5.2%	7.3%	9.8%
SMB	1.3%	2.7%	5.2%	7.8%	9.9%
HML	1.1%	2.7%	5.0%	7.7%	10.2%

  - Test Power:

Risk Factor	Test Power
MKT	83.8%
SMB	51.8%
HML	91.5%
MKT or SMB or HML	99.6%

- Similar simulation results are obtained with time-varying betas.

Tests of the CAPM and FF3M

- Sample period: 1956 through 2012
- Average number of stocks: $N = 2425$

	(1)	(2)	(3)	(4)
Constant	**1.02**	**0.725**	**2.906**	**3.007**
	(7.89)	**(6.00)**	**(4.31)**	**(4.92)**
MKT Beta	-0.246	-0.288	-0.09	-0.018
	(-1.36)	(-1.60)	(-0.51)	(-0.10)
SMB Beta		**0.301**		-0.043
		(2.20)		(-0.42)
HML Beta		**0.344**		0.242
		(2.55)		(1.88)
SIZE			**-0.120**	**-0.118**
			(-3.49)	**(-3.93)**
BM			**0.196**	**0.180**
			(4.40)	**(4.50)**

- **Main Takeaways**
  - Pricing evidence of SMB and HML risks exists
    - When Size and BM are not controlled for.
    - Indicates that the IV-tests have reasonable power.
  - The pricing evidence disappears
    - When Size and BM are included.
    - Size and BM seem to dominate the SMB and HML factor loadings.
  - It is puzzling to have sharply contradictory results
    - Between individual stocks and portfolios as test assets.
  - Similar results are obtained from sub-period analysis.

Tests of the FF5M

- FF5M = FF3M + Two additional factors
  - RMW (Robust-Minus-Weak): Profitability factor
  - CMA (Conservative-Minus-Aggressive): Investment factor
- Sample period: 1964 through 2012
- Average number of stocks: $N = 2811$
  - Pricing evidence of HML risks exists w/o controlling for characteristics.
  - Characteristics seem to dominate factor risks in FM-CSR.
  - Similar results are obtained from sub-period analysis.
- When testing the FF3M and FF5M, among factor loadings,
  - HML risk seems the most robust.

	(1)	(2)	(3)	(4)	(5)	(6)
MKT Beta			-0.198			0.367
			(-0.57)			(1.06)
SMB Beta			0.453			-0.095
			(1.85)			(-0.50)
HML Beta			0.766			0.354
			(1.83)			(0.87)
RMW Beta	0.121		-0.237	0.207		-0.051
	(0.59)		(-0.46)	(1.07)		(-0.11)
CMA Beta		0.030	0.159		-0.043	0.049
		(0.13)	(0.29)		(-0.18)	(0.11)
Size						**-0.153**
						(-4.38)
BM						**0.178**
						(4.12)
OP				-0.01		**0.649**
				(-0.00)		**(6.10)**
INV					**-0.963**	**-0.709**
					(-6.62)	**(-9.25)**

Tests of the q-factor Model by HXZ

- The expected excess return is

$$E[r_t^i] = \gamma_{MKT}\beta^i_{MKT} + \gamma_{ME}\beta^i_{ME} + \gamma_{I/A}\beta^i_{I/A} + \gamma_{ROE}\beta^i_{ROE}$$

  - $\beta^i_{I/A}$: Beta for investment factor
  - β^i_{ROE}: Beta for profitability factor

- Sample period: 1972 through 2012
- Average number of stocks: $N = 3162$
  - Pricing evidence of factor loadings seems very weak.
  - Characteristics seem to dominate factor risks.
  - Similar results are obtained from sub-period analysis.

	(1)	(2)	(3)	(4)
Constant	**0.856**	**1.120**	**0.809**	**4.268**
	(3.68)	**(4.56)**	**(3.55)**	**(5.39)**
MKT Beta	-0.247			0.437
	(-0.86)			(1.20)
ME Beta	0.222			-0.118
	(0.67)			(-0.28)
I/A Beta	0.001	0.247		-0.547
	(0.01)	(0.89)		(-0.83)
ROE Beta	-0.400		-0.100	-0.632
	(-0.77)		(-0.46)	(-0.83)
Size				**-0.202**
				(-5.41)
Inv		**-0.651**		**-0.579**
		(-5.16)		**(-6.54)**
Profit			**2.699**	**3.734**
			(3.74)	**(4.97)**

The Liquidity-adjusted CAPM (LCAPM)

- Proposed by Acharya and Pedersen (2005)
- Incorporate the illiquidity-based trading friction
- The unconditional expected excess return is

$$E[r_t^i] = E[c_t^i] + \lambda \left(\beta_1^i + \beta_2^i - \beta_3^i - \beta_4^i\right)$$

  - β_1^i is related to $\text{Cov}\left(r_t^i, r_t^m\right)$.
  - β_2^i is related to $\text{Cov}\left(c_t^i, c_t^m\right)$.
  - β_3^i is related to $\text{Cov}\left(r_t^i, c_t^m\right)$.
  - β_4^i is related to $\text{Cov}\left(c_t^i, r_t^m\right)$.
  - $\beta_{LMKT}^i = \beta_1^i + \beta_2^i - \beta_3^i - \beta_4^i$.
- Pricing of β_{LMKT}^i implies that the market beta and three liquidity betas affect the expected returns.

Test of the LCAPM

- Sample period: 1956 through 2012
- Average number of stocks: $N = 1283$

	(1)	(2)
Constant	**0.559**	**0.503**
	(3.85)	**(3.48)**
LMKT Beta	0.150	0.085
	(0.66)	(0.38)
Amihud Illiquidity		**0.220**
		(4.21)

- Liquidity-adjusted market beta is not priced.
- Illiquidity level is priced.
- Similar results are obtained from sub-period analysis.

Conclusions

- Develop an IV estimator of risk premiums and a test of asset pricing models with individual stocks.
  - Overcome the low dimensionality of portfolios as test assets.
- The IV estimator and associated *t*-test
  - Have consistency and asymptotic normal distributions.
  - Have good small sample properties.
    - Nearly unbiased / low RMSEs
    - Correct test size / powerful enough.
  - Easy to implement
- Test various asset pricing models recently proposed.
  - Characteristics seem to paly more important roles than betas.
    - Pricing evidence of SMB and HML risk w/o BM
    - No pricing evidence when characteristics are included.