Social Discounting and Intergenerational Pareto

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Which Social Discount Rate Should We Use?

Many economic decisions are dynamic and affect multiple individuals

- Corporate/household long-term investments
- Durable public good investments
- Intertemporal tax transfers
- Environmental projects

These decisions depend on one number, the social discount rate

- The society's trade-off between current benefit and future benefit
- No consensus on which social discount rate should be used

The Stern Review

"...if we don't act, the overall costs and risks of climate change will be equivalent to losing at least 5% of global GDP each year, now and forever."

— The Stern Review on the Economics of Climate Change

"...(the Stern Review) depends decisively on the assumption of a near-zero time discount rate..."

— William Nordhaus

"...(using discount rates ranging from 3-5%) is ethically indefensible."

— Lord Nicholas Stern

Questions

1. In what sense is a social discount rate reasonable?

2. What are the reasonable social discount rates?

Social Discounting Depends on Individual Discounting

Social discounting should be more patient than individual discounting (Caplin & Leahy 2004, Farhi & Werning 2007)

- Pure time-preference discounting, rather than consumption discounting
- Social discounting should take into account how future generations value their consumption
- Future generations value future more than the current generation value future
- Thus, social discounting also values future more than the current generation does
- However, these theories only have one individual (representative agent)

A Negative Result

Common in these situations...

- A benevolent planner chooses for multiple generations
- Uncertainty about payoffs

Widely used assumptions in economics:

1. Planner has an exponential discounting expected utility function
2. Some Pareto property

Gollier & Zeckhauser (2005), Zuber (2011), Jackson & Yariv (2014, 2015): even when individuals also discounts exponentially

$$1 + 2 \Rightarrow \text{Dictatorship}$$

Preferences

Model Setup

- $2 < T \leq +\infty$ generations/periods
- $N < \infty$ individuals in each generation who live for one period
- One risky public consumption $p_t \in \Delta(X)$ in each period t
- Consumption sequence: $\mathbf{p} = (p_1, \ldots, p_T) \in \Delta(X)^T$

Individual Preferences

- Generation-t individual i's preference over $\mathbf{p}$'s: $\succsim_{i,t}$
- Generation-t individual i's discounting utility function:

$$U_{i,t}(\mathbf{p}) = \sum_{\tau=t}^{T} \delta_i(\tau - t) u_i(p_\tau)$$

  - Discount function $\delta_i(\cdot)$: $\delta_i(0) = 1$, $\delta_i > 0$; if $T = +\infty$, $\delta_i \in \ell^1$
  - Instantaneous (expected) utility function $u_i : \Delta(X) \to \mathbb{R}$

1. $U_{i,t}$ only depends on current and future consumption
   - can be relaxed when δ_i's are exponential
2. The offspring inherits the parent's δ_i
   - They rank $\mathbf{p}$'s differently ($\delta_i(\cdot)$ is shifted forward)
   - can be relaxed
3. Instantaneous utility does not depend on time
   - can be relaxed

The Planner's Preference

As in the negative results, we first focus on exponential discounting

- In each period t, the planner's preference over $\mathbf{p}$'s: $\succsim_t$
- In each period t, the planner's utility function:

$$U_t(\mathbf{p}) = \sum_{\tau=t}^{T} \delta^{\tau-t} u(p_\tau)$$

  - Social discount factor $\delta > 0$; $0 < \delta < 1$ if $T = +\infty$
  - Instantaneous utility function $u : \Delta(X) \to \mathbb{R}$

1. U_t only depends on current and future consumption
2. The discount factor and instantaneous utility do not depend on time
3. Normalization of expected utility functions: for some x_* and x^*, $u_i(x_*) = u(x_*) = 0$ and $u_i(x^*) = u(x^*) = 1$

Intergenerational Pareto

A Variant of the Negative Result

- In a dynamic setting, there are different ways to define Pareto

The planner is current-generation Pareto if for each t,

$\mathbf{p} \succsim_{i,t} \mathbf{q}$ for all i implies $\mathbf{p} \succsim_t \mathbf{q}$,

and $\mathbf{p} \succ_{i,t} \mathbf{q}$ for all i implies $\mathbf{p} \succ_t \mathbf{q}$.

- An generation-t individual i has an exponential discounting utility (EDU) function if

$$U_{i,t}(\mathbf{p}) = \sum_{\tau=t}^{T} \delta_i^{\tau-t} u_i(p_\tau)$$

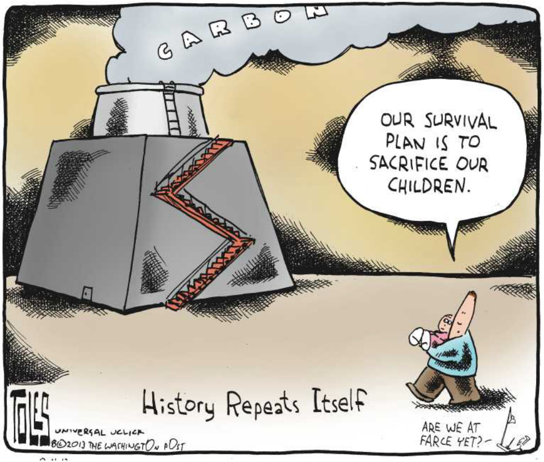
A Variant of the Negative Result

Proposition Suppose each generation-t individual i has an EDU function with (δ_i, u_i). For a generic N-tuple of discount factors $(\delta_i)_{i \in N}$, the planner is current-generation Pareto if and only if for each t, there exists a unique i such that $U_t = U_{i,t}$.

Sketch of the proof:

- Example: $N = 2$ and $u_1 = u_2 = u$
- Harsanyi 1955: Pareto $\Leftrightarrow$ Utilitarian, i.e., $U = \omega U_1 + (1 - \omega) U_2$
- $\begin{cases} \omega \delta_1 u_1 + (1 - \omega) \delta_2 u_2 = \delta u, \\ \omega \delta_1^2 u_1 + (1 - \omega) \delta_2^2 u_2 = \delta^2 u. \end{cases} \Rightarrow \omega = 0, 1$

Intergenerational Pareto



Intergenerational Pareto

The planner is intergenerationally Pareto if for each $t \in T$,

$\mathbf{p} \succsim_{i,s} \mathbf{q}$ for all i and all $s \geq t$ implies $\mathbf{p} \succsim_t \mathbf{q}$,

and $\mathbf{p} \succ_{i,s} \mathbf{q}$ for all i and all $s \geq t$ implies $\mathbf{p} \succ_t \mathbf{q}$.

- The planner can disagree with a selfish current generation
- The planner ignores past generations whose utility can no longer be changed

Intergenerational Pareto allows the planner to make rather discretionary decisions?

Intergenerational Pareto and Utilitarianism

Lemma Suppose $U_{i,t}(\mathbf{p}) = \sum_{\tau=t}^{T} \delta_{i,t}(\tau - t) u_i(p_\tau, \tau)$, and $U_t(\mathbf{p}) = \sum_{\tau=t}^{T} \delta_t(\tau - t) u_t(p_\tau, \tau)$. Suppose $T < +\infty$. The planner is intergenerationally Pareto if and only if for each t, there exists a finite sequence of nonnegative numbers $(\omega_t(i,s))_{i \in N, s \geq t}$ such that $\sum_{i=1}^{N} \sum_{s=t}^{T} \omega_{i,t}(s) > 0$ and

$$U_t = \sum_{i=1}^{N} \sum_{s=t}^{T} \omega_{i,t}(s) U_{i,s}.$$

Social Discounting

and Individual Long-Run Discounting:

The Benchmark Case

Strongly Non-Dictatorial

The planner is strongly non-dictatorial if for each t,

$$U_t(\mathbf{p}) = f_t\left(U_{1,t}(\mathbf{p}), \ldots, U_{1,T}(\mathbf{p}), U_{2,t}(\mathbf{p}), \ldots, U_{2,T}(\mathbf{p}), \ldots, U_{N,T}(\mathbf{p})\right)$$

for some strictly increasing function f_t.

- Negative results: The only way for a time-consistent planner to be current-generation Pareto is dictatorship
- Non-dictatorial: The planner cares about more than one individual

Individual Average and Relative Discounting

- $\delta_i(\cdot)$ is defined on $\mathbb{N}$; T may vary

Average discounting: $\sqrt[\tau]{\delta_i(\tau)}$ Relative discounting: $\frac{\delta_i(\tau+1)}{\delta_i(\tau)}$

A1: $\lim_{\tau\to\infty} \sqrt[\tau]{\delta_i(\tau)}$ exists

A2: $\frac{\delta_i(\tau+1)}{\delta_i(\tau)}$ is bounded

A3: $\frac{\delta_i(\tau+1)}{\delta_i(\tau)}$ is increasing

- A2 and A3 $\Rightarrow \lim_{\tau\to\infty} \frac{\delta_i(\tau+1)}{\delta_i(\tau)}$ exists
  $\Rightarrow \lim_{\tau\to\infty} \sqrt[\tau]{\delta_i(\tau)} = \lim_{\tau\to\infty} \frac{\delta_i(\tau+1)}{\delta_i(\tau)}$
- $\sqrt[\tau]{\delta_i(\tau)} = \sqrt[\tau]{\frac{\delta_i(\tau)}{\delta_i(\tau-1)} \times \frac{\delta_i(\tau-1)}{\delta_i(\tau-2)} \times \cdots \times \frac{\delta_i(1)}{\delta_i(0)}}$

Benchmark Case

The benchmark case assumes that $T < +\infty$ and $u_i = u$

The main results will highlight how individual instantaneous utility affects the range of "reasonable" social discount rates

Benchmark Case

Theorem Suppose $T < +\infty$, and each generation-t individual i's discounting utility function satisfies A1, A2, and $u_i = u$. Then,

1. if $\delta > \min_i \max_{\tau \in \{0,\ldots,T-1\}} \frac{\delta_i(\tau+1)}{\delta_i(\tau)}$, the planner is intergenerationally Pareto and strongly non-dictatorial;
2. For each $\delta < \min_i \lim_{\tau \to \infty} \sqrt[\tau]{\delta_i(\tau)}$, there exists some $T^* > 0$ such that if $T \geq T^*$, the planner is not intergenerationally Pareto.

- The first part fixes the negative result, and can be used to check whether a planner satisfies intergenerational Pareto
- The second part: if δ is too low, there exist $\mathbf{p}$ and $\mathbf{q}$ such that all individuals from all generations prefer $\mathbf{p}$ to $\mathbf{q}$, but the planner disagrees
- In many examples, the two cutoffs are identical

Individual Long-Run Discounting

- In both examples, two cutoffs coincide

A1: $\lim_{\tau\to\infty} \sqrt[\tau]{\delta_i(\tau)}$ exists

A2: $\frac{\delta_i(\tau+1)}{\delta_i(\tau)}$ is bounded

A3 (present bias): $\frac{\delta_i(\tau+1)}{\delta_i(\tau)}$ is increasing

- A2 and A3 $\Rightarrow \lim_{\tau\to\infty} \frac{\delta_i(\tau+1)}{\delta_i(\tau)} = \lim_{\tau\to\infty} \sqrt[\tau]{\delta_i(\tau)}$

Define

$$\delta_i^* := \lim_{\tau\to\infty} \frac{\delta_i(\tau+1)}{\delta_i(\tau)} = \lim_{\tau\to\infty} \sqrt[\tau]{\delta_i(\tau)}$$

as individual i's long-run discount factor

Individual Long-Run Discounting

Corollary Suppose $T < +\infty$ and each generation-t individual i's discounting utility function satisfies A2, A3, and $u_i = u$. Then,

1. if $\delta > \min_i \delta_i^*$, the planner is intergenerationally Pareto and strongly non-dictatorial;
2. For each $\delta < \min_i \delta_i^*$, there exists some $T^* > 0$ such that if $T \geq T^*$, the planner is not intergenerationally Pareto.

- Social discounting literature: social discouning should be more patient than individual discounting, but which individual and what individual discount factor?
- Benchmark case: the individual with the least patient long-run discount factor
- However, this does not contribute much to the debate on social discounting, because $\min_i \delta_i^*$ can be quite low

Social Discounting and Individual Instantaneous Utility Functions

Instantaneous Utility Functions

$(u_i)_{i \in N}$ is said to be linearly independent if there are no constants $(\alpha_i)_{i \in N}$ such that they are not all zero and $\sum_i \alpha_i u_i(p) = 0$ for all $p \in \Delta(X)$.

- Generically, $(u_i)_{i \in N}$ is linearly independent

Instantaneous Utility Functions

Theorem Suppose $T < +\infty$, each generation-t individual i's discounting utility function satisfies A2 and A3, and $(u_i)_{i \in N}$ is linearly independent. Let the planner's u be any strict convex combination of $(u_i)_{i \in N}$. Then,

1. For each $\delta > \max_i \delta_i^*$, the planner is intergenerationally Pareto and strongly non-dictatorial;
2. For each $\delta < \max_i \delta_i^*$, there exists some $T^* > 0$ such that if $T \geq T^*$, the planner is not intergenerationally Pareto.

Remarks

- If A1 and A2 are assumed, rather than A2 and A3, we again have two cutoffs defined analogously
- The benchmark case is not robust: a small perturbation of $u_i = u$ moves the cutoff from $\min_i \delta_i^*$ to $\max_i \delta_i^*$
- The choice of δ is independent of the choice of u
- This result provides support for the use of near-zero discount rate
- Robustness: (i) T can be $+\infty$; (ii) the offspring does not have to inherit the parent's preference parameters; (iii) intergenerational Pareto can be strengthened...

Sketch of the Proof: Part 2

- Consider a special case where individuals have exponential discounting. In period 1,

$$\begin{aligned} U &= \sum_{s=1}^{T}\sum_{i=1}^{N}\omega(i,s)U_{i,s} \\ \sum_{\tau=1}^{T}\delta^{\tau-1}u(p_\tau) &= \sum_{s=1}^{T}\sum_{i=1}^{N}\omega(i,s)\sum_{\tau=s}^{T}\delta_i^{\tau-s}u_i(p_\tau) \end{aligned}$$

- There is a unique way to write u as a convex combination of u_i's: $\sum_i \lambda_i u_i = u$

- First period: $u = \sum_i \omega(i,1)u_i \Rightarrow \lambda_i = \omega(i,1)$

- Second period: $\delta u = \sum_i \omega(i,1)\delta_i u_i + \sum_i \omega(i,2)u_i \Rightarrow \lambda_i\delta = \omega(i,1)\delta_i + \omega(i,2)$

- $\omega(i,1)\delta = \omega(i,1)\delta_i + \omega(i,2)$

Gradual Transition of the Cutoff

- An individual's instantaneous utility function describes his risk attitude
- $(u^\theta)_{\theta=1}^{\Theta}$ is a linearly independent Θ-tuple of instantaneous utility functions—Θ generic types of risk attitude
  - $\Theta = 1$: $u_i = u$; $\Theta = N$: $(u_i)_{i \in N}$ is linearly independent
- Define

$$\delta^*_{\mathsf{maxmin}} := \max_{\theta} \min_{k \in \{i \in N : u_i = u^\theta\}} \delta^*_k.$$

Gradual Transition of the Cutoff

Theorem Suppose $T < +\infty$ and each generation-t individual i's discounting utility function has an instantaneous utility function $u_i \in \{u^\theta\}_{\theta=1}^{\Theta}$ for some linearly independent Θ-tuple of instantaneous utility functions $(u^\theta)_{\theta=1}^{\Theta}$ such that $\{u_i\}_{i \in N} = \{u^\theta\}_{\theta=1}^{\Theta}$, and has a discount function δ_i that satisfies A2 and A3. Let the planner's u be an arbitrary strict convex combination of $(u_i)_{i \in N}$. Then,

1. if $\delta > \delta^*_{\mathsf{maxmin}}$, the planner is are intergenerationally Pareto and strongly non-dictatorial;
2. for each $\delta < \delta^*_{\mathsf{maxmin}}$, there exists some $T^* > 0$ such that if $T \geq T^*$, the planner is not intergenerationally Pareto.