Structural Breaks in the Variance Process and the Pricing Kernel Puzzle

Main findings

- Structural breaks in the variance process (GARCH) can explain why many researchers find a puzzling S-shaped pricing kernel derived from S&P 500 option prices.
- A GARCH model with structural breaks leads to the estimation of only U-shaped pricing kernels. This is theoretically very useful, as the S-shaped is hard to explain rationally.
- The reason for the finding is that the standard GARCH model, that is typically used in the estimation, can not model extended periods of high or low volatility. It produces volatility forecasts that are biased towards the long-run mean.
- The empirical results can be explained economically by a variance-dependent pricing kernel, with structural breaks as a necessary component.

Introduction

- The pricing kernel is the central object of interest in asset pricing. It conveys valuable information about risks, prices of risks and preferences.
- A natural approximation is the projection of the pricing kernel on the market index.
- In the literature on empirical pricing kernels derived from option prices, puzzling shapes have been documented (stylized: S-shaped and U-shaped).
- These shapes are theoretically incompatible. The S-shaped can be explained only with behavioral biases, while rational explanations exists for the U-shape. Hence the empirical shape is very important for theory.

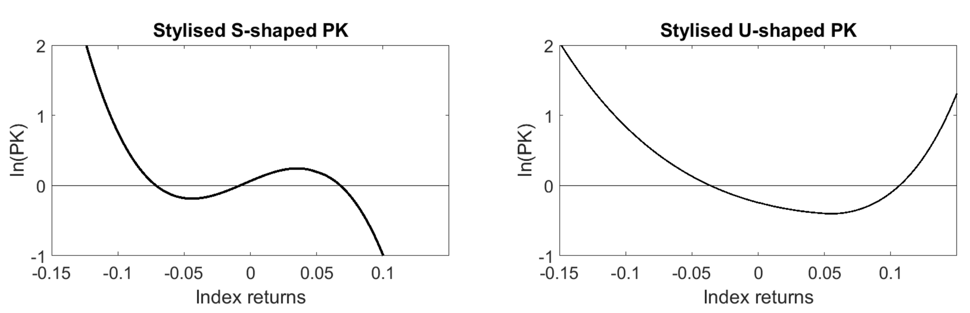

Illustration of S-shaped and U-shaped pricing kernel.

Aim

- Understand why researchers find different shapes of the pricing kernel
- Find the correct shape of the pricing kernel
- Understand the economics behind the shape

Method

- The pricing kernel is the ratio of risk-neutral to physical return density: dQ/dP.
- Risk-neutral return density can be obtained from option prices. Standard methods exist.
- For the physical return density, the key ingredient is the volatility forecast. Almost all studies use GARCH models for this.
- Volatility is time-varying and clustered. Standard GARCH forecasts can not capture the clustering.
- I propose a new GARCH model with structural breaks.
- Change-point Heston-Nandi GARCH model, dynamics:

$$ln\left(\frac{S_t}{S_{t-1}}\right) - r_t = \left(\mu_{y_t} - \frac{1}{2}\right)h_t + \sqrt{h_t}z_t,$$

$$h_t = \omega_{y_t} + \beta_{y_t}h_{t-1} + \alpha_{y_t}\left(z_{t-1} - \gamma_{y_t}\sqrt{h_{t-1}}\right)^2,$$

$z_t \sim N(0,1)$, y_t is an integer variable that numbers the regimes.

- I develop an algorithm to estimate the change-point model via maximum likelihood.

Data

- S&P 500 daily returns from 1.1.1992-31.8.2015.
- Five regimes identified. Break dates: 28.10.1996, 12.08.2003, 07.06.2007 and 29.11.2011.
- Options data on monthly S&P 500 index options from 1.1.1996-31.8.2015.

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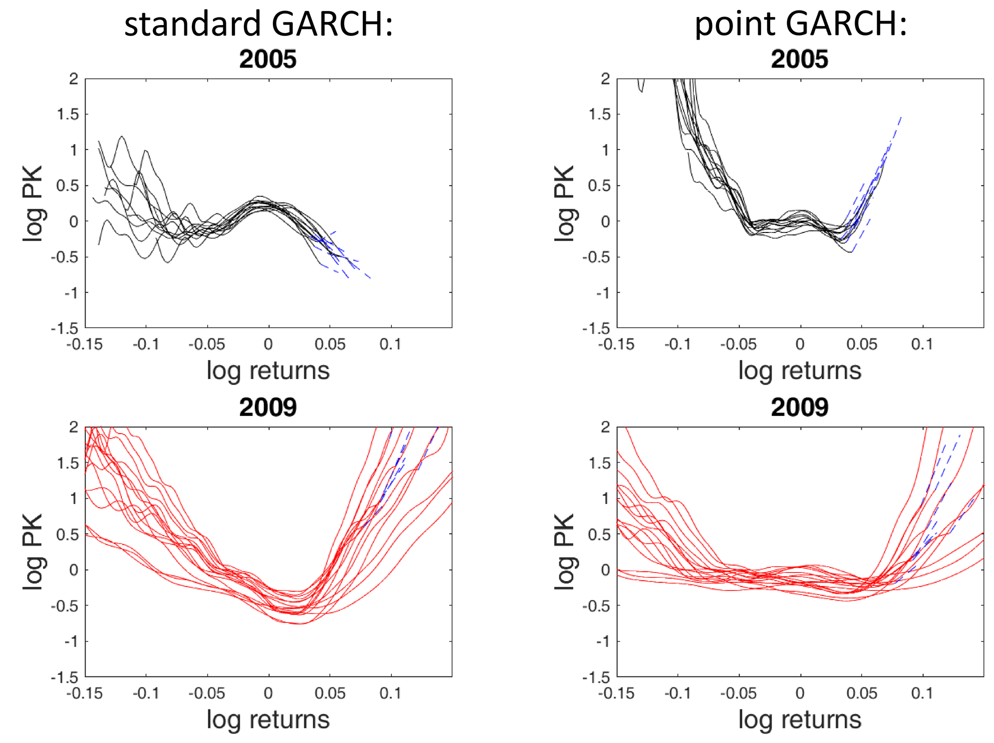
Results

- Standard estimation with standard GARCH finds S-shaped pricing kernels in times of low volatility, and U-shaped pricing kernels in times of high volatility.
- My estimation method with change-point GARCH finds U-shaped pricing kernel in the full sample.
- The results are illustrated for two representative years: 2005, a typical low volatility year, and 2009, a typical high volatility year.

Estimation results with standard GARCH: | Estimation results with change-point GARCH:



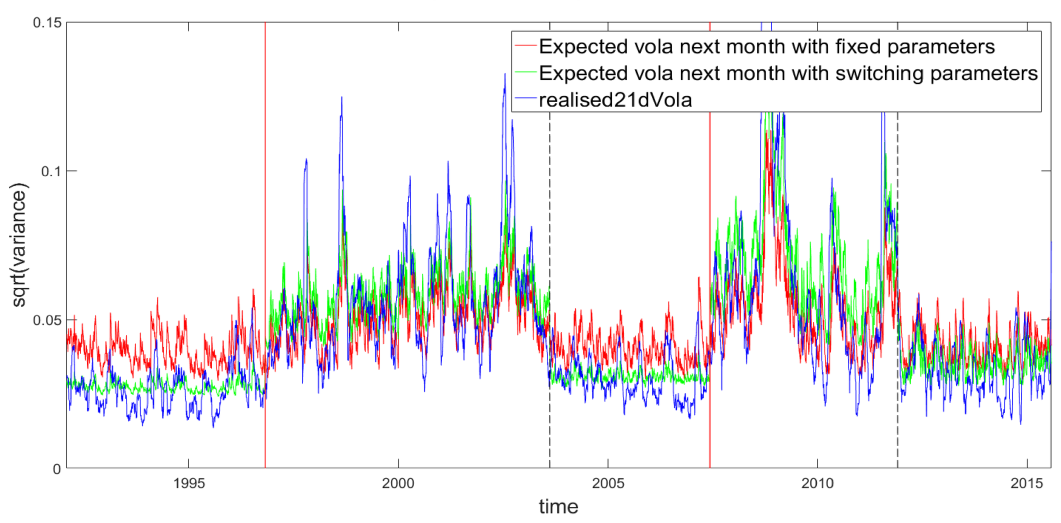
Results for two representative years. The plots show the natural logarithm of estimated pricing kernels. The horizon is one month.

- The results are robust to numerous variations in the method.

Explanation for results

The volatility forecasts of standard GARCH models are biased towards the long run mean. Therefore, they overestimate (underestimate) volatility significantly in times of low (high) volatility.

Predicted vs. realized 21 day volatility:



	'92-'96	'96-'03	'03-'07	'07-'11	'11-'15
Average realized 21d vola	0.0268	0.0573	0.0300	0.0683	0.0334
FP Avg. predicted 21d vola	0.0391	0.0511	0.0409	0.0543	0.0413
CP Avg. predicted 21d vola	0.0270	0.0586	0.0308	0.0701	0.0354

The figure shows the 21 day rolling window realized volatility, as well as the ex ante expected volatility implied by the FP and CP-GARCH model.
The table shows the average realized 21 day volatility across the different regimes, as well as the average ex ante predicted volatility by both the FP and CP-GARCH model.

Mechanism how volatility drives results

Only the volatility forecasts drive the results. In particular, the overestimated volatility in times of low volatility cause the estimation of S-shaped pricing kernels.

Oct 2005 (typical low volatility year):

Oct 2009 (typical high volatility year):

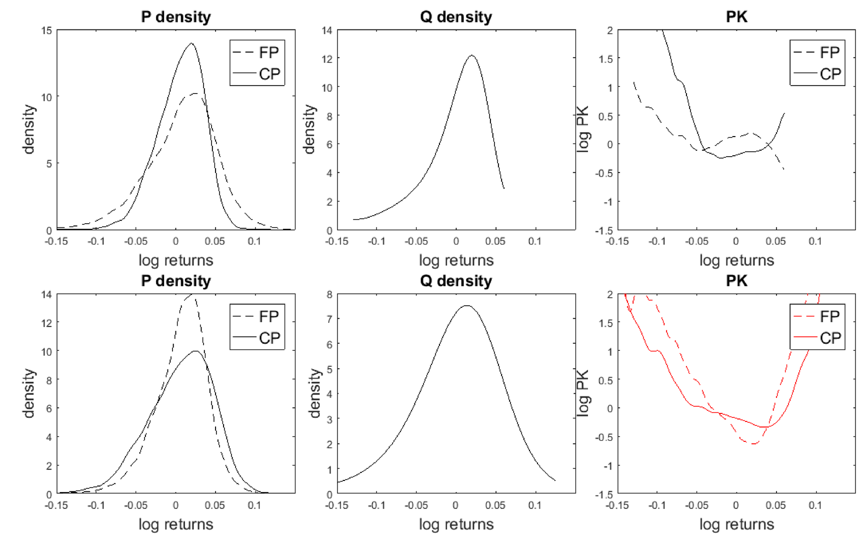


Illustration of main mechanism, how volatility estimates drive the results.

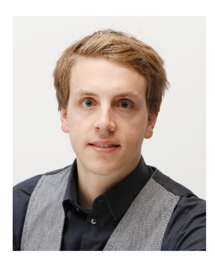
Model

- My new model is a modified version of the variance-dependent pricing kernel of Christoffersen, Heston & Jacobs (RFS, 2013). It explains the empirical results well.
- Pricing kernel is decreasing in returns and increasing in volatility, and projection on returns is U-shaped.
- Structural breaks are necessary to obtain a good model fit to the data.
- The breaks also considerably improve the option pricing fit.

Conclusion

- Inclusion of structural breaks in GARCH model is economically important in several settings.
- S-shaped pricing kernels appears to be the consequence of a measurement error due to a misspecification of the volatility process.
- Pricing kernel is likely to be U-shaped.
- Valid explanation: priced variance risk.

Contact



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