Verifiability and Fraud in a Dynamic Credence Goods Market*

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Abstract

Complementary to the existing literature that extensively studied credence goods markets in static settings, we develop a dynamic model in which a durable good breaks down stochastically after treatments, and the customer meets the expert recurrently. We assume that the minor treatment alleviates the symptom of the major problem but fails to cure it, increasing the future failure rate. In contrast to the literature, we show that the truth-telling equilibrium never exists under the verifiability assumption, because the standard equal-margin condition fails. In our dynamic setting, the expert has a stronger incentive to undertreat since undertreatment induces more future business. But on the other hand, the customer becomes less willing to pay for the minor treatment for fear of increased future payments. Therefore, depending on the relative magnitude of these two opposing forces, either undertreatment or overtreatment can emerge in equilibrium. Surprisingly, the expert's incentive to undertreat weakens as the increment of failure rates rises.

Keywords: Credence Goods, Verifiability, Dynamic Games
JEL Codes: C73, D82

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1 Introduction

In the credence goods market, the expert has superior knowledge about the customer's needs. And even after consuming the good, the customer learns nothing about whether the purchased quantity/quality is the most desirable for her. Medical treatments and car repair services are popular examples of credence goods. A medical doctor knows what the appropriate treatment is following a diagnosis, but the patient has no idea whether the prescribed treatment is the most suitable even when the symptom disappears after the treatment. A car mechanic knows whether replacing a nail or the whole engine is sufficient to fix the breakdown while the driver without any expertise has to rely entirely on the recommendation. Enjoying such an informational advantage, the expert is often suspected of behaving dishonestly in recommending inappropriate treatments (*undertreatment* or *overtreatment*) or charging high prices for simple procedures (*overcharging*), which leads to market inefficiencies.

It is observed that a consumer would repeatedly purchase credence goods because credence goods are often associated with durables, such as automobiles, computers, health, etc. A durable good tends to break down multiple times over its lifetime and thus maintenance services, credence goods, are frequently required. Referring back to the car mechanic example, the vehicle as a durable is supposed to break down again after the repair. And often times, as long as the car is good to drive, the driver would not know whether only the symptom is alleviated or the problem is fully fixed. For example, replacing a wire may still bring a seriously damaged car back to the street when fully fixing the problem requires the replacement of the whole engine. However, the car would more easily break down when it is undertreated, increasing the future demand for repair. In this paper, we explicitly model this dynamic incentive of the expert and investigate its implications on potential frauds.

The existing literature on credence goods since Darby and Karni (1973), who coined this term, typically models the interaction between the expert and the customer by a one-shot game, which we consider insufficient to capture the abovementioned dynamic incentive. Dulleck and Kerschbamer (2006) provided a comprehensive review and a unified framework of the one-shot models. They argued that under different assumptions, various types of expert's misbehavior may be of concern.[1] In particular, if liability rules are absent, under the *verifiability* assumption where the type of treatment is observable to the customer and verifiable to the court, *undertreatment* and *overtreatment* are potential misbehaviors. However, with the *commitment* assumption, i.e., the customer is committed to accepting the treatment recommendations, it was shown that market forces

[1]Emons (1997) showed the existence of equilibrium with nonfraudulent behavior under capacity constraint and verifiability assumption. Wolinsky (1993) examined the role of second opinion in disciplining the expert's behavior. Fong (2005) showed that liability does not necessarily imply frauds and that heterogeneity among customers can give rise to selective cheating.

successfully discipline the expert to be honest, because he would find it optimal to set an equal profit margin for both treatments and hence has no incentive to lie. This is also confirmed in Fong, Liu, and Wright (2014) who dropped the *commitment* assumption. Truth-telling coupled with equal-margin prices is always an equilibrium when prices are exogenous although it may not be the most profitable with endogenous prices. In contrast, the equal-margin condition falls apart in our model once we incorporate the dynamic incentive for undertreating, and thus the truth-telling equilibrium is completely destroyed even under exogenous prices. Intuitively, in our model, honestly treating the major problem requires a sufficiently higher profit margin for the major treatment to compensate for the loss of future business which would otherwise be induced through undertreatment. This dynamic incentive is absent in the static models without recurrent interactions between the expert and the customer.

Other papers studying the dynamics of the credence goods markets are mostly models of repeated games. Those models typically consist of a long-lived expert and a sequence of short-lived customers, and investigate how the reputation concern would prevent the expert from lying (e.g., Fong and Liu (2018), Fong, Liu, and Meng (2017)). Our model differs from those repeated games in that the expert and the customer are both long-lived, and more importantly, their frequency of interactions is affected by the expert's behavior. These differences are essential because they are the source of the expert's incentive for undertreatment that we consider as an important feature of the credence goods markets. This dynamic incentive is absent in either the static models or the repeated game models. The most related theory to this paper is Taylor (1995). Our model shares a similar setup with his, in which a durable good transits from some good state to a bad one, and that the time spent in a particular state is governed by an exponential distribution. But we focus on the potential fraudulent behaviors of the expert whereas Taylor (1995) essentially assumes away the expert's misbehavior by making liability assumptions and studies the ex-ante optimal choice of the maintenance contracts.

The rest of the paper is organized as follows. The dynamic model is introduced in the next section. We first describe the physical environment and then characterize the first-best policy in the absence of information asymmetry. Before characterizing the equilibria with information asymmetry, we examine whether the first-best policy is implementable. In the characterization of the equilibria, we first consider the case where the expert plays the stationary recommendation strategy and then relax this assumption to consider more general strategies. In Section 3, we discuss the case where personalized pricing is not possible. In the last section, we conclude.

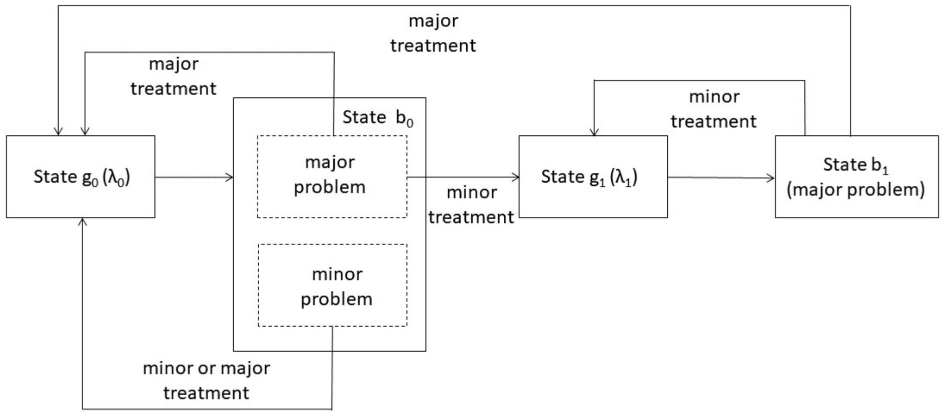


Figure 1: Dynamic Credence Goods Market

2 The Dynamic Model

2.1 Setup

We now set up a dynamic model in which the durable good stochastically breaks down after treatments and the customer recurrently interacts with the expert.

Time is continuous, starting at $\tau = 0$ and lasts forever. A durable good K starts its life at time $\tau = 0$ in a good state g_0 and generates a flow of benefits B for its owner until it breaks down (see Figure 1). The failure of Good K is stochastic. The time spent in State g_0 is governed by an exponential distribution with hazard rate λ_0. When it breaks down from State g_0, it enters a bad state b_0 in which the good suffers from either the major problem with probability α, or the minor problem with probability $1 - \alpha$. As in the standard static models, there are two types of treatments, a minor treatment and a major treatment. The major treatment fully fixes both problems, and brings Good K to State g_0. The minor treatment fully fixes the minor problem and returns Good K to State g_0, while it only partially fixes the major problem and the good transits to another good state g_1. In State g_1, the good produces the same instantaneous benefits B, and the time spent in this state is also exponentially distributed but with a higher hazard rate $\lambda_1 (> \lambda_0)$. If the good breaks down from g_1, it enters a bad state b_1 with the major problem for sure. Denote the state by $s \in \{g_0, b_0, g_1, b_1\}$.

In this main analysis, we consider the interaction between a monopolist expert and a representative customer.[2] Whenever Good K breaks down, the owner or the customer (she) does not know the exact problem, but she can bring it to the expert (he) who then diagnoses the problem and provides either treatment. If no treatment is performed, Good K stays in the bad state forever, providing no more services. The expert can

[2]Formally, there is a continuum of customers and the expert is able to inter-temporal price discriminate among the customers. In Section 3, we consider the case where inter-temporal price discrimination is not possible.

perfectly and costlessly diagnose the problem, but has to incur a cost of $\underline{c}$ for the minor treatment and $\bar{c}$ for the major treatment, where $\underline{c} < \bar{c}$. We assume that the performance of the treatment is observable to the customer and verifiable to the court (the *verfiability* assumption). In other words, the expert cannot charge the major treatment price while performing the minor treatment. Hence, overcharging is ruled out, but undertreatment and overtreatment remain to be potential sources of inefficiencies.

The timing of the dynamic game is the following:

0. At time $\tau = 0$, the durable good K starts its life in State g_0.

1. When it breaks down, the expert posts the price list $(\underline{p}, \bar{p})$. After observing the prices, the customer decides whether to visit the expert.

2. If the customer comes to visit the expert, the expert then diagnoses the problem, and decides whether to recommend a treatment or to refuse to treat at all.

3. If a recommendation is provided, the customer decides whether to accept or reject it. If she accepts, the customer pays the corresponding price and the expert performs the prescribed treatment. Good K returns to some good state according to the rule described above.

4. When Good K breaks down the next time, the same interactions (1, 2 & 3) ensue.

We assume that the customer and the expert share the same instantaneous discount rate r, and that both the customer and the expert have a full record of the customer's treatment history (including the number of treatments, the types of each treatment, and the time lapse between every two treatments).

In this dynamic setting, full warranty, such as a long-term maintenance contract, may solve the incentive problem. With full warranty, the expert has the correct incentive since he bears all the marginal costs whereas the prices are not to be paid upon each interaction. However, full warranty is prone to the moral hazard problem from the customer side. In the equilibrium with severe customer moral hazard, the customer may not want to purchase the warranty. In our model, we do not explicitly model warranties and customer moral hazard, but consider the case where warranties are not provided or not purchased. Our focus is on the price mechanism alone.

Equilibrium Concept. The player's strategy space allows for history dependent mixed strategies. Denote the history until the n^{th} breakdown by $h_n = \{(\tau_1, \theta_1), (\tau_2, \theta_2), ..., (\tau_n, \theta_n)\}$, where θ_i is the treatment type of the i^{th} treatment, and τ_i is the time lapse between the i^{th} and the $(i-1)^{th}$ treatments. Let (γ_m, γ_M) denote the probabilities of accepting the minor and the major treatment recommendations respectively by the customer, and let (β_m, β_M) denote the probabilities of recommending the major treatment by the expert given the minor problem and the major problem respectively. All these probabilities as well as the price list $(\underline{p}, \bar{p})$ are functions of the history h_n. Let μ_n be the customer's belief of Good K being in State g_1 at the n^{th} breakdown, and let μ'_n be the post-treatment belief. Following the literature on dynamic games, we adopt *Perfect Markovian Equilibrium* as the equilibrium concept and consider strategies contingent on the customer's belief μ'_n. The law of motion of the customer's belief can be described by

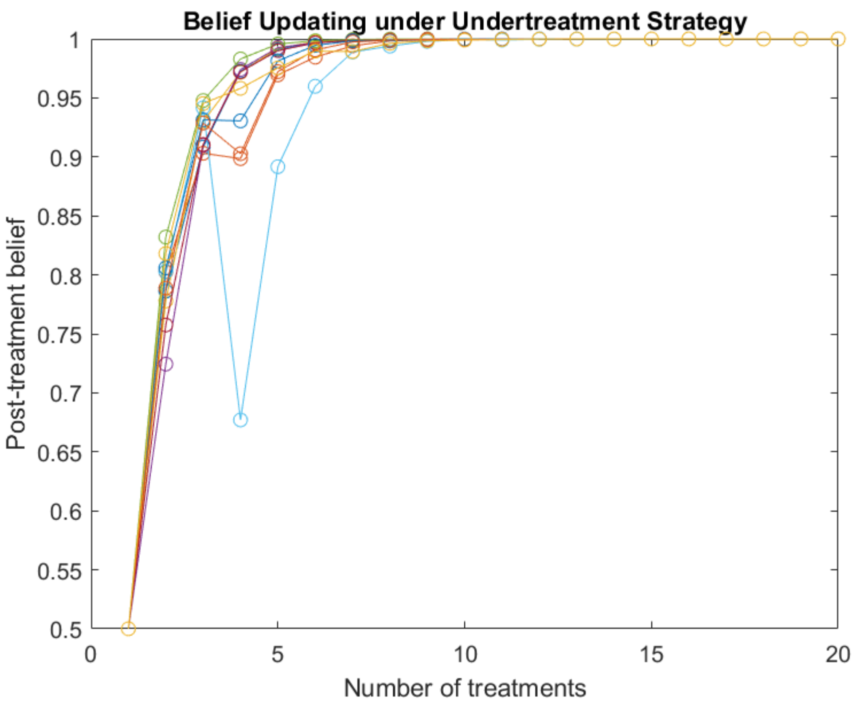
$$\mu'_n = \begin{cases} 0, & \text{if } \theta_n = \text{major treatment}, \\ \mu'_{m,n} = \frac{(\mu_n+(1-\mu_n)\alpha)(1-\beta_M(\mu_n))}{(\mu_n+(1-\mu_n)\alpha)(1-\beta_M(\mu_n))+(1-\mu_n)(1-\alpha)(1-\beta_m(\mu_n))}, & \text{if } \theta_n = \text{minor treatment}, \end{cases} \tag{2.1}$$

and

$$\mu_{n+1} = \frac{\mu'_n \lambda_1 e^{-\lambda_1 \tau_{n+1}}}{\mu'_n \lambda_1 e^{-\lambda_1 \tau_{n+1}} + (1-\mu'_n)\lambda_0 e^{-\lambda_0 \tau_{n+1}}}, \tag{2.2}$$

with initial conditions $\mu'_0 = 0$ and $\mu_1 = 0$.

The following graph depicts sample paths of beliefs when the expert always recommends the minor treatment, i.e., the expert follows the undertreatment strategy.



parameter values: $\alpha = 0.5, \lambda_0 = 1, \lambda_1 = 2$

Note that the customer's belief converges to $\mu'_\infty = 1$ but there are places where non-monotonicity is observed. Formally, non-monotonicity occurs when $\mu'_{n+1} < \mu'_n$, i.e.,

$$\begin{aligned}
& \mu_{n+1} + (1-\mu_{n+1})\alpha < \mu'_n \\
\Rightarrow\ & \frac{\mu'_n \lambda_1 e^{-\lambda_1 \tau_{n+1}}}{\mu'_n \lambda_1 e^{-\lambda_1 \tau_{n+1}} + (1-\mu'_n)\lambda_0 e^{-\lambda_0 \tau_{n+1}}}(1-\alpha) + \alpha < \mu'_n \\
\Rightarrow\ & \tau_{n+1} > \frac{1}{\lambda_1 - \lambda_0} \ln \Big[\frac{\lambda_1}{\lambda_0}\frac{\mu'_n}{\mu'_n - \alpha}\Big]. \qquad (2.3)
\end{aligned}$$

That is, when the time lapse between two treatments are sufficiently large, the belief reversal happens. This is an intuitive result since after a long enough time lapse, the customer would be convinced that the state is more likely to be g_0.

The customer's expected payoff after the n^{th} treatment, given her post-treatment belief μ'_n, is given by

$$U(\mu'_n) = \mu'_n U_1(\mu'_n) + (1-\mu'_n) U_0(\mu'_n), \qquad (2.4)$$

where $U_0(\mu'_n)$ is the expected payoff with the belief μ'_n in State g_0:

$$\begin{aligned}
U_0(\mu'_n) = \int_0^\infty \Big\{ & (1-e^{r\tau_{n+1}})\frac{B}{r} \\
& + e^{-r\tau_{n+1}}\Big[\alpha\big(\beta_M \gamma_M (U_0(0) - \bar{p}) + (1-\beta_M)\gamma_m (U_1(\mu'_{m,n+1}) - \underline{p})\big) \\
& + (1-\alpha)\big(\beta_m \gamma_M (U_0(0) - \bar{p}) + (1-\beta_m)\gamma_m (U_0(\mu'_{m,n+1}) - \underline{p})\big)\Big]\Big\}\lambda_0 e^{-\lambda_0 \tau_{n+1}} d\tau_{n+1},
\end{aligned} \qquad (2.5)$$

and $U_1(\mu'_n)$ is the expected payoff with the belief μ'_n in State g_1:

$$\begin{aligned}
U_1(\mu'_n) = \int_0^\infty \Big\{ & (1-e^{r\tau_{n+1}})\frac{B}{r} + e^{-r\tau_{n+1}}\Big[\beta_M \gamma_M (U_0(0) - \bar{p}) \\
& + (1-\beta_M)\gamma_m (U_1(\mu'_{m,n+1}) - \underline{p})\Big]\Big\}\lambda_1 e^{-\lambda_1 \tau_{n+1}} d\tau_{n+1}.
\end{aligned} \qquad (2.6)$$

Thus, the customer's ex-ante expected payoff is given by $U(\mu'_0) = U_0(0)$.

Similarly, the expert's expected payoff is given by

$$\Pi(\mu'_n) = \mu'_n \Pi_1(\mu'_n) + (1-\mu'_n)\Pi_0(\mu'_n), \qquad (2.7)$$

where

$$\begin{aligned}
\Pi_0(\mu'_n) = \int_0^\infty & e^{-r\tau_{n+1}}\Big\{\alpha\big[\beta_M \gamma_M \big(\bar{p} - \bar{c} + \Pi_0(0)\big) + (1-\beta_M)\gamma_m \big(\underline{p} - \underline{c} + \Pi_1(\mu'_{m,n+1})\big)\big] \\
& + (1-\alpha)\big[\beta_m \gamma_M \big(\bar{p} - \bar{c} + \Pi_0(0)\big) + (1-\beta_m)\gamma_m \big(\underline{p} - \underline{c} + \Pi_0(\mu'_{m,n+1})\big)\big]\Big\}\lambda_0 e^{-\lambda_0 \tau_{n+1}} d\tau_{n+1},
\end{aligned} \qquad (2.8)$$

and

$$\Pi_1(\mu'_n) = \int_0^\infty e^{-r\tau_{n+1}} \Big\{ \beta_M \gamma_M \big(\bar{p} - \bar{c} + \Pi_0(0)\big) + (1-\beta_M)\gamma_m \big(\underline{p} - \underline{c} + \Pi_1(\mu'_{m,n+1})\big) \Big\} \lambda_1 e^{-\lambda_1 \tau_{n+1}} d\tau_{n+1}. \tag{2.9}$$

2.2 The First-Best Policy

Before investigating the interactions between the expert and the customer, we derive the first-best policy where there is no information asymmetry between the two players, or equivalently, we consider the case where the customer can diagnose the problem and perform the treatments by herself. We make the following two assumptions on parameter values:

Assumption 1. $B \geq \max\{(r + \lambda_1)\underline{c}, (r + \lambda_0)\bar{c}\}$.

Assumption 2. $\underline{c} < \bar{c} < \left(1 + \frac{\lambda_1 - \lambda_0}{r + \alpha\lambda_0}\right) \underline{c}$.

Assumption 1 means that the services from Good K are sufficiently valuable. This condition ensures that any problem is worth fixing, even using the inappropriate treatment. It also guarantees the feasibility of all strategies analyzed in the next section. Assumption 2 ensures that the relative cost of the major treatment is not too high, making it suboptimal to undertreat the major problem. Intuitively, performing the major treatment on the major problem saves a substantial amount of future expenditures which would otherwise be induced by undertreating, and the benefit of future cost savings would dominate when the major treatment is only moderately more costly. Hence, under these two conditions, which will be assumed throughout the paper, performing appropriate treatments yields the highest expected surplus. The following proposition formalizes the result.

Proposition 1. *Suppose Assumptions 1 and 2 hold, performing appropriate treatments is the first-best solution. The maximal social surplus in flow terms is* $rW^* = B - \lambda_0 [\alpha\bar{c} + (1 - \alpha)\underline{c}]$.

Proof. We compare the ex-ante expected surpluses achieved by all possible treatment plans. There are nine treatment plans in total: one plan of treating neither problem, four plans for treating only one type of problem (e.g., treating only the major problem with the minor treatment), and four plans for treating both types of problem (e.g., treating the major problem with the major treatment and the minor problem with the minor treatment). We denote the ex-ante expected surplus of each treatment plan as:

Notation	Treatment Plan
V^n	Treating neither problem
V_M^M	Treating only major problem with major treatment
V_m^M	Treating only major problem with minor treatment
V_M^m	Treating only minor problem with major treatment
V_m^m	Treating only minor problem with minor treatment
V^t	Treating major problem with major treatment and minor problem with minor treatment
V^o	Treating major problem with major treatment and minor problem with major treatment
V^u	Treating major problem with minor treatment and minor problem with minor treatment
V^f	Treating major problem with minor treatment and minor problem with major treatment

It is clear that $V_m^m > V_M^m$ because both treatments fully fix the minor problem but the minor treatment is less costly. For the same reason, $V^u > V^f$ and $V^t > V^o$. Hence, it suffices to consider the remaining six treatment plans.

Treating neither problem yields an expected surplus equal to the expected value of services produced by Good K until the first time it breaks down. In flow terms, it is given by

$$rV^n = r\int_0^\infty \left\{(1-e^{-r\tau})\frac{B}{r}\right\}\lambda_0 e^{-\lambda_0\tau}d\tau = \frac{r}{r+\lambda_0}B, \tag{2.10}$$

where the term in the curly brackets is the present value of services produced by Good K until it fails at time τ.

The expected surplus of treating only the major problem with the major treatment can be written recursively as

$$V_M^M = \int_0^\infty \left\{(1-e^{-r\tau})\frac{B}{r} + e^{-r\tau}\alpha\left(V_M^M - \bar{c}\right)\right\}\lambda_0 e^{-\lambda_0\tau}d\tau, \tag{2.11}$$

where the second term in the curly brackets is the present expected continuation value when Good K breaks down at time τ. Note that only the major problem is treated, and this happens with probability α. Performing the major treatment incurs a cost of $\bar{c}$ and returns Good K to State g_0, which generates a surplus of V_M^M again. The above equation can be solved for

$$rV_M^M = \frac{r}{r+(1-\alpha)\lambda_0}B - \frac{r\lambda_0}{r+(1-\alpha)\lambda_0}\alpha\bar{c}. \tag{2.12}$$

Similarly, we have

$$rV_m^m = \frac{r}{r+\alpha\lambda_0}B - \frac{r\lambda_0}{r+\alpha\lambda_0}(1-\alpha)\underline{c}. \tag{2.13}$$

Now consider treating only the major problem with the minor treatment. The expected surplus can be written as

$$V_m^M = \int_0^\infty \left\{(1-e^{-r\tau})\frac{B}{r} + e^{-r\tau}\alpha\left(\tilde{V}_m^M - \underline{c}\right)\right\}\lambda_0 e^{-\lambda_0\tau}d\tau. \tag{2.14}$$

The continuation value is now the expected surplus in State g_1 minus the cost of the minor treatment, $\tilde{V}_m^M - \underline{c}$. Since Good K remains in State g_1 when the minor treatment is applied repeatedly, we can write $\tilde{V}_m^M$ recursively as

$$\tilde{V}_m^M = \int_0^\infty \left\{ (1-e^{-r\tau})\frac{B}{r} + e^{-r\tau}\left(\tilde{V}_m^M - \underline{c}\right) \right\} \lambda_1 e^{-\lambda_1 \tau} d\tau. \tag{2.15}$$

Combining the above two equations, one can obtain

$$rV_m^M = \frac{r+\alpha\lambda_0}{r+\lambda_0}B - \frac{\lambda_0(r+\lambda_1)}{r+\lambda_0}\alpha\underline{c}. \tag{2.16}$$

Now we turn to treatment plans in which both types of problems are treated. As argued above, we only need to calculate V^t and V^u since V^o is dominated by V^t and V^f by V^u. As appropriate treatments return Good K to State g_0, V^t can be written recursively as

$$V^t = \int_0^\infty \left\{ (1-e^{-r\tau})\frac{B}{r} + e^{-r\tau}\left[V^t - \alpha\bar{c} - (1-\alpha)\underline{c}\right] \right\} \lambda_0 e^{-\lambda_0 \tau} d\tau, \tag{2.17}$$

which can be solved for

$$rV^t = B - \lambda_0\left[\alpha\bar{c} + (1-\alpha)\underline{c}\right]. \tag{2.18}$$

In contrast, treating both problems with the minor treatment could possibly transfer Good K into State g_1 and it would never return to State g_0. We denote the expected surplus in State g_1 by $\tilde{V}^u$, which can be written as

$$\tilde{V}^u = \int_0^\infty \left\{ (1-e^{-r\tau})\frac{B}{r} + e^{-r\tau}\left(\tilde{V}^u - \underline{c}\right) \right\} \lambda_1 e^{-\lambda_1 \tau} d\tau. \tag{2.19}$$

And the expected surplus in State g_0 is given by

$$V^u = \int_0^\infty \left\{ (1-e^{-rt})\frac{B}{r} + e^{-r\tau}\left[(1-\alpha)V^u + \alpha\tilde{V}^u - \underline{c}\right] \right\} \lambda_0 e^{-\lambda_0 \tau} d\tau. \tag{2.20}$$

Combining the above two equations yields

$$rV^u = B - \frac{(r+\alpha\lambda_1)\lambda_0}{r+\alpha\lambda_0}\underline{c}. \tag{2.21}$$

We calculate the difference in surpluses between providing appropriate treatments and every other treatment plan:

$$rV^t - rV^u = \alpha\lambda_0\left[\left(1 + \frac{\lambda_1-\lambda_0}{r+\alpha\lambda_0}\right)\underline{c} - \bar{c}\right], \tag{2.22}$$

$$rV^t - rV_m^m = \frac{\alpha\lambda_0}{r+\alpha\lambda_0}\left\{B - \left[r\bar{c} + \lambda_0(\alpha\bar{c} + (1-\alpha)\underline{c})\right]\right\} \equiv \frac{\alpha\lambda_0}{r+\alpha\lambda_0}(B - B_1), \tag{2.23}$$

$$rV^t - rV_M^M = \frac{(1-\alpha)\lambda_0}{r+(1-\alpha)\lambda_0}\{B - [r\underline{c} + \lambda_0(\alpha\bar{c} + (1-\alpha)\underline{c})]\} \equiv \frac{(1-\alpha)\lambda_0}{r+(1-\alpha)\lambda_0}(B - B_2), \tag{2.24}$$

$$rV^t - rV_m^M = \frac{(1-\alpha)\lambda_0}{r+\lambda_0}\left\{B - \left[\frac{r+\lambda_0}{1-\alpha}(\alpha\bar{c} + (1-\alpha)\underline{c}) - \frac{\alpha(r+\lambda_1)}{1-\alpha}\underline{c}\right]\right\} \equiv \frac{(1-\alpha)\lambda_0}{r+\lambda_0}(B - B_3), \tag{2.25}$$

$$rV^t - rV^n = \frac{\lambda_0}{r+\lambda_0}\{B - (r+\lambda_0)(\alpha\bar{c} + (1-\alpha)\underline{c})\} \equiv \frac{\lambda_0}{r+\lambda_0}(B - B_4). \tag{2.26}$$

We then show that all these differences are greater than zero.

Note first that Assumption 2 immediately implies $rV^t > rV^u$.

Also note that $B_1 > B_2$, and $B_1 - B_4 = r(1-\alpha)(\bar{c} - \underline{c}) > 0$. It then suffices to prove $\max\{(r+\lambda_1)\underline{c}, (r+\lambda_0)\bar{c}\} \geq \max\{B_1, B_3\}$. In fact, we can show that $\min\{(r+\lambda_1)\underline{c}, (r+\lambda_0)\bar{c}\} > \max\{B_1, B_3\}$ under Assumption 2:

$$B_1 - (r+\lambda_1)\underline{c} = (r+\alpha\lambda_0)(\bar{c} - \underline{c}) - (\lambda_1 - \lambda_0)\underline{c} < 0; \tag{2.27}$$

$$B_1 - (r+\lambda_0)\bar{c} = -\lambda_0(1-\alpha)(\bar{c} - \underline{c}) < 0; \tag{2.28}$$

$$B_3 - (r+\lambda_1)\underline{c} = \frac{\alpha(r+\lambda_0)}{1-\alpha}(\bar{c} - \underline{c}) - \frac{\lambda_1 - \lambda_0}{1-\alpha}\underline{c} \leq -\frac{r(\lambda_1 - \lambda_0)}{(r+\alpha\lambda_0)}\underline{c} < 0; \tag{2.29}$$

$$B_3 - (r+\lambda_0)\bar{c} = \frac{\alpha}{1-\alpha}[(r+\lambda_0)\bar{c} - (r+\lambda_1)\underline{c}] - (r+\lambda_0)(\bar{c} - \underline{c}) \leq -[r + (1-\alpha)\lambda_0](\bar{c} - \underline{c}) < 0, \tag{2.30}$$

where the last line uses the fact that $(r+\lambda_0)\bar{c} - (r+\lambda_1)\underline{c} \leq (1-\alpha)\lambda_0(\bar{c} - \underline{c})$, which is implied by Assumption 2.

Note that if it is optimal to undertreat the major problem for at least one time, then it would be optimal to always undertreat. Hence, it is sufficient here to consider the treatment plan of always performing the minor treatment.

Therefore, performing appropriate treatments yields the highest ex-ante expected surplus under Assumptions 1 and 2. □

2.3 Impossibility of First-Best

In this subsection, we show that in the decentralized problem, the First-Best policy where appropriate treatment is performed is never an equilibrium outcome.

Proposition 2. *There exists no "truth-telling equilibrium" in which the expert plays the truth-telling strategy.*

Proof. Suppose to the contrary that such an equilibrium exists. Then the expert strategy is $\beta_M = 1, \beta_m = 0, \forall \mu_n$. By the evolution of the customer's belief, we have $\mu_n = \mu_n' = 0, \forall n$. Then we can simply write the expert's pricing strategy and the customer's strategy as $\bar{p}(\mu_n) = \bar{p}, \underline{p}(\mu_n) = \underline{p}$ and $\gamma_M(\mu_n) = \gamma_M, \gamma_m(\mu_n) = \gamma_m, \forall \mu_n$ respectively.

Consider the expert incentive compatibility constraints conditioning on the major problem. His equilibrium payoff is

$$\Pi_{\text{on}}^t = \gamma_M(\bar{p} - \bar{c} + \Pi_0^t), \tag{2.31}$$

where

$$\Pi_0^t = \int_0^\infty e^{-r\tau}\left\{\alpha\gamma_M\left(\bar{p} - \bar{c} + \Pi_0^t\right) + (1-\alpha)\gamma_m\left(\underline{p} - \underline{c} + \Pi_0^t\right)\right\}\lambda_0 e^{-\lambda_0\tau}d\tau, \tag{2.32}$$

which can be solved for

$$\Pi_0^t = \frac{\alpha\lambda_0\gamma_M}{\alpha\lambda_0(1-\gamma_M)+r}(\bar{p} - \bar{c}) + \frac{(1-\alpha)\lambda_0}{\alpha\lambda_0(1-\gamma_M)+r}(\underline{p} - \underline{c}). \tag{2.33}$$

Consider a one-shot deviation and his off-equilibrium payoff is given by

$$\Pi_{\text{off}}^t = \gamma_m(\underline{p} - \underline{c} + \Pi_1^t), \tag{2.34}$$

where

$$\Pi_1^t = \int_0^\infty e^{-r\tau}\left\{\gamma_M\left(\bar{p} - \bar{c} + \Pi_0^t\right)\right\}\lambda_1 e^{-\lambda_1\tau}d\tau = \frac{\lambda_1\gamma_M}{r+\lambda_1}(\bar{p} - \bar{c} + \Pi_0^t). \tag{2.35}$$

Hence, the incentive compatibility for truth-telling requires $\Pi_{\text{on}}^t \geq \Pi_{\text{off}}^t$, i.e.,

$$\begin{aligned} &\gamma_M\left[(\lambda_1 + r) - \lambda_1\gamma_m\right]\left[\lambda_0 + r - (1-\alpha)\gamma_m\lambda_0\right](\bar{p} - \bar{c}) \\ \geq&\gamma_m\left\{\left[(1-\gamma_M)\lambda_0 + r\right](\lambda_1 + r) - (1-\alpha)\gamma_m\lambda_0\left[\lambda_1(1-\gamma_M) + r\right]\right\}\left(\underline{p} - \underline{c}\right). \end{aligned} \tag{2.36}$$

Similarly, when facing the minor problem, the expert's equilibrium payoff is

$$\Pi_{\text{on}}^t = \gamma_m(\underline{p} - \underline{c} + \Pi_0^t), \tag{2.37}$$

and his off-equilibrium payoff is

$$\Pi_{\text{off}}^t = \gamma_M(\bar{p} - \bar{c} + \Pi_0^t). \tag{2.38}$$

Hence the incentive compatibility requires that $\Pi_{\text{on}}^t \geq \Pi_{\text{off}}^t$, i.e.,

$$\gamma_m\left[\lambda_0(1-\gamma_M) + r\right]\left(\underline{p} - \underline{c}\right) \geq \gamma_M\left[\lambda_0(1-\gamma_m) + r\right](\bar{p} - \bar{c}). \tag{2.39}$$

We now show that conditions (2.36) and (2.39) are not compatible. Note that γ_M and γ_m cannot be zero simultaneously, otherwise, the customer always rejects recommendations and the market breaks down. Suppose first that $\gamma_M > 0$, then we can write (2.36) as

$$(\bar{p}-\bar{c}) \geq \frac{\gamma_m \{[(1-\gamma_M)\lambda_0 + r](\lambda_1 + r) - (1-\alpha)\gamma_m\lambda_0[\lambda_1(1-\gamma_M)+r]\}}{\gamma_M[(\lambda_1+r)-\lambda_1\gamma_m][\lambda_0+r-(1-\alpha)\gamma_m\lambda_0]}(\underline{p}-\underline{c}) \equiv \Phi_1(\underline{p}-\underline{c}), \tag{2.40}$$

and (2.39) as

$$(\bar{p}-\bar{c}) \leq \frac{\gamma_m[\lambda_0(1-\gamma_M)+r]}{\gamma_M[\lambda_0(1-\gamma_m)+r]}(\underline{p}-\underline{c}) \equiv \Phi_2(\underline{p}-\underline{c}). \tag{2.41}$$

But

$$\Phi_1 - \Phi_2 = \frac{\gamma_m^2[(1-\alpha)(1-\gamma_m)\lambda_0 r + \alpha(1-\gamma_M)\lambda_0 r + r^2](\lambda_1 - \lambda_0)}{\gamma_M[(\lambda_1+r)-\lambda_1\gamma_m][\lambda_0+r-(1-\alpha)\gamma_m\lambda_0][\lambda_0(1-\gamma_m)+r]}, \tag{2.42}$$

which is equal to zero if $\gamma_m = 0$ and greater than zero if $\gamma_m > 0$. If $\gamma_m = 0$, then the customer rejects the minor treatment recommendation, and the expert sets $\bar{p} = \bar{c}$ and earns zero profits. If $\gamma_m > 0$, then the expert sets $\underline{p} = \underline{c}$ and $\bar{p} = \bar{c}$ which again yields zero profits.

The case when $\gamma_m > 0$ follows similarly. □

Intuitively, the impossibility result is due to the fact that the incremental failure rate adds to the attractiveness of the minor treatment and thus the equal-margin condition ceases to be incentive compatible. In order for the expert to honestly recommend the minor treatment to the minor problem, same as in the standard static models, it requires the minor margin to be weakly higher than the major margin. However, in order for the expert to honestly recommend the major treatment to the major problem, it requires not just a weakly larger profit margin for the major treatment but a sufficiently larger one. This is because undertreating the major problem increases the failure rate of Good K ($\lambda_1 > \lambda_0$) and brings more future business for the expert. For the expert to behave honestly, the profit margin of the major treatment has to compensate for the reduction of future revenues. Therefore, the usual equal-margin condition does not guarantee the honest behavior of the expert in our model. And no prices can satisfy the above two incentive constraints simultaneously. Thus we conclude that there exists no truth-telling equilibrium, and that the first-best cannot be achieved.

2.4 Equilibria with Stationary Pure Recommendation Strategy

In this subsection, we focus on the stationary pure recommendation strategy where the expert plays the same pure recommendation strategy in each interaction.

There are four such recommendation strategies: truth-telling (t), overtreatment (o), undertreatment (u) and falsehood (f). The expert recommends the appropriate treatment with the truth-telling strategy, i.e., the minor treatment for the minor problem and the major treatment for the major problem. Under the overtreatment strategy, the expert recommends the major treatment regardless of the problem types. Conversely, under the undertreatment strategy, the expert always recommends the minor treatment. Finally, under the falsehood strategy, the expert recommends the minor treatment for the major problem and the major treatment for the minor problem. We denote the strategy set as $\sigma \in \{t, o, u, f\}$. Let $\Pi^\sigma(\tilde{\Pi}^\sigma)$ be the expert's expected profit in State $g_0(g_1)$ under strategy σ, and $U^\sigma(\tilde{U}^\sigma)$ be the corresponding expected payoff of the customer.

In the previous subsection, we ruled out the truth-telling strategy (t). We then turn to characterize the equilibria in which the expert plays each of the remaining three recommendation strategies. And we name these equilibria after the corresponding strategy.

Lemma 1. *There exists an "overtreatment equilibrium" in which the expert sets the price list* ($\underline{p} > \frac{1}{r+\lambda_1}B, \bar{p} = \frac{1}{r+\lambda_0}B$) *in each interaction, and plays the overtreatment strategy (o). The customer always accepts the recommendation. The expert's expected profit is* $r\Pi^o = \frac{\lambda_0}{r+\lambda_0}B - \lambda_0\bar{c}$. *The customer's expected payoff is* $rU^o = \frac{r}{r+\lambda_0}B$. *The social surplus is* $rW^o = B - \lambda_0\bar{c}$.

Proof. See Appendix. □

Lemma 1 characterizes "overtreatment equilibrium" where expert always recommends the major treatment. Knowing that the problems are overtreated and that Good K always returns to State g_0, the customer's maximum willingness to pay is the expected present value of services provided by Good K in State g_0, or

$$\bar{U}_0 = \int_0^\infty \left\{(1 - e^{-r\tau})\frac{B}{r}\right\} \lambda_0 e^{-\lambda_0 \tau} d\tau = \frac{1}{r + \lambda_0}B. \tag{2.43}$$

This is also the customer's expected payoff if she never consults the expert, and we call it her reservation value in g_0. Therefore, to attract the customer by playing the overtreatment strategy, the price $\bar{p}$ at each interaction must not exceed this reservation value. The expert's expected profit is equal to the profit margin of the major treatment multiplied by the frequency of the customer's visits represented by the failure rate λ_0. The maximal profit is thus obtained by setting $\bar{p}$ to the highest willingness to pay. Consequently, the customer only retains the expected value of services produced by Good K until its first breakdown, which is the lowest payoff she could obtain regardless of the expert's strategy. This equilibrium is supported by the customer's off-equilibrium belief that Good K had the major problem if the minor treatment was ever recommended, and that the expert would thereafter always recommend the minor treatment. Given this belief, the condition $\underline{p} > \frac{1}{r+\lambda_1}B$ guarantees that the recommendation of the minor treatment would be rejected by the customer as the price exceeds the reservation value in State g_1.

Lemma 2. *There exists a "falsehood equilibrium" in which the expert plays the falsehood strategy (f) and the customer always accepts the recommendation. The optimal price list is the same across time. With different parameter values, the price lists vary.*

- *Case I:* $\bar{c} - \underline{c} < \frac{\lambda_1 - \lambda_0}{r+\lambda_0}\underline{c}$.

  The optimal prices are

$$\bar{p} = \frac{1}{r+\lambda_0}B + \left(\bar{c} - \frac{r+\lambda_1}{r+\lambda_0}\underline{c}\right), \ \underline{p} = \frac{1}{r+\lambda_1}B. \tag{2.44}$$

  The expert's expected profit is

$$r\Pi^f = \frac{\lambda_0}{r+\lambda_0}B - \frac{\lambda_0(r+\lambda_1)}{r+\lambda_0}\underline{c}. \tag{2.45}$$

  The customer's expected payoff is

$$rU^f = \frac{r}{r+\lambda_0}B + \frac{r\lambda_0(1-\alpha)}{r+\alpha\lambda_0}\left(\frac{r+\lambda_1}{r+\lambda_0}\underline{c} - \bar{c}\right). \tag{2.46}$$

- *Case II:* $\frac{\lambda_1 - \lambda_0}{r+\lambda_0}\underline{c} \leq \bar{c} - \underline{c} \leq \frac{\lambda_1 - \lambda_0}{(r+\lambda_0)(r+\lambda_1)}B$.

  The optimal prices are

$$\bar{p} = \frac{1}{r+\lambda_0}B, \ \underline{p} = \frac{1}{r+\lambda_1}B. \tag{2.47}$$

  The expert's expected profit is

$$r\Pi^f = \frac{\lambda_0}{r+\lambda_0}B - \frac{r\lambda_0(1-\alpha)}{r+\alpha\lambda_0}\bar{c} - \frac{\alpha\lambda_0(r+\lambda_1)}{r+\alpha\lambda_0}\underline{c}. \tag{2.48}$$

  The customer's expected payoff is

$$rU^f = \frac{r}{r+\lambda_0}B. \tag{2.49}$$

  In this case, the expert obtains his highest possible profit by the falsehood strategy since the customer receives her lowest possible payoff.

- *Case III:* $\bar{c} - \underline{c} > \frac{\lambda_1 - \lambda_0}{(r+\lambda_0)(r+\lambda_1)}B$.

  The optimal prices are

$$\begin{aligned} \bar{p} &= \frac{r+\alpha\lambda_0}{r(r+\lambda_0)+\alpha\lambda_0(r+\lambda_1)}B + \frac{\alpha\lambda_0(r+\lambda_1)}{r(r+\lambda_0)+\alpha\lambda_0(r+\lambda_1)}(\bar{c} - \underline{c}), \\ \underline{p} &= \frac{r+\alpha\lambda_0}{r(r+\lambda_0)+\alpha\lambda_0(r+\lambda_1)}B - \frac{r(r+\lambda_0)}{r(r+\lambda_0)+\alpha\lambda_0(r+\lambda_1)}(\bar{c} - \underline{c}). \end{aligned} \tag{2.50}$$

  The expert's expected profit is

$$r\Pi^f = \frac{\lambda_0(r+\alpha\lambda_1)}{r(r+\lambda_0)+\alpha\lambda_0(r+\lambda_1)}\left[B - \frac{r(r+\lambda_0)}{r+\alpha\lambda_0}\bar{c} - \frac{\alpha\lambda_0(r+\lambda_1)}{r+\alpha\lambda_0}\underline{c}\right]. \tag{2.51}$$

The customer's expected payoff is

$$rU^f = \frac{r(r+\alpha\lambda_0)}{r(r+\lambda_0)+\alpha\lambda_0(r+\lambda_1)}B + \frac{\alpha r\lambda_0(r+\lambda_1)}{r(r+\lambda_0)+\alpha\lambda_0(r+\lambda_1)}(\bar{c}-\underline{c}). \tag{2.52}$$

For all of the above three cases, the expected social surplus is

$$rW^f = B - \frac{\lambda_0}{r+\alpha\lambda_0}\left[\alpha(r+\lambda_1)\underline{c} + (1-\alpha)r\bar{c}\right] = rV^f. \tag{2.53}$$

Proof. See Appendix. □

To find the falsehood equilibrium prices, we again need the customer's participation constraints and the expert's incentive compatibility constraints to be satisfied. Knowing that the expert always "lies" about the problem, the customer, in fact, learns the exact state of Good K. Thus we may write the customer's expected payoff in State g_0 as

$$U^f = \int_0^\infty \left\{(1-e^{-r\tau})\frac{B}{r} + e^{-r\tau}\left[(1-\alpha)(U^f-\bar{p}) + \alpha(\tilde{U}^f - \underline{p})\right]\right\}\lambda_0 e^{-\lambda_0\tau}d\tau, \tag{2.54}$$

where $\tilde{U}^f$ is the expected payoff in State g_1:

$$\tilde{U}^f = \int_0^\infty \left\{(1-e^{-r\tau})\frac{B}{r} + e^{-r\tau}(\tilde{U}^f - \underline{p})\right\}\lambda_1 e^{-\lambda_1\tau}d\tau. \tag{2.55}$$

Thus, the customer's participation requires $\bar{p} \leq U^f$ and $\underline{p} \leq \tilde{U}^f$. On the other hand, the expert's incentive compatibility of playing the falsehood strategy requires $\underline{p}-\underline{c} \leq \bar{p}-\bar{c} \leq \frac{r+\lambda_1}{r+\lambda_0}(\underline{p}-\underline{c})$ (See Appendix for details). In other words, the profit margin of the major treatment should be at least as large as that of the minor treatment so that the major treatment is profitable to recommend for the minor problem. However, it should not be too large so that the higher future profits from undertreating the major problem exceed the higher immediate profit from the major treatment. Now that the problem boils down to an optimization program. The expert chooses prices to maximize his expected profit subject to his own incentive constraints and the customer's participation constraints. As the parameter values vary, not all constraints would be binding. The highest possible profit is obtained in the second case of the above lemma, in which both of the customer's participation constraints are binding and thus she is left with the lowest possible ex-ante expected payoff.

Lemma 3. *There exists an "undertreatment equilibrium" in which the expert sets the* *price list* $(\underline{p}, \bar{p})$ *according to*

$$\underline{p} = \mu'_{m,n} \frac{1}{r+\lambda_1} B + (1-\mu'_{m,n}) \frac{1}{r+\lambda_0} B,$$
$$\bar{p} > \frac{1}{r+\lambda_0} B,$$

where $\mu'_{m,n}$ *is the customer's post-treatment belief defined in Equation (2.1). The expert plays the undertreatment strategy (u). The customer always accepts the recommendation. The expert's expected profit is* $r\Pi^u = \frac{\lambda_0}{r+\lambda_0} B - \frac{(r+\alpha\lambda_1)\lambda_0}{r+\alpha\lambda_0} \underline{c}$. *The customer's expected payoff is* $rU^u = \frac{r}{r+\lambda_0} B$. *The social surplus is* $rW^u = B - \frac{(r+\alpha\lambda_1)\lambda_0}{r+\alpha\lambda_0} \underline{c}$.

Proof. See Appendix. □

Lemma 3 characterizes the "undertreatment equilibrium" in which the expert always recommends the minor treatment. Since undertreatment still brings Good K back to functioning, the customer does not know if it has been treated appropriately, but she might infer Good K's state given the record of the treatment history. Formally, the customer updates her belief according to Equation (2.1) and Equation (2.2) with initial conditions $\mu'_0 = 0$ and $\mu_1 = 0$. Since the expert also has the record of treatment history, the expert is able to extract all future surplus by setting $\underline{p}$ according to the customer's belief so that the customer will just participate while earning the value of her outside option 0. From the ex-ante point of view, the customer only earns her reservation value at g_0, which is the expected value until the first break down. All the remaining social surplus goes to the expert.

2.4.1 Expert Optimality

The following proposition characterizes the expert optimal stationary pure strategy equilibrium.

Proposition 3. *The expert optimal stationary pure strategy equilibrium is as follows:*

(i) If $\bar{c} - \underline{c} \in \left(0, \alpha \frac{\lambda_1 - \lambda_0}{r+\alpha\lambda_0} \underline{c}\right)$, *then the highest expected profit for the expert is*

$$r\Pi^o = \frac{\lambda_0}{r+\lambda_0} B - \lambda_0 \bar{c}, \tag{2.56}$$

which is obtained in "overtreatment equilibrium". The optimal price list is $(\underline{p} > \frac{1}{r+\lambda_1} B, \bar{p} = \frac{1}{r+\lambda_0} B)$. *The customer's payoff is* $rU^o = \frac{r}{r+\lambda_0} B$. *The social surplus is* $rW^o = B - \lambda_0 \bar{c}$.

(ii) *if* $\bar{c} - \underline{c} \in \left(\alpha \frac{\lambda_1 - \lambda_0}{r + \alpha\lambda_0}\underline{c}, \frac{\lambda_1 - \lambda_0}{r + \alpha\lambda_0}\underline{c}\right)$*, then the highest profit is*

$$r\Pi^u = \frac{\lambda_0}{r + \lambda_0}B - \frac{(r + \alpha\lambda_1)\lambda_0}{r + \alpha\lambda_0}\underline{c}, \tag{2.57}$$

which is obtained in "undertreatment equilibrium". The optimal price list is $(\underline{p} = \mu'_{m,n}\frac{1}{r+\lambda_1}B + (1 - \mu'_{m,n})\frac{1}{r+\lambda_0}B, \bar{p} > \frac{1}{r+\lambda_0}B)$*. The customer's payoff is* $rU^u = \frac{r}{r+\lambda_0}B$*. The social surplus is* $rW^u = B - \frac{(r+\alpha\lambda_1)\lambda_0}{r+\alpha\lambda_0}\underline{c}$*.*

Proof. See Appendix. □

Corollary 1. *In terms of the failure rate differential,*

(i) *If* $\lambda_1 - \lambda_0 > \frac{(\bar{c}-\underline{c})(r+\alpha\lambda_0)}{\alpha\underline{c}}$*, the optimal stationary pure strategy equilibrium is "overtreatment equilibrium".*

(ii) *If* $\lambda_1 - \lambda_0 \in \left(\frac{(\bar{c}-\underline{c})(r+\alpha\lambda_0)}{\underline{c}}, \frac{(\bar{c}-\underline{c})(r+\alpha\lambda_0)}{\alpha\underline{c}}\right)$*, the optimal stationary pure strategy equilibrium is "undertreatment equilibrium".*

Proposition 3 establishes the stationary pure strategy equilibrium for the entire game in which the expert's profits are maximized. Corollary 1 reinterprets the result in terms of the failure rate differential. Some implications are worth mentioning. The most surprising result is that when the failure rate in state g_1 is substantially higher than that in g_0, overtreatment actually becomes more profitable. This is counterintuitive at the first glance because one would expect the expert to have stronger incentives to undertreat when undertreatment would bring more future business. However, a closer look at the prices charged in "undertreatment equilibrium" reveals that a higher λ_1 forces the expert to lower the price of the minor treatment. This is because the customer is less willing to accept the minor treatment since she knows undertreatment leads to more future expenses. Therefore, the increased failure rate, although brings more future business, squeezes the profit margin at the same time. In the end, since the expert is able to extract all the surplus except for the customer's reservation value in both equilibria, the expert prefers the equilibrium that maximizes social welfare.

Second, in terms of the price list, the expert does not change the price list across time in "overtreatment equilibrium" while he constantly changes the price list in "undertreatment equilibrium". This is because after the major treatment, the customer's belief restored to $\mu'_n = 0$, whereas after the minor treatment, the customer's belief updates according to Equation (2.1) and Equation (2.2) with initial conditions $\mu'_0 = 0$ and $\mu_1 = 0$. In "undertreatment equilibrium", the customer's belief in general deteriorates but is not necessarily so. After a rare event of long enough time interval between treatments, the customer belief improves. In the limit, the customer's belief goes to $\mu'_\infty \to 1$.

Third, the peculiar "falsehood equilibrium" is always dominated by "undertreatment equilibrium". So a profit maximizing monopolist expert will never want to adopt the falsehood strategy.

2.4.2 Customer Optimality

Despite the fact the expert would not opt for "falsehood equilibrium", "falsehood equilibrium" is the only equilibrium that might leave some rent to the customer. According to Case I and Case III of Lemma 2, to satisfy the expert's incentive compatibility constraints and the customer's participation constraints, the expert must set $\bar{p}$ below $\frac{1}{\lambda_0+r}B$ (Case I) or $\underline{p}$ below $\frac{1}{\lambda_1+r}B$ (Case III) and thus cannot extract all surplus.

In "overtreatment equilibrium" and "undertreatment equilibrium", one of the prices is never used. The expert could set the never-to-be-used price to be sufficiently high so that if the treatment with the high price is ever recommended, the customer would reject this and all future treatments. Customer's rejection would in turn discipline the expert from recommending the treatment. With one treatment never recommended, the expert could set the price for the intended treatment to be the maximum willingness to pay of the customer and thus leave the customer with no rent.

2.5 The Cutoff Equilibrium

In the previous subsection, we consider the case where the expert plays the same strategy consistently over time. In this subsection, we relax this constraint and allow the expert to change strategies across interactions. From the analysis of the stationary strategy, we know that the optimal strategy is either stationary undertreatment strategy or stationary overtreatment strategy. Since undertreatment is more appealing in the initial interactions when the customer is more likely to be endowed with the minor problem and overtreatment would be more efficient when the customer becomes more pessimistic, we conjecture that the following cutoff strategy might outperform both stationary undertreatment and overtreatment equilibria. In the cutoff strategy, the expert always undertreats the problem whenever the customer's belief about the condition of Good K is sufficiently positive, i.e., the customer's belief that the probability of Good K in State g_1 is below some cutoff value, and the expert always overtreats the problem whenever the customer's belief is beyond the cutoff value. Proposition 4 below shows that such strategy constitutes an equilibrium.

Lemma 4. *There exists a "cutoff equilibrium" with the cutoff $\bar{\mu} \in (\alpha, 1]$ where the expert recommends the minor treatment whenever the post-treatment belief $\mu'_n \leq \bar{\mu}$ and recommends the major treatment otherwise. The price list while recommending the minor* *treatment which we call the minor price list is given by*

$$\underline{p} = \mu_n' U_1 + (1-\mu_n')U_0 = \mu_n' \frac{1}{r+\lambda_1}B + (1-\mu_n')\frac{1}{r+\lambda_0}B;$$
$$\bar{p} > \frac{1}{r+\lambda_0}B,$$

and the price list while recommending the major treatment which we call the major price list is given by

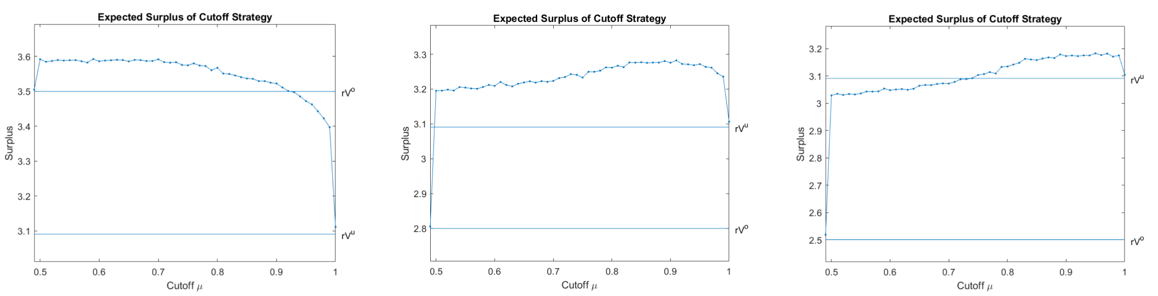
$$\underline{p} > \mu_n' U_1 + (1-\mu_n')U_0 = \mu_n' \frac{1}{r+\lambda_1}B + (1-\mu_n')\frac{1}{r+\lambda_0}B;$$
$$\bar{p} = \frac{1}{r+\lambda_0}B.$$

The customer always accepts the expert's recommendation on the equilibrium path.

Proof. See Appendix. □

Two points are worth noticing. First, "undertreatment equilibrium" and "overtreatment equilibrium" are two extreme cases of "cutoff equilibrium". "Undertreatment equilibrium" is equivalent to $\bar{\mu} = 1$ whereas "overtreatment equilibrium" is equivalent to $\bar{\mu} < \alpha$. Second, in "cutoff equilibrium", same as in "undertreatment equilibrium" and "overtreatment equilibrium", the customer obtains her reservation value at g_0 only.

Generally, to characterize the value of "cutoff equilibrium" is rather difficult. There are a continuum of states. Strategies are state dependent and would also in turn affect the state. The non-monotonicity of the state updating process as indicated in Equation (2.3) further complicates the problem. The following graphs simulate the values of "cutoff equilibrium" with parameter values $\alpha = 0.5, \lambda_0 = 1, \lambda_1 = 2, \underline{c} = 1, B = 5, r = 0.05$ and different values of $\bar{c}$ indicated below the graphs.



$\bar{c} = 1.5$ $\bar{c} = 2.2$ $\bar{c} = 2.5$

Figure 2: Cutoff Equilibrium

Despite the fact that the analytical solutions are generally not possible, we could find "cutoff equilibrium" that strictly dominates both "undertreatment equilibrium" and "overtreatment equilibrium" under certain parameter values.

Proposition 4. *When* $\frac{\alpha(\lambda_1-\lambda_0)}{r+(1-\alpha)\lambda_1+\alpha\lambda_0}\underline{c} < \bar{c}-\underline{c} < \frac{\lambda_1-\lambda_0}{\lambda_0}\underline{c}$*, there exists* $\bar{\mu}$ *such that "cutoff equilibrium" strictly dominates "undertreatment equilibrium" and "overtreatment equilibrium".*

Proof. See Appendix. □

For the case where "overtreatment equilibrium" is more profitable, the alternating equilibrium ("cutoff equilibrium" with cutoff α) where the expert alternates between the undertreatment strategy and the overtreatment strategy dominates "overtreatment equilibrium" when $\bar{c}-\underline{c} > \frac{\alpha(\lambda_1-\lambda_0)}{r+(1-\alpha)\lambda_1+\alpha\lambda_0}\underline{c}$. For the case where "undertreatment equilibrium" is more profitable, the equilibrium with $\bar{\mu} \to 1$ dominates "undertreatment equilibrium" when $\tilde{V^u} < V^o$, i.e., $\bar{c}-\underline{c} < \frac{\lambda_1-\lambda_0}{\lambda_0}\underline{c}$.

3 Discussion: No Inter-temporal Price Discrimination

In the extension, we consider the case where there is a continuum of customers and the expert is not able to price discriminate among the customers. More specifically, the expert could not charge personalized prices based on the customer's beliefs as we discussed in the main model.

Formally, the main model is modified as follows. Instead of setting price lists at each interaction, the expert has to set the price list $(\underline{p}, \bar{p})$ at time $\tau = 0$. Once the price list is set, the expert could not alter it.

Without inter-temporal price discrimination, truth-telling is still not sustainable as an equilibrium outcome. To see this, the analysis in Section 2.3 is applicable to the current setting since in the original analysis, the expert would not change the price list over time. Proposition 2 is modified as follows:

Proposition 2'. *Without inter-temporal price discrimination, there still exists no "truth-telling equilibrium" in which the expert plays the truth-telling strategy.*

Since the expert does not change prices even if he could in "overtreatment equilibrium" and "falsehood equilibrium", Without inter-temporal price discrimination, the equilibrium outcomes would not change in these two equilibria. Therefore, Lemma 1 and Lemma 2 still apply.

In "undertreatment equilibrium", since the customer's belief converges to $\mu'_\infty = 1$, to invite customer participation, the expert needs to set $\underline{p} = \frac{1}{\lambda_1+r}B$. In such an equilibrium, the customer is left with some rent. Lemma 3 is modified as follows:

Lemma 3'. *There exists an "undertreatment equilibrium" in which the expert sets the price lists* $(\underline{p} = \frac{1}{r+\lambda_1}B, \bar{p} > \frac{1}{r+\lambda_0}B)$ *and plays the undertreatment strategy* (u)*. The* *expert's expected profit* $r\Pi^u$ *is* $\frac{\lambda_0(r+\alpha\lambda_1)}{r+\alpha\lambda_0}\left(\frac{1}{r+\lambda_1}B-\underline{c}\right)$. *The customer's expected payoff is* $rU^u=\left[1-\frac{\lambda_0(r+\alpha\lambda_1)}{(r+\alpha\lambda_0)(r+\lambda_1)}\right]B$. *The social surplus is* $rW^u=B-\frac{\lambda_0(r+\alpha\lambda_1)}{r+\alpha\lambda_0}\underline{c}$.

Proposition 3 is modified as follows:

Proposition 3'. *The expert optimal stationary pure strategy equilibrium is as follows:*

(i) If $\bar{c}-\underline{c}\in\left(0,\frac{\lambda_1-\lambda_0}{r+\lambda_0}\underline{c}\right)$, *then the highest expected profit for the expert is*

$$r\Pi^o=\frac{\lambda_0}{r+\lambda_0}B-\lambda_0\bar{c}, \tag{3.1}$$

which is obtained in "overtreatment equilibrium" by charging $\bar{p}=\frac{1}{r+\lambda_0}B$ *and* $\underline{p}>\frac{1}{r+\lambda_1}B$. *The customer's payoff is* $rU^o=\frac{r}{r+\lambda_0}B$. *The social surplus is* $rW^o=B-\lambda_0\bar{c}$.

(ii) If $B>\frac{(r+\lambda_0)(r+\lambda_1)}{r+\alpha\lambda_0}\underline{c}$ *and* $\bar{c}-\underline{c}\in\left(\frac{\lambda_1-\lambda_0}{r+\lambda_0}\underline{c},\frac{\lambda_1-\lambda_0}{r+\alpha\lambda_0}\underline{c}\right)$, *or if* $B<\frac{(r+\lambda_0)(r+\lambda_1)}{r+\alpha\lambda_0}\underline{c}$ *and* $\bar{c}-\underline{c}\in\left(\frac{\lambda_1-\lambda_0}{r+\lambda_0}\underline{c},\frac{\lambda_1-\lambda_0}{(r+\lambda_0)(r+\lambda_1)}B\right)$ *then the highest expected profit is*

$$r\Pi^f=\frac{\lambda_0}{r+\lambda_0}B-\frac{r\lambda_0(1-\alpha)}{r+\alpha\lambda_0}\bar{c}-\frac{\alpha\lambda_0(r+\lambda_1)}{r+\alpha\lambda_0}\underline{c}, \tag{3.2}$$

which is obtained in "falsehood equilibrium" by charging $\bar{p}=\frac{1}{r+\lambda_0}B$ *and* $\underline{p}=\frac{1}{r+\lambda_1}B$. *The customer's payoff is* $rU^f=\frac{r}{r+\lambda_0}B$. *The social surplus is* $rW^f=B-\frac{r\lambda_0(1-\alpha)}{r+\alpha\lambda_0}\bar{c}-\frac{\alpha\lambda_0(r+\lambda_1)}{r+\alpha\lambda_0}\underline{c}$.

(iii) if $B<\frac{(r+\lambda_0)(r+\lambda_1)}{r+\alpha\lambda_0}\underline{c}$ *and* $\bar{c}-\underline{c}\in\left(\frac{\lambda_1-\lambda_0}{(r+\lambda_0)(r+\lambda_1)}B,\frac{\lambda_1-\lambda_0}{r+\alpha\lambda_0}\underline{c}\right)$, *then the highest profit is*

$$r\Pi^u=\frac{\lambda_0(r+\alpha\lambda_1)}{r+\alpha\lambda_0}\left[\frac{1}{r+\lambda_1}B-\underline{c}\right], \tag{3.3}$$

which is obtained in "undertreatment equilibrium" by charging $\underline{p}=\frac{1}{r+\lambda_1}B$ *and* $\bar{p}>\frac{1}{r+\lambda_0}B$. *The customer's payoff is* $rU^u=\frac{\lambda_0(r+\alpha\lambda_1)}{r+\alpha\lambda_0}\frac{r}{r+\lambda_1}B$. *The social surplus is* $rW^u=B-\frac{r+\alpha\lambda_1}{r+\alpha\lambda_0}\lambda_0\underline{c}$.

Surprisingly, "falsehood equilibrium" would now appear as an optimal equilibrium for the expert. It only happens when the difference in treatment costs or the increment of failure rate is within a moderate range.

Regarding the welfare of the expert optimal equilibrium, both "overtreatment equilibrium" and "falsehood equilibrium" leave the customer with her reservation value in g_0, while she obtains more in "undertreatment equilibrium". This is because as time goes by, the customer tends to have the more pessimistic belief that the good is more likely to be in State g_1 and hence she is more likely to quit the market if the price is too high. Hence, to keep the customer around, the expert has to surrender some rents to her.

4 Conclusions

We study a dynamic model of a credence goods market where the expert's performance of treatment is observable to the customer and verifiable to the court. Previous literature argues that in a one-shot setting, the expert would set an equal profit margin for both treatments, which leads to "truth-telling equilibrium". However, our dynamic model tells a different story. Here, the expert has additional incentive to undertreat since undertreatment increases the frequency of the customer's visits and hence brings more future business. The equal-margin condition fails because the minor treatment is now more attractive to the expert, and thus in order for the expert to honestly recommend the major treatment for the major problem, the profit margin of the major treatment has to be sufficiently higher than that of the minor treatment to compensate for the less future revenue. Consequently, "truth-telling equilibrium" no longer exists under the verifiability assumption.

Nevertheless, we also show that the expert is not necessarily encouraged to undertreat. In fact, "undertreatment equilibrium" is only optimal for a limited region of parameter values. This is due to the fact that the customer is less willing to pay for the minor treatment given a higher increment in the failure rate, which lowers the profit margin from performing the minor treatment. In our main model where the expert is able to offer personalized pricing, since the expert is able to extract all surplus in both undertreatment and overtreatment equilibria, the expert chooses the equilibrium that maximizes social welfare. In the extended model without inter-temporal price discrimination, the optimal region for "undertreatment equilibrium" is further reduced, since to induce customer participation, the expert has to leave some rent to the customer in "undertreatment equilibrium". Moreover, "falsehood equilibrium", where the expert offers the inappropriate treatments to both problems, would now appear as an optimal equilibrium for the expert.

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5 Appendix

5.1 Proof of Lemma 1

Proof. Note that the largest social value under the overtreatment plan can be written recursively as

$$V^o = \int_0^\infty \left\{ (1 - e^{-r\tau})\frac{B}{r} + e^{-r\tau}\left[V^o - \bar{c}\right] \right\} \lambda_0 e^{-\lambda_0 \tau} d\tau, \tag{5.1}$$

which can be solved for

$$V^o = B - \lambda_0 \bar{c}. \tag{5.2}$$

Consider now the customer's incentive of accepting the major treatment recommendation. Suppose the customer always does so, and given the expert's overtreatment strategy, her expected payoff can be written recursively as

$$U^o = \int_0^\infty \left\{ (1 - e^{-r\tau})\frac{B}{r} + e^{-r\tau}\left[U^o - \bar{p}\right] \right\} \lambda_0 e^{-\lambda_0 \tau} d\tau, \tag{5.3}$$

which can be solved for

$$rU^o = B - \lambda_0 \bar{p} = \frac{r}{r + \lambda_0} B. \tag{5.4}$$

Since $\bar{p} \leq U^o$, the customer would indeed accept the major treatment recommendation. Assume that the customer has the off-equilibrium belief that if the minor treatment is recommended, then Good K suffers from the major problem, and the expert would thereafter always recommend the minor treatment. Given that $\underline{p} > \frac{1}{r+\lambda_1} B$, the customer would reject the minor treatment.

Now consider the expert's incentive of the recommendation. He would not recommend the minor treatment since it would be rejected. Hence, he would always recommend the major treatment. Given that the customer always accepts the major treatment recommendation, the expert's expected profit can be written as

$$\Pi^o = \int_0^\infty e^{-r\tau}\left[\Pi^o + (\bar{p} - \bar{c})\right] \lambda_0 e^{-\lambda_0 \tau} d\tau, \tag{5.5}$$

which can be solved for

$$r\Pi^o = \lambda_0(\bar{p} - \bar{c}) = \frac{\lambda_0}{r + \lambda_0} B - \lambda_0 \bar{c}. \tag{5.6}$$

This profit is the highest among all overtreatment strategies since the social surplus achieves its highest possible level $rW^o = rU^o + r\Pi^o = B - \lambda_0 \bar{c} = rV^o$, and the customer retains her lowest possible payoff $U^o = \bar{U}_0 = \frac{1}{r+\lambda_0} B$.

□

5.2 Proof of Lemma 2

Proof. First, note that the social value under the falsehood treatment plan can be written as

$$V^f = \int_0^\infty \left\{(1-e^{-r\tau})\frac{B}{r} + e^{-r\tau}\left[\alpha(\tilde{V}^f - \underline{c}) + (1-\alpha)(V^f - \bar{c})\right]\right\}\lambda_0 e^{-\lambda_0\tau}d\tau, \tag{5.7}$$

where

$$\tilde{V}^f = \int_0^\infty \left\{(1-e^{-r\tau})\frac{B}{r} + e^{-r\tau}\left[\tilde{V}^f - \underline{c}\right]\right\}\lambda_1 e^{-\lambda_1\tau}d\tau. \tag{5.8}$$

Combining the above two equations yields

$$r\tilde{V}^f = B - \lambda_1\underline{c}, \tag{5.9}$$

and

$$rV^f = B - \frac{\lambda_0}{r+\alpha\lambda_0}\left[\alpha(r+\lambda_1)\underline{c} + (1-\alpha)r\bar{c}\right]. \tag{5.10}$$

Now we solve for the most profitable falsehood strategy. Consider first the customer's incentive to accept recommendations. Suppose the customer always accepts the recommendations under the expert's falsehood strategy, her expected payoff in State g_0 is given by

$$U^f = \int_0^\infty \left\{(1-e^{-r\tau})\frac{B}{r} + e^{-r\tau}\left[(1-\alpha)(U^f - \bar{p}) + \alpha(\tilde{U}^f - \underline{p})\right]\right\}\lambda_0 e^{-\lambda_0\tau}d\tau, \tag{5.11}$$

where

$$\tilde{U}^f = \int_0^\infty \left\{(1-e^{-r\tau})\frac{B}{r} + e^{-r\tau}(\tilde{U}^f - \underline{p})\right\}\lambda_1 e^{-\lambda_1\tau}d\tau \tag{5.12}$$

is the expected payoff in State g_1. Combining the above two equations, one can obtain

$$r\tilde{U}^f = B - \lambda_1\underline{p}, \tag{5.13}$$

and

$$rU^f = B - \left(\frac{r\lambda_0}{r+\alpha\lambda_0}(1-\alpha)\bar{p} + \frac{\lambda_0(r+\lambda_1)}{r+\alpha\lambda_0}\alpha\underline{p}\right). \tag{5.14}$$

Given the minor treatment recommendation, inferring that Good K has the major problem and will enter State g_1 after which all future treatments will be minor, the customer is willing to accept if and only if the price is no greater than her State g_1 reservation value, i.e.,

$$\underline{p} \leq \frac{1}{r+\lambda_1}B. \tag{5.15}$$

Given the major treatment recommendation, inferring that Good K has the minor problem and will return to State g_0, the customer is willing to accept it if and only if $\bar{p} \leq U^f$, or

$$\bar{p} \leq \frac{r+\alpha\lambda_0}{r(r+\lambda_0)}B - \frac{\alpha\lambda_0(r+\lambda_1)}{r(r+\lambda_0)}\underline{p}. \tag{5.16}$$

Consider now the expert's incentive of playing falsehood strategy. Given the minor problem, since both treatments return Good K to State g_0, the expert would have no incentive to recommend the minor treatment if and only if the profit margin of the major treatment is higher:

$$\bar{p} - \bar{c} \geq \underline{p} - \underline{c}. \tag{5.17}$$

Given the major problem, the expert's on-the-path expected profit in State g_0 is

$$\Pi^f_{\text{on}} = \underline{p} - \underline{c} + \tilde{\Pi}^f, \tag{5.18}$$

where $\tilde{\Pi}^f$ is his expected profit in State g_1 which can be obtained from

$$\tilde{\Pi}^f = \int_0^\infty e^{-r\tau}\left\{\tilde{\Pi}^f + (\underline{p} - \underline{c})\right\}\lambda_1 e^{-\lambda_1\tau}d\tau, \tag{5.19}$$

which implies $r\tilde{\Pi}^f = \lambda_1(\underline{p} - \underline{c})$. Suppose now the expert deviates one-time to recommending the major treatment, then his off-the-path expected profit is

$$\Pi^f_{\text{off}} = (\bar{p} - \bar{c}) + \Pi^f, \tag{5.20}$$

where Π^f is his equilibrium expected profit in State g_0 which can be obtained from

$$\Pi^f = \int_0^\infty e^{-r\tau}\left\{(1-\alpha)[\Pi^f + (\bar{p} - \bar{c})] + \alpha[\tilde{\Pi}^f + (\underline{p} - \underline{c})]\right\}\lambda_0 e^{-r\lambda_0}d\tau, \tag{5.21}$$

which implies

$$r\Pi^f = \frac{r\lambda_0}{r+\alpha\lambda_0}(1-\alpha)(\bar{p} - \bar{c}) + \frac{\lambda_0(r+\lambda_1)}{r+\alpha\lambda_0}\alpha(\underline{p} - \underline{c}). \tag{5.22}$$

Hence no deviation requires $\Pi^f_{\text{on}} \geq \Pi^f_{\text{off}}$, or

$$\bar{p} - \bar{c} \leq \frac{r+\lambda_1}{r+\lambda_0}(\underline{p} - \underline{c}). \tag{5.23}$$

In summary, the expert chooses p to maximize (5.22) subject to (5.15) (5.16) (5.17) and (5.23). We solve this maximization problem graphically under three cases of parameter values (see Figure 3). The two positively sloped lines represent the two incentive constraints for the expert, (5.17) and (5.23), holding in equality, and hence the shaded area represents all incentive compatible prices. The negatively sloped black line represents the customer's participation constraint (5.16) holding in equality. The other participation constraint is represented by the vertical line at $\underline{p} = \frac{1}{r+\lambda_1}$. Hence, the shaded area represents all admissible equilibrium prices. Finally, the y-intercept of the other negatively

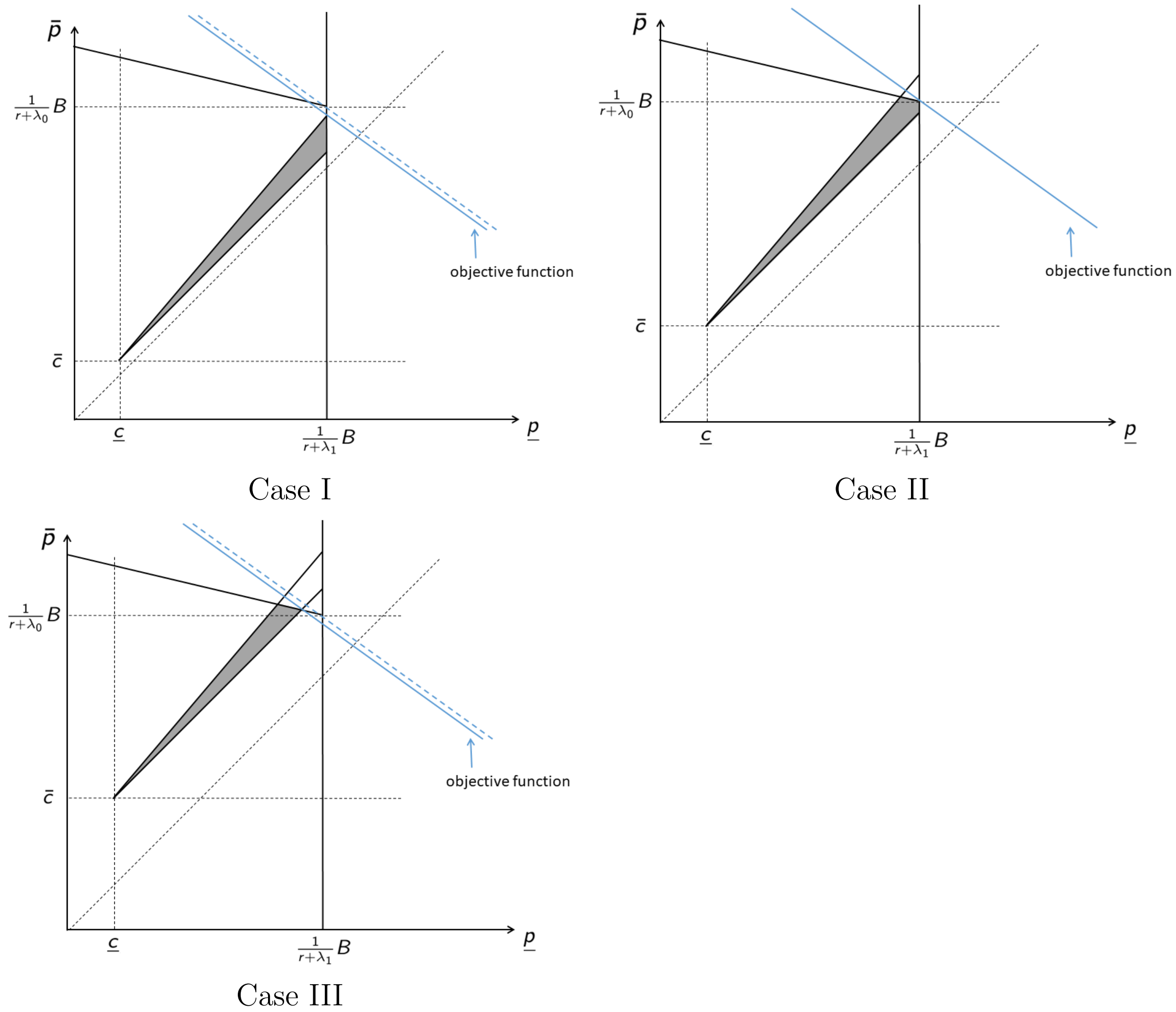

Figure 3: Optimal Falsehood Equilibrium

sloped blue line indicates the value of the objective function. We have the following three scenarios.

- Case I: $\bar{c} - \underline{c} < \frac{\lambda_1 - \lambda_0}{r+\lambda_0}\underline{c}$.
  The optimal prices are $\bar{p} = \frac{1}{r+\lambda_0}B + \left(\bar{c} - \frac{r+\lambda_1}{r+\lambda_0}\underline{c}\right), \underline{p} = \frac{1}{r+\lambda_1}B$, and the maximal profit is $r\Pi^f = \frac{\lambda_0}{r+\lambda_0}B - \frac{\lambda_0(r+\lambda_1)}{r+\lambda_0}\underline{c}$. The customer's expected payoff is $rU^f = \frac{r}{r+\lambda_0}B + \frac{r\lambda_0(1-\alpha)}{r+\alpha\lambda_0}\left(\frac{r+\lambda_1}{r+\lambda_0}\underline{c} - \bar{c}\right)$. The expected social surplus is
  $rW^f = B - \frac{\lambda_0}{r+\alpha\lambda_0}\left[\alpha(r+\lambda_1)\underline{c} + (1-\alpha)r\bar{c}\right] = rV^f$.

- Case II: $\frac{\lambda_1 - \lambda_0}{r+\lambda_0}\underline{c} \leq \bar{c} - \underline{c} \leq \frac{\lambda_1 - \lambda_0}{(r+\lambda_0)(r+\lambda_1)}B$.
  The optimal prices are $\bar{p} = \frac{1}{r+\lambda_0}B, \underline{p} = \frac{1}{r+\lambda_1}B$, and the maximal profit is $r\Pi^f = \frac{\lambda_0}{r+\lambda_0}B - \frac{r\lambda_0(1-\alpha)}{r+\alpha\lambda_0}\bar{c} - \frac{\alpha\lambda_0(r+\lambda_1)}{r+\alpha\lambda_0}\underline{c}$. The customer's expected payoff is $rU^f = \frac{r}{r+\lambda_0}B$. The expected social surplus is $rW^f = B - \frac{\lambda_0}{r+\alpha\lambda_0}\left[\alpha(r+\lambda_1)\underline{c} + (1-\alpha)r\bar{c}\right] = rV^f$. This is the highest possible profit that the expert can obtain by the falsehood strategy since the customer obtains her lowest possible payoff.

- Case III: $\bar{c} - \underline{c} > \frac{\lambda_1 - \lambda_0}{(r+\lambda_0)(r+\lambda_1)} B$.
  The optimal prices are $\bar{p} = \frac{r+\alpha\lambda_0}{r(r+\lambda_0)+\alpha\lambda_0(r+\lambda_1)} B + \frac{\alpha\lambda_0(r+\lambda_1)}{r(r+\lambda_0)+\alpha\lambda_0(r+\lambda_1)} (\bar{c} - \underline{c})$, $\underline{p} = \frac{r+\alpha\lambda_0}{r(r+\lambda_0)+\alpha\lambda_0(r+\lambda_1)} B - \frac{r(r+\lambda_0)}{r(r+\lambda_0)+\alpha\lambda_0(r+\lambda_1)} (\bar{c} - \underline{c})$, and the maximal profit is $r\Pi^f = \frac{\lambda_0(r+\alpha\lambda_1)}{r(r+\lambda_0)+\alpha\lambda_0(r+\lambda_1)} \left[B - \frac{r(r+\lambda_0)}{r+\alpha\lambda_0} \bar{c} - \frac{\alpha\lambda_0(r+\lambda_1)}{r+\alpha\lambda_0} \underline{c} \right]$.
  The customer's expected profit is $rU^f = \frac{r(r+\alpha\lambda_0)}{r(r+\lambda_0)+\alpha\lambda_0(r+\lambda_1)} B + \frac{\alpha r\lambda_0(r+\lambda_1)}{r(r+\lambda_0)+\alpha\lambda_0(r+\lambda_1)} (\bar{c} - \underline{c})$.
  The expected social surplus is $rW^f = B - \frac{\lambda_0}{r+\alpha\lambda_0} [\alpha(r+\lambda_1)\underline{c} + (1-\alpha) r\bar{c}] = rV^f$.

□

5.3 Proof of Lemma 3

Proof. The largest social value under the undertreatment plan is given by

$$rV^u = B - \frac{\lambda_0(r + \alpha\lambda_1)}{r + \alpha\lambda_0}\underline{c}. \tag{5.24}$$

Consider now the customer's incentive of accepting the minor treatment recommendation. Given the expert's undertreatment strategy and the customer's post-treatment belief $\mu'_{m,n}$, the customer obtains

$$U^u(\mu'_{m,n}) = \mu'_{m,n}\frac{1}{r + \lambda_1}B + (1 - \mu'_{m,n})\frac{1}{r + \lambda_0}B \tag{5.25}$$

if the customer expects to obtain no surplus from all future interactions. The expert then sets

$$\underline{p}(\mu'_{m,n}) = U^u(\mu'_{m,n}) = \mu'_{m,n}\frac{1}{r + \lambda_1}B + (1 - \mu'_{m,n})\frac{1}{r + \lambda_0}B \tag{5.26}$$

so that the customer indeed obtains no surplus from this and all future interactions. By extracting all surplus from the first interaction onwards, this pricing strategy also gives the expert the highest possible profit.

Assume that the customer has the off-equilibrium belief that if the major treatment is recommended, then the expert would thereafter always recommend the major treatment. Given that $\bar{p} > \frac{1}{r+\lambda_0}B$, the customer would reject the major treatment recommendation. Knowing this, the expert would never recommend the major treatment.

In this equilibrium, the customer retains her reservation value at g_0, i.e., $rU_0 = \frac{r}{r+\lambda_0}B$, and the expert obtains

$$r\Pi^u = rV^u - rU_0 = \frac{\lambda_0}{r + \lambda_0}B - \frac{\lambda_0(r + \alpha\lambda_1)}{r + \alpha\lambda_0}\underline{c}. \tag{5.27}$$

□

5.4 Proof of Proposition 3

Proof. First, we show that "falsehood equilibrium" is always dominated by "undertreatment equilibrium". In terms of social welfare,

$$\begin{aligned} rW^u - rW^f &= \Big[B - \frac{(r+\alpha\lambda_1)\lambda_0}{r+\alpha\lambda_0}\underline{c}\Big] - \Big[B - \frac{r\lambda_0(1-\alpha)}{r+\alpha\lambda_0}\bar{c} - \frac{\alpha\lambda_0(r+\lambda_1)}{r+\alpha\lambda_0}\underline{c}\Big] \\ &= \frac{r\lambda_0(1-\alpha)}{r+\alpha\lambda_0}(\bar{c}-\underline{c}) > 0. \end{aligned} \tag{5.28}$$

Also,

$$\begin{aligned} rU^f &\geq \frac{r}{r+\lambda_0}B, \\ rU^u &= \frac{r}{r+\lambda_0}B. \end{aligned}$$

Therefore,

$$r\Pi^u - r\Pi^f = [rW^u - rU^u] - [rW^f - rU^f] \geq rW^u - rW^f > 0.$$

So "falsehood equilibrium" is dominated.

For "undertreatment equilibrium" and "overtreatment equilibrium", since the expert's extract all surplus except for the customer's reservation value in both equilibria, we only need to compare the expected surplus in State g_0.

$$\begin{aligned} rW^u - rW^o &= \lambda_0\bar{c} - \frac{(r+\alpha\lambda_1)\lambda_0}{r+\alpha\lambda_0}\underline{c} \\ &= \lambda_0\Big\{\bar{c} - \big[1+\alpha\frac{(\lambda_1-\lambda_0)}{r+\alpha\lambda_0}\big]\underline{c}\Big\} \end{aligned} \tag{5.29}$$

Therefore, when $\bar{c} > \big[1+\alpha\frac{(\lambda_1-\lambda_0)}{r+\alpha\lambda_0}\big]\underline{c}$, "undertreatment equilibrium" is more profitable than "overtreatment equilibrium" and vice versa. Together with the cutoff established by Assumption (2), we arrive at Proposition 3. □

5.5 Proof of Lemma 4

Proof. The equilibrium is supported by the customer's off-equilibrium belief described as follows. For a deviation in treatment strategies, we assume that if the major treatment is recommended whenever $\mu'_{m,n} \leq \bar{\mu}$, the customer believes that the expert would always recommend the major treatment thereafter with price list $(\underline{p}, \bar{p} > \frac{1}{r+\lambda_0}B)$; if the minor treatment is recommended whenever $\mu'_{m,n} > \bar{\mu}$, the customer believes that Good K had the major problem and that the expert would thereafter always recommend the minor treatment with price list $(\underline{p} > \frac{1}{r+\lambda_1}B, \bar{p})$. This off-equilibrium belief ensures that the customer would reject the expert's off-equilibrium treatment recommendation at the undertreatment stage if $\bar{p} > \frac{1}{r+\lambda_0}B$ and at the overtreatment stage if $\underline{p} > \frac{1}{r+\lambda_1}B$. These two conditions on prices are satisfied with the equilibrium price lists proposed in Proposition 4. For a deviation in prices, if the expert deviates to a lower price, the customer would accept the recommendation as long as the expert adopts the on-equilibrium-path recommendation strategy whereas if the expert deviates to a higher price, the customer would assume that the high price strategy would be recommended and thus would restrain from visiting the expert.

Given the belief of the customer that in the future interactions, the expert would set price lists and use recommendation strategies such that the customer obtains no surplus, the maximum price the customer would accept for the minor treatment recommendation when $\mu'_{m,n} \leq \bar{\mu}$ is

$$\underline{p} = \mu'_n U_1 + (1 - \mu'_n) U_0 = \mu'_n \frac{1}{r + \lambda_1} B + (1 - \mu'_n) \frac{1}{r + \lambda_0} B$$

and the maximum price for the major treatment recommendation when $\mu'_{m,n} > \bar{\mu}$ is

$$\bar{p} = \frac{1}{r + \lambda_0} B.$$

At the same time, similar to the proof in the "overtreatment equilibrium" and "undertreatment equilibrium", the expert needs to set the never-to-be-used price to be high enough such that the customer would reject the treatment if the treatment is ever recommended. □

5.6 Proof of Proposition 4

Proof. **Case I:** $\bar{c} = \left[1 + \alpha \frac{(\lambda_1 - \lambda_0)}{r + \alpha\lambda_0}\right]\underline{c}$, i.e., $\bar{c} - \underline{c} = \alpha \frac{(\lambda_1 - \lambda_0)}{r + \alpha\lambda_0}\underline{c}$

Under this constraint, "undertreatment equilibrium" gives the same value as "overtreatment equilibrium", i.e., $V^u = V^o$.

Consider an equilibrium where the expert performs the minor treatment until some $\bar{\mu}$ and performs the major treatment forever thereafter, and call this equilibrium **"$\bar{\mu}_1$ equilibrium"**. In general, we define **"$\bar{\mu}_n$ equilibrium"** as an equilibrium where the minor treatment until $\bar{\mu}$ happens n rounds. More specifically, for the first $n-1$ rounds, the expert performs the minor treatment until $\bar{\mu}$ and performs the major treatment exactly once to bring Good K back to State g_0, and in the n^{th} round, the expert performs the minor treatment until $\bar{\mu}$ and performs the major treatment forever thereafter.

When $\bar{c} - \underline{c} = \alpha \frac{(\lambda_1 - \lambda_0)}{r + \alpha\lambda_0}\underline{c}$, for any $\bar{\mu} \in [\alpha, 1]$, "$\bar{\mu}_1$ equilibrium" has the same value as the hypothetical case where the expert performs undertreatment until $\bar{\mu}$, then Good K restores to g_0 and the expert performs the minor treatment thereafter. The value of this hypothetical case is higher than that of "undertreatment equilibrium" since the two values before $\bar{\mu}$ are the same while after $\bar{\mu}$, the hypothetical case has a higher value. Specifically,

$$V^u > (1 - \bar{\mu})V^u + \bar{\mu}\tilde{V}^u.$$

Therefore, "$\bar{\mu}_1$ equilibrium" has a higher value compared to both "undertreatment equilibrium" and "overtreatment equilibrium". Now consider "$\bar{\mu}_2$ equilibrium". since "$\bar{\mu}_1$ equilibrium" has a higher value than "overtreatment equilibrium", then "$\bar{\mu}_2$ equilibrium" has a higher value than "$\bar{\mu}_1$ equilibrium". We can repeatedly apply this logic and "cutoff equilibrium" obtains a higher value compared to "undertreatment equilibrium", "overtreatment equilibrium" and "$\bar{\mu}_n$ equilibrium" for any $n < \infty$.

Case II: $\bar{c} - \underline{c} < \alpha \frac{(\lambda_1 - \lambda_0)}{r + \alpha\lambda_0}\underline{c}$

Under this constraint, "overtreatment equilibrium" is more profitable than "undertreatment equilibrium", i.e., $V^o > V^u$.

Consider "α_1 equilibrium". This is an equilibrium where the minor treatment is performed exactly once before the major treatment is performed forever. Let the value of such equilibrium be V^{α_1}, and the value of the later overtreatment forever part be $\tilde{V}^{\alpha_1}$.

$$\begin{aligned}
\tilde{V}^{\alpha_1} =&(1-\alpha)\int_0^{\infty}[(1-e^{-r\tau})\frac{B}{r}+e^{-r\tau}(V^o-\bar{c})]\lambda_0 e^{-\lambda_0\tau}d\tau \\
&+\alpha\int_0^{\infty}[(1-e^{-r\tau})\frac{B}{r}+e^{-r\tau}(V^o-\bar{c})]\lambda_1 e^{-\lambda_1\tau}d\tau \\
=&(1-\alpha)V^o+\alpha[\frac{r}{\lambda_1+r}\frac{B}{r}+\frac{\lambda_1}{\lambda_1+r}(V^o-\bar{c})] \\
=&\frac{B}{r}-[(1-\alpha)\frac{\lambda_0}{r}+\alpha\frac{\lambda_1}{\lambda_1+r}\frac{\lambda_0+r}{r}]\bar{c}
\end{aligned}$$

$$\begin{aligned}
V^{\alpha_1} =&\int_0^{\infty}[(1-e^{-r\tau})\frac{B}{r}+e^{-r\tau}(\tilde{V}^{\alpha_1}-\underline{c})]\lambda_0 e^{-\lambda_0\tau}d\tau \\
=&\frac{r}{r+\lambda_0}\frac{B}{r}+\frac{\lambda_0}{\lambda_0+r}(\tilde{V}^{\alpha_1}-\underline{c}) \\
=&\frac{B}{r}-[(1-\alpha)\frac{\lambda_0}{\lambda_0+r}+\alpha\frac{\lambda_1}{\lambda_1+r}]\frac{\lambda_0}{r}\bar{c}-\frac{\lambda_0}{\lambda_0+r}\underline{c}
\end{aligned}$$

$$\begin{aligned}
V^{\alpha_1}-V^o =&\Big\{\frac{B}{r}-[(1-\alpha)\frac{\lambda_0}{\lambda_0+r}+\alpha\frac{\lambda_1}{\lambda_1+r}]\frac{\lambda_0}{r}\bar{c}-\frac{\lambda_0}{\lambda_0+r}\underline{c}\Big\}-\Big\{\frac{B}{r}-\frac{\lambda_0}{r}\bar{c}\Big\} \\
=&\Big\{1-[(1-\alpha)\frac{\lambda_0}{\lambda_0+r}+\alpha\frac{\lambda_1}{\lambda_1+r}]\Big\}\frac{\lambda_0}{r}\bar{c}-\frac{\lambda_0}{\lambda_0+r}\underline{c} \\
=&\frac{\lambda_0}{\lambda_0+r}\Big\{[1-\alpha\frac{\lambda_1-\lambda_0}{\lambda_1+r}]\bar{c}-\underline{c}\Big\} \\
=&\frac{\lambda_0}{\lambda_0+r}[1-\alpha\frac{\lambda_1-\lambda_0}{\lambda_1+r}]\Big\{\bar{c}-\frac{\lambda_1+r}{r+(1-\alpha)\lambda_1+\alpha\lambda_0}\underline{c}\Big\} \\
=&\frac{\lambda_0}{\lambda_0+r}[1-\alpha\frac{\lambda_1-\lambda_0}{\lambda_1+r}]\Big\{(\bar{c}-\underline{c})-\frac{\alpha(\lambda_1-\lambda_0)}{r+(1-\alpha)\lambda_1+\alpha\lambda_0}\underline{c}\Big\}
\end{aligned}$$

Therefore, $V^{\alpha_1}-V^o>0$ if and only if

$$\bar{c}-\underline{c}>\frac{\alpha(\lambda_1-\lambda_0)}{r+(1-\alpha)\lambda_1+\alpha\lambda_0}\underline{c}.$$

Note

$$\frac{\alpha(\lambda_1-\lambda_0)}{r+(1-\alpha)\lambda_1+\alpha\lambda_0}\underline{c}<\alpha\frac{(\lambda_1-\lambda_0)}{r+\alpha\lambda_0}\underline{c}.$$

Case III: $\bar{c}-\underline{c}>\alpha\frac{(\lambda_1-\lambda_0)}{r+\alpha\lambda_0}\underline{c}$

Under this constraint, "undertreatment equilibrium" is more profitable than "overtreatment equilibrium", i.e., $V^u>V^o$.

Consider $\tilde{V^u}<V^o$, i.e., $\bar{c}-\underline{c}<\frac{\lambda_1-\lambda_0}{\lambda_0}\underline{c}$ and the equilibrium with $\bar{\mu}\to 1$. Under this case, at $\bar{\mu} \to 1$, undertreatment forever gives lower value compared to overtreatment forever. Therefore, the value of "undertreatment equilibrium" is lower than the value of "$\bar{\mu}_1$ equilibrium". Since "undertreatment equilibrium" is more profitable than "overtreatment equilibrium", the value of "$\bar{\mu}_1$ equilibrium" is also higher than that of "overtreatment equilibrium". Then consider "$\bar{\mu}_2$ equilibrium". The profit in "$\bar{\mu}_2$ equilibrium" is higher than that in "$\bar{\mu}_1$ equilibrium" since "$\bar{\mu}_1$ equilibrium" has a higher value than "overtreatment equilibrium". We can repeatedly apply this logic and "cutoff equilibrium" obtains a higher value compared to "undertreatment equilibrium", "overtreatment equilibrium" and "$\bar{\mu}_n$ equilibrium" for any $n < \infty$. □

5.7 Proof of Lemma 3'

Proof. The expert sets $\underline{p} = \frac{1}{r+\lambda_1}B$ to induce the customers always accept the minor treatment recommendation even when they become extremely pessimistic, then the customer's expected payoff in State g_0 can be written as with

$$U^u = \int_0^\infty \left\{(1-e^{r\tau})\frac{B}{r} + e^{-r\tau}\left[\alpha(\tilde{U}^u - \underline{p}) + (1-\alpha)(U^u - \underline{p})\right]\right\}\lambda_0 e^{-\lambda_0\tau}d\tau, \tag{5.30}$$

where

$$\tilde{U}^u = \int_0^\infty \left\{(1-e^{r\tau})\frac{B}{r} + e^{-r\tau}(\tilde{U}^u - \underline{p})\right\}\lambda_1 e^{-\lambda_1\tau}d\tau \tag{5.31}$$

is the expected payoff in State g_1. Note that the value functions does not depend on the customer's belief but only on the state of Good K since the expert would always recommend the minor treatment and the customer would always accept. Combining the above two equations, one can solve for

$$r\tilde{U}^u = B - \lambda_1\underline{p} = \frac{r}{r+\lambda_1}B \geq r\underline{p}, \tag{5.32}$$

and

$$rU^u = B - \frac{\lambda_0(r+\alpha\lambda_1)}{r+\alpha\lambda_0}\underline{p} = \left[1 - \frac{\lambda_0(r+\alpha\lambda_1)}{(r+\alpha\lambda_0)(r+\lambda_1)}\right]B \geq r\underline{p}. \tag{5.33}$$

Hence, the customer indeed will always accept the minor treatment recommendation.

Similarly, the expert's expected profit is given by

$$\Pi^u = \int_0^\infty e^{-r\tau}\left\{(1-\alpha)\left[\Pi^u + (\underline{p} - \underline{c})\right] + \alpha\left[\tilde{\Pi}^u + (\underline{p} - \underline{c})\right]\right\}\lambda_0 e^{-\lambda_0\tau}d\tau, \tag{5.34}$$

where

$$\tilde{\Pi}^u = \int_0^\infty e^{-r\tau}\left\{\tilde{\Pi}^u + (\underline{p} - \underline{c})\right\}\lambda_1 e^{-\lambda_1\tau}d\tau. \tag{5.35}$$

Combining the above two equations, one can obtain

$$r\tilde{\Pi}^u = \lambda_1(\underline{p} - \underline{c}) = \lambda_1\left(\frac{1}{r+\lambda_1}B - \underline{c}\right) \tag{5.36}$$

and

$$r\Pi^u = \frac{\lambda_0(r+\alpha\lambda_1)}{r+\alpha\lambda_0}(\underline{p} - \underline{c}) = \frac{\lambda_0(r+\alpha\lambda_1)}{r+\alpha\lambda_0}\left(\frac{1}{r+\lambda_1}B - \underline{c}\right), \tag{5.37}$$

which is the expert's expected profit.

We assume that the customer has an off-equilibrium belief that if the major treatment is recommended, the expert would always recommend the major treatment thereafter. Given this belief, the customer rejects the major treatment recommendation as long as $\bar{p} > \frac{1}{r+\lambda_0}B$. □

5.8 Proof of Proposition 3'

Step 1. *$r\Pi^o > r\Pi^u$ if and only if*

$$\bar{c} - \underline{c} < \frac{\lambda_1 - \lambda_0}{r + \alpha\lambda_0}\left[\frac{r(1-\alpha)}{(r+\lambda_0)(r+\lambda_1)}B + \alpha\underline{c}\right]. \tag{5.38}$$

Proof. Since

$$\begin{aligned} r\Pi^o - r\Pi^u &= \left\{\frac{\lambda_0}{r+\lambda_0}B - \lambda_0\bar{c}\right\} - \left\{\frac{\lambda_0(r+\alpha\lambda_1)}{r+\alpha\lambda_0}\left[\frac{1}{r+\lambda_1}B - \underline{c}\right]\right\} \\ &= \frac{r\lambda_0(1-\alpha)(\lambda_1-\lambda_0)}{(r+\alpha\lambda_0)(r+\lambda_0)(r+\lambda_1)}B - \lambda_0\bar{c} + \frac{\lambda_0(r+\alpha\lambda_1)}{r+\alpha\lambda_0}\underline{c} \\ &= \frac{\lambda_0(\lambda_1-\lambda_0)}{r+\alpha\lambda_0}\left[\frac{r(1-\alpha)}{(r+\lambda_0)(r+\lambda_1)}B + \alpha\underline{c}\right] - \lambda_0(\bar{c}-\underline{c}), \end{aligned} \tag{5.39}$$

the result follows immediately. □

Step 2. $\frac{\lambda_1-\lambda_0}{r+\lambda_0}\underline{c} < \frac{\lambda_1-\lambda_0}{r+\alpha\lambda_0}\left[\frac{r(1-\alpha)}{(r+\lambda_0)(r+\lambda_1)}B + \alpha\underline{c}\right] < \frac{\lambda_1-\lambda_0}{(r+\lambda_0)(r+\lambda_1)}B.$

Proof. By Assumption 1, direct calculation yields

$$\begin{aligned} \frac{\lambda_1-\lambda_0}{r+\alpha\lambda_0}\left[\frac{r(1-\alpha)}{(r+\lambda_0)(r+\lambda_1)}B + \alpha\underline{c}\right] - \frac{\lambda_1-\lambda_0}{r+\lambda_0}\underline{c} &> \frac{\lambda_1-\lambda_0}{r+\alpha\lambda_0}\left[\frac{r(1-\alpha)}{r+\lambda_0}\underline{c} + \alpha\underline{c}\right] - \frac{\lambda_1-\lambda_0}{r+\lambda_0}\underline{c} \\ &= \frac{\alpha(\lambda_1-\lambda_0)}{r+\alpha\lambda_0}(\bar{c}-\underline{c}) \\ &> 0, \end{aligned} \tag{5.40}$$

and

$$\begin{aligned} \frac{\lambda_1-\lambda_0}{(r+\lambda_0)(r+\lambda_1)}B - \frac{\lambda_1-\lambda_0}{r+\alpha\lambda_0}\left[\frac{r(1-\alpha)}{(r+\lambda_0)(r+\lambda_1)}B + \alpha\underline{c}\right] &= \frac{\alpha(\lambda_1-\lambda_0)}{(r+\alpha\lambda_0)(r+\lambda_1)}[B - (r+\lambda_1)\underline{c}] \\ &> 0. \end{aligned} \tag{5.41}$$

□

Step 3. *If $\bar{c} - \underline{c} < \frac{\lambda_1-\lambda_0}{r+\lambda_0}\underline{c}$, then $r\Pi^o > r\Pi^f$ and $r\Pi^o > r\Pi^u$.*

Proof. $r\Pi^o > r\Pi^u$ follows immediately from Steps 1 and 2. And direct calculation gives

$$\begin{aligned} r\Pi^o - r\Pi^f &= \left\{\frac{\lambda_0}{r+\lambda_0}B - \lambda_0\bar{c}\right\} - \left\{\frac{\lambda_0}{r+\lambda_0}B - \frac{\lambda_0(r+\lambda_1)}{r+\lambda_0}\underline{c}\right\} \\ &= \lambda_0\left[\frac{\lambda_1-\lambda_1}{r-\lambda_0}\underline{c} - (\bar{c}-\underline{c})\right] \\ &> 0. \end{aligned} \tag{5.42}$$

□

Suppose first that $\frac{\lambda_1-\lambda_0}{(r+\lambda_0)(r+\lambda_1)}B > \frac{\lambda_1-\lambda_0}{r+\alpha\lambda_0}\underline{c}$, or $B > \frac{(r+\lambda_0)(r+\lambda_1)}{r+\alpha\lambda_0}\underline{c}$. Then Case III of "falsehood equilibrium" does not exist, and Case II reduces to $\frac{\lambda_1-\lambda_0}{r+\lambda_0}\underline{c} \le \bar{c}-\underline{c} \le \frac{\lambda_1-\lambda_0}{r+\alpha\lambda_0}\underline{c}$

Step 4. *If* $\bar{c}-\underline{c} \in \left(\frac{\lambda_1-\lambda_0}{r+\lambda_0}\underline{c}, \frac{\lambda_1-\lambda_0}{r+\alpha\lambda_0}\underline{c}\right)$, *then* $r\Pi^f > r\Pi^o$ *and* $r\Pi^f > r\Pi^u$.

Proof. Direct calculation gives

$$\begin{aligned} r\Pi^f - r\Pi^o &= \left\{\frac{\lambda_0}{r+\lambda_0}B - \frac{r\lambda_0(1-\alpha)}{r+\alpha\lambda_0}\bar{c} - \frac{\alpha\lambda_0(r+\lambda_1)}{r+\alpha\lambda_0}\underline{c}\right\} - \left\{\frac{\lambda_0}{r+\lambda_0}B - \lambda_0\bar{c}\right\} \\ &= \frac{\alpha\lambda_0(r+\lambda_0)}{(r+\alpha\lambda_0)}\left(\bar{c}-\underline{c}-\frac{\lambda_1-\lambda_1}{r+\lambda_0}\underline{c}\right) \\ &> 0, \end{aligned} \tag{5.43}$$

and

$$\begin{aligned} r\Pi^f - r\Pi^u &= \left\{\frac{\lambda_0}{r+\lambda_0}B - \frac{r\lambda_0(1-\alpha)}{r+\alpha\lambda_0}\bar{c} - \frac{\alpha\lambda_0(r+\lambda_1)}{r+\alpha\lambda_0}\underline{c}\right\} - \left\{\frac{\lambda_0(r+\alpha\lambda_1)}{r+\alpha\lambda_0}\left[\frac{1}{r+\lambda_1}B - \underline{c}\right]\right\} \\ &= \frac{\lambda_0(1-\alpha)}{r+\alpha\lambda_0}\left[\frac{\lambda_1-\lambda_0}{(r+\lambda_0)(r+\lambda_1)}B - (\bar{c}-\underline{c})\right] \\ &> \frac{\lambda_0(1-\alpha)}{r+\alpha\lambda_0}\left[\frac{\lambda_1-\lambda_0}{r+\alpha\lambda_0}\underline{c} - (\bar{c}-\underline{c})\right] \\ &> 0. \end{aligned} \tag{5.44}$$

□

Suppose now that $\frac{\lambda_1-\lambda_0}{(r+\lambda_0)(r+\lambda_1)}B < \frac{\lambda_1-\lambda_0}{r+\alpha\lambda_0}\underline{c}$, or $B < \frac{(r+\lambda_0)(r+\lambda_1)}{r+\alpha\lambda_0}\underline{c}$.

Step 5. *If* $\bar{c}-\underline{c} \in \left(\frac{\lambda_1-\lambda_0}{r+\lambda_0}\underline{c}, \frac{\lambda_1-\lambda_0}{(r+\lambda_0)(r+\lambda_1)}B\right)$, *then* $r\Pi^f > r\Pi^o$ *and* $r\Pi^f > r\Pi^u$.

Proof. This follows immediately from Step 4. □

Step 6. *If* $\bar{c}-\underline{c} > \frac{\lambda_1-\lambda_0}{(r+\lambda_0)(r+\lambda_1)}B$, *then* $r\Pi^u > r\Pi^f$ *and* $r\Pi^u > r\Pi^o$.

Proof. Direct calculation gives

$$\begin{aligned} r\Pi^u - r\Pi^f =& \left\{\frac{\lambda_0(r+\alpha\lambda_1)}{r+\alpha\lambda_0}\left[\frac{1}{r+\lambda_1}B - \underline{c}\right]\right\} - \\ & \left\{\frac{\lambda_0(r+\alpha\lambda_1)}{r(r+\lambda_0)+\alpha\lambda_0(r+\lambda_1)}\left[B - \frac{r(r+\lambda_0)}{r+\alpha\lambda_0}\bar{c} - \frac{\alpha\lambda_0(r+\lambda_1)}{r+\alpha\lambda_0}\underline{c}\right]\right\} \\ =& \frac{r\lambda_0(r+\alpha\lambda_0)(r+\lambda_0)}{[r(r+\lambda_0)+\alpha\lambda_0(r+\lambda_1)](r+\alpha\lambda_0)}\left[\bar{c}-\underline{c}-\frac{\lambda_1-\lambda_0}{(r+\lambda_0)(r+\lambda_1)}B\right] \\ >& 0. \end{aligned} \tag{5.45}$$

Then by Steps 1 and 2, $r\Pi^u > r\Pi^o$. □