Knowing What Matters to Others: Information Selection in Auctions.*

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Abstract The valuation of bidders for an object consists of a common value component (which matters to all bidders) and a private value component (which is relevant only to individual bidders). Bidders select about which of these two components they want to acquire noisy information. Learning about a private component yields independent estimates, whereas learning about a common component leads to correlated information between bidders. In a second-price auction, I show when bidders only learn about their private component, so an independent private value framework and efficient outcome arises endogenously. In a first-price auction, every robust equilibrium is inefficient under certain conditions. In the all-pay auction, in any equilibrium bidders prefer information about the more relevant component.

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1 Introduction

Bidding preparation for auctions usually involves evaluating multiple characteristics. This paper delves into which characteristics bidders should gather information about and how such decision is influenced by the auction format in cases wherein people cannot take into account all existing information.

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These issues are relevant to, for example, corporate takeovers, in which acquiring firms have access to a variety of information about a target company. This information encompasses the R&D activities and the book value. A reasonable assumption is that firms cannot perfectly process or uncover all existing information, and are thus driven to select elements to focus on before bidding takes place. Should an acquiring firm conduct research on aspects that are specific to them, such as their R&D overlap? Or should they focus on factors that also matter to other acquiring firms, such as the book value of a target?

Another example are resource rights auctions for oil fields or timber. Each bidder derives the same monetary value from an unknown volume of oil or timber on a site, and this value stems from the market price. Bidders may incur different costs in extracting the resources from a site because of the use of different drilling or logging technologies and variances in experience levels. I inquire into whether a bidder prefers to perform an exploratory drilling to learn about oil volume (i.e., the common component) or to learn about the costs of extracting the resource through estimations of the drilling costs specific to him (i.e., the private component).

The contribution of this paper is to investigate the incentives provided by a variety of auction formats regarding information selection,[1] in particular in a second price auction (SPA), a first price auction (FPA) and an all-pay auction (APA). The novelty of this paper lies in its illumination of *which* random variables bidders seek to learn (*information selection*) instead of what level of accuracy of information they favor about a given real-valued random variable (*information acquisition*). Holding the level of accuracy constant, do bidders prefer their private information to be dependent (information about a common component) or independent (information about a private component)?

The independent private values setting (IPV) and the interdependent values setting (IntV) lead to different theoretical predictions and vary significantly in their implications for auction design and policy. The literature on auctions typically assumes either IPV or IntV at the outset of the analysis. By restricting the ability of bidders to learn about more than one attribute, I study which value setting arises endogenously.

For a brief outline of the model, consider two bidders who compete for one indivisible object in a SPA. They share the same common component (e.g., the book value of a firm) and have independent private value components (e.g., match-specific R&D

[1] See Porter (1995) for a survey of oil and gas lease auctions and Hendricks and Porter (2014) for an analysis of the auction mechanisms in selling resource rights in the U.S. See Gentry and Stroup (2017) for an analysis of auctions and negotiation procedures commonly used in mergers and acquisition.

overlap). Bidders select how much to learn about a common and a private component. Information selection is simultaneous and covert.

Let the valuation of each bidder is the sum of two value components. Learning about the common or the private component has equal accuracy. In a single agent problem, an agent would be indifferent between learning about either component, as the two experiments are equally informative about the total value of the object. In an auction, information about the object plays a dual role. Beyond containing information about the object's worth, it is also informative about the signal of the opponent and his bid. Moreover, a rational bidder conditions his estimate of the object not only on his own information, but also on what he learns from the event of winning.

In my model, the extent of this winner's curse and the interdependence between bidders' information are endogenous and depend on the bidders' information choice. The signals of bidders become more affiliated if they learn more about the common component. The winner's curse exacerbates. If other bidders learn only about their private component, their information bears no relevance for other bidders and there is no winner's curse. Two standard valuation settings are nested in my model. An IPV setting arises if both bidders learn only about their private components. A pure common value setting emerges if both bidders learn only about the same common component.

The result for the SPA with two bidders is that in any symmetric equilibrium, information selection is unique: There is only learning about the private component, and an IPV setting is the unique equilibrium outcome. The SPA induces the ex-ante efficient outcome. No resources are wasted by learning about the common component which is irrelevant for efficiency, and the object is allocated to the bidder with the highest estimate of his private component. This result holds in a general class of value functions, as long as the private component has a weakly higher impact on overall value than the common component.

In the FPA, if the common component has a weakly higher impact on the overall value than the private component, any robust equilibrium is inefficient. Any outcome in which bidders learn to some extent about the private component (that is, not a pure CV) cannot be sustained. In the APA, looking at the more relevant component is the unique equilibrium candidate.

For a sketch of the argument, fix a symmetric bidding strategy for the bidders, and consider the effect of increasing or decreasing the correlation of his private signal with that of his opponent. Hence, while keeping the marginal distribution of signals fixed, vary the joint distribution of their signals. A higher correlation increases the second-order statistic of the signals and the distribution of the loosing bid. On the contrary, a higher correlation decreases the first-order statistic of the signals and the distribution of the winning bid. Conditional on winning, a bidder pays the second-order statistic in the SPA, and the first-order statistic in the FPA. By decreasing correlation by learning more about the private component, the distribution of the second order statistic puts more weight on lower bids, and expected payment strictly decreases in the SPA. In the FPA, as a winning bidder pays his own bid, he does not want to "leave money on the table" by overbidding his opponent by too much. Having a better estimate of the opponent's bid by increasing correlation reduces the expected payment conditional on a win as it reduces the first order statistic of winning bids. In the APA, as the marginal bid distribution does not vary, the expected payment conditional on winning is the same irrespective of the correlation between the bidders.

The approach is to find a deviation strategy that does not decrease the expected gain and keeps the winning probability constant, but strictly decreases the expected payment. The deviation strategy varies the correlation for a fixed marginal bid distribution as described in the last paragraph. For example, let the value of the object being the sum of the two components, and consider a candidate equilibrium of the SPA in which both bidders learn only about the common component. Then, the following deviation is strictly profitable for a bidder: *Learn about the private component, but bid as if it were a signal about the common component.* This strategy eliminates interdependence in private signals but employs the same bidding function as the candidate equilibrium for tractability. The expected gain from this deviation is the same as in the candidate equilibrium. For every realization of the total value of the object, the probability of placing the highest bid is the same with the candidate equilibrium and the deviation strategy. However, given a total value for the object, winning probability for different compositions of the two components changes with deviating. In the candidate equilibrium, as both bidders learn about the common component, they win with equal probability for each realization of it. In the deviation strategy, a deviating bidder is more likely to win in states that involve a high private and a low common component, and vice versa. The existence of a deviation strategy that leads to the same expected gain for a strictly lower payment pushes the incentives of bidders in the SPA towards independence, and yields a unique information choice in equilibrium of the SPA.

Section 1.1 describes the related literature. Section 2 introduces the model and the informational framework. In Section 3 I show how information selection impacts the the joint signal distribution and the resulting value framework. Section 4 narrows down the information choice of bidders in any equilibrium, and provides conditions for a unique information choice. I provide sufficient conditions for existence and uniqueness of the equilibrium of the SPA in Section 4.3. I show the impact of information selection on the revenue of the designer in Section 5.1, and extend some results to the case of more than two bidders in Section 5.2, before concluding in Section 6.

1.1 Related Literature

In the classic literature in Auction Theory, the distribution of private information of bidders is exogenous and does not depend of the choice of the auction format.[2] In their seminal work, Milgrom and Weber (1982) introduce a theory of affiliation in signals, and derive the equilibrium for the SPA, the FPA and English auction. The all-pay auction for affiliated signals has been analyzed by Krishna and Morgan (1997) and Chi et al. (2017).

The literature on information acquisition in auctions[3] endogenizes the private information of bidders, by asking *how much* costly information they seek to acquire about a single-dimensional payoff relevant variable. Persico (2000) considers costly information acquisition in an IntV model in the FPA and the SPA. Before bidding, bidders choose the *accuracy* (a statistical order by Lehmann (1988)) of their signal about a one-dimensional random variable. In the model of Persico (2000), learning with higher accuracy has two effects: first, the information about the own valuation becomes more precise; second, bidders obtain a better estimate of the signals of other bidders. Therefore, a higher accuracy inextricably links these two effects. Persico (2000) shows that incentives for information acquisition are stronger in the FPA than in the SPA.

In contrast to Persico (2000), in my model the accuracy of information is fixed and equal for each available signal. Bidders in my model have to select the variable about which they prefer to learn. The results in Persico (2000) are of a relative nature: given a level of accuracy acquired in the SPA, the level of accuracy in a FPA is higher. In contrast, my framework provides an absolute prediction: about *which* component do bidders learn.

[2]For an IPV setup, see Vickrey (1961) and Riley and Samuelson (1981). For a common value model, see Wilson (1969) and Milgrom (1981).

[3]Endogenous information acquisition has been analyzed in other areas of Economics. E.g., see Bergemann and Välimäki (2002), Crémer et al. (2009), Shi (2012) and Bikhchandani and Obara (2017) in optimal and efficient mechanism design, Martinelli (2006) and Gerardi and Yariv (2007) in committees, Crémer and Khalil (1992) and Szalay (2009) in principal-agent-settings, and Rösler and Szentes (2017) in bilateral trade.

In Bergemann et al. (2009), the value of an object is a weighted sum of everybody's payoff type. Information acquisition is binary: either learn perfectly about the own payoff-type, or learn nothing. Note that in this formulation, learning cannot introduce any dependence between the signal of bidders, as all payoff types are distributed independently (although they matter to other bidders). With positive interdependencies in payoff types, Bergemann et al. (2009) show that in a generalized Vickrey-Clarke-Groves mechanism[4] bidders acquire more information than would have been socially efficient.

In the IPV setup of Hausch and Li (1991), the SPA and FPA induce equal incentives to acquire information about the one-dimensional value. Stegeman (1996) shows that the incentives to acquire information in an IPV setting coincides in FPA and SPA, and with the incentives of a planner, making information acquisition efficient.

The above literature on information acquisition in auctions considers *covert* information acquisition. That is, bidders do not know how much information their competitors acquire before the auction. Another strand of the literature also analyzes *overt* information acquisition. Hausch and Li (1991) show that the SPA and the FPA induce different incentives to acquire information when information acquisition is overt, and revenue equivalence fails. Compte and Jehiel (2007) show in an IPV setup that an ascending dynamic auction induces more overt information acquisition and higher revenues than a sealed-bid auction. Hernando-Veciana (2009) compares the incentives to overtly acquire information in the English auction and the SPA, when bidders can learn about a common component or about a private component. In his model, it is exogenous which component information acquisition is about, while in my model, I endogenize the decision of information selection between the two components.

My paper also relates to the literature on *information choice* in games with strategic complementarities or substitutes. Hellwig and Veldkamp (2009) ask whether bidders want to coordinate on the same or on different information channels about the same one-dimensional state of the world in a beauty contest game. They show that the choice of information relates to the complementarity of actions in their model: if actions are strategic complements, agents want to know what others know. If actions are strategic substitutes, agents want different information channels. In a beauty contest game in Myatt and Wallace (2012), agents choose between multiple information channels about a common state variable. Agents decide how clearly (endogenous noise) to listen to

[4] See Dasgupta and Maskin (2000) for a generalized Vickrey-Clarke-Groves mechanism in the context of auctions, and Jehiel and Moldovanu (2001) for a general mechanism design setting with externalities in information and allocations.

which of many available signals, that vary in accuracy (exogenous noise).

Gendron-Saulnier and Gordon (2017) fix the informativeness of signals, similar to my approach. In their paper, agents have the choice between multiple information channels, that all have the same informativeness: they are all Blackwell sufficient for each other. Information channels vary in the level of dependence they induce between the signals of agents. Actions exhibit strategic complementarities, as in the framework of Hellwig and Veldkamp (2009) and Myatt and Wallace (2012).

There are two major differences between my model and the papers Hellwig and Veldkamp (2009), Myatt and Wallace (2012) and Gendron-Saulnier and Gordon (2017):[5] bidding functions do not exhibit strategic complementarities in the auction formats in my model (see e.g., Athey, 2002) which leads to a fundamentally different strategic problem. Further, in the above models, all channels contain information about the same single-dimensional payoff-relevant random variable (the one-dimensional state of the world). In contrast, in my model bidders choose about which component of the multidimensional state of the world to learn. Learning about their private component leaves them with an independent signal realization, irrespective of the information acquired by their opponent.

2 Model

Two risk/neutral bidders, indexed by $i \in \{1, 2\}$, compete for one indivisible object in an auction. The reservation value of the auctioneer and the outside options of the bidders are zero.

The valuation for the object of bidder i, denoted by V_i, depends on two attributes: a common value component S distributed with full support on $[0, 1]$, that is identical for all bidders, and a private value component T_i distributed with full support on $[0, 1]$, the idiosyncratic taste parameter of bidder i.[6] The common value component and the two private value components $\{S, T_1, T_2\}$ are drawn mutually independent, and T_1 and T_2 are drawn identically. It is without loss to assume that $\{S, T_1, T_2\}$ are drawn from a

[5]See also Yang (2015) for flexible information acquisition in investment games and Denti (2017) for an unrestricted information acquisition technology in potential games.

[6]The assumption of full support is for clarity of exposition. Results hold if there are two realizations in the support.

uniform distribution on $[0,1]$, given the assumption of full support.[7]

The value function for each bidder i is $V_i \coloneqq u(S, T_i)$ and has the same functional form for both bidders. The private component T_j of the other agent $j \neq i$ has no impact on the valuation V_i of bidder i.

Assumption 1. *The value function $u : [0,1]^2 \to \mathbb{R}$ satisfies $u(0,0) \geq 0$, $u(1,1) < \infty$, and is continuous and strictly increasing in both arguments in $(0,1)^2$.*

Due to Assumption 1, it is efficient to sell the object. From now on, I assume that every value function u satisfies this assumption. I consider value functions in the three classes of the following definition.

Definition 1. *The value function u is*

1. ***symmetric*** *if $u(a,b) = u(b,a)$,*
2. ***t-preferred*** *if $u(a,b) > u(b,a)$,*
3. ***s-preferred*** *if $u(a,b) < u(b,a)$,*

for all tuples $\{a,b\} \in (0,1)^2$ with $a < b$.[8]

A simple symmetric value function is $V_i = S + T_i$. If the value function is t-preferred, the bidder prefers to have a higher quantile of the private component than the common component (or, if s-preferred, the other way around). For example, the function

$$u(S, T_i) = S^{\alpha} T_i^{1-\alpha}$$

is t-preferred if $\alpha \in (0, \frac{1}{2})$, s-preferred if $\alpha \in (\frac{1}{2}, 1)$, and symmetric if $\alpha = \frac{1}{2}$.

Information Selection. The realizations of the random variables S, T_1, T_2 are unobservable to neither the auctioneer, nor the bidders. Instead, bidders engage in information gathering about their valuations. The information choice of bidder i is one of *information selection*: about which component should he learn how much.

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[7]Let $H_s(S)$ and $H_t(T_i)$ be two arbitrary strictly increasing distribution functions of S and T_i. Then, by a standard probability integral transformation, relabel each realization with its quantile, that is, $\hat{S} = H_s(S)$ and $\hat{T}_i = H_t(T_i)$. Then, $\hat{S}$ and $\hat{T}_i$ is distributed uniformly on $[0,1]$. Then, relabel the value function in terms of quantiles, such that $\hat{u}(\hat{S}, \hat{T}_i) \coloneqq u(H_s^{-1}(\hat{S}), H_t^{-1}(\hat{T}_i)) = u(S, T_i)$.

[8]This can be relaxed, such that the strict inequality holds for a non-zero measure of $(0,1)^2$.

For each bidder $i \in \{1, 2\}$, there are two potential sources of information: random variable X_i^S about S, and random variable X_i^T about T_i. Random variable X_i^T contains information only about the private component T_i, random variable X_i^S is only informative about the common component S.

Both signals X_i^T and X_i^S have support $[0, 1]$. The marginal distribution of the random variables X_i^T or X_i^S, conditional on the state $T_i = r$ or $S = r$, have a cumulative distribution function $F^T(\cdot|r)$ or $F^S(\cdot|r)$ for $r \in [0, 1]$. Learning X_i^S (X_i^T) is the most accurate signal that is available about S (T_i) in this environment.

Assumption 2. *For all $r \in [0, 1]$, the distribution $F^S(x_i|r)$ and $F^T(x_i|r)$ admit a density $f^S(x_i|r)$ and $f^T(x_i|r)$, such that:*

(2A) $\forall x_i \in [0, 1] : f^S(x_i|r) = f^T(x_i|r) =: f(x_i|r)$.

(2B) $\forall x'_i > x_i$, $\frac{f(x'_i|r)}{f(x_i|r)}$ *is strictly increasing in r.*

Assumption 2A implies the same informativeness on each available signal about its component. Let $F(x) := F^S(x_i|r) = F^T(x_i|r)$. The signals X_i^S and X_i^T satisfy a strong monotone likelihood ratio property (MLRP) in Assumption 2B such that higher signal realizations are more indicative of higher realizations of a component. Moreover, let $f(x_i|r)$ be continuously differentiable in x_i for all r.

Due to the following assumptions, the private signals of bidders can only be interdependent via learning about the common variable S:

Assumption (CI)**.** $X_i^S \perp\!\!\!\perp X_j^S \mid S$.

Assumption (IN)**.** $X_i^T \perp\!\!\!\perp X_j^T$*, and* $X_i^T \perp\!\!\!\perp X_j^S$.

By Assumption CI, X_i^S and X_j^S are independent conditional on S.[9] According to Assumption IN, signal X_i^T is independent from both signals X_j^S and X_j^T available to his opponent j.

Bidders cannot observe both signal realizations X_i^S and X_i^T, due to physical constraints or time limitations. Instead, they face a trade-off between learning about the common or the private component.

Bidders learn one random variable X_i with support on $[0, 1]$, that is some combination of X_i^S and X_i^T. It is a a mixture distribution over two random variables: bidders choose the probability ρ_i of learning $X_i = X_i^S$. With probability $1 - \rho_i$, bidder i learns

[9]As X_i^S and S are affiliated by Assumption 2A, the random variables X_1^S and X_2^S are affiliated.

X_i^T. E.g., with $\rho = 0.5$, it is equally likely that $X_i = X_i^s$ as $X_i = X_i^t$. Let $\boldsymbol{\rho} = \{\rho_1, \rho_2\}$ be the vector of information selection variables.

The information selection variable $\rho_i \in [0, 1]$ captures a continuous learning choice between the two components.[10] This learning process resembles a truth-or-noise technology as in Johnson and Myatt (2006). It creates a signal that contains information about both components, where the agent decide how to split attention between his two value components without changing the marginal distribution of X_i. The only costs of learning more about one component are the opportunity costs of not learning more about the other component.

The game consists of two stages:

1. After an auction format is announced, bidders simultaneously select how to split their attention between X_i^S and X_i^T by choosing ρ_i.

2. Bidders privately observe their signal X_i and bid in the auction.

Information selection is *covert* throughout both stages: bidders do not observe the information selection of their opponent (but anticipate it correctly in equilibrium). Moreover, bidders select their information *after* the auction format is announced. This enables an analysis of the incentives of various auctions on information selection.

3 The Impact of Information Selection

The *marginal* distribution of bidder i's signal X_i with information choice ρ_i is

$$\Pr(X_i \leq x | \rho_i) = (1 - \rho_i) F^T(x) + \rho_i F^S(x) = F(x),$$

where $F^S(x) \coloneqq \Pr(X_i^S \leq x) = \int_0^1 F(x|r)dr =\colon F^T(x)$ is the unconditional distribution function of a bidders' private signal when he learns about either component. It is not a function of ρ_i, as applying the signal to both components results in the same distribution of signals due to $F^S(x) = F^T(x)$.

The *joint* distribution of X_1 and X_2 is endogenous as it depends on the information

[10] An interior ρ_i is not a mixed strategy. See a previous version of this paper, Bobkova (2017), for a discrete learning model: bidders learn either X_i^S or X_i^T perfectly, or mix between the two signals for interior ρ_i. The agent observes which experiment his signal stems from for any randomization.

choices ρ_1 and ρ_2 of the bidders. The joint density is

$$g(x_i, x_j|\rho_i, \rho_j) = (1 - \rho_i\rho_j)f(x_i)f(x_j) + \rho_i\rho_j \int_0^1 f(x_i|s)f(x_j|s)ds. \tag{1}$$

With probability $(1 - \rho_i\rho_j)$ at least one bidder observes a signal about his private attribute T_i and signals are independent by Assumption IN. With the remaining probability $\rho_i\rho_j$, bidders observe correlated (and conditionally independent) signals about the same realization of the common attribute S.

Let bidder i have a signal realization x_i, given a vector of information choice $\{\rho_i, \rho_j\}$. Then, his probability of having a higher signal than his opponent is

$$G_j(x_i|x_i, \rho_i, \rho_j) := \Pr(X_j \leq x_i|X_i = x_i, \rho_i, \rho_j) \tag{2}$$

$$= \frac{\int_0^{x_i} g(x_i, x_j|\rho_i, \rho_j)dx_j}{f(x_i)}. \tag{3}$$

The degree of the winner's curse is endogenous in my model. Let the expected value of the object, given two signal realizations and information choices be

$$v[x_i, x_j; \rho_i, \rho_j] := \mathbb{E}\left[V_i|X_i = x_i, X_j = x_j; \rho_i, \rho_j\right].$$

If bidder j learns only about his private component T_j (by setting $\rho_j = 0$), his signal $X_j = X_j^T$ contains no payoff relevant information for i, and there is no winner's curse for bidder i from winning. If $\rho_j = 1$, the signal $X_j = X_j^S$ is as informative for bidder i as it is for j, as it only contains information about the common component S. The following two value settings are nested in my model:

1. **Independent private values (IPV).** If $\rho_1 = \rho_2 = 0$, private signals X_1 and X_2 are independent. Bidder i's expected value does not depend on bidder j's signal:

$$v[x_i, x_j; 0, 0] = \mathbb{E}\left[V_i|X_i = x_i, \rho_i = 0\right].$$

2. **Pure Common values/ mineral rights model (CV).** If $\rho_1 = \rho_2 = 1$, expected value of the bidders is symmetric in the two private signals X_1 and X_2:

$$v[x_i, x_j; 1, 1] = v[x_j, x_i; 1, 1].$$

For any interior information selection $\rho \in (0, 1)$, a general IntV framework applies

(Milgrom and Weber, 1982).

Define the (random) sum of the two value components for bidder i, the variable $\omega_i \in \Omega_i = S + T_i$ with $\omega_i \in [0, 2]$. Being the sum of two uniform random variables, the random variable Ω_i is distributed with a symmetric triangular distribution with density function $h(\omega_i) = \begin{cases} \omega_i \text{ if } 0 < \omega_i < 1, \\ 2 - \omega_i \text{ if } 1 \leq \omega_i < 2, \\ 0 \text{ otherwise.} \end{cases}$

4 Analysis

Let M be an auction mechanisms in which the highest bid wins the object, and ties are broken evenly. I consider a symmetric Bayes Nash equilibrium, where bidders select the same pure $\rho_i = \rho^M$, and follow a pure and non-decreasing bidding function $\beta^M(X_i)$. The remainder of this section is devoted to proving the following main result.

Theorem 1 (Main Result)**.**

1. *In the SPA, in any equilibrium $\rho^{II} = 0$ if the value function is symmetric or t-preferred.*
2. *In the FPA, in any equilibrium $\rho^I = 1$ if the value function is s-preferred, and $\rho^I \in \{1, 0\}$ if it is symmetric.*[11]
3. *In the APA, in any equilibrium $\rho^A = 0$ if the value function is t-preferred, and $\rho^A = 1$ if s-preferred.*

IPV is the unique equilibrium outcome of the SPA in the symmetric or t-preferred framework. Then, there is no learning about the common component in any equilibrium of the SPA, if the bidders weakly prefer the private over the common component.

Pure CV is the unique equilibrium outcome in the FPA if the value function is s-preferred, and an equilibrium if it is symmetric (in Section 4.3 I show that the other equilibrium $\rho^I = 0$ for a symmetric value function is trivial and non-robust).

Let $CE^M := \{\rho^M, \beta^M\}$ be a *candidate equilibrium* in an auction mechanism M. In what follows, I mostly use the following *class of deviation strategies* to rule out candidate equilibria: $\{\rho_i, \beta^M\}$. That is, a bidder uses the same bidding function as in the candidate equilibrium, but change for a different information selection strategy. The

[11]See Section 4.3 for an argument why the $\rho^I = 0$ equilibrium is trivial and not robust.

advantage of this deviation strategy is tractability, as with such a deviation strategy, the event of winning occurs if and only if $X_i \geq X_j$.[12]

Let $U_i^M(\rho_i|CE^M)$ be the expected utility of bidder i with strategy $\{\rho_i, \beta^{CE}\}$ in auction M, whose opponent plays according to the candidate equilibrium. It can be separated into his the expected gain $EG(\rho_i|CE^M)$ minus his payment conditional on winning $W(\rho_i|CE^M)$ times the probability of winning $P(\rho_i|CE^M)$:

$$U_i(\rho_i|CE) \coloneqq EG(\rho_i|CE^M) - W(\rho_i|CE^M)P(\rho_i|CE^M). \tag{4}$$

In the following, I first show the impact of this deviation strategy on the expected gain and the winning probability, and thereafter on the expected conditional payment.

4.1 Expected Gain and Winning Probability

First, consider the winning probability of bidder i point-wise at every $\omega_i = S + T_i$. If the opponent j follows the candidate equilibrium CE^M and bidder i deviates to $\{\rho_i, \beta^M\}$,

$$\Pr(i \text{ wins}|\omega_i, \rho_i, CE^M) = \Pr(X_i \geq X_j|\omega_i, \rho_i, \rho^M).$$

Lemma 1. *For any $a, b \in [0,1]$,*

$$\frac{\partial \Pr(X_i \geq X_j|S = a, T_i = b, \rho_i, \rho_j)}{\partial \rho_i} = -\frac{\partial \Pr(X_i \geq X_j|S = b, T_i = a, \rho_i, \rho_j)}{\partial \rho_i} \begin{cases} > 0 \text{ if } a > b, \\ = 0 \text{ if } a = b, \\ < 0 \text{ if } a < b. \end{cases}$$

If bidder i learns more about S and less about T_i by increasing ρ_i, his probability of winning (with deviation strategy $\{\rho_i, \beta^M\}$) increases if the common component is higher than his private component, and decreases otherwise. Hence, increasing ρ_i shifts winning probability into S-T_i-combinations with a higher common component.

Lemma 2. *Let $CE^M = \{\rho^M, \beta^M\}$. For any ω_i, any information selection $\rho_i \in [0,1]$ with bidding function β^M yields the same winning probability,*

$$\frac{\partial \Pr(i wins|\omega_i, \rho_i, \beta_i^M; CE^M)}{\partial \rho_i} = 0.$$

[12] Ties have zero probability and are ignored.

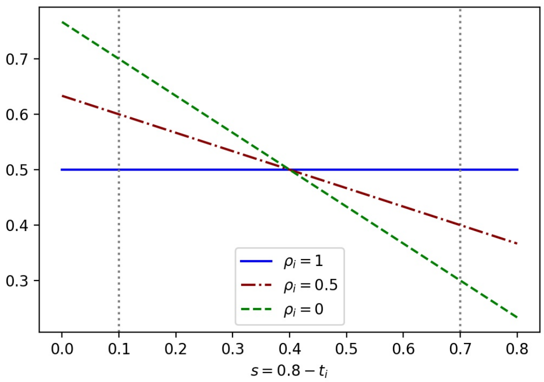

Figure 1: Winning probability with $\omega_i = s + t_i = 0.8$ and $\rho_j = 1$, for different ρ_i. Signals have densities $f(x|r) = (2 - 2r) + (4r - 2)x$.

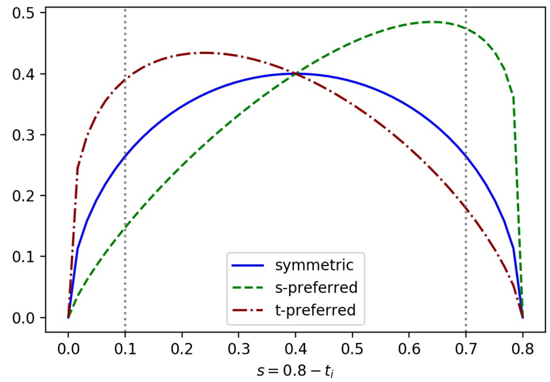

Figure 2: Utilities for $\omega_i = 0.8$ with $u(s, t_i) = s^{0.5}t_i^{0.5}$ (blue solid), $u(s, t_i) = s^{0.8}t_i^{0.2}$ (green dashed) and $u(s, t_i) = s^{0.3}t_i^{0.7}$ (purple dotted dashed).

As long as the bidder follows the same bidding function β^M as in the candidate equilibrium, his information selection ρ_i has no impact on the probability of winning for every realization ω_i.

Figure 1 shows the probability of bidder i having the highest signal realization (and hence, winning with $\{\rho_i, \beta^M\}$) on the y-axis when $\omega_i = s + t_i = 0.8$. His opponent chooses $\rho_j = 1$ and learns only about S. The x-axis shows the realization of the common component S that can arise with $\omega_i = 0.8$; the corresponding private component at each point of the x-axis is $t_i = 0.8 - s$. If bidder i chooses $\rho_i = 1$, both bidders learn only about the common component. As they have access to the same information technology, winning probability is one-half for any realization of s, given ω_i (blue solid line).

If bidder i learns more about his private component T_i and less about S_i, winning probability changes (green and purple lines), as shown in Lemma 1. The two vertical grey lines indicate how bidder i gains winning probability in state $(s = 0.7, t_i = 0.1)$, and looses the same mass of winning probability in state $(s = 0.7, t_i = 0.1)$. Due to the MLRP, higher signals are more likely for higher realizations of the value components. Hence, the lower ρ_i, the more likely bidder i wins in states with a high *private* component realization, and the less likely he wins with a high common component realization. If the components coincide, $s = t_i = 0.4$, varying ρ_i has no effect as it results in the same joint signal density.

As the two realizations $(s = 0.7, t_i = 0.1)$ and $(s = 0.7, t_i = 0.1)$ are equally likely, their overall effect on winning probability cancels out. The same argument holds for any feasible pair (a, b) and (b, a) such that $a + b = 0.8$. For any $\rho_i \in [0, 1]$, and given

$\omega_i = 0.8$, bidder i has the same winning probability as proven in Lemma 2. Hence, information selection shuffles the combinations of S-T_i states in which bidder i wins, while keeping the overall probability of winning at some ω_i fixed.

The total probability of winning is

$$P(\rho_i|CE^M) := \int_{\Omega_i} \Pr(i \text{ wins}|\omega_i, \rho_i, \beta^M; CE^M) h(\omega_i) d\omega_i. \tag{5}$$

Corollary 1 (Constant Winning Probability)**.** *Let $CE^M = \{\rho^M, \beta^M\}$. For any $\rho_i \in [0,1]$, strategy $\{\rho_i, \beta^M\}$ yields the same probability of winning $P(\rho_i|CE^M) = \frac{1}{2}$.*

This is an immediate corollary of Lemma 2. For any ω_i, winning probability does not depend on ρ_i. Hence, total winning probability is constant for any ρ_i.

The impact of a deviation (ρ_i, β^{CE}) on the expected gain depends on the class of value function.

Proposition 1 (Expected Gain)**.** *Let $CE^M = \{\rho^M, \beta^M\}$ and bidder i unilaterally deviate to $\{\rho_i \neq \rho^M, \beta^M\}$. The expected gain $EG(\rho_i|CE^M)$ is*

1. *constant for all $\rho_i \in [0,1]$ if u is symmetric,*
2. *strictly decreasing in ρ_i if u is t-preferred,*
3. *strictly increasing in ρ_i if u is s-preferred.*

The proof proceeds by showing that point-wise for any $\omega_i = s + t_i$, expected utility is increasing in ρ_i if $u(s, t_i)$ is s-preferred, decreasing in ρ_1 if it is t-preferred, and constant if it is symmetric.

As established in Lemma 1 and in Figure 1, an increase in ρ_i symmetrically shifts winning utility from $(S = 0.1, T_i = 0.7)$ to $(S = 0.7, T_i = 0.1)$. Figure 2 plots three different value function for a fixed sum of the two components $\omega_i = 0.8$, with S on the x-axis. The effect of increasing ρ_1 can be seen at the two vertical lines at $(S = 0.1, T_i = 0.7)$ and $(S = 0.7, T_i = 0.1)$.

For a symmetric value function, the blue solid line in Figure 1 shows that a bidder is indifferent between $(0.1, 0.7)$ to $(0.7, 0.1)$ (and any other symmetric pair of S-T_i-combination). Therefore, an increase in ρ_i will not have any effect on the expected gain. If the value is s-preferred (green dashed line), bidder i wins with a higher probability at $(S, T_i) = (0.7, 0.1)$ if he increases ρ_i, which he prefers over $(0.1, 0.7)$. Hence, an increase in ρ_i raises overall expected gain: a bidder is more likely to win when he values the object more. For a t-preferred value function (purple dotted-dashed line), an increase in ρ_i shifts winning probability into less favourable states with a higher common than private component (e.g., into $(S, T_i) = (0.7, 0.1)$ instead of $(0.1, 0.7)$).

4.2 Expected Payment

The first- and second-order statistics of the bids vary within the class $\{\rho_i, \beta^M\}_{\rho_i \in [0,1]}$. Given an information selection vector $\{\rho_i, \rho_j\}$, let $G_{(1)}(x|\rho_i, \rho_j)$ denote the first-order statistic of the two signals X_1 and X_2, and $G_{(2)}(x|\rho_i, \rho_j)$ the second-order statistic.

For two distribution functions, write $F \succeq_{FOSD} G$ if distribution F first-order stochastically dominates G (i.e., $F(x) \leq G(x)$ for all x). Write $F \succ_{FOSD} G$ if distribution $F \succeq_{FOSD} G$ and $E_F[x] > E_G[x]$. Write $F =_{FOSD} G$ if $F \succeq_{FOSD} G$ and $G \succeq_{FOSD} F$. The following holds for the first- and second-order statistic of signals in this information structure.

Lemma 3 (Order Statistics)**.** *Let $\rho_i > \rho_i'$.*

1. *If $\rho_j \neq 0$, then $G_{(2)}(.|\rho_i, \rho_j) \succ_{FOSD} G_{(2)}(.|\rho_i', \rho_j)$ and $G_{(1)}(.|\rho_i', \rho_j) \succ_{FOSD} G_{(1)}(.|\rho_i, \rho_j)$.*
2. *If $\rho_j = 0$, then $G_{(2)}(.|\rho_i, \rho_j) =_{FOSD} G_{(2)}(.|\rho_i', \rho_j)$ and $G_{(1)}(.|\rho_i', \rho_j) =_{FOSD} G_{(1)}(.|\rho_i, \rho_j)$.*

Bidder i can influence the correlation between his signal X_i and that of his opponent X_j if $\rho_j > 0$.[13] Increasing correlation by increasing ρ_i results in a higher second-order statistic and a lower first-order statistic of the signals in the sense of FOSD. This becomes apparent with $\rho_j = 1$ and as signals become perfectly informative: if bidder i also sets $\rho_i = 1$, he observes the same signal realization as his opponent. In this case, under perfectly informative signals, the first- and second-order statistics coincide. As the positive correlation decreases (by decreasing ρ_i), the wedge between the first-order and second-order statistic increases.

If a bidder plays the same bidding function β^M as his opponent, he wins if and only if he has a higher signal realization X_i than his opponent, irrespective of his information choice ρ_i. That is, for any ρ_i and conditional on winning, bidder i pays the bid of the second-order statistic in the SPA, and the bid of the first-order statistic in the FPA,

$$W^{II}(\rho_i|CE^{II}) = \int_0^1 \beta^{II}(x) dG_{(2)}(x|\rho_i, \rho^{II}). \tag{6}$$

$$W^{I}(\rho_i|CE^{I}) = \int_0^1 \beta^{I}(x) dG_{(1)}(x|\rho_i, \rho^{I}). \tag{7}$$

[13] If $\rho_j = 0$, due to Assumption (IN), bidder i's signal is independent of X_j for any $\rho_i \in [0, 1]$.

The bidding functions β^I and β^{II} are non-decreasing, and by Lemma 3 the order statistics $G_{(1)}$ and $G_{(2)}$ can be FOSD-ordered in ρ_i. This translated into the following effect on expected payment conditional on winning $W^M(\rho_i|CE^M)$.

Proposition 2. *For an auction mechanism $M \in \{II, I, A\}$, let $CE^M = \{\rho^M, \beta^M\}$ be a candidate equilibrium with β^M any non-decreasing bidding function. Let $\rho^M > 0$.*

1. *In the SPA, $W^{II}(\rho_i|CE^{II})$ is strictly increasing in ρ_i.*
2. *In the FPA, $W^I(\rho_i|CE^I)$ is strictly decreasing in ρ_i.*
3. *In the all-pay auction, $W^A(\rho_i|CE^A)$ is constant for any $\rho_i \in [0, 1]$.*

Let $\rho^M = 0$. Then, $W^M(\rho_i|CE^M)$ is constant for any $\rho_i \in [0, 1]$.

In the SPA, decreasing correlation leads to a lower second-order statistic, and hence, a lower expected payment conditional on winning. This effect is reversed for the FPA, as the effect of an increase in correlation on the first-order statistic is reversed to the second-order statistic. In the APA, a bidder pays irrespective of winning, and the marginal distribution of X_i, $F(.)$, does not depends on the information choice ρ_i or ρ_j. Hence, if he bids with the same bidding function β^A, his expected payment is the same.

Overall effect of a (ρ_i, β^M)-deviation. The main Theorem 1 follows by combining Corollary 1, Proposition 1 with Proposition 2. For example, for a symmetric value function $u(S, T_i)$, decreasing ρ_i while keeping β^M fixed has the following effect: the deviation yields the same expected gain (Proposition 1), the same winning probability of one half (Corollary 1), and a strictly lower (higher) payment in the SPA (FPA) (Proposition 2). Hence, a decrease (increase) in ρ_i constitutes a strictly profitable deviation in the SPA (FPA) for any $\rho^{CE} > 0$.

Social surplus is maximized if a bidder with the highest expected private component T_i receives the object. All bidders share the same common component S, which therefore plays no role for the social surplus. Information about the common component is not socially valuable, and available only by incurring the opportunity costs of not learning about the private component. For a symmetric or t-preferred value function, Theorem 1 establishes any equilibrium of the SPA is ex-ante efficient as it induces $\rho^{II} = 0$ and allocates efficiently.

4.3 Equilibrium Existence

Theorem 1 focuses on which information selection *cannot* be part of an equilibrium. The next result shows when an equilibrium exists.

Definition 2. *A value function satisfies increasing differences in T_i if $u(a,b) - u(b,a)$ is non-decreasing in b for every a.*

The value functions $u(S,T_i) = \alpha S + (1-\alpha)T_i$ for $\alpha < \frac{1}{2}$ or the product value $u(S,T_i) = ST_i$ satisfy increasing differences in T_i. Any symmetric value function u satisfies increasing differences. If a value function is s-preferred, it cannot satisfy increasing differences. If a value function has increasing differences in T_i, the difference $E[V_i|X_i^T = x_i] - E[V_i|X_i^S = x_i]$ crosses zero exactly once from below as the signal x_i increases, as shown in the proof of the following.

Proposition 3. *Let the value function u satisfy increasing differences in T_i. Then, there exists an equilibrium with $\rho = 0$ in the SPA, FPA and APA.*

With increasing differences in T_i, IPV is always an equilibrium outcome of the three auctions. This is easiest seen for a symmetric value function. If bidder 2 learns only about his private component T_2, his signal X_2 is always independent from the signal of bidder 1, X_1. This holds irrespective of bidder 1's information choice ρ_1. Then, the information choice of bidder 1 serves only the purpose to learn in the most informative way about his total value, not to vary the correlation between the signals or to mitigate the winner's curse. As each value component is equally informative about the total value with a symmetric value function, the IPV outcome is sustainable as an equilibrium.

Corollary 2. *Let the value function u satisfy increasing differences in T_i. Then, there exists an essentially*[14] *unique equilibrium in the SPA in which $\rho^{II} = 0$.*

Hence, for the SPA and a value function with increasing differences in T_i, my analysis yields the existence of a unique information choice equilibrium.

Next, consider the unique possible information choice for the FPA, if the value function is s-preferred. For the FPA, it is straightforward to see that $\rho^I = 1$ cannot be the equilibrium under some conditions. Let $\rho^I = 1$ and consider fully revealing signals about the components. Then, if both bidders choose $\rho_i = 1$, they both observe the same signal realization $X_1^S = X_2^S = S$. In this case, the bidders do not obtain any

[14] It is unique up to the bid of the lowest signal realization bidder who never wins.

information rent as their signal is essentially public, and bid their true estimate of the good $E[V_i|X_i^S = S]$. Both bidders have an expected utility of zero. A simple deviation of a bidder to $\rho_i = 0$ and bidding $E[V_i|X_i^T = x, S = 0]$ is a strictly profitable deviation. By introducing a sufficiently small amount of noise in the private signals, bidders bid so close to their true value that their expected utility can be made arbitrarily close to zero. Then, deviating to the private component is strictly profitable and a CE with $\rho^I = 1$ cannot be sustained for sufficiently precise signals. Then, the equilibrium of the FPA might only exist in mixed strategies.

In the APA, bidders always pay their bid, irrespective of the event of winning. They win if they submitted a higher bid than their opponents. Krishna and Morgan (1997) analyze the all-pay auction in a symmetric interdependent value framework. They find a condition such that a symmetric equilibrium in increasing strategies exists.

4.4 Robustness

For the remainder of this section, let the value function be symmetric. By Theorem 1, in any equilibrium of the SPA it holds that $\rho^{II} = 0$. In the FPA, there are two candidates for an equilibrium, $\rho^I \in \{0, 1\}$. In the following, I show that $\rho^I = 0$ can be ruled out by a slight perturbation of the model: I introduce a small degree of interdependence between the bidders by introducing a small tremble into their information choice. Then, there is a force in the FPA pushing the bidders towards higher correlation.

With probability $\epsilon > 0$, a bidder 'trembles' when choosing his information and his signal is $X_i = X_i^S$ and contains only information about the common component.[15] With probability $1 - \epsilon$, his signal X_i contains information about S and T_i as in the model depending on his chosen ρ_i.[16]

This formulation guarantees a strictly positive correlation between the signals of the bidders for any information choice. Hence, if $\epsilon > 0$, a bidder can vary the degree of correlation with his opponent even if his opponent chooses $\rho_j = 0$. If $\epsilon = 0$, the original model applies: $\rho_j = 0$ results in independent signals $X_i \perp\!\!\!\perp X_j$ by Assumption (IN).

Proposition 4. *Let $\epsilon > 0$, and $u(S, T_i)$ symmetric. In any equilibrium of the SPA, $\rho^{II} = 0$. In any equilibrium of the FPA, $\rho^I = 1$.*

[15] The following results will also hold for any exogenous mixing where the bidder learns about the common component with strictly positive probability.

[16] An alternative formulation is that for each bidder, with probability $\epsilon > 0$, his private and common components are perfectly correlated. While this significantly complicates the notation, this does not change the result. A similar formulation can be found in a previous version (Bobkova, 2017).

Hence, any equilibrium of the SPA is IPV and ex-ante efficient. Any robust equilibrium in the FPA is inefficient. A strictly profitable deviation in the FPA is to increase correlation $\rho_i > \rho^I$ and bid with the same bidding strategy β^I as in the candidate equilibrium. This way, a bidder obtains the same expected gain for a strictly lower payment. The lower payment stems from a strictly lower distribution of winning bids (via a strictly lower first-order statistic of signals). If $\epsilon > 0$, bidder i can increase correlation with his opponent for any $\rho^I < 1$. This rules out an equilibrium with $\rho^I = 0$ in the FPA.

5 Extensions

5.1 Revenue

In this section, I analyze the effect of ρ on the expected revenue for the auctioneer in the SPA. Increasing ρ increases the correlation between the two bidders' private information, hence, links their private information. While intuition suggests that there is similarity to the Linkage Principle from Milgrom and Weber (1982), there are multiple effects at play and the overall effect on revenue is ambiguous.

Let $\Pi^{SPA}(\rho)$ be the expected revenue in the SPA in a symmetric equilibrium, where both bidders chose ρ (exogenously) and bid optimally. The expected revenue for the auctioneer in the SPA can be computed as follows.

$$\Pi^{SPA}(\rho) = \int_0^1 \beta^{SPA}(x|\rho) f_{(2)}(x|\rho, \rho) dx \tag{8}$$

$$= \int_0^1 E[V_i|X_1 = x, X_2 = x|\rho] f_{(2)}(x|\rho, \rho) dx. \tag{9}$$

First, I provide numerical examples. Let $S, T_i \in \{0, 1\}$, and a symmetric additive value function $V_i = S + T_i$. Figure 3 plots the expected revenue as a function of ρ, for three signal distributions: (i) $f(x|0) = 3(x-1)^2$ and $f(x|1) = 1$; (ii) $f(x|0) = 1$ and $f(x|1) = 3x^2$; (iii) $f(x|0) = 1$ and $f(x|1) = 2x$.

In Examples (i) and (iii), the highest revenue is achieved with $\rho = 0$. In Example (ii), $\rho = 1$ maximizes revenue. Furthermore, Examples (i) and (iii) show that revenue can be non-monotonic in ρ. Hence, the relation between revenue and ρ depends on the parametric framework of the signal distributions. To provide further insights why the overall effect on revenue is ambiguous, I de-construct the change in revenue from an

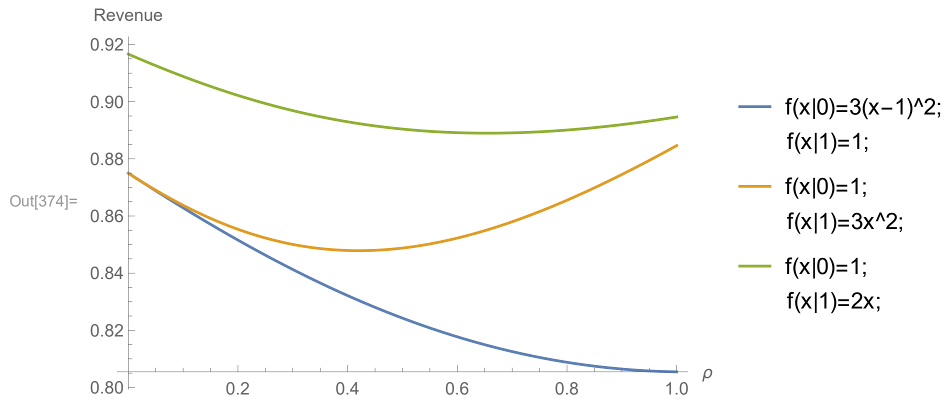

Figure 3: Revenue as a function of ρ for three numerical examples with binary uniform $S, T_i \in \{0, 1\}$.

increase in ρ.[17] Let $\rho < \rho'$, and let the change in the bid and the second-order statistic be defines as

$$\Delta_{\beta}(x|\rho', \rho) = \beta^{SPA}(x|\rho') - \beta^{SPA}(x|\rho)$$
$$\Delta_{f_{(2)}}(x|\rho', \rho) = f_{(2)}(x|\rho', \rho') - f_{(2)}(x|\rho, \rho)$$

Then, the difference in revenue can be expressed as

$$\Pi^{SPA}(\rho') - \Pi^{SPA}(\rho) = \int_0^1 \Delta_{\beta}(x|\rho', \rho) f_{(2)}(x|\rho, \rho) dx \tag{10}$$
$$+ \int_0^1 \beta^{SPA}(x|\rho') \Delta_{f_{(2)}}(x|\rho', \rho) dx. \tag{11}$$

The effect of a change in ρ can be separated into two effects: the first summand in Expression 10 isolates the impact on bidding, given a second-order statistic (*linkage effect*); the second summand in Expression 11 isolates the effect on the second order statistic, given a bidding function (*second-order statistic effect*). This second-order statistic effect is absent in the classical framework of the Linkage Principle in Milgrom and Weber (1982). In their model, disclosing information (publicly) or choosing between different auction formats has an effect on the bidding function (linkage effect), but not on the second-order statistic.[18]

In my model, the second-order statistic effect is present and unambiguously non-

[17] If revenue, the bidding function and the second order statistic are differentiable in ρ, the following expressions can be simplified into a marginal statement.

[18] Varying the linkage in Milgrom and Weber (1982) has no effect on the joint distribution of the initial private signals, X_1 and X_2, and hence, no effect on the second-order statistic.

negative for $\rho' > \rho$,

$$\int_0^1 \beta^{SPA}(x|\rho')\Delta_{f_{(2)}}(x|\rho',\rho)dx \geq 0.$$

This is because the fixed bidding function $\beta^{SPA}(x|\rho^*)$ is non-decreasing, and by Lemma 3, $F_{(2)}(.|\rho',\rho') \geq_{FOSD} F_{(2)}(.|\rho,\rho') \geq_{FOSD} F_{(2)}(.|\rho,\rho)$. Increasing the correlation between the private information of the bidders raises revenues, given a fixed bidding distribution.

Given a symmetric information selection ρ, by the argument in Milgrom and Weber (1982), a symmetric equilibrium is to bid $\beta^{SPA}(x|\rho) = v_i[x,x,\rho,\rho]$. Define bidder i's change in expected value from a change in ρ_i or ρ_j as

$$\Delta_{\rho_i} v_i[\rho_i', \rho_i; x, \rho_j] := v_i[x,x,\rho_i',\rho_j] - v_i[x,x,\rho_i,\rho_j],$$
$$\Delta_{\rho_j} v_i[\rho_j', \rho_j; x, \rho_i] := v_i[x,x,\rho_i,\rho_j'] - v_i[x,x,\rho_i,\rho_j].$$

The linkage effect in Expression 10 can be decomposed further using this notation, to isolate the effect of a bidder's own information selection ρ_1 and his opponent's ρ_2 on equilibrium bidding.

$$\Delta_\beta(x|\rho',\rho) = \Delta_{\rho_i} v_i[\rho',\rho;x,\rho] + \Delta_{\rho_j} v_i[\rho',\rho;x,\rho']$$

I show that the effect of both terms on revenue are ambiguous, by means of a numerical example. Let $S, T_i \in \{0,1\}$ binary with equal probability, and signal densities be $f(x|0) = 2-2x$ and $f(x|1) = 2x$. For this symmetric setup, any signal $x < 0.5$ is bad news about the value of the object, and any signal $x > 0.5$ good news.[19] Thus, for any ρ_1 and ρ_2, $E[V_i|X_1 = 0.5, X_2 = 0.5, \rho_1, \rho_2] = E[V_i] = 1$, as $x = 0.5$ is neutral news. For any ρ, $\beta(0.5|\rho) = 1$, similar to a boundary condition in this framework.

Figure 4 plots $E[V_1|X_1 = x, X_2 = x, \rho_1, \rho_2]$ for three levels of $\rho_2 \in \{0, 0.5, 1\}$, while keeping $\rho_1 = 0.5$ fixed. The higher ρ_2, the more relevant is the signal of the opponent about S, and the stronger his impact on the expected value. Bad news ($X_2 < 0.5$) is worse news, the higher ρ_2. A higher ρ_2 *links* the bid of the opponent (and hence the own payment conditional on winning) stronger with the true value of the object. Hence, an increase in ρ_2 rotates the expected value counter-clockwise around $x = 0.5$. Due to this rotation, in contrast to the framework in Milgrom and Weber (1982), a higher linkage does not lead to a higher expected payment, as it elevates the payment for low signal realizations. This is easily seen for a signal below 0.5, as any possible bid of the loosing

[19] This can be easily computed by observing that the likelihood ratio is strictly monotonic due to the monotone likelihood ratio property, and exactly equal to one at $x = 0.5$.

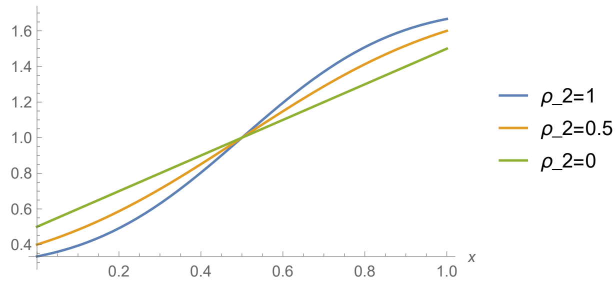

Figure 4: Expected value with $X_1 = X_2 = x$, if $\rho_1 = 0.5$, varying ρ_2.

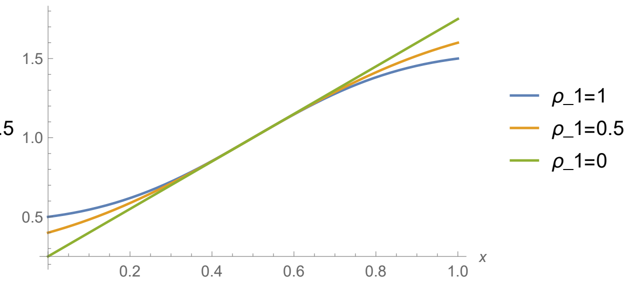

Figure 5: Expected value with $X_1 = X_2 = x$, if $\rho_2 = 0.5$, varying ρ_1.

opponent is higher with a lower (and hence, less informative and less 'linked') ρ_2. For high enough signals above 0.5, this effect might reverse. Hence, the effect of a higher linkage due to an increase in ρ_2 has an ambiguous effect on overall revenue.

Figure 5 plots $E[V_1|X_1 = x, X_2 = x, \rho_1, \rho_2]$ for $\rho_1 \in \{0, 0.5, 1\}$ with a fixed $\rho_2 = 0.5$. The higher ρ_1, the higher the redundancy in bidder 1's information to the information of bidder 2.[20] Hence, bad news ($X_1 < 0.5$) is worse news and good news ($X_1 > 0.5$) is better news, if ρ_1 is lower. A *decrease* in ρ_1 leads to a higher linkage between the true value and the price paid. However, this effect in total is also ambiguous, as a lower linkage leads to a higher expected payment for negative news, but this effect possibly reverts for high enough signals.

In sum, as ρ_1 and ρ_2 increase, the two rotations of the bidding function around $x = 0.5$ are opposing forces. The two linkage effects in Expression 10 go in opposite directions, and the effect of a higher linkage on revenue is ambiguous. If the auctioneer can pick the (symmetric)[21] information selection variable ρ for both bidders, his choice will depend on the parametric assumptions of the informational framework.

5.2 Many Bidders and the All Pay Auction

With $N = 2$, a deviation strategy of the form $\{\rho_i < \rho^{CE}, \beta^{CE}\}$ established, that the equilibrium in a SPA is efficient if the value function is symmetric or t-preferred. Unfortunately, with more than two bidders, this class of deviation strategies $\{\rho_i, \beta^{CE}\}$

[20]For intuition, let signals be almost perfectly revealing and consider $\rho_2 = 1$. For bidder 1, $\rho_1 = 1$ yields almost no additional information, while $\rho_1 = 0$ yields almost perfect information about T_1.

[21]In this section, I considered a symmetric information choice $\rho_1 = \rho_2$ and a symmetric equilibrium. In the context of the classical set-up of the Linkage Principle, Mares and Harstad (2003) show that a seller might derive a higher revenue from disclosing information privately and not publicly. In light of these results, the optimal choice of ρ_1 and ρ_2 remains an open question.

	$(s=0.7, t_i=0.1)$	$(s=0.1, t_i=0.7)$	total winning prob.
CE	$1/N$	$1/N$	$1/N$
DS	0	1	1/2

Table 1: Probability of bidder i winning in DS and CE with $\rho^* = 1$, conditional on $v_i = 1$. Both state combinations have equal probability of $h(0.1)h(0.7)$. Overall winning probability is higher with DS.

is not able to rule out as before inefficient equilibria of the SPA with $\rho > 0$.

I show the complications that arise with $N > 2$ for a candidate equilibrium with $\rho^{CE} = 1$. In the candidate equilibrium, and in a deviation strategy of the form $\{\rho_i, \beta^{CE}\}$, bidder i wins if and only if he has a higher signal realization than all of his opponents, where ties can be ignored. Let $Y_i = \max_{j \neq i} X_j$ be the highest signal realization of all opponent bidders of bidder i.

For each total value realization v_i for bidder i the following theorem pins down the probability of winning under DS or CE, depending on whether he observes X_i^T or X_i^S.

Proposition 5. *Let $N > 2$ and $\rho^{CE} = 1$. Then, for all $\omega_i \in [0, 2]$, the probability of bidder i having the highest signal is $\Pr(X_i \geq Y_i | \omega_i, \rho_i = 1) = \frac{1}{N}$. For all $\omega_i \in (0, 2)$, $\Pr(X_i \geq Y_i | \omega_i, \rho_i)$ is strictly decreasing in ρ_i.*

Let all other bidders learn about the common component S. The proposition says that, by decreasing ρ_i and learning more private component T_i instead of S, bidder i can strictly increase his probability of having the highest signal for all ω_i.

The intuition is best conveyed by an example with fully revealing signals, i.e., $\Pr(X_i^K = x | K = r) = \begin{cases} 1 & \text{if} \quad x = r, \\ 0 & \text{otherwise.} \end{cases}$ for $K \in \{S, T_i\}$. Fix $\omega_i = 0.8$ and consider two S-T_i-combinations that are compatible with it for bidder i, $(s = 0.1, t_i = 0.7)$ and $(s = 0.7, t_i = 0.1)$. Both combinations occur with equal density. If multiple bidders have the same highest signal realization, ties are broken evenly about who wins.[22]

If $(s = 0.7, t_i = 0.1)$, all $N - 1$ other bidders learn a signal X_j^S with realization $x_j = 0.7$. If bidder i learns X_i^S as well, he has signal realization 0.7, and wins with probability $\frac{1}{N}$. If bidder i observes signal X_i^T instead about his private component, his signal realization is 0.1 and he has zero probability of winning. These probabilities are summarized in the first column of Table 1.

If $(s = 0.1, t_i = 0.7)$, all other bidders observe a signal realization $x_j = 0.1$. If bidder i learns about S, he also observes realization 0.1 and wins with probability $\frac{1}{N}$.

[22]In the continuous version of my model, ties have zero probability. In this discrete example, ties occur with strictly positive probability, which requires a tie-breaking rule.

If bidder i learns about his private component, his signal realization is 0.7 and he wins with probability 1. This is summarized in the second column of the Table 1.

Winning probability overall in DS is higher than in CE. In $(s = 0.1, t_i = 0.7)$, bidder i has a lot of probability mass of winning to gain by learning about T_i. In state $(s = 0.7, t_i = 0.1)$, even if bidder i learns about S, his probability of a win is not very high, since the first order statistic of the other bidders is elevated by the high realization of S. The gain in probability mass of winning in $(s = 0.1, t_i = 0.7)$ is larger than the loss in $(s = 0.7, t_i = 0.1)$. This argument becomes stronger $N \to \infty$. As the number of bidders increases and all other bidders learn about the common component, bidder i's probability of winning with CE approaches zero in both $(s = 0.1, t_i = 0.7)$ and $(s = 0.7, t_i = 0.1)$. On the other hand, playing DS always guarantees bidder i a win in state $(s = 0.1, t_i = 0.7)$. It is easy to see that when there are only two bidders, gain and loss in the two states are exactly equal: learning about either component yields the same overall probability $\frac{1}{2}$ of having the highest signal for bidder i in above two state realizations. This is evident in the third column of Table 1 for $N = 2$.

In the general setting with noisy signals, a bidder with a deviation strategy $\{\rho_i < 1, \beta^{CE}\}$ will therefore have a higher expected gain. This is because at each ω_i, bidder i wins with a higher probability.

However, the effect on the expected payment in the SPA is ambiguous. First, a higher winning probability at every ω_i corresponds also to a higher probability of having to pay. In contrast, for two bidders, overall winning probability remained unchanged. Second, the effect on the expected payment conditional on winning is also uncertain. With $N = 2$, the marginal distribution of the own signal and of Y_i was identically distributed. Therefore, the effect or more or less correlation manifested by a first-order stochastic dominance on the order statistics when varying ρ_i (see Proposition 3). With $N > 2$, the marginal distributions of X_i and Y_i do no longer correspond, and no such order as in Proposition 3 can be established.[23]

These effects are clearer in the all-pay auction, as the following result shows.

Proposition 6. *For $N > 2$ and $u(s, t_i)$ being symmetric, there exists no equilibrium of the all-pay auction with $\rho^A = 1$. There exists an equilibrium with $\rho^A = 0$.*

In a candidate equilibrium where all bidders learn about S, a bidder has a strictly profitable deviation: deviate to $\{\rho_i = 0, \beta^A\}$. This results in a strictly higher expected

[23] For example, let S, T_i be binary with equal probability, and let $f(x|0) = 2 - 2x$ and $f(x|1) = 2x$. Let $\omega_i = 1$ and $N = 4$. It can be easily computed, that the distribution of Y_i conditional on bidder i winning cross for different ρ_i, i.e., there is no FOSD in ρ_i.

gain as his winning probability increases at every value realization v_i, as Proposition 5 shows. As the bidding function is preserved in the deviation from the candidate equilibrium, the deviation's expected payment is unchanged in an APA.

Moreover, if $\rho^A = 0$, no other bidder knows anything of relevance to other bidders. Any information choice of a bidder results in an independent signal from all other bidders, and the overall information content does not depend on the information choice. This establishes existence of an equilibrium with $\rho^A = 0$.

6 Conclusion

If bidders cannot consider all possible information, a question of *which* variables to learn about arises. I analyze this question in the context of auctions. In takeover auctions, out of all the multidimensional information available about the target, which characteristics do bidders choose to focus on? Do they want to know what matters to others – a common variable like the book value – which induces interdependence in private information? Or do bidders prefer to focus on a private component like their specific R&D synergies and receive independent private signals?

The focus of this paper is on information selection, specifically *which* payoff-relevant variable to learn about. This contrasts with the literature on information acquisition, which usually asks *how much* information about a single payoff relevant variable a bidder acquires.

In the SPA, information selection in equilibrium is unique if the private component matters at least as much as the common component. Any candidate equilibrium in which bidders learn with non-zero weight about the common component cannot be sustained. I construct deviation strategy, such that a bidder strictly decreases his expected payment but retains his overall gain and winning probability. By decreasing correlation via learning about the private component, a bidder is more likely to win in states with a high *private* component, and less likely to win in states with a high *common* component, while there is no effect on the overall winning probability. In the FPA, if the common component matters at least as much as the private component to a bidder, there is a force towards more correlation. Under certain conditions, any candidate equilibrium but the pure CV outcome can be ruled out.

This paper explores the impact of a selling mechanism on the type of information bidders select. Information about the common component simplifies coordination and is informative about other bidder's bids. However, learning about a common component that matters equally for all bidders is socially wasteful, as this information comes at the opportunity cost of not learning socially valuable information about the private components. A designer who wishes to maximize efficiency should take into consideration, that his auction choice might affect about which value components bidders learn. My analysis suggests that, in such a simplified setting, the SPA is a good choice, as it is ex-ante efficient. It induces learning only about the socially relevant variable and allocates the good efficiently. An IPV setup arises endogenously.

A Appendix

(The proof of Theorem 1 follows after the proof of Proposition 2.)

Proof of Lemma 1. The probability of winning for bidder i with ρ_i is

$$\Pr(X_i \geq X_j|S, T_i, \rho_i, \rho_j) = \rho_i \Pr(X_i^S \geq X_j|S, T_i, \rho_j) + (1-\rho_i)\Pr(X_i^T \geq X_j|S, T_i, \rho_j).$$

As this is differentiable in ρ_i, it holds that

$$\frac{\partial \Pr(X_i \geq X_j|S, T_i, \rho_i, \rho_j)}{\partial \rho_i} = \Pr(X_i^S \geq X_j|S, T_i, \rho_j) - \Pr(X_i^T \geq X_j|S, T_i, \rho_j). \tag{12}$$

If bidder i learns X_i^S, his probability of having the highest signal is

$$\begin{aligned}\Pr(X_i^S \geq X_j|S=a, T_i=b, \rho_j) &= \rho_j \int_0^1 f(x|a)F(x|a)dx + (1-\rho_j)\int_0^1 f(x|a)F(x)dx \\ &= \rho_j \frac{1}{2} + (1-\rho_j)\int_0^1 f(x|a)F(x)dx.\end{aligned}$$

If bidder i learns X_i^T, it holds that

$$\Pr(X_i^T \geq X_j|S=a, T_i=b, \rho_j) = \rho_j \int_0^1 f(x|b)F(x|a)dx + (1-\rho_j)\int_0^1 f(x|b)F(x)dx.$$

First, I show that the derivative with respect to ρ_i in Equation 12 at $S=a, T_i=b$

is the additive inverse of the derivative at $S=b, T_i=a$. Using integration by parts,

$$\begin{aligned}&\frac{\partial \Pr(X_i \geq X_j|a,b,\rho_i,\rho_j)}{\partial \rho_i}+\frac{\partial \Pr(X_i \geq X_j|b,a,\rho_i,\rho_j)}{\partial \rho_i}\\ =&\rho_j-\rho_j\int_0^1 f(x|b)F(x|a)dx-\rho_j\int_0^1 f(x|a)F(x|b)dx\\ =&\rho_j\left[1-\left(1-\int_0^1 f(x|a)F(x|b)dx\right)-\int_0^1 f(x|a)F(x|b)dx\right]=0.\end{aligned}$$

Next, I pin down the sign of the derivative in Equation 12. First, let $a=b$. With $\int_0^1 f(x|b)F(x|a)dx=\left[\frac{1}{2}F(x|a)\right]_0^1=\frac{1}{2}$, it is immediate that $\frac{\partial \Pr(X_i\geq X_j|a,b,\rho_i,\rho_j)}{\partial\rho_i}=0$.

Let $a>b$. The strict MLRP implies FOSD, hence, $F(x|a)<F(x|b)$ for all $x\in(0,1)$. Using this inequality and integration by parts repeatedly, it follows that

$$\begin{aligned}\Pr(X_i^T \geq X_j|S=a,T_i=b,\rho_j)=&\rho_j\int_0^1 f(x|b)F(x|a)dx+(1-\rho_j)\int_0^1 f(x|b)F(x)dx\\ <&\rho_j\int_0^1 f(x|b)F(x|b)dx+(1-\rho_j)\left[1-\int_0^1 f(x)F(x|a)dx\right]\\ =&\rho_j\frac{1}{2}+(1-\rho_j)\int_0^1 f(x|a)F(x)dx\\ =&\Pr(X_i^S\geq X_j|S=a,T_i=b,\rho_j).\end{aligned}$$

Hence, $\frac{\partial \Pr(X_i\geq X_j|a,b,\rho_i,\rho_j)}{\partial\rho_i}>0$ for $a>b$. For $a<b$, by the MLRP, it holds that $F(x|a)>F(x|b)$, all inequalities reverse and $\frac{\partial \Pr(X_i\geq X_j|a,b,\rho_i,\rho_j)}{\partial\rho_i}<0$, concluding the proof of the lemma. □

Proof of Lemma 2. Fix $\omega_i\in(0,2)$. Define the subset of S that is feasible with ω_i as $\mathcal{S}(\omega_i):=\{s\in S:\exists t_i\in[0,1]:\omega_i=s+t_i\}=[\max\{0,\omega_i-1\},\min\{1,\omega_i\}]$.[24] Let $\underline{s}(\omega_i)=\max\{0,\omega_i-1\}$ and $\overline{s}(\omega_i)=\min\{1,\omega_i\}$. Define $\hat{s}(\omega_i)$ that bisects this interval: $\hat{s}(\omega_i):=\frac{\underline{s}(\omega_i)+\overline{s}(\omega_i)}{2}=\frac{\omega_i}{2}$. Let $h_{\omega_i}(.)$ be the density of a component, conditional on a realization ω_i. It coincides for both components.

Increasing the information selection ρ_i yields the following change in winning probability conditional on ω_i:

[24] If $\omega_i\geq 1$, we have $\mathcal{S}(\omega_i)=[\omega_i-1,1]$. If $\omega_i<1$, we have $\mathcal{S}(\omega_i)=[0,\omega_i]$.

$$
\begin{aligned}
&\frac{\partial \Pr(i\text{wins}|\omega_i, \rho_i, \beta_i^M; CE^M)}{\partial \rho_i} \\
=&\frac{\partial}{\partial \rho_i}\int_{\mathcal{S}(\omega_i)} \Pr(X_i \geq X_j | S = s, T_i = \omega_i - s, \rho_i, \rho^M) h_{\omega_i}(s) ds \\
=&\int_{\underline{s}(\omega_i)}^{\frac{\omega_i}{2}} \frac{\partial}{\partial \rho_i} \Pr(X_i \geq X_j | S = s, T_i = \omega_i - s, \rho_i, \rho^M) h_{\omega_i}(s) ds \\
&+ \int_{\frac{\omega_i}{2}}^{\overline{s}(\omega_i)} \frac{\partial}{\partial \rho_i} \Pr(X_i \geq X_j | S = s, T_i = \omega_i - s, \rho_i, \rho^M) h_{\omega_i}(s) ds.
\end{aligned}
$$

As S and T_i are distributed uniformly, $h_{\omega_i}(s) = h_{\omega_i}(\omega_i - s)$. Further, using Proposition 1, the second summand can be expressed as

$$
\begin{aligned}
&\int_{\frac{\omega_i}{2}}^{\overline{s}(\omega_i)} \frac{\partial}{\partial \rho_i} \Pr(X_i \geq X_j | S = s, T_i = \omega_i - s, \rho_i, \rho^M) h_{\omega_i}(s) ds \\
=& -\int_{\frac{\omega_i}{2}}^{\overline{s}(\omega_i)} \frac{\partial}{\partial \rho_i} \Pr(X_i \geq X_j | S = \omega_i - s, T_i = s, \rho_i, \rho^M) h_{\omega_i}(\omega_i - s) ds \\
=& -\int_{\underline{s}(\omega_i)}^{\frac{\omega_i}{2}} \frac{\partial}{\partial \rho_i} \Pr(X_i \geq X_j | S = s, T_i = \omega_i - s, \rho_i, \rho^M) h_{\omega_i}(s) ds.
\end{aligned}
$$

Hence, $\frac{\partial \Pr(i\text{wins}|\omega_i, \rho_i, \beta_i^M; CE^M)}{\partial \rho_i} = 0$ for all $\omega_i \in (0, 2)$.

Let $\omega_i \in \{0, 2\}$. Then, the components coincide $S = T_i = \frac{\omega_i}{2} \in \{0, 1\}$. By Proposition 1 (for $a = b = \frac{\omega_i}{2}$), $\frac{\partial \Pr(i\text{wins}|\omega_i, \rho_i, \beta_i^M; CE^M)}{\partial \rho_i} = \frac{\partial \Pr(X_i \geq X_j | S = \frac{\omega_i}{2}, T_i = \frac{\omega_i}{2}, \rho_i, \rho^M)}{\partial \rho_i} = 0$. □

Proof of Proposition 1. Take any candidate equilibrium (ρ^M, β^M). Without loss, let bidder 1 deviate to (ρ_1, β^M) with $\rho_1 \neq \rho^M$ while bidder 2 follows the candidate equilibrium.

Fix any $\omega_1 \in \Omega_1$. Define $\underline{s}(\omega_1)$, $\overline{s}(\omega_1)$, $\hat{s}(\omega_1)$ and $h_{\omega_1}(.)$ as in the proof of Lemma 2. Let $EU(\rho_1|\omega_1, CE)$ be the utility of bidder 1 from this deviation strategy for this realization of ω_i. It can be expressed as

$$
\begin{aligned}
EU(\rho_1|\omega_1, CE) &= \int_{\underline{s}(\omega_1)}^{\overline{s}(\omega_1)} u(a, \omega_1 - a) \Pr(i \ wins | S = a, T = \omega_1 - a, \rho_1, CE) h_{\omega_1}(a) da \\
&= \int_{\underline{s}(\omega_1)}^{\overline{s}(\omega_1)} u(a, \omega_1 - a) \Pr(X_1 \geq X_2 | S = a, T = \omega_1 - a, \rho_1, \rho^M) h_{\omega_1}(a) da
\end{aligned}
$$

The derivative with respect to ρ_1 yields

$$\begin{aligned}\frac{\partial EU(\rho_1|\omega_1, CE)}{\partial\rho_1} &= \int_{\underline{s}(\omega_1)}^{\overline{s}(\omega_1)} u(a,\omega_1-a)\frac{\partial\Pr(X_1\geq X_2|S=a,T=\omega_1-a,\rho_1,\rho^M)}{\partial\rho_1}h_{\omega_1}(a)da\\ &= \int_{\underline{s}(\omega_1)}^{\hat{s}(\omega_1)} u(a,\omega_1-a)\frac{\partial\Pr(X_1\geq X_2|S=a,T=\omega_1-a,\rho_1,\rho^M)}{\partial\rho_1}h_{\omega_1}(a)da\\ &\quad+\int_{\hat{s}(\omega_1)}^{\overline{s}(\omega_1)} u(a,\omega_1-a)\frac{\partial\Pr(X_1\geq X_2|S=a,T=\omega_1-a,\rho_1,\rho^M)}{\partial\rho_1}h_{\omega_1}(a)da\end{aligned}$$

Using Lemma 1, a change of variables and $h_{\omega_1}(a)=h_{\omega_1}(\omega_1-a)$ yields

$$\begin{aligned}&\int_{\underline{s}(\omega_1)}^{\hat{s}(\omega_1)} u(a,\omega_1-a)\frac{\partial\Pr(X_1\geq X_2|S=a,T=\omega_1-a,\rho_1,\rho^M)}{\partial\rho_1}h_{\omega_1}(a)da\\ &-\int_{\hat{s}(\omega_1)}^{\overline{s}(\omega_1)} u(a,\omega_1-a)\frac{\partial\Pr(X_1\geq X_2|S=\omega_1-a,T=a,\rho_1,\rho^M)}{\partial\rho_1}h_{\omega_1}(a)da\\ =&\int_{\underline{s}(\omega_1)}^{\hat{s}(\omega_1)} u(a,\omega_1-a)\frac{\partial\Pr(X_1\geq X_2|S=a,T=\omega_1-a,\rho_1,\rho^M)}{\partial\rho_1}h_{\omega_1}(a)da\\ &-\int_{\underline{s}(\omega_1)}^{\hat{s}(\omega_1)} u(\omega_1-a,a)\frac{\partial\Pr(X_1\geq X_2|S=a,T=\omega_1-a,\rho_1,\rho^M)}{\partial\rho_1}h_{\omega_1}(\omega_1-a)da\\ =&\int_{\underline{s}(\omega_1)}^{\hat{s}(\omega_1)} \left[u(a,\omega_1-a)-u(\omega_1-a,a)\right]\frac{\partial\Pr(X_1\geq X_2|S=a,T=\omega_1-a,\rho_1,\rho^M)}{\partial\rho_1}h_{\omega_1}(a)da.\end{aligned}$$

In the interval of integration $(\underline{s}(\omega_1),\hat{s}(\omega_1))$, it holds that $a<\omega_1-a$. Thus, due to Lemma 1, $\frac{\partial\Pr(X_1\geq X_2|S=a,T=\omega_1-a,\rho_1,\rho^M)}{\partial\rho_1}<0$. Furthermore, $[u(a,\omega_1-a)-u(\omega_1-a,a)]$ is zero if the value function is symmetric, positive if it is t-preferred, and negative if it is s-preferred. Hence, for any ω_1,

$$\frac{\partial EU(\rho_1|\omega_1, CE)}{\partial\rho_1}\begin{cases}=0 \text{ if } u \text{ is symmetric,}\\ <0 \text{ if } u \text{ is t-preferred,}\\ >0 \text{ if } u \text{ is s-preferred.}\end{cases}$$

Overall, the sign of the derivative carries over to the total expected gain $EG(\rho_1|CE^M)$,

$$\frac{\partial EG(\rho_1|CE^M)}{\rho_1}=\int_{\Omega_1}\frac{\partial EU(\rho_1|\omega_1, CE)}{\partial\rho_1}h(w_1)dw_1.$$

$\square$

Proof of Lemma 3. Without loss, fix ρ_2 and let ρ_1 vary. Using Corollary 1, $\Pr(X_1\geq$

$X_2|\rho_1, \rho_2) = \frac{1}{2}$, the first-order statistic can be expressed as

$$\begin{aligned} G_{(1)}(x|\rho_1, \rho_2) =& \Pr(X_1 \le x | X_1 \ge X_2, \rho_1, \rho_2) \\ =& 2 \Pr(X_1 \le x, X_1 \ge X_2, \rho_1, \rho_2) \\ =& 2(1 - \rho_1\rho_2) \int_0^x f(x)F(x)dx + 2\rho_1\rho_2 \int_0^1 \int_0^x f(x|s)F(x|s)dxds \\ =& \frac{1}{2}(1 - \rho_1\rho_2)F(x)^2 + \frac{1}{2}\rho_1\rho_2 \int_0^1 F(x|s)^2 ds. \end{aligned}$$

As it holds by definition that $F(x_j) = \int_0^1 F(x_j|s)ds$, we have

$$\frac{\partial G_{(1)}(x|\rho_1, \rho_2)}{\partial \rho_1} = \frac{1}{2}\rho_2 \left[\int_0^1 F(x|s)^2 ds - \left(\int_0^1 F(x|s)ds \right)^2 \right].$$

Let $\rho_2 = 0$. Then it is immediate that $\frac{\partial G_{(1)}(x|\rho_1, \rho_2)}{\partial \rho_1} = 0$. Let $\rho_2 > 0$. Then, the strict Cauchy-Bunyakovsky-Schwartz inequality[25] and strong MLRP yields for all $x_j \in (0, 1)$,

$$\left(\int_0^1 F(x|s)ds \right)^2 < \int_0^1 ds \int_0^1 F(x|s)^2 ds.$$

Hence, $\frac{\partial G_{(1)}(x|\rho_1, \rho_2)}{\partial \rho_1} > 0$ for all $x \in (0, 1)$, which concludes the proof of FOSD for the first-order statistic $G_{(1)}$.

The following lemma establishes a reversed FOSD order relationship between the first-order and second-order statistic.

Lemma 4. *Let $\mathcal{R} \in \{\succ_{FOSD}, \succeq_{FOSD}, =_{FOSD}, \}$ be a FOSD relation and $\rho_1' \neq \rho_1''$. Then, $G_{(1)}(.|\rho_1', \rho_2)\mathcal{R}G_{(1)}(.|\rho_1'', \rho_2)$ if and only if $G_{(2)}(.|\rho_1'', \rho_2)\mathcal{R}G_{(2)}(.|\rho_1', \rho_2)$.*

Proof. Decomposing $F(x)$ into a first-order and a second-order statistic yields

$$\begin{aligned} F(x) =& \Pr(X_i \le x | \rho_i, \rho_j) \\ =& \Pr(X_i \ge X_j, X_i \le x | \rho_i, \rho_j) + \Pr(X_i < X_j, X_i \le x | \rho_i, \rho_j) \\ =& \frac{1}{2} \left(G_{(1)}(x|\rho_i, \rho_j) + G_{(2)}(x|\rho_i, \rho_j) \right), \end{aligned} \tag{13}$$

where $\Pr(X_i \ge X_j | \rho_i, \rho_j) = \Pr(X_i < X_j | \rho_i, \rho_j) = \frac{1}{2}$ followed from Corollary 1.

[25]The Cauchy-Bunyakovsky-Schwartz inequality $\left[\int_a^b c(s)d(s)ds\right]^2 \le \int_a^b c(s)^2 ds \cdot \int_a^b d(s)^2 ds$ is strict unless $c(s) = \alpha \cdot d(s)$ for some constant α (see Hardy et al., 1934, Chapter VI). In above argument, $c(s) = 1$, and $d(s) = F(x|s)$. Due to the strong MLRP, unless $x \in \{0, 1\}$, $F(x|s)$ is not constant in s.

$F(x)$ does not depend on ρ_i. Hence, if $G_{(1)}(x|\rho_i', \rho_j) > (=) G_{(1)}(x|\rho_i'', \rho_j)$, then $G_{(2)}(x|\rho_i'', \rho_j) > (=) G_{(1)}(x|\rho_i', \rho_j)$ to satisfy Equation 13. □

Hence, the result for the first-order statistic proven above together with above Lemma 4 establishes the result for the second-order statistic. □

Proof of Proposition 2. Let $\rho^M = 0$. Then, varying ρ_i has no effect on neither the first-order nor the second-order statistic by Lemma 3. Hence, expected payment conditional on winning in the SPA (Equation 6) and FPA (Equation 7) is constant for any $\rho_i \in [0, 1]$.

Let $\rho^M > 0$. Then, increasing ρ_i elevates the second-order statistic in a FOSD order, and the first-order statistic decreases. As the bidding functions β^{II} and β^I are increasing, this yields the result.

Finally, consider the APA for any $\rho^A \in [0, 1]$. A bidder pays his bid β^A irrespective of winning, and overall winning probability is $\frac{1}{2}$ when using β^A (Corollary 1). Hence, $W^A(\rho_i|CE^A)\frac{1}{2} = \int_0^1 \beta^A(x)dF(x)$. Hence, $W^A(\rho_i|CE^A)$ does not depend on ρ_i. □

Proof of Theorem 1. The proof follows immediately from combining Proposition 1, Corollary 1, and Proposition 2.

First, consider the SPA and let the valuation function be symmetric or t-preferred. If $\rho^{II} > 0$, then $\{\rho_1 < \rho^{II}, \beta^{II}\}$ is a strictly profitable deviation: it yields a weakly higher expected gain (Proposition 1, 1. and 2.), the same winning probability (Corollary 1) for a strictly lower expected payment (Proposition 2, 1.).

Next, consider a candidate equilibrium of the FPA with $\rho^I \in (0, 1)$ and let the valuation function be symmetric or t-preferred. A deviation $\rho_1 > \rho^I$ and β^I is strictly profitable, as it results in a weakly higher expected utility for a strictly lower payment. Consider a candidate equilibrium of the FPA with $\rho^I = 0$ and a s-preferred valuation function. Then, a deviation strategy $\rho_1 > \rho^I$ and β^I is strictly profitable, as it yields a strictly higher expected gain (Proposition 1, 3.) for the same expected payment (Proposition 2).

Finally, consider the APA. If $u(.,.)$ is t-preferred, for any candidate equilibrium with $\rho^A > 0$, $\{\rho_1 < \rho^A, \beta^A\}$ is a strictly profitable deviation (higher expected gain for the same expected payment). Accordingly, for a s-preferred value function u, $\{\rho_1 > \rho^A, \beta^A\}$ is a strictly profitable deviation for a candidate equilibrium with $\rho < 1$. □

Proof of Proposition 3. Let the candidate equilibrium be $\rho^{CE} = 0$, and the bidders follow an optimal symmetric, pure and strictly increasing bidding function β^{CE}, given this information choice. Bidders are in an IPV setup and the standard IPV bidding functions for SPA, FPA and APA constitute a fix point, given this fixed information choice $\rho^{CE} = 0$.

The proof proceeds as follows: in Part A, I show that no bidder has a profitable deviation from choosing $\rho_i = 1$ and learning X_i^S instead of X_i^T. In Part B, I prove that any interior $\rho_i \in (0, 1)$ cannot lead to a strictly profitable deviation. Without loss, I construct deviations for bidder 1.

Part A. Let bidder 1 deviate to $\rho_1 = 1$ and learn X_1^S. Let β^{DS} be the optimal deviation strategy of bidder 1 *after* the deviation. It is without loss to assume that β^{DS} is pure and non-decreasing in the SPA, FPA and APA.

Next, I show that the strategy $\rho_1 = 0, \beta^{DS}$ yields a weakly higher payoff than the initial deviation strategy $\rho_1 = 1, \beta^{DS}$.

Bidder 2 learns about T_2, and thus $X_2 = X_2^T$ is independent of X_1^S and X_1^T. Thus, the density of bidder 2's signal is independent of the signal X_1, $g_2(x_2|x_1, \rho_1 = 0, \rho_1 \in \{0, 1\}) = f(x_2)$. Hence, the expected payments in all three auction formats with β^{DS} and signal realization x_1 do not depend on the information choice ρ_1:

$$\begin{aligned} SPA: &\qquad \int_{x_2:\beta^{CE}(x_2)<\beta^{DS}(x_1)} \beta^{CE}(x_2) f(x_2) dx_2. \\ FPA: &\qquad \int_{x_2:\beta^{CE}(x_2)<\beta^{DS}(x_1)} \beta^{DS}(x_1) f(x_2) dx_2. \\ APA: &\qquad \int_0^1 \beta^{DS}(x_1) f(x_1) dx_1. \end{aligned}$$

Thus, overall expected payment is the same with $\rho_1 \in \{0, 1\}$, as long as bidder 1 follows the deviation strategy β^{DS}. The winning probability with β^{DS} and signal realization x_1 (when bidder 2 follows the CE) is $P(x_1) := \int_{x_2:\beta^{CE}(x_2)<\beta^{DS}(x_1)} f(x_2)dx_2$. Winning probability for each x_1 also does not depend ρ_1.

The overall difference in expected utility from learning with $\rho_1 = 0$ and $\rho = 1$ while bidding with β^{DS} can be expressed as

$$\int_0^1 \Big(E[V_1|X_1^T = x_1] - E[V_1|X_1^S = x_1] \Big) P(x_1) f(x_1) dx_1. \tag{14}$$

Next, I show that the expression in Equation 14 is non-negative.

Definition 3 (Karamardian and Schaible, 1990)**.** *A function $H(x)$ is quasi-monotone if $x' > x$ and $H(x) > 0$ imply $H(x') \geq 0$.*

Lemma 5. *Let $u(.,.)$ satisfy increasing differences in T_i. Then, the expression $\left(E[V_1|X_1^T = x_1] - E[V_1|X_1^S = x_1]\right) f(x_1)$ is quasi-monotone.*

Proof. As $f(x_1)$ is non-negative, it is sufficient to show that $E[V_1|X_1^T = x_1] - E[V_1|X_1^S = x_1]$ is quasi-monotone. A signal realization x_1 induces the same posterior distribution over a component, irrespective of whether it is the private or common component. The other component is distributed with a uniform distribution on $[0, 1]$, as a signal is only informative about one component. Hence, $E[V_1|X_1^T = x_1] = \int_0^1 \int_0^1 u(a, b) dF(b|x_1) da$ and $E[V_1|X_1^S = x_1] = \int_0^1 \int_0^1 u(b, a) dF(b|x_1) da$.

The difference in the expected value between the two information sources can thus be expressed as

$$E[V_1|X_1^T = x_1] - E[V_1|X_1^S = x_1] = \int_0^1 \int_0^1 [u(a, b) - u(b, a)] \, dF(b|x_1) da.$$

By assumption, $u(a, b) - u(b, a)$ is non-decreasing in b for any a. Further, as signal distributions satisfy the MLRP, for any $x_1' > x_1$, we have $F(.|x_1') \succeq_{FOSD} F(.|x_1)$. □

The following result is Lemma 1 in Persico (2000) (for the proof, see his Appendix).

Lemma 6. *For $x \in [0, 1]$, let $J(x)$ be a non-decreasing function, and $H(x)$ be quasi-monotone. If $\int_0^1 H(x)dx = 0$, then $\int_0^1 H(x)J(x)dx \geq 0$.*

Let $H(x_1) := \left(E[V_1|X_1^T = x_1] - E[V_1|X_1^S = x_1]\right) f(x_1)$, which is quasi-monotone by Lemma 5. Let $J(x) := P(x_1)$ be the winning probability which is non-decreasing in x_1 as the bidding function β^{DS} is non-decreasing. Finally, by the law of iterated expectations

$$\int_0^1 \left(E[V_1|X_1^T = x_1] - E[V_1|X_1^S = x_1]\right) f(x_1) dx_1 = E[V_1] - E[V_1] = 0.$$

Hence, by Lemma 6, the integral in Equation 14 is non-negative. Learning with ρ_1 and bidding with β^{DS} yields a weakly higher payoff than the initial deviation strategy $\rho_1 = 1$ and β^S. However, by construction, β^{CE} is the optimal bidding strategy for $\rho_1 = 0$. This contradicts that the initial deviation $\rho_1 = 1$ and β^{DS} was strictly profitable.

Part B Let the candidate equilibrium be $\rho^{CE} = 0$ and bidders bid optimally with β^{CE}, given this information choice only about the private component. By contradiction, assume that bidder 1 has a strictly profitable deviation by deviating to $\rho_1 \in (0, 1)$ and bidding according to β^{DS}. By construction of the learning technology, for an interior ρ_1, bidder 1 does not observe the source of his signal, X_1^T or X_1^S.

Bidder 1 would be weakly better of by learning whether his observed signal is about the common component or the private component: the deviation strategy β^{DS} would still feasible, but now he could adapt his bidding to the source of his signal. If he observed X_1^T, his optimal payoff would be exactly his candidate equilibrium payoff. If he observed X_1^S, his optimal payoff is weakly lower than in the candidate equilibrium, as shown in Part A. Thus, this is a contradiction to the strategy $\rho_1 \in (0,1)$ and β^{DS} being a strictly profitable deviation. $\square$

Proof of Proposition 4. Bidder i's signal is

$$X_i = \begin{cases} X_i^S \text{ with probability } \left[\epsilon + (1-\epsilon)\rho_i\right], \\ X_i^T \text{ with probability } (1-\epsilon)(1-\rho_i). \end{cases} \tag{15}$$

Define $\rho_i^\epsilon := \epsilon + (1-\epsilon)\rho_i$. If $\epsilon > 0$, the effective information choice of bidder i is reduced to the interval $\rho_i^\epsilon \in [\epsilon, 1]$ and bounded away from zero.

Combining Proposition 1 (same expected gain for any ρ_i) with Corollary 1 and Proposition 2 (strictly lower (higher) payment with lower ρ_i in the SPA (FPA)) yields the result. In the FPA, $\rho^I = 0$ can be ruled out in equilibrium, because it results in $\rho_j^\epsilon = \epsilon > 0$, and hence if bidder i increases his ρ_i, he can exploit a strictly lower second order statistic of signals and a strictly lower expected payment by Proposition 2. $\square$

Proof of Proposition 5. Let $\omega_i = 0$. Then $s = 0$ and $t_i = 0$. For any ρ_i, bidder i's signal X_i has density $f(x|0)$. Irrespective of ρ_i, the probability of bidder i having the highest signal if $\omega_i = 0$ is $\int_0^1 f(x_i|0)F(x|0)^{N-1}dx_i = \frac{1}{N}$. Similarly, for $\omega_i = 2$ (i.e., $s = 1$ and $t_i = 1$), winning probability of bidder i is $\frac{1}{N}$.

Next, let $\omega_i \in (0,2)$. Define the feasible set of the common component by $\mathcal{S}(\omega_i)$, and let $\hat{s}(\omega_i) = \frac{\min \mathcal{S}(\omega_i) + \max \mathcal{S}(\omega_i)}{2}$ be the dissection of $\mathcal{S}(\omega_i)$ into two equidistant intervals.

The density of S given ω_i is $h(s|\omega_i) = \frac{1}{h(\omega_i)}$ if $s \in \mathcal{S}(\omega_i)$ and 0 otherwise. If bidder i chooses ρ_i, his probability of having the highest signal is

$$\Pr(X_i \geq Y_i|\omega_i, \rho_i, \rho^{CE}) = \rho_i \Pr(X_i^S \geq Y_i|\omega_i, \rho^{CE}) + (1-\rho_i)\Pr(X_i^T \geq Y_i|\omega_i, \rho^{CE}).$$

If learning X_i^S, it holds that

$$\Pr(X_i^S \geq Y_i|\omega_i, \rho^{CE}) = \int_{\mathcal{S}(\omega_i)} \int_0^1 f(x|s)F(x|s)^{N-1} h_{\omega_i}(s)dxds = \int_{\mathcal{S}(\omega_i)} \frac{1}{N} h_{\omega_i}(s)ds = \frac{1}{N}.$$

Due to the uniform distribution of components, $h_{\omega_i}(s) = \frac{1}{\overline{s}(\omega_i)-\underline{s}(\omega_i)} = \frac{1}{\overline{t}(\omega_i)-\underline{t}(\omega_i)}$. If learning X_i^T, it holds that

$$\begin{aligned}
\Pr(X_i^T \geq Y_i|\omega_i, \rho^{CE}) =& \int_{\mathcal{S}(\omega_i)} \int_0^1 f(x|\omega_i - s)F(x|s)^{N-1} h_{\omega_i}(s) dxds \\
=& \frac{1}{\overline{s}(\omega_i) - \underline{s}(\omega_i)} \int_{\mathcal{S}(\omega_i)} \int_0^1 f(x|\omega_i - s)F(x|s)^{N-1} dxds \\
=& \frac{1}{\overline{s}(\omega_i) - \underline{s}(\omega_i)} \int_{\mathcal{S}(\omega_i)} \int_0^1 \frac{N-1}{N} f(x|\omega_i - s)F(x|s)^{N-1} dxds \\
&+ \frac{1}{\overline{s}(\omega_i) - \underline{s}(\omega_i)} \int_{\mathcal{S}(\omega_i)} \int_0^1 \frac{1}{N} f(x|\omega_i - s)F(x|s)^{N-1} dxds. \quad (16)
\end{aligned}$$

Integrating the inner integral of the second summand by parts yields

$$\begin{aligned}
&\frac{1}{\overline{s}(\omega_i) - \underline{s}(\omega_i)} \int_{\mathcal{S}(\omega_i)} \int_0^1 \frac{1}{N} f(x|\omega_i - s)F(x|s)^{N-1} dxds \\
=& \frac{1}{\overline{s}(\omega_i) - \underline{s}(\omega_i)} \int_{\mathcal{S}(\omega_i)} \frac{1}{N} \left(1 - \int_0^1 (N-1) f(x|s)F(x|s)^{N-2} F(x|\omega_i - s) dx\right) ds \\
=& \frac{1}{N} - \int_{\mathcal{S}(\omega_i)} \int_0^1 \frac{N-1}{N} f(x|s)F(x|s)^{N-2} F(x|\omega_i - s) dx \frac{1}{\overline{s}(\omega_i) - \underline{s}(\omega_i)} ds \\
=& \frac{1}{N} - \int_{\mathcal{S}(\omega_i)} \int_0^1 \frac{N-1}{N} f(x|s)F(x|\omega_i - s)F(x|s)^{N-2} dx \frac{1}{\overline{s}(\omega_i) - \underline{s}(\omega_i)} ds.
\end{aligned}$$

Plugging this back into equation 16, and using $\mu(s, x|\omega_i) \coloneqq f(x|\omega_i - s)F(x|s) - f(x|s)F(x|\omega_i - s)$, gives the following expression

$$\begin{aligned}
&\Pr(X_i^T \geq Y_i|\omega_i) \\
=& \frac{1}{N} + \int_{\mathcal{S}(\omega_i)} \int_0^1 \frac{N-1}{N} \left[f(x|\omega_i - s)F(x|s) - f(x|s)F(x|\omega_i - s)\right] F(x|s)^{N-2} \frac{1}{\overline{s}(\omega_i) - \underline{s}(\omega_i)} dxds. \\
=& \frac{1}{N} + \frac{1}{\overline{s}(\omega_i) - \underline{s}(\omega_i)} \int_0^1 \int_{\mathcal{S}(\omega_i)} \frac{N-1}{N} \mu(s, x|\omega_i) F(x|s)^{N-2} dsdx \\
=& \frac{1}{N} + \frac{1}{\overline{s}(\omega_i) - \underline{s}(\omega_i)} \frac{N-1}{N} \int_0^1 \Big(\int_{\max\{\omega_i - 1, 0\}}^{\hat{s}(\omega_i)} \mu(s, x|\omega_i) F(x|s)^{N-1} ds \quad (17) \\
&+ \int_{\hat{s}(\omega_i)}^{\min\{\omega_i, 1\}} \mu(s, x|\omega_i) F(x|s)^{N-2} ds\Big) dx. \quad (18)
\end{aligned}$$

Using $\mu(s, x|\omega_i) = -\mu(\omega_i - s, x|\omega_i)$, we have

$$\int_{\hat{s}(\omega_i)}^{\min\{\omega_i,1\}} \mu(s,x|\omega_i)F(x|s)^{N-2}ds = \int_{\max\{\omega_i-1,0\}}^{\hat{s}(\omega_i)} \mu(\omega_i - s, x|\omega_i)F(x|\omega_i - s)^{N-2}ds$$
$$= -\int_{\max\{\omega_i-1,0\}}^{\hat{s}(\omega_i)} \mu(s,x|\omega_i)F(x|\omega_i - s)^{N-2}.$$

Plugging this back into equation 18 yields

$$\frac{1}{N} + \frac{1}{\overline{s}(\omega_i) - \underline{s}(\omega_i)} \frac{N-1}{N} \int_0^1 \int_{\max\{\omega_i-1,0\}}^{\hat{s}(\omega_i)} \mu(s,x|\omega_i) \left[F(x|s)^{N-2} - F(x|\omega_i - s)^{N-2}\right] dsdx. \tag{19}$$

For $N = 2$, the expression in square brackets and the double integral is zero. For $N > 2$, the strong MLRP and thus, FOSD[26] imply: for all $a < b$ and for all $x \in (0,1)$, we have $F(x|a) > F(x|b)$. As the integral is below $\hat{s}(\omega_i)$, we have $s < \omega_i - t$. Therefore, for $x \in (0,1)$,

$$\left[F(x|s)^{N-2} - F(x|\omega_i - s)^{N-2}\right] > 0.$$

A well-known implication of the MLRP is that for all $a < b$, we have reverse hazard rate dominance

$$\frac{f(x|a)}{F(x|a)} \leq \frac{f(x|b)}{F(x|b)}.$$

Due to $s \leq \omega_i - s$ in the reverse hazard rate, $\mu(s,x|\omega_i) \geq 0$ in the entire domain of integration. This establishes the non-negativity in the second summand of Equation 19. Thus, for $N > 2$ and $\omega_i \in (0,2)$ we have $\Pr(X_i^T \geq Y_i|\omega_i) > \frac{1}{N}$. □

Proof of Proposition 6. First, I show that $\rho^A = 1$ cannot be an equilibrium for $N > 2$ bidders. The proof is by contradiction. Let $\{\rho^A = 1, \beta^A\}$ be an equilibrium. Then, consider the following deviation for (without loss) bidder 1: $\{\rho_1 = 0, \beta^A\}$.

The marginal signal distribution of bidder 1 is $F(x)$, irrespective of his choice of ρ_1. Thus, his expected payment in the APA does not depend on ρ_i, as he foregoes his bid irrespective of the event of winning, $\int_0^1 \beta^A(x)dF(x)$.

Next, consider the expected gain. With the candidate equilibrium strategy and in

[26] For implications of the MLRP, like FOSD and reverse hazard rate dominance, see Milgrom and Weber (1982).

the deviation, a bidder wins if and only if he has a higher signal than his opponent,

$$EG(\rho_1|CE^A) = \int_0^1 v_1 \Pr(X_1 > Y_1|v_1, \rho_1, \rho^A)h(v_1)dv_1.$$

Fix a value v_1 for bidder 1. In the candidate equilibrium, he wins if $X_1^S > Y_1$. With the deviation, he wins if $X_1^T > Y_1$. Thus, due to Proposition 5, the probability of winning with X_1^T is (strictly) higher for any (interior) v_1. Hence, a bidder's expected gain is strictly higher with $\rho_1 = 0$ than $\rho_1 = 1$ when bidding with β^A, but the expected payment is the same. Hence, $\{\rho^A = 1, \beta^A\}$ cannot be an equilibrium.

Next, I establish existence of an equilibrium with $\rho^A = 0$. Let $\rho_j = 0$ for all $j \neq 1$, and follow a symmetric pure increasing bidding function β^A. For any $\rho_1 \in [0, 1]$, X_1 is independent of Y_i by Assumption (CI). In this IPV setup, after any signal the optimal bid with signal X_i coincides for any information choice ρ_i, $\beta(x_i) = E[V_i|X_i^S = x_i] = E[V_i|X_i^T = x_i]$. A bidder is indifferent between learning about S or T_i as both leads to the same informativeness overall, the same marginal distribution of his private information, and no interdependence with his opponent. □

References

Athey, Susan (2002): Monotone Comparative Statics under Uncertainty. *The Quarterly Journal of Economics*, 117:187–223.

Bergemann, Dirk; Xianwen Shi; and Juuso Välimäki (2009): Information Acquisition in Interdependent Value Auctions. *Journal of the European Economic Association*, 7:61–89.

Bergemann, Dirk and Juuso Välimäki (2002): Information Acquisition and Efficient Mechanism Design. *Econometrica*, 70:1007–1033.

Bikhchandani, Sushil and Ichiro Obara (2017): Mechanism Design with Information Acquisition. *Economic Theory*, 63:783–812.

Bobkova, Nina (Nov 2017): Knowing what matters to others: information selection in auctions. Discussion paper.

Chi, Chang Koo; Pauli Murto; and Juuso Välimäki (2017): All-Pay Auctions With Affiliated Values. Working paper.

Compte, Olivier and Philippe Jehiel (2007): Auctions and Information Acquisition: Sealed Bid or Dynamic Formats? *The Rand Journal of Economics*, 38:355–372.

Crémer, Jacques and Fahad Khalil (1992): Gathering Information Before Signing a Contract. *American Economic Review*, 82:566–578.

Crémer, Jacques; Yossi Spiegel; and Charles Z. Zheng (2009): Auctions With Costly Information Acquisition. *Economic Theory*, 38:41–72.

Dasgupta, Partha and Eric Maskin (2000): Efficient Auctions. *The Quarterly Journal of Economics*, 115:341–388.

Denti, Tommaso (2017): Unrestricted Information Access. Working Paper.

Gendron-Saulnier, Catherine and Sidartha Gordon (2017): Choosing Between Similar and Dissimilar Information: the Role of Strategic Complementarities. Working Paper.

Gentry, Matthew and Caleb Stroup (2017): Entry and Competition in Takeover Auctions. Working Paper.

Gerardi, Dino and Leeat Yariv (2007): Information Acquisition in Committees. *Games and Economic Behavior*, 62:436–459.

Hardy, Godfrey H.; John E. Littlewood; and George Pólya (1934): *Inequalities*. Cambridge University Press.

Hausch, Donald B. and Lode Li (1991): Private Values Auctions with Endogenous Investment. Working Paper.

Hellwig, Christian and Laura Veldkamp (2009): Knowing What Others Know: Coordination Motives in Information Acquisition. *The Review of Economic Studies*, 76:223–251.

Hendricks, Kenneth and Robert H. Porter (2014): Auctioning Resource Rights. *Annual Review of Resource Economics*, 6:175–190.

Hernando-Veciana, Angel (2009): Information Acquisition in Auctions: Sealed Bids vs. Open Bids. *Games and Economic Behavior*.

Jehiel, Philippe and Benny Moldovanu (2001): Efficient design with interdependent valuations. *Econometrica*, 69(5):1237–1259.

Johnson, Justin P. and David M. Myatt (2006): On the Simple Economics of Advertising, Marketing, and Product Design. *American Economic Review*, 96:756–784.

Karamardian, S. and S. Schaible (1990): Seven kinds of monotone maps. *Journal of optimization Theory and Applications*, 66:37–46.

Krishna, Vijay and John Morgan (1997): An Analysis of the War of Attrition and the All Pay Auction. *Journal of Economic Theory*, 71:343–362.

Lehmann, Erich L. (1988): Comparing Location Experiments. *The Annals of Statistics*, 16:521–533.

Mares, Vlad and Ronald M. Harstad (Apr 2003): Private information revelation in common-value auctions. *Journal of Economic Theory*, 109(2):264–282.

Martinelli, César (2006): Would Rational Voters Acquire Costly Information? *Journal of Economic Theory*, 129:225–251.

Milgrom, Paul (1981): Rational Expectations, Information Acquisition and Competitive Bidding. *Econometrica*, 49:921–943.

Milgrom, Paul R. and Robert J. Weber (1982): A Theory of Auctions and Competitive Bidding. *Econometrica*, 50:1089.

Myatt, David P. and Chris Wallace (2012): Endogenous Information Acquisition in Coordination Games. *The Review of Economic Studies*, 79:340–374.

Persico, Nicola (2000): Information Acquisition in Auctions. *Econometrica*, 68:135–148.

Porter, Robert H. (1995): The Role of Information in U.S. Offshore Oil and Gas Lease Auctions. *Econometrica*, 63:1–27.

Riley, John and William Samuelson (1981): Optimal Auctions. *American Economic Review*, 71:381–392.

Rösler, Anne-Katrin and Balázs Szentes (2017): Buyer-Optimal Learning and Monopoly Pricing. *American Economic Review*, 107:2072–2080.

Shi, Xianwen (2012): Optimal Auctions with Information Acquisition. *Games and Economic Behavior*, 74:666–686.

Stegeman, Mark (1996): Participation Costs and Efficient Auctions. *Journal of Economic Theory*, 71(1):228–259.

Szalay, Dezsö (2009): Contracts with Endogenous Information. *Games and Economic Behavior*, 65:586–625.

Vickrey, William (1961): Counterspeculation, Auctions and Competitive Sealed Tenders. *Journal of Finance*, 16:8–37.

Wilson, Robert (1969): Competitive Bidding With Asymmetric Information. *Management Science*, 15:446–448.

Yang, Ming (2015): Coordination with Flexible Information Acquisition. *Journal of Economic Theory*, 158:721–738.