The Optimality of Liquidity Requirements and Central Bank Interventions during Banking Crises

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This paper studies optimal policy during financial crises using a general equilibrium model in which agents are subject to liquidity risk and a fraction of banks become insolvent. Agents with high liquidity needs and deposits at insolvent banks face a binding liquidity constraint, whose tightness depends on the price of liquid assets. A pecuniary externality arises because other agents do not internalize that their choices affect prices, tightening those constraints even more. The optimal policy—a combination of liquidity requirements on banks and asset purchases by the central bank—increases welfare by affecting the cross-sectional allocation of real resources.

1 Introduction

The 2008 financial meltdown and its disruptive consequences have left academics and policymakers wondering how to deal with the next possible crisis. Insolvencies and runs on financial intermediaries not covered by deposit insurance were widespread in 2008 (Duygan-Bump et al., 2013; Gorton and Metrick, 2012; Schmidt, Timmermann, and Wermers, 2016). The Federal Reserve responded at that time by injecting liquidity and implementing large asset purchase programs, and these interventions might be used again in the event of another acute crisis. However, the financial landscape has changed because of new policies. In particular, new requirements to hold safe and liquid assets—the liquidity coverage ratio and the net stable funding ratio—have been imposed on financial institutions.

The literature has not yet provided a definite answer about the optimality of liquidity requirements and central bank interventions during financial crises. Are these policies beneficial and, if not, what is the optimal policy? With respect to central bank interventions, only a few papers study these policies in models in which banks take and repay deposits—and their modern equivalents, such as repo—using financial assets. Most of the literature that

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studies financial crises assumes that banks take and repay deposits using real goods.[1] While this assumption is useful for many purposes, accounting for the role of financial assets might give rise to non-negligible interactions with policy interventions. With respect to liquidity regulation, there are even fewer analyses, to the point where Allen and Gale (2017) conclude that "[w]ith liquidity regulation, we do not even know what to argue about."[2] An analysis of optimal policy that includes both liquidity requirements and central bank interventions is, to date, missing.

This paper fills this gap by showing that liquidity requirements and asset purchases are optimal during episodes of banking crises. This result is derived in a general equilibrium model in which financial intermediaries take and repay deposits using liquid financial assets rather than goods. The key financial assets in the model are government bonds and central bank reserves. These assets played a central role in the 2008 crisis: their demand from investors and supply from the central bank increased dramatically in 2008, as occurred with fiat money during the Great Depression.

I use a simple three-period structure ($t = 0, 1, 2$) in which depositors are subject to idiosyncratic liquidity risk, in the spirit of Diamond and Dybvig (1983), and financial crises are generated by exogenous shocks so that the equilibrium is unique, as in Allen and Gale (1998). A fraction of the banks in the economy become insolvent at $t = 1$, and agents holding deposits at those banks face losses and do not receive insurance against their liquidity shocks.[3] The losses can be interpreted as a haircut on deposits, so that the model is consistent with the new regulatory approach at winding down insolvent financial institutions during crises.[4]

Without policy interventions, the equilibrium is constrained inefficient. The inefficiency occurs even if banks offer the optimal contract and arises because of a novel pecuniary externality acting through the price of liquid assets. In particular, agents with deposits at solvent banks have access to an abundant amount of liquidity. These agents sell liquid assets to deal with their own liquidity shocks, thereby putting downward pressure on the equilibrium price of such assets. They do not internalize that a low price hurts agents with price-dependent, binding liquidity constraints (i.e., agents with high liquidity needs and deposits at insolvent banks) because it tightens such constraints even more. Crucially, the

[1]This is the case for the papers on bank runs that build on Diamond and Dybvig (1983) and Allen and Gale (1998), but also for a more recent literature that studies banking crises in macroeconomic models (e.g., Gertler and Kiyotaki, 2010).

[2]A few other papers deal with these observations and study central banks' interventions or liquidity regulations. I review this literature in the next section.

[3]In the literature on bank runs, the effect of an exogenous shock that makes a bank insolvent and forces haircuts on deposits is often referred to as a fundamental-based run (Keister and Narasiman, 2015).

[4]Both the FDIC Orderly Liquidation Authority in the United States and the Single Resolution Mechanism in Europe call for losses on investors holding deposits and other liabilities of troubled banks.

pecuniary externality would not arise if banks were to repay deposits using goods rather than financial assets.

This pecuniary externality has some important differences in comparison to the classic fire-sale externality (Bianchi, 2011; Dávila and Korinek, 2017; Lorenzoni, 2008). Unlike those models, a key role here is played by the combination of liquidity shocks, the inability of the banking system to provide full insurance against such shocks, and a limited supply of liquid assets. Nonetheless, as in those models, financial constraints are price-dependent and can be microfounded by limited commitment.

Policy interventions—liquidity requirements and central bank interventions—increase welfare through a general equilibrium channel that causes a cross-sectional redistribution of real resources. This is different from classic models that study liquidity, in which policy interventions that improve welfare do so by increasing investments (Holmström and Tirole, 1998) or consumption (Lagos and Wright, 2005; Lucas and Stokey, 1987) without changing the cross-sectional distribution.

Liquidity requirements increase welfare by addressing the pecuniary externality. These requirements are imposed on solvent banks and reduce the amount of liquidity distributed by such banks to their depositors. As a result, fewer liquid assets are in circulation and, thus, their relative price increases. Liquidity-constrained agents (i.e., those with high liquidity needs and deposits at insolvent banks) benefit from this effect because their price-dependent liquidity constraint is relaxed by the higher price, thereby resulting in a first-order welfare gain. This gain more than offsets the welfare loss of agents with deposits at banks subject to the requirements, which is second-order because their liquidity constraints are not binding in the unregulated equilibrium. Diamond and Kashyap (2016) interpret this type of liquidity requirement as the liquidity coverage ratio introduced by Basel III.

The rationale for liquidity requirements is related to the novel pecuniary externality and thus differs from the conventional wisdom. A typical view of liquidity requirements is that they make banks resilient to sudden and large withdrawals. My novel contribution with respect to liquidity requirements is that this policy is beneficial in avoiding further negative welfare effects even if some banks do become insolvent. In addition, the model suggests that the tightness of liquidity requirements should increase with the size of central bank interventions, indicating that the optimal size of this policy depends on the overall policy stance of the central bank and, possibly, of other policymakers.

Central bank interventions (i.e., asset purchases coupled with liquidity injections) increase welfare by expanding the total supply of liquid assets, thus closing the gap between the constrained first best and the first best. The effects are disproportionately beneficial to agents with binding liquidity constraints (i.e., those with high liquidity needs and deposits at insolvent banks), which are relaxed by the policy intervention. However, this policy exposes the central bank to the risk that the asset purchased will lose value. Large central bank losses must be covered by the Treasury through higher taxes, but this might not be feasible if fiscal capacity is limited. As a result, a limited fiscal capacity imposes a constraint on the size of the asset purchase interventions.

I derive these results in a simple model in which the equilibrium can be solved analytically. I first study a baseline model in which output and investments are exogenous and liquid assets are provided only by the government. Then, I provide two extensions: a model in which a financial crisis generates a recession, and another in which agents can invest in safe capital which is used as collateral for (costly) privately supplied liquidity. In these extensions, a financial crisis is associated with a contraction in real activity and, similar to Gorton and Ordoñez (2014, 2016), an increase in resources devoted to the production of safe private liquidity. Policy interventions have the same effects as in the baseline model, but they also interact with the endogenous output and liquidity supply. In particular, asset purchases always offset the contraction in real activity, whereas liquidity requirements might increase or reduce it. In case liquidity requirements reduce real activity, however, the welfare impact is negligible in comparison to the benefit of improving the cross-sectional misallocation of resources which is at the source of the financial crisis. Overall, the extensions show that the logic of the results is robust and thus likely to hold even in richer models.

1.1 Additional comparison with the literature

A classic paper that studies the role of liquidity and government policies in the presence of idiosyncratic risk is Holmström and Tirole (1998). My core environment without banks (Section 2) has a flavor similar to Holmström and Tirole (1998)'s full general equilibrium model without aggregate uncertainty (Section III.A in their paper); in both environments, the optimum cannot be achieved. While Holmström and Tirole (1998) introduce well-functioning intermediaries (Section III.C in their paper), I am instead interested in financial crises and thus consider an intermediation sector that is not well functioning because some banks become insolvent. As a result, in my model, there is not enough liquidity provided by banks, and public liquidity (i.e., asset purchases) improves welfare. In addition, a novel pecuniary externality arises and motivates liquidity requirements.

In the literature on bank runs, a few other papers analyze central bank interventions when banks take and repay deposits using financial assets, but they differ in important ways. Allen, Carletti, and Gale (2013), Allen and Gale (1998), and Diamond and Rajan (2006) study the monetary policy response to aggregate shocks under nominal deposit contracts.

Skeie (2008) shows that multiple equilibria in a monetary version of Diamond and Dybvig (1983) cannot arise unless currency is withdrawn and stored outside of the banking system. Carapella (2012), Cooper and Corbae (2002), Martin (2006), and Robatto (2018) show that a central bank's liquidity injection can eliminate bank runs driven by panics, in the sense of multiple equilibria.[5] In my model, however, the focus is on how policy tools address the cross-sectional misallocation in liquid assets driven by exogenous shocks (rather than panics) without imposing any restriction on the optimal contract.

The literature on liquidity requirements is surveyed by Allen and Gale (2017), who conclude that it is still at an early stage. Diamond and Kashyap (2016) study a real model of bank runs with multiple equilibria in which incomplete information justifies some liquidity regulation, even though the optimal one differs from the rules implemented in practice. The approach of Farhi, Golosov, and Tsyvinski (2009) is closer to mine in the sense that they identify a pecuniary externality that can be corrected with liquidity regulation. However, their externality is driven by hidden trades, which I do not consider, and their focus is on normal times rather than on crises. Bech and Keister (2017) show that the liquidity coverage ratio can affect interest rates and the effects of a central bank's open market operations, but their focus is on normal times as well.

Some of my policy results are related to Levine (1991). In his infinite-horizon model, the optimal monetary policy is expansionary (unlike classic monetary models in which the Friedman rule is optimal) and improves the cross-sectional distribution of real resources, similar to my model. However, he includes neither banks nor financial crises.

2 Core environment without banks

The objective of this section is to present the core environment without banks and to highlight the policy analysis. The results of this section provide some simple insights for the analysis of the full model with banks and financial crises in Section 3. Indeed, some results of the full model are in an intermediate case between the bankless economy of this section and an economy with banks and no crisis.

Time is discrete with three periods indexed by $t \in \{0, 1, 2\}$. The economy is populated by a continuum of households born at $t = 0$ and consuming at $t = 1$ and $t = 2$, and a continuum of producers born at $t = 1$ and consuming at $t = 2$ only. Preference shocks hit households at $t = 1$, in the spirit of Diamond and Dybvig (1983), and generate liquidity needs. The liquid asset here is a government-issued interest-bearing security, which can be

[5] Malherbe (2014) warns against the risk of such large interventions on liquidity hoarding and adverse selection in financial markets.

interpreted as Treasury debt or interest-bearing central bank reserves.

I follow a large banking and monetary literature that assumes that liquid assets are used by households to finance consumption expenditure. This literature includes Allen and Gale (1998), Diamond and Dybvig (1983), Lagos and Wright (2005), and Lucas and Stokey (1987). This approach not only keeps the model simple and tractable but also allows for an easy comparison with these classic models. One could reformulate the analysis in a different framework (e.g., by assuming that firms are the agents that need liquidity, as in Bigio (2015), Holmström and Tirole (1998), or Jermann and Quadrini (2012)), but the main results and their logic would be unchanged.

The monetary theory literature has identified some underlying frictions that give rise to the need for liquid assets, such as limited commitment and lack of record keeping, which I am implicitly assuming (Kocherlakota, 1998). Other financial instruments, such as claims on productive physical capital, cannot be used for transactions. This assumption can be microfounded by an adverse selection problem in the spirit of Akerlof (1970), which I formalize following the approach of Lester, Postlewaite, and Wright (2012) (see Section 2.5).

In this bankless economy and in the full model with banks in Section 3, I use two stark assumptions: output is fixed at $t = 1$, and government debt is the only liquid asset. Section 4 relaxes both assumptions, showing that the main results and their logic are unchanged.

2.1 Households

A unit mass of households consume at $t = 1$ and $t = 2$, and their utility depends on a preference shock ε that is realized at the beginning of $t = 1$,

$$\mathbb{E}_{\varepsilon}\left\{\varepsilon\, u\left(C_1\right) + C_2\right\}, \tag{1}$$

where

$$\varepsilon = \begin{cases} \varepsilon^h & \text{(impatient household)} \quad \text{with probability } \kappa \\ \varepsilon^l & \text{(patient household)} \quad \text{with probability } 1-\kappa \end{cases} \tag{2}$$

and $\varepsilon^h > 1 > \varepsilon^l > 0$ to capture the patience of the second group of households. I impose the normalization $\mathbb{E}\left(\varepsilon\right) = 1$. The realization of the preference shock is private information and is i.i.d. across households. The law of large numbers holds so that the fraction of impatient agents in the economy equals κ. For future reference, it is useful to denote $F\left(\cdot\right)$ to be the c.d.f. of households' preference shocks implied by (2). Utility at $t = 1$ is given by[6]

$$u(C) = \log C. \tag{3}$$

The utility function in (1) differs from that used in standard models of bank runs, such as Diamond and Dybvig (1983). To clarify this difference, recall that the key role of banks in Diamond and Dybvig is to allocate consumption efficiently between time $t = 1$ and time $t = 2$. Here, the key role of banks will be to allocate consumption efficiently at $t = 1$ between patient and impatient households (i.e., to allocate consumption so that the marginal utilities of patient and impatient households are equalized at $t = 1$). Thus, it is crucial that ε^l is strictly positive, so that patient households want to consume some positive amount at $t = 1$.

At $t = 0$, each household is endowed with an amount k of a productive asset, which I refer to as capital. The amount k is exogenously given and fixed. Capital produces output at $t = 2$, and its productivity is influenced by an aggregate shock realized at $t = 2$. The role of this aggregate shock is to generate some limitations on the central bank's ability to implement large asset purchases, but it will not affect the results of the equilibrium without policy interventions.[7] Output produced at $t = 2$ by each unit of capital is

$$R_2 = \begin{cases} \overline{a} & \text{with probability } 1 - \pi \\ \underline{a} & \text{with probability } \pi, \end{cases}$$

where $0 < \underline{a} < 1 < \overline{a}$. Since I will normalize the time-0 price of capital to one, R_2 will also denote the gross return on capital. I impose the normalization $(1 - \pi)\,\bar{a} + \pi\underline{a} = 1$, so that the average productivity of capital is one: $\mathbb{E}\{R_2\} = 1$. When introducing banks in Section 3, I will specify a slightly richer stochastic process for R_2, which will be required to generate banks' insolvencies. This richer formulation does not affect the results of the bankless economy, and thus I introduce it later.

2.2 Producers

The second set of agents is a unit mass of producers born at $t = 1$. Producers work at $t = 1$ to supply perishable consumption goods to households, and they have linear utility from consumption at $t = 2$, denoted by X_2. In particular, I assume that producers work one unit of time, which allows them to inelastically supply one unit of consumption goods at

[6]The results can be generalized to strictly concave utility functions for consumption at $t = 1$, but the exposition would become more complicated.

[7]More precisely, limitations on the central bank's power come from the combination of the aggregate shocks and of the inability of the central bank to suffer losses, as described in Section 2.7.

$t = 1$. Anticipating some of the results, producers will sell consumption goods to households at $t = 1$ in exchange for liquid financial assets, and will use the payoff of such assets to consume at $t = 2$.

Goods supplied by producers are the only resources available for consumption at $t = 1$, and thus they correspond to time-1 output. Since producers inelastically supply one unit of goods, output at $t = 1$ is fixed and independent of financial conditions. Nonetheless, in Section 4, I extend the model to endogenize output at $t = 1$. In the extension, producers endogenously choose the amount of time they work at $t = 1$, and thus time-1 output will be derived endogenously as a function of policy and parameters.

2.3 First-best allocation

This section derives the first best, which is then used as a benchmark against the decentralized equilibrium. I consider a planner that has the ability to observe the realization of households' preference shocks and gives equal weight to the welfare of households and producers (recall that producers consume X_2 at $t = 2$).

The planner's problem is to choose the consumption of each agent at $t = 1$ and $t = 2$ subject to the feasibility and non-negativity constraints. At the optimum, all impatient households (i.e., those hit by the preference shock ε^h) consume the same amount at $t = 1$, and their individual consumption is denoted by C_1^h. Similarly, all patient households (i.e., those hit by the preference shock ε^l) consume the same amount at $t = 1$, and their individual consumption is denoted by C_1^l. Thus, the planner maximizes the ex ante welfare $\mathcal{W}$ by solving the following problem:[8]

$$\mathcal{W} = \max_{C_1^h, C_1^l, C_2, X_2} \left[\kappa \varepsilon^h u\left(C_1^h\right) + (1-\kappa)\, \varepsilon^l u\left(C_1^l\right)\right] + \mathbb{E}_R\left\{C_2 + X_2\right\} \tag{4}$$

subject to a non-negativity constraint on all choice variables and to the resource constraint at $t = 1$ and $t = 2$, which are given by

$$\begin{aligned} \kappa\, C_1^h + (1-\kappa)\, C_1^l &\leq 1, & (5)\\ C_2 + X_2 &\leq R_2 k. & (6) \end{aligned}$$

At $t = 1$, the consumption of impatient and patient households (i.e., C_1^h and C_1^l) is provided by the goods supplied by producers, which are normalized to one. At $t = 2$, the consumption

[8] Without loss of generality, I assume that the planner distributes the time-2 consumption allocated to households equally to patient and impatient households. This assumption does not affect the results because of the linearity of households' utility at $t = 2$.

of households and producers (i.e., C_2 and X_2) is given by the output produced by capital, that is, $R_2 k$.

The key optimality condition for the social planner problem is the first-order condition that describes households' time-1 consumption allocation, which is given by

$$\varepsilon^h u'\left(C_1^h\right) = \varepsilon^l u'\left(C_1^l\right). \tag{7}$$

The planner equalizes the marginal utility of impatient households (i.e., the left-hand side of (7)) to that of patient households (i.e., the right-hand side of (7)). Given the log-utility specification in (3), the first-best allocation at $t = 1$ is

$$\left(C_1^h\right)^* = \varepsilon^h, \quad \left(C_1^l\right)^* = \varepsilon^l. \tag{8}$$

2.4 Government debt

I now introduce government debt. This security will be used by households to finance their consumption expenditure at $t = 1$ and will be the only means of payment. That is, households will purchase consumption goods from producers and will pay for their purchases with government debt. Some financial frictions will prevent the use of other means of payment at $t = 1$, as I describe in more details in the next section.

In addition to the endowment k of capital described in Section 2.1, households are also endowed with b units of government debt at $t = 0$. This debt is a liability of the government and is modeled as a zero-coupon security with a payoff of one at $t = 2$. For the moment, b can be interpreted as a Treasury security, but I will return to its interpretation in Section 2.7 and will argue that it can also be viewed as central bank reserves.

The government repays the debt at $t = 2$ by imposing lump-sum taxes T_2 on households. The budget constraint of the government at $t = 2$ is

$$b = T_2. \tag{9}$$

I assume that the initial supply of government debt is in an intermediate range:

$$1 \leq b < \varepsilon^h. \tag{10}$$

The lower bound on b, given by $b \geq 1$ is introduced mostly for expositional reasons. This assumption removes any distortionary effect that would arise in an economy without preference shocks. Indeed, as $\left\{\varepsilon^h, \varepsilon^l\right\} \rightarrow \{1, 1\}$, the first best is achieved without any regulation, and liquidity premia on government debt are zero. As a result, the asumption

$b \geq 1$ helps to clarify the effects of preference shocks.[9]

The upper bound on b, given by $b < \varepsilon^h$, is crucial and implies that the bankless equilibrium does not achieve the first best, thereby opening up a role for banks and policy interventions. If instead the supply of government debt were large (i.e., if $b \geq \varepsilon^h$), households could fully insure against preference shocks by holding government debt, without any role for banks to provide such an insurance or without the need for any policy intervention. The case $b \geq \varepsilon^h$ corresponds to the Friedman rule in monetary models. Note that, because of Equation (9), a supply of government debt $b \geq \varepsilon^h$ would require the government to impose large taxes T_2 at $t = 2$. Stepping outside the model for a moment, a large supply of public debt might not be feasible or optimal if the government faces a limit on lump-sum taxes or is instead restricted to imposing distortionary taxes, and this observation provides some justification for the restriction $b < \varepsilon^h$.[10]

2.5 Equilibrium with liquidity constraint at $t = 1$

I now solve for the decentralized equilibrium, showing that it does not implement the first-best allocation. In the decentralized equilibrium, households have access to a Walrasian market at $t = 0$ in which they can trade their endowments of capital and government bonds, and to a centralized market in which they can purchase consumption goods at $t = 1$ from producers. Consumption expenditures at $t = 1$ must be paid with government bonds, as I describe in more detail below.

At $t = 0$, households start with endowments b and k of government debt and physical capital, respectively, and they can adjust their portfolio by trading in a Walrasian market. The variables B_0 and K_0 denote the amount of government debt and physical capital held by a representative household at the end of $t = 0$, which are subject to the budget constraint

$$K_0 + Q_0 B_0 \leq k + Q_0 b, \tag{11}$$

where Q_0 is the price of government debt. The price of capital is normalized to one.[11]

At $t = 1$, households' consumption must be purchased using government debt B_0 as a means of payment. Some remarks about this assumption are in order. First, even though government debt might not be used directly as a means of payment in practice, such debt

[9] One could easily extend the analysis to economies with $b < 1$. Some derivations are slightly different, but the main results are unchanged, including those of the policy analysis.

[10] See Andolfatto (2013) on the connection between lump-sum taxes and the government supply of liquidity in monetary models.

[11] Since two assets are traded in the Walrasian market at $t = 0$ (i.e., capital and government debt), I can normalize the price of one of the two assets to one. I choose to normalize the price of capital and thus to keep track of the price of government bonds because of the liquidity role played by bonds in the model.

has a liquidity premium, as shown by Krishnamurthy and Vissing-Jorgensen (2012), and my model can capture this fact. Second, there is a simple way to interpret the assumption that government bonds are used as a means of payment and to make it more consistent with how transactions happen in practice. One could introduce money in this model as a pure unit of account and express the consumption expenditures in dollar value. Households would then costlessly sell their bonds for money at $t = 1$ and use money as payment. Then, the producer that sold the goods to the household would immediately convert the money into liquid government bonds, still at $t = 1$. This is equivalent to my model, in which transactions happen directly with government bonds, without the need to buy or sell bonds in exchange for money. Third, neither the physical capital purchased at $t = 0$ nor a claim on it can be used as a means of payment at $t = 1$, as I discuss next.

To microfound why physical capital (or a claim on it) is not used as a payment instrument, one can appeal to an adverse selection problem as in Akerlof (1970). I discuss here an approach based on Lester, Postlewaite, and Wright (2012). Absent financial frictions, households could purchase consumption goods and pay by handing over not only government debt B_0 but also the capital K_0 or a claim on it. However, if households can costlessly create low-quality claims, a lemon problem arises and K_0 is not traded at $t = 1$. More specifically, the households must be able to create, at no cost, a bad financial asset with a zero payoff at $t = 2$, and producers must be able to recognize government debt but be unable to distinguish between good claims on capital and bad financial assets.[12]

Given the above discussion, consumption expenditures are subject to the liquidity constraint $C_1 \leq Q_1 B_0$, where Q_1 denotes the price at which government debt is traded in exchange for consumption goods—trades at $t = 1$ take place in a centralized market. The liquidity constraint can be rearranged as

$$\frac{1}{Q_1} C_1 \leq B_0, \tag{12}$$

so that $1/Q_1$ can be interpreted as the price of consumption goods in terms of government bonds. This constraint resembles a cash-in-advance constraint, as in Lucas and Stokey (1987), although liquidity here is provided by government debt.

At $t = 2$, consumption C_2 is not subject to any liquidity constraint, but it must satisfy

[12] Lester, Postlewaite, and Wright (2012) note that there are two interpretations of a worthless financial claim. One interpretation is that it is a bad claim to a good unit of capital, namely, a counterfeit. Another possible interpretation is that it is a good claim to a unit of capital that has zero productivity, that is, a good claim to a lemon.

the budget constraint

$$C_2 \leq R_2 K_0 + \left(B_0 - \frac{C_1}{Q_1} \right) - T_2, \tag{13}$$

where $R_2 K_0$ is the return on capital purchased at $t = 0$, $B_0 - C_1/Q_1$ denotes the time-2 value of the government debt not used at $t = 1$ as a means of payment (which is positive if the liquidity constraint (12) is not binding), and T_2 denotes lump-sum taxes from the government.

The problem of households is to maximize their own utility subject to the budget constraint at $t = 1$, (11), the liquidity constraint, (12), and the budget constraint at $t = 2$, (13). The problem of investors is simpler; at $t = 1$, they inelastically supply one unit of consumption goods in exchange for government bonds, and at $t = 2$, their consumption X_2 is equal to the payoff of the bonds received at $t = 1$, which is given by $X_2 = 1/Q_1$. Given b and households' and investors' optimal choices, the equilibrium is then determined by requiring that the markets for capital and government bonds clear at $t = 0$, the goods market clears at $t = 1$, and the budget constraint of the government holds at $t = 2$.

There exists a unique equilibrium. In the rest of this section, I describe the qualitative and key features of the equilibrium. I also solve for the equilibrium variables in closed form as a function of the parameters, but the details of the solution are provided in Appendix A.1.

At $t = 0$, all households are alike and thus hold the same amount of capital and government bonds. In particular, market clearing implies that K_0 and B_0 are equal to the respective endowments: $K_0 = k$ and $B_0 = b$.

At $t = 1$, the liquidity constraint (12) is binding for impatient households (i.e., households hit by the preference shocks ε^h). This is the case because the the time-0 price of government debt, Q_0, includes a liquidity premium in equilibrium (i.e., government debt trades at a premium in comparison to capital). As a result, an impatient household uses all its holdings of government debt at $t = 1$ to purchase consumption goods; if this were not the case, the household would be better off by purchasing fewer bonds and more capital at $t = 0$, thereby earning a higher return. Thus, the time-1 consumption of an impatient household, C_1^h, is determined by (12) holding with equality:

$$C_1^h = Q_1 B_0. \tag{14}$$

Even though C_1^h is given by (14), it is useful to state the first-order condition of impatient households, in order to compare it with that of patient ones, which is derived later. Denoting μ_1^h to be the Lagrange multiplier of the liquidity constraint (12) of an impatient household, the optimality condition at $t = 1$ is

$$\varepsilon^h u'\left(C_1^h\right) = \frac{1 + \mu_1^h}{Q_1}. \tag{15}$$

For patient households (i.e., those hit by the preference shock ε^l), the liquidity constraint is not binding.[13] Thus, their optimal time-1 consumption, C_1^l, solves the first-order condition

$$\varepsilon^l u'\left(C_1^l\right) = \frac{1}{Q_1}. \tag{16}$$

At the optimum, the marginal utility of consuming at $t = 1$, given by $\varepsilon^l u'\left(C_1^l\right)$, is equal to the marginal utility of not spending an extra unit of government debt and, thus, carrying such an asset to $t = 2$. The gross return on the bonds between $t = 1$ and $t = 2$ is $1/Q_1$, and this amount can be spent at $t = 2$, when the marginal utility of consumption is one.

Since the liquidity constraint is binding only for impatient households, the marginal utilities of impatient and patient households are not equalized in equilibrium. That is, Equations (15) and (16) imply

$$\varepsilon^h u'\left(C_1^h\right) > \varepsilon^l u'\left(C_1^l\right). \tag{17}$$

A comparison between (17) and the planner's first-order condition (7) shows that the decentralized equilibrium does not achieve the first best. Impatient households consume too little in comparison to the first best (i.e., their marginal utilities are too high) because they face a binding liquidity constraint. Patient households face a nonbinding liquidity constraint and consume too much. I elaborate further on this inefficiency in the next section.

At $t = 0$, households' demand for capital and government debt pins down the time-0 price Q_0 of government debt (recall that the price of capital is normalized at one):

$$Q_0 = 1 + \kappa \mu_1^h. \tag{18}$$

If impatient households did not face a binding liquidity constraint (i.e., if the Lagrange multiplier of their liquidity constraint were $\mu_1^h = 0$), the price of government debt would be the same as that of capital, that is, one. This is because the two assets have the same unitary payoff at $t = 2$ (i.e., physical capital produces on average one unit of output at $t = 2$, and government debt is modeled as a zero-coupon bond). However, the government debt includes the liquidity premium $\kappa \mu_1^h$ because it relaxes the time-1 liquidity constraint

[13] The assumption $b \geq 1$ in (10) is sufficient to imply that the liquidity constraint of patient households is not binding in equilibrium.

of impatient households, and thus it trades at a higher price in comparison to the illiquid physical capital. Note that $\mu_1^h > 0$ implies $Q_0 > 1$, and thus government debt trades above par (i.e., its return is negative). This result follows from the fact that I have not included any discounting between $t = 0$ and $t = 2$. If instead I add discounting, the equilibrium value of Q_0 can be less than one.[14] However, all the results would be unchanged, and thus I maintain the normalization of no discounting.

2.6 The inefficiency of the bankless equilibrium

In comparison to the first best, the inefficiency of the bankless equilibrium is a cross-sectional dispersion in the marginal utility of consumption across households, which is in turn related to a cross-sectional dispersion of the Lagrange multiplier of the liquidity constraint. This result is the by-product of two assumptions: (i) the preference shocks and (ii) a limited amount of liquidity. Both assumptions (i) and (ii) are necessary to obtain such an inefficiency, as I discuss in this section.

To clarify the role of preference shocks, consider the limit in which such shocks disappear, that is, $\left\{\varepsilon^h, \varepsilon^l\right\} \to \{1, 1\}$. In this case, all households are identical at $t = 1$, and they all consume the same amount, $C_1^h = C_1^l = 1$, which corresponds to the first best.[15] In addition, there is no liquidity premium on government bonds, and thus $Q_0 = Q_1 = 1$.

As a second step, I clarify the role played by the supply of liquid assets. If the supply of government debt is so large that (10) is violated (i.e., if $b \geq \varepsilon^h$), impatient households enter $t = 1$ with enough liquidity to finance the first-best level of consumption. This result is akin to the Friedman rule in monetary models, in which a sufficiently large amount of liquidity implements the first best.

2.7 Central bank interventions: asset purchases

Under the asset purchase policy, the government issues interest-bearing reserves through the central bank and uses the proceeds to buy capital on the market at $t = 0$. The central bank keeps the capital until maturity (i.e., until $t = 2$), and at that point, the gross return paid by the capital is used to "buy back" the reserves and reduce the balance sheet of the central bank, similar to the approach used by the Federal Reserve to reduce its balance sheet.[16]

[14] More precisely, Q_0 would be less than one as long as the discount factor is sufficiently small in comparison to the liquidity premium.

[15] The result in (8) implies that the first best converges to $C_1^h = C_1^l = 1$ as $\left\{\varepsilon^h, \varepsilon^l\right\} \to \{1, 1\}$.

[16] The approach at modeling the asset purchase policy follows Benigno and Robatto (2018), which in turn builds on Magill, Quinzii, and Rochet (2016).

The extra liquidity injected through this policy relaxes the liquidity constraint of impatient households and thus increases welfare.

Although the mechanics of this policy is just to increase the amount of liquidity, its effects on welfare are different in comparison to standard models without preference shocks, such as Lucas and Stokey (1987) and Lagos and Wright (2005). In those models, the extra liquidity closes the welfare gap with the first best by increasing the amount of goods that are traded in exchange for liquid assets. In this model, however, the amount of goods traded at $t = 1$ is essentially fixed at one by design. The role played by the extra liquidity is to improve the *cross-sectional allocation of goods*, reducing the cross-sectional dispersion in households' marginal utilities.

In what follows, I use the term "government debt" and the same notation to refer to the initial debt b of the Treasury and the newly issued central bank reserves. Indeed, if one consolidates the Treasury and the central bank into one single government entity, there is no difference between the debt of the Treasury and that of the central bank, and Treasury securities are identical to interest-bearing central bank reserves (Sargent and Wallace, 1981). This is why the government-supplied liquid security can be interpreted not only as Treasury securities but also as the central bank's interest-bearing reserves.

To describe the asset purchase policy, I amend the model to allow the government to increase the supply of government debt to $b_0 \geq b$ at $t = 0$.[17] The government uses the resources collected by issuing $b_0 - b$ units of new debt to purchase capital k_0, and the budget constraint of the government at $t = 0$ is

$$Q_0 \left(b_0 - b\right) = k_0. \tag{19}$$

At $t = 2$, the government repays the debt b_0 (i.e., the initial debt b and the new debt $b_0 - b$). To do so, the government uses not only taxes but also the return on the capital k_0 purchased at $t = 0$. Thus, the government budget constraint at $t = 2$, which is given by (9) under no interventions, is now given by

$$b_0 = T_2 + R_2 k_0. \tag{20}$$

Unlike the model with no interventions, taxes T_2 are not constant anymore and are instead contingent on the return on capital purchased by the central bank, which is given by R_2. Depending on R_2 and on the amount of debt and capital b_0 and k_0, taxes T_2 might be higher or lower (or the same) in comparison to the model with no interventions. If one wants to

[17]If one wants to interpret the asset purchase policy as implemented by the central bank, then the additional debt $b_0 - b$ must be interpreted as central bank reserves.

interpret the newly issued debt $b_0 - b$ as central bank reserves, then the central bank could either make a profit or incur a loss (or break even). If the central bank makes a profit, it rebates it to the Treasury, which in turn needs to raise fewer taxes from households to repay the initial debt b. If instead the central bank incurs a loss, the Treasury recapitalizes the central bank, which in turn requires that higher taxes T_2 be imposed in comparison to the economy with no interventions.[18]

Asset purchases increase the liquidity available in the economy and relax the binding liquidity constraint of impatient households (i.e., that of households with preference shocks ε^h). Thus, consumption C_1^h of impatient households increases and, because of market clearing, consumption C_1^l of patient households decreases. The ultimate effect of the intervention is to improve the cross-sectional distribution of consumption, thereby "bringing" the economy closer to the first best. In addition, when the economy is not at the first best, any extra dollar of asset purchases reduces the price of government debt by shrinking the liquidity premium, $\partial Q_0 / \partial k_0 < 0$.

Without any limit or costs associated with taxes T_2, the government could implement a sufficiently large asset purchase program that achieves the first best. This is because any loss incurred by the central bank can be offset simply by raising more taxes and because taxes are nondistortionary. Appendix A.2 formalizes the effects of asset purchases absent any restrictions on T_2.

To limit the power of the central bank and the size of the asset purchase program it can implement, I impose a limit on the amount of taxes that the government can raise. To keep the analysis simple, I assume that the government cannot raise taxes above the level that arises without interventions, and thus T_2 must satisfy

$$T_2 \leq b. \tag{21}$$

This is a simple assumption to model a limit on fiscal capacity. In a richer model in which the government can use only distortionary taxes, the government might not find it optimal to implement large asset purchases. If there are states of the world in which the central bank needs to be recapitalized, distortionary taxes must be raised, which might be very costly if such states are also associated with low output. However, modeling distortionary taxes

[18]This asset purchase program can also be reinterpreted in a way that captures the purchase of agency mortgage-backed securities that were implemented by the Federal Reserve during the 2008 financial crisis. One could extend the model by separating the central bank and the Treasury, adding a Treasury guarantee to the capital in order to create an asset that resembles an agency mortgage-backed security, and by having the central bank purchase this security. However, if one consolidates the Treasury and the central bank again, the Treasury guarantee would disappear because it is just a transfer between the Treasury and the central bank, resulting in the same exact formulation that I use.

would substantially complicate the model and the analysis.

The next proposition formalizes the asset purchase policy under the restriction in (21). The full characterization of the equilibrium under the asset purchase policy and the proof of Proposition 2.1 are provided in Appendix A.3.

Proposition 2.1. *(Optimal asset purchases with limit on taxes) Assume $k \geq \varepsilon^h - b$. Under the limit on taxes in (21), the optimal asset purchase program is a purchase of capital and a supply of government debt*

$$\{k_0,\, b_0\} = \begin{cases} \{0, b\} & \text{if } \underline{a} \leq \frac{b}{b(1-\kappa)+\kappa\varepsilon^h} \\ \left\{\frac{\kappa\varepsilon^h}{1-\underline{a}(1-\kappa)} - \frac{b}{\underline{a}},\ \frac{\kappa\varepsilon^h\underline{a}}{1-\underline{a}(1-\kappa)}\right\} & \text{if } \underline{a} > \frac{b}{b(1-\kappa)+\kappa\varepsilon^h}. \end{cases}$$

The limit in (21) is equivalent to a non-negativity constraint on the state-contingent profits earned through the asset purchase intervention. Notwithstanding such a limit, the government can implement the policy as long as the return $R_2 = \underline{a}$ earned by the risky capital in the low state is sufficiently large. In this case, a moderate asset purchase does not violate (21) because it exploits an arbitrage opportunity. The newly issued government debt pays a return $1/Q_0 < 1$ because of the liquidity premium, whereas the new capital has an expected return $\mathbb{E}\{R_2\} = 1$. As long as $1/Q_0$ is sufficiently small (in particular, as long as $1/Q_0 \leq \underline{a}$), the arbitrage opportunity implies no losses even in the low state. As a result, the government issues debt and purchases capital up to the point at which the limit on taxes (21) is binding in the low state (i.e., when $R_2 = \underline{a}$).[19] If instead $1/Q_0$ is large (which is the case if $\underline{a}$ is small), the losses incurred in the low state by purchasing the risky capital offset the gains from issuing government debt at a premium, and thus asset purchases cannot be implemented without violating (21).

The asset purchase policy increases welfare but does not implement the first best, in general. The first best can be achieved only in the limit as the capital becomes riskless. In this case, the central bank increases government debt up to the point of satiating the liquidity needs of the economy, and the riskless capital backs the repayment of the debt at $t = 2$.

Corollary 2.2. *In the limit in which $\underline{a} \to 1$, the optimal asset purchase program is $\{k_0,\, b_0\} \to \left\{\varepsilon^h - b,\, \varepsilon^h\right\}$ and implements the first best. However, for any $\underline{a} < 1$, the optimal asset purchase program does not implement the first best.*

[19] If the limit were not binding when $R_2 = \underline{a}$, the central bank could increase its asset purchase program. Note also that, in the high state, the asset purchase policy generates profits, and thus (21) is slack because the government needs to collect fewer taxes in comparison to the baseline case with no intervention.

2.8 Liquidity requirements: a preview

The asset purchase policy studied in Section 2.7 increases welfare but cannot, in general, implement the first best. In this bankless economy, no further intervention is possible, but in the full model with banks of Section 3, an additional policy intervention that imposes liquidity requirements on some banks at $t = 1$ will increase welfare. The objective of this section is to use the simpler bankless economy to give a preview of the logic and the welfare effects of liquidity requirements, even though the full analysis of liquidity requirements will be undertaken in the full model of Section 3.

In the full model with banks of Section 3, liquidity requirements will reduce the cross-sectional dispersion of households' marginal utilities at $t = 1$. This will be achieved by reducing the consumption of liquidity-unconstrained households and increasing that of liquidity-constrained households. I use the bankless economy to study the effect of a similar change in the consumption allocation at $t = 1$. That is, I slightly tilt the consumption allocation away from the equilibrium by reducing the consumption of households with a nonbinding liquidity constraint—here, the patient households—and increasing the consumption of households with a binding liquidity constraint—here, the impatient households. Let me stress that the analysis that follows is conditional on the ability to implement these changes in the consumption allocation. While I do not take a stand on how this is possible in the bankless economy, the full model with banks of Section 3 will show that this effect can be achieved with liquidity requirements imposed on some banks.

The change in the consumption allocation that I consider increases welfare. That is not surprising, given that patient households consume too much and impatient households consume too little in comparison to the first best. Nonetheless, this exercise is useful because it clarifies how this change in the consumption allocation interacts with prices and the liquidity constraints.

A marginal reduction in the consumption of patient households, C_1^l, is equivalent to a reduction in the consumption *expenditures* of these agents. That is, I am implicitly forcing patient households to use fewer government bonds to purchase C_1^l at $t = 1$. Since fewer government bonds are traded at $t = 1$, the remaining ones that are traded—including those held by impatient households—become more valuable because they are traded against the same amount of consumption goods. Thus, the purchasing power of impatient households goes up, generating a first-order welfare gain because these agents face a binding liquidity constraint. Patient households face a welfare loss because of the lower C_1^l, but the loss is only second-order because their liquidity constraint is not binding in the unregulated equilibrium.

The effect of a marginal change in the consumption C_1^l of patient households on welfare

$\mathcal{W}$ can be characterized using (4):

$$\frac{\partial \mathcal{W}}{\partial C_1^l} = \kappa \varepsilon^h u'\left(C_1^h\right) \frac{\partial C_1^h}{\partial Q_1} \frac{\partial Q_1}{\partial C_1^l} + (1-\kappa)\, \varepsilon^l u'\left(C_1^l\right).$$

Crucially, a change in C_1^l affects the price Q_1 of government bonds. This, in turn, affects impatient households' consumption C_1^h because these households face a binding, price-dependent liquidity constraint. Indeed, the term $\partial C_1^h / \partial Q_1$ can be computed using the binding liquidity constraint of impatient households, Equation (14), whereas the term $\partial Q_1 / \partial C_1^l$ can be computed using the goods market clearing condition at $t = 1$, Equation (5).[20] It follows that

$$\frac{\partial \mathcal{W}}{\partial C_1^l} = (1-\kappa)\left[\varepsilon^l u'\left(C_1^l\right) - \varepsilon^h u'\left(C_1^h\right)\right] < 0, \tag{22}$$

where the inequality uses the fact that the marginal utility of impatient households, $\varepsilon^h u'\left(C_1^h\right)$, is higher in comparison to that of patient households, $\varepsilon^l u'\left(C_1^l\right)$, as noted in (17). Thus, a marginal reduction in the consumption C_1^l of patient households generates an increase in welfare. (The derivative in (22) is negative because it describes the effects of a marginal *increase* in C_1^l.)

The marginal twist in the consumption allocation that is studied here is effective because the allocation differs from the first best. If consumption were at the first best, Equations (7) and (22) would imply that $\partial \mathcal{W} / \partial C_1^l = 0$. More generally, this exercise increases welfare only if there is an inefficient cross-sectional dispersion in the marginal utilities of consumption across households. In standard models that study liquid assets without preference shocks, this cross-sectional dispersion does not arise, and thus this exercise does not affect welfare.

Let me emphasize again that the objective of this section is just to provide a simple intuition for the role and effects of liquidity regulation in the full model of Section 3. In that model, a fraction of the banks will become insolvent at $t = 1$, and the liquidity regulation will be imposed on banks that remain solvent at $t = 1$. Thus, the liquidity regulation in the full model can easily be implemented by a planner or regulator that has the ability to observe whether a bank is insolvent or not, without the need to be able to directly alter the consumption allocation or consumption expenditure of households.

3 Full model with banks

The results of the bankless economy open up a role for banks to redistribute liquid assets between households after the realization of the preference shocks. This section studies the

[20]More precisely, the goods market clearing condition (5) needs to be evaluated at $C_1^h = Q_1 b$.

optimal arrangement that is offered by competitive banks to insure against preference shocks, and the implications for policy interventions and welfare. The results about asset purchases extend to this economy with banks and are qualitatively unchanged. In addition, the optimal policy includes a liquidity requirement imposed on banks.

If banks are introduced in the environment of Section 2, the equilibrium implements the first best without the need for any policy interventions.[21] To generate banking crises, I make a simple extension by adding idiosyncratic shocks to the return on physical capital, in addition to the aggregate shocks. These idiosyncratic shocks generate fundamental insolvencies of a fraction of the banks in the economy, similar to the fundamental-based runs in Allen and Gale (1998).[22]

I derive the optimal deposit contract under the assumption that banks take prices as given and thus do not internalize the possible externalities associated with the deposit contract that they offer. To do so, I assume that households are spatially separated across many *locations*. Each location is small with respect to the whole economy, but large enough so that the law of large numbers about households' preference shocks holds at each location. In addition, I assume that each location hosts at most one bank, although multiple banks can compete to offer the best contract in each location. Crucially, households have access only to the bank in their own location.

Although each bank offers the optimal deposit contract, the equilibrium is generically constrained inefficient because of a pecuniary externality. This is because each bank takes prices as given, but prices enter the binding liquidity constraint of households with high liquidity needs and deposits at insolvent banks. Solvent banks do not internalize that their distribution of liquidity at $t = 1$ affects the purchasing power of the liquidity-constrained households through the price of liquid assets.

More formally, the pecuniary externality is related to the generic inefficiency in incomplete market economies (Geanakoplos and Polemarchakis, 1986). The market incompleteness in this model is the inability, for banks, to insure against the idiosyncratic shocks to physical capital; these are the shocks that generate the insolvencies of a fraction of banks in the economy. A regulator can address the externality and improve welfare by forcing banks to offer a different contract, which boils down to a liquidity requirement at $t = 1$ on banks that remain solvent.

[21]The logic is the same as in the good equilibrium of Diamond and Dybvig (1983), in which banks fully insure households against preference shocks.

[22]All the results of the bankless economy are unchanged if one solves for the bankless equilibrium in the economy with idiosyncratic shocks to capital.

3.1 Extended environment with idiosyncratic shocks to capital

The environment is very similar to that of the bankless economy, with the addition of some assumptions that describe the interactions between households and banks and of idiosyncratic shocks that hit physical capital and generate the insolvencies of a fraction of banks.

3.1.1 Households, banks, and producers

Households are located on a continuum of spatially separated locations. Each location hosts a continuum of households, and thus there is a double continuum of households in the economy. The total mass of households is normalized to one, and the law of large numbers about the preference shocks holds in each location and in the economy as a whole, in line with the results of Al-Najjar (2004) about the law of large numbers in large economies.

In each location, banks compete to offer the best deposit contract, although only one bank operates in each location in equilibrium. Thus, banks offer the deposit contract that maximizes households' utility, earning zero profits. Moreover, each bank operates only in one location (or at most in finitely many locations), and thus takes prices as given.

Households have access only to one bank—namely, the bank that operates in their own location—but they can also access the centralized market where they can purchase consumption goods using government bonds at $t = 1$. I also allow banks to access the Walrasian market at $t = 0$.[23]

Producers are the same as in the baseline model. That is, they inelastically sell one unit of consumption goods to households at $t = 1$ and have linear utility at $t = 2$.

3.1.2 Idiosyncratic shocks to capital

Recall from Section 2.1 that the productivity of the k units of capital that each household is endowed with is subject to aggregate shocks. In this full model, I assume that capital is subject not only to such aggregate shocks but also to idiosyncratic shocks whose realization differs across locations.

The overall productivity of capital is still denoted by R_2, but now $R_2 = a_2\psi_2$, where a_2 denotes the aggregate shock as in Section 2.1, whereas ψ_2 is an idiosyncratic component whose realization differs across location. The aggregate shock has the same stochastic process described in Section 2.1, that is, $a_2 = \bar{a} > 1$ with probability $1 - \pi$ and $a_2 = \underline{a} < 1$

[23] Similar to the bankless economy, no trade takes place in the unregulated equilibrium at $t = 0$. Thus, access to the Walrasian market at $t = 0$ matters only to implement the asset purchase policy. Indeed, the results of the unregulated equilibrium and those associated with the liquidity regulation can be generalized to models in which there is no trade at $t = 0$, and the only market is that of consumption goods at $t = 1$.

with probability π, subject to the normalization $\mathbb{E}\{a_2\} = 1$. Similar to the bankless economy, the aggregate shock is realized at $t = 2$ and only affects the ability of the central bank to implement asset purchases, without interacting with the results of the unregulated equilibrium.

The term ψ_2 denotes the idiosyncratic component of the shock and differs across locations. Even though the shock ψ_2 affects the output of capital at $t = 2$, its realization becomes common knowledge at $t = 1$, without any noise. At each location, the idiosyncratic shock takes the value

$$\psi_2 = \begin{cases} \overline{\psi} > 1 & \text{with probability } 1 - \alpha \\ \underline{\psi} = 0 & \text{with probability } \alpha. \end{cases}$$

I impose the normalization $\mathbb{E}\{\psi_2\} = 1$, so that $\mathbb{E}\{R_2\} = 1$ still holds.[24] The baseline model of Section 2 corresponds to the case $\alpha = 0$.[25]

The role of the idiosyncratic shocks to capital is to generate insolvencies of some banks. More precisely, a fraction α of the banks will be hit by the bad idiosyncratic shock to capital, $\underline{\psi}$, and will become insolvent. The equilibrium results will resemble those in Allen and Gale (1998). A bank in a location with shock $\psi_2 = 0$ earns zero gross return from investing in capital, $R_2 = 0$. This bank is insolvent and thus is forced to pay liquid assets to all households—including patient ones—to elicit truthful revelation of the preference shocks. This result can be interpreted as a haircut on deposits (i.e., a bail-in) or a run (i.e., all depositors get the same amount of liquidity at $t = 1$, independently of their preference shock). For a bank in a location hit by the good idiosyncratic shock $\overline{\psi} > 1$, however, it is feasible to distribute more liquidity to impatient households at $t = 1$ and compensate patient households by giving them more goods at $t = 2$, thereby providing insurance against the preference shocks.

One of the players that can hold physical capital is the government, which can acquire the capital through asset purchases at $t = 0$. The government is a large player in the economy, in the sense that it can acquire a large amount of assets in comparison to a single bank.[26] For simplicity, I assume that the government is large enough to the point that it is able to differentiate its holdings of physical capital. As a result, the government is not

[24] I assume that markets are exogenously incomplete with respect to the idiosyncratic shocks to capital. That is, it is not possible to write contracts that insure against the realization of the bad shock $\underline{\psi}$. This incompleteness might also be derived endogenously. For instance, one can assume that the incompleteness arises from a lack of commitment to delivering insurance payments to locations hit by the bad idiosyncratic shock to capital $\underline{\psi}$.

[25] If $\alpha = 0$ (and thus $\overline{\psi} = 1$ according to the normalization $\mathbb{E}\{\psi_2\} = 1$), all banks remain solvent and there is no financial crisis in equilibrium.

[26] Formally, the government can acquire an amount of capital with positive measure, whereas each bank is atomistic and thus its holdings have zero measure.

subject to idiosyncratic shocks (unlike private banks, which are subject to that risk). This assumption is not crucial but dramatically simplifies the analysis. As long as the government is subject to aggregate shocks and to the restriction on taxes imposed in Section 2.7, the government cannot achieve the first best using asset purchases, and a role for banks and liquidity regulation arises. Exposing the government to idiosyncratic shocks would require the specification of a more complicated process for ψ_2, but it would not change the qualitative message of the policy analysis.[27] The time-2 return on capital earned by the government is thus $R_2 = \mathbb{E}\{\psi_2\} a_2$, or, using the normalization $\mathbb{E}\{\psi_2\} = 1$, $R_2 = a_2$.

3.2 Deposit contracts and government budget constraint

At $t = 0$, households give all their endowments of physical capital k and government liquidity b to the bank in their own location, following a standard approach in deriving optimal contracts in the banking literature.[28] The bank sets up the optimal mechanism to distribute government bonds to households at $t = 1$ and the output produced by the physical capital at $t = 2$.

At $t = 0$, each bank faces the budget constraint

$$K_0 + Q_0 B_0 \leq k + Q_0 b, \tag{23}$$

where now K_0 and B_0 denote the amount of capital and government bonds held by the bank, and Q_0 still denotes the price of government bonds.

At $t = 1$, the realization of the idiosyncratic shock to capital at each location becomes common knowledge, households privately learn the realization of their own preference shocks, and then banks distribute liquid assets to households. I consider direct, incentive-compatible mechanisms in which households reveal their true type (i.e., the true realization of their shock ε) to the bank. The bank distributes to households liquid assets based on such reporting and on the idiosyncratic shock to capital ψ_2. Thus, where necessary, I use the notation (ε, ψ_2) to refer to a household with preference shock ε in a location hit by the idiosyncratic shock to capital ψ_2. The aggregate shock a_2, which is realized at $t = 2$, does not affect the distribution of liquidity at $t = 1$ by banks.[29]

Unlike classic banking models where the bank distributes goods at $t = 1$, each bank here

[27] To keep the shock ψ_2 idiosyncratic at the economy-wide level, one would need to require the measure of locations hit by the good or bad shocks (which is given by the parameter α in the model) to be a function of the assets held by the government, which varies depending on the policy implemented by the government.

[28] See, for instance, Allen and Gale (1998) and Diamond and Dybvig (1983).

[29] This is because households are risk neutral with respect to time-2 consumption. Thus, when a household decides between revealing its true preference shock or pretending to be of a different type, only the expectation $\mathbb{E}\{a_2\} = 1$ affects the decision, as if there were no risk at $t = 2$.

distributes financial assets (i.e., government bonds), and then households choose whether and how much of these assets to sell in exchange for consumption goods at $t = 1$ versus how much to carry to $t = 2$. Let $B_1(\varepsilon, \psi_2)$ be the amount of government bonds distributed to a household that reports the preference shock ε in a location with shock ψ_2. Feasibility requires that, for each ψ_2,

$$\int B_1(\varepsilon, \psi_2)\, dF(\varepsilon) \leq B_0, \tag{24}$$

where $F(\cdot)$ denotes the c.d.f. of households' preference shocks. That is, the total amount of liquidity distributed to households at $t = 1$ cannot exceed the time-0 holdings B_0. After liquidity is distributed, each household chooses consumption $C_1(\varepsilon, \psi_2)$ to maximize

$$\max_{C_1(\varepsilon,\psi_2)} \quad \varepsilon u\left[C_1(\varepsilon, \psi_2)\right] + \underbrace{B_1(\varepsilon, \psi_2) - \frac{C_1(\varepsilon, \psi_2)}{Q_1}}_{\text{liquidity carried to } t=2} \tag{25}$$

subject to a liquidity constraint similar to (12):

$$C_1(\varepsilon, \psi_2)\frac{1}{Q_1} \leq B_1(\varepsilon, \psi_2). \tag{26}$$

The objective function (25) takes into account that the liquidity unused at $t = 1$, which is given by $B_1(\varepsilon, \psi_2) - C_1(\varepsilon, \psi_2)/Q_1$, is carried to $t = 2$ and used to finance consumption at that time, yielding a linear unitary marginal utility.

Finally, at $t = 2$, the bank distributes to households the goods produced by capital. Let $Y_2(\varepsilon, \psi_2)$ denote the goods distributed to a household that reports the preference shock ε in a location with the idiosyncratic shock to capital ψ_2. Without loss of generality, I restrict the analysis to contracts such that impatient households do not obtain any goods at $t = 2$: $Y_2\left(\varepsilon^h, \psi_2\right) = 0$.[30] Thus, the goods distributed to each patient household are

$$Y_2\left(\varepsilon^l, \psi_2\right) = \frac{a_2\psi_2 K_0 + \left[B_0 - \int B_1(\varepsilon, \psi_2)\, dF(\varepsilon)\right] - T_2(\psi_2)}{1-\kappa}, \tag{27}$$

where $a_2\psi_2 K_0 = R_2 K_0$ is the output produced by the capital K_0 purchased at $t = 0$, $B_0 - \int B_1(\varepsilon, \psi_2)\, dF(\varepsilon)$ denotes the government bonds not distributed by the bank at $t = 1$, and $T_2(\psi_2)$ are taxes, which can be contingent on the realization of ψ_2 (see the discussion after Equation (29) for why taxes are contingent on ψ_2). All these resources are distributed equally to each of the $1 - \kappa$ patient agents. Note that total household consumption at $t = 2$

[30] Formally, there exist two constants k^{min} and k^{max} such that the normalization $Y_2\left(\varepsilon^h, \psi_2\right) = 0$ is without loss of generality if $k^{min} < k < k^{max}$. Appendix B.3 characterizes the two constants k^{min} and k^{max} as a function of the parameters.

is

$$C_2(\varepsilon, \psi_2) = Y_2(\varepsilon, \psi_2) + \left[B_1(\varepsilon, \psi_2) - \frac{C_1(\varepsilon, \psi_2)}{Q_1}\right]. \tag{28}$$

That is, total consumption at $t = 2$ includes not only the goods $Y_2(\varepsilon, \psi_2)$ received at $t = 2$ from the bank but also the payoff earned by carrying the amount $B_1(\varepsilon, \psi_2) - C_1(\varepsilon, \psi_2)/Q_1$ of government liquidity from $t = 1$ to $t = 2$.

Finally, since I am allowing for time-2 taxes to depend on the realization of the idiosyncratic shock, the government budget constraint (9) is replaced by

$$b = (1 - \alpha) T_2\left(\overline{\psi}\right) + \alpha T_2\left(\underline{\psi}\right). \tag{29}$$

I restrict taxes to be non-negative, that is, $T_2(\psi_2) \geq 0$. Otherwise, the government could indirectly provide insurance against the idiosyncratic shocks to capital by setting $T_2\left(\underline{\psi}\right) < 0$. Nonetheless, taxes are contingent on the idiosyncratic shock to capital ψ_2 because banks hit by the bad shock $\underline{\psi}$ might not have resources available to pay such taxes. Indeed, in equilibrium, $T_2\left(\underline{\psi}\right) = 0$ and $T_2\left(\overline{\psi}\right) > 0$ will be the case.

The equilibrium definition is similar to that of the bankless economy, with the additional requirements related to the optimal banking contract. An equilibrium is a collection of: prices of government debt at $t = 0$ and $t = 1$, Q_0 and Q_1; a deposit contract that specifies withdrawals at $t = 1$, $B_1(\varepsilon, \psi_2)$, and goods distributed at $t = 2$, $Y_2(\varepsilon, \psi_2)$, for each (ε, ψ_2), and that is optimal for households taking Q_0 and Q_1 as given; and a consumption allocation, $C_t(\varepsilon, \psi_2)$ for $t = 1, 2$ and for each (ε, ψ_2) and X_2 for producers, that is feasible, maximizes households' and producers' utility, and satisfies market clearing.

The requirement that the deposit contract must be optimal implies that such a contract must be consistent with truth-telling (i.e., households must find it convenient to truthfully report their type). The general incentive-compatibility constraint is provided in Appendix B.1. However, the incentive-compatibility constraint will be binding in equilibrium only for households in locations hit by the bad idiosyncratic shock to capital $\underline{\psi}$ (i.e., in locations in which banks become insolvent). In these locations, the constraint dramatically simplifies because of the assumption $\underline{\psi} = 0$, as I discuss in the next section.

3.3 Equilibrium

This section describes the qualitative and key features of the equilibrium, and Appendix B provides a full characterization of all the endogenous variables in closed form as a function of the parameters. Similar to the bankless economy of Section 2, the equilibrium is unique.

At $t = 0$, all banks are alike and thus hold the same amount of capital and government debt. As in the bankless economy of Section 2, market clearing implies that their holdings must be equal to the initial endowments: $B_0 = b$ and $K_0 = k$.

To analyze the equilibrium outcome at $t = 1$ and $t = 2$, consider first the case in which $\alpha = 0$ and thus the idiosyncratic shock to capital ψ_2 is essentially shut down (this case corresponds to the environment of the bankless economy). All locations will be hit by the good idiosyncratic shock to capital $\overline{\psi}$, and thus no bank will become insolvent at $t = 2$. As a result, banks can offer a contract that provides full insurance against preference shocks, which is based on the standard logic of the optimal deposit contract in banking models with preference shocks. At $t = 1$, banks distribute more liquidity to impatient households (i.e., to households with preference shock ε^h) and less liquidity to patient ones (i.e., those with shock ε^l). At $t = 2$, banks distribute goods to patient households, thereby compensating them for the fewer resources received at $t = 1$.

Next, consider the case $\alpha > 0$. The equilibrium is at an intermediate case between the economy with $\alpha = 0$, in which all banks offer full insurance against preference shocks, and the bankless economy, in which households receive no insurance at all. More precisely, banks in the mass $1 - \alpha$ of locations hit by the good idiosyncratic shock to capital, $\overline{\psi}$, provide full insurance against preference shocks, whereas banks in the mass α of locations hit by the bad shock $\underline{\psi}$ provide no insurance. Banks in locations hit by $\underline{\psi}$ have no goods available at $t = 2$ to be distributed to patient households because $\underline{\psi} = 0$. As a result, patient households at these locations will truthfully report their type only if they get, at $t = 1$, the same amount of liquid assets that the bank distributes to impatient households. This implies that all households in these locations have the same amount of liquid assets at $t = 1$, independently of their type, much like in the bankless economy. This outcome can be interpreted either as a run (because all households withdraw liquidity at $t = 1$, independently of their type) or as a haircut on deposits (because the value of the liquidity distributed at $t = 1$ is less than the total value of the endowments that the household gave to the bank at $t = 0$).

A bank in a location hit by the good idiosyncratic shock to capital, $\overline{\psi}$, is solvent. At $t = 1$, this bank provides insurance against the preference shocks, distributing more liquid assets to impatient households and less liquidity to patient ones. In equilibrium, the constraint (24) is slack for a bank hit by $\overline{\psi}$.[31] That is, under the optimal deposit contract, the desired amount of time-1 consumption for depositors of this banks can be financed by distributing only a fraction of the government bonds $B_0 = b$ carried by the bank from $t = 0$. Without loss of generality, I consider a contract in which solvent banks distribute only the amount of government debt that is necessary for the optimal amount of consumption $C_1\left(\varepsilon, \overline{\psi}\right)$; the remaining government debt is carried by the bank to $t = 2$, and its payoff is distributed

[31] This result follows from the lower bound on government debt in (10).

to households at $t = 2$ together with the output produced by the capital.[32] The liquidity constraint (26) of households with deposits at a solvent bank is not binding as well. That is, their first-order conditions are

$$\varepsilon u' \left[C_1 \left(\varepsilon, \overline{\psi}\right)\right] = \frac{1}{Q_1} \quad , \quad \varepsilon \in \left\{\varepsilon^h, \varepsilon^l\right\}. \tag{30}$$

Solvent banks provide full insurance against preference shocks and thus equalize the marginal utilities of time-1 consumption among their depositors:

$$\varepsilon^h u' \left[C_1 \left(\varepsilon^h, \overline{\psi}\right)\right] = \varepsilon^l u' \left[C_1 \left(\varepsilon^l, \overline{\psi}\right)\right]. \tag{31}$$

While this condition is similar to the planner's optimality, Equation (7), there is a subtle but important difference. Because financial distress generates a liquidity premium on the price of government debt (i.e, on Q_1), the levels of consumption $C_1 \left(\varepsilon^h, \overline{\psi}\right)$ and $C_1 \left(\varepsilon^l, \overline{\psi}\right)$ are not equal to the first-best ones. In equilibrium,

$$C_1 \left(\varepsilon^h, \overline{\psi}\right) > \left(C_1^h\right)^* \quad \text{and} \quad C_1 \left(\varepsilon^l, \overline{\psi}\right) > \left(C_1^l\right)^*, \tag{32}$$

where $\left(C_1^h\right)^*$ and $\left(C_1^l\right)^*$ are the first-best levels of consumption, defined in Section 2.3. That is, households with deposits at solvent banks consume too much, in comparison to the first best. As discussed next, the households that consume too little are those with deposits at insolvent banks (i.e., banks hit by $\underline{\psi}$) and that are impatient (i.e., with preference shock ε^h).

A bank in a location hit by the bad idiosyncratic shock to capital, $\underline{\psi}$, is insolvent. Since $\underline{\psi} = 0$, this bank has no goods available to be distributed to patient depositors at $t = 2$ and thus $Y_2 \left(\varepsilon^l, \underline{\psi}\right) = 0$. As a result, patient depositors truthfully reveal their own type at $t = 1$ only if the distribution of liquidity at $t = 1$ satisfies the incentive-compatibility constraint

$$B_1 \left(\varepsilon^h, \underline{\psi}\right) \geq B_1 \left(\varepsilon^l, \underline{\psi}\right). \tag{33}$$

That is, patient households reveal their preference shock $\varepsilon = \varepsilon^h$ only if they obtain at least the same amount of liquidity as impatient households (i.e., those hit by $\varepsilon = \varepsilon^l$).[33] In equilibrium, (33) is binding, and thus banks hit by $\underline{\psi}$ distribute the same liquidity to all households: $B_1 \left(\varepsilon^h, \underline{\psi}\right) = B_1 \left(\varepsilon^l, \underline{\psi}\right)$. The fact that banks hold $B_0 = b$ government bonds at $t = 0$ implies that each household obtains b units of government debt at $t = 1$.

[32] If the bank distributes all its holdings of government debt at $t = 1$, households would spend only a fraction of it at $t = 1$ and carry the difference to $t = 2$, but consumption allocation and prices would be unchanged.

[33] The full specification of the incentive-compatibility constraint is provided in Appendix B.1.

Since insolvent banks distribute the same amount of liquidity to all their depositors, the behavior of such depositors resembles what happens in the bankless economy, in which all households have the same amount of liquid assets at $t = 1$ as well. Impatient households (i.e., those with preference shocks ε^h) use all the liquidity they have to purchase consumption at $t = 1$, and thus their binding liquidity constraint implies

$$C_1\left(\varepsilon^h, \underline{\psi}\right) = Q_1 b. \tag{34}$$

As in the bankless economy, it is useful to state the impatient households' first-order condition. Denoting $\mu_1(\varepsilon, \psi)$ to be the Lagrange multiplier of the liquidity constraint (26):

$$\varepsilon^h u'\left[C_1\left(\varepsilon^h, \underline{\psi}\right)\right] = \frac{1 + \mu_1\left(\varepsilon^h, \underline{\psi}\right)}{Q_1}. \tag{35}$$

Patient households (i.e., those with preference shocks ε^l) spend only a fraction of the liquidity they have, and their liquidity constraint is not binding. Their first-order condition is similar to that of households with deposits at solvent banks:

$$\varepsilon^l u'\left[C_1\left(\varepsilon^l, \underline{\psi}\right)\right] = \frac{1}{Q_1}. \tag{36}$$

Comparing (35) and (36) yields an expression that resembles (17) derived in the bankless economy:

$$\varepsilon^h u'\left[C_1\left(\varepsilon^h, \underline{\psi}\right)\right] > \varepsilon^l u'\left[C_1\left(\varepsilon^l, \underline{\psi}\right)\right]. \tag{37}$$

Banks' inability to insure against preference shocks implies that the marginal utility of impatient households is higher in comparison to that of patient households. Similar again to the bankless economy, impatient households consume too little and patient households consume too much:

$$C_1\left(\varepsilon^h, \underline{\psi}\right) < \left(C_1^h\right)^* \quad \text{and} \quad C_1\left(\varepsilon^l, \underline{\psi}\right) > \left(C_1^l\right)^*. \tag{38}$$

The welfare implications are crucially different from models in which banks repay deposits using goods rather than financial assets. In addition to the standard inability of insolvent banks to insure against preference shocks, a novel inefficiency arises through a general equilibrium channel. Consumption of all households with deposits at solvent banks is higher in comparison to the first-best level, as noted by (32). This is because the crisis is associated with a price of government debt, Q_1, which is higher in comparison to an economy without bank insolvencies. This higher price benefits households with deposits at solvent banks disproportionately more because solvent banks allocate the liquid government debt more efficiently than insolvent ones. The flip side is that impatient households with deposits at insolvent banks consume even less than in the bankless economy. That is, $C_1\left(\varepsilon^h, \underline{\psi}\right)$ is not just below the first best, it is also below the equilibrium consumption C_1^h of impatient households that arises in the bankless economy.

The general equilibrium effect that reduces welfare even further than in a real model is driven by a pecuniary externality, as I will discuss in more detail in the next section. Liquidity requirements will address this distortion and correct the negative effects of the externality.

Finally, note that the results of this economy with bank insolvencies and runs can also be reinterpreted as those of an economy with no financial crisis but segmented access to financial markets. That is, the same results arise in an economy in which there are no idiosyncratic shocks to capital but a fraction α of households have no access to either markets or financial intermediaries at $t = 0$. These households would just hold their endowment and thus would have an amount b of liquidity available to be spent at $t = 1$, similar to the depositors of the insolvent banks.[34]

3.4 Optimal policy: asset purchases and liquidity requirements

This section studies a combination of asset purchases and liquidity requirements in the full model with banks. The mechanics and logic of asset purchases are similar to that described in Section 2.7 for the bankless economy, and thus I refer the reader to that section for more explanations. Here, the agents that face a binding liquidity constraint are the impatient households (i.e., those with shocks ε^h) at the fraction α of locations with insolvent banks, and the asset purchase policy relaxes their constraint by increasing the supply of government debt at $t = 0$. If asset purchases do not implement the first best, liquidity requirements improve welfare further by addressing a pecuniary externality. The rest of the section focuses primarily on the description of liquidity requirements, and then characterizes the optimal combination of asset purchases and liquidity requirements.

Recall from Section 3.3 that, in the unregulated equilibrium, households with deposits at solvent banks consume too much with respect to the first best, and impatient households with deposits at insolvent banks consume even less than in the bankless economy. Liquidity requirements eliminate this effect, reestablishing the results that would arise if banks repaid

[34] The results of the policy interventions can be extended to this interpretation if the fraction of households with no access to financial markets and intermediaries is randomly selected at $t = 0$ after the policy is announced, that is, if households are ex ante identical. Otherwise, if there is ex ante heterogeneity, the welfare analysis of policy interventions should include considerations about redistribution.

deposits using goods rather than financial assets.

To understand the source of the pecuniary externality, recall that the contract offered by banks and analyzed in Section 3.3 maximizes the utility of households in each location taking the price of government liquidity as given. However, the price Q_1 of government bonds at $t = 1$ enters the binding liquidity constraint of impatient households in locations with insolvent banks. Households with deposits at solvent banks do not internalize that any additional unit of government debt used to purchase consumption goods at $t = 1$ decreases the price Q_1, thereby making the liquidity constraint of the impatient households with deposits at insolvent banks even more binding.

A planner (or regulator) that forces banks to offer a different contract can address the pecuniary externality, improving welfare. Different from individual banks, the planner internalizes the effect of the banking contract on the price of liquid assets at $t = 1$.[35] The planner's optimal contract can then be implemented with a liquidity requirement on banks that remain solvent at $t = 1$. Because of this requirement, some liquid assets—which would otherwise be sold in exchange for consumption goods—remain idle in the vault of solvent banks at $t = 1$ and thus are removed from circulation. As a result, the fewer liquid assets that are still traded become more valuable relative to the time-1 consumption goods. That is, the price Q_1 of such assets increases.

The liquidity requirement has two effects on welfare. On the one hand, it hurts households with deposits at solvent banks because it limits the liquidity that they can withdraw from their own banks (and, thus, limits their consumption expenditure). On the other hand, the liquidity requirement boosts the price Q_1 of government bonds, relaxing the liquidity constraint of impatient households with deposits at insolvent banks. The welfare gain of the latter set of households (which face a tight liquidity constraint) offsets the welfare loss of the former ones, and thus liquidity requirements improve welfare.

To compute the optimal liquidity requirements, I consider a planner that restricts the amount of government bonds that can be distributed at $t = 1$ by solvent banks, that is, banks hit by the good idiosyncratic shock to capital $\overline{\psi}$.[36] More precisely, the planner forces a solvent bank, at $t = 1$, to retain a fraction ζ of the government bonds B_0 purchased at $t = 0$. The government bonds that are not distributed at $t = 1$ by a bank are held until $t = 2$. This policy does not require the government to violate market incompleteness because it does not directly redistribute resources to locations hit by the bad idiosyncratic shock $\underline{\psi}$, and it does not require the government to observe the realization of the households' preference

[35] I consider a planner with limiting planning abilities, that is, a planner that can force banks to offer a different deposit contract but cannot control how households trade liquid assets at $t = 1$.

[36] Rather than analyzing the full planner problem, I solve directly for the optimal liquidity requirement that maximizes welfare and then verify that it is binding in equilibrium.

shocks. Nonetheless, the policy generates a redistribution of real resources through a general equilibrium channel.

To model the liquidity requirement, Equation (24) for banks hit by the good shock $\overline{\psi}$ is replaced by

$$\int B_1\left(\varepsilon, \overline{\psi}\right) dF\left(\varepsilon\right) \leq B_0\left(1-\zeta\right), \tag{39}$$

where ζ parameterizes the liquidity requirement and, thus, denotes the fraction of government bonds that cannot be distributed at $t = 1$ and must be held until $t = 2$. All the other equations that describe the equilibrium are the same as in Section 3.3. Note that under the liquidity requirements studied here, solvent banks must hold some liquid assets even after withdrawals have taken place at $t = 1$. Diamond and Kashyap (2016) interpret this type of regulation as the liquidity coverage ratio introduced by Basel III.

The next proposition characterizes the optimal combination of asset purchases and liquidity requirements. Proofs of the results of this section are provided in Appendix B.4.

Proposition 3.1. *Under the optimal asset purchase and liquidity requirement policy, the optimal asset purchases of the central bank are*

$$\{k_0,\, b_0\} = \begin{cases} \{0, b\} & \text{if } \underline{a} \leq \frac{b}{b(1-\kappa)+\kappa\varepsilon^H} \\ \left\{ \frac{1-\alpha\left(1-\kappa\varepsilon^h\right)}{1-\underline{a}\alpha(1-\kappa)} - \frac{b}{\underline{a}},\ \frac{\underline{a}\left[1-\alpha\left(1-\kappa\varepsilon^h\right)\right]}{1-\underline{a}\alpha(1-\kappa)} \right\} & \text{if } \underline{a} > \frac{b}{b(1-\alpha\kappa)+\alpha\kappa\varepsilon^H}, \end{cases}$$

and the optimal liquidity requirement is

$$\zeta^* = \frac{b_0 - 1 + \kappa\left(\varepsilon^h - b_0\right)}{b_0}.$$

While liquidity requirements correct the effect of the pecuniary externality, asset purchases close the gap with the first best. The optimal asset purchase intervention is very similar to that of the bankless economy. Indeed, not only is it qualitatively identical, but as α approaches one, the size of the intervention approaches that of the bankless economy (i.e., k_0 and b_0 approach that of Proposition 2.1).

The optimal liquidity requirement is a function of the liquidity supplied by the government and, thus, it is interlinked with the asset purchase policy. The optimal liquidity requirement is increasing in the size of the asset purchase policy, $\partial\zeta^*/\partial b_0 > 0$, but the total amount of liquidity distributed by solvent banks increases with the asset purchase policy:

$$\frac{\partial\left[\int B_1\left(\varepsilon, \overline{\psi}\right) dh\Big|_{\zeta=\zeta^*}\right]}{\partial b_0} = \kappa \in (0, 1).$$

As the central bank purchases more assets (i.e., as k_0 and b_0 go up), the liquidity requirement increases but banks enter $t = 1$ with more liquidity because, at $t = 0$, they sell physical capital to the central bank in exchange for government debt. This second force more than offsets the increase in ζ^*, and thus the total liquidity distributed by banks goes up with b_0, even though less than proportionally.

Finally, I characterize the equilibrium under the optimal policy, focusing on the consumption allocation at $t = 1$. The liquidity regulation eliminates the distortive effects of the pecuniary externality. Under the optimal liquidity regulation, households with deposits at solvent banks consume the first-best amount. As a result, the distortive effect of financial crises remains confined to households with deposits at banks subject to runs, without rippling through the rest of the economy.

Proposition 3.2. *Under the optimal asset purchase and liquidity requirement policy, consumption at* $t = 1$ *is at the first-best level for households with deposits at solvent banks,*

$$C_1\left(\varepsilon^h, \overline{\psi}\right) = \varepsilon^h, \qquad C_1\left(\varepsilon^l, \overline{\psi}\right) = \varepsilon^l,$$

and consumption at $t = 1$ *is the same as in the bankless economy for households with deposits at insolvent banks,*

$$C_1\left(\varepsilon^h, \underline{\psi}\right) = \frac{b_0}{1 - \kappa\left(\varepsilon^h - b_0\right)}, \qquad C_1\left(\varepsilon^l, \underline{\psi}\right) = \frac{\varepsilon^l}{1 - \kappa\left(\varepsilon^h - b_0\right)}.$$

4 Extensions and robustness analysis

The model of Sections 2 and 3 uses some stark assumptions to convey the key results of the paper. In this section, I relax two of these assumptions: that output at $t = 1$ is exogenous (i.e., total consumption of households is fixed at one) and that the only liquid assets in the economy are those supplied by the government, with no role for privately supplied liquid securities. The main elements and results of the two extensions are provided in Sections 4.1 and 4.2, respectively, and the full details of the analysis are provided in Appendix C.

The main takeaway is that the key results of the baseline model are robust to adding these additional features. In addition, some new results arise: a financial-driven recession and an interaction of policy interventions with output and investments.

In the extended models, a financial crisis reduces real aggregate activity, that is, both output and investments contract. In particular, the effects on investments are related to demand for privately supplied liquid assets, which increases during a crisis. Such assets must be backed by safe projects (i.e., safe collateral), and the costs associated with investing in these safe projects reduce investments. This resembles the results of Gorton and Ordoñez (2014, 2016), in which crises are associated with costly information production about collateral, which might in turn reduce investments.

Asset purchases and liquidity requirements can offset the contraction in real activity, even though with some caveats. Asset purchases alone always counteract the drop in output and investments. The effect of liquidity requirements, however, is more nuanced. Fixing investments (Section 4.1), liquidity requirements reduce the amount of liquid assets used for trades and thus worsen the financial-driven recession. Nonetheless, the welfare effect of liquidity regulation remains positive because of the same benefits highlighted in Section 3.4. In addition, liquidity requirements reduce the demand for costly privately supplied liquid assets (Section 4.2), boosting investments and, thus, output. Overall, when liquidity requirements are combined with asset purchases, they can increase both investments and output.

4.1 Endogenous output at $t = 1$ and financial-driven recessions

In this extension, I generalize the preferences of producers to endogenize output and consumption at $t = 1$. Producers must now exert effort to supply goods at $t = 1$. As a result, a banking crisis generates not only a misallocation of consumption across households but also a demand-driven drop in output at $t = 1$, that is, a recession. Liquidity requirements still reduce the cross-sectional dispersion in households' marginal utilities, but now they also further depress output. Nonetheless, liquidity requirements are welfare improving as long as producers' supply has a finite elasticity, however large. This result follows from the fact that the welfare loss from the reduction in consumption is exactly offset by the welfare gain due to the lower effort exerted by producers. Asset purchases, in contrast, boost output and thus offset the financial-driven recession. This section highlights the key element of the extended model and the most important results, and the full analysis is provided in Appendix C.1.

To supply goods, producers must work at $t = 1$, and this work now generates disutility. Each producer supplies N_1 units of work, and each unit of time spent working produces one unit of consumption goods. The assumption of linear utility from consumption X_2 at $t = 2$ is unchanged. Thus, producers' utility is

$$\frac{(n - N_1)^{1-\sigma} - 1}{1 - \sigma} + X_2,$$

where n is an upper bound on the amount of work that each producer can supply. Thus, $n - N_1$ is producers' leisure, which gives them utility $(n - N_1)^{1-\sigma} / (1 - \sigma)$. This structure is similar to Lagos and Wright (2005), in which sellers produce using labor, even though consumption goods are traded here in a competitive market.

I distinguishing between three cases: $\sigma \to \infty$, $\sigma \in (0, \infty)$, and $\sigma \to 0$. The case $\sigma \to \infty$ delivers the same equilibrium as in the baseline model. Indeed, as $\sigma \to \infty$, producers inelastically supply $N_1 = 1$ for any price Q_1. As a result, the policy analysis in this case is the same as in the baseline model. If instead $\sigma \in (0, \infty)$ and $\sigma \to 0$, producers' supply responds to the amount of liquidity expended by households. Nonetheless, the benefits of asset purchases and liquidity regulation are unchanged.

When $\sigma \in (0, \infty)$, producers supply an amount of goods that is a nontrivial function of the price of liquid assets Q_1. The equilibrium is qualitatively similar to that of the baseline model, and it can be solved in closed form in the special case $\sigma \to 1$ (see Appendix C.1). Nonetheless, one can easily verify numerically that the equilibrium exists even for other values of σ. The key novelty here is that bank insolvencies now generate a demand-driven drop in output at $t = 1$—that is, a recession. With less liquidity in the hands of impatient households with deposits at insolvent banks, households' demand drops and so does output supplied by producers.

With no asset purchases, liquidity requirements generate two effects in this richer model. The first effect is the same as in the baseline model, that is, to reduce the cross-sectional dispersion of the marginal utilities of households. The new, second effect is to reduce even further the demand for consumption goods by households. Liquidity requirements reduce the purchasing power of households with deposits at solvent banks, exacerbating the demand-driven recession. Nonetheless, liquidity requirements still increase welfare, as formalized next.

Proposition 4.1. *Assume that $\alpha > 0$ and, given an asset purchase policy $\{k_0, b_0\}$, there exists an equilibrium with $\sigma \in (0, \infty)$ that does not implement the first best. The optimal liquidity requirement is $\zeta^* > 0$ and strictly reduces the distribution of government bonds by banks hit by $\overline{\psi}$ (i.e., solvent banks) in comparison to the unregulated equilibrium.*

Even if liquidity requirements reduce output, they are beneficial for welfare. On the one hand, forcing households with deposits at solvent banks to marginally reduce their consumption generates a welfare loss for such households. However, this loss is exactly offset by the welfare gain of producers, who now enjoy higher utility because they reduce production and enjoy more leisure. Indeed, the proof of Proposition 4.1 relies on the fact that these two effects cancel out. In addition, liquidity requirements eliminate the negative welfare effects of the pecuniary externality, similar to Section 3.4. Thus, the total effect on welfare is unambiguously positive.

When $\sigma \to 0$, the optimal policy does not include liquidity requirements. This is because producers' supply of goods is infinitely elastic at $Q_1 = 1$, thereby pinning down the price of government bonds at $t = 1$.[37] As a result, liquidity requirements do not alter the price of liquid assets, which is fixed at $t = 1$, and thus do not reduce the cross-sectional dispersion of households' marginal utilities. More precisely, as $\sigma \to 0$, the pecuniary externality described in Section 3.4 disappears because the consumption expenditure of each household does not affect the price of liquid assets, which is fixed at $Q_1 = 1$.

More generally, the positive welfare effect of liquidity requirements should hold even in richer models, as long as σ is sufficiently large (i.e., as long as producers' supply is sufficiently inelastic). For instance, one can think of an extension in which other frictions induce a wedge between the marginal utility of consumption of households and the marginal disutility of effort by producers. In this case, the welfare loss from the reduction in consumption will not be entirely offset by the welfare gain from the reduction in the work required for production. The final welfare effect will be a horse race between this loss and the gain from the improvement of the cross-sectional distribution of households' consumption. As $\sigma \to \infty$, the second effect must always prevail because output becomes fixed. Thus liquidity requirements must be beneficial if producers' supply is sufficiently inelastic (i.e., if σ is sufficiently large).

When liquidity requirements and asset purchases are combined together, the total effect on output can instead be positive. This is because asset purchases relax the liquidity constraint of impatient households with deposits at insolvent banks, thereby boosting their demand. Thus, a combination of liquidity requirements and asset purchases not only improves the cross-sectional distribution of consumption across households but might also counteract the recession driven by the financial crisis.

4.2 Endogenous production of safe and liquid assets

In this extension, I maintain the assumption of a fixed amount of goods traded at $t = 1$, but I allow agents, at $t = 0$, to invest more in the risky capital or invest in a less productive but risk-free physical capital. This second type of capital can be used as collateral for privately supplied liquid assets. Asset purchases and liquidity requirements have effects that are similar to those of the baseline model, but they also reduce the production of private liquidity, so that agents invest more resources in the more-productive risky capital. This section highlights the key elements of the extended model and the most important results, and the full analysis is provided in Appendix C.2.

At $t = 0$, in addition to the endowment of risky capital (now denoted by k^{risky}) and government bonds (still denoted by b), households are also endowed with an amount e of goods

[37] Alternatively, one could interpret the result $Q_1 = 1$ by saying that producers' *demand for liquid assets* is infinitely elastic. In any case, the price of liquid assets at $t = 1$ must be $Q_1 = 1$.

that can be invested in two technologies. The first technology has the same productivity of the risky capital, and thus investing in it is equivalent to increasing the initial stock of k^{risky}. The other technology is safe but entails a proportional cost ϕ of investment. The cost ϕ can be motivated by the resources devoted to screening projects to make sure that the investment is safe, even though the equilibrium results suggest an additional interpretation, as I discuss at the end of this section.

If a fraction $S_0 \in [0,1]$ of the endowment e is invested in the safe technology, it generates an amount $S_0 e (1-\phi)$ of safe capital, and each unit of safe capital produces one unit of output at $t=2$. The remaining amount $(1-S_0)\, e$ is invested in the risky capital, and the amount $S_0 e \phi$ is lost. In addition, I assume that the safe capital $S_0 e (1-\phi)$ is not plagued by the adverse selection problem discussed in Section 2.5 that makes the risky capital illiquid at $t=1$.[38]

At $t=1$, the liquid assets held by a bank include not only the government debt B_0 carried from $t=0$ but also the safe and liquid capital K_0^{safe} that a bank can purchase at $t=0$. Thus, Equations (24) and (39) are replaced by

$$\int B_1\left(\varepsilon, \overline{\psi}\right) dF(\varepsilon) \leq \left(B_0 + K_0^{safe}\right)(1-\zeta),$$

where $B_1\left(\varepsilon, \overline{\psi}\right)$ now denotes the amount of liquid assets (i.e., either government bonds or safe liquid capital) distributed by a solvent bank to a household with preference shock ε; the term ζ still parameterizes the liquidity requirement.

The equilibrium is qualitatively identical to that of Section 3, but now liquid assets can also be supplied privately, in addition to being supplied by the government. If no bank becomes insolvent (i.e., if $\alpha = 0$), then no resources are invested in the safe technology. If instead some banks become insolvent (i.e., $\alpha > 0$), then some resources are invested in the safe technology and thus a fraction of the endowment is wasted because of the cost ϕ, reducing investments. In addition, the endogenous production of liquid assets at $t=0$ has important interactions with the analysis of both asset purchases and liquidity requirements, even though both policies remain beneficial.

Asset purchases crowd out the production of private liquidity, either partially or entirely. A small asset purchase reduces one-for-one the production of private safe and liquid capital, reducing the resources wasted because of the cost ϕ and thus increasing investments. As the

[38] If both the risky and safe capital are liquid at $t=1$, the results are unchanged. For locations hit by the bad idiosyncratic shock to capital $\underline{\psi}$, the capital is worthless because $\underline{\psi}=0$; thus, none of the features of the deposit contract related to insolvent banks changes. For locations hit by the good idiosyncratic shock to capital $\overline{\psi}$, banks do not distribute all their liquidity at $t=1$ (i.e., (24) is not binding), and thus nothing changes in the unregulated equilibrium either. The only difference is that liquidity requirements must now be imposed on all the liquid assets held by a solvent bank, including the liquid capital.

asset purchase gets larger and larger, it eventually entirely crowds out the supply of privately provided liquidity. At this point, the effect of any additional dollar injected in the economy is the same as in Section 3.

The liquidity regulation is broadly unchanged. Different from the baseline model, the planner takes into account that producing safe capital at $t = 0$ is costly, and thus the objective function of the planner accounts for this cost. In addition to the effects highlighted in Section 3.4, the liquidity requirement reduces the production of private liquidity. This is because this policy somehow reduces the liquidity needs at $t = 1$, which in turn reduces the demand for liquid assets at $t = 0$. While this effect only reduced the liquidity premium in Section 3.4, here it generates a drop in the production of private liquid assets. This effect saves resources because the safe capital that backs private liquidity is costly, and thus the increase in welfare due to the liquidity requirement is even higher than in the baseline model.

Finally, I note that the effects of the cost ϕ are reminiscent of those obtained by Gorton and Ordoñez (2014, 2016) in models in which costly production of information reduces investments. In those models, good times are associated with high investments and information-insensitive assets (here, when no bank becomes insolvent, no private liquidity is supplied, and investment is maximal), and bad times are associated with low investments and the production of information (here, when some banks are expected to become insolvent, some private liquidity is supplied and the associated costs reduce investments).

5 Conclusion

I have presented a general equilibrium model of financial crises with government-supplied liquid assets to study the optimal liquidity policies in the event of a financial crisis. If the banking system is not well functioning, asset purchases and liquidity regulations improve welfare. In particular, liquidity regulations address a pecuniary externality that arises because solvent banks and their depositors do not internalize the effects of their choices on the price of liquid assets, which in turn tightens the already binding constraint of agents with high liquidity needs and with deposits at insolvent banks.

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Appendix (For Online Publication)

A Bankless economy

This appendix presents the full details of the equilibrium in the bankless economy.

A.1 Equilibrium with no policy intervention

The equilibrium can be solved using Equations (11), (13)-(16), (18), the government budget constraint (9), the market clearing conditions for capital and government bonds at $t = 0$ (which imply that the amount of government bonds and capital held by households is equal to the respective endowments, $K_0 = k$ and $B_0 = b$), the equation that pins down consumption of producers, $X_2 = 1/Q_1$, and the market clearing conditions for consumption goods at $t = 1$ (i.e., Equation (5)). These equations can be solved in closed form, implying the following unique equilibrium:

- The price of government debt at $t = 0$ and $t = 1$ is

$$Q_0 = 1 + \kappa \left(\frac{\varepsilon^h}{b} - 1 \right), \quad Q_1 = \frac{1}{1 - \kappa \left(b - \varepsilon^h \right)};$$

- Consumption at $t = 1$ is

$$C_1^h = \frac{b}{1 - \kappa \left(b - \varepsilon^h \right)}, \quad C_1^l = \frac{\varepsilon^l}{1 - \kappa \left(b - \varepsilon^h \right)},$$

  and the Lagrange multiplier of impatient households is

$$\mu_1^h = \frac{\varepsilon^h}{b} - 1;$$

- Consumption at $t = 2$ of households, which is a function of the preference shock at $t = 1$ and thus is denoted by $C_2(\varepsilon)$, is

$$C_2(\varepsilon) = \begin{cases} R_2 k - b & \text{if } \varepsilon = \varepsilon^h \\ R_2 k - \varepsilon^l & \text{if } \varepsilon = \varepsilon^l; \end{cases}$$

- Consumption at $t = 2$ of producers, given by $X_2 = 1 - \kappa\left(b - \varepsilon^h\right)$;
- Taxes collected at $t = 2$ by the government are $T_2 = b$.

One can verify that the liquidity constraint of patient households is not binding because $Q_1 B_0 - C_1^l = \frac{b-\varepsilon^l}{1-\kappa\left(b-\varepsilon^h\right)} > 0$, where the inequality follows from the restrictions on the stochastic process of ε and from (10).

A.2 Asset purchases with no restrictions on T_2: equilibrium

If the government can use lump-sum taxes without any restriction, asset purchases implement the first best, as formalized by the next proposition.

Proposition A.1. *Assume that $k \geq \varepsilon^h - b$ and there are no restrictions on T_2. Then, the asset purchase policy $\{k_0, b_0\} = \left\{\varepsilon^h - b, \varepsilon^h\right\}$ implements the first best with taxes*

$$T_2 = \begin{cases} \varepsilon^h\left(1 - \underline{a}\right) + b\underline{a} & \textit{if } R_2 = \underline{a} \\ \varepsilon^h\left(1 - \bar{a}\right) + b\bar{a} & \textit{if } R_2 = \bar{a}, \end{cases}$$

and it drives to zero the liquidity premium on government debt (i.e., $Q_0 = Q_1 = 1$).

Proof. The allocation stated in the proposition is the unique solution to the equations that characterize the equilibrium, namely, Equations (11)-(16); the government budget constraints (19) and (20); and the market clearing conditions for capital and bonds, which are now given by $K_0 + k_0 = k$ and $B_0 = b_0$ to account for the fact that the supply k of capital is now held not only by households (i.e., K_0) but also by the government (i.e., k_0), and that the supply of government debt is b_0. The assumption $k \geq \varepsilon^h - b$ guarantees that the amount of capital purchased by the central bank, k_0, does not exceed the exogenous supply k. □

A.3 Asset purchases with restrictions on T_2: proof of Proposition 2.1

Conjecture that $k_0 > 0$. Combining the binding limit on taxes (21) with the government budget constraint at $t = 0$, (19), and the constraint at $t = 2$, (20), evaluated in the low state (i.e., evaluated at $R_2 = \underline{a}$) implies that $Q_0 = 1/\overline{a}$. Thus, I can solve for the equilibrium that implies $Q_0 = 1/\overline{a}$ using Equations (11), (13)-(16), (18), the government budget constraints (19) and (20), and the market clearing conditions for capital and bonds, which are now given by $K_0 + k_0 = k$ and $B_0 = b_0$ to account for the fact that the supply k of capital is now held not only by households (i.e., K_0) but also by the government (i.e., k_0), and that the supply of government debt is b_0. These equations can be solved in closed form, implying the following unique equilibrium:

- The asset purchase policy k_0 and b_0 is as stated in the proposition;
- The price of government debt at $t = 0$ and $t = 1$ is

$$Q_0 = \frac{1}{\overline{a}}, \quad Q_1 = \frac{1 - \underline{a}(1-\kappa)}{1 - \underline{a}(1-\kappa) - \kappa\varepsilon^h (1 - \underline{a})};$$

- Capital held by the central bank is given by Proposition 2.1, and capital held by households is

$$K_0 = k - \frac{1}{\underline{a}}\left[\frac{\underline{a}\kappa\varepsilon^h}{1 - \underline{a}(1-\kappa)} - b\right];$$

- Consumption at $t = 1$ is

$$C_1^h = \frac{\underline{a}\kappa\varepsilon^h}{1 - \underline{a}(1-\kappa) - \kappa\varepsilon^h(1-\underline{a})}, \quad C_1^l = \frac{\varepsilon^l(1 - \underline{a}(1-\kappa))}{1 - \underline{a}(1-\kappa) - \kappa\varepsilon^h(1-\underline{a})},$$

  and the Lagrange multiplier of impatient households is

$$\mu_1^h = \frac{1-\underline{a}}{\kappa\underline{a}};$$

- Consumption of households at $t = 2$ is

$$C_2(\varepsilon) = \begin{cases} kR_2 - \frac{\underline{a}\varepsilon^h\kappa}{1-\underline{a}(1-\kappa)} & \text{if } \varepsilon = \varepsilon^h \\ kR_2 - \varepsilon^l & \text{if } \varepsilon = \varepsilon^l, \end{cases}$$

  and consumption of producers is $X_2 = \frac{1-\underline{a}(1-\kappa)-\kappa\varepsilon^h(1-\underline{a})}{1-\underline{a}(1-\kappa)}$;
- Taxes collected at $t = 2$ by the government are

$$T_2 = \frac{bR_2}{\underline{a}} - \frac{(R_2 - \underline{a})\kappa\varepsilon^h}{1 - \underline{a}(1-\kappa)}.$$

Finally, given the solution for k_0, one can set $k_0 = 0$ and solve for the value of $\underline{a}$ such that $k_0 > 0$.

B Full model with banks and financial crises

This appendix discusses the incentive-compatibility (IC) constraints that must be satisfied by the optimal deposit contract (Appendix B.1), provides the list of equations that must be used to solve for the equilibrium (Appendix B.2), solves for the equilibrium under no policy intervention (Appendix B.3), and provides the proofs of Proposition 3.1 and Proposition 3.2 (Appendix B.4).

B.1 Incentive-compatibility (IC) constraints

Unlike standard banking models, I have to specify an incentive-compatibility (IC) constraint not only for patient households but also for impatient ones. This specification is necessary because of the different structure of preference; in Diamond and Dybvig (1983) and Allen and Gale (1998), impatient households do not have utility at $t = 2$ and thus have no incentive to misreport their type. Here, in contrast, banks distribute some liquidity at $t = 1$ to all households, and impatient households enjoy utility from time-2 consumption, and thus I must make sure that it is not profitable for them to misreport their own type.

For patient households (i.e., those with preference shock ε^l), the IC constraint requires that the utility of truthfully reporting the true type ε^l must be greater than the utility from misreporting and claiming to be of type ε^h. This can be stated as

$$\begin{aligned} \max_{C_1\left(\varepsilon^l,\psi_2\right)\leq Q_1 B_1\left(\varepsilon^l,\psi_2\right)} & \left\{\varepsilon^l u\left[C_1\left(\varepsilon^l,\psi_2\right)\right] + B_1\left(\varepsilon^l,\psi_2\right) - C_1\left(\varepsilon^l,\psi_2\right)/Q_1 + Y_2\left(\varepsilon^l,\psi_2\right)\right\} \\ & \geq \max_{C\leq Q_1 B_1\left(\varepsilon^h,\psi_2\right)} \left\{\varepsilon^l u\left(C\right) + B_1\left(\varepsilon^h,\psi_2\right) - C/Q_1\right\}. \end{aligned} \tag{40}$$

The utility from reporting the true type ε^l (left-hand side) is the utility of receiving liquidity $B_1\left(\varepsilon^l,\psi_2\right)$ at $t = 1$ and allocating it optimally between time-1 consumption $C_1\left(\varepsilon^l,\psi_2\right)/Q_1$ and savings $B_1\left(\varepsilon^l,\psi_2\right) - C_1\left(\varepsilon^l,\psi_2\right)/Q_1$; the savings can be used for consumption at $t = 2$, together with the goods $Y_2\left(\varepsilon^l,\psi_2\right)$ received at $t = 2$ from the bank. The utility from misreporting and pretending to be high type ε^h (right-hand side) is the utility that can be obtained by receiving liquidity $B_1\left(\varepsilon^h,\psi_2\right)$ and allocating such liquidity optimally between $t = 1$ and $t = 2$. For banks hit by $\underline{\psi}$ and, thus, with $Y_2\left(\varepsilon^l,\underline{\psi}\right) = 0$, the constraint simplifies to (33).

Similarly, for impatient households (i.e., those with preference shock ε^h), the IC constraint is

$$\max_{C_1\left(\varepsilon^h,\psi_2\right)\le Q_1 B_1\left(\varepsilon^h,\psi_2\right)} \left\{\varepsilon^h u\left[C_1\left(\varepsilon^h,\psi_2\right)\right] + B_1\left(\varepsilon^h,\psi_2\right) - C_1\left(\varepsilon^h,\psi_2\right)/Q_1\right\}$$
$$\ge \max_{C\le Q_1 B_1\left(\varepsilon^l,\psi_2\right)} \left\{\varepsilon^h u\left(C\right) + B_1\left(\varepsilon^l,\psi_2\right) - C/Q_1 + Y_2\left(\varepsilon^l,\psi_2\right)\right\}. \quad (41)$$

B.2 Equilibrium equations

To solve for the equilibrium, conjecture that the IC constraints (40) and (41) are satisfied. Then, the equilibrium solves (23), (24) evaluated at $\psi_2 = \underline{\psi}$, (27) evaluated at $\psi_2 \in \left\{\overline{\psi}, \underline{\psi}\right\}$,(26) evaluated at $\varepsilon \in \left\{\varepsilon^h, \varepsilon^l\right\}$ and $\psi_2 = \overline{\psi}$, (28) evaluated at $(\varepsilon, \psi_2) \in \left\{\left\{\varepsilon^h, \underline{\psi}\right\}, \left\{\varepsilon^l, \underline{\psi}\right\}, \left\{\varepsilon^h, \underline{\psi}\right\}, \left\{\varepsilon^l, \underline{\psi}\right\}\right\}$, (29), (30) evaluated at $\varepsilon \in \left\{\varepsilon^h, \varepsilon^l\right\}$, (33) evaluated with equality, (34)-(36), the market clearing for bonds and capital at $t = 0$ (which are given by $B_0 = b$ and $K_0 = k$), the market clearing condition for consumption goods at $t = 1$, which is given by

$$(1-\alpha)\int C_1\left(\varepsilon,\overline{\psi}\right) dF\left(\varepsilon\right) + \alpha \int C_1\left(\varepsilon,\underline{\psi}\right) dF\left(\varepsilon\right) = 1, \quad (42)$$

the first-order condition for time-0 holdings of government debt by banks, B_0, which is given by

$$Q_0 = 1 + \kappa\alpha\mu_1\left(\varepsilon^h, \underline{\psi}\right)$$

(this last condition is similar to that of the baseline economy, Equation (18), but the fraction of households with a binding liquidity constraint is only $\kappa\alpha$ in the full model because it includes only impatient households with deposits at insolvent banks), and the non-negativity constraint for taxes, $T_2(\psi_2) \ge 0$, evaluated with equality for $\psi_2 = \underline{\psi}$. This is a system of 22 equations in 21 unknowns—one equation can be dropped by Walras' law—which can be solved in closed form as functions of the parameters. Finally, one needs to verify that the IC constraints (40) and (41) are indeed not binding.

B.3 Equilibrium without policy intervention

The following proposition characterizes the unregulated equilibrium in the full model with banks.

Proposition B.1. *Assume that* (10) *holds and*

$$k^{min} < k < k^{max},$$

where

$$k^{max} = 1 + \alpha\left(b-1\right) + \left(1-\alpha\right)\left(1-\kappa\right)\varepsilon^h \log\left(\varepsilon^h/\varepsilon^l\right)$$

$$k^{min} \quad = \quad b\alpha + \varepsilon^h (1-\alpha) \, .$$

Then, there exists a unique equilibrium given by:

- *Time-0 portfolio: each bank holds capital $K_0 = k$ and bonds $B_0 = b$;*
- *Price of government debt: $Q_0 = 1 + \alpha\kappa \left(\varepsilon^h / b - 1\right) > 1$, $Q_1 = \left[1 - \alpha\kappa \left(\varepsilon^h - b\right)\right]^{-1} > 1$;*
- *Deposit contract:*
    - *Banks hit by the bad idiosyncratic shock to capital, $\psi_2 = \underline{\psi}$, distribute liquidity equally among all agents at $t = 1$, $B_1 \left(\varepsilon, \underline{\psi}\right) = b$ for $\varepsilon \in \left\{\varepsilon^h, \varepsilon^l\right\}$, and have no resources at $t = 2$ to be distributed to patient households: $Y_2 \left(\varepsilon^l, \underline{\psi}\right) = 0$;*
    - *Banks hit by the good idiosyncratic shock to capital, $\psi_2 = \overline{\psi}$, distribute more liquidity to impatient households and less liquidity to patient ones at $t = 1$:*

$$B_1 \left(\varepsilon, \overline{\psi}\right) = \begin{cases} \varepsilon^h & \text{if } \varepsilon = \varepsilon^h \\ \varepsilon^l & \text{if } \varepsilon = \varepsilon^l; \end{cases}$$

    and distribute goods $Y_2 \left(\varepsilon^l, \overline{\psi}\right) = \frac{a_2 \overline{\psi} K_0 - b\left(\frac{\alpha}{1-\alpha}\right) - 1}{1-\kappa}$ to patient households;

- *Time-1 and time-2 consumption allocation:*
    - *Time-1 consumption at locations hit by the good idiosyncratic shock to capital, $\psi_2 = \overline{\psi}$:*

$$C_1 \left(\varepsilon^h, \overline{\psi}\right) = \frac{\varepsilon^h}{1 - \alpha\kappa \left(\varepsilon^h - b\right)} > \left(C_1^h\right)^*, \quad C_1 \left(\varepsilon^l, \overline{\psi}\right) = \frac{\varepsilon^l}{1 - \alpha\kappa \left(\varepsilon^h - b\right)} > \left(C_1^l\right)^*,$$

    where $\left(C_1^h\right)^*$ and $\left(C_1^l\right)^*$ are the first-best level of consumption defined in Section 2.3;

    - *Time-1 consumption at locations hit by the bad idiosyncratic shock to capital, $\psi_2 = \underline{\psi}$:*

$$C_1 \left(\varepsilon^h, \underline{\psi}\right) = \frac{b}{1 - \alpha\kappa \left(\varepsilon^h - b\right)} < \left(C_1^h\right)^*, \quad C_1 \left(\varepsilon^l, \underline{\psi}\right) = \frac{\varepsilon^l}{1 - \alpha\kappa \left(\varepsilon^h - b\right)} > \left(C_1^l\right)^*;$$

    - *Time-2 consumption of households: $C_2 \left(\varepsilon^h, \psi\right) = 0$ for $\psi \in \left\{\overline{\psi}, \underline{\psi}\right\}$, $C_2 \left(\varepsilon^l, \underline{\psi}\right) = b - \varepsilon^l$, and*

$$C_2 \left(\varepsilon^l, \overline{\psi}\right) = \frac{a_2 \overline{\psi} K_0 - b\left(\frac{\alpha}{1-\alpha}\right) - 1}{1-\kappa};$$

    - *Time-2 consumption of producers: $X_2 = 1 - \alpha\kappa \left(\varepsilon^h - b\right)$;*

- *Government taxes: $T_2 \left(\underline{\psi}\right) = 0$ and $T_2 \left(\overline{\psi}\right) = \frac{b}{1-\alpha}$.*

Proof. The equilibrium can be solved following the approach described in Appendix B.2. The equations listed there can be solved in closed form, and it can be verified that the solution is unique. The conjecture that the incentive-compatibility constraints, (40) and (41), are satisfied for households with deposits in locations hit by $\underline{\psi}$ follows from the restriction $k^{min} < k < k^{max}$. □

B.4 Proof of Proposition 3.1 and Proposition 3.2

The results can be derived in two steps: solving for the optimal liquidity requirement given any asset purchase policy $\{k_0, b_0\}$ and then deriving the optimal asset purchase policy.

Given $\{k_0, b_0\}$, the optimal liquidity requirement is computed as follows. I consider the subset of equations used to solve for the equilibrium with no interventions to solve for selected endogenous variables. These equations are:

- The market clearing condition for bonds at $t = 0$, which is given by $B_0 = b_0$ under the asset purchase policy;
- The goods market clearing condition at $t = 1$, which is not affected by the policy and thus is given by (42);
- The equations that pin down the optimal level of consumption: for households with deposits at solvent banks, (39) holding with equality and combined with (24) holding with equality, which implies

$$\int C_1 \left(\varepsilon, \overline{\psi}\right) dF\left(\varepsilon\right) = B_0 \left(1 - \zeta\right),$$

  and (31); for households with deposits at insolvent banks, (34) and (36).

These equations pin down five endogenous variables: $Q_1(\zeta)$ and $C_1(\varepsilon, \psi; \zeta)$ for each $(\varepsilon, \psi_2) \in \left\{\left\{\varepsilon^h, \underline{\psi}\right\}, \left\{\varepsilon^l, \underline{\psi}\right\}, \left\{\varepsilon^h, \underline{\psi}\right\}, \left\{\varepsilon^l, \underline{\psi}\right\}\right\}$, where I have emphasized that these quantities are a function of the liquidity requirement ζ. To determine the optimal ζ, I consider the component of the welfare function that depends on the allocation at $t = 1$, that is,

$$\mathcal{W}_1(\zeta) = (1 - \alpha) \int C_1 \left(\varepsilon, \overline{\psi}; \zeta\right) dF(\varepsilon) + \alpha \int C_1 \left(\varepsilon, \underline{\psi}; \zeta\right) dF(\varepsilon),$$

take the first-order condition with respect to ζ, and solve for the optimal value of the liquidity requirement ζ^*. Then, plugging ζ^* back into the values of $C_1(\varepsilon, \psi; \zeta)$, I can solve for the level of consumption stated in Proposition 3.2.

As a second step, I compute the optimal size of asset purchases. To do so, I note that the limit on taxes (21) implies $Q_0 = 1/\underline{a}$, similar to the asset purchase policy in the baseline economy. In addition to $Q_0 = 1/\underline{a}$, I use the time-1 consumption just computed (i.e., those stated in Proposition 3.2), the government budget constraint (19), the first-order condition (36) for patient households with deposits at insolvent banks, and the first-order condition for time-0 holdings of government bonds, which now implies

$$Q_0 = 1 + (1-\alpha)\left[\kappa\mu_1\left(\varepsilon^h, \overline{\psi}\right) + (1-\kappa)\,\mu_1\left(\varepsilon^l, \underline{\psi}\right)\right] + \alpha\kappa\mu_1\left(\varepsilon^h, \underline{\psi}\right)$$

because the liquidity requirement ζ^* implies that the liquidity constraint (26) is now binding for the mass $1-\alpha$ of households with deposits at solvent banks. These equations pin down Q_0, Q_1, and the values of b_0, and k_0 stated in the proposition.

C Extensions and robustness: detailed analysis

C.1 Endogenous output at $t = 1$ and financial-driven recessions: detailed analysis

Model and equilibrium definition. The households' and banks' building blocks of the model are the same as in Section 3. To determine the supply of consumption goods by producers, recall that each unit of work N_1 produces one unit of consumption goods, which in turn is sold in exchange for $1/Q_1$ units of zero-coupon bonds. Thus, producers' consumption at $t = 2$ is subject to the budget constraint $X_2 \leq N_1/Q_1$, and the optimal time-1 supply of goods by producers is

$$(n - N_1)^{-\sigma} = \frac{1}{Q_1}. \tag{43}$$

The equilibrium concept is the same as in Section 3, with the addition of the supply of goods by producers, which is given by N_1 and must satisfy (43). The list of equations that pin down the equilibrium is thus the same as those of the model of Section 3 (see Appendix B.2) with the addition of (43), even though (42) must be modified as follows to account for the endogenous output at $t = 1$:

$$(1-\alpha)\int C_1\left(\varepsilon, \overline{\psi}\right) dF(\varepsilon) + \alpha \int C_1\left(\varepsilon, \underline{\psi}\right) dF(\varepsilon) = N_1.$$

Special case: $\sigma \to 1$. Under this assumption, several results can be characterized in closed form. For this case, it is useful to impose the normalization

$$n = 2, \tag{44}$$

so that the first best will be similar to (2.3).

Consider the first best. A planner that observes households' preference shocks maximizes welfare, which is now given by

$$\mathcal{W}=\left[\kappa\varepsilon^{h}u\left(C_{1}^{h}\right)+\left(1-\kappa\right)\varepsilon^{l}u\left(C_{1}^{l}\right)+C_{2}\right]+\left[\frac{\left(n-N_{1}\right)^{1-\sigma}-1}{1-\sigma}+X_{2}\right]$$

subject to the resource constraints at $t=1$,

$$\kappa C_{1}^{h}+\left(1-\kappa\right)C_{1}^{l}\leq N_{1},$$

and the resource constraint at $t=2$, which is still given by Equation (6). The optimal allocation at $t=1$ equalizes the marginal utility among households, and it equalizes households' marginal utility to that of producers:

$$\varepsilon^{h}u'\left(C_{1}^{h}\right)=\varepsilon^{l}u'\left(C_{1}^{n}\right)=\left(n-N_{1}\right)^{-\sigma}.$$

The normalization in (44) implies that the first-best allocation at $t=1$ is

$$\left(C_{1}^{h}\right)^{*}=\varepsilon^{h},\quad\left(C_{1}^{l}\right)^{*}=\varepsilon^{l},\quad\left(N_{1}\right)^{*}=1.$$

When banks are active and a fraction α of them are subject to the bad idiosyncratic shock to capital $\underline{\psi}$, the key equilibrium objects are:

- Prices of government debt $Q_0=1+\alpha\kappa\left(\varepsilon^{h}/b-1\right)$ and $Q_1=\frac{2}{2-\alpha\kappa\left(\varepsilon^{h}-b\right)}$;
- Supply by producers: $N_1=\frac{2\left[1-\alpha\kappa\left(\varepsilon^{h}-b\right)\right]}{2-\alpha\kappa\left(\varepsilon^{h}-b\right)}<\left(N_1\right)^{*}$;
- Consumption by households with deposits at solvent banks:

$$C_{1}\left(\varepsilon^{h},\overline{\psi}\right)=\varepsilon^{h}\frac{2}{2-\alpha\kappa\left(\varepsilon^{h}-b\right)}>\left(C_{1}^{h}\right)^{*},\qquad C_{1}\left(\varepsilon^{l},\overline{\psi}\right)=\varepsilon^{l}\frac{2}{2-\alpha\kappa\left(\varepsilon^{h}-b\right)}>\left(C_{1}^{h}\right)^{*};$$

  Consumption by households with deposits at insolvent banks:

$$C_{1}\left(\varepsilon^{h},\underline{\psi}\right)=\frac{2b}{2-\alpha\kappa\left(\varepsilon^{h}-b\right)}<\left(C_{1}^{h}\right)^{*},\qquad C_{1}\left(\varepsilon^{l},\underline{\psi}\right)=\varepsilon^{l}\frac{2}{2-\alpha\kappa\left(\varepsilon^{h}-b\right)}>\left(C_{1}^{h}\right)^{*}.$$

Proof of Proposition 4.1. In the extended model with endogenous output and banking crises, welfare is given by

$$\begin{aligned}\mathcal{W}&=\left(1-\alpha\right)\left[\kappa\varepsilon^{h}u\left(C_{1}\left(\varepsilon^{h},\overline{\psi}\right)\right)+\left(1-\kappa\right)\varepsilon^{l}u\left(C_{1}\left(\varepsilon^{l},\overline{\psi}\right)\right)\right]\\&\quad+\alpha\left[\kappa\varepsilon^{h}u\left(C_{1}\left(\varepsilon^{h},\underline{\psi}\right)\right)+\left(1-\kappa\right)\varepsilon^{l}u\left(C_{1}\left(\varepsilon^{l},\underline{\psi}\right)\right)\right]+\frac{\left(n-N_{1}\right)^{1-\sigma}-1}{1-\sigma}+\mathbb{E}_{R_{2}}\left\{C_{2}+X_{2}\right\}.\end{aligned}$$

Consider the effect of a liquidity requirement that reduces the distribution of liquid assets to the households with deposits at $1-\alpha$ banks not subject to runs. This requirement reduces consumption $C_1\left(\varepsilon^h,\overline{\psi}\right)$ and $C_1\left(\varepsilon^l,\overline{\psi}\right)$. Thus, we can compute

$$\begin{aligned}
\frac{d\mathcal{W}}{dC_1\left(\varepsilon^h,\overline{\psi}\right)} &= (1-\alpha)\left[\kappa\varepsilon^h u'\left(C_1\left(\varepsilon^h,\overline{\psi}\right)\right)+(1-\kappa)\,\varepsilon^l u'\left(C_1\left(\varepsilon^l,\overline{\psi}\right)\right)\frac{dC_1\left(\varepsilon^l,\overline{\psi}\right)}{dC_1\left(\varepsilon^h,\overline{\psi}\right)}\right] \\
&\quad +\alpha\left[\kappa\varepsilon^h u'\left(C_1\left(\varepsilon^h,\underline{\psi}\right)\right)\frac{dC_1\left(\varepsilon^h,\underline{\psi}\right)}{dQ_1}\frac{dQ_1}{dC_1\left(\varepsilon^h,\overline{\psi}\right)}\right. \\
&\qquad \left.+(1-\kappa)\,\varepsilon^l u'\left(C_1\left(\varepsilon^l,\underline{\psi}\right)\right)\frac{dC_1\left(\varepsilon^l,\underline{\psi}\right)}{dQ_1}\frac{dQ_1}{dC_1\left(\varepsilon^h,\overline{\psi}\right)}\right] \qquad (45) \\
&\quad -(n-N_1)^{-\sigma}\frac{dN_1}{dQ_1}\frac{dQ_1}{dC_1\left(\varepsilon^h,\overline{\psi}\right)}. \qquad (46)
\end{aligned}$$

To make progress, note the following. Equation (31) implies $C_1\left(\varepsilon^l,\overline{\psi}\right)=\frac{\varepsilon^l}{\varepsilon^h}C_1\left(\varepsilon^h,\overline{\psi}\right)$ and thus $dC_1\left(\varepsilon^l,\overline{\psi}\right)/dC_1\left(\varepsilon^h,\overline{\psi}\right)=\varepsilon^l/\varepsilon^h$. The binding liquidity constraint of impatient households with deposits at insolvent banks, $C_1\left(\varepsilon^h,\underline{\psi}\right)=Q_1b$, implies $dC_1\left(\varepsilon^h,\underline{\psi}\right)/dQ_1=b$. The optimality condition for $C_1\left(\varepsilon^l,\underline{\psi}\right)$, given by Equation (36), implies, together with (3), that $C_1\left(\varepsilon^l,\underline{\psi}\right)=\varepsilon^l Q_1$ and thus $dC_1\left(\varepsilon^l,\underline{\psi}\right)/dQ_1=\varepsilon^l$. The optimality condition of producers, (43), implies $N_1=\bar{n}-Q_1^{1/\sigma}$ and thus $dN_1/dQ_1=(-1/\sigma)\,Q_1^{\frac{1-\sigma}{\sigma}}$. And finally, the market clearing condition for consumption goods at $t=1$, given by

$$(1-\alpha)\left[\kappa C_1\left(\varepsilon^h,\overline{\psi}\right)+(1-\kappa)\,C_1\left(\varepsilon^l,\overline{\psi}\right)\right]+\alpha\left[\kappa C_1\left(\varepsilon^h,\underline{\psi}\right)+(1-\kappa)\,C_1\left(\varepsilon^l,\underline{\psi}\right)\right]=N_1,$$

implies, when evaluated at the values of the endogenous variables discussed before (i.e., $C_1\left(\varepsilon^l,\overline{\psi}\right)=\frac{\varepsilon^l}{\varepsilon^h}C_1\left(\varepsilon^h,\overline{\psi}\right)$, $C_1\left(\varepsilon^h,\underline{\psi}\right)=Q_1b$, $C_1\left(\varepsilon^l,\underline{\psi}\right)=\varepsilon^l Q_1$, and $N_1=\bar{n}-Q_1^{1/\sigma}$),

$$\frac{dQ_1}{dC_1\left(\varepsilon^h,\overline{\psi}\right)}=-\frac{(1-\alpha)/\varepsilon^h}{\frac{1}{\sigma}Q_1^{\frac{1-\sigma}{\sigma}}+\alpha\left[\kappa b+(1-\kappa)\,\varepsilon^l\right]}.$$

Thus, plugging all these results into (46) and rearranging:

$$\frac{d\mathcal{W}}{dC_1\left(\varepsilon^h,\overline{\psi}\right)}=-\sigma\frac{(1-\alpha)}{\varepsilon^h}\frac{\alpha\kappa b\left[u'\left(C_1\left(\varepsilon^h,\underline{\psi}\right)\right)-1\right]}{Q_1^{\frac{1-\sigma}{\sigma}}+\sigma\alpha\left[\kappa b+(1-\kappa)\,\varepsilon^l\right]}<0,$$

where the inequality follows from the assumption $\sigma>0$ and $\alpha>0$ and from the fact that, in equilibrium, consumption of impatient households with deposits at insolvent banks is not at the first best, and thus $u'\left(C_1\left(\varepsilon^h,\underline{\psi}\right)\right)>1$. The result $d\mathcal{W}/dC_1\left(\varepsilon^h,\overline{\psi}\right)<0$ means that an increase in $C_1\left(\varepsilon^h,\overline{\psi}\right)$ reduces welfare, and thus a reduction in $C_1\left(\varepsilon^h,\overline{\psi}\right)$ increase welfare. Since the liquidity requirement reduces $C_1\left(\varepsilon^h,\overline{\psi}\right)$ by constraining the liquidity distributed by banks hit by the good idiosyncratic shock to capital $\overline{\psi}$, the result follows.

C.2 Endogenous production of safe and liquid assets: detailed analysis

The amount $S_0e\left(1-\phi\right)$ of safe capital trades at the same price Q_0 and Q_1 at which government bonds are traded because the two assets have the same payoff and the same liquidity properties. I focus on an economy with banks, in which households hand all their endowments to the bank in their own location, as in Section 3. Thus, the budget constraint of a bank is

$$K_0^{risky}+Q_0\left(B_0+K_0^{safe}\right)\leq\left[Q_0b+k^{risky}\right]+\left[Q_0S_0e\left(1-\phi\right)+\left(1-S_0\right)e\right], \tag{47}$$

where K_0^{risky} and K_0^{safe} are the amount of risky and safe capital held at the end of $t=0$, and similar to the baseline model, B_0 is the amount of government bonds.[39] The right-hand side of (47) distinguishes between two set of resources available to the household. The first term, Q_0b+k^{risky}, is the same as in the baseline economy. The second term, $Q_0S_0e\left(1-\phi\right)+\left(1-S_0\right)e$, denotes the resources available because of the new endowment e. The amount $S_0e\left(1-\phi\right)$ invested in the safe capital trades at Q_0 as discussed before, whereas the amount $\left(1-S_0\right)e$ trades at the same price of risky capital, which is normalized at one.

The first-order condition with respect to S_0 implies

$$S_0=\begin{cases}1 & \text{if } Q_0>1+\phi\\ \text{(any amount)}\in[0,1] & \text{if } Q_0=1+\phi\\ 0 & \text{if } Q_0<1+\phi.\end{cases}$$

Because of the linearity of the cost ϕ, the solution is to go to the corners if the liquidity premium is too high or too low. If instead the liquidity premium is equal to the cost ϕ (i.e., if $Q_0=1+\phi$), then the bank is indifferent about investing any fraction in the safe capital, and S_0 will be determined in equilibrium so that the liquidity premium is indeed equal to ϕ. To make the analysis interesting, I impose the following parameter restriction, which

[39] Note that Equation (47) could also represent the time-0 budget constraint of a household in a bankless economy.

guarantees an interior solution (i.e., $S_0 \in (0,1)$) when there are runs:

$$\max\left\{1,\ \varepsilon^h \frac{\alpha\kappa}{\alpha\kappa+\phi} - e\left(1-\phi\right)\right\} < b < \varepsilon^h \frac{\alpha\kappa}{\alpha\kappa+\phi}, \tag{48}$$

thereby replacing Equation (10). If public liquidity is too abundant (i.e., greater than the upper bound in (48)), the liquidity premium is less than ϕ, and thus $S_0 = 0$ and no endowment is invested the safe technology. If instead public liquidity is too scarce, or if the endowment e is small, then $S_0 = 1$ and the entire endowment e is invested in the safe capital.[40]

To analyze the equilibrium, consider first the case in which the fraction of endowment invested in safe capital is exogenously set at $S_0 = s$.[41] In this case, the equilibrium is identical to that of the baseline model, with the only difference being that the stock of liquid assets is now $b + se\left(1-\phi\right)$ rather than just b, where $se\left(1-\phi\right)$ account for the privately supplied liquid assets. In particular, Proposition B.1 implies that the price of liquid assets Q_0 is

$$Q_0 = 1 + \alpha\kappa\left(\frac{\varepsilon^h}{b + se\left(1-\phi\right)} - 1\right). \tag{49}$$

Once S_0 is allowed to be chosen endogenously, condition (48) (which implies $S_0 \in (0,1)$ and $Q_0 = 1+\phi$) and (49) imply

$$S_0 = \frac{\alpha\kappa\varepsilon^h}{e\left(1-\phi\right)\left(\alpha\kappa+\phi\right)} - \frac{b}{e\left(1-\phi\right)}.$$

Under the asset purchase policy, the supply of government debt is b_0, and thus the equilibrium value of S_0 becomes

$$S_0 = \min\left\{0,\ \frac{\alpha\kappa\varepsilon^h}{e\left(1-\phi\right)\left(\alpha\kappa+\phi\right)} - \frac{b_0}{e\left(1-\phi\right)}\right\}.$$

It is then straightforward to verify that, as long as $S_0 \in (0,1)$, asset purchases reduce one-for-one the supply of privately produced liquidity:

$$\left.\frac{\partial\left[S_0 e\left(1-\phi\right)\right]}{\partial b_0}\right|_{S_0\in(0,1)} = -1.$$

[40]The lower bound in (48) is set to be at least as large as one for the same motivation discussed in relation to Equation (10).

[41]I consider the relevant case in which s is not too large so that $b + se\left(1-\phi\right) < \varepsilon^h$; otherwise, there would be enough liquidity to satiate the economy. The requirement $b + se\left(1-\phi\right) < \varepsilon^h$ must be imposed when taking $S_0 = s$ to be exogenously given, but it will not be necessary when allowing S_0 to be chosen endogenously because, in that case, the restriction in (48) implies $S_0 \in (0,1)$.

In this model in which the cost to produce private liquidity is a constant ϕ, a small asset purchase program changes one-for-one the amount of liquid assets, but it has no effect on prices nor on the consumption allocation at $t = 1$. This is because each dollar injected by the central bank simply replaces a dollar of privately supplied liquid assets. Nonetheless, this small asset purchase program is beneficial for welfare because it reduces the resources wasted on the cost ϕ. If instead the asset purchase is large, it can entirely crowd out the supply of privately provided liquidity, driving S_0 to zero. At this point, any additional dollar injected into the economy reduces the liquidity premium, as discussed in Section 2.7.

To compute the optimal capital requirement, note that welfare in the economy with banking crises is given by

$$\begin{aligned}\mathcal{W} \;=\; & (1-\alpha)\left[\kappa\varepsilon^h u\left(C_1\left(\varepsilon^h,\overline{\psi}\right)\right)+(1-\kappa)\,\varepsilon^l u\left(C_1\left(\varepsilon^l,\overline{\psi}\right)\right)\right] \\ & +\alpha\left[\kappa\varepsilon^h u\left(C_1\left(\varepsilon^h,\underline{\psi}\right)\right)+(1-\kappa)\,\varepsilon^l u\left(C_1\left(\varepsilon^l,\underline{\psi}\right)\right)\right]+\mathbb{E}_{R_2}\left\{C_2+X_2\right\}-\phi S_0 e,\end{aligned}$$

where $\phi S_0 e$ are the resources wasted to make the capital riskless. Then, I proceed as follows. First, fixing $S_0 = s$, I solve for the liquidity requirement $\zeta(s)$ as discussed in Appendix B.4. Then, using condition (48) (which implies $Q_0 = 1 + \phi$), I solve for the optimal value of S_0, under the assumption that asset purchases are not large enough (i.e., formally, I consider the equilibrium under an asset purchase policy such that b_0 is in a neighborhood of b). Finally, I can recover the optimal capital requirement $\zeta^* = \zeta(S_0)$, which is given by

$$\zeta^* = \frac{\kappa\left(\varepsilon^h-1\right)-\phi\left(1-\kappa\varepsilon^h\right)}{1-\alpha\left(1-\kappa\varepsilon^h\right)}.$$