Less competition, more meritocracy?*

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Abstract

Uncompetitive contests for grades, promotions, and job assignments, which feature lax standards or consider only limited talent pools, are often criticized for being unmeritocratic. We show that, when contestants are strategic, lax standards and exclusivity can make selection *more* meritocratic. Strategic contestants take more risks in more competitive contests. Risk taking reduces the correlation between selection and ability. By reducing the noise engendered by strategic risk taking, dialing down competition can produce outcomes that better conform with the meritocratic ideal of selecting the best and only the best.

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1 Introduction

Competitions to identify and select "the best and the brightest"—e.g., educational tests, worker performance evaluations, league-table rankings of mutual funds, are a pervasive feature of modern life. Given the growing labor-income share of "working rich" (Piketty, 2005; David et al., 2006; Goldin and Katz, 2007), an elite selected through meritocratic selection contests, and their dominant position in multinational corporations and global institutions (Brezis and Temin, 2008), such selection contests clearly have profound social and economic effects.

The design of selection contests is frequently shaped by the perspective that competition and high standards are fundamental features of meritocratic selection or even its defining characteristic (Frost, 2017). In fact, "meritocratic society" is sometimes even treated as a synonym for "competitive society" (Ekins, 2014).

However, this paper shows that, when contestants are strategic, making contests more competitive can make selection *less* meritocratic. Making contests more competitive, by increasing the number of competitors or raising selection standards, has not only the direct effect of adding contestants who might be better than the incumbent contestants or of excluding a marginal candidate unlikely to merit selection, but also an indirect equilibrium effect: making contests more competitive changes contestants' equilibrium strategies.

We show that, when contests become too competitive, contestants choose riskier strategies that reduce the correlation between ability and contest performance, thereby making selection less meritocratic. When this occurs, meritocratic selection can often be furthered by anti-competitive policies such as low selection bars and restricted candidate fields.

These implications are developed in a parsimonious model of contest design. In the model, n contestants compete for selection. The number of contestants selected, m, is determined by the contest's *selection quota*. The m quota places are assigned to the m best-performing contestants. Contestants prefer selection to deselection.

Each contestant is endowed with *ability* and *contest ability*. Ability represents the characteristics of the contestant valued by the contest designer. Contest ability represents a contestant's ability to perform in the contest. There are two types of contestants, strong and weak. Strong contestants have greater ability and contest ability. Each contestant knows his own type, but not the types of other contestants. In the contest, if a contestant takes no risk, his contest performance simply equals his contest ability and thus fully reflects his ability. However, a contestant can take risky activities that add noise to his contest performance. We assume that such additive noise has a zero mean and we allow for all "fair gambles," i.e., a contestant is free to choose any distribution of nonnegative performance with mean equal to his contest ability.[1] The contestant's realized performance is independently drawn from his performance distribution.

[1] In our setting, there is no explicit cost of risk taking. The only cost of risk taking is implicit: given fixed expected performance, to increase the probability of attaining high performance levels, a contestant also has to increase the probability of having low performance levels. Low performance levels are less likely to be sufficient for selection.

The fair-gambles framework has been adopted in many studies of contests, including political campaigns (Myerson, 1993; Lizzeri, 1999), status contests (Robson, 1992; Becker et al., 2005; Ray and Robson, 2012), and fund manager competitions (Bell and Cover, 1980; Seel and Strack, 2013; Fang and Noe, 2016; Strack, 2016). Strategic risk-taking in highly competitive environments has also been documented empirically in a number of contexts (Chevalier and Ellison, 1997; Khorana, 2001; Bothner et al., 2007; Beaudoin and Swartz, 2010; Genakos and Pagliero, 2012) and in laboratory experiments utilizing professional subjects (Kirchler et al., 2018). Thus, the mechanism we posit, strategic risk taking, is plausible, as is the effect it produces in our analysis, weakening the link between ability and contest performance.

A contest designer, who rationally anticipates contestant strategies, has some control over contest design. The designer's welfare function is purely meritocratic, i.e., welfare depends positively on the number of strong contestants selected and negatively on the number of weak contestants selected, and is not affected by any other aspect of contestant performance.

First, consider situations in which both the number of contestants and the selection quota are beyond the designer's control, e.g., a university admissions contest in which the selection quota is fixed by the university's capacity and the number of contestants by the number of applicants. Our analysis shows that, if the number of applicants relative to the university's capacity is sufficiently large, risk taking can be reduced without lowering admitted student quality by adopting a relaxed selection policy which "approves" more applicants than can be admitted, and fills the selection quota with approved students using a random lottery.[2] This mechanism coincides with the elite-university admission scheme proposed in Schwartz (2007).[3]

Next, consider situations in which the number of places is fixed but the number of contestants is under the designer's control, e.g., a competition for a firm's CEO position. Firms have only one CEO, so the selection quota is fixed. However, firms can vary the number of contestants either by excluding external candidates, through an "in-house" competition, or including external candidates, through an "open competition." Our results in Sections 4 and 6.4 imply that, when competition between internal candidates is sufficiently intense, considering external candidates does not increase expected CEO quality but does increase risk taking. When external candidates are less likely to exhibit high ability than internal candidates, an in-house competition can yield strictly higher expected CEO quality, even though it is possible that an excluded external candidate is better than any internal candidate (cf. Section 6.3).

Finally, consider situations in which the number of contestants is fixed but the designer controls the selection quota, e.g., a promotion contest within a firm in which the firm can choose the promotion rate. Our analysis in Section 5 yields two key results. First, when contestants

[2] See Section 6.4.

[3] Using a large all-pay contest setting, which focuses on contestants' effort strategies but abstracts from risk taking, Olszewski and Siegel (2018) show that making contests less competitive by pooling intervals of performance rankings can improve student welfare in a Pareto sense via reduced student effort, even though pooling reduces the correlation between selection and ability. Our result implies that, if students strategically take risks, reducing competition need not reduce the correlation between selection and ability.

have low prior quality, then regardless of how large the contestant pool is, risk taking leads to *contest mechanism failure*, in the sense that designer welfare is maximized by setting a zero quota. Consequently, risk taking reduces the scope of contest selection, which can be costly given many relative advantages of contests over other selection mechanisms.[4]

Second, if the contest mechanism does not fail, selection is biased toward "Peter Principle" selection: the selection quota is inflated so much that some selected candidates are expected to be unworthy of selection. Quota inflation occurs when the benefit from quota inflation—reducing contestant risk taking and thereby increasing the correlation between ability and contest performance—exceeds the cost of inflation—selecting some candidates who are expected to be unworthy.

Social promotions and lax grading by schools can be interpreted as awards to sub-marginal performance ranks. In our analysis, such policies can make selection more meritocratic. For the same reasons, motivational promotions by firms can be rationalized even when such "motivational promotions" have no motivational effect.[5] In retention contests, where not being dismissed is the contest reward, our results predict that dismissal rates will be lower under strategic risk taking than when the dismissal rate is fixed purely on the basis of the distribution of contestant ability. This conclusion appears to be consistent with empirical studies of dismissals in mutual funds.[6]

We conclude by extending our baseline analysis to show that its key implications—that risk taking caps the gains from increasing contest participation and biases meritocratic contest designers toward quota inflation—are robust to various modifications, which include but are not limited to (a) endogenous contest ability acquired through costly effort, (b) ex post discretionary filling of the selection quota, and (c) scoring caps that bound contestant performance.

Related literature

This paper is probably most closely related to the literature on the effect of risk on selection. Lazear (2004) studies promotion in a model where risk is generated by exogenous noise affecting the performance/ability relation. His analysis, like ours, identifies a Peter Principle effect: because a component of worker performance is produced by luck, i.e., a high realization of a random noise term affecting contest performance, the expected future performance of promoted workers is less than their performance in the promotion contest. However, the

[4]A number of researchers have argued that contest selection is more advantageous than selection based on absolute performance when performance is hard to verify (Che and Gale, 2003) or affected by common time-varying shocks (Lazear and Rosen, 1981; Knoeber and Thurman, 1994), as well as when relative performance is easier to measure (Lazear and Rosen, 1981), and when firms have a strong preference for offering a fixed amount of total compensation to employees (Gürtler and Kräkel, 2010).

[5]For a discussion of social promotion in schools, see Jimerson et al. (2006). For a discussion of motivational promotion in the workplace, see Deeprose (2006).

[6]For example, Khorana (1996, Table 4) finds that only 14% of managers in the lowest performance decile are replaced despite the fact that, as Khorana (2001) documents, replacing low performing managers improves mutual fund returns.

implications of our analysis differ quite dramatically from Lazear (2004). In Lazear (2004), the contest designer, realizing that part of contestant performance is the product of luck, adjusts upward the performance cutoff required for promotion. Thus, in Lazear (2004), exogenous risk leads to an increase in promotion standards relative to the no-noise case. In our analysis, endogenous strategic risk leads the designer to lower standards by expanding the selection quota relative to the quota which would have been selected in the absence of risk taking.

Ryvkin and Ortmann (2008), Ryvkin (2010), and Hvide and Kristiansen (2003) also consider the effect of risk on selection. These papers fix the selection quota at one, and thus, unlike our paper and Lazear (2004), they do not address the effect of risk on the selection quota. However, they do consider the effect of expanding the contestant pool.

In selection contests with exogenous risk, Ryvkin (2010) demonstrates that expanding the contestant pool always strictly increases the expected ability of the winner.[7] In contrast, we show that, when contestants are strategic risk takers, pool expansion beyond a threshold number of contestants never strictly increases the expected ability of the winner(s).

Hvide and Kristiansen (2003) develop a restricted risk-taking contest model in which a contestant can only choose/mix between two fixed strategies. They find an example in which pool expansion reduces the expected ability of the winner. This example is consistent with the idea that contestant risk taking can nullify the gains from pool expansion. However, the example depends on the specific risk-taking strategies imposed on the contestants. In contrast, our analysis shows that when contestants are free to choose any fair-gamble risk-taking strategy, beyond a threshold number of contestants, strategic risk taking always nullifies the gains from pool expansion.

More generally, our paper is related to research showing that meritocracy can be furthered by seemingly unmeritocratic policies. Meyer (1991) and Kawamura and Moreno de Barreda (2014) find that biasing the contest selection mechanism toward certain contestants can increase selection efficiency.[8]

In some research on the benefits of seemingly unmeritocratic policies, "meritocratic" is defined very differently than it is in our analysis. For example, Morgan et al. (2018) model contests where contest performance equals output and the designer's objective is to maximize expected total output. Prizes are allocated based on measured output and measured output is affected by an exogenous noise term. In their framework, more meritocratic means less noise and thus, "meritocratic" is a property of the contest mechanism not, as in our analysis, a property of the designer's objective function. They show that too much meritocracy, i.e., too little noise, can reduce expected total output.

[7] Ryvkin (2010) also verifies a similar result when dynamic tournament mechanisms, such as binary elimination tournaments and round-robin tournaments, are used for selection.

[8] Meyer (1991) considers sequential selection of non-strategic agents when performance is a noisy signal of ability and shows that biasing contests in favor of early leaders can increase selection efficiency. Kawamura and Moreno de Barreda (2014) show, in an all-pay contest setting in which contestant abilities are known to every contestant but not to the designer, that selection efficiency can be increased by biasing the contest in favor of one of the contestants even if the contestants are ex ante identical to the designer.

In addition, the large literature on the effects of strategic risk taking is also relevant to understanding the policy implications of our analysis. Researchers studying risk taking have documented many situations in which strategic risk taking in extremely competitive contests imposes social costs, quite independent of its effect on meritocratic selection, e.g., portfolio volatility (Chevalier and Ellison, 1997; Khorana, 2001; Kaniel and Parham, 2017), military aggression by politicians (Hess and Orphanides, 1995), and fraud by bureaucrats, accountants, and academics (Weinstein, 1979; Shleifer, 2004; Ghanem and Zhang, 2014; Serrato et al., 2016).

Our results complement this analysis. We show that, even when these costs are absent, extremely competitive contests make selection less correlated with ability. Thus, in many situations, the standard tradeoff determining the optimal degree of contest competition—the risk-taking costs of competition vs. competition's selection benefits—may not be present. In addition, we address the concerns of this literature in Section 6.4, where we consider mechanisms that reduce contestant risk taking without sacrificing meritocratic selection.

Our results on quota inflation complement other explanations for "Peter Principle" hiring polices. In the tournament models of Lazear and Rosen (1981) and Gürtler and Kräkel (2010), over-promotion results from its effect on employee effort. In the job assignment models of Prendergast (1992) and Fairburn and Malcomson (2001), over-promotion results because the labor market cannot observe the ability of individual employees, but can observe the average ability of employees assigned to a given task. Because of employee risk aversion, reducing the gap between expected ability conditioned on the two job assignments through over-promotion increases employee welfare and thus permits risk-neutral employers to attract employees at lower cost.

The questions considered in these paper are quite different from the one we address. These papers consider situations in which employers are willing to make hiring less meritocratic, through over-promotion, in order to increase the incentive efficiency of compensation. We examine whether over-promotion can be motivated by the objective of making hiring more meritocratic.

Our model is also broadly related, at a technical level, to a large literature on unrestricted risk-taking contests. These contests have been extensively analyzed in compete-information settings (Bell and Cover, 1980; Myerson, 1993; Becker et al., 2005; Hart, 2008, 2016; Fang and Noe, 2016; Strack, 2016) and are closely related to an even more extensively analyzed mechanism, all-pay auctions (Baye et al., 1996; Barut and Kovenock, 1998).[9] In contrast to the existing studies of unrestricted risk-taking contests, our analysis is developed in an incomplete-information setting and we focus on the effect of risk taking on meritocratic selection, an issue

[9]As pointed out by Hart (2016), an all-pay auction, with linear bidding costs, can be considered as a two-stage game in which each player chooses his expected bid in the first stage followed by a randomization of his bid subject to the constraint that the mean of his second-stage bid distribution equals the expected bid chosen in the first stage. See Sahuguet and Persico (2006), Fang and Noe (2016), and Hwang et al. (2018) for further discussions of how risk-taking contests relate to all-pay auctions.

that has not been addressed in unrestricted risk-taking contest settings.

2 Risk taking

2.1 Risk-taking selection contests

Consider a contest with $n \geq 2$ contestants; m of them will be *selected* to fill a *place*, and the remaining $n-m$ contestants will be *deselected* and not receive a place, where $0 < m < n$. The number of places, m, which we call the *selection quota*, and the number of contestants, n, which we call *contest size*, are fixed before the contest and are common knowledge.

The contestants are of two possible types, t: strong, S, and weak, W. Whether a contestant is strong or weak is determined by an independent draw from a Bernoulli distribution which assigns probability θ to S, and probability $1-\theta$ to W. A contestant's type is the contestant's private information.

Selection is based on performance in the contest. Every type-t contestant can take risky activities in the contest that add noise to his otherwise fixed performance $\mu_t > 0$, $t \in \{S,W\}$. We call μ_t a type-t contestant's *contest ability* and assume that strong contestants have higher contest ability than weak contestants, i.e., $\mu_S > \mu_W$. In Section 6.2, we endogenize contest ability by allowing each contestant to acquire contest ability through costly effort. As we will show, the qualitative conclusions of our analysis are fairly robust to this extension. We assume that the additive noise has a zero mean and we allow for all "fair gambles," i.e., a contestant can costlessly choose any distribution of nonnegative performance subject to the *contest ability constraint* that the expected performance of a type-t contestant must equal the type-t contestant's contest ability, μ_t, $t \in \{S, W\}$. As we will show in Section 6.4, restricting fair gambles to distributions with a bounded support, $[0, \bar{x}]$, $\bar{x} \geq \mu_S$, does not change any of our welfare results.

Each contestant's realized performance is independently drawn from his performance distribution. The m contestants with the highest realized performances are selected and the remaining contestants are deselected, with ties broken randomly. Contestants are expected utility maximizers who strictly prefer selection to deselection. Thus, given that risk taking is costless, each contestant chooses his performance distribution to maximize his *probability of winning* a place given his rivals' strategies and the contest's parameters.

2.2 Best reply and the probability of winning function

To determine the effect of contest design on meritocracy, we first need to characterize equilibrium contestant behavior. We focus on symmetric equilibria in which contestants of the same type all play the same strategy, i.e., each type-t contestant chooses performance distribution F_t with support Supp_t, $t \in \{S,W\}$.

A contestant's probability of winning function maps the contestant's realized performance, x, to his probability of being selected and is thus determined endogenously by his rivals' strategies. Because, each contestant faces the same distribution of rivals, and strategies are symmetric, all of the contestants face the same *probability of winning function*, $P : \mathbb{R}_+ \to [0,1]$.

In this section, we provide a characterization of equilibrium probability of winning functions that will be the foundation of our subsequent analysis. Our derivation assumes that (a) equilibrium performance distributions, F_t, $t \in \{S,W\}$, are continuous, and (b) the probability of winning function, P, is continuous, and increasing when $P(x) < 1$. Because many authors have established properties analogous to (a) and (b) in symmetric unrestricted risk-taking contests and symmetric all-pay auctions (Barut and Kovenock, 1998; Fang and Noe, 2016), our verifications of (a) and (b) are not very original. So, we defer these verification to the Online Appendix.

To initiate this derivation, a few definitions are required: for any two points (x_1, p_1), (x_2, p_2) in $\mathbb{R}^2_+$, we define the *interval between the points*, $[(x_1, p_1),(x_2, p_2)]$ by

$$[(x_1, p_1),(x_2, p_2)] = \{\lambda\,(x_1, p_1) + (1-\lambda)\,(x_2, p_2) : \lambda \in [0,1]\}.$$

A *gamble* between performance levels x' and x'' represents a performance distribution that randomizes between x' and x''. A *fair gamble* between x' and x'' for a contestant of type t is a gamble between x' and x'' with the property that the probability of choosing x', π, satisfies $\pi x' + (1-\pi)x'' = \mu_t$. Because fair gambles are feasible performance distributions, if performance levels x' and x'' are in the support of the equilibrium performance distribution of type t, then a type-t contestant's payoff from a fair gamble between x' and x'' equals his equilibrium payoff. Thus, in equilibrium, no fair gamble produces a higher payoff to a type-t contestant than the fair gamble between x' and x''.

Next, note that, for performance levels x_1, x_2, and x_3 satisfying $x_1 < \mu_t < x_2$ and $x_1 < \mu_t < x_3$, if $(P(x_3) - P(x_1))/(x_3 - x_1) < (P(x_2) - P(x_1))/(x_2 - x_1)$, the interval $[(x_1, P(x_1)),(x_3, P(x_3))]$ lies below the interval $[(x_1, P(x_1)),(x_2, P(x_2))]$. Because $\mu_t \in [x_1, x_2] \cap [x_1, x_3]$, this implies that a payoff to a type-t contestant from a fair gamble between x_1 and x_2 exceeds the payoff from a fair gamble between x_1 and x_3. This result is illustrated by Figure 1.

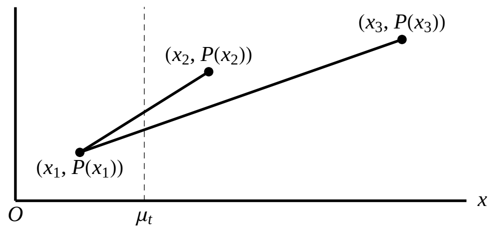

Figure 1: *Fair gambles and best replies.* In the figure, for a contestant of type $t \in \{S,W\}$, the payoff from a fair gamble between x_1 and x_3, given by the intersection of the dashed line and the interval $[(x_1, P(x_1)),(x_3, P(x_3))]$, yields a lower payoff than a fair gamble between x_1 and x_2, given by the intersection of the dashed line and the interval $[(x_1, P(x_1)),(x_2, P(x_2))]$.

Because all fair gambles in the support of a type-t contestant's performance distribution must produce the same payoff, x_3 cannot be in the support of t's performance distribution if x_1 and x_2 are in its support. Similarly, if $(P(x_3) - P(x_1))/(x_3 - x_1) > (P(x_2) - P(x_1))/(x_2 - x_1)$, x_2 cannot be in the support of t's performance distribution if x_1 and x_3 are in the support. Thus, the slope of the line joining any two points $(x_1, P(x_1))$ and $(x_2, P(x_2))$ in the support of t's performance distribution is constant, and hence all performance/probability-of-winning pairs $(x, P(x))$ such that $x \in \mathrm{Supp}_t$ are collinear.[10]

For each contestant type t, we can represent this line in the performance/probability-of-winning space by an affine function with intercept α_t and slope β_t. Expressed in this fashion, the collinearity condition is $\alpha_t + \beta_t x = P(x)$, $x \in \mathrm{Supp}_t$. Moreover, it must be the case that all $x \geq 0$ satisfy the condition $\alpha_t + \beta_t x \geq P(x)$. To see this, suppose that a point $x^* \geq 0$ did not satisfy this condition. Then a fair gamble between x^* and some point in the support would produce a higher payoff than fair gambles over performance levels contained in the support. The formal expression of these conditions is[11]

$$\forall t \in \{S, W\}, \qquad x \in \mathrm{Supp}_t \Longrightarrow \alpha_t + \beta_t x = P(x), \tag{1}$$

$$x \geq 0 \Longrightarrow \alpha_t + \beta_t x \geq P(x). \tag{2}$$

Because the probability of winning produced by any performance level is nonnegative, (2) implies that $\alpha_t \geq 0$. Because, by assumption, P is increasing when $P(x) < 1$, (1) implies that $\beta_t > 0$. Because P can only increase at points in the support of at least one type's performance distribution, and because the support of a distribution is, by definition, a closed set,

$$\mathrm{Supp}_W \cup \mathrm{Supp}_S = [0, \min\{x \geq 0 : P(x) = 1\}]. \tag{3}$$

In terms of the graph of the performance distribution, equation (2) and the fact that the probability of winning is never greater than 1, imply that, if P is an equilibrium probability of winning function, then

$$(x, P(x)) \in \mathscr{P} = \{(x,p) : \alpha_S + \beta_S x \geq p\} \cap \{(x,p) : \alpha_W + \beta_W x \geq p\} \cap \{(x,p) : p \leq 1\}. \tag{4}$$

For all x such that $P(x) < 1$, equation (3) implies that equation (1) is satisfied for at least one type. Thus, equations (1), (3), and (4) imply that

$$P(x) = \max\{p \geq 0 : (x,p) \in \mathscr{P}\} = \min\left[\alpha_W + \beta_W x,\, \alpha_S + \beta_S x,\, 1\right]. \tag{5}$$

The last equality implies that P is concave. The lines $x \hookrightarrow \alpha_t + \beta_t x$, $t = S, W$, must meet at some x such that $0 < P(x) < 1$, since otherwise by (1), the support of one type's performance distribution would be empty, inconsistent with the existence of an equilibrium, or a single point, contradicting the performance distribution being continuous.

Thus, there are only two possibilities. In the first, $(\alpha_S, \beta_S) = (\alpha_W, \beta_W) = (\alpha, \beta)$. By the hypothesis that equilibrium performance distributions are continuous, the probability of zero

[10] Collinearity for the single performance level where $x = \mu_t$ follows from the continuity of P.

[11] These conditions can also be derived using the concavification argument *à la* Aumann et al. (1995) and Kamenica and Gentzkow (2011).

performance is 0 for both types, this implies that $P(0) = 0$. Thus, equation (5) implies that $\alpha = 0$. Thus, in this case, the probability of winning function is given by $P(x) = \min[\beta x, 1]$.

The second case is $(\alpha_S, \beta_S) \neq (\alpha_W, \beta_W)$. In this case, the lines $x \hookrightarrow \alpha_t + \beta_t x$, $t = S, W$, meet at a single point. By (1) and (5), this implies that the supports of the two types' performance distributions are adjacent intervals. Because the contest ability of S is greater than the contest ability of W, the upper interval must coincide with Supp_S and the lower interval with Supp_W. By the concavity of P, the slope of P over the upper interval is less than the slope of P over the lower interval. Thus, $\beta_S < \beta_W$, which implies, because the lines cross, that $\alpha_S > \alpha_W$. Hence, given the fact that $P(0) = 0$, (5) implies that $\alpha_W = 0$, Thus, in this case, the probability of winning function is given by $P(x) = \min[\beta_W x, \alpha_S + \beta_S x, 1]$. These results are summarized in the next lemma and illustrated in Figure 2. All of the formal proofs of our results are relegated to the Online Appendix.

Lemma 1. *In any symmetric equilibrium, the probability of winning function, P, satisfies one and only one of the following conditions:*

(i) There exists $\beta > 0$*, such that the probability of winning function is given by*

$$P(x) = \min[\beta x, 1].$$

(ii) There exist β_S*,* β_W*, and* α_S*, with* $0 < \beta_S < \beta_W$ *and* $\alpha_S > 0$*, such that the probability of winning function is given by*

$$P(x) = \min[\beta_W x, \alpha_S + \beta_S x, 1].$$

In both (i) *and* (ii), $\mathrm{Supp}_W \cup \mathrm{Supp}_S = [0, \min\{x \geq 0 : P(x) = 1\}]$. *In* (ii), $\max \mathrm{Supp}_W = \min \mathrm{Supp}_S$.

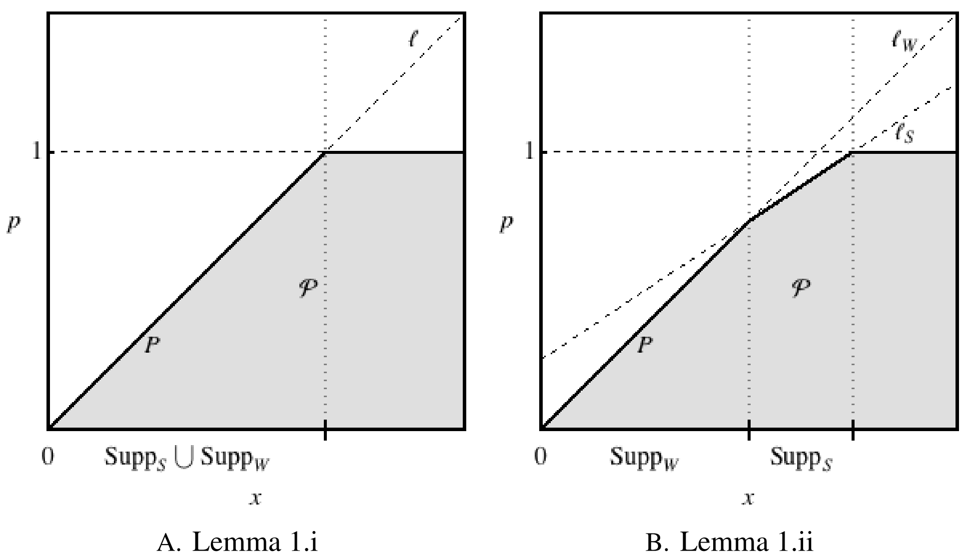

Figure 2: *The possible forms of probability of winning function, P, defined in Lemma 1.* In the figure, $\ell_W = \{(x, p) : \beta_W x = p\}$, $\ell_S = \{(x, p) : \alpha_S + \beta_S x = p\}$, and $\ell = \{(x, p) : \beta x = p\}$.

Figure 2.A illustrates the satisfaction of condition (i) of Lemma 1, i.e., the case in which

P is linear over its support. In Figure 2.A, the union of Supp_S and Supp_W equals the interval between 0 and the upper endpoint of the support of P. When (i) of Lemma 1 is satisfied, except for the non-generic boundary case in which the upper bound of Supp_W exactly equals the lower bound of Supp_S, the upper bound of Supp_W lies strictly above the lower bound of Supp_S, which implies that a weak contestant's performance sometimes tops a strong contestant's. We refer to any pair of performance distributions, (F_S, F_W) in which the upper bound of Supp_W lies strictly above the lower bound of Supp_S as *challenge configurations* and refer to equilibria in which the performance distributions are challenge configurations as *challenge equilibria*.

Figure 2.B illustrates the contrasting case in which P satisfies condition (ii) of Lemma 1. The equilibrium strategy of each type of contestant places all probability weight on one of the two adjacent intervals. Thus, weak contestants concede to strong contestants and concentrate their contest ability on beating other weak contestants. We refer to any pair of performance distributions, (F_S, F_W) in which the upper bound of Supp_W equals the lower bound of Supp_S as *concession configurations*, and refer to equilibria in which the performance distributions are concession configurations as *concession equilibria*. Concession configurations include all configurations satisfying condition (ii) of Lemma 1 as well as the non-generic boundary case in which P is linear over its support (i.e., P satisfies condition (i) of Lemma 1) but the supports of the two types' performance distributions are non-overlapping.

2.3 Equilibrium configurations

The question that remains is determining the conditions under which each of these configurations can be sustained in equilibrium. We first show that, for each parameterization of the model, either concession or challenge equilibria exist (but never both). The key to establishing this assertion as well as to identifying the conditions under which a configuration sustains an equilibrium is provided by considering probability of winning to a weak contestant. First, consider concession configurations. Let p_t^C be the probability of winning for a contestant of type $t \in \{S, W\}$ in concession configurations. For a given contestant i, let $\tilde{S}_n^{-i}$ be the number of strong rivals to i. Note that $\tilde{S}_n^{-i}$ is Binomially distributed with parameters $n-1$ and θ, i.e., $\tilde{S}_n^{-i} \sim \text{Binom}(n-1, \theta)$. Because, in concession configurations, weak contestants never outperform strong contestants, and contestants of the same type have the same probability of winning, if contestant i is weak, i has no chance of winning if $\tilde{S}_n^{-i} \geq m$ and has a probability of winning equal to $(m - \tilde{S}_n^{-i})/(n - \tilde{S}_n^{-i})$ if $\tilde{S}_n^{-i} < m$.[12] Thus,

$$p_W^C = \mathbb{E}\left[\max\left[0, \frac{m - \tilde{S}_n^{-i}}{n - \tilde{S}_n^{-i}}\right]\right]. \tag{6}$$

For concession configurations to sustain an equilibrium, it must be that a given weak con-

[12] Suppose contestant i is weak and a concession configuration is played. If $\tilde{S}_n^{-i} < m$, then after the $\tilde{S}_n^{-i}$ strong rivals all win a place, there are still $m - \tilde{S}_n^{-i} > 0$ places left to be assigned to the $n - \tilde{S}_n^{-i}$ weak contestants, including i. By symmetry, these $n - \tilde{S}_n^{-i}$ weak contestants have the same probability of winning. Thus, each of these weak contestants has a probability of winning equal to $(m - \tilde{S}_n^{-i})/(n - \tilde{S}_n^{-i})$.

testant has no incentive to challenge strong contestants. A simple and feasible way for a weak contestant to challenge strong contestants is to mimic strong contestants' strategy with probability μ_W/μ_S and choose 0 with probability $1-(\mu_W/\mu_S)$. Under this "mimicking strategy," a weak contestant's expected performance equals $\frac{\mu_W}{\mu_S}\times\mu_S+\left(1-\frac{\mu_W}{\mu_S}\right)\times 0=\mu_W$ and, hence, the weak type's contest ability constraint is satisfied. For concession configurations to sustain an equilibrium, it must be that a weak contestant has no incentive to deviate from a concession strategy to the prescribed mimicking strategy. This requires that

$$p_W^C \geq \frac{\mu_W}{\mu_S} p_S^C, \tag{7}$$

where the right-hand side is the probability of winning for a weak contestant if he deviates to the prescribed mimicking strategy.[13] Note that, in any equilibrium, the expected number of places filled must equal the selection quota, i.e.,

$$m = n\left(\theta\, p_S + (1-\theta)\, p_W\right), \tag{8}$$

where p_t represents a type-$t \in \{S,W\}$ contestant's equilibrium probability of winning. Thus, by equations (7) and (8), concession equilibria exist only if

$$p_W^C \geq \frac{m}{n}\left(\frac{1}{\theta r + 1 - \theta}\right), \tag{9}$$

where $r = \mu_S/\mu_W$ represents the *strength asymmetry* between strong and weak contestants.

In fact, the right-hand side of (9) is just a weak contestant's probability of winning in challenge configurations. To see this, let p_t^G be the probability of winning for a contestant of type $t \in \{S,W\}$ in challenge configurations. Because the probability of winning function, P, is concave, by Jensen's inequality, choosing a deterministic performance level equal to contest ability is always a weakly optimal strategy for each type. Hence, we can evaluate each type's probability of winning in challenge configurations simply by evaluating P at the type's contest ability. Thus, given that P is linear over its support and meets the origin in challenge configurations, it must be that, in challenge configurations, the ratio between strong and weak types' probabilities of winning, p_S^G/p_W^G, equals their strength asymmetry, $r = \mu_S/\mu_W$, i.e.,

$$\frac{p_S^G}{p_W^G} = \frac{\mu_S}{\mu_W} = r. \tag{10}$$

Equation (10), combined with identity (8), implies that

$$p_W^G = \frac{m}{n}\left(\frac{1}{\theta r + 1 - \theta}\right). \tag{11}$$

Equations (9) and (11) thus imply that concession equilibria exist only if $p_W^C \geq p_W^G$.

If $p_W^C < p_W^G$, only challenge equilibria can exist. In fact, the condition that $p_W^C < p_W^G$ is necessary for challenge equilibria to exist. This is because, in challenge equilibria, weak contestants not only have a chance of winning by beating weak rivals but also by beating strong

[13] By deviating, the weak contestant's probability of winning equals the strong type's probability of winning in concession configurations, p_S^C, with probability μ_W/μ_S, and equals 0 with the complementary probability.

rivals. Because the necessary condition for the existence of challenge equilibria, $p_W^C < p_W^G$, and the one for the existence of concession equilibria, $p_W^C \geq p_W^G$, are complementary, and because, as we show in the Online Appendix, an equilibrium always exists, these necessary conditions are also sufficient conditions. We thus obtain the following proposition.

Proposition 1. *Let p_t be a type-$t \in \{S, W\}$ contestant's equilibrium probability of winning.*

(a) Concession (challenge) configurations sustain an equilibrium if and only if $p_W^C \geq p_W^G$ ($p_W^C < p_W^G$), where p_W^C and p_W^G are given by (6) *and* (11) *respectively. (The construction of equilibrium strategies are given by Lemmas A-3 and A-4 in the Online Appendix.)*[14]

(b) A weak contestant's equilibrium probability of winning, p_W, is given by

$$p_W = \max\left[p_W^C, p_W^G\right]. \tag{12}$$

(c) A strong contestant's equilibrium probability of winning, p_S, is determined by p_W through equation (8).

Proposition 1 implies that which configuration is played, for a given parameterization of the model, is determined by weak contestants' preferences. Through adopting high-risk strategies, weak contestants are able to sometimes challenge strong contestants for places. However, because of the contest ability constraint, such challenges require increasing the probability of low performance, performance that is likely to be topped even by weak rivals. High-risk strategies can be sustained in equilibrium only when the benefits of such high-risk strategies outweigh their costs.

The next lemma shows that increasing competition tends to induce weak contestants to challenge strong contestants, and that weak contestants will challenge strong contestants if the selection contest is sufficiently competitive.

Lemma 2. *Everything else being equal, challenge configurations will be played in equilibrium if (a) contest size, n, is sufficiently large, or (b) strength asymmetry, r, is sufficiently small (i.e., sufficiently close to 1).*

If challenge configurations are played in equilibrium, challenge configurations will also be played in equilibrium if (a) contest size, n, increases, (b) the selection quota, m, decreases, or (c) strength asymmetry, r, decreases.

Increasing contest size or decreasing the selection quota increases the proportion of rivals that must be topped to win a place. Both these parameter changes make it less likely that besting only weak rivals is sufficient for a weak contestant to be selected. This increases weak contestants' incentives to challenge strong contestants through high-risk strategies. Reducing strength asymmetry increases weak contestants' contest ability relative to strong contestants',

[14] The construction is unique for concession but not for challenge equilibria. However, the multiplicity of challenge equilibria does not affect our welfare analysis, because all of the challenge equilibria produce the same probability of winning for each type and thus have the same implications for meritocracy.

making it easier for weak contestants to best strong contestants. This also increases weak contestants' incentives to challenge strong contestants.

3 Risk taking and meritocracy

For a fixed contest design—contest size, n and the selection quota, m—the designer's welfare is determined by the ability (type) of contestants selected. The designer's welfare function is meritocratic, i.e., she prefers to select strong contestants and deselect weak contestants. However, the designer does not know, ex ante, which contestants are strong and which are weak. Moreover, as we have seen from the results in Section 2, the outcome of the contest may not perfectly reveal ability. Thus, whether the selection of a contestant of unknown type will increase designer welfare depends on the tradeoff between the increase in welfare that results if the contestant turns out to be strong versus the decrease in welfare that results if the contestant turns out to be weak. We incorporate this tradeoff into the analysis with a symmetric linear specification—we assume that the designer maximizes the expectation of

$$\#\text{Strong Selected Contestants} - \#\text{Weak Selected Contestants}. \tag{13}$$

Thus, the designer gains one utile by selecting a strong contestant and loses one utile by selecting a weak contestant.[15]

For a given selection quota, m, and contest size, n, let $u(m,n)$ represent the designer's welfare and let $\Pi(m,n)$ be a selected contestant's probability of being strong, which we interpret as *winner quality*. Consistent with equation (13), the designer's welfare, u, is given by

$$u(m,n) = m\Pi(m,n) - m(1-\Pi(m,n)) = m\,(2\Pi(m,n)-1)\,. \tag{14}$$

An application of Bayes rule shows that, in the risk-taking contest, the probability that a selected contestant is strong, Π, is given by

$$\Pi(m,n) = \frac{\theta\, p_S}{\theta p_S + (1-\theta)p_W} = 1 - \frac{n}{m}(1-\theta)p_W = 1 - \frac{n}{m}(1-\theta)\max\left[p_W^C,\, p_W^G\right], \tag{15}$$

where the second equality follows from equation (8) and the last from (12).

The fundamental question we aim to address is how contestant risk-taking affects the attainment of the meritocratic ideal embodied in the designer's objective function. Because holding a contest requires the designer to commit to a fixed number of places, and the realized number of strong contestants is random, there is always a possibility that some places will be filled by weak contestants and some strong contestants will not receive a place. Subject to the constraint imposed by this commitment, the best possible selection strategy for the designer is to prioritize strong contestants, i.e., select weak contestants to fill the quota only after all strong contestants have been selected. We term this policy *merit-based selection*. Designer welfare under merit-

[15] The assumption of equal gains and losses simplifies the presentation. Extending the analysis by introducing asymmetry between gains and losses does not result in any qualitative change of our results. This extension is provided by an earlier version of the paper under the title "Lowering the bar and limiting the field: The effect of strategic risk-taking on selection contests" and is available from the authors upon request.

based selection, represented by u_M, will be our benchmark for measuring the welfare effects of risk taking.

Under concession configurations, weak contestants never outperform strong contestants. Thus, because places are allocated based on performance ranking, concession equilibria implement merit-based selection. In contrast, under challenge configurations, it is possible for a weak contestant to best strong contestants and, hence, strong contestants are not always prioritized. Given the designer's meritocratic preferences, this implies that the designer's welfare in challenge equilibria is lower than her welfare under merit-based selection. These observations are formalized below.

Lemma 3. *For a given selection quota, m, and contest size, n, the designer's welfare, $u(m,n)$, and welfare under merit-based selection, $u_M(m,n)$, satisfy the following conditions:*

i. if $p_W^C \geq p_W^G$, only concession equilibria exist, and $u(m,n) = u_M(m,n)$;

ii. if $p_W^C < p_W^G$, only challenge equilibria exist, and $u(m,n) < u_M(m,n)$,

where p_W^C and p_W^G are defined by equations (6) *and* (11) *respectively.*

Lemma 3 shows that, for a fixed contest design, the noise generated by strategic risk taking can lower designer welfare by making contest selection less meritocratic. In the following two sections, we examine how meritocratic contest designers respond to contestants' risk taking when designing contests.

4 Risk taking and contest size

In this section, we consider the effect on designer welfare of varying contest size, n, for a fixed selection quota, m. Note that the number of strong selected contestants and the number of weak selected contestants add up to m, which is fixed when m is fixed. Inspection of equation (13) then shows that, when m is fixed, the designer's problem is equivalent to maximizing the expected number of strong selected contestants.

To identify the effect of risk taking, we first examine the effect of varying n under merit-based selection. Suppose we add a new contestant to the contestant pool. If the added contestant is strong, and if before the contestant's addition, less than m contestants were strong, the new contestant will be selected, and the number of strong selected contestants will increase. Otherwise, i.e., if the new contestant is weak or the selection quota has already been filled by strong contestants, the number of strong selected contestants will not change. Because the ability of each contestant is drawn independently, the probability that the pool contains less than m strong contestants is always positive. Thus, fixing the quota, under merit-based selection, adding contestants increases the expected number of strong selected contestants and thus designer welfare.

Now consider the effect of pool expansion in the risk-taking contest. As shown by Lemma 3, in concession configurations, the designer's welfare, u, equals her welfare under merit-based selection, u_M. Given that $n \hookrightarrow u_M(m,n)$ is strictly increasing, pool expansion increases the designer's welfare as long as concession configurations are played. However, by Lemma 2, once contest size is sufficiently large, challenge configurations will be played. By equations (11) and (15), in challenge configurations, winner quality satisfies

$$\Pi = \frac{\theta r}{\theta r + 1 - \theta}, \tag{16}$$

which is independent of contest size. Thus, given that the expected number of strong selected contestants equals $m \times \Pi$, equation (16) reveals that, once contest size becomes sufficiently large such that challenge configurations are triggered, adding further contestants will not increase the expected number of strong selected contestants and, hence, will not increase designer welfare. This argument yields the following characterization.

Theorem 1. *For any fixed selection quota, m, there exists n^c, such that the marginal effect of increasing contest size on designer welfare is positive, i.e., $u(m,n+1) > u(m,n)$, only if $n < n^c$. Moreover,*

i. if $n \geq n^c$, increasing contest size does not change designer welfare, i.e., $u(m,n+1) = u(m,n)$.

ii. If $n > n^c$, designer welfare in the risk-taking contest is less than designer welfare under merit-base selection, i.e., $u(m,n) < u_M(m,n)$.

The basic implication of Theorem 1 is that risk taking caps the gains from inclusivity. When making the contestant pool more inclusive is costly because of outreach, advertisement, or search costs, the optimal contest size under risk-taking contest selection will tend to be smaller than under merit-based selection. As we will show in Section 6.3, even when increasing contest size is costless, when the pool of potential new contestants is, on average, of lower quality than the incumbent candidate pool, the designer may strictly gain from excluding the potential contestants from the contest. In such cases, the gain from expanding the pool produced by increasing the expected number of strong candidates is overwhelmed by the cost of increased risk taking. In contrast, under merit-based selection, the designer always strictly gains from inclusion because adding contestants increases the expected number of strong candidates.

Thus, in risk-taking contests, even meritocratic designers who are not biased toward specific candidates have little incentive to expand candidate fields and sometimes will deliberately restrict consideration to candidates who, ex ante, look promising, even if considering a wider field is costless.

In contrast, exogenous-noise contest models and all-pay contest models generally predict winner quality to be increasing in contest size. For example, Ryvkin (2010, Corollary 3.1) shows that, in selection contests where contestant performance equals ability plus an exogenous i.i.d. noise, expected winner ability is always increasing in contest size.[16] In all-pay contests,

[16]While Ryvkin (2010) restricts the selection quota to one, his result can be easily extended to the case with an arbitrary fixed quota.

where contestant performance equals contestant effort and effort costs depend on contestant ability, merit-based selection is implemented if contestants are ex ante homogenous with ability being private information (Moldovanu and Sela, 2001), and is approximately implemented if contest size is large (Olszewski and Siegel, 2016). Thus, in these all-pay contests, adding contestants also increases expected winner ability.

5 Risk taking and the selection quota

In this part, we fix contest size, n, and consider the welfare effects of varying the selection quota, m. Given that n is fixed, we suppress the dependence of the designer's welfare on n. Note that, because n is fixed, the expected number of weak contestants is fixed. This implies that the sum of the expected number of weak selected contestants and the expected number of weak deselected contestants is fixed. Inspection of (13) then shows that the designer's problem is equivalent to maximizing the expectation of

$$\text{\#Strong Selected Contestants} + \text{\#Weak Deselected Contestants}. \tag{17}$$

Thus, for fixed n, the selection problem is equivalent to the classic task assignment problem. In this problem, the designer assigns a fixed pool of contestants either to a more desirable "selection task" or a less desirable "deselection task." The marginal product of strong (weak) contestants is higher when performing the selection (deselection) task. For this reason, the results in this section can either be interpreted as a designer setting the number of places allocated to a fixed pool of contestants or as a designer fixing the performance rank required for "promotion" to more desirable task.

We first determine the optimal quota under merit-based selection. The number of strong contestants is Binomially distributed with parameters n and θ. Under merit-based selection, it is optimal for the designer to set the quota such that the marginal contestant selected (deselected) is more (less) likely to be strong than to be weak. Thus, given that strong contestants are prioritized for selection under merit-based selection, it is optimal for the designer to set the quota, m, such that the m-th highest ability contestant has a probability of being strong no less than one half and the $(m+1)$-th highest ability contestant has a probability of being strong no greater than one half, i.e., it is optimal to set the quota equal to a median of the $\text{Binom}(n, \theta)$ distribution.

Binomial distributions have either one or two medians. However, for any fixed n, the $\text{Binom}(n, \theta)$ distribution has two medians only for a finite set of θ. Thus, generically, the $\text{Binom}(n, \theta)$ distribution has a unique median. In the non-generic case where the $\text{Binom}(n, \theta)$ distribution has two medians, these two medians differ by one and are both optimal under merit-based selection.[17] For expositional convenience, in the subsequent analysis, we assume

[17]In such a non-generic case of merit-based selection, when the quota equals the lower median of the $\text{Binom}(n, \theta)$ distribution, the marginal contestant deselected has a probability of being strong equals exactly one half. In this case, setting the quota to the higher median is equally optimal as setting the quota to the lower median

that, whenever the designer is indifferent between two quotas, she chooses the larger quota. Let m_M^* be the optimal quota selected by the designer under merit-based selection and call m_M^* the *merit-based optimal quota*. The next lemma, which characterizes m_M^*, is thus straightforward.

Lemma 4. *The merit-based optimal quota, m_M^*, is given by*

$$m_M^*(n, \theta) = \min\{m \in \{0, 1, \ldots, n\} : B(m; n, \theta) > 1/2\},$$

where $B(\cdot; n, \theta)$ denotes the CDF of the Binom(n, θ) distribution. In other words, m_M^* equals the median number of strong contestants (i.e., the median of the Binom(n, θ) distribution) if the Binom(n, θ) distribution has a unique median, and equals the larger median if the Binom(n, θ) distribution has two medians.

If the Binom(n, θ) distribution has a median equal to 0 (n), then the merit-based optimal quota selects none (all) of the contestants. In this case, even in the absence of risk taking, the contest mechanism does not further the goal of meritocratic selection. The examination of the effects of risk taking on contests when contests cannot further merit-based selection is not a very interesting exercise. Thus, in the subsequent analysis, we impose the following restriction:

Assumption 1. The Binom(n, θ) distribution has no median equal to 0 or n.

Now consider the designer's optimal quota when selection is determined by the risk-taking contest. Lemma 3 implies that, for any selection quota, the designer's welfare, $u(m)$, is bounded above by her welfare under merit-based selection, $u_M(m)$, and thus, *a fortiori*, by her welfare under optimal merit-based selection, $u_M(m_M^*)$. When does designer welfare in the risk-taking contest attain this upper bound, $u_M(m_M^*)$, and how does contestant risk taking affect the designer's optimal selection quota? The following proposition answers these questions.

Theorem 2. *Let m^* be the optimal selection quota in the risk-taking contest and let m_M^* be the merit-based optimal quota.*

i. Designer welfare in the risk-taking contest equals designer welfare under merit-based selection, i.e., $u(m^*) = u_M(m_M^*)$, if and only if a concession configuration is played in the risk-taking contest at $m = m_M^*$; otherwise, designer welfare is lower in the risk-taking contest, i.e., $u(m^*) < u_M(m_M^*)$.

ii. If $r \le (1-\theta)/\theta$ (in which case, challenge configurations will be played at $m = m_M^*$), then $m^* = 0$.

iii. If $r > (1-\theta)/\theta$, then (a) if a concession configuration is played at $m = m_M^*$, $m^* = m_M^*$, whereas (b) if challenge configurations are played at $m = m_M^*$, $m^* \ge m_M^*$ and $m^* = \bar{m}$ or $\bar{m} +$ 1, where $\bar{m}$ is the largest quota at which challenge configurations are played, i.e.,

$$\bar{m} = \max\left\{m \in \{m_M^*, \ldots, n-1\} : \, p_W^C(m) < p_W^G(m)\right\}, \tag{18}$$

where p_W^C and p_W^G are defined by equations (6) *and* (11) *respectively.*

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under merit-based selection.

The logic behind Theorem 2 is fairly straightforward. Part (i) asserts that the necessary and sufficient condition for designer welfare to attain its merit-based upper bound is that a concession configuration is played at the merit-based optimal quota, m_M^*. This follows because, if challenge configurations are played at m_M^*, designer welfare will fall below its merit-based upper bound either due to the reduction in the correlation between selection and ability caused by risk taking or due to the designer's use of a "distorted" quota to accommodate risk taking (or even both).

Part (ii) shows that, if the strength asymmetry between strong and weak contestants, r, is smaller than the threshold, ${}^{(1-\theta)}/_{\theta}$, it is optimal to set a zero quota, even though doing so is not optimal under merit-based selection. This result follows because, when weak contestants are only marginally weaker than strong contestants, weak contestants will not concede. In this case, only challenge configurations sustain an equilibrium. Inspection of equation (16) shows that, when $r \leq {}^{(1-\theta)}/_{\theta}$, a selected contestant's probability of being strong under challenge configurations is less than one half. Hence, by (17), each contestant selected lowers the designer's welfare. This tends to induce the designer to reduce the quota in an attempt to raise the quality of selected contestants. However, reducing the quota makes the selection even more competitive and, consequently, challenge configurations will continue to be played. By (16), winner quality in challenge configurations is independent of the quota, m. Thus, reducing the quota cannot improve winner quality and it is optimal to set a zero quota, or equivalently not conduct a selection contest. Thus, selection through risk-taking contests will not be implemented when the relation between contest ability and ability, measured by r, is sufficiently weak.

Note that the condition in part (ii) highlights a fundamental difference between selection in risk-taking contests and merit-based selection: the merit-based optimal quota is nonzero when candidate pool is sufficiently large. In contrast, under contest selection, the condition for setting a zero quota in part (ii) is independent of n and, for any fixed level of strength asymmetry, is always satisfied for sufficiently small prior quality of candidates, θ. Thus, part (ii) implies that, when the designer is faced with a pool of candidates with low average prior quality, the designer has no incentive to run a competition for selection/assignment regardless of the size of pool of candidates the designer can tap. Risk taking blocks using selective contests to identify a few high-ability agents hidden in a large pool of weak candidates.

Part (iii) shows that, if the strength asymmetry, r, is larger than the threshold, ${}^{(1-\theta)}/_{\theta}$, and if challenge configurations are played at the merit-based optimal quota, m_M^*, contestant risk taking can induce *quota inflation*, setting quotas greater than the merit-based optimal quota.

When $r > {}^{(1-\theta)}/_{\theta}$, the marginal gain from adding the first quota place to a zero quota is positive. As long as the selection quota is small enough to sustain a challenge configuration, the marginal gain from adding another quota place is constant. In this case, when challenge configurations are played at m_M^*, the designer will always inflate the quota up to $\bar{m} \geq m_M^*$, where $\bar{m}$ denotes the highest quota that supports a challenge configuration.

Whether the designer will inflate the quota even further to $\bar{m}+1$ depends on a tradeoff specific to the parameters of the contest: inflating the quota from $\bar{m}$ to $\bar{m}+1$ will result in a concession configuration being sustained. The concession configuration implements merit-based selection at the quota $\bar{m}+1$ while, because of high-risk challenge strategies played by weak contestants, selection at $\bar{m}$ is not merit-based. This risk-mollification effect favors expanding the quota to $\bar{m}+1$. However, $\bar{m}+1$ exceeds the merit-based optimal quota, m_M^*, even more than $\bar{m}$. Thus, expanding the quota even further requires filling marginal quota places with contestants whose expected quality is even more deficient than under $\bar{m}$. If the risk-mollification effect is dominant, then the quota will be inflated further, to $\bar{m}+1$. Otherwise, the quota will be set at $\bar{m}$.

In any case, expansion of the quota past $\bar{m}+1$ is never optimal: expansion to $\bar{m}+1$ eliminates the adverse selection effects of strategic risk taking and the $\bar{m}+1$ quota already is inflated relative to the merit-based optimal quota, m_M^*. The following example provides an illustration of these results.

Example 1. Consider a contest with $n = 8$ contestants, where ex ante, each contestant's probability of being strong equals $\theta = 1/2$. The contest ability of strong contestants is $\mu_S = 2$ while the contest ability of weak contestants is $\mu_W = 1$. Designer welfare under merit-based selection and under risk-taking contest selection are presented in Table 1.

Consistent with Lemma 4, the merit-based optimal quota, m_M^*, equals 4, the median of the Binom$(n = 8, \theta = 1/2)$ distribution. Also note that, if the performance of each contestant simply equaled contest ability plus an i.i.d. noise, i.e., if risk were exogenous, the optimal selection quota would also equal 4, provided that the noise term was Normally distributed, or more generally, satisfied the standard restrictions, i.e., the density of the noise distribution was symmetric and log-concave (i.e., strongly unimodal). Symmetry and the fact that $\theta = 1/2$ imply that the gain from increasing the quota from m to $m+1$ equals the loss from increasing the quota from $n-(m+1)$ to $n-m$. The fact that the error law is strongly unimodal implies that designer welfare is quasi-concave in the quota; thus its optimum over $\{0, 1, \ldots 8\}$ is also attained at 4, the merit-based optimal quota. Hence, absent strategic risk taking, in this example, we should not expect quota inflation.

Now consider the risk-taking contest. Challenge configurations are played for $m \le \bar{m} = 5$. In challenge configurations, the odds of a strong versus a weak contestant being selected equal the strength asymmetry, $r = \mu_S/\mu_W = 2$ regardless of the size of the quota. Thus, as reported in Table 1, the marginal gain from increasing the quota is constant when the quota is below $\bar{m}$. Although, in the contest, the designer's welfare is lower than under merit-based selection, at the merit-based optimal quota, $m_M^* = 4$, the marginal gain from adding a quota place is higher. This is not too surprising: at $m = m_M^*$, challenge configurations are played, and thus the noise produced by strategic risk taking reduces the expected winner quality relative to merit-based selection. Hence, given that selection does not affect the quality of the entire pool of contestants, the expected loser quality under contest selection is higher than under merit-based selection, and thus the marginal gain from increasing the quota is larger. Hence, the quota will

	merit-based selection			*contest selection*			
	$\mathbb{E}$[# Selected]				$\mathbb{E}$[# Selected]		
m	S	W	welfare, u_M	Configuration	S	W	welfare, u
1	0.996	0.004	0.992	Challenge	0.667	0.333	0.333
2	1.961	0.039	1.922	Challenge	1.333	0.667	0.667
3	2.816	0.184	2.632	Challenge	2.000	1.000	1.000
4	3.453	0.547	**2.906**	Challenge	2.667	1.333	1.333
5	3.816	1.184	2.632	Challenge	3.333	1.667	1.667
6	3.961	2.039	1.922	Concession	3.961	2.039	**1.922**
7	3.996	3.004	0.992	Concession	3.996	3.004	0.992

Table 1: *The selection quota, m, and designer welfare in Example 1*

be inflated at least until it reaches the highest quota that supports a challenge configuration, $\bar{m} = 5$.

Whether it is optimal to inflate the quota even more, from $m = \bar{m} = 5$ to $m = \bar{m} + 1 = 6$, involves a tradeoff between further quota distortion and risk mollification. At $m = 5$, challenge configurations are played; at $m = 6$, a concession configuration is played. The mollification of risk taking in concession configurations makes selection at $m = 6$ merit-based. This effect encourages inflating the quota further to $m = 6$. However, the $\bar{m} = 5$ selection quota already exceeds the merit-based optimal quota. This implies that the additional place created by further increasing the quota is very likely to be filled by a weak contestant. In fact, in this example, the probability that a sixth quota place will be filled by a weak contestant is approximately 85%.[18] This effect discourages further inflating the quota.

However, as Table 1 reveals, in this example, the benefit of risk mollification exceeds the cost of further quota distortion, and increasing the quota from $m = \bar{m} = 5$ to $m = \bar{m} + 1 = 6$ is optimal. Thus, the meritocratic contest designer is willing to offer a place to a contestant who is very likely to be unworthy in order to mollify weak contestants' risk-taking incentives and thereby, on net, further the goal of meritocratic selection. Further increases of the quota beyond six are clearly suboptimal because increasing the quota to $m = \bar{m} + 1 = 6$ eliminates the distortions in selection caused by risk taking and further increases in the quota will lead to marginal quota places being filled by contestants who are even more likely to be weak, and thus unworthy of selection.

Figure 3 illustrates the relation between the strength asymmetry, r, and the optimal contest selection quota, m^*, when $n = 10$ and $\theta = 0.5$. As the figure shows, the optimal quota is highly inflated for small r. As the strength asymmetry increases, m^* decreases and eventually con-

[18] If selection is merit-based, the probability that the sixth contestant selected is weak equals the probability that the number of strong contestants is less than or equal to five, i.e., $B(5; n = 8, \theta = 1/2) \approx 0.85$, where $B(\cdot; n, \theta)$ denotes the CDF of the $\text{Binom}(n, \theta)$ distribution.

verges to the merit-based optimal quota, m^*_M. As the next proposition reveals, the convergence from above exhibited in Figure 3 is a general property of optimal contest selection quotas.

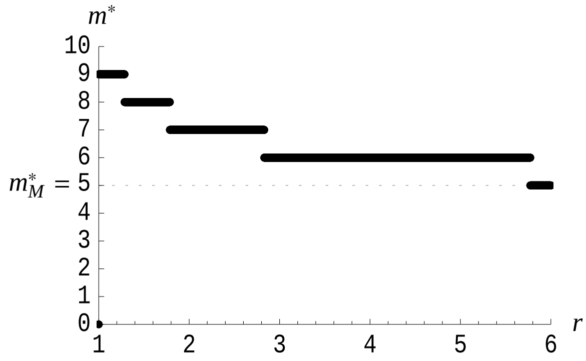

Figure 3: Optimal quota in the risk-taking contest, m^*, given strength asymmetry, r, when $n = 10$ and $\theta = 0.5$. The merit-based optimal quota, m^*_M, equals 5.

Proposition 2. *Suppose that $r > {}^{(1-\theta)}/_{\theta}$ (otherwise, by Theorem 2, setting a zero quota is optimal). The optimal quota in the risk-taking contest, m^*, is nonincreasing in strength asymmetry, r, and equal to m^*_M, the merit-based optimal quota, for r sufficiently large.*

Proposition 2 implies that over-selection will be most pronounced in contests where contest ability is only weakly related to ability. This case is very likely to occur, as we will show in Section 6.2, where we endogenize contest ability through costly effort, when the cost of acquiring contest ability is highly convex.

6 Extensions

In this section, we consider various modifications of our baseline model. These extensions show that our results are quite robust and also lead to new implications.

6.1 Ex post discretionary selection

In our baseline model, we assumed that the designer commits to fill the quota places by best performers. Such commitment, however, can be hard to enforce in practice, because contest performance sometimes depends on complex evaluations that are difficult for outsiders to verify.[19] Thus, an important question to address is whether our game has an equilibrium in which the designer, even if she is ex post filling the quota at her discretion, has no incentive to fill the quota with contestants who are not the best performers. The corollary to the next lemma provides an affirmative answer to this question.

[19] In contrast, the number of contestants selected is easy to verify in practice, which justifies the assumption that the designer can commit to her choice of the selection quota.

Lemma 5. *There always exists an equilibrium in which F_S and F_W satisfy the* monotone likelihood ratio property *(MLRP), i.e.,*

$$\textit{for every } x'' > x' \geq 0, \quad \frac{f_S(x'')}{f_W(x'')} \geq \frac{f_S(x')}{f_W(x')},$$

where f_S and f_W denote the density functions for F_S and F_W, respectively, and we treat $f_S(x)/f_w(x) = \infty$ if $f_W(x) = 0$.

Lemma 5 implies that there always exists an equilibrium in which better performance is a stronger signal of ability. To see this, simply note that, by Bayes rule, a contestant's probability of being strong conditional on his performance level x is given by

$$\mathbb{P}[S|x] = \frac{\theta\, f_S(x)}{\theta\, f_S(x) + (1-\theta)\, f_W(x)} = \frac{\theta\,(f_S(x)/f_W(x))}{\theta\,(f_S(x)/f_W(x)) + 1 - \theta},$$

which is nondecreasing in performance level x if the MLRP holds. Given that better performance more reflects ability, the next result is straightforward.

Corollary 1. *Filling the selection quota, m, by the m best performers is a credible commitment.*

6.2 Endogenous contest ability

In our baseline model, we assumed that contest ability of each contestant was fixed and positively related to contestant ability. In this subsection, we endogenize contest ability by allowing each contestant to acquire contest ability through costly effort. To do so, we assume that, after the selection quota and contest size are announced to the contestants, the contestants first simultaneously exert effort, which determines their contest ability. We assume that the effort cost function is a strictly convex power function. Specifically, the cost of choosing contest ability μ for a type-$t \in \{S, W\}$ contestant is $c_t(\mu) = \mu^\alpha/a_t$, where $\alpha > 1$ and a_t is an ability parameter satisfying $0 < a_W < a_S$. After the contestants acquire their contest ability, the contestants, without knowing each other's contest ability, simultaneously choose nonnegative random performance subject to their contest ability constraints. Selection is still based on the ranking of realized performance. Without loss of generality, we assume that the reward from being selected equals 1 and the reward from being deselected equals 0.[20] A contestant's payoff equals the reward he receives less his effort cost.

The next proposition shows that this modified game has symmetric equilibria in which contestants of the same type choose the same level of contest ability and play the same performance distribution. These equilibria still feature either concession or challenge configurations, and the conditions for the play of concession/challenge configurations are similar to those in our baseline model except that now the strength asymmetry, $r = \mu_S/\mu_W$, is endogenized.

[20]Assuming that rewards are functions of contest size and/or the selection quota does not change any result or implication of Proposition 3, as long as the reward from being selected is greater than the reward from being deselected.

Proposition 3. *Define* p_W^C *as in* (6) *and define* $p_W^G(r)$ *as in* (11) *and as a function of strength asymmetry, r. The modified game in which contest ability is acquired through costly effort has either concession or challenge equilibria.*

i. A concession equilibrium exists if and only if

$$p_W^C \geq p_W^G(r^*), \quad \text{where } r^* = \left(\frac{a_S}{a_W}\right)^{\frac{1}{\alpha-1}}. \tag{19}$$

ii. If $p_W^C < p_W^G(r^*)$, *challenge equilibria exist with endogenous strength asymmetry equal to* r^* *given in equation* (19).

iii. Everything else being equal, if challenge configurations are played in a contest with $\alpha = \alpha'$, *challenge configurations will also be played in contests with* $\alpha > \alpha'$.

In concession configurations, a weak contestant's probability of winning equals p_W^C given by equation (6), which is independent of contest ability. In challenge configurations, a weak contestant's probability of winning is given by $p_W^G(r^*)$, where r^*, given in equation (19), is the endogenous strength asymmetry in a challenge configuration.[21] Thus, Proposition 3 implies that which configuration is played, for a given parameterization of our modified game with endogenous contest ability, is still determined by weak contestants' preferences. Because r^* depends neither on contest size, n, nor on the selection quota, m, our previous analysis of how contest size and the selection quota affect contestant risk taking and how risk taking in turn affects the design of selection contests is robust to this extension.

Moreover, part (iii) of Proposition 3 leads to a new implication. Note that, α, the power coefficient of the effort cost function, measures effort cost convexity. Increasing α reduces r^* given in (19), and the reduction in r^* increases $p_W^G(r^*)$, making it less likely that condition (19) holds. Thus, consistent with part (iii), increasing effort cost convexity makes it less likely that weak contestants will concede to strong contestants and, conditional on weak contestants challenging strong contestants, a reduction in strength asymmetry due to an increase in cost convexity will further increase weak contestants' chance of besting strong contestants. Thus, the performance/ability relation is most noisy if effort costs are highly convex. In this case, when the prior quality of the contestant pool is not too low (otherwise, by Theorem 2.ii, setting a zero quota is optimal), to reduce strategic noise, selection contests tend to be highly "clubby," featuring limited candidate pools and low selection standards for pool members.[22] As is well known that, in the mutual fund industry, it is hard for fund managers to generate "alpha," i.e., risk-adjusted abnormal returns (Fama and French, 2010), this fact suggests that the cost of improving mean performance is highly convex for mutual fund managers. Thus, our result might offer a rational explanation for why retention contests in the mutual fund industry are

[21] If condition (19) holds, then a concession equilibrium exists. In this concession equilibrium, endogenous strength asymmetry is different from r^* but does not enter p_W^C.

[22] If the power coefficient α tends to 1, in which case the cost function lacks convexity, r^* will tend to infinity and thus, by equation (11), $p_W^G(r^*)$ will tend to 0. In this case, by Proposition 3, weak contestants will always concede, which makes risk-taking contest selection merit-based.

highly clubby—only 14% of managers in the lowest performance decile are replaced (Khorana, 1996, Table 4).

6.3 Pool expansion by including less promising candidates

In Section 4, we studied the effect of risk-taking on the optimal size of the contestant pool. We showed that, if the contestant pool is sufficiently large, adding more contestants who are as likely to exhibit ability as the contestants in the original pool does not affect the expected ability of contest winners. Thus, a meritocratic contest designer has no incentive to expand the contestant pool if the pool is already sufficiently large. The next result shows that, in fact, expanding a large pool makes selection less meritocratic if the external candidates are less likely to exhibit ability than the contestants in the original pool.

Proposition 4. *Suppose that the designer can only expand the contestant pool by including external candidates whose prior quality (measured by θ) is lower than the internal candidates'. If the contest with only the internal candidates has challenge equilibria, pool expansion strictly reduces designer welfare in any symmetric equilibrium.*[23]

Proposition 4 implies that, even without any direct cost of pool expansion, as long as the external candidates are ex ante less promising than internal ones, a meritocratic designer strictly prefers "limiting the field" only to internal candidates if the internal competition already triggers the play of challenge configurations. Proposition 4 might shed some light on why many real-world selection contests limit participation by requiring, sometimes in a *de facto* way, candidates to have certain qualifications to be eligible for contest participation.

6.4 Scoring caps and risk mitigation

Throughout our earlier analysis, we focused on the problem of a meritocratic designer who only cares about the effect of risk taking on meritocracy. As we mentioned in the introduction, risk taking in many selection contests also imposes social costs other than the cost of reducing meritocracy. Given these unwanted social costs of risk taking, a natural question to ask is whether it is possible for a meritocratic designer to reduce risk taking without sacrificing meritocracy. Our answer is affirmative and we propose three mechanisms.

The first is to use a scoring cap. Many real-life contests naturally have a scoring cap, such as a full score in examinations, that caps the highest performance a contestant can possibly obtain. Even in cases, e.g., mutual fund tournaments, in which performance is unbounded, a designer can impose a scoring cap if she can credibly specify that all performance levels no less than a threshold will be treated the same for the purpose of determining contest winners. Under this specification, that threshold will effectively be the scoring cap. The second is to limit

[23] Because, by assumption, the external candidates are *ex ante* different from the internal candidates, the concept of symmetric equilibria in such a case refers to equilibria in which all type-t internal candidates play the same strategy and all type-t external candidates play the same strategy, $t \in \{S, W\}$.

contest size when the quota is fixed. The third is to randomly allocate the quota places over an expanded set of "contest winners" *à la* Schwartz (2007) when both the quota and contest size are fixed. For these mechanisms to work, we require certain conditions. Details are provided by the next proposition.

Proposition 5. *Let $F = \theta F_S + (1-\theta)F_W$ be the equilibrium performance distribution chosen by a contestant of unknown type. All of the following induce F to undergo a mean-preserving contraction (i.e., to be larger in the sense of second-order stochastic dominance with a fixed mean) without changing meritocratic designer welfare (specified by equation* (14)*):*

i. impose a scoring cap $\bar{x} \in [\mu_S, \hat{x})$, where $\hat{x}$ represents the upper bound of the support of F in the contest without the scoring cap (imposing any cap $\bar{x} \geq \hat{x}$ does not affect F).

ii. For fixed quota, m, reduce contest size from n to $n' < n$, provided that the n'-contestant/m-winner risk-taking contest has challenge equilibria.

iii. For fixed quota, m, and fixed contest size, n, use a "relaxed" selection policy by first approving $m' > m$ contestants based on performance ranks and then randomly select m out of these m' approved contestants, provided that the n-contestant/m'-winner risk-taking contest has challenge equilibria.

Part (i) of Proposition 5 implies that imposing a scoring cap no less than strong contestants' contest ability, μ_S, has no effect on meritocracy but tends to reduce contestant risk taking.[24] As we show in the proof of Proposition 5, if a scoring cap, no less than μ_S, is lower than the upper endpoint of the union of the supports of weak and strong contestants' performance distributions chosen in the contest without the cap, imposing such a scoring cap will lead contestants to move any probability weight originally placed on performance levels above the cap and some weight below the cap to the cap, thereby reducing performance riskiness.

Parts (ii) and (iii) imply that making highly competitive selection contests less competitive by either limiting contestant fields or randomly allocating places over an expanded group of best performers reduces performance riskiness without sacrificing meritocracy, provided that contests with reduced competitiveness are still fairly selective (such that challenge configurations are still played in less competitive contests). Thus, if performance riskiness *per se* causes direct social costs or if fierce competition imposes psychological costs on contestants, our result implies that, for competitions that are naturally fierce, e.g., competitions for a CEO position and elite-university admissions, reducing competition by, e.g., running an "in-house" competition or using a "relaxed" selection policy followed by a lottery process as proposed by Schwartz (2007), can reduce the side effect of contest selection without sacrificing meritocracy.

7 Conclusion

In this paper, we studied selection contests in which contestants of private types are strategic risk takers. We showed that increasing competition, either by expanding the contestant

[24] Imposing a scoring cap less than μ_S will handicap strong contestants and reduce meritocracy.

pool or reducing the selection quota, increases weak contestants' tendency to play high-risk strategies to challenge potential strong contestants, which limits the gains in selected applicant quality produced by intensifying competition. Consequently, even meritocratic designers have an incentive to limit competition by adopting "clubby" contests, contests that feature less inclusive contestant pools and over-promotion of marginal candidates. Our model implies that many seemingly unmeritocratic practices and proposals, in fact, further meritocracy, such as the use of "Peter Principle" promotion policies in companies and organizations (Peter and Hull, 1969), the running of "in-house" competition instead of "open competition" for leader selection, and the advocate of using a "relaxed" selection policy followed by a lottery process for elite-university admissions (Schwartz, 2007).

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Online Appendix to "Less competition, more meritocracy?:" Proofs of results

Proof of Lemma 1. In the main text, we provided an informal but intuitive discussion of the proof of Lemma 1. In what follows, we give a formal proof based on duality theory. The proof consists of several steps.

Step 1: The first step, which is the key, is to develop an algorithm that can be used to characterize each type's best reply in a symmetric equilibrium. The algorithm is given in Lemma A-1. Note that, for every type-$t \in \{S, W\}$ contestant, his problem is to choose a performance distribution, F_t, for his nonnegative random performance, X_t, so as to maximize $\mathbb{E}[P(X_t)]$, subject to the contest ability constraint, i.e., $\mathbb{E}(X_t) = \mu_t$. More conveniently, we can formulate the problem as one of choosing a performance measure, dF_t, to use against P. The performance measure, dF_t, has to satisfy two constraints: (a) it has to be a probability measure and (b) its expectation equals μ_t. The solution to this problem coincides with the solution to the following relaxed problem of choosing a measure dF_t supported by $[0, \infty)$:

$$\max_{dF_t \geq 0} \int_{0-}^{\infty} P(x)\, dF_t(x) \qquad \text{s.t. (i)} \int_{0-}^{\infty} dF_t(x) \leq 1 \ \& \ \text{(ii)} \int_{0-}^{\infty} x\, dF_t(x) \leq \mu_t. \tag{P$_\text{F}$}$$

The Lagrangian associated with problem (P_F) is given by

$$\mathscr{L}(dF_t, \alpha_t, \beta_t) = \int_{0-}^{\infty} P(x) dF_t(x) - \alpha_t \left(\int_{0-}^{\infty} dF_t(x) - 1 \right) - \beta_t \left(\int_{0-}^{\infty} x\, dF_t(x) - \mu_t \right), \tag{A-1}$$

where α_t and β_t are nonnegative dual variables. Rewrite equation (A-1) as

$$\mathscr{L}(dF_t, \alpha_t, \beta_t) = \int_{0-}^{\infty} [P(x) - (\alpha_t + \beta_t x)]\, dF_t(x) + \alpha_t + \beta_t \mu_t. \tag{A-2}$$

Our next result shows the existence of a solution to problem (P_F).

Result A-1. Suppose that P is nonnegative, nondecreasing, bounded, and upper semicontinuous, with $P(\mu_t) < P(\infty)$ (which we will show to be the case in any symmetric equilibrium). Problem (P_F) has a solution and the support of this solution is bounded.

Proof. Define

$$v_t^* = \sup \left\{ \int_{0-}^{\infty} P(x)\, dF_t(x) : \int_{0-}^{\infty} dF_t(x) \leq 1 \ \& \int_{0-}^{\infty} x\, dF_t(x) \leq \mu_t \right\}. \tag{A-3}$$

Because P is bounded and the unit mass constraint, (P_F–i), and the contest ability constraint, (P_F–ii), are not mutually exclusive, v_t^* clearly exists. The map $dF_t \hookrightarrow \int_{0-}^{\infty} P(x)\, dF_t(x)$ is linear. The set of nonnegative measures satisfying the contest ability and the unit mass constraints is convex. Thus, by basic duality theory (Luenberger, 1969, §8.3, Theorem 1), there exist $\alpha_t^o \geq 0$ and $\beta_t^o \geq 0$ such that

$$\sup_{dF_t \geq 0} \mathscr{L}(dF_t, \alpha_t^o, \beta_t^o) = v_t^*.\text{[25]} \tag{A-4}$$

[25] The other conditions for the existence of the nonnegative dual variables are clearly satisfied as the constraint

Because P is nondecreasing and $P(\mu_t) < P(\infty)$, increased contest ability has value. Hence, the contest ability constraint cannot be slack, which implies that $\beta_t^o > 0$. Because P is bounded, $P(\infty)$ exists. Thus, given that $\alpha_t^o \geq 0$ and $P(x) \leq P(\infty)$ for all $x \geq 0$, we must have that, for all $x > x^* = P(\infty)/\beta_t^o$, $P(x) - (\alpha_t^o + \beta_t^o x) < 0$. Inspection of equation (A-2) then shows that placing positive weight on any performance level $x > x^*$ lowers the Lagrangian, $\mathscr{L}$. Hence, restricting the probability measure to $[0,x^*]$ will not lower its supremum. Thus,

$$\sup\{\mathscr{L}(dF_t, \alpha_t^o, \beta_t^o) : dF_t \geq 0\} = \sup\{\mathscr{L}(dF_t, \alpha_t^o, \beta_t^o) : dF_t \geq 0 \ \& \ dF_t\{(x^*, \infty)\} = 0\}. \quad \text{(A-5)}$$

The set of measures with support in $[0,x^*]$ is compact in the weak topology and P is upper semicontinuous. Thus, the supremum of the Lagrangian is attained over the restricted set of measures, i.e., there exists dF_t^o such that

$$\mathscr{L}(dF_t^o, \alpha_t^o, \beta_t^o) = \sup\{\mathscr{L}(dF_t, \alpha_t^o, \beta_t^o) : dF_t \geq 0 \ \& \ dF_t\{(x^*, \infty)\} = 0\}. \quad \text{(A-6)}$$

Thus, equations (A-3), (A-4), (A-5), (A-6), and basic duality theory (Luenberger, 1969, §8.4, Theorem 1) imply that dF_t^o solves problem ($\mathrm{P_F}$). □

The next lemma presents the algorithm that can be used to characterize the solution to problem ($\mathrm{P_F}$).

Lemma A-1. *Suppose that P satisfies all the conditions in Result A-1. A probability distribution function, F_t, solving problem* ($\mathrm{P_F}$) *exists. For any such solution, its support is bounded and there exist dual variables $\alpha_t \geq 0$ and $\beta_t > 0$ such that α_t and β_t satisfy*

$$P(x) \leq \alpha_t + \beta_t x \quad \forall\, x \geq 0 \quad \& \quad dF_t\{x \geq 0 : P(x) < \alpha_t + \beta_t x\} = 0, \quad \text{(A-7)}$$

and, if $\mathbf{v}(P, \mu_t)$ represents the optimal value of problem ($\mathrm{P_F}$),

$$\mathbf{v}(P, \mu_t) = \alpha_t + \beta_t \mu_t. \quad \text{(A-8)}$$

Conversely, if a probability distribution, F_t, satisfies (A-7) *and makes the contest ability constraint, ($\mathrm{P_F}$–ii), bind, it is a solution to* ($\mathrm{P_F}$).

Proof. The dual variables associated with an optimal solution are the solutions to the following dual problem:

$$\min_{\alpha_t, \beta_t \geq 0} \sup_{dF_t \geq 0} \mathscr{L}(dF_t, \alpha_t, \beta_t), \quad (\mathrm{D_F})$$

where $\mathscr{L}$ is given by (A-2). Equation (A-2) implies that the optimal dual variables that solve ($\mathrm{D_F}$) must satisfy

$$P(x) - (\alpha_t + \beta_t x) \leq 0 \quad \forall\, x \geq 0, \quad \text{(A-9)}$$

since otherwise $\sup_{dF_t \geq 0} \mathscr{L}(dF_t, \alpha_t, \beta_t)$ tends to positive infinity. Thus, $\alpha_t + \beta_t x$ is an upper bound for $P(x)$.

space is simply $\Re^2$ and thus its nonnegative cone has a nonempty interior and the feasible set contains a point where the contest ability and the unit mass constraints are strictly satisfied.

When condition (A-9) is satisfied, by (A-2), the value of the dual problem ($\mathrm{D_F}$) equals

$$\alpha_t + \beta_t \mu_t,$$

which is strictly increasing in both α_t and β_t. Thus, the nonnegative optimal dual variables must minimize $\alpha_t + \beta_t \mu_t$ subject to condition (A-9). Hence, given that P is upper semicontinuous, condition (A-9) must be binding for some $x \geq 0$, i.e., there exists some point(s) $x' \geq 0$ such that $P(x') - (\alpha_t + \beta_t x') = 0$. Thus, $\alpha_t + \beta_t x$ is an *upper support line* for P. To distinguish this line from other upper support lines that P might have, we call this line the *type-t's upper support line*. Placing any probability weight on points at which $P(x) - (\alpha_t + \beta_t x) < 0$ lowers the Lagrangian. Thus, the optimal performance distribution for type-t must place no weight on such points. Therefore, the optimal performance measure for type-t is always concentrated on points at which the type-t's support line, $\alpha_t + \beta_t x$, meets P. Thus, the optimal performance measure for type-t, dF_t, and the associated optimal dual variables, α_t and β_t, must satisfy (A-7).

Note that the maps $dF_t \hookrightarrow \int_{0-}^{\infty} dF_t(x)$ and $dF_t \hookrightarrow \int_{0-}^{\infty} x\, dF_t(x)$ are linear. Thus, the set of nonnegative measures satisfying the unit mass constraint, ($\mathrm{P_F}$–i), and the contest ability constraint, ($\mathrm{P_F}$–ii), is convex. Given that the map $dF_t \hookrightarrow \int_{0-}^{\infty} P(x)\, dF_t(x)$ is linear, by basic duality theory (Luenberger, 1969, §8.6, Theorem 1), strong duality holds. Thus, the primal problem ($\mathrm{P_F}$) must have its optimal value equal to that of the dual problem ($\mathrm{D_F}$). Hence, the optimal value of problem ($\mathrm{P_F}$) satisfies equation (A-8). Relaxing the unit mass constraint, ($\mathrm{P_F}$–i), by $\varepsilon > 0$ increases the type-t contestant's probability of winning by at least $\varepsilon \times P(0) \geq 0$. Thus, a solution to problem ($\mathrm{P_F}$) in which the unit mass constraint is satisfied as an equality (i.e., the optimal measure is a probability measure) always exists and α_t, the dual variable associated with the unit mass constraint, is at least equal to 0. Similarly, since $P(\mu_t) < P(\infty)$, the type-t contestant does not have sufficient contest ability to guarantee the largest possible payoff. Thus, the contest ability constraint, ($\mathrm{P_F}$–ii), must be binding at the optimum and hence, $\beta_t > 0$. □

Step 2: The second step is to argue that

Lemma A-2. *In any symmetric equilibrium, the probability of winning function, P, is nonnegative, nondecreasing, bounded, and continuous, with* $P(\mu_t) < P(\infty) = 1$ *and* $P(0) = 0$.

Proof. First, note that in any symmetric equilibrium, no contestant places any point mass. This is because, if a contestant of type-t^o placed point mass on some performance level, say x^o, then by symmetry, all of the contestants of type-t^o would place point mass on x^o. Thus, given that contestant types are independently drawn from a Bernoulli distribution, there would exist a positive probability that all of the contestants are of type-t^o and all of them tie at x^o. Then a type-t^o contestant would be strictly better off transferring mass away from x^o to $x^o + \varepsilon$, for $\varepsilon > 0$ sufficiently small. The transfer's effect on satisfying the contest ability constraint could be made arbitrarily small by shrinking ε to zero while, for all positive ε, no matter how small, the transfer would generate a gain that is bounded below by a positive number, a contradiction.

Next, given that no contestant places any point mass in any symmetric equilibrium, P must be continuous and, given that performances are nonnegative, it must be that $P(0) = 0$.

Finally, the result that $P(\mu_t) < 1$ follows from the fact that no contestant can ensure winning in any symmetric equilibrium. The rest is obvious. $\square$

Because continuity implies upper semicontinuity, Lemma A-2 implies that, in any symmetric equilibrium, P satisfies all the conditions in Lemma A-1. This enables us to apply Lemma A-1. Lemma A-1 implies that the optimal performance measure for type-t is always concentrated on points at which the type-t's upper support line, $\alpha_t + \beta_t x$, meets P. Because P is continuous, the set of points at which P meets the upper support line must be closed. Thus, given that supports of distributions are by definition closed, equation (1) must hold.

Define ψ as the *concave lower envelope* of the two upper support lines, $\{\alpha_t + \beta_t x\}_{t=S,W}$, associated with the two types of contestants, i.e.,

$$\psi(x) = \min[\alpha_S + \beta_S x,\ \alpha_W + \beta_W x]. \tag{A-10}$$

By Lemma A-1 and the definition of the concave lower envelope, ψ, ψ is increasing and

$$\forall t \in \{S,W\}, \quad \alpha_t + \beta_t x \geq \psi(x) \geq P(x). \tag{A-11}$$

Step 3: The third step in the proof is to establish the following technical result:

Result A-2. Define ψ as in (A-10). In any symmetric equilibrium, for any $a > 0$, if P is continuous over $[0, a)$ and if there exists $x' \in (0,a)$ such that $P(x') = \psi(x')$, it must be that $P(x) = \psi(x)$ for all $x \in [0,x']$.

Proof. We prove the result by way of contradiction. Let $Z = \{x \in [0,x'] : P(x) \neq \psi(x)\}$. Suppose, contrary to the result, that $Z \neq \varnothing$. Then Z must contain a point, say x_o. Let $x_1 = \min\{x \in (x_o,x'] : P(x) = \psi(x)\}$. Because ψ, given by (A-10), is continuous and because, by hypothesis, $P(x') = \psi(x')$ and P is continuous for $x \leq x'$, x_1 is well defined. By the definition of x_1 and equation (A-11), $P(x) < \psi(x)$ for all $x \in (x_o,x_1)$. Thus, by (A-11), $P(x) < \alpha_t + \beta_t x$, $t \in \{S,W\}$, for all $x \in (x_o,x_1)$. Hence, by Lemma A-1, no contestant places any weight on (x_o,x_1). Thus, $P(x) = P(x_o)$ for all $x \in (x_o,x_1)$. Thus, by continuity of P for $x \leq x'$, $P(x_o) = P(x_1)$. However, because ψ, given by (A-10), is increasing, $\psi(x_o) < \psi(x_1)$. Thus, given that $P(x_o) \leq \psi(x_o)$ and $P(x_o) = P(x_1)$, we must have $P(x_1) < \psi(x_1)$, which contradicts the definition of x_1. Thus, $Z = \varnothing$ and the result follows. $\square$

Step 4: The last step is to use Result A-2 to show that

$$P(x) = \min[\psi(x),\ 1], \quad x \geq 0, \tag{A-12}$$

which, given (A-10), will imply equation (5). Then Lemma 1 will follow immediately from equation (5) and the argument between equation (5) and Lemma 1 in the main text.

Thus, to complete the proof Lemma 1, it suffices to show equation (A-12). Let $\hat{x} = \max\{x \geq 0 : P(x) = \psi(x)\}$. Because (i) both P and ψ are continuous, (ii) P is bounded while ψ is unbounded, and (iii) there exists $x' \geq 0$ such that $P(x') = \psi(x')$, $\hat{x}$ must exist. By equation (A-11) and the definition of $\hat{x}$, $P(x) < \alpha_t + \beta_t x$, $t \in \{S, W\}$, for all $x > \hat{x}$. Thus, by Lemma A-1, no contestant places any weight over $(\hat{x}, \infty)$. Thus, given that no one places point mass in any symmetric equilibrium, a contestant ensures winning if his performance is no less than $\hat{x}$, i.e., $P(x) = 1$ for all $x \geq \hat{x}$. Thus, by the definition of $\hat{x}$, $P(\hat{x}) = \psi(\hat{x}) = 1$. Hence, by Result A-2 and continuity of P, $P(x) = \psi(x)$ for all $x \in [0, \hat{x}]$. Because ψ is increasing, the fact that $\psi(\hat{x}) = 1$ implies that $\min[\psi(x), 1] = \psi(x)$ for $x \in [0, \hat{x}]$ and $\min[\psi(x), 1] = 1$ for $x \geq \hat{x}$. Equation (A-12) thus follows. This completes the proof of Lemma 1. □

Proof of Proposition 1. The "only if" part and each type's equilibrium payoff are both established in the main text.

Below we establish the "if" part by constructing a symmetric equilibrium. Note that the performance distribution chosen by a contestant of unknown type is given by

$$F(x) = \theta F_S(x) + (1 - \theta) F_W(x). \tag{A-13}$$

In any symmetric equilibrium, no one places point mass. Thus, if a contestant has performance equal to x, his probability of besting any given rival of unknown type equals $F(x)$. To win a place, the contestant has to best at least $(n - m)$ out of his $(n - 1)$ rivals, whose types are unknown to him and whose performances are independent. Thus, in any symmetric equilibrium, a contestant's probability of winning function, P, has a relation to F given by

$$P(x) = \sum_{i=n-m}^{n-1} \binom{n-1}{i} F(x)^i (1 - F(x))^{n-1-i}. \tag{A-14}$$

The next lemma shows the existence of a concession equilibrium when $p_W^C \geq p_W^G$, and gives a construction of P and F_t, $t \in \{S, W\}$, in this concession equilibrium.

Lemma A-3. *When $p_W^C \geq p_W^G$, there exists a unique concession equilibrium. In this equilibrium, the probability of winning function, P, is given by*

$$P(x) = \begin{cases} \beta_W x, & x \in [0, \tilde{x}] \\ \alpha_S + \beta_S x, & x \in [\tilde{x}, \hat{x}] \\ 1, & x \geq \hat{x} \end{cases}, \tag{A-15}$$

where β_W, $\tilde{x}$, α_S, β_S, and $\hat{x}$ are determined by contest parameters as follows:

$$\beta_W = \frac{p_W^C}{\mu_W} \tag{A-16}$$

$$\tilde{x} = \frac{\tilde{p}\,\mu_W}{p_W^C} \tag{A-17}$$

$$\alpha_S = \frac{\tilde{p}\left(\mu_S - \left(p_S^C \mu_W / p_W^C\right)\right)}{\mu_S - (\tilde{p}\mu_W / p_W^C)} \tag{A-18}$$

$$\beta_S = \frac{p_S^C - \tilde{p}}{\mu_S - (\tilde{p}\mu_W / p_W^C)} \tag{A-19}$$

$$\hat{x} = \frac{(1-\tilde{p})\mu_S - (1-p_S^C)(\tilde{p}\mu_W / p_W^C)}{p_S^C - \tilde{p}} \tag{A-20}$$

with p_W^C given by (6)*, p_S^C determined by p_W^C through equation* (8)*, and $\tilde{p}$ given by*

$$\tilde{p} = \sum_{i=n-m}^{n-1} \binom{n-1}{i} (1-\theta)^i \theta^{n-1-i}. \tag{A-21}$$

The constants, β_W, α_S, and β_S, given by (A-16), (A-18), *and* (A-19)*, respectively, satisfy the following: if $p_W^C = p_W^G$, then $\alpha_S = 0$ and $\beta_W = \beta_S > 0$. If $p_W^C > p_W^G$, then $\alpha_S > 0$ and $\beta_W > \beta_S > 0$.*

Define

$$\phi(y) = \frac{1}{\beta_W} \sum_{i=n-m}^{n-1} \binom{n-1}{i} [(1-\theta)y]^i [1-(1-\theta)y]^{n-1-i}, \quad y \in [0,1] \tag{A-22}$$

$$\zeta(y) = \frac{1}{\beta_S} \left(\sum_{i=n-m}^{n-1} \binom{n-1}{i} [1-\theta+\theta y]^i [\theta(1-y)]^{n-1-i} - \alpha_S \right), \quad y \in [0,1]. \tag{A-23}$$

$\phi : [0,1] \to [0,\tilde{x}]$ and $\zeta : [0,1] \to [\tilde{x},\hat{x}]$ are both increasing, smooth, and continuous, where $\tilde{x}$ and $\hat{x}$ are given by (A-17) *and* (A-20)*, respectively. Thus, their inverse functions, $\phi^{-1} : [0,\tilde{x}] \to [0,1]$ and $\zeta^{-1} : [\tilde{x},\hat{x}] \to [0,1]$ exist. In the concession equilibrium,* $\mathrm{Supp}_W = [0,\tilde{x}]$ *and* $\mathrm{Supp}_F = [\tilde{x},\hat{x}]$*, and over the corresponding support, F_W and F_S are given by*

$$F_W(x) = \phi^{-1}(x), \quad x \in [0,\tilde{x}]; \qquad F_S(x) = \zeta^{-1}(x), \quad x \in [\tilde{x},\hat{x}]. \tag{A-24}$$

Proof. The proof consists of several steps.

Step 1: By Lemma 1 and the definition of a concession configuration, P must have the form given by (A-15) in a concession configuration.

Step 2: Now we show that the five constants, β_W, $\tilde{x}$, α_S, β_S, and $\hat{x}$, must satisfy equations (A-16)–(A-20) in a concession configuration. First, continuity of P, combined with (A-15), implies that

$$\beta_W \tilde{x} = \alpha_S + \beta_S \tilde{x} \tag{A-25}$$

$$\alpha_S + \beta_S \hat{x} = 1. \tag{A-26}$$

Next, by Lemma A-1, in a concession configuration, it must be that $p_t^C = \alpha_t + \beta_t \mu_t$, $t \in \{S, W\}$. Thus, given that $\alpha_W = 0$, we must have

$$\beta_W \, \mu_W = p_W^C \tag{A-27}$$

$$\alpha_S + \beta_S \, \mu_S = p_S^C. \tag{A-28}$$

Third, by equations (1) and (A-15) and Lemma 1, $\text{Supp}_W = [0, \tilde{x}]$ and $\text{Supp}_S = [\tilde{x}, \hat{x}]$. Thus, given that no one places point mass in any symmetric equilibrium, for a given contestant, if his performance equals $\tilde{x}$, he will outperform all weak rivals but be outperformed by all strong rivals. Given that each rival is strong with probability θ and rival types are independent, the given contestant's probability of winning by having performance equal to $\tilde{x}$ in a concession configuration equals $\tilde{p}$ given by (A-21). Thus, it must be that $P(\tilde{x}) = \tilde{p}$. Because, by (A-15), $P(\tilde{x}) = \beta_W \tilde{x}$, we must have

$$\beta_W \, \tilde{x} = \tilde{p}. \tag{A-29}$$

Equations (A-25)–(A-29) imply equations (A-16)–(A-20).

Step 3: Next, we show that the values of β_W, α_S, and β_S, given by (A-16), (A-18), and (A-19), respectively, satisfy that

$$p_W^C > (=) p_W^G \implies \alpha_S > (=) 0 \;\&\; \beta_W > (=) \beta_S > 0. \tag{A-30}$$

Note that, p_S^C is the probability of winning if a contestant always bests weak rivals and ties with strong rivals. $\tilde{p}$, given by (A-21), is the probability of winning if a contestant always bests weak rivals but is always beaten by strong rivals. p_W^C is the probability of winning if a contestant ties with weak rivals and is always beaten by strong rivals. It is thus clear that

$$p_S^C > \tilde{p} > p_W^C > 0. \tag{A-31}$$

Also note that, by identity (8), $p_W^C > (=) p_W^G$ implies that $p_S^C < (=) p_S^G$. Thus, given equation (10) that $p_W^G / p_S^G = \mu_W / \mu_S$, we must have

$$p_W^C > (=) \, p_W^G \implies p_W^C > (=) \, p_S^C \, \mu_W / \mu_S. \tag{A-32}$$

By (A-31), $p_S^C > \tilde{p}$. Thus, $p_W^C \geq p_S^C \, \mu_W / \mu_S$ implies that $p_W^C > \tilde{p} \, \mu_W / \mu_S$. Thus, by (A-32),

$$p_W^C \geq p_W^G \implies p_W^C > \tilde{p} \, \mu_W / \mu_S. \tag{A-33}$$

By (A-16) and (A-19),

$$\beta_W - \beta_S = \frac{p_W^C \mu_S - p_S^C \mu_W}{\mu_W \left(\mu_S - (\tilde{p} \mu_W / p_W^C) \right)}. \tag{A-34}$$

Suppose $p_W^C \geq p_W^G$. By (A-32), the numerators of the right-hand sides of (A-18) and (A-34) are both nonnegative and are zero if and only if $p_W^C = p_W^G$. By (A-33), the denominators of the right-hand sides of (A-19), (A-18), and (A-34) are positive. By (A-31), the numerator of the right-hand side of (A-19) is positive. These facts imply (A-30). (A-30) implies that, if $p_W^C \geq p_W^G$, then P, constructed in (A-15), is increasing over its support and weakly concave. Moreover, if $p_W^C = p_W^G$, we obtain the boundary case in which P satisfies the linearity condition for challenge configurations but weak contestants still concede to strong contestants.

Step 4: The above analysis shows that, when $p_W^C \geq p_W^G$, we can always construct, according to (A-15), a continuous, piecewise linear, weakly concave P that intersects the origin with β_W, $\tilde{x}$, α_S, β_S, and $\hat{x}$ given by equations (A-16)–(A-20), respectively. The final step is to show that F_W and F_S, constructed in (A-24), are CDFs that jointly produce such a P and satisfy their contest ability constraints.

First, note that, by construction, $\mathrm{Supp}_W = [0,\tilde{x}]$ and $\mathrm{Supp}_S = [\tilde{x},\hat{x}]$. Thus, by (A-13), $F(x) = (1-\theta)F_W(x)$ for $x \in [0,\tilde{x}]$, while $F(x) = 1-\theta+\theta F_S(x)$ for $x \in [\tilde{x},\hat{x}]$. Thus, if $\hat{P}$ represents the probability of winning function produced via equation (A-14) by the two CDFs constructed in (A-24), $\hat{P}$ satisfies that

$$\hat{P}(x) = \begin{cases} \beta_W\, \phi \circ F_W(x), & x \in [0,\tilde{x}] \\ \alpha_S + \beta_S\, \zeta \circ F_S(x), & x \in [\tilde{x},\hat{x}] \\ 1, & x \geq \hat{x} \end{cases},$$

where ϕ and ζ are given by (A-22) and (A-23), respectively. Because, by construction, $F_W(x) = \phi^{-1}(x)$ for $x \in [0,\tilde{x}]$ and $F_S(x) = \zeta^{-1}(x)$ for $x \in [\tilde{x},\hat{x}]$, it is clear that $\hat{P}$, jointly produced by F_W and F_S given in (A-24), equals the probability of winning function given by (A-15).

Next, we show that F_W, constructed in (A-24), satisfies W's contest ability constraint. Let $\hat{\mu}_W$ be the mean of F_W constructed in (A-24). Note that

$$\begin{aligned} \hat{\mu}_W &= \int_0^{\tilde{x}} x dF_W(x) = \int_0^1 F_W^{-1}(y)dy = \int_0^1 \phi(y)dy \\ &= \frac{1}{\beta_W}\int_0^1 \sum_{i=n-m}^{n-1} \binom{n-1}{i} [(1-\theta)y]^i [1-(1-\theta)y]^{n-1-i} dy, \end{aligned} \tag{A-35}$$

where the third equality follows from the construction of F_W in (A-24) and the last from (A-22). Also note that

$$\int_0^1 \sum_{i=n-m}^{n-1} \binom{n-1}{i} [(1-\theta)y]^i [1-(1-\theta)y]^{n-1-i} dy = p_W^C, \tag{A-36}$$

where p_W^C is given by (6). To see why (A-36) holds, note that the left-hand side of (A-36) is a weak contestant's probability of winning if he concedes to all strong rivals and both he and his weak rivals play a uniform performance distribution on [0, 1]. In this hypothetical contest, this weak contestant has no chance of winning if the number of strong rivals, denoted by $\tilde{S}_n^{-i}$, is no less than m. If $\tilde{S}_n^{-i} < m$, given that this weak contestant and all his weak rivals play the same strategy, after the $\tilde{S}_n^{-i}$ strong contestants each win a place, each of the $n-\tilde{S}_n^{-i}$ weak contestants will have the same chance of winning one of the remaining $m-\tilde{S}_n^{-i}$ places. Thus, if $\tilde{S}_n^{-i} < m$, this weak contestant's probability of winning equals $(m-\tilde{S}_n^{-i})/(n-\tilde{S}_n^{-i})$. Hence, by the definition of p_W^C given in (6), this weak contestant's probability of winning simply equals p_W^C. Equations (A-35) and (A-36) imply that $\hat{\mu}_W = p_W^C/\beta_W$. Thus, by (A-27), we must have $\hat{\mu}_W = \mu_W$ and, hence, the construction of F_W satisfies W's contest ability constraint.

Finally, to show that F_S, constructed in (A-24), satisfies S's contest ability constraint, we can follow the argument analogous to the one used above. We thus omit its proof. $\square$

Lemma A-3 shows the construction of the concession equilibrium when $p_W^C \geq p_W^G$. Now we turn to the case in which $p_W^C < p_W^G$. In the next lemma and its proof, we construct a challenge equilibrium when $p_W^C < p_W^G$.

Lemma A-4. *When $p_W^C < p_W^G$, there exist challenge equilibria. All of these challenge equilibria produce the same probability of winning function, P, given by*

$$P(x) = \min\left[\frac{m}{n\bar{\mu}}x,\, 1\right], \tag{A-37}$$

where $\bar{\mu} \equiv \theta\mu_S + (1-\theta)\mu_W$ is the expected contest ability of a contestant of unknown type.

There exist positive constants, x^o, ρ_W, and ρ_S, where $x^o < n\bar{\mu}/m$ and $\rho_W > \rho_S$, and distributions, F_W^o and F_S^o, with F_W^o supported by $[0, x^o]$ and F_S^o supported by $[x^o, n\bar{\mu}/m]$, such that, in one of these challenge equilibria, each weak contestant plays F_W^o with probability ρ_W and plays F_S^o with probability $1-\rho_W$ and each strong contestant plays F_W^o with probability ρ_S and plays F_S^o with probability $1-\rho_S$ (the construction of such a challenge equilibrium is provided in the proof).

Proof. First, we establish equation (A-37). By Lemma 1, in challenge configurations, $P(x) = \min[\beta x,\, 1]$. Given concavity of P, playing safe by concentrating all mass on μ_W is a best reply to P for a weak contestant. Thus, $P(\mu_W) = p_W^G$, where p_W^G is given by (11). Thus, $\beta = {}^{m}\!/_{(n\bar{\mu})}$ and equation (A-37) follows.

The next step is to find a pair of CDFs, F_S and F_W, that produce P constructed in (A-37) and satisfy the specific characterization given in Lemma A-4 and their contest ability constraints under the condition $p_W^C < p_W^G$. The argument next to the following lemma gives such a construction.

Result A-3. For fixed n, m, and θ, define $r^o \equiv p_S^C/p_W^C$, where p_W^C is given by (6) and p_S^C is determined by p_W^C via (8) and neither p_W^C nor p_S^C depends on r. Define $p_W^G(r)$ as in (11) and as a function of r. Then $p_W^C > (=)(<) p_W^G(r)$ if and only if $r > (=)(<) r^o$.

Proof. By equation (8),

$$p_W^C = p_W^G(r) \iff p_S^C = p_S^G(r) \iff p_S^C/p_W^C = p_S^G(r)/p_W^G(r),$$

where, by (10), $p_S^G(r)/p_W^G(r) = \mu_S/\mu_W = r$. Thus, $p_W^C = p_W^G(r)$ if and only if $p_S^C/p_W^C = r$. The result then follows from the fact that p_W^C is independent of r while p_W^G is decreasing in r. $\square$

Step 1: Identify the equilibrium performance distributions in the boundary case. Note that, given μ_S, μ_W, and $r^o \equiv p_S^C/p_W^C$, there uniquely exist μ_S^o and μ_W^o such that

$$\theta\mu_S^o + (1-\theta)\mu_W^o = \theta\mu_S + (1-\theta)\mu_W \equiv \bar{\mu} \tag{A-38}$$

$$\mu_S^o/\mu_W^o = r^o. \tag{A-39}$$

By construction and the fact that $\mu_W < \mu_S$,

$$r = \frac{\mu_S}{\mu_W} < r^o \quad \Longrightarrow \quad \mu_W^o < \mu_W < \mu_S < \mu_S^o. \tag{A-40}$$

Now let F_S^o and F_W^o be the equilibrium performance distributions played in the case where the strong and the weak type's contest abilities equal μ_S^o and μ_W^o, respectively. Because, by construction, $\mu_S^o/\mu_W^o = r^o$, and because, by Result A-3, $p_W^C = p_W^G(r^o)$, this case is a boundary case, in which F_S^o and F_W^o jointly produce a uniform probability of winning function while F_S^o and F_W^o have adjacent supports and, hence, represent a concession configuration. The construction of equilibrium in this boundary case follows from Lemma A-3, which shows the construction of a concession equilibrium. By Lemma A-3, in this boundary case, there exists a unique pair of CDFs, F_S^o and F_W^o, such that the mean of F_S^o equals μ_S^o, the mean of F_W^o equals μ_W^o, the lower endpoint of the support of F_S^o equals the upper endpoint of the support of F_W^o, denoted by x^o, F_S^o and F_W^o jointly produce a uniform probability of winning function through equations (A-13) and (A-14), and the upper endpoint of the support of this probability of winning function, given by (A-20), satisfies

$$\hat{x} = \frac{(1-\tilde{p})\mu_S^o - (1-p_S^C)(\tilde{p}\mu_W^o/p_W^C)}{p_S^C - \tilde{p}}.$$

By (A-39) and the definition of r^o, $p_S^C/p_W^C = r^o = \mu_S^o/\mu_W^o$. We can thus simplify the expression for $\hat{x}$ into $\hat{x} = \mu_S^o/p_S^C$. By the definition of r^o, Result A-3, and equation (8), $p_S^C = p_S^G(r^o)$. Thus, $\hat{x} = \mu_S^o/p_S^G(r^o) = n\bar{\mu}/m$, where the last equality follows from (8) and (11). Thus, by construction, the upper endpoint of the support of the probability of winning function produced by F_W^o and F_S^o equals $n\bar{\mu}/m$. Hence, the probability of winning function produced by F_W^o and F_S^o equals P given by (A-37).

Step 2: Mix the boundary equilibrium performance distributions. By Result A-3, if $p_W^C < p_W^G(r)$, it must be that $r < r^o$. In this case, by (A-40), there must exist $\rho_W \in (0,1)$ and $\rho_S \in (0,1)$ such that

$$\rho_W \mu_W^o + (1-\rho_W)\mu_S^o = \mu_W \quad \& \quad \rho_S \mu_W^o + (1-\rho_S)\mu_S^o = \mu_S. \tag{A-41}$$

Hence,

$$\rho_W = \frac{\mu_S^o - \mu_W}{\mu_S^o - \mu_W^o} \quad \& \quad \rho_S = \frac{\mu_S^o - \mu_S}{\mu_S^o - \mu_W^o}. \tag{A-42}$$

Equations (A-38) and (A-41) imply that ρ_W and ρ_S satisfy that

$$\theta\rho_S + (1-\theta)\rho_W = 1-\theta. \tag{A-43}$$

Now we argue that, when $p_W^C < p_W^G(r)$, there exists a challenge equilibrium in which strong and weak contestants play as follows:

$$\text{S-strategy} = \begin{cases} F_W^o & \text{w. p. } \rho_S \\ F_S^o & \text{w. p. } 1-\rho_S \end{cases} \quad \text{W-strategy} = \begin{cases} F_W^o & \text{w. p. } \rho_W \\ F_S^o & \text{w. p. } 1-\rho_W \end{cases}, \tag{A-44}$$

where ρ_W and ρ_S are given in (A-42). Note that, by (A-41), the prescribed strategies satisfy each type's contest ability constraint. Also, by construction, a contestant of unknown type will play F_W^o with probability $\theta\rho_S + (1-\theta)\rho_W = 1-\theta$, where the equality follows from (A-43), and play F_S^o with the complementary probability, θ. Thus, by construction, the performance distribution played by a contestant of unknown type equals the one in the boundary case, i.e., $\theta F_S + (1-\theta)F_W = \theta F_S^o + (1-\theta)F_W^o$. Because, by (A-14), the shape of P only depends on the performance distribution played by a contestant of unknown type, and because $\theta F_S^o + (1-\theta)F_W^o$ produces a uniform P given by (A-37), it must be that $\theta F_S + (1-\theta)F_W$ also produces such a uniform P. Thus, the prescribed strategies constitute a challenge equilibrium. □

Lemmas A-3 and A-4 establish the "if" part. This completes the proof of the proposition. □

Proof of Lemma 2. To establish the first part, note that equation (6) implies that

$$p_W^C = \mathbb{E}\left[\frac{m-\tilde{S}_n^{-i}}{n-\tilde{S}_n^{-i}}\Big|\tilde{S}_n^{-i} \le m-1\right]\mathbb{P}[\tilde{S}_n^{-i} \le m-1] \le \frac{m}{n}\mathbb{P}[\tilde{S}_n^{-i} \le m-1]. \tag{A-45}$$

Because $r = {}^{\mu_S}/_{\mu_W} > 1$, equation (11) implies that

$$p_W^G = \frac{m}{n}\frac{1}{r\theta + (1-\theta)} > \frac{m}{n}\frac{1}{r}. \tag{A-46}$$

By Proposition 1, challenge configurations will be played in equilibrium if and only if $p_W^G > p_W^C$. Thus, by (A-45) and (A-46), challenge configurations will be played whenever

$$\mathbb{P}[\tilde{S}_n^{-i} \le m-1] \le {}^{1}/_{r}. \tag{A-47}$$

Everything else being equal, $\mathbb{P}[\tilde{S}_n^{-i} \le m-1] \to 0$ as $n \to \infty$, and ${}^{1}/_{r} \to 1$ as $r \to 1$. Thus, by (A-47) and by the fact that $\mathbb{P}[\tilde{S}_n^{-i} \le m-1] < 1$, challenge configurations will be played if either n is sufficiently large or r is sufficiently close to 1. This establishes the first part of the lemma.

To establish the second part, let $\tilde{S}_n \sim \text{Binom}(n,\theta)$. Note that, by (6), we can rewrite p_W^C as

$$\begin{aligned} p_W^C &= \sum_{s=0}^{n-1} \max\left[0, \frac{m-s}{n-s}\right]\binom{n-1}{s}\theta^s(1-\theta)^{n-1-s} \\ &= \frac{1}{n(1-\theta)}\sum_{s=0}^{n}\max[0, m-s]\binom{n}{s}\theta^s(1-\theta)^{n-s} = \frac{1}{n(1-\theta)}\mathbb{E}\left[\max\left[0, m-\tilde{S}_n\right]\right], \end{aligned} \tag{A-48}$$

where the second equality follows from the binomial coefficient identity, $\binom{n-1}{s} = \left(\frac{n-s}{n}\right)\binom{n}{s}$, and the fact that $\max[0, m-n] = 0$. Thus, by equations (11) and (A-48),

$$\frac{p_W^C}{p_W^G} = \mathbb{E}\left[\max\left[1-\frac{\tilde{S}_n}{m}, 0\right]\right]\left(\frac{r\theta + 1 - \theta}{1-\theta}\right). \tag{A-49}$$

Equation (A-49) allows us to evaluate the effect of a parameter change on equilibrium configurations. First consider a change of n. Note that $s \hookrightarrow \max[1-(s/m), 0]$ is nonincreasing. Also note that, when n increases, the distribution of $\tilde{S}_n$ after the increase stochastically dominates the one before the increase. Thus, $n \hookrightarrow \mathbb{E}\left[\max\left[1-\frac{\tilde{S}_n}{m}, 0\right]\right]$ is nonincreasing. Hence, by equation (A-49), $n \hookrightarrow p_W^C(n)/p_W^G(n)$ must be nonincreasing. Thus, by Proposition 1, an increase in n favors the play of challenge configurations.

Next, consider a change of m. Note that, for any fixed s, $m \hookrightarrow \max[1-(s/m), 0]$ is nondecreasing. Because a change in m does not change the distribution of $\tilde{S}_n$, $m \hookrightarrow \mathbb{E}\left[\max\left[1-\frac{\tilde{S}_n}{m}, 0\right]\right]$ must be nondecreasing. Hence, by equation (A-49), $m \hookrightarrow p_W^C(m)/p_W^G(m)$ must be nondecreasing. Thus, by Proposition 1, a decrease in m favors the play of challenge configurations.

Finally, consider a change of r. Because p_W^C is independent of r while $r \hookrightarrow p_W^G(r)$ is decreasing, it is clear that a decrease in r increases $p_W^G(r)$ relative to p_W^C. Thus, by Proposition 1, a decrease in r favors the play of challenge configurations. □

Proof of Lemma 3. Follows immediately from Proposition 1 and the fact that only concession equilibria implement merit-based selection. □

Proof of Theorem 1. Established by the argument in the main text before Theorem 1. □

Proof of Lemma 4. Let $\tilde{S}_n$ be the random number of strong contestants in a pool of n contestants. Note that $\tilde{S}_n \sim \text{Binom}(n, \theta)$. Consider the effect of reducing the quota by one from $m+1$ to m under merit-based selection. If there are at least $m+1$ strong contestants in the pool, which happens with probability $\mathbb{P}\left[\tilde{S}_n > m\right] = 1 - \mathbb{P}\left[\tilde{S}_n \leq m\right]$, then reducing the quota will lower designer welfare by 1, because, under merit-based selection, a strong contestant would have been selected had the quota equaled $m+1$. If there are less than $m+1$ strong contestants, which happens with probability $\mathbb{P}\left[\tilde{S}_n \leq m\right]$, lowering the quota will increase designer welfare by σ, because, had the quota equaled $m+1$, the designer would have been forced to fill the $(m+1)$-th place with a weak contestant. Thus, the marginal gain from reducing the quota from $m+1$ to m is given by

$$\Delta(m) = \mathbb{P}\left[\tilde{S}_n \leq m\right] - \left(1 - \mathbb{P}\left[\tilde{S}_n \leq m\right]\right) = 2B(m; n, \theta) - 1, \tag{A-50}$$

where the last equality follows from the fact that $\tilde{S}_n \sim \text{Binom}(n, \theta)$.

Thus, when $\Delta(m) > 0$, it is strictly optimal to reduce the quota from $m+1$ to m. Note that Δ is increasing in m. Thus, m_M^*, defined in Lemma 4, represents the smallest $m \geq 0$ at which $\Delta(m) > 0$. Hence, it is strictly optimal to reduce the quota from above m_M^* to m_M^* and any further reduction in the quota will not strictly increase designer welfare. In the non-generic case where $\Delta(m_M^* - 1) = 0$, both m_M^* and $m_M^* - 1$ are optimal under merit-based selection. Because, by assumption, when the designer is indifferent between two quotas, she chooses the larger quota, the designer chooses m_M^* in the non-generic case. The lemma thus follows. □

Proof of Theorem 2. We first prove part (i). Because concession configurations implement merit-based selection, if a concession configuration is played at $m = m_M^*$, it must be that $u(m_M^*) = u_M(m_M^*)$. Because $u(m) \leq u_M(m_M^*)$, it is then optimal to choose $m = m_M^*$. Thus, $u(m^*) = u(m_M^*) = u_M(m_M^*)$. This establishes the "if" part. To establish the "only if" part, suppose that challenge configurations are played at $m = m_M^*$. Then by Lemma 2, challenge configurations will also be played at $m < m_M^*$. Note that only m_M^* and (in the non-generic case) also $m_M^* - 1$ (see the discussion in the proof of Lemma 4) can be optimal under merit-based selection. Thus, if m^* equals either m_M^* or $m_M^* - 1$, given that challenge configurations are played at $m = m_M^*$ and, *a fortiori*, played at $m = m_M^* - 1$, Lemma 3 implies that $u(m^*) < u_M(m^*) \leq u_M(m_M^*)$. If m^* differs from m_M^* and $m_M^* - 1$ and, thus, differs from any optimal quota under merit-based selection, by Lemma 3, it must be that $u(m^*) \leq u_M(m^*) < u_M(m_M^*)$. Thus, in both cases, $u(m^*) < u_M(m_M^*)$. This establishes the "only if" part.

Next, we prove part (ii). Note that winner quality, $\Pi(m,n)$, has the following upper bound:

$$\Pi(m,n) \leq 1 - \frac{n}{m}(1-\theta)p_W^G = \frac{\theta r}{\theta r + 1 - \theta},$$

where the inequality follows from (15) and the equality from (11). Thus, by (14), designer welfare in the risk-taking contest, u, has the following upper bound:

$$u(m,n) \leq m\left(\frac{2\theta r}{\theta r + 1 - \theta} - 1\right) = m\left(\frac{\theta r - 1 + \theta}{\theta r + 1 - \theta}\right). \tag{A-51}$$

The last expression is nonpositive if $r \leq {}^{(1-\theta)}/_{\theta}$. Thus, if $r \leq {}^{(1-\theta)}/_{\theta}$, $u(m,n) \leq 0$ and, clearly, this zero upper bound will be attained by choosing $m = 0$. This establishes part (ii).

Finally, consider part (iii). If a concession configuration is played at m_M^*, then by part (i), which has been proved, it is optimal to choose $m = m_M^*$. Now suppose that challenge configurations are played at $m = m_M^*$ and that $r > {}^{(1-\theta)}/_{\theta}$. Note that, in challenge configurations, designer welfare, u, is given by the last expression in (A-51), which, given the hypothesis that $r > {}^{(1-\theta)}/_{\theta}$, is increasing in m. By Lemma 2, challenge configurations are played at any $0 < m \leq \bar{m}$, where $\bar{m}$ is the largest quota at which challenge configurations are played. Thus, choosing $m = \bar{m}$ strictly dominates choosing $m < \bar{m}$. By the definition of $\bar{m}$, a concession configuration will be played at $m > \bar{m}$. Concession configurations implement merit-based selection. Thus, by (A-50), the marginal gain from increasing the quota from $m > \bar{m}$ to $m+1$ is $1 - 2B(m;n,\theta)$. By the definition of m_M^* given in Lemma 4, $1 - 2B(m;n,\theta) < 0$ for $m > m_M^*$. Thus, given that $\bar{m} \geq m_M^*$, the marginal gain from increasing the quota from $m > \bar{m}$ to $m+1$ is negative. Thus, choosing $m = \bar{m}+1$ strictly dominates choosing $m > \bar{m}+1$. Hence, if challenge configurations are played at $m = m_M^*$ and $r > {}^{(1-\theta)}/_{\theta}$, either $m = \bar{m}$ or $m = \bar{m}+1$ is optimal. This completes the proof of the theorem. □

Proof of Proposition 2. Throughout, we fix n and θ. Thus, the merit-based optimal quota, m_M^*, defined in Lemma 4, is fixed. Let $m^*(r)$ be the optimal contest selection quota conditional on r. Define $\bar{m}(r)$ as in (18) and as a function of r. Note that $\bar{m}(r)$ represents the largest quota under which challenge configurations are played. Let $u(m,r)$ be designer welfare under risk-taking contest selection when the quota is m and the strength asymmetry is r. Let $u_M(m)$ be designer welfare under merit-based selection when the quota is m. Note that u_M does not depend on r. Let $p_W^G(m,r)$ be a weak contestant's probability of winning under challenge configurations, given by (11), when the quota is m and the strength asymmetry is r. Let $p_W^C(m)$ be a weak contestant's probability of winning under concession configurations, given by (6), when the quota is m. Note that p_W^C does not depend on r.

First, we show that $m^*(r) = m_M^*$ for r sufficiently large. By Result A-3, for any fixed m, $p_W^C \geq p_W^G(r)$ for r sufficiently large. Thus, by Proposition 1, for r sufficiently large, a concession configuration will be played at $m = m_M^*$. Thus, by Theorem 2, $m^*(r) = m_M^*$ for r sufficiently large.

Now we prove that m^* is nonincreasing in r by way of contradiction. Suppose, to the contrary, that there exist $r'' > r' > {}^{(1-\theta)}/_{\theta}$ such that $m^*(r'') > m^*(r')$. Then by Theorem 2 and the hypothesis that $r' > {}^{(1-\theta)}/_{\theta}$, $m^*(r') \geq m_M^*$. Thus, the hypothesis that $m^*(r'') > m^*(r')$ implies that $m^*(r'') > m_M^*$. Thus, by Theorem 2, it must be that, under r'', challenge configurations are played at $m = m_M^*$. Thus, by the definition of $\bar{m}$, $\bar{m}(r'') \geq m_M^*$.

Next, note that the definition of $\bar{m}$ given by (18), combined with the facts that p_W^C is independent of r whereas p_W^G is decreasing in r and the hypothesis that $r'' > r'$, implies that $\bar{m}(r') \geq \bar{m}(r'')$. Thus, given that $\bar{m}(r'') \geq m_M^*$, it must be that $\bar{m}(r') \geq \bar{m}(r'') \geq m_M^*$. Thus, Theorem 2 and the hypotheses that $r'' > r' > {}^{(1-\theta)}/_{\theta}$ and $m^*(r'') > m^*(r')$ imply that

$$\bar{m}(r'') = \bar{m}(r') \tag{A-52}$$

$$m^*(r'') = \bar{m}(r'') + 1 \tag{A-53}$$

$$m^*(r') = \bar{m}(r'). \tag{A-54}$$

Thus,

$$\begin{aligned} u(\bar{m}(r'), r'') \leq u(m^*(r''), r'') = u(\bar{m}(r'') + 1, r'') = u_M(\bar{m}(r'') + 1) \\ = u_M(\bar{m}(r') + 1) = u(\bar{m}(r') + 1, r') \leq u(m^*(r'), r') = u(\bar{m}(r'), r'), \end{aligned} \tag{A-55}$$

where in the first line, the inequality follows from the fact that $m^*(r'')$ is the optimal contest selection quota when $r = r''$, the first equality from (A-53), and the last from the fact that, by the definition of $\bar{m}(r'')$, a concession configuration is played under $(m = \bar{m}(r'') + 1,\ r = r'')$, which implements merit-based selection at $m = \bar{m}(r'') + 1$. In the second line, the first equality follows from (A-52), the second equality from the fact that, by the definition of $\bar{m}(r')$, a concession configuration is played under $(m = \bar{m}(r') + 1,\ r = r')$, which implements merit-based selection at $m = \bar{m}(r') + 1$, the inequality from the fact that $m^*(r')$ is the optimal contest selection quota when $r = r'$, and the last equality from (A-54).

The first and the last term in (A-55), combined with the inequalities between the two terms, imply that $u(\bar{m}(r'), r'') \leq u(\bar{m}(r'), r')$, which, by (14) and (15), further implies that

$$\max\left[p_W^C(\bar{m}(r')),\, p_W^G(\bar{m}(r'), r'')\right] \geq \max\left[p_W^C(\bar{m}(r')),\, p_W^G(\bar{m}(r'), r')\right]. \tag{A-56}$$

Equation (11) and the hypothesis that $r'' > r'$ imply that $p_W^G(\bar{m}(r'), r'') < p_W^G(\bar{m}(r'), r')$. This result, combined with (A-56), implies that $p_W^C(\bar{m}(r')) \geq p_W^G(\bar{m}(r'), r')$. However, the definition of $\bar{m}(r')$ implies that $p_W^C(\bar{m}(r')) < p_W^G(\bar{m}(r'), r')$, a contradiction. The proposition then follows.

□

Proof of Lemma 5. In the challenge equilibrium constructed in Lemma A-4, each contestant plays F_S^o with some probability and plays F_W^o with the complementary probability, where the lower endpoint of the support of F_S^o equals the upper endpoint of the support of F_W^o. Because, by construction, strong contestants play F_W^o with a lower probability and play F_S^o with a higher probability compared to weak contestants, the constructed challenge equilibrium clearly satisfies the MLRP. □

Proof of Corollary 1. Established by the argument in the main text before Corollary 1. □

Proof of Proposition 3. In a symmetric equilibrium, at the effort stage, every weak contestant chooses the same contest ability μ_W and every strong contestant chooses the same contest ability μ_S. Note that, in any symmetric equilibrium, it must be that $\mu_W > 0$ and $\mu_S > 0$. This is because choosing zero contest ability would imply placing point mass on 0. By the same argument used in the proof of Lemma A-2 for showing the continuity of P and by the fact that the cost of choosing $\varepsilon > 0$ contest ability can be made arbitrarily small by shrinking ε to zero, it is clear that placing point mass on 0 cannot be sustained in a symmetric equilibrium.

Given $\mu_W > 0$ and $\mu_S > 0$, weak and strong contestants' performance distributions, F_W and F_S, and the probability of winning function, P, at the risk-taking stage are characterized by Lemmas A-3 and A-4. Let $P(\cdot;\mu_W,\mu_S)$ be the probability of winning function at the risk-taking stage when, at the effort stage, weak contestants choose μ_W and strong contestants choose μ_S. By Lemma 1, in any symmetric equilibrium, P is concave.[26] Because taking no risk is a best reply to a concave P, by choosing contest ability μ, a contestant's probability of winning is given by $P(\mu;\mu_W,\mu_S)$. In a symmetric equilibrium, it must be that a type-$t \in \{S,W\}$ contestant's expected payoff, $P(\mu;\mu_W,\mu_S) - (\mu^\alpha/a_t)$ is maximized at $\mu = \mu_t$. Because P is concave while the cost functions are strictly convex and because both P and the cost functions are continuous, μ_W and μ_S are best replies to $P(\cdot;\mu_W,\mu_S)$ if and only if they satisfy the first-order conditions:

$$P'(\mu_W;\mu_W,\mu_S) = \frac{\alpha\mu_W^{\alpha-1}}{a_W} \quad \& \quad P'(\mu_S;\mu_W,\mu_S) = \frac{\alpha\mu_S^{\alpha-1}}{a_S}. \tag{A-57}$$

We first argue that, in any symmetric equilibrium, it must be that $\mu_S > \mu_W$. This is because, if, to the contrary, $\mu_S \le \mu_W$, the fact that $a_W < a_S$ would imply that $\alpha\mu_W^{\alpha-1}/a_w > \alpha\mu_S^{\alpha-1}/a_S$. Thus, by (A-57), it would have to be that $P'(\mu_W;\mu_W,\mu_S) > P'(\mu_S;\mu_W,\mu_S)$, which, given concavity of P, could only happen if $\mu_W < \mu_S$, contradicting the hypothesis that $\mu_S \le \mu_W$.

Next, by Proposition 1, either concession or challenge equilibria exist for any subgame starting from the risk-taking stage with contestants of the same type having the same contest ability and with $\mu_S > \mu_W > 0$. In challenge configurations, by Lemma A-4, $P'(x) = m/(n\bar{\mu})$ over $[0, n\bar{\mu}/m]$, the support of P, where $\bar{\mu} = \theta\mu_S + (1-\theta)\mu_W$. Thus, by (A-57), in challenge

[26] Concavity of P holds even if $\mu_S \le \mu_W$, because if $\mu_S < \mu_W$, we can simply treat the high-ability type as the weak type and the low-ability type as the strong type at the risk-taking stage and all the arguments used in the proof of Lemma 1 apply. If $\mu_S = \mu_W$, one can treat every contestant as of the same type at the risk-taking stage by treating either $\theta = 0$ or $\theta = 1$. The proof of Lemma 1 does not rely on the value of θ. Thus, concavity of P still holds under $\mu_S = \mu_W$.

configurations, it must be that

$$\frac{m}{n\bar{\mu}} = \frac{\alpha\mu_t^{\alpha-1}}{a_t}, \quad t \in \{S, W\}, \tag{A-58}$$

which implies that

$$r = \frac{\mu_S}{\mu_W} = \left(\frac{a_S}{a_W}\right)^{\frac{1}{\alpha-1}} \equiv r^*. \tag{A-59}$$

By Proposition 1, equation (A-59), and the fact that p_W^C is independent of r, challenge equilibria exist only if $p_W^C < p_W^G(r^*)$.

To show that $p_W^C < p_W^G(r^*)$ is sufficient for the existence of challenge equilibria, simply note that, when each type-$t \in \{S, W\}$ contestant chooses μ_t according to (A-58), in the subgame starting from the risk-taking stage, $\mu_S/\mu_W = r^*$. Given that $p_W^C < p_W^G(r^*)$, by Lemma A-4, this subgame produces a uniform P with $P' = m/(n\bar{\mu})$ over the support of P. Thus, the choice of μ_t given by (A-58) satisfies the first-order condition, which, by the argument right before equation (A-57), ensures that the choice of μ_t given by (A-58) is a best reply to such a uniform P for any type-t contestant.

Now we show that there are no concession equilibria if $p_W^C < p_W^G(r^*)$. Suppose, to the contrary, that a concession equilibrium exists given $p_W^C < p_W^G(r^*)$. By Lemma A-3, in any concession equilibrium, the marginal benefit of contest ability is weakly higher for weak contestants than for strong contestants, i.e., $\beta_W \geq \beta_S$. By the first-order conditions,

$$\frac{\alpha\mu_W^{\alpha-1}}{a_W} = \beta_W \quad \& \quad \frac{\alpha\mu_S^{\alpha-1}}{a_S} = \beta_S, \tag{A-60}$$

which implies, given $\beta_W \geq \beta_S$, that

$$r = \frac{\mu_S}{\mu_W} = \left(\frac{a_S\beta_S}{a_W\beta_W}\right)^{\frac{1}{\alpha-1}} \leq \left(\frac{a_S}{a_W}\right)^{\frac{1}{\alpha-1}} = r^*. \tag{A-61}$$

Because $r \hookrightarrow p_W^G(r)$ is decreasing, the hypothesis $p_W^C < p_W^G(r^*)$ and equation (A-61) imply that $p_W^C < p_W^G(r)$. Thus, by Proposition 1, at the risk-taking stage, only challenge configurations can sustain an equilibrium, a contradiction. This contradiction implies that concession equilibria exist only if $p_W^C \geq p_W^G(r^*)$.

Finally, we show that there exists a concession equilibrium if $p_W^C \geq p_W^G(r^*)$. Suppose that $p_W^C \geq p_W^G(r^*)$. Define r^o as in Result A-3, i.e., $r^o = p_S^C/p_W^C$. By Proposition 1 and Result A-3, to show that $p_W^C \geq p_W^G(r^*)$ is sufficient for the existence of a concession equilibrium, it suffices to verify that there exists a pair, (μ_S, μ_W), such that $\mu_S/\mu_W \geq r^o$ and a type-$t \in \{S, W\}$ contestant's choice of μ_t in the effort stage is a best reply to the probability of winning function, $P(\cdot; \mu_W, \mu_S)$, produced by μ_S and μ_W according to the concession equilibrium construction given in Lemma A-3.

Note that the optimal dual variables, β_W and β_S, in concession equilibria are given by (A-16) and (A-19), respectively. Thus, by (A-60), the choices of μ_W and μ_S are best replies to $P(\cdot; \mu_W, \mu_S)$ constructed in Lemma A-3 for W and S, respectively, if and only if they satisfy the following first-order conditions:

$$\frac{\alpha\mu_W^{\alpha-1}}{a_W} = \frac{p_W^C}{\mu_W} \tag{A-62}$$

$$\frac{\alpha\mu_S^{\alpha-1}}{a_S} = \frac{p_S^C - \tilde{p}}{\mu_S - (\tilde{p}\mu_W/p_W^C)}, \tag{A-63}$$

where p_W^C and $\tilde{p}$ are given by (6) and (A-21), respectively, and p_S^C is determined by p_W^C through (8).

Now we show that, when $p_W^C \geq p_W^G(r^*)$, (μ_S, μ_W) that solves (A-62) and (A-63) satisfies that $\mu_S/\mu_W \geq r^o$. Note that the value of $\mu_W > 0$ that satisfies (A-62) is uniquely given by

$$\mu_W = \mu_W' \equiv \left(\frac{a_W p_W^C}{\alpha}\right)^{\frac{1}{\alpha}} > 0. \tag{A-64}$$

Define

$$\mathscr{K}(\mu_S) = \frac{\alpha\mu_S^{\alpha-1}}{a_S} - \frac{p_S^C - \tilde{p}}{\mu_S - (\tilde{p}\mu_W'/p_W^C)}. \tag{A-65}$$

By (A-31), $p_S^C > \tilde{p}$. Thus, given that p_S^C, p_W^C, and $\tilde{p}$ do not depend on μ_S, $\mu_S \hookrightarrow \mathscr{K}(\mu_S)$ is increasing. Note that, when $\mu_S = \mu_W' r^o$, where μ_W' is given by (A-64), we have

$$\begin{aligned}\mathscr{K}(\mu_S = \mu_W' r^o) &= \frac{\alpha(\mu_W' r^o)^{\alpha-1}}{a_S} - \frac{p_S^C - \tilde{p}}{\mu_W' r^o - (\tilde{p}\mu_W'/p_W^C)} \\ &= \frac{p_W^C}{\mu_W'}\left(\frac{(r^o)^{\alpha-1}a_W}{a_S} - \frac{p_S^C - \tilde{p}}{p_W^C r^o - \tilde{p}}\right),\end{aligned} \tag{A-66}$$

where the second line follows from substituting $\alpha(\mu_W')^{\alpha-1}$ using $\alpha(\mu_W')^{\alpha-1} = a_W p_W^C/\mu_W'$ implied by (A-62) and collecting the common factor in the resulting expression. By Result A-3 and the hypothesis that $p_W^C \geq p_W^G(r^*)$, we have $r^* \geq r^o$. Thus, given that, by definition, $r^* = (a_S/a_W)^{\frac{1}{\alpha-1}}$, we must have

$$\frac{(r^o)^{\alpha-1}a_W}{a_S} \leq 1. \tag{A-67}$$

Because, by definition, $r^o = p_S^C/p_W^C$, we must also have

$$\frac{p_S^C - \tilde{p}}{p_W^C r^o - \tilde{p}} = 1. \tag{A-68}$$

Equations (A-66), (A-67), and (A-68) imply that $\mathscr{K}(\mu_W' r^o) \leq 0$. It is obvious that $\mathscr{K}(\mu_S) \to \infty$ as $\mu_S \to \infty$. Thus, given that $\mu_S \hookrightarrow \mathscr{K}(\mu_S)$ is increasing, there exists a unique $\mu_S' \geq \mu_W' r^o$ such that $\mathscr{K}(\mu_S = \mu_S') = 0$. Thus, there exists a pair $(\mu_S = \mu_S', \mu_W = \mu_W')$ that solves (A-62) and (A-63) simultaneously and satisfies that $\mu_S/\mu_W \geq r^o$. The satisfaction of (A-62) and (A-63) implies that choosing $\mu_t = \mu_t'$ is a best reply to $P(\cdot; \mu_S = \mu_S', \mu_W = \mu_W')$. By Proposition 1 and Result A-3, the fact that $\mu_S'/\mu_W' \geq r^o$ implies the play of a concession configuration when $\mu_S = \mu_S'$ and $\mu_W = \mu_W'$. Thus, there exists a concession equilibrium if $p_W^C \geq p_W^G(r^*)$. □

Proof of Proposition 4. Let θ (θ') be the probability of being strong for every internal (external) candidate, where $\theta > \theta'$. Consider adding $n' > 0$ external candidates to the contest with $n > m$ internal candidates. Throughout, suppose that the contest with only the n internal candidates produces challenge equilibria.

We show that, in any symmetric equilibrium of the expanded contest ("symmetric" in the sense that every type-$t \in \{S, W\}$ internal candidate plays the same strategy and every type-$t \in \{S, W\}$ external candidate plays the same strategy), the expanded contest has lower winner quality than the contest with only the n internal candidates.

Consider the expanded contest. Let $\hat{p}_t$ and $\hat{p}'_t$ be the equilibrium probability of winning for a type-t internal candidate and for a type-t external candidate, respectively, in the expanded contest, $t \in \{S, W\}$. Because a weak internal candidate always has the option of mimicking a strong internal candidate's strategy with probability μ_W/μ_S and choosing zero performance with the complementary probability, it must be that

$$\hat{p}_W \geq \frac{\mu_W}{\mu_S}\hat{p}_S = \frac{\hat{p}_S}{r}. \tag{A-69}$$

Analogously,

$$\hat{p}'_W \geq \frac{\mu_W}{\mu_S}\hat{p}'_S = \frac{\hat{p}'_S}{r}. \tag{A-70}$$

Let $\hat{\Pi}(n, n')$ be winner quality in the expanded contest. Note that

$$\begin{aligned}\hat{\Pi}(n,n') &= \frac{n\theta\hat{p}_S + n'\theta'\hat{p}'_S}{n\theta\hat{p}_S + n'\theta'\hat{p}'_S + n(1-\theta)\hat{p}_W + n'(1-\theta')\hat{p}'_W} \\ &\leq \frac{n\theta\hat{p}_W r + n'\theta'\hat{p}'_W r}{n\theta\hat{p}_W r + n'\theta'\hat{p}'_W r + n(1-\theta)\hat{p}_W + n'(1-\theta')\hat{p}'_W}, \end{aligned} \tag{A-71}$$

where the first line follows from the fact that winner quality equals the expected number of strong winners divided by the sum of expected number of strong winners and the expected number of weak winners, and the second line follows from (A-69), (A-70), and the fact that, for any fixed $b > 0$, $f(a) = a/(a+b)$ is increasing in a for $a > 0$. Let $\Pi(n)$ be winner quality in the contest with only the n internal candidates. If the contest with only the internal candidates has challenge equilibria, $\Pi(n)$ is given by (16). By (16) and (A-71), for any $\theta' < \theta$ and $n, n' > 0$,

$$\hat{\Pi}(n,n') - \Pi(n) \leq \frac{n\theta\hat{p}_W r + n'\theta'\hat{p}'_W r}{n\theta\hat{p}_W r + n'\theta'\hat{p}'_W r + n(1-\theta)\hat{p}_W + n'(1-\theta')\hat{p}'_W} - \frac{r\theta}{r\theta + 1 - \theta} =$$
$$\frac{n'\hat{p}'_W r}{\left(n\theta\hat{p}_W r + n'\theta'\hat{p}'_W r + n(1-\theta)\hat{p}_W + n'(1-\theta')\hat{p}'_W\right)\left(r\theta + 1 - \theta\right)}(\theta' - \theta) < 0.$$

The result then follows immediately from the fact that, fixing m, designer welfare is maximized by maximizing winner quality. □

Proof of Proposition 5. We first prove part (i). Let $\bar{x} \geq \mu_S$ be the scoring cap. Let $P(\cdot;\bar{x})$ be the probability of winning function under a scoring cap $\bar{x}$, and let $P(\cdot;\infty)$ be the probability of winning function without any scoring cap. Applying the argument for continuity of P used in the proof of Lemma A-2 to all $x \in [0, \bar{x})$ establishes the following result:

Lemma A-5. *In any symmetric equilibrium, the probability of winning function, $P(\cdot;\bar{x})$, intersects the origin and is continuous over $[0,\bar{x})$, where $\bar{x}$ represents the scoring cap.*

We first argue that imposing the cap $\bar{x} \geq \mu_S$ does not affect designer welfare. Note that, although a discontinuity of $P(\cdot;\bar{x})$ can occur at $\bar{x}$, by Lemma A-5 and the fact that $P(x;\bar{x}) = P(\bar{x};\bar{x})$ for all $x \geq \bar{x}$, $P(\cdot;\bar{x})$ is upper semicontinuous. By Result A-1 and the fact that $P(\cdot;\bar{x})$ is nondecreasing and bounded, the upper semicontinuity of $P(\cdot;\bar{x})$ guarantees the existence of a best reply to $P(\cdot;\bar{x})$ and makes Lemma A-1 applicable here. By Lemma A-1, equations (1) and (2) still hold, which implies that

$$\forall t \in \{S,W\}, \quad \mathrm{Supp}_t \in \{x \geq 0 : P(x;\bar{x}) = \psi(x;\bar{x})\} \quad \& \quad P(x;\bar{x}) \leq \psi(x;\bar{x}),\ x \geq 0, \quad \text{(A-72)}$$

where $\psi(\cdot;\bar{x})$, defined according to (A-10), denotes the concave lower envelope of the upper support lines, $\{\alpha_t + \beta_t x\}_{t=S,W}$, when the cap is $\bar{x}$.[27] In this case, still either concession or challenge configurations are played. W's configuration-conditioned payoffs are still given by equations (6) and (11) and are unaffected by the scoring cap $\bar{x} \geq \mu_S$. Thus, given that S's payoff is determined by W's payoff through (8), imposing the cap $\bar{x} > \mu_S$ does not change any type's configuration-conditioned payoff. By the same argument used in the proof of Proposition 1 for showing that, between concession and challenge, equilibrium configurations are determined by W's preferences, after imposing $\bar{x} \geq \mu_S$, equilibrium configurations are still determined by W's preferences. Thus, given that each type's configuration-conditioned payoffs are unaffected by $\bar{x} \geq \mu_S$, each type's equilibrium payoff is unaffected by $\bar{x} \geq \mu_S$ and, hence, designer welfare is unaffected by $\bar{x} \geq \mu_S$.

Next, we show that imposing the cap $\bar{x} \geq \mu_S$ reduces risk taking. Note that the optimal dual variables, α_t and β_t, $t \in \{S,W\}$, are constant in $\bar{x} \in [\mu_S,\infty)$.[28] This is because, as discussed above, whether weak contestants challenge strong contestants or not is unaffected by the cap $\bar{x} \in [\mu_S,\infty)$. The optimal dual variables in challenge equilibria are given by $\alpha_W = \alpha_S = 0$ and $\beta_W = \beta_S = m/(n(\theta\mu_S + (1-\theta)\mu_W))$ (see the argument in the proof of Lemma A-4), which are unaffected by the cap $\bar{x} \in [\mu_S,\infty)$. The optimal dual variables in concession equilibria are given by equations (A-16), (A-18), and (A-19) and $\alpha_W = 0$ (see the argument in the proof of Lemma A-3), which are again unaffected by the cap $\bar{x} \in [\mu_S,\infty)$. Thus, the optimal dual variables must be constant in $\bar{x} \in [\mu_S,\infty)$.

Then, note that, constant optimal dual variables imply that, for $\bar{x} \geq \mu_S$,

$$\psi(x;\bar{x}) = \psi(x;\infty), \quad x \geq 0. \quad \text{(A-73)}$$

Thus, when the cap constraint is not binding, i.e., when $\bar{x} \geq \hat{x}$, where $\hat{x}$ is defined as the upper bound of the support of F in the contest without the cap, equilibrium distributions are unaf-

[27] With a scoring cap, tie might occur at the scoring cap. Because we have assumed a symmetric tie-breaking rule, there are still only two contestant types, distinguished by contest ability but not by the tie-breaking rule. Thus, there are still only two upper support lines, S-support line and W-support line.

[28] When $\bar{x} = \mu_S$, the optimal dual variables are not unique. However, S's strategy is unique: S places all the mass on μ_S when $\bar{x} = \mu_S$. Thus, without loss of generality, we redefine the values of α_S and β_S when $\bar{x} = \mu_S$ by their limiting values when $\bar{x} \downarrow \mu_S$.

fected.

Now consider the case in which the cap constraint is binding, i.e., $\bar{x} \in [\mu_S, \hat{x})$. By the definition of $\hat{x}$ and equation (A-12), $\psi(\hat{x};\infty) = P(\hat{x};\infty) = 1$. Thus, when $\bar{x} \in [\mu_S, \hat{x})$,

$$P(\bar{x};\bar{x}) \leq \psi(\bar{x};\bar{x}) = \psi(\bar{x};\infty) < \psi(\hat{x};\infty) = 1,$$

where the first inequality follows from (A-72), the first equality from (A-73), and the second inequality from the fact that $\psi(\cdot;\infty)$ is increasing. The result that $P(\bar{x};\bar{x}) < 1$ implies that contestants cannot ensure winning by having performance equal to the cap. Thus, there must be point mass on $x = \bar{x}$ when $\bar{x} \in [\mu_S, \hat{x})$. Thus, by the random resolution of ties, $P(\cdot;\bar{x})$ is discontinuous at $x = \bar{x}$ when $\bar{x} \in [\mu_S, \hat{x})$. Moreover, given point mass on $x = \bar{x}$, (A-72) implies

$$P(\bar{x};\bar{x}) = \psi(\bar{x};\bar{x}), \quad \bar{x} \in [\mu_S, \hat{x}). \tag{A-74}$$

Given that P is discontinuous at $x = \bar{x}$ while ψ is increasing and continuous, (A-74) implies

$$P(\bar{x}-;\bar{x}) < \psi(\bar{x}-;\bar{x}), \quad \bar{x} \in [\mu_S, \hat{x}). \tag{A-75}$$

Note that P must also meet ψ at some point $x' \in (0, \bar{x})$, because otherwise, P could only meet ψ at 0 and at $\bar{x} \geq \mu_S$, which, by Lemma A-1, would imply that weak contestants place point mass on 0, contradicting Lemma A-5. Thus, given that P meets ψ at some point $x' \in (0, \bar{x})$ and given the continuity of P over the interval $[0, \bar{x})$, Result A-2 and equations (A-73), (A-74), and (A-75) imply the existence of $x^o \in (0, \bar{x})$ such that, when $\bar{x} \in [\mu_S, \hat{x})$,

$$P(x;\bar{x}) = \begin{cases} \psi(x;\infty) & \text{if} \quad x \in [0, x^o] \\ \psi(x^o;\infty) & \text{if} \quad x \in [x^o, \bar{x}) \\ \psi(\bar{x};\infty) & \text{if} \quad x \geq \bar{x} \end{cases}.$$

Thus, by the one-to-one relation between P and F, imposing the cap $\bar{x} \in [\mu_S, \hat{x})$ induces contestants to transfer mass over $(x^o, \hat{x}]$ to the point mass on $\bar{x}$, without changing F over $[0, x^o]$ or the mean of F. Thus, it is clear that imposing the cap $\bar{x} \in [\mu_S, \hat{x})$ induces F to undergo a mean-preserving contraction. This completes the proof of part (i).

Now we establish part (ii). We first show that designer welfare is unchanged. By Lemma 2, everything else being equal, if challenge configurations are played under n', challenge configurations will also be played under $n > n'$. Winner quality under challenge configurations, given by (15), is independent of contest size. Thus, given that, fixing m, designer welfare is measured by winner quality, reducing contest size from n to $n' < n$ does not affect designer welfare if challenge configurations are played in the n'-contestant/m-winner contest.

Next, we show the risk-taking result in part (ii). Let $F_{m:n}$ be the equilibrium performance distribution played by a contestant of unknown type when n contestants compete for m places. By equation (A-14) and Lemma A-4, if challenge configurations are played, $F_{m:n}$ is given, over its support $[0, {}^{n\bar{\mu}}/_{m}]$, by

$$\sum_{i=n-m}^{n-1} \binom{n-1}{i} F_{m:n}(x)^i \left(1 - F_{m:n}(x)\right)^{n-1-i} = \frac{m}{n\bar{\mu}} x, \tag{A-76}$$

where $\bar{\mu} = \theta\mu_S + (1-\theta)\mu_W$. Thus, by Jones (2002), $F_{m:n}$ in challenge equilibria is a Complementary Beta distribution. Complementary Beta distributions are smooth and have positive derivatives on the interior of their supports. Thus, the inverse function, $F_{m:n}^{-1}$, in challenge equilibria is smooth and has positive derivatives over the open interval $(0,1)$. Given that $F_{m:n}$ and $F_{m:n'}$, $n > n'$, are two non-identical distributions with the same mean, to show that $F_{m:n'}$ is a mean-preserving contraction of $F_{m:n}$ (or equivalently, $F_{m:n}$ is a mean-preserving spread of $F_{m:n'}$), it suffices to show that $F_{m:n}$ and $F_{m:n'}$ satisfy a single-crossing condition (Diamond and Stiglitz, 1974): there exists x' such that $F_{m:n}(x) - F_{m:n'}(x) \leq (\geq) 0$ when $x \geq (\leq) x'$. This single-crossing condition can be equivalently expressed in terms of the quantile functions: there exists $\hat{q} \in (0,1)$ such that $F_{m:n}^{-1}(q) - F_{m:n'}^{-1}(q) \geq (\leq) 0$, when $q \geq (\leq) \hat{q}$. Below, we show that $F_{m:n}^{-1}$ and $F_{m:n'}^{-1}$ satisfy this single-crossing condition.

Note that equation (A-76) implies that

$$F_{m:n}^{-1}(q) = \frac{n\bar{\mu}}{m} \sum_{i=n-m}^{n-1} \binom{n-1}{i} q^i (1-q)^{n-1-i}, \quad q \in (0,1). \tag{A-77}$$

Thus, for $q \in (0,1)$,

$$F_{m:n}^{-1}(q) - F_{m:n'}^{-1}(q) = \frac{n\bar{\mu}}{m} \sum_{i=n-m}^{n-1} \binom{n-1}{i} q^i (1-q)^{n-1-i} - \frac{n'\bar{\mu}}{m} \sum_{i=n'-m}^{n'-1} \binom{n'-1}{i} q^i (1-q)^{n'-1-i}. \tag{A-78}$$

Differentiate (A-78) with respect to q, apply the result that $(i+1)\binom{n-1}{i+1} = (n-1-i)\binom{n-1}{i}$ to cancel the common terms, and combine the common factors. This yields

$$\frac{d(F_{m:n}^{-1}(q) - F_{m:n'}^{-1}(q))}{dq} = \frac{\bar{\mu} q^{n'-1-m}(1-q)^{m-1}}{m} K(q), \tag{A-79}$$

$$K(q) = n(n-m)\binom{n-1}{m-1} q^{n-n'} - n'(n'-m)\binom{n'-1}{m-1}.$$

When $q \in (0,1)$, the sign of (A-79) is determined by the sign of K. Note that $K(0) < 0$, $K(1) > 0$, and K is continuous and increasing for $q \geq 0$. Thus, there exists $q^* \in (0,1)$ such that K single crosses the horizontal axis from below at $q = q^*$. This implies, by (A-79), that $F_{m:n}^{-1} - F_{m:n'}^{-1}$ is decreasing for $q \in (0, q^*)$ and increasing for $q \in (q^*, 1)$. Because $F_{m:n}^{-1}(0) = F_{m:n'}^{-1}(0) = 0$, it follows that $F_{m:n}^{-1}(q) - F_{m:n'}^{-1}(q) < 0$ for $q \in (0, q^*]$. This result, together with the facts that $F_{m:n}^{-1}(1) - F_{m:n'}^{-1}(1) = (n-n')\bar{\mu}/m > 0$ and $F_{m:n}^{-1} - F_{m:n'}^{-1}$ is continuous and increasing for $q \in (q^*, 1)$, implies the satisfaction of the single-crossing condition. Thus, $F_{m:n}$ is a mean-preserving spread of $F_{m:n'}$, i.e., $F_{m:n'}$ is a mean-preserving contraction of $F_{m:n}$.

Part (iii) can be proved in a similar manner. To see that designer welfare is unchanged by the "relaxed" policy, note that, by Lemma 2, everything else being equal, if challenge configurations are played under m', challenge configurations will also be played under $m < m'$. Thus, by (15), winner quality is unaffected by the relaxed policy. Because the assignment of the m places over the m' best performers under the relaxed policy is random, the average quality of the m selected contestants under the relaxed policy will be the same as the average quality of the m winners in the original contest. Thus, given that contest size and the selection quota are both fixed, designer welfare is unaffected by the adoption of the relaxed policy if challenge configurations are played in the n-contestant/m'-winner contest.

To establish the risk-taking result in part (iii), note that, as just discussed, the play of challenge configurations under m' implies the play of challenge configurations under $m < m'$. Thus, $F_{m,n}$ and $F_{m',n}$ both satisfy equation (A-76). Thus, by (A-77),

$$F_{m:n}^{-1}(q) - F_{m':n}^{-1}(q) = \frac{n\bar{\mu}}{m} \sum_{i=n-m}^{n-1} \binom{n-1}{i} q^i (1-q)^{n-1-i} - \frac{n\bar{\mu}}{m'} \sum_{i=n-m'}^{n-1} \binom{n-1}{i} q^i (1-q)^{n-1-i}. \quad \text{(A-80)}$$

Differentiate (A-80) with respect to q, apply the result that $(i+1)\binom{n-1}{i+1} = (n-1-i)\binom{n-1}{i}$ to cancel the common terms, and combine the common factors. This yields

$$\frac{d(F_{m:n}^{-1}(q) - F_{m':n}^{-1}(q))}{dq} = n\bar{\mu} q^{n-1-m'} (1-q)^{m-1} J(q), \quad \text{(A-81)}$$

$$J(q) = \frac{n-m}{m}\binom{n-1}{n-m} q^{m'-m} - \frac{n-m'}{m'}\binom{n-1}{n-m'} (1-q)^{m'-m}.$$

When $q \in (0,1)$, the sign of (A-81) is determined by the sign of J. Note that $J(0) < 0$, $J(1) > 0$, and J is continuous and increasing for $q \in [0,1]$. Thus, there exists $q^o \in (0,1)$ such that J single crosses the horizontal axis from below at $q = q^o$. This result implies, by (A-81), that $F_{m:n}^{-1} - F_{m':n}^{-1}$ is decreasing for $q \in (0, q^o)$ and increasing for $q \in (q^o, 1)$. Because $F_{m:n}^{-1}(0) = F_{m':n}^{-1}(0) = 0$, it follows that $F_{m:n}^{-1}(q) - F_{m':n}^{-1}(q) < 0$ for $q \in (0, q^o]$. This result, together with the facts that $F_{m:n}^{-1}(1) - F_{m':n}^{-1}(1) = n\bar{\mu}((1/m) - (1/m')) > 0$ and $F_{m:n}^{-1} - F_{m':n}^{-1}$ is continuous and increasing for $q \in (q^o, 1)$, implies the satisfaction of the single-crossing condition. Thus, $F_{m:n}$ is a mean-preserving spread of $F_{m':n}$, i.e., $F_{m':n}$ is a mean-preserving contraction of $F_{m:n}$. This establishes part (iii) and completes the proof of Proposition 5. □