Informative fundraising: The signaling value of seed money and matching gifts *

Silvana Krasteva
Department of Economics
Texas A&M University
Allen 3054
College Station, TX 77843
E-mail: ssk8@tamu.edu

Piruz Saboury
Department of Economics
Texas A&M University
Allen 3010
College Station, TX 77843
E-mail: piruz_saboury@tamu.edu

December 31, 2018

Abstract

While existing theory predicts that a matching leadership gift raises more donations than seed money, recent experiments find otherwise. We aim to reconcile the two by studying a model of sequential fundraising under incomplete information about the charity's quality. Both the fundraising scheme employed by the charity and the contribution decision of the lead donor may signal the charity's quality to subsequent donors. With exogenously informed lead donor, the charity optimally solicits the lead donor for a matching gift independent of its quality and the size of the gift credibly reveals the charity's quality to the follower donors. Under costly information acquisition, the lead donor becomes less reliable in conveying the charity's quality as she might choose to remain uninformed. Consequently, the charity employs the fundraising scheme itself to credibly signal its quality. In particular, the high quality charity solicits the lead donor for seed money more often and for matching gift less often than the low quality charity. As a result, seed money becomes a signal of high quality and matching-a signal of low quality. Thus, consistent with experimental data, seed money is associated with higher quality and raises more donations relative to matching.

JEL Classifications: H00,H41

Keywords: fundraising, charity's quality, matching gift, seed money, information acquisition

*****Preliminary and incomplete. Please do not cite.**

1 Introduction

The non-profit sector is known for significant presence of highly inefficient organizations. Apart from the well-publicized fraudulent non-profits, such as the Cancer Fund of America[1], there are many other organizations that may not be blatantly scamming donors, but are nevertheless doing a poor job in providing public benefits. For instance, Charity Navigator, the largest charity rating agency in the USA, has classified close to one third of rated charities in years 2007-2010 as having exceptionally poor or poor performance (Yörük, 2016). Donors' lack of information is likely a contributing factor. According to Money for Good 2015 report, "49% [of donors] don't know how nonprofits use their money" [2]. While the presence of rating agencies such as Charity Navigator, CharityWatch, and GiveWell can help donors, the mere number of charitable organizations[3] makes the available information imperfect and costly to obtain. Consequently, a big challenge for well-run non-profits is finding ways to credibly inform donors of their quality and distinguish themselves from their poorly performing counterparts.

In this paper, we investigate the role that leadership giving plays in conveying information to donors. Leadership giving refers to a fundraising strategy by charities of soliciting a large donation by a wealthy donor, whose donation announcement aims to incentivize giving by other donors. Leadership gifts can be in the form of an unconditional lump sum called "seed money" or a promise of matching small donations by a fixed ratio called "matching gift".

The impact of the size and the form of the leadership gift has attracted significant interest in the theoretical and empirical literature. Theoretically, most of the focus has been in analyzing leadership giving under complete information. In this environment, seed money is equivalent to sequential fundraising, which results in significant free-riding by downstream donors on the lead donor's gift (Varian 1994). In contrast, a matching gift is associated with weaker free-riding incentives as the lead donor's contribution is contingent on the subsequent donors' giving. Thus, matching leadership gifts should be more effective in increasing contributions compared to seed money.

In light of this theoretical prediction, the recent experimental evidence (e.g, Karlan et al.

[1]The Cancer Fund of America and their leader James Reynolds Sr. has notoriously bilked more than $187 million from donors under the pretense of serving people with cancer. See `https://www.ftc.gov/news-events/press-releases/2016/03/ftc-states-settle-claims-against-two-entities-claiming-be-cancer`.

[2]At first glance, this lack of information might be attributed to the donors' lack of interest. However, survey evidence suggests otherwise. Money for Good 2015 reveals that donors "want clearer communication with nonprofits" regarding the charitable services that their money provides. The lack of information is attributed to the fact that "[donors] are often uncertain where to start, don't have the information they want, feel pressed for time, ...". For the full survey conducted by Camber Collective, visit `http://www.cambercollective.com/moneyforgood/`.

[3]According to National Center for Charitable Statistics, there are more than 1.5 million tax-exempt organizations in USA. For more information, visit `http://nccs.urban.org/data-statistics/quick-facts-about-nonprofits`.

(2011), List and Lucking-Reiley (2002), Rondeau and List (2008)) has been puzzling. It suggests that donors' giving is not very responsive to a matching gift, while seed money significantly increases giving. One plausible explanation for this finding is that the structure of the leadership gift itself conveys information about the charity's quality. In particular, if seed money is associated with high quality, while matching gift with lower quality, donors may respond favorably to an announcement of seed money, but would respond little or even reduce donations in response to a matching gift.

To investigate the signaling impact of leadership giving, we propose a model of charitable fundraising with a large donor population, in which the charity is privately informed about its quality. It chooses its fundraising mechanism to maximize donations. In particular, the charity chooses whether to solicit the lead donor for seed money or a matching gift. Subsequently, given the fundraising strategy of the charity, the lead donor decides whether to acquire costly information about the charity's quality before making a donation decision. Under leadership giving, the information acquired not only benefits the lead donor directly, as it results in more informed giving, but it enables the lead donor to signal the charity's quality to downstream donors through the size of her contribution.

We find that the charity's choice of a fundraising mechanism depends crucially on the lead donor's information. In general, both the charity's fundraising strategy and the lead donor's donation size has the potential of conveying information about the charity's quality. If the large donor is exogenously informed about the quality, the charity can always rely on the lead donor to reveal the quality to subsequent donors through the size of her donation. As a result, the charity finds it optimal to solicit the lead donor for a matching gift independent of the charity's quality. This is because under either leadership scheme, the charity's quality will be revealed, but the matching gift has the advantage of reducing the free-rider incentives by downstream donors. Thus, to understand the use of seed money, one needs to consider a model of costly information acquisition by the lead donor.

Under costly information, the lead donor acquires information only if the value of information exceeds the cost. The value of information varies not only with the prior quality distribution, but also with the charity's equilibrium fundraising strategy. Thus, it is possible for fully informed and fully uninformed equilibria to co-exist. However, we show that these two extremes of no information acquisition and full information acquisition cannot explain the experimental evidence. With full information acquisition by the lead donor, the high quality charity would successfully separate from the low quality charity as the lead donor's contribution amount will credibly signal the charity's quality. Thus, analogous to the exogenously informed lead donor, the fully informed equilibrium results in each charity type choosing matching gift fundraising. Without information acquisition by the lead donor, different charity types are indistinguishable and must raise the same amount of donations independent of the fundraising scheme.

In order to explain the presence of seed money, we focus on partially informed equilibria with seed money on the equilibrium path, which we refer to as *SPI* equilibria. *SPI* equilibria require less than perfect information acquisition under matching in order to make seed money appealing for the high quality type, and strictly positive information acquisition under seed money in order to reduce the ability of the low type to pool with the high type under seed money. More importantly, the equilibrium fundraising behavior is consistent with the experimental findings. In particular, we show that in *every SPI* equilibrium, the high quality charity is more likely to choose seed money fundraising, and thus less likely to choose matching, compared to the low quality charity. Intuitively, as the lead donor becomes less reliable at signaling the charity's quality, the high quality charity engages in costly signaling through the fundraising scheme by choosing to solicit for seed money.

Our theoretical finding establishes a plausible mechanism through which leadership gifts may convey information to donors. It predicts that seed money is used as a costly signal of higher quality as it exposes the charity to stronger free-rider's incentives. Interestingly, the credibility of seed money as a signal relies on the lead donor verifying the quality with positive probability in order to make it costly for the low quality charity to fully mimic the high quality charity. In this respect, both the charity and the lead donor play a role of conveying information to downstream donor. While the quality uncertainty is not fully resolved in equilibrium, downstream donors associate seed money with better charities and thus may respond more favorably to seed money compared to matching gift.

Our finding further suggests that the optimal structure of the leadership gift crucially depends on the donors' information about the charity. In particular, it predicts that established charities that are well-known to potential donors would find matching an attractive fundraising scheme since they have little need to convince donors of their value. In contrast, newer charities of high quality are likely to find seed money more attractive as they are trying to credibly inform donors of their value[4]. Moreover, our model also suggests that the low propensity of information acquisition, evidenced by recent survey data, may be partially attributable to the informational value of the fundraising mechanism employed by charities. These predictions provide powerful testable hypotheses to be further explored in a controlled lab environment and the field.

Related Literature Our theoretical model builds upon a large theoretical literature. Early theoretical work on private provision of public goods, such as Warr (1983) and Bergstrom et al. (1986), have focused on simultaneous contributions. They show the equivalence of

[4]Indeed, Rondeau and List (2008) compare the two types of leadership gift in the field and the lab and find that seed money is more effective in the field. They conjecture that this difference is likely due to the fact that in the lab the individuals' benefits from the public good are known and thus the leadership gift does not carry quality information.

the non-cooperative equilibrium from the simultaneous contributions game to the Lindhal equilibrium. Admati and Perry (1991) expand the analysis to a mechanism of alternating sequential contributions towards a threshold public good. They find that this can lead to an inefficient outcome. Similarly, Varian (1994) considers sequential fundraising and finds that it results in lower public good provision compared to simultaneous contributions due to donors' incentives to free-ride on earlier contributions. However, the possibility of a donor subsidizing others' contributions can alleviate this problem. The implication of these findings is that matching leadership gift is more effective in encouraging contributions by downstream donors compared to seed money.

In the context of complete and symmetric information, the use of seed money can be rationalized by the presence of threshold public good or other-regarding preferences. In particular, Andreoni (1998) shows that charities can use seed money to avoid zero-contributions equilibrium, in which no donor contributes due to an expectation that the threshold will not reached. Romano and Yildirim (2001) show that other-regarding preferences can give rise to upward-sloping best respose functions, making sequential fundraising more effective than simultaneous fundraising. In the context of standard altruistic preferences, Gong and Grundy (2014) illustrate that rapidly diminishing returns to the public good could make high match ratios ineffective. As a result, the lead donor may find it optimal to choose a low match ratio, leading to low overall donations under a matching gift.[5]

There is sparse theoretical literature that has considered incomplete information about the public good. Bag and Roy (2011) show that when donors have independent private valuations of the public good, free-riding incentives could diminish with sequential giving and thus sequential contributions might result in higher total donations compared to simultaneous ones. Krasteva and Yildirim (2013) consider an independent value threshold public good, in which each donor can choose whether to contribute informed or uninformed. They find that announcing seed money discourages informed giving while a matching gift encourages it. However, in both studies, the independence of donors' valuations precludes the possibility of signaling through the scheme choice or the contribution amount by the lead donor. In this respect, the closest papers to ours are Vesterlund (2003) and Andreoni (2006).

Similar to our model, Vesterlund (2003) and Andreoni (2006) consider the use of seed money as a signaling device to convey the charity's quality. They demonstrate that leadership gifts in the form of seed money may result in larger total donations compared to simultaneous contributions since seed money enables the lead donor to signal the charity's quality

[5]While rapidly diminishing returns to the public good may provide an alternative explanation for the low effectiveness of matching leadership giving, matching still emerges as the dominant scheme in a large economy as long as donors are primarily driven by altruistic motives. Then seed money is ineffective in increasing total donations as it fully crowds out giving by other donors (see Yildirim (2014)), while matching impacts downstream donors' marginal benefit of giving and results in higher total contributions.

to subsequent donors. The intuition behind this finding is that the information provided to potential donors through the signal has a positive effect on the donors' giving incentives and can outweigh their incentives to free-ride on the large initial donation. However, an important distinction between these papers and ours is that they only allow for the seed money leadership scheme and ignore the possible signaling value of a matching gift. By enabling charities to choose between seed money and matching, we allow them to use the structure of the leadership gift itself to convey quality information to donors. In particular, such quality signaling through the scheme becomes an important tool of information transmission when acquiring information about the charity's quality is costly for donors.

In the realm of experimental studies, Silverman et al. (1984), Frey and Meier (2004), Soetevent (2005), Croson and Shang (2008), and Shang and Croson (2009) find that donors respond positively to information about other donors' gift, and Güth et al. (2007) show the positive impact of leadership gifts in particular. Furthermore, field experiments by List and Lucking-Reiley (2002), and Landry et al. (2006) demonstrate that both the probability and size of donations significantly increase with the seed money amount. More interestingly, Potters et al. (2005) find that when some donors are informed and others are not, they are likely to endogenously choose to donate sequentially rather than simultaneously with more informed donors donating first, resulting in higher total donations compared to simultaneous contributions. All of these findings support the theory of seed money having a signaling value. Potters et al. (2007) confirm this in their experiment[6].

The impact of matching gifts has also been studied experimentally. Eckel and Grossman (2003), Meier and Frey (2004), Eckel and Grossman (2006a), Eckel and Grossman (2006b), Eckel et al. (2007), Falk (2007), Eckel and Grossman (2008) find evidence in support of matching gifts being effective in boosting donations. However, Meier (2007) illustrates that this effect is short-lived and reverses in the long run. Moreover, Karlan and List (2007) find that total donations are unresponsive to increasing the match ratio.

The result of recent experiments is more surprising. For example Alpizar et al. (2008) show that knowledge about others' donations increases individual donations, but the price of giving has little impact on donations. Similarly, Karlan et al. (2011) find little response to a matching gift and Adena and Huck (2017) find a negative response by donors. Rondeau and List (2008), Huck and Rasul (2011), and Huck et al. (2015) compare seed money to matching gifts directly in the context of field experiments and find that seed money has positive impact on giving, but matching has little to no effect. These findings are consistent with the prediction of our model and suggest that the two types of leadership giving may carry different

[6]Other related empirical work (e.g. Khanna and Sandler (2000), Okten and Weisbrod (2000), Andreoni and Payne (2003), Andreoni and Payne (2011)) studies the impact of government grants of private contributions. They find mixed results, which could be attributed to the differential impact of government grants on the fundraising effort by charities.

quality information.

In the following sections, we present our model and findings. Section 2 describes the theoretical model. Section 3.1 considers the benchmark case of complete information and describes how the fundraising schemes rank in terms of total contributions. Section 3.2 considers the signaling benchmark, in which the lead donor is exogenously informed and shows that in this case the charity will always choose a matching gift fundraising. Section 3.3 presents the case of endogenously informed donor and discusses the possibility of signaling through the fundraising scheme. Section 4 presents a few extensions to the base model and Section 5 concludes.

2 Model description

A single charity, C, aims to maximize the amount of money raised, G, to a continuous public good. The quality q takes two values, $q \in \{q_l, q_h\}$ with $0 < q_l < q_h$. The prior distribution over quality is given by $\pi = \{\pi_l, \pi_h\}$ where $\pi_h \in (0,1)$ denotes the likelihood of high quality.

There are $n \geq 2$ potential donors. Each donor i is endowed with wealth $w_i \in [\underline{w}, \overline{w}]$ drawn from an iid distribution with continuous density $f(w_i)$ and domain $[\underline{w}, \overline{w}]$ where $0 \leq \underline{w} < \overline{w} < \infty$. Donor i has the following preferences over private and public consumption:

$$u(g_i, G, q) = h(w_i - g_i) + qv(G), \qquad i = L, F \tag{1}$$

where $h'(\cdot) > 0, h''(\cdot) \leq 0, v'(\cdot) > 0, v''(\cdot) < 0$, and $qv'(0) > h'(\underline{w})$[7]. Moreover, we assume that $\left| \frac{Gv''(G)}{v'(G)} \right| < 1$ so that the donors' marginal utility from the public good is not diminishing too rapidly as provision increases.[8]

The charity solicit donors by employing leadership giving, in which it first solicits a lead donor, denoted by L. We let $w_L \geq \max_{i \in F} w_i$ so that the lead donor is the richest individual in the economy. This is consistent with Andreoni (2006) who finds that the wealthy individuals have stronger incentives to become leaders in charitable campaigns[9]. The leadership gift scheme, Z, chosen by C can be either seed money, S, or matching gift, M. Under S, L makes an unconditional contribution g_L^S that is publicly announced prior to the follower donors' contribution decisions. Under M, L commits to a match ratio m, which is publicly announced, and results in a contribution $g_L^M = m \sum_{i \in F} g_i^M = mG_F^M$ by L.

[7] The condition $v'(0) > qh'(\underline{w})$ ensures that there will be positive amount of the public good provided in equilibrium for all quality realizations.

[8] This condition is a sufficient condition for the matching scheme to eliminate the free-riding incentives by the follower donors. It is satisfied by a large class of utility functions commonly used in economics, such as the CES utility function. For a discussion about the consequence of violating this condition, see Gong and Grundy (2014).

[9] The lead donor being the wealthiest individual also guarantees that in a limit economy with $n \to \infty$, the matching scheme converges to a strictly higher giving compared to seed money as the lead donor will have incentives to offer a positive match.

The timing of the game is as follows. First, C privately observes q and commits to a fundraising scheme $Z \in \{S, M\}$. Then, it solicits L for a donation, g_L^Z. L decides whether to learn q at cost k and then makes her contribution decision g_L^Z. All follower donors then observe Z and g_L^Z, and simultaneously choose their donations g_i^Z for $i \in F$.

In the following section, we provide the equilibrium analysis of the game. We focus on characterizing the sequential equilibria of this dynamic signalling game. Moreover, as commonly adopted in the literature, we refine equilibria using the Cho-Kreps intuitive criterion (Cho and Kreps, 1987).

3 Equilibrium characterization

As as starting point of our analysis, we provide two benchmarks that are instructive in understanding how the fundraising scheme affects donors' giving. Section 3.1 discusses the benchmark case of complete information about the charity's quality, q, and establishes that under complete information, the matching gift maximizes total contributions. Section 3.2 expands the analysis to an uninformed follower, but an exogenously informed leader. This introduces the possibility of the lead donors' contribution amount signaling the charity's quality to the follower donors. We show that with a large contributing donor base, the matching gift is still the only mechanism that emerges in equilibrium. These two benchmarks lay the foundation for introducing endogenous information acquisition in Section 3.3 as providing incentives for the charity to engage in costly signaling though the scheme choice.

3.1 Benchmark: observable quality

Given an observable quality and a fundraising scheme Z by the charity, each follower donor chooses her donation to maximize her payoff given by eq. (1). Consider the best response of a follower donor $i \in F$. For seed money, equating the marginal cost and benefit of donating results in

$$h'(w_i - g_i^S) = qv'(G^S) \tag{2}$$

Inverting $h'(\cdot)$ in eq. (2) and rearranging terms results in

$$g_i^S(q, G^S) = \max\{w_i - \phi(qv'(G^S)), 0\} \tag{3}$$

where $\phi(\cdot) = [h']^{-1}(\cdot)$ is a strictly decreasing function. Clearly, i's contribution is increasing in her individual wealth w_i and thus the set of contributors F^S comprises of the wealthiest individuals. Allowing n^S to denote the number of contributing donors in equilibrium, the aggregate follower donors best response is derived by summing eq. (3) across all contributors:

$$G_F^S(q, G^S) = \sum_{i \in F^S} w_i - n^S \phi(qv'(G^S)) \tag{4}$$

Analogously, for the matching scheme, the optimal donation by a contributing follower donor solves

$$g_i^M(q, G^M) = \max\{w_i - \phi\left(qv'(G^M)\frac{G^M}{G_F^M}\right), 0\}, \tag{5}$$

where $\frac{G^M}{G_F^M} = 1 + m$ corresponds to the marginal contribution of the followers' gift towards increasing total donations G^M. Summing across all contributing donors F^M results in the following equation for the aggregate best response function:

$$G_F^M(q, G^M) = \sum_{i \in F^M} w_i - n^M \phi\left(qv'(G^M)\frac{G^M}{G_F^M(q, G^M)}\right) \tag{6}$$

where n^M denotes the number of contributing donors under M. Comparing eq. (4) and eq. (6), it is easy to verify that for the same amount of total giving, i.e. $G^M = G^S$, the follower donors must be contributing more under a matching scheme relative to a seed money scheme[10]. Intuitively, while the marginal cost of giving for a follower donor is the same across the two types of leadership gift, the marginal benefit of donating an additional dollar is higher under the matching grant due to the lead donor's commitment to match each donation. As a result, the follower donor has stronger incentives to give under a matching scheme compared to a seed money scheme.

Turning to the lead donor's contribution choice, her utility function given by eq. (1) for a fundraising scheme Z can be re-written as

$$u_L(q, G^Z) = h(w_L - G^Z + G_F^Z(q, G^Z)) + qv(G^Z). \tag{7}$$

Thus, the lead donor's contribution choice can be re-defined as choosing the total contributions G^Z given the follower donor's equilibrium best response $G_F^Z(q, G^Z)$. The equilibrium total donation amount then solves

$$h'(w_L - G^Z + G_F^Z(q, G^Z))\left(-1 + \frac{dG_F^Z(q, G^Z)}{dG^Z}\right) + qv'(G^Z) = 0 \tag{8}$$

Eq. (8) reveals that the marginal cost of increasing total donations depends not only on the follower donors' total contributions G_F^Z to the public good, but also on how the follower

[10]For a formal proof, see Lemma A1 in the appendix.

donors' contributions change with the rise in the total contributions, i.e. $\frac{dG_F^Z(q,G^Z)}{dG^Z}$. The following lemma describes the differential impact of increasing total contributions by the lead donor on the follower donors' response under the two fundraising schemes.

Lemma 1 *The follower donors' contributions decrease as total donations increase under seed money (i.e. $\frac{dG_F^S(q,G^S)}{dG^S} < 0$), while they increase under a matching gift (i.e. $\frac{dG_F^M(q,G^M)}{dG^M} \in (0,1)$).*

Lemma 1 highlights the standard free-rider problem present in public good provision. Under seed money, the incentives for the follower donors to give as total donations increase are diminishing due to the decreasing marginal utility of the public good. Under matching gift, the free-rider incentives are mitigated since the lead donor's giving hinges on the contributions by the followers. This causes the follower donors' contributions to increase with the lead donors' match ratio and thus with the rise in total donations[11].

The weaker free rider incentives under matching makes the lead donor more willing to contribute to the public good herself, leading to the following observation.

Proposition 1 *Equilibrium total donations satisfy $G^{M,*}(q) > G^{S,*}(q)$ for all q and all n with $\lim_{n\to\infty} G^{M,*}(q) > \lim_{n\to\infty} G^{S,*}(q)$. As a result, the equilibrium fundraising scheme $Z^*(q) = M$ for all q and n.*

Proposition 1 states that matching dominates seed money from the charity's point of view for any quality level q. Moreover, as the economy grows ($n \to \infty$), the matching scheme converges to a strictly higher total giving compared to the seed money scheme, i.e. $\lim_{n\to\infty} G^{M,*}(q) > \lim_{n\to\infty} G^{S,*}(q)$. This is because the lead donor's giving converges to 0 under seed money as the follower donors completely free-ride on the lead donor's giving (see Andreoni, 1988). In contrast, the matching scheme induces positive amount of giving by the lead donor since even in the limit economy the follower donors are responsive to a positive match ratio. Therefore, in absence of asymmetric information about the quality of the charity, both charity types would prefer to fundraise for a matching gift.

To understand the use of seed money, we next extend the model to include incomplete information about the charity's quality. The next section presents the case, in which only the lead donor is informed about the charity's quality, turning the contribution decision of the lead donor into a signaling game.

[11]The follower donors' giving is increasing in the match ratio as long as $\left|\frac{Gv''(G)}{v'(G)}\right| \leq 1$, guaranteeing that the matching scheme raises more total donations. Gong and Grundy (2014) show that violating this condition may result in seed money raising more donations as the follower donors' giving may be hump-shaped in the match ratio- increasing for low match ratios and decreasing for high match ratios. Then the lead donor may find it optimal to choose a low match ratio in order to mitigate the free-rider problem.

3.2 Benchmark: exogenously informed leader and uninformed follower

Given that the charity and the lead donor are privately informed about the charity's quality, both the fundraising scheme as well as the lead donor's donation decision may convey information to the follower donors. Thus, in this section, we are considering a dynamic signaling game with two channels of information- Z and g_L^Z. The solution concept we use is sequential equilibrium and, consistent with Andreoni (2006), we employ Cho-Kreps intuitive criterion to rule out equilibria with unreasonable off-equilibrium beliefs.

In the last stage of the game, the follower donors make simultaneous donation decisions corresponding to $G_F^Z(\overline{q}^Z, G^Z)$ derived in Section 3.1, where $\overline{q}^Z$ denotes the follower donors' belief about the charity's quality. To pin down the equilibrium value of $\overline{q}^Z$, we consider the lead donor's contribution decision. Similar to Section 3.1, the lead donor's objective can be written as choosing G^Z to maximize

$$\overline{u}_L(q, \overline{q}^Z, G^Z) = h(w_L - G^Z + G_F^Z(\overline{q}^Z, G^Z)) + qv(G^Z). \tag{9}$$

Since L is endowed with private information about the charity's quality, from the point of view of the follower donors, the lead donor's type $\overline{q}_L^Z \in \{q_l, q_h\}$ with a corresponding probability of type q_j for $j = \{l, h\}$ denoted by $\overline{\eta}_j^Z$. Since the lead donor and the charity possess symmetric information about the charity's quality, $\overline{q}_L^Z = q$, and the posterior probability of the charity being of type q_j, denoted by π_j^Z, exactly coincides with $\overline{\eta}_j^Z$ and depends on the charity's fundraising strategy. In particular, let $\beta^Z(q_j)$ denote the probability that a charity of type q_j chooses scheme Z. Then, by Bayes' rule, π_j^Z for $j = \{l, h\}$ satisfies:

$$\pi_j^Z = \frac{\beta^Z(q_j)\pi_j}{\sum_{y \in \{l,h\}} \beta^Z(q_y)\pi_y} \quad for\ y = \{l, h\}. \tag{10}$$

π_j^Z is the likelihood of quality q_j conditional only on the observed scheme Z, prior to the lead donor's contribution decision. Since G^Z can also serve as an informative signal about the charity's quality, the follower donor's belief $\overline{q}^Z$ may also be impacted by the lead donors' strategy. While the strategy set by the leader is infinite, to apply the notion of sequential equilibrium, we will restrict attention to strategies that are Borel probability distributions $\mathcal{D}$ over the strategy set (e.g., Manelli, 1996). An admissable strategy for type $j \in \{l, h\}$ of the lead donor is a probability distribution in $\mathcal{D}$ where $d_j(G^Z)$ is measurable and denotes the probability of type j choosing contribution G^Z. Then, given $\overline{\eta}_j^Z = \pi_j^Z$ the followers' posterior belief of quality q_j is formed using Bayes' rule:

$$\overline{\mu}_j^Z(G^Z) = \frac{\overline{\eta}_j^Z d_j(G^Z)}{\sum_{y \in \{l,h\}} \overline{\eta}_y^Z d_y(G^Z)} \; for\ y = \{l, h\}. \tag{11}$$

That is, the posterior probability of quality q_j upon observing G^Z depends on the relative likelihood of G^Z coming from a leader of type q_j. Then, $\overline{q}^Z$ solves the following equation:

$$\overline{q}^Z = \sum_{j \in \{l,h\}} \overline{\mu}_j^Z(G^Z) q_j \tag{12}$$

Given the above definition of the strategy space and the corresponding belief structure, we formally define a sequential equilibrium as follows.

Definition 1 *Sequential equilibrium consists of an equilibrium strategy set* $\mathcal{S}_j = (\overline{\beta}_j^{Z,*}(q), \overline{d}_j^*(G^Z))$ *and corresponding belief structure* $\mathcal{B} = \left\{\overline{\pi}_j^{Z,*}, \overline{\mu}_j^{Z,*}(G^Z), \overline{q}^{Z,*}\right\}$ *satisfying:*

1. *(Sequential rationality) Given* $\mathcal{B}$, $\overline{\beta}_j^{Z,*}(q)$ *maximizes total expected donations* $\sum_Z \sum_{\mathbf{supp}(\overline{d}_j^*)} \overline{\beta}_j^Z(q) \overline{d}_j^*(G^Z))$; *and* $\overline{d}_j^*(G^Z)$ *maximizes the lead donors' payoff given by eq. (9).*

2. *(Consistent beliefs)* $\mathcal{B}$ *satisfies eq. (10)-(12) and is a limit of a convergent sequence* $(\mathcal{S}^m, \mathcal{B}^m) \to (\mathcal{S}, \mathcal{B})$ *with* $\beta^{Z,m}(q) > 0$ *for all* Z *and* q; $d_j^m(G^Z) > 0$ *in the support of* d_j^m; *and a belief structure* $\mathcal{B}^m$ *satisfying eq. (10)-(12).*

The above definition ensures that the equilibrium donation game correspond to the limit of a signaling game, in which each type of lead donor is present in each scheme Z, i.e. $\pi_j^{Z,m} > 0$ for all $j \in \{l, h\}$. This simplies the equilibrium characterization of the contribution game as it implies that in each scheme the lead donor's contribution will have a signaling value. Note that the lead donor's objective function given by eq. (9) satisfies the single crossing property $\frac{\partial^2 \overline{u}_L(q, \overline{q}^Z, G^Z)}{\partial G^Z \partial q} > 0$, which ensures the existence of a separating equilibrium in pure strategies. Moreover, by a standard application of Cho-Kreps intuitive criterion (Cho and Kreps, 1987) selects the least costly separating equilibrium, in which the equilibrium contributions $(\overline{G}^{Z,*}(q_j), \overline{G}^{Z,*}(q_{-j}))$ where $-j = \{l, h\} \setminus \{j\}$ by the two types of lead donor uniquely solve:

$$\begin{aligned}
&\overline{G}^{Z,*}(q_j) = \underset{G^Z}{argmax} \quad \overline{u}_L(q_j, q_j, G^Z) \\
&s.t. \quad \overline{u}_L(q_j, q_{-j}, \overline{G}^{Z,*}(q_{-j})) \leq \overline{u}_L(q_j, q_j, \overline{G}^{Z,*}(q_j))
\end{aligned}$$

The above incentive constraint is non-binding for q_l, implying that equilibrium contributions coincide with the complete information level, i.e. $\overline{G}^{Z,*}(q_l) = G^{Z,*}(q_l)$. For q_h, the incentive constraint may be binding with equilibrium contributions $\overline{G}^{Z,*}(q_h)$ exceeding $G^{Z,*}(q_h)$ in order to dissude imitation by the low quality type[12].

Since under either scheme the charity's quality will be revealed with certainty by the lead donor, similar to the complete information benchmark, the charity will choose the scheme that raises the highest donation amount. Clearly, for q_l matching dominates seed money since the donation amount coincides with the complete information outcome from Section 3.1. For the charity of high quality, the comparison is less clear since the lead donor may engage in costly signaling. However, as pointed out by Andreoni (2006), in a large economy the equilibrium donations under seed money $\overline{G}^{S,*}(q)$ approach the full information amount $G^{S,*}(q)$. This implies that with sufficiently large donor base, the equilibrium contributions under matching will always exceed their seed money counterpart, causing all charity types to opt for matching[13].

Proposition 2 *There exists $\overline{n} \in [2, \infty)$ such that if $n \geq \overline{n}$, the matching scheme raises more contributions, i.e. $\overline{G}^{M,*}(q) > \overline{G}^{S,*}(q)$ for all q. Consequently, in equilibrium both types of charities pool on a matching scheme, $\overline{Z}^*(q) = M$ for all q.*

Intuitively, since the lead donor has incentives to credibly reveal the charity's type to the downstream donor, there is no need for the high quality charity to employ seed money and expose itself to stronger free-riding incentives. Thus, Proposition 2 predicts that seed money is the inferior fundraising scheme if the lead donor is already informed about the quality. This raises the question of whether reducing the lead donor's information by making it costly for her to learn the charity's quality may induce the charity to choose seed money. Intuitively, by weakening the lead donor's reliability in conveying information about the charity's quality, the high quality charity may be forced to use the fundraising mechanism itself to signal its quality. The following section analyzes this possibility.

[12]One can verify that the incentive constraint is binding for q_h with $\overline{G}^{Z,*}(q_h) > G^{Z,*}(q_h)$ as long as q_h is not too high relative to q_l. An interesting consequence of costly separation is that the high quality charity benefits from limited information about its quality since it results in strictly higher contributions.

[13]Our focus on a large economy is consistent with the size of the charitable giving market in the USA, in which 72% of contributions come from individual donations (see `https://www.nptrust.org/philanthropic-resources/charitable-giving-statistics/`). In general, with a relatively small donor base, it is possible for contributions under seed money to exceed the ones under matching for the high type, i.e. $\overline{G}^{S,*}(q_h) > \overline{G}^{M,*}(q_h)$. Intuitively, with costly quality signaling, the low type of lead donor may find it more costly to pull with the high type under matching since the resulting higher donation amounts by the follower donors also increases the lead donor's contributions through the match. This can make separation by the high type less costly under match than under seed money and thus result in lower overall contributions. While this finding is consistent with the anecdotal evidence alluded to in the Introduction, it is limited in its insight and thus not the focus of our analysis.

3.3 Endogenous information acquisition by the lead donor

Suppose that instead of costlessly observing the charity's quality, the lead donor has to pay $k > 0$ in order to learn q. This introduces the possibility that the lead donor is uninformed about the charity's quality. As a consequence, the charity's type q and the lead donor's type, q_e may no longer coincide. In particular, let α^Z denote the lead donor's probability of information acquisition upon observing Z. Then, from the point of view of the follower donors, the contribution game consits of a signaling game, in which the lead donor can be one of three types, $\widetilde{q}_L^Z \in \{q_l, q_U^Z, q_h\}$, where q_U^Z denotes the expected quality by an uninformed lead donor. This expected quality, in turn, depends on the posterior quality distribution $\pi^Z = \{\pi_l^Z, \pi_h^Z\}$ where π_j^Z for $j = \{l, h\}$ is defined by eq. (10). Then, q_U^Z solves

$$q_U^Z = \sum_j \pi_j^Z q_j \tag{13}$$

An interesting feature of the endogenously informed donor is that the type space of the lead donor is endogenous and depends on the charity's equilibrium choice of scheme $\beta^Z(q)$ through the posterior distribution of π^Z. We denote the corresponding probability of type $q_e \in \widetilde{q}_L^Z$ for $e = \{l, U, h\}$ by $\widetilde{\eta}_e^Z$ where

$$\widetilde{\eta}_e^Z = \begin{cases} \pi_e^Z \alpha^Z & \text{for } e = \{l, h\} \\ 1 - \alpha^Z & \text{for } e = U. \end{cases} \tag{14}$$

Eq. (14) highlights that the distribution over types for the lead donor depends both on the charity's fundraising choice, $\beta^Z(q)$, and the lead donor's information acquisition strategy α^Z. In addition, the follower donors' equilibrium belief about the charity's quality depends also on the lead donor's donation strategy. Analogous to Section 3.2, let $d_e(G^Z)$ denote the lead donor's donation strategy of type e. Then, the posterior likelihood of quality q_e upon observing G^Z, and the corresponding expected quality $\widetilde{q}^Z$ satisfy the following equations:

$$\widetilde{\mu}_e^Z(G^Z) = \frac{\widetilde{\eta}_e^Z d_e(G^Z)}{\sum_y \widetilde{\eta}_y^Z d_e(G^Z)}, \; for \; y = \{l, U, h\} \tag{15}$$

$$\widetilde{q}^Z = \sum_e \widetilde{\mu}_e^Z(G^Z) q_e. \tag{16}$$

A sequential equilibrium of this game consists of a strategy set $\mathcal{S}^E = (\widetilde{\beta}^{Z,*}(q), \widetilde{\alpha}^{Z,*}, \widetilde{d}_e^*(G^Z)$ and a correspoding belief $\mathcal{B}^E = (\widetilde{\pi}_j^{Z,*}, \widetilde{\eta}_e^{Z,*}, \widetilde{\mu}_e^{Z,*}(G^Z), \widetilde{q}^{Z,*})$ satisfying sequential rationality and consistency as defined in Section 3.2.

Analoglous to the exogenously informed lead donor, let $(\widetilde{G}^{Z,*}(q_l), \widetilde{G}^{Z,*}(q_U^Z), \widetilde{G}^{Z,*}(q_h))$ denote the contribution amounts corresponding to the least costly separating equilibrium. It is worth noting that while the low type's total contributions coincide with the ones from exogenously informed leader, the presence of an uninformed donor with q_U^Z changes the equilibrium condition for the total donations of the high quality charity. This is because the lead donor of high type is choosing her donation to separate both from the low and the uninformed type. Since our focus is on a market with a large number of contributors, the equilibrium giving satisfies the following relationship.

Condition 2 *n is sufficiently large such that* $\widetilde{G}^{M,*}(q_e) > \widetilde{G}^{S,*}(q_e)$ *for all* $q_e \in \{l, h\}$ *and* $\widetilde{G}^{M,*}(q_U^M) > \widetilde{G}^{S,*}(q_U^S)$ *for* $q_U^M > q_U^S$[14].

As discussed in Section 3.2, the large donor base guarantees that the equilibrium total donations under seed money are not too far from the optimal amounts under symmetric information. This, in turn, implies that the matching scheme should raise more donations compared to seed money for both the high and the low quality type whenever the lead donor is informed. Moreover, since the equilibrium donations are increasing in quality, matching donations would exceed seed money donations under uninformed lead donor as long as the expected quality under matching is higher.

To obtain an expression for the value of information, let $\overline{V}_L(q_e) = \overline{u}_L(q_e, q_e, \widetilde{G}^{Z,*}(q_e))$ denote the corresponding equilibrium utility for the lead donor of type q_e. Then, the value of information for the lead donor is simply the difference between the expected informed and uninformed utility:

$$\Delta^Z(\pi^Z) = \pi_h^Z \overline{V}_L(q_h) + \pi_l^Z \overline{V}_L(q_l) - \overline{V}_L(q_U^Z). \tag{17}$$

The value of information depends crucially on the charity's equilibrium fundraising strategy. In particular, the following lemma points out that $\Delta^Z(\pi^Z)$ is positive if and only if the two charity types (partially) pool in equilibrium, thus leaving the lead donor uncertain of the charity's quality.

Lemma 2 $\Delta^Z(\pi^Z)$ *is continuous in* $\pi_h^Z \in [0, 1]$. *Moreover,* $\Delta^Z(\pi^Z) = 0$ *for* $\pi_h^Z \in \{0, 1\}$ *and* $\Delta^Z(\pi^Z) > 0$ *for all* $\pi_h^Z \in (0, 1)$.

Clearly, if the two charity types follow a fully separating fundraising strategy, the fundraising scheme would be perfectly informative, i.e. $\pi_j^Z = 1$ for some $j \in \{l, h\}$. Then, by eq. (13), $q_U^Z = q_j$ and the value of information will be zero. Since the scheme would be perfectly revealing of the charity's type, it must be the case that the low quality charity chooses a matching

[14] The proof is analogous to the proof of Proposition 2 and thus is omitted here

scheme, while the high quality charity chooses seed money. Then, by eq. (13), the uninformed lead donor under seed money coincides with the high quality type, i.e. $\widetilde{q}_U^{S,*} = q_h$, implying that the only consistent belief in the donation subgame corresponds to the two type case under exogenous information, i.e. $\widetilde{\mu}_U^S(G^S) = \overline{\mu}_h^S(G^S)$. [15] Therefore, $\widetilde{G}^{S,*}(q_U^{S,*}) = \overline{G}^{S,*}(q_h)$. Moreover, given a fully uninformed lead donor, to prevent deviation by either type of charity, the two schemes must raise the same amount of money ($\widetilde{G}^{S,*}(q_h) = \widetilde{G}^{M,*}(q_l)$). Therefore, while in a fully separating equilibrium seed money emerges as a signal of high quality, such equilibrium is purely incidental and requires that each scheme is equally attractive for the charity.

Apart from the fully separating equilibrium, there are other possible equilibria, in which the lead donor chooses to stay uninformed either due to a high information cost or low value of information. A common feature of such equilibria is that both charity types raise the same amount of money, as the following Proposition highlights.

Proposition 3 *(Fully uninformed equilibria) In every equilibrium with no information acquisition on the equilibrium path, i.e., $\widetilde{\alpha}^{Z,*} = 0$ for all Z with $\sum_q \widetilde{\beta}^{Z,*}(q) > 0$, each scheme on the equilibrium path results in the same total donations and each charity raises the same amount of money.*

In absence of information acquisition, the high quality charity is not able to effectively separate from the low quality charity since imitation by the low type is always possible. Consequently, the two charities will either pool on the scheme or the two schemes would be equally attractive to prevent profitable deviation. Since this is not consistent with the experimental evidence alluded to in the Introduction, we instead focus on equilibria, in which information acquisition occurs with positive probability.

In order for information acquisition to take place, the value of information should be sufficiently high relative to the cost. In particular, if the value of information at the prior distribution $\Delta^M(\pi)$ exceeds the cost k, fully informed equilibrium always exists. In such equilibrium, the two charity types necessarily pool on the matching fundraising scheme.

Proposition 4 *(Fully informed equilibrium) Fully informed equilibrium with $\widetilde{\alpha}^{Z,*} = 1$ for all Z on the equilibrium path ($\sum_q \widetilde{\beta}^{Z,*}(q) > 0$) exists if and only if $\Delta^M(\pi) \geq k$. Moreover, the fully informed equilibrium is unique with $\widetilde{Z}^*(q) = M$ for all q, $\widetilde{\alpha}^{M,*} = 1$, and $\widetilde{G}^{M,*}(q_h) \geq \overline{G}^{M,*}(q_h)$.*

[15] More technically, recall that the equilibrium beliefs in a sequential equilibrium are derived as a limit of fully mixed strategies at every information set. Consequently, the type space under seed money is the converging limit of a sequence $\widetilde{q}_L^{S,m}(S) = \left\{q_l, q_U^{S,m}, q_h\right\}$ with corresponding probability distribution $\widetilde{\eta}^{S,m} = \left\{\alpha^{S,m}\pi_l^{S,m}, (1-\alpha^{S,m}), \alpha^{S,m}\pi_h^{S,m}\right\}$ where $\alpha^{S,m} \to 0$, $\beta^{S,m}(q_h) \to 1$, and $\beta^{S,m}(q_l) \to 0$. Therefore, $\widetilde{q}_L^{S,m} \to \{q_l, q_h\}$ and $\widetilde{\eta}^{S,m} \to \{0,1\}$. Thus, the corresponding equilibrium belief about the likelihood of high quality given the lead donor's contribution, $\widetilde{\mu}_U^S(G^S) \to \overline{\mu}_h^S(G^S)$.

The intuition behind Proposition 4 coincides with the one under exogenously informed donor- as long as the lead donor obtains information with certainty, the high quality charity can rely on the lead donor to signal the charity's quality through her donation size. As a result, matching will be preferred by the charity since it incentivizes more giving. Interestingly, the amount of money raised by the high quality charity in equilibrium exceeds the amount raised under an exogenously informed donor ($\widetilde{G}^{M,*}(q_h) \geq \overline{G}^{M,*}(q_h)$). This is because the donation is tailored to not only signal away from the low quality type, but also the uninformed type who is more willing to mimic the high type.

Proposition 4 implies that the lead donor must have reduced incentives to acquire information in order for the high quality charity to find seed money attractive. However, Proposition 3 indicates that the other extreme of no information acquisition also does not provide strict incentives for seed money fundraising. Thus, we next turn to partial information acquisition. In particular, we focus on equilibria with partial information acquisition, in which seed money is on the equilibrium path[16]. We refer to such equilibria as *SPI* (seed-partial info) equilibria. More formally, the likelihood of scheme Z emerging in equilibrium, $E[\widetilde{\beta}^{Z,*}]$, and the corresponding expected likelihood of information acquisition, $E[\widetilde{\alpha}^{*}]$, are given by

$$E[\widetilde{\beta}^{Z,*}] = \pi_h \widetilde{\beta}^{Z,*}(q_h) + (1 - \pi_h)\widetilde{\beta}^{Z,*}(q_l), \tag{18}$$

$$E[\widetilde{\alpha}^{*}] = \sum_{Z \in \{S,M\}} E[\widetilde{\beta}^{Z,*}]\widetilde{\alpha}^{Z,*}. \tag{19}$$

The following provides a formal definition of an *SPI* equilibrium.

Definition 3 *SPI equilibrium satisfies* $E[\widetilde{\beta}^{S,*}] > 0$ *and* $E[\widetilde{\alpha}^{*}] \in (0,1)$.

SPI equilibrium requires both that seed money is chosen with positive probability by some quality type and that there is limited information acquisition on the equilibrium path. Note that limited information may arise as a result of randomization in the information acquisition strategy by the lead donor for a given scheme or the lead donor's asymmetric information acquisition strategy under the two schemes.

The following Lemma provides sufficient conditions for the existence of an *SPI* equilibrium and additional equilibrium properties.

[16] As typical for most signaling games, there is multiplicity of equilibria, including an equilibrium, in which seed money is off the equilibrium path due to very pessimistic beliefs about the charity's quality. For our purposes, however, the more relevant equilibria involve seed money being utilized by charities in equilibrium since it allows us to address the question of which type of charity is more likely to employ seed money fundraising.

Lemma 3 *(Existence of an SPI equilibrium) If $\Delta^S(\pi) \geq k$ and $\widetilde{G}^{S,*}(E[q]) > \widetilde{G}^{M,*}(q_l)$, there exists an SPI equilibrium. Moreover, every SPI equilibrium satisfies 1) $\beta^{S,*}(q) > 0$ for all q; 2) $\alpha^{M,*} < 1$ and $\alpha^{S,*} \in (0,1)$.*

Lemma 3 states that an *SPI* equilibrium exists as long as the cost of information is low relative to the value of information at the prior belief π (i.e, $\Delta^S(\pi) \geq k$) and the prior expected quality is high enough so that the uninformed seed money fundraising at the prior is sufficiently attractive for the low type (i.e, $\widetilde{G}^S(E[q]) > \widetilde{G}^M(q_l)$). This is because, as stated by the first property, both charity types must be present in seed money and high uninformed giving is necessary to make seed money attractive for the low type. To understand the first property, note that the low type would never unilaterally choose seed money since it would perfectly reveal its quality. The high type, on the other hand, may find seed money attractive if it is perfectly revealing of its quality, but the resulting zero value of information and no quality verification by the lead donor, would make seed money also attractive for the low type. Thus, in equilibrium, both types need to utilize seed money, resulting in strictly positive value of information (Lemma 2).

Given the presence of both types in seed money, the second property stated in Lemma 3 requires that information acquisition is less than perfect under the matching scheme and the lead donor strictly randomizes in her information acquisition strategy under seed money. Less than perfect information acquisition under matching ($\alpha^{M,*} < 1$) and some information acquisition under seed money ($\alpha^{S,*} > 0$) is necessary in order for the high type to consider seed money fundraising. In addition, limited information acquisition under seed money $\alpha^{S,*} < 1$ is required in order to make seed money attractive for the low type.

Lemma 3 establishes that with partial information acquisition, seed money cannot be a perfectly revealing signal of quality. Nevertheless, we are interested in how seed money compares to matching in conveying quality information to donors. The following Proposition delivers a sharp prediction by establishing that in any *SPI* equilibrium, seed money is a stronger signal of high quality compared to matching.

Proposition 5 *In every SPI equilibrium, the seed money scheme is associated with higher expected quality, i.e. $\widetilde{q}_U^{S,*} > \widetilde{q}_U^{M,*}$, and higher expected donations, i.e. $E_q[\widetilde{G}^{S,*}(q)] > E_q[\widetilde{G}^{M,*}(q)]$.*

Proposition 5 is consistent with the experimental evidence alluded to in the Introduction. It reveals that in every *SPI* equilibrium, the high quality charity chooses seed money with higher probability relative to the low quality charity. This leads to more optimistic beliefs regarding the quality type under seed money and higher expected quality. Consequently, the expected donations under seed money are also higher. Intuitively, the attraction of seed money for the high quality type is in its ability to signal the charity's type more reliability.

Thus, seed money must be either associated with higher expected quality for the uninformed lead donor or induce more information acquisition by the lead donor relative to matching. However, if the benefit is coming purely from more information acquisition, such that $\widetilde{\alpha}^{S,*} > \widetilde{\alpha}^{M,*}$ and $\widetilde{q}_U^{M,*} > \widetilde{q}_U^{S,*}$, then the low quality charity would strictly prefer to fund-raise for matching. This is because unlike the high type, the low type dislikes information acquisition and would find matching more attractive if it is less informative and associated with more optimistic belief regarding its type. Thus, a necessary condition for both types to find seed money attractive is for seed money to signal higher quality to the donors.

In terms of fundraising strategies, the *SPI* equilibrium is not necessarily unique. While both types need to be present in seed money (Lemma 3), this is not true for the matching scheme. The possible equilibrium strategies vary with both types pooling on seed money, only the low type being present in matching, or each type being present in both schemes. The more interesting equilibria involve both schemes being on the equilibrium path. Thus, in the remainder of this section, we focus on characterizing this set of *SPI* equilibria.

For any equilibrium with strict mixing in information acquisition under Z, it must be the case that the value of information is equal to the cost. Let $(\hat{\pi}^S, \hat{\pi}^M)$ denote the pair of posterior beliefs that satisfy the following conditions:

Definition 4 *The set of posterior beliefs $(\hat{\pi}^S, \hat{\pi}^M)$ with a corresponding expected quality $(\hat{q}_U^S, \hat{q}_U^M)$ satisfy:*

$$\begin{aligned} C2: &\quad \Delta^Z(\hat{\pi}^Z) = k \quad \textit{for} \quad Z = \{S, M\} \\ C3: &\quad \hat{\pi}_h^S > \hat{\pi}_h^M \end{aligned} \tag{20}$$

In the Appendix, we show that as long as the value of information under the prior exceeds the cost for each scheme, i.e. $\Delta^Z(\pi) \geq k$, there always exits a (non)degenerate strategy by the two types of charities that guarantees a pair of posterior beliefs that satisfy C2 and C3. Using this property, the following Proposition describes the equilibrium strategies by the two charities that emerge under an *SPI* equilibrium.

Proposition 6 *Consider SPI equilibria, in which M is on the equilibrium path.*

- *If $\Delta^S(\pi) \geq k$ and $\widetilde{G}^S(E[q]) > \widetilde{G}^M(q_l)$, there exists an equilibrium with $\widetilde{\beta}^{S,*}(q_h) = 1$ and $\beta^{S,*}(q_l) \in (0,1)$ satisfying $\Delta^S\left(\widetilde{\pi}^{S,*}\right) = k$.*
- *If $\Delta^Z(\pi) > k$ for all Z and $\widetilde{G}^S(\hat{q}_U^S) > \widetilde{G}^M(\hat{q}_U^M)$, there exists a fully non-degenerate equilibrium with*
$$\widetilde{\beta}^{S,*}(q_h) = \frac{\hat{\pi}_h^S}{\pi_h}\frac{\pi_h - \hat{\pi}_h^M}{\hat{\pi}_h^S - \hat{\pi}_h^M}; \widetilde{\beta}^{S,*}(q_l) = \frac{1-\hat{\pi}_h^S}{1-\pi_h}\frac{\pi_h - \hat{\pi}_h^M}{\hat{\pi}_h^S - \hat{\pi}_h^M}$$
where $0 < \widetilde{\beta}^{S,*}(q_l) < \widetilde{\beta}^{S,*}(q_h) < 1$.

Proposition 6 characterizes two types of equilibria. In the first one, only the low quality type chooses matching, making matching a sure signal of low quality, while both types are present in seed money. Note that in such an equilibrium, the low type of charity is indifferent between the two schemes and in equilibrium randomizes to make the lead donor indifferent in her information acquisition strategy under seed money. To guarantee the existence of such an equilibrium, the cost of information should be sufficiently low to ensure some information acquisition in equilibrium. Moreover, the low quality charity should raise significant donations under seed money when the lead donor is uniformed to compensate for the lower donations when she is informed.

In the second equilibrium, both charities are randomizing between matching and seed money. This equilibrium is important since it illustrates that both schemes could be used by the two charity types. Thus, neither schemes is perfectly informative, but rather in equilibrium the follower donors use both the fundraising scheme and the size of the lead donor's gift to infer information about the charity's quality. This equilibrium requires not only that seed money is sufficiently lucrative for the low type when the lead donor is uninformed, but also that uninformed donations raised under matching are low enough to make seed money an attractive option for the high type.

Overall, the analysis in this section illustrates that with costly information acquisition, seed money is likely used by the high quality charity to credibly signal its quality. More importantly, we illustrate that with both schemes being utilized in equilibrium, the seed money scheme is always indicative of a higher expected quality compared to the matching scheme. This is a rather strong result that provides a feasible explanation for the recent experimental findings. In the next Section, we discuss a few extensions and variations of the model to both highlight the robustness of this finding and inform how the signaling by scheme is affected by factors such as the option to not announce the lead donor's contribution, the presence of an alternative credible signal of quality, and warm glow incentives for giving among donors.

4 Model extensions and variations

4.1 Opting out of leadership giving

So far, we have assumed that the charity always chooses to reveal the lead donor's gift and thus the only decision that the charity makes is whether to ask for seed or matching leadership gift. One may wonder how the relative appeal of the two leadership schemes may change if we allow the charity to opt out of leadership giving completely. In spirit of Vesterlund (2003), suppose that the charity can commit not to announce (N) the lead donor's contribution and instead to solicit each donor for an unconditional gift. This turns the contribution game into a simultaneous game, precluding the possibility of signaling by the lead donor and leaving the scheme choice as the only possible source of information.

It is important to note that unlike Vesterlund (2003), who allows the lead donor to donate multiple times, we model the lead donor's decision as a one-time contribution. However, this distinction becomes immaterial in a large economy. As pointed out by Vesterlund (2003), under symmetric quality information, sequential and simultaneous contributions raise the same amount of money if the lead donor is allowed to contribute multiple times. Interestingly, this equivalence between the two schemes also holds in a large economy with purely altruistic donors. This is because, as pointed out by Andreoni (1988) and more recently Yildirim (2014), individual contributions converge to 0 in a large economy, making seed leadership giving inconsequential under complete information. Thus, in a large economy, the main distinction between seed money and non-announcement must come from the quality information conveyed to donors.

The equivalence of seed and non-announcement under complete information implies that matching is still the dominant scheme. This dominance is preserved if the lead donor is exogenously informed since then the high quality charity can always rely on the lead donor to convey its quality to downstream donors and thus it will opt for the scheme that raises the most donations, mainly matching. Given this choice by the high quality charity, non-announcement will be construed as evidence of low quality, making matching more attractive for the low quality charity as well.

Under endogenous information acquisition, the comparison of the three schemes is less clear since the use of a leadership scheme does not guarantee informed contributions. Similar to the benchmark model, in absence of information acquisition, the two charity types must raise the same amount of money under any scheme on the equilibrium path since the lack of verification makes it costless for the low type to mimic the high type. Interestingly, however, fully informed equilibrium, in which the lead donor acquires information with probability one, no longer guarantees the use of matching. Recall from Section 3.2 that the low quality charity favors matching over seed money if his type is fully revealed in equilibrium. Non-announcement, however, provides means for the low quality charity to pool with the high even if the lead donor chooses to acquire information. The high quality charity may also favor non-announcement if matching is associated with sufficiently pessimistic beliefs about the charity's quality and no information acquisition. Nevertheless, a fully informed equilibrium precludes the possibility of seed money as stated by the following Lemma.

Lemma 4 *In any fully informed equilibrium (i.e., $\tilde{\alpha}^{Z,*} = 1$ for all Z on the equilibrium path), seed money is chosen with zero probability (i.e. $\tilde{\beta}^{S,*}(q) = 0$ for all q). Moreover, the two types of charities pull either on matching ($\tilde{\beta}^{M,*}(q) = 1$ for all q) or non-announcement ($\tilde{\beta}^{N,*}(q) = 1$ for all q).*

The intuition behind Proposition 4 is straightforward. Given an informed leader, matching always dominates seed money for the low charity type. Thus, seed money can be chosen only by the high type, which in turn results in no verification under seed money. Thus, fully informed equilibrium precludes the use of seed money[17].

Similar to our base model, Lemma 4 implies that seed money should be associated with partial information acquisition in order to be attractive to both charity types. The following Proposition establishes the possibility that seed money conveys the strongest signal of quality in equilibrium.

Proposition 7 *Every SPI equilibrium satisfies $q_U(\pi^{\tilde{S},*}) > q_U(\pi^{\tilde{M},*})$. Moreover, if $\Delta^S(\pi) \geq k$ and $\widetilde{G}^S(E[q]) > \widetilde{G}^M(q_l)$, there exists a SPI equilibrium, in which seed money is associated with the highest expected quality, $q_U(\pi^{\tilde{S},*}) > \max\{q_U(\pi^{\tilde{M},*}), q_U(\pi^{\tilde{N},*})\}$.*

The first part of Proposition 7 generalizes our main finding and establishes that the presence of non-announcement does not impact the relative quality comparison of seed money and matching gift. This is intuitive in light of the earlier discussion. The second part of the Proposition establishes the possibility that seed money is also a stronger quality signal relative to non-announcement. In fact, an equilibrium, in which non-announcement is off the equilibrium path and construed as a signal of low quality clearly meets this description. However, the comparison between non-announcement and seed money is less clear-cut. Similar to Vesterlund (2003), we cannot rule out the existence of equilibria, in which both schemes are used and non-announcement is a signal of higher quality than seed money. Such equilibrium requires that seed money results in little verification, preventing the high quality charity from generating significant separation from the low quality charity under seed. Intuitively, the only advantage of seed money over non-announcement in a large economy is in its signaling potential through the leader's gift. Thus, the lack of significant verification on the leader's part would remove this advantage of seed money over non-announcement, opening the possibility for either scheme to emerge as a signal of high quality.

4.2 Multiple quality types

Consider an extension of the base model to finite quality levels where $q \in \{q_1, q_2, ..., q_t\}$ with $t > 2$ and $q_{j-1} < q_j$ for all $j \in (2, t]$. The corresponding distribution of types $\pi = (\pi_1, \pi_2, ..., \pi_t)$ denotes the likelihood of each type prior to any action being taken by the players. The information structure and timing of the game is identical to the base model.

[17] Proposition 4 stands in contrast to Vesterlund (2003), who shows the existence of an equilibrium, in which seed money results in full information acquisition by the lead donor. The possibility of matching and the fact that the low quality is non-zero ($q_l > 0$) precludes such equilibrium in our setting since the inability of the low quality charity to pool with the high under seed money makes matching strictly more attractive for the low quality charity.

Analogous to the base model, the lead donor's type space in the donation game $\widetilde{\mathcal{Q}}_L \in \{q_1, ...q_j, q_U(\pi^Z), q_{j+1}, ..., q_t\}$ can take $t+1$ values as it includes the possibility of the lead donor choosing to remain uninformed, in which case her type is the expected quality $q_U(\pi^Z) = \sum_{j=1}^{t} \pi^Z(q_j)q_j$. Given the probability of information acquisition, α^Z, and letting $e = \{1, 2, ..., t\} \cup \{U\}$, the prior belief, $\widetilde{\eta}_e^Z$, and posterior belief, $\widetilde{\mu}_e^Z(G^Z)$, of type q_e are given by eq. (14) and eq. (15), respectively.

Similar to the two type case, in the least costly separating equilibrium the lead donor's contribution amount is perfectly informative of her type with $\widetilde{G}^M(q_e) > \widetilde{G}^S(q_e)$ for all q_e. The corresponding value of information is

$$\Delta^Z(\pi^Z) = \sum_{j=1}^{t} \pi^Z(q_j)\overline{V}_L(q_j) - \overline{V}_L(q_U(\pi^Z)). \tag{21}$$

It is straightforward to verify that the fully informed equilibrium exists as long as $\Delta^M(\pi) \geq k$ and necessitates pooling on matching. The other extreme of fully uninformed equilibrium requires each charity and each scheme on the equilibrium path to raise the same amount of money. Thus, similar to the two-type case, we focus our analysis on *SPI* equilibria defined by Definition 3. The following Lemma states that in any *SPI* equilibrium, information acquisition has to be limited under matching and positive under seed money to induce seed money fundraising by some charity types.

Lemma 5 *Every SPI equilibrium satisfies 1) $\widetilde{\alpha}^{M,*} < 1$ and $\widetilde{\alpha}^{S,*} > 0$; 2) $\widetilde{\pi}^{S,*}(q_j) < 1$ for all $j \in \{1, t\}$.*

Limited information acquisition under matching ($\widetilde{\alpha}^{M,*} < 1$) is necessary to prevent unraveling, in which each charity deviates to matching. To see this, note that with full information under matching, total donations under matching $\widetilde{G}^{M,*}(q_e)$ must dominate the expected donations under seed money for each charity with above average quality $q_e > q_U(\widetilde{\pi}^{S,*})$ since $\widetilde{G}^{M,*}(q_e) > \widetilde{G}^{S,*}(q_e) > \widetilde{G}^{S,*}(\widetilde{\pi}^{S,*})$. Intuitively, a charity is willing to solicit for seed money only if it generates more favorable beliefs about its type under seed money. This implies that any charity above the average quality $q_U(\widetilde{\pi}^{S,*})$ would prefer to avoid seed money. This would reduce the expected quality under seed money, causing further unraveling, in which all charities gravitate towards matching. Thus, to induce seed money fundraising, matching should be associated with less than perfect information acquisition.

Similar dynamics as the one described above would take place if there is no information acquisition under seed money. Then, the expected giving under matching $\alpha(M)\widetilde{G}^{M,*}(q_e) + (1-\alpha(M))\widetilde{G}^{M,*}(q_U(\widetilde{\pi}^{M,*}))$ is strictly increasing in the quality, while the expected giving under seed money is uniform across the charities, $\widetilde{G}^{S,*}(q_U(\widetilde{\pi}^{S,*}))$. This implies that the highest quality types would choose matching and thus the expected quality under matching,

$q_U(\widetilde{\pi}^{M,*})$, should exceed the one under seed money, $q_U(\widetilde{\pi}^{S,*})$. Consequently, any type $q_e > q_U(\widetilde{\pi}^{S,*})$ would have strict incentives to deviate to matching, further reducing $q_U(\widetilde{\pi}^{M,*})$ and causing all charity types to gravitate towards matching. Thus, some information acquisition under seed money ($\alpha^{S,*} > 0$) is necessary to make seed money fundraising attractive.

The second property in Lemma 5 follows immediately from the first one. In order for information acquisition to take place under seed money, it must be the case that the value of information is positive, which necessitates (partial) pooling, i.e. $\widetilde{\pi}^{S,*}(q_e) < 1$. Even though seed money is only partially informative about the charity's quality in equilibrium, the following Proposition states that it is associated with higher expected quality relative to matching.

Proposition 8 *In every SPI equilibrium, the seed money scheme is associated with higher expected quality, i.e.,* $q_U(\widetilde{\pi}^{S,*}) > q_U(\widetilde{\pi}^{M,*})$.

Proposition 8 generalizes our main result by showing that for arbitrary discrete distribution of types, seed money on the equilibrium path must be associated with higher expected quality relative to matching. To glean more insight into the equilibrium forces that drive this result, note that the highest type present in seed money $\overline{q}^S > q_U(\widetilde{\pi}^{S,*})$ must necessarily exceed the expected quality under match, i.e. $\overline{q}^S > q_U(\widetilde{\pi}^{M,*})$ to prevent $\overline{q}^S$ from deviating to matching[18]. Moreover, $\overline{q}^S$ finds seed money attractive either because it leads to higher uninformed giving, $q_U(\widetilde{\pi}^{S,*}) > q_U(\widetilde{\pi}^{M,*})$, or has an informational advantage over matching, $\widetilde{\alpha}^{S,*} > \widetilde{\alpha}^{M,*}$. However, if the advantage is coming purely from information acquisition, then the lowest type under seed money $\underline{q}^S$ must have strict incentives to deviate to matching. To see this, note that by definition $\underline{q}^S < q_U(\widetilde{\pi}^{S,*}) < q_U(\widetilde{\pi}^{M,*})$ implying that information acquisition is never good news for $\underline{q}^S$. Thus, matching would be a more attractive option for $\underline{q}^S$ as it is both less informative and associated with more optimistic beliefs about its type. This shows that $q_U(\widetilde{\pi}^{S,*}) > q_U(\widetilde{\pi}^{M,*})$ is a necessary condition to prevent deviation by both the highest ($\overline{q}^S$) and the lowest ($\underline{q}^S$) type under seed money.

Unlike the two-type case, characterizing the set of *SPI* equilibria can be challenging. In the two-type model, the higher expected quality under seed requires that the high quality charity chooses seed money more often than the low quality charity. Thus, the likelihood of choosing seed money has to be monotonically increasing in the quality of the charity. This monotonic relationship does not necessarily hold for arbitrary large quality set. This is because the contribution gap under match and seed given an informed donor, i.e. $\widetilde{G}^{M,*}(q_j) - \widetilde{G}^{S,*}(q_j)$, is not necessarily monotonic in quality as the following example illustrates.

Example 5 *Let* $t = 4$ *with quality set* $q = \{0.01, 0.95, 1, 2.5\}$ *and corresponding distribution* $\pi = \{0.6, 0.25, 0.05, 0.1\}$. *Moreover, suppose that* $n = 2$, *with* $w_L = 1000$ *for the lead donor and* $w_F = 100$

[18] Note that if $q_U(\widetilde{\pi}^{M,*}) > \overline{q}^S > q_U(\widetilde{\pi}^{S,*})$, then $\widetilde{G}^{M,*}(q_U(\widetilde{\pi}^{M,*})) > \widetilde{G}^{M,*}(\overline{q}^S) > \widetilde{G}^{S,*}(\overline{q}^S) > \widetilde{G}^{S,*}(\widetilde{\pi}^{S,*})$.

for the follower donor. Then, the following strategies constitute a sequential equilibrium:

$$\beta^{S,*}(q) = (0, 0.997, 0, 1),\ \widetilde{\alpha}^{S,*} = 1,\ \widetilde{\alpha}^{M,*} = 0.900$$

giving rise to posterior beliefs:

$$\widetilde{\eta}^{S}_{e}(q) = (0, 0.71, 0, 0.29), \widetilde{\eta}^{M}_{e}(q) = (0.92, 0.005, 0.075, 0).$$

Then, the expected qualities are $q_U(\widetilde{\pi}^{S,*}) = 1.394$ *and* $q_U(\widetilde{\pi}^{M,*}) = 0.087$ *with corresponding informed and uninformed giving*

$$\widetilde{G}^{S,*}(q_j) = (0.1, 474, 500, 862),\ \widetilde{G}^{S,*}(\widetilde{\pi}^{S,*}) = 660$$

$$\widetilde{G}^{M,*}(q_j) = (0.13, 526, 561, 947),\ \widetilde{G}^{M,*}(\widetilde{\pi}^{M,*}) = 10$$

The expected equilibrium contributions are $E[\widetilde{G}^{S,*}] = 585$ *and* $E[\widetilde{G}^{M,*}] = 40$.

5 Concluding remarks

Our analysis provides a theoretical foundation to understand the recent empirical findings in favor of seed money fundraising. It suggests that seed money can be used as a signaling tool for high quality charities to differentiate themselves from lower quality ones. This conclusion is rather robust since we find that in every equilibrium, in which seed money is on the equilibrium path, seed money leadership gift necessarily emerges as a signal of higher quality compared to a matching gift. This finding is also robust to a few notable variations and extensions of our model. In particular, the model easily extends to arbitrary number of finite types and reveals that, while the incentives for seed money are not necessarily monotonically increasing in the charity's quality, the equilibrium expected quality is always higher under seed money compared to a matching gift in any *SPI* equilibrium. This finding is also robust to extending the charity's choice set to include no leadership giving similar to Vesterlund (2003).

On the empirical side, our model provides a few testable hypotheses. First, it suggest that newer charities may be more eager to seek seed money financing compared to established charities since the former are more concerned with reputation building among donors. Second, it predicts that donors would respond differently to an announcement of a matching gift if a charity is less established compared to a more established one. This could help explain the mixed results in the existing literature on the impact of a matching gift announcement and could be a profitable avenue for future experimental study.

References

Adena, Maja and Steffen Huck, "Matching donations without crowding out? Some theoretical considerations, a field, and a lab experiment," *Journal of Public Economics*, 2017, *148*, 32–42.

Admati, Anat R and Motty Perry, "Joint projects without commitment," *The Review of Economic Studies*, 1991, *58* (2), 259–276.

Alpizar, Francisco, Fredrik Carlsson, and Olof Johansson-Stenman, "Anonymity, reciprocity, and conformity: Evidence from voluntary contributions to a national park in Costa Rica," *Journal of Public Economics*, 2008, *92* (5), 1047–1060.

Andreoni, James, "Privately provided public goods in a large economy: the limits of altruism," *Journal of public Economics*, 1988, *35* (1), 57–73.

_ , "Toward a theory of charitable fund-raising," *Journal of Political Economy*, 1998, *106* (6), 1186–1213.

_ , "Leadership giving in charitable fund-raising," *Journal of Public Economic Theory*, 2006, *8* (1), 1–22.

_ **and A Abigail Payne**, "Do government grants to private charities crowd out giving or fund-raising?," *The American Economic Review*, 2003, *93* (3), 792–812.

_ **and** _ , "Is crowding out due entirely to fundraising? Evidence from a panel of charities," *Journal of Public Economics*, 2011, *95* (5), 334–343.

Bag, Parimal Kanti and Santanu Roy, "On sequential and simultaneous contributions under incomplete information," *International Journal of Game Theory*, 2011, *40* (1), 119–145.

Bergstrom, Theodore, Lawrence Blume, and Hal Varian, "On the private provision of public goods," *Journal of public economics*, 1986, *29* (1), 25–49.

Cho, In-Koo and David M Kreps, "Signaling games and stable equilibria," *The Quarterly Journal of Economics*, 1987, *102* (2), 179–221.

Croson, Rachel and Jen Yue Shang, "The impact of downward social information on contribution decisions," *Experimental Economics*, 2008, *11* (3), 221–233.

Eckel, Catherine C and Philip J Grossman, "Rebate versus matching: does how we subsidize charitable contributions matter?," *Journal of Public Economics*, 2003, *87* (3), 681–701.

— **and** — , "Do donors care about subsidy type? An experimental study," in "Experiments Investigating Fundraising and Charitable Contributors," Emerald Group Publishing Limited, 2006, pp. 157–175.

— **and** — , "Subsidizing charitable giving with rebates or matching: Further laboratory evidence," *Southern Economic Journal*, 2006, pp. 794–807.

— **and** — , "Subsidizing charitable contributions: a natural field experiment comparing matching and rebate subsidies," *Experimental Economics*, 2008, *11* (3), 234–252.

— **,** — **, and Angela Milano**, "Is more information always better? An experimental study of charitable giving and Hurricane Katrina," *Southern Economic Journal*, 2007, *74* (2), 388–412.

Falk, Armin, "Gift exchange in the field," *Econometrica*, 2007, *75* (5), 1501–1511.

Frey, Bruno S and Stephan Meier, "Social comparisons and pro-social behavior: Testing" conditional cooperation" in a field experiment," *The American Economic Review*, 2004, *94* (5), 1717–1722.

Gong, Ning and Bruce D Grundy, "The design of charitable fund-raising schemes: Matching grants or seed money," *Journal of Economic Behavior & Organization*, 2014, *108*, 147–165.

Güth, Werner, M Vittoria Levati, Matthias Sutter, and Eline Van Der Heijden, "Leading by example with and without exclusion power in voluntary contribution experiments," *Journal of Public Economics*, 2007, *91* (5), 1023–1042.

Huck, Steffen and Imran Rasul, "Matched fundraising: Evidence from a natural field experiment," *Journal of Public Economics*, 2011, *95* (5), 351–362.

— **,** — **, and Andrew Shephard**, "Comparing Charitable Fundraising Schemes: Evidence from a Natural Field Experiment and a Structural Model," *American Economic Journal: Economic Policy*, 2015, *7* (2), 326–69.

Karlan, Dean and John A List, "Does price matter in charitable giving? Evidence from a large-scale natural field experiment," *The American economic review*, 2007, *97* (5), 1774–1793.

— **,** — **, and Eldar Shafir**, "Small matches and charitable giving: Evidence from a natural field experiment," *Journal of Public Economics*, 2011, *95* (5), 344–350.

Khanna, Jyoti and Todd Sandler, "Partners in giving:: The crowding-in effects of UK government grants," *European Economic Review*, 2000, *44* (8), 1543–1556.

Krasteva, Silvana and Huseyin Yildirim, "(Un) Informed charitable giving," *Journal of Public Economics*, 2013, *106*, 14–26.

Landry, Craig E, Andreas Lange, John A List, Michael K Price, and Nicholas G Rupp, "Toward an Understanding of the Economics of Charity: Evidence from a Field Experiment," *The Quarterly Journal of Economics*, 2006, pp. 747–782.

List, John A and David Lucking-Reiley, "The effects of seed money and refunds on charitable giving: Experimental evidence from a university capital campaign," *Journal of Political Economy*, 2002, *110* (1), 215–233.

Manelli, Alejandro M, "Cheap talk and sequential equilibria in signaling games," *Econometrica: Journal of the Econometric Society*, 1996, pp. 917–942.

Meier, Stephan, "Do subsidies increase charitable giving in the long run? Matching donations in a field experiment," *Journal of the European Economic Association*, 2007, *5* (6), 1203–1222.

_ and Bruno S Frey, "Matching donations: subsidizing charitable giving in a field experiment," 2004.

Okten, Cagla and Burton A Weisbrod, "Determinants of donations in private nonprofit markets," *Journal of Public Economics*, 2000, *75* (2), 255–272.

Potters, Jan, Martin Sefton, and Lise Vesterlund, "After youendogenous sequencing in voluntary contribution games," *Journal of public Economics*, 2005, *89* (8), 1399–1419.

_ , _ , and _ , "Leading-by-example and signaling in voluntary contribution games: an experimental study," *Economic Theory*, 2007, pp. 169–182.

Romano, Richard and Huseyin Yildirim, "Why charities announce donations: a positive perspective," *Journal of Public Economics*, 2001, *81* (3), 423–447.

Rondeau, Daniel and John A List, "Matching and challenge gifts to charity: evidence from laboratory and natural field experiments," *Experimental economics*, 2008, *11* (3), 253–267.

Shang, Jen and Rachel Croson, "A field experiment in charitable contribution: The impact of social information on the voluntary provision of public goods," *The Economic Journal*, 2009, *119* (540), 1422–1439.

Silverman, Wendy K, Steven J Robertson, Jimmy L Middlebrook, and Ronald S Drabman, "An investigation of pledging behavior to a national charitable telethon," *Behavior Therapy*, 1984, *15* (3), 304–311.

Soetevent, Adriaan R, "Anonymity in giving in a natural contexta field experiment in 30 churches," *Journal of public Economics*, 2005, *89* (11), 2301–2323.

Varian, Hal R, "Sequential contributions to public goods," *Journal of Public Economics*, 1994, *53* (2), 165–186.

Vesterlund, Lise, "The informational value of sequential fundraising," *Journal of Public Economics*, 2003, *87* (3), 627–657.

Warr, Peter G, "The private provision of a public good is independent of the distribution of income," *Economics letters*, 1983, *13* (2-3), 207–211.

Yildirim, Huseyin, "Andreoni–McGuire algorithm and the limits of warm-glow giving," *Journal of Public Economics*, 2014, *114*, 101–107.

Yörük, Barış K, "Charity ratings," *Journal of Economics & Management Strategy*, 2016, *25* (1), 195–219.

Appendix

Proof of Lemma 1

Let

$$F^S = \{i | w_i \geq \phi(qv'(G^S))\};\ \ F^M = \{i | w_i \geq \phi\left(qv'(G^M)\frac{G^M}{G^M_F}\right)\} \tag{A-1}$$

denote the set of contributing donors under $Z = \{S, M\}$ and $n^Z = |F^Z| -$ the number of contributing donors for a fixed G^Z.

Consider $Z = S$. By eq. (3), $\frac{dg_i^S}{dG^S} = \phi'(qv'(G^S))qv''(G^S) < 0$ whenever $g_i^S > 0$ since $\phi'(\cdot) < 0$ and $v''(\cdot) < 0$. Thus, there exists a unique $G_i^S > 0$ such that $w_i = \phi(qv'(G_i^S))$ and by eq. (3), $g_i(G_i^S) = 0$. This implies that $G_F^S(q, G_i^S | F^S) = G_F^S(q, G_i^S | F^S \setminus \{i\})$ and thus $\dfrac{dG_F^Z(q, G^Z)}{dG^Z} = \dfrac{\partial G_F^Z(q, G^Z)}{\partial G^Z}$. Then, differentiating eq. (4) with respect to G^S results in

$$\frac{dG_F^S(q, G^S)}{dG^S} = -n^S \phi'(qv'(G^S))qv''(G^S) < 0 \tag{A-2}$$

Analogously, for $Z = M$, $\dfrac{dG_F^M(q, G^M)}{dG^M} = \dfrac{\partial G_F^M(q, G^M)}{\partial G^M}$ where implicit differentiation of eq. (6) with respect to G^M results in

$$\frac{dG_F^M(q, G^M)}{dG^M} = \frac{-qv''(G^M)G^M - qv'(G^M)}{\frac{[G_F^M]^2}{n^M G^M} h''(qv'(G^M)\frac{G^M}{G_F^M}) - qv'(G^M)} \frac{G_F^M}{G^M} \in (0, \frac{G_F^M}{G^M}) \tag{A-3}$$

since $\phi'(\cdot) = \frac{1}{h''(\cdot)}$, $v'(\cdot) > 0$ and $\left|\frac{Gv''(G)}{v'(G)}\right| \leq 1$. ■

Lemma A-1 *Let* $\underline{G}^Z = G_F^Z(q, \underline{G}^Z) > 0$. *Then, for any* $q > 0$ *and total donations* $G^Z \geq \underline{G}^Z$, $G_F^S(q, G^Z) \leq G_F^M(q, G^Z)$ *with strict inequality for* $G^Z > \underline{G}^Z$.

Proof of Lemma A-1

By definition, $\frac{\underline{G}^M}{G_F^M(q, \underline{G}^M)} = 1$ and thus comparing eq. (3) and eq. (5), $g_i^M(q, \underline{G}^M) = g_i^S(q, \underline{G}^M)$ for all i. This, in turn, implies that $G_F^S(q, \underline{G}^M) = G_F^M(q, \underline{G}^M) = \underline{G}^M$. Therefore, $\underline{G}^S = \underline{G}^M = \underline{G}^Z$. Moreover, $\underline{G}^Z > 0$ since $q_l v'(0) > h'(\underline{w})$, which by eq. (2) implies that $g_i^S(q, 0) > 0$, contradicting $\underline{G}^Z = 0$.

For $G^Z > \underline{G}^Z$, note that $\frac{d}{dG^Z}\left[\frac{G^Z}{G_F^M(q, G^Z)}\right] = \frac{1}{G_F^M(q, G^Z)}\left[1 - \frac{G^Z}{G_F^M(q, G^Z)}\frac{dG_F^M(q, G^Z)}{dG^Z}\right] > 0$ since by eq. (A-3), $\frac{dG_F^M(q, G^Z)}{dG^Z} < \frac{G_F^M(q, G^Z)}{G^Z}$. Thus, $\frac{G^Z}{G_F^M(q, G^Z)} > 1$. This, in turn, implies that $\phi(qv'(G^Z)\frac{G^Z}{G_F^M}) < \phi(qv'(G^Z))$ since $\phi(\cdot)$ is a decreasing function of its argument. Then, by eq. (3) and eq. (5),

$g_i^M(q, G^Z) \geq g_i^S(q, G^Z)$ for all i with strict inequality whenever $g_i^M(q, G^Z) > 0$. This, in turn, implies that $F^S \subseteq F^M$ and $G_F^S(q, G^Z) = \sum_{i \in F^S} g_i^S(q, G^Z) < \sum_{i \in F^M} g_i^M(q, G^Z) = G_F^M(q, G^Z)(> 0)$. ■

Proof of Proposition 1

To show that $G^{M,*}(q) > G^{S,*}(q)$ for all $n < \infty$, note that $G^{S,*}(q)$ satisfies eq. (8). By Lemma A-1, $G_F^M(q, G^{S,*}) \geq G_F^S(q, G^{S,*})$ and by Lemma 1, $\frac{dG_F^S(q,G^{S,*})}{dG^S} < 0 < \frac{dG_F^M(q,G^{S,*})}{dG^M} < 1$. Therefore, since $h''(\cdot) < 0$

$$h'(w_L - G^{S,*} + G_F^M(q, G^{S,*})) \left(-1 + \frac{dG_F^M(q, G^{S,*})}{dG^M} \right) - qv'(G^{S,*}) > 0,$$

implying that $G^{M,*}(q) > G^{S,*}(q)$.

For the remainder of the proof, let

$$G^{Z,\infty}(q) = \lim_{n \to \infty} G^{Z,*}(q). \tag{A-4}$$

To show that $G^{M,\infty}(q) > G^{S,\infty}(q)$, first note that $G^{Z,\infty}(q) < \infty$ for all Z since $\lim_{G^Z \to \infty} \phi(qv'(G^Z)) = \infty$ due to the fact that $\phi(\cdot) > 0$ and is strictly increasing in G^Z. As a result eq. (3) and eq. (5) imply that $g_i^Z(q, G^Z) = 0$ for some finite $G^Z < \infty$. To determine $G^{S,\infty}(q)$, let $w^S(n) = \phi(qv'(G^{S,*}(q)))$, implying that $g_i(q, G^{S,*}(q))) = w_i - w^S(n)$. As shown by Andreoni (1998), $w^S(\infty) = \lim_{n \to \infty} w^S(n) = \overline{w}$. Otherwise, adding k new followers in an infinite economy, will results in average new donations $\frac{1}{k} G_F^S(q, G^{S,\infty}(q)) = \frac{1}{k} \sum_{w_i > w^S(\infty)} [w_i - w^S(\infty)]$. By the law of large numbers, $\lim_{k \to \infty} \frac{1}{k} G_F^S(q, G^{S,\infty}(q)) = \int_{w^S(\infty)}^{\overline{w}} (w_i - w^S(\infty)) f(w_i) dw_i > 0$, contradicting $G^{S,\infty}(q)$ being a finite asymptote. Thus, given $w^S(\infty) = \overline{w}$, $G^{S,\infty}(q)$ uniquely solves

$$-h'(\overline{w}) + qv'(G^{S,\infty}(q)) = 0. \tag{A-5}$$

To show that $G^{M,\infty}(q) > G^{S,\infty}(q)$ note that by eq. (A-3)

$$\lim_{n \to \infty} \frac{dG_F^M(q, G^M)}{dG^M} = \frac{-qv''(G^{M,\infty}(q))G^{M,\infty}(q) - qv'(G^{M,\infty}(q))}{-qG^{M,\infty}(q)} \frac{G_F^{M,\infty}(q)}{G^{M,\infty}(q)} > 0$$

since $G^{M,\infty}(q) < \infty$. Therefore, by eq. (8) and eq.(A-5)

$$h'(\overline{w}) \left(-1 + \frac{dG_F^M(q, G^{S,\infty}(q))}{dG^M} \right) + qv'(G^{S,\infty}(q)) > 0.$$

This, in turn, implies that $G^{M,\infty}(q) > G^{S,\infty}(q)$.

■

Proof of Proposition 2

To prove that $\overline{G}^{S,*}(q_h) < \overline{G}^{M,*}(q_h)$ for sufficiently large n, it suffices to show that $\lim_{n\to\infty} \overline{G}^{S,*}(q_h) = G^{S,\infty}(q_h)$ (defined by eq. (A-4)) since by definition $\overline{G}^{M,*}(q_h) \geq G^{M,*}(q_h)$ and by Proposition 1 $G^{S,\infty}(q_h) < G^{M,\infty}(q_h)$. Then, by continuity of $\overline{G}^{Z,*}(q_h)$, it follows that there exists $\overline{n}$ such that $\overline{G}^{S,*}(q_h) < \overline{G}^{M,*}(q_h)$ for $n > \overline{n}$.

To show that $\lim_{n\to\infty} \overline{G}^{S,*}(q_h) = G^{S,\infty}(q_h)$, we first establish that $\overline{G}^{S,*}(q_h) \leq G^{S,\infty}(q_h)$ for all n. Suppose by contradiction that $\overline{G}^{S,*}(q_h) > G^{S,\infty}(q_h)$ for some n. By eq. (A-5), $\overline{w} = \phi(q_h v'(G^{S,\infty}(q_h)))$. Since $\phi(\cdot)$ is increasing in G^S, eq. (3) implies that $\overline{G}^S_F(q_h, \overline{G}^{S,*}(q_h)) = 0$ and thus the lead donor is the sole contributor in equilibrium. Then, the lead donor's gift satisfies eq. (3) and results in $\overline{G}^{S,*}(q_h) < G^{S,\infty}(q_h)$, leading to a contradiction. Therefore, $\overline{G}^{S,*}(q_h) \leq G^{S,\infty}(q_h)$ for all n. Moreover, since $\overline{G}^{S,*}(q_h) \geq G^{S,*}(q_h)$ and $G^{S,*}(q_h)$ converges to $G^{S,\infty}(q_h)$, it follows that $\lim_{n\to\infty} \overline{G}^{S,*}(q_h) = G^{S,\infty}(q_h)$. This establishes $\overline{G}^{S,*}(q_h) < \overline{G}^{M,*}(q_h)$ for sufficiently large $n > \overline{n}$. $\overline{Z}^*(q) = M$ follows immediately. ∎

Proof of Lemma 2

Given $\overline{V}_L(q_e) = \overline{u}_L(q_e, q_e, \widetilde{G}^{Z,*}(q_e))$, we can re-write eq. (17) as

$$\Delta^Z(\pi^Z) = \sum_{j=\{l,h\}} \widetilde{\pi}_j^{Z,*} \left[\overline{u}_L(q_j, q_j, \widetilde{G}^{Z,*}(q_j)) - \overline{u}_L(q_j, \widetilde{q}_U^{Z,*}, \widetilde{G}^{Z,*}(\widetilde{q}_U^{Z,*}))\right] \tag{A-6}$$

Clearly, $\Delta^Z(\pi^Z)$ is continuous in $\widetilde{\pi}_h^{Z,*}$ since $\widetilde{G}^{Z,*}(q_e)$ and $\widetilde{q}_U^{Z,*}$ are continuous functions.

If $\widetilde{\pi}_j^{Z,*} = 1$ for some j, eq. (13) reduces to $\widetilde{q}_U^{Z,*} = q_j$. Thus, $\Delta^Z(1,0) = \Delta^Z(0,1) = 0$ follows immediately from $\widetilde{\pi}_h^{Z,*} + \widetilde{\pi}_l^{Z,*} = 1$.

If $\widetilde{\pi}_j^{Z,*} \in (0,1)$, it suffices to show that $\overline{u}_L(q_h, q_h, \widetilde{G}^{Z,*}(q_h)) > \overline{u}_L(q_h, \widetilde{q}_U^{Z,*}, \widetilde{G}^{Z,*}(\widetilde{q}_U^{Z,*}))$. If $\widetilde{G}^{Z,*}(q_h) = G^{Z,*}(q_h)$, this inequality follows immediately from the fact that $G^{Z,*}(q_h)$ maximizes $\overline{u}_L(q_h, q_h, G^Z)$ and $\overline{u}_L(q_h, \widetilde{q}^Z, G^Z)$ is strictly decreasing in $\widetilde{q}^Z$. For $\widetilde{G}^{Z,*}(q_h) > G^{Z,*}(q_h)$, it must be true that $\overline{u}_L(\widetilde{q}_U^{Z,*}, \widetilde{q}_U^{Z,*}, \widetilde{G}^{Z,*}(\widetilde{q}_U^{Z,*})) = \overline{u}_L(\widetilde{q}_U^{Z,*}, q_h, \widetilde{G}^{Z,*}(q_h))$. Note that $\overline{u}_L(q_h, q_h, \widetilde{G}^{Z,*}(q_h)) = \overline{u}_L(\widetilde{q}_U^{Z,*}, q_h, \widetilde{G}^{Z,*}(q_h)) + (q_h - \widetilde{q}_U^{Z,*})v(\widetilde{G}^{Z,*}(q_h))$ and $\overline{u}_L(q_h, \widetilde{q}_U^{Z,*}, \widetilde{G}^{Z,*}(\widetilde{q}_U^{Z,*})) = \overline{u}_L(\widetilde{q}_U^{Z,*}, \widetilde{q}_U^{Z,*}, \widetilde{G}^{Z,*}(\widetilde{q}_U^{Z,*})) + (q_h - \widetilde{q}_U^{Z,*})v(\widetilde{G}^{Z,*}(\widetilde{q}_U^{Z,*})))$. Then, $\overline{u}_L(q_h, q_h, \widetilde{G}^{Z,*}(q_h)) > \overline{u}_L(q_h, \widetilde{q}_U^{Z,*}, \widetilde{G}^{Z,*}(\widetilde{q}_U^{Z,*}))$ follows immediately from the fact that $v'(\cdot) > 0$ and $\widetilde{G}^{Z,*}(q_h) > \widetilde{G}^{Z,*}(\widetilde{q}_U^{Z,*})$. ∎

Proof of Proposition 3

Let

$$\widetilde{G}_E^{Z,*}(q, \widetilde{\alpha}^{Z,*}) = \widetilde{\alpha}^{Z,*}\widetilde{G}^{Z,*}(q) + (1 - \widetilde{\alpha}^{Z,*})\widetilde{G}^{Z,*}(\widetilde{q}_U^{Z,*}) \tag{A-7}$$

It is immediately obvious that $\widetilde{G}_E^{Z,*}(q_h, 0) = \widetilde{G}_E^{Z,*}(q_l, 0)$. Moreover, by definition $\widetilde{\beta}^{M,*}(q) = 1 - \widetilde{\beta}^{S,*}(q)$ with

$$\widetilde{\beta}^{Z,*}(q) = \underset{\beta^Z}{argmax} \sum_Z \beta^Z \widetilde{G}_E^{Z,*}(q, \widetilde{\alpha}^{Z,*}).$$

The linearly of the above objective function implies that $\widetilde{\beta}^{Z,*}(q) \in (0,1)$ if and only if $\widetilde{G}_E^{M,*}(q, \widetilde{\alpha}^{M,*}) = \widetilde{G}_E^{S,*}(q, \widetilde{\alpha}^{S,*})$. Together with $\widetilde{G}_E^{Z,*}(q_h, 0) = \widetilde{G}_E^{Z,*}(q_l, 0)$, this implies that $\widetilde{G}_E^{M,*}(q_h, 0) = \widetilde{G}_E^{M,*}(q_l, 0) = \widetilde{G}_E^{S,*}(q_l, 0) = \widetilde{G}_E^{S,*}(q_h, 0)$, completing the proof. ■

Proof of Proposition 4

First, we show that in any equilibrium with $\widetilde{\alpha}^{Z,*} = 1$ for all Z on the equilibrium path, $Z = S$ must be off the path, i.e. $\widetilde{\beta}^{S,*}(q) = 1 - \widetilde{\beta}^{M,*}(q) = 0$ for all q. By means of contradiction, suppose that $\widetilde{\beta}^{S,*}(q) > 0$ for some q. Then $\widetilde{\alpha}^{S,*} = 1$, $\Delta^S(\widetilde{\pi}^{S,*}) \geq k > 0$, which by Lemma 2 implies that $\widetilde{\pi}_j^{S,*} \in (0,1)$ for all j. This, in turn, by eq. (10) requires that $\widetilde{\beta}^{S,*}(q) > 0$ for all q. Moreover, by eq. (A-7) the expected equilibrium contributions are $\widetilde{G}_E^{S,*}(q) = \widetilde{G}^{S,*}(q)$ for $\widetilde{\alpha}^{S,*} = 1$. However, this results in a profitable deviation by q_l to $\beta^S(q_l) = 0$ since $\widetilde{G}^{S,*}(q_l) < \widetilde{G}^{M,*}(q_l)$. Therefore, in equilibrium $\widetilde{\beta}^{S,*}(q) = 0$ for all q.

To establish the existence of an equilibrium with $\widetilde{\alpha}^{M,*} = 1$, note that by eq. (10), $\widetilde{\beta}^{M,*}(q) = 1$ implies that $\widetilde{\pi}^{M,*} = \pi$. Therefore, $\widetilde{\alpha}^{M,*} = 1$ requires $\Delta^M(\pi) \geq k$. No deviation incentives to $\beta^S(q) > 0$ is guaranteed by an off-equilibrium belief $\widetilde{\alpha}^{S,*} = 1$ since $\widetilde{G}^{S,*}(q) < \widetilde{G}^{M,*}(q)$ for all q. ■

Proof of Lemma 3

We first show the existence of an *SPI* equilibrium for $\Delta^S(\pi) \geq k$ and $\widetilde{G}^{S,*}(E[q]) > \widetilde{G}^{M,*}(q_l)$ by constructing such an equilibrium. By Lemma 2, $\Delta^S(\pi^S)$ is continuous in π_h^S and reaches a minimum at $\pi_h^S = 1$ with $\Delta^S(0,1) = 0$. This implies that there exists $\check{\pi}^S$ with $\check{\pi}_h^S > \pi_h$ such that $\Delta^S(\check{\pi}^S) = k$. Consider an equilibrium with $\widetilde{\pi}^{S,*} = \check{\pi}^S$ and $\widetilde{\beta}^{S,*}(q_h) = 1$. Then, by eq. (10), $\widetilde{\beta}^{S,*}(q_l) = \frac{\pi_h}{\pi_l}\left(\frac{1}{pi_h} - 1\right) \leq 1$ and $\widetilde{\pi}_l^{M,*} = 1$. It follows that $\widetilde{q}_U^{M,*} = q_l$ (by eq. (13)), and $\Delta^M(\widetilde{\pi}^{M,*}) = 0$ (by Lemma 2), implying that $\widetilde{\alpha}^{M,*} = 0$. Then, by eq. (A-7), $\widetilde{G}_E^{M,*}(q,0) = \widetilde{G}^{M,*}(q_l)$. Since $\Delta^S(\widetilde{\pi}^{S,*}) = k$, the lead donor is indifferent in her information acquisition strategy α^S. To prevent deviation from $\widetilde{\beta}^{S,*}(q_l)$, it suffices that $\widetilde{G}_E^{S,*}(q_l, \widetilde{\alpha}^{S,*}) = \widetilde{G}^{M,*}(q_l)$. Substituting for $\widetilde{G}_E^{S,*}(q_l, \widetilde{\alpha}^{S,*})$ from eq. (A-7) and solving for $\widetilde{\alpha}^{S,*}$ results in $\widetilde{\alpha}^{S,*} = \frac{\widetilde{G}^{S,*}(\widetilde{q}_U^{S,*}) - \widetilde{G}^{M,*}(q_l)}{\widetilde{G}^{S,*}(\widetilde{q}_U^{S,*}) - \widetilde{G}^{S,*}(q_l)} \in (0,1)$ since by eq. (13) $\widetilde{q}_U^{S,*} > E[q]$ as a result of $\widetilde{\pi}_h^{S,*} > \pi_h$, which implies that $\widetilde{G}^{S,*}(\widetilde{q}_U^{S,*}) > \widetilde{G}^{S,*}(E[q]) > \widetilde{G}^{M,*}(q_l) > \widetilde{G}^{S,*}(q_l)$.

To establish property 1), note that if $\widetilde{\beta}^{S,*}(q_j) = 0$ for some q_j then by eq. (10) $\widetilde{\pi}_j^{S,*} = 0$ and thus $\Delta^S(\widetilde{\pi}^{S,*}) = 0$ with $\widetilde{\alpha}^{S,*} = 0$ and $\widetilde{\alpha}^{M,*} > 0$ (by Definition 3). Then, by eq. (13) $\widetilde{q}_U^{S,*} = q_{-j}$ where $q_{-j} = \{l, h\} \setminus \{j\}$ and by eq. (A-7) $\widetilde{G}_E^{S,*}(q, \widetilde{\alpha}^{S,*}) = \widetilde{G}^{S,*}(q_{-j})$ for all q. If $q_j = q_h$ and $q_{-j} = q_l$, then there is strict deviation incentives to $\beta^M(q_l) = 1$ since $\widetilde{G}^{S,*}(q_l) < \widetilde{G}^{M,*}(q_l)$. If $q_j = q_l$ and $q_{-j} = q_h$, $\widetilde{\beta}^S(q_h) > 0$ implies that $\widetilde{G}^{M,*}(q_h) = \widetilde{G}_E^{M,*}(q_h, \widetilde{\alpha}^{M,*}) > \widetilde{G}_E^{M,*}(q_l, \widetilde{\alpha}^{M,*})$ due to the fact that $\widetilde{G}^{M,*}(q_h) > \widetilde{G}^{M,*}(q_l)$. This, in turn implies a profitable deviation to $\beta^S(q_l) = 1$. It follows that $\widetilde{\beta}^{S,*}(q_j) = 0$ for some q_j cannot be supported as an equilibrium and thus in any *SPI* equilibrium $\widetilde{\beta}^{S,*}(q) > 0$ for all q.

To show that $\widetilde{\alpha}^{M,*} < 1$, note by eq. (A-7) that $\widetilde{G}_E^{M,*}(q_h, 1) = \widetilde{G}^{M,*}(q_h) > \widetilde{G}^{S,*}(q_h) \geq \widetilde{G}_E^{M,*}(q_h, \widetilde{\alpha}^{M,*})$. Therefore, $\widetilde{\alpha}^{M,*} = 1$ results in $\widetilde{\beta}^{S,*}(q_h) = 0$, contradicting property 1). Analogously, $\widetilde{\alpha}^{S,*} = 1$ implies that $\widetilde{G}_E^{S,*}(q_l, 1) = \widetilde{G}^{S,*}(q_l) < \widetilde{G}^{M,*}(q_l) \leq \widetilde{G}_E^{M,*}(q_l, \widetilde{\alpha}^{M,*})$, which in turn implies that $\widetilde{\beta}^{S,*}(q_l) = 0$, contradicting property 1).

Finally, to establish that $\widetilde{\alpha}^{S,*} > 0$, note that by Definition 3, $\widetilde{\alpha}^{S,*} = 0$ implies that $\widetilde{\alpha}^{M,*} > 0$ and $\widetilde{\beta}^{M,*}(q) > 0$ for some q. Then, by eq. (A-7) $\widetilde{G}_E^{M,*}(q_h, \widetilde{\alpha}^{M,*}) > \widetilde{G}_E^{M,*}(q_l, \widetilde{\alpha}^{M,*})$ and $\widetilde{G}_E^{S,*}(q, \widetilde{\alpha}^{M,*}) = \widetilde{G}^{S,*}(\widetilde{q}_U^{S,*})$ for all q. Since $\widetilde{\beta}^{S,*}(q) > 0$ for all q, it must be true that $\widetilde{G}^{S,*}(\widetilde{q}_U^{S,*}) \geq \widetilde{G}_E^{M,*}(q_h, \widetilde{\alpha}^{M,*}) > \widetilde{G}_E^{M,*}(q_l, \widetilde{\alpha}^{M,*})$, which implies that $\widetilde{\beta}^{S,*}(q_l) = 1$ and thus $\widetilde{\beta}^{M,*}(q_h) \in (0,1)$. This, in turn, implies that $\widetilde{\pi}_h^{M,*} = 1$, which results in $\Delta^{M,*}(\widetilde{\pi}^{M,*}) = 0$ (by Lemma 2), contradicting $\alpha^{M,*} > 0$. Thus, $\widetilde{\alpha}^{S,*} > 0$. This completes the proof. ■

Proof of Proposition 5

By means of contradition, suppose that $\widetilde{q}_U^{S,*} \leq \widetilde{q}_U^{M,*}$. By Lemma 3, $\widetilde{\beta}^{S,*}(q) > 0$ for all q, which requires that $\widetilde{G}_E^{S,*}(q, \widetilde{\alpha}^{S,*}) \geq \widetilde{G}_E^{M,*}(q, \widetilde{\alpha}^{M,*})$, where $\widetilde{G}_E^{Z,*}(q, \widetilde{\alpha}^{Z,*})$ is defined by eq. (A-7). Let $\widetilde{\alpha}^{M,*} = \widetilde{\alpha}^{S,*} + \epsilon$. Then, $\widetilde{G}_E^{M,*}(q, \widetilde{\alpha}^{M,*})$ can be rewitten as

$$\widetilde{G}_E^{M,*}(q, \widetilde{\alpha}^{M,*}) = \widetilde{G}_E^{M,*}(q, \widetilde{\alpha}^{S,*}) + \epsilon \left[\widetilde{G}^{M,*}(q) - \widetilde{G}^{M,*}(\widetilde{q}_U^{M,*}) \right]$$

Notice that $\widetilde{G}_E^{M,*}(q, \widetilde{\alpha}^{S,*}) > \widetilde{G}_E^{S,*}(q, \widetilde{\alpha}^{S,*})$ since by Condition 2, $\widetilde{G}^{M,*}(q) > \widetilde{G}^{S,*}(q)$ and $\widetilde{G}^{M,*}(\widetilde{q}_U^{M,*}) > \widetilde{G}^{S,*}(\widetilde{q}_U^{S,*})$ for $\widetilde{q}_U^{M,*} > \widetilde{q}_U^{S,*}$. Thus, $\widetilde{G}_E^{S,*}(q_h, \widetilde{\alpha}^{S,*}) \geq \widetilde{G}_E^{M,*}(q_h, \widetilde{\alpha}^{M,*})$ requires $\epsilon > 0$ since $\widetilde{G}^{M,*}(q_h) > \widetilde{G}^{M,*}(\widetilde{q}_U^{M,*})$. This, however, results in $\widetilde{G}_E^{S,*}(q_l, \widetilde{\alpha}^{S,*}) < \widetilde{G}_E^{M,*}(q_l, \widetilde{\alpha}^{M,*})$ since $\widetilde{G}^{M,*}(q_l) < \widetilde{G}^{M,*}(\widetilde{q}_U^{M,*})$, contradicting $\widetilde{\beta}^{S,*}(q_l) > 0$. Therefore, $\widetilde{q}_U^{S,*} \leq \widetilde{q}_U^{M,*}$ cannot be supported in an *SPI* equilibrium, establishing that in any *SPI* equilibrium satisfies $\widetilde{q}_U^{S,*} > \widetilde{q}_U^{M,*}$. ■

Proof of Proposition 6

1) is established in the proof of Lemma 3. To establish 2), note that by Lemma 4, $\Delta^Z(\pi^Z)$ is continuous in π_h^Z and satisfies $\Delta^Z(1,0) = \Delta^Z(0,1) = 0$. Therefore, since $\Delta^Z(\pi) > k$,There exist unique values $0 < \underline{\pi}_h^Z < \pi_h < \overline{\pi}_h^Z$ that $\Delta^Z(\underline{\pi}_h^Z, 1 - \underline{\pi}_h^Z) = \Delta^Z(\overline{\pi}_h^Z, 1 - \overline{\pi}_h^Z)$. Let $\hat{\pi}_h^S = \overline{\pi}_h^S$ and $\hat{\pi}_h^S = \underline{\pi}_h^S$. Subsituting for $\hat{\pi}_h^S$ and $\hat{\pi}_h^M$ in eq. (10) yields $\widetilde{\beta}^{S,*}(q)$, where $0 < \widetilde{\beta}^{S,*}(q_l) < \widetilde{\beta}^{S,*}(q_h) < 1$ follow immediately from $\hat{\pi}_h^M < \pi_h < \hat{\pi}_h^S$. Lastly, we need to ensure that there is no deviation incentives from $\widetilde{\beta}^{S,*}(q)$, which requires $\widetilde{G}_E^{S,*}(q, \widetilde{\alpha}^{S,*}) = \widetilde{G}_E^{M,*}(q, \widetilde{\alpha}^{M,*})$ for all q, where $\widetilde{G}_E^{Z,*}(q, \widetilde{\alpha}^{Z,*})$ is defined by eq. (A-7). Solving for $\widetilde{\alpha}^{M,*}$ and $\widetilde{\alpha}^{S,*}$ yields:

$$\widetilde{\alpha}^{M,*} = \frac{\widetilde{G}^{S,*}(\widetilde{q}_U^{S,*}) - \widetilde{G}^{M,*}\widetilde{q}_U^{M,*})}{\widetilde{G}^{M,*}(q_h)\frac{\widetilde{G}^{S,*}(\widetilde{q}_U^{S,*}) - \widetilde{G}^{S,*}(q_l)}{\widetilde{G}^{S,*}(q_h) - \widetilde{G}^{S,*}(q_l)} + \widetilde{G}^{M,*}(q_l)\frac{\widetilde{G}^{S,*}(q_h) - \widetilde{G}^{S,*}(\widetilde{q}_U^{S,*})}{\widetilde{G}^{S,*}(q_h) - \widetilde{G}^{S,*}(q_l)} - \widetilde{G}^{M,*}(\widetilde{q}_U^{S,*})} \in (0,1)$$

$$\widetilde{\alpha}^{S,*} = \frac{\widetilde{G}^{S,*}(\widetilde{q}_U^{S,*}) - \widetilde{G}^{M,*}(\widetilde{q}_U^{M,*})}{\widetilde{G}^{S,*}(\widetilde{q}_U^{S,*}) - \widetilde{G}^{S,*}(q_h)\frac{\widetilde{G}^{M,*}(\widetilde{q}_U^{M,*}) - \widetilde{G}^{M,*}(q_l)}{\widetilde{G}^{M,*}(q_h) - \widetilde{G}^{M,*}(q_l)} - \widetilde{G}^{S,*}(q_l)\frac{\widetilde{G}^{M,*}(q_h) - \widetilde{G}^{M}(\widetilde{q}_U^{M,*})}{\widetilde{G}^{M,*}(q_h) - \widetilde{G}^{M,*}(q_l)}} \in (0,1)$$

where $\widetilde{\alpha}^{Z,*} \in (0,1)$ follows from

$$\widetilde{G}^{S,*}(q_l) < \widetilde{G}^{M,*}(q_l) < \widetilde{G}^{M}(\hat{q}_U^{M}) < \widetilde{G}^{S,*}(\hat{q}_U^{S}) < \widetilde{G}^{S,*}(q_h) < \widetilde{G}^{M,*}(q_h)$$

This completes the proof. ■