The International Medium of Exchange*

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Abstract

We propose a model of endogenous, persistent coordination on the international medium of exchange. An asset becomes the dominant medium because it is widely held, and remains widely held because it is dominant. The country issuing the dominant asset is a net debtor, but earns an "exorbitant privilege" on its position. In a calibrated version of the model, steady states with one dominant asset are stable, whereas multipolar steady states are not. The dominant country experiences a significant welfare gain, most of which is accrued in transition. Trade wars reduce the size of privilege, especially harming the dominant country.

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1 Introduction

International trade, both in goods and in financial assets, is largely intermediated by US dollars.[1] At the same time, the US has a unique external position: it is the world's largest debtor country, but earns positive net income from its positions vis-a-vis the rest of the world.[2] These two phenomena, trade dollarization and exorbitant privilege, have each received significant attention in recent research, but have rarely been analyzed jointly.

In this paper, we propose a new theory of endogenous coordination on an international medium of exchange, in which exorbitant privilege emerges as a consequence. The essential insight of our theory is that asset availability matters: when more than one asset might serve as a medium of exchange, traders tend to coordinate on using an asset that is readily available to all counterparties. In the context of international trade, when safe dollar assets are more widely held worldwide, more of world trade will be facilitated by dollars. Moreover, if dollars are known to be useful for trade, then foreigners will be willing to pay a premium to hold substantial stocks of dollar-denominated assets. The effects of asset availability thus serve to enforce continued coordination on an already-established medium of exchange.

Our analysis begins with a stylized model that includes the essential ingredients of our theory. In the model, trading firms from a continuum of countries seek to engage in a profitable international transaction with firms from other countries. Contracting frictions require trading firms to collateralize their cross-country transactions with safe assets.[3] Trading firms may choose collateral in the form of safe dollar or euro assets, which they seek in domestic funding markets. These funding markets are characterized by search frictions between trading firms who seek funding and households who offer intra-period loans of their safe asset holdings. In the event a firm trades with a counterparty who has incompatible collateral, both firms forfeit a "mismatch cost" that is proportional to the surplus created by the transaction.

[1] Gopinath (2015) shows that dollar-invoiced trade is five times larger than trade directly involving US counterparties, and the dollar is far and away the largest third party currency used for trade invoice. Goldberg (2011) and Maggiori et al. (2017) show that 85% of foreign exchange rate transactions involve the USD, 60% international debt securities are issued in dollars, and dollar debt is virtually the only debt asset that foreign investors buy denominated in its own currency.

[2] Estimates of the foreign asset/liability return differential range from below 1% to 3% (e.g. Gourinchas and Rey, 2007). Applied to gross positions well in excess of 100% of GDP, even the lower range of these estimates implies a sizable transfer to the US.

[3] Many different financing strategies have emerged to serve the need for collateralization in international trade. See Amiti and Weinstein (2011), Antràs (2013), Ahn (2015) and Antras and Foley (2015) for more detailed evidence.

Because of search frictions, firms prefer ceteris paribus to search for an asset that is relatively plentiful locally. Households, conversely, are aware that an asset that is actively used in trade is more likely to be loaned in collateral search markets, earning a liquidity premium. Thus, the incentives of household and firms in this context are mutually reinforcing, and lead to a strong feedback effect: wide availability of an asset drives higher adoption of it as the medium of exchange, while higher adoption drives world households to hold a larger quantity of the asset.

We characterize equilibrium in the stylized model in a series of propositions. In particular, we show that the availability channel described above leads the model to have three steady-states: a dollar-dominant equilibrium in which all trade is denominated in dollars, a mirror-image euro-dominant equilibrium, and a symmetric multipolar equilibrium in which each currency intermediates one half of transactions. Importantly, all three steady states obtain regardless of the mismatch cost accrued when firms trade with partners using different collateral.

We next analytically demonstrate several key features of the steady states of the stylized economy. First, we show that, while the asset availability channel is sufficient to generate three steady states, *cross-country* coordination incentives are crucial for analyzing their stability. In particular, when the coordination incentives generated by mismatch costs are strong enough, only dominant currency steady states can be stable. This is true because substantial complementarity in currency adoption means that even a small initial deviation from symmetry both pushes firms to adopt the more dominant currency and encourages households to hold more of it. This implication seems consistent with the historical experience of persistently dominant currencies, including the recent reign of the US dollar and the earlier dominance of the British pound.

Second, we find that the coordinated steady states fit several puzzling empirical features of currency returns and international portfolio positions. The country with a dominant currency earns a liquidity premium, derived from the fees paid by trading firms who use the asset as collateral. This premium corresponds to the failure (unconditionally) of uncovered interest parity. Holdings of the dominant asset are more widely distributed around the world, implying a persistent negative net foreign asset position for the issuing country. The portfolio holdings of countries also exhibit significant home bias, with the dominant country being substantially less biased than the other major country, despite there being no frictions in asset trade. Qualitatively, this situation describes the status quo in the data quite well.

We embed our model of currency choice within a fully-fledged open economy model consisting of two big countries, the 'US' and the 'EU', and a continuum of small open economies that form the 'rest-of-the-world' (RW).[4] The governments of the two big countries are the only ones that can issue safe (collateral-eligible) assets, each of which is denominated in their respective home good, but is otherwise ex-ante identical. Each country has an endowment of a differentiated tradable good and households consume both domestic and foreign consumption goods, sold domestically by importing firms. Moreover, households freely trade the bonds issued by the two big countries' governments.

Though more realistic, our three country model incorporates the same feedback effect between firms' currency choice and the long-run asset position of households, generating several steady states. To study dynamics away from the steady-states of the economy, we introduce modest portfolio adjustment costs. These costs ensure that conjectured deviations cannot lead to jumps in asset holdings and, therefore, to jumps in equilibrium. With this adjustment cost in place, equilibrium paths of the economy are determinate (i.e. not subject to sunspot shocks) and the currency regime at any point in time is uniquely determined by economic fundamentals.[5] In this case, dynamics in the model are easy to characterize from any initial starting point, allowing us to determine the long-run currency regime to which the economy converges. In addition, we can study the effects of unexpected economic shocks or changes in policy on the attraction regions of the different steady states, and also analyze the transition paths that the economy takes after such shocks.

To quantify our mechanism, we select parameters to match several target moments from the data on the size of government debt and trade, exorbitant privilege, as well as a modest cost of currency mismatch. We fix these parameters so that the model achieves its target values at the empirically relevant, dollar-dominant, steady-state. The model is able to match realistic values for trade flows and exorbitant privilege, with implications for portfolio compositions that are qualitatively consistent with the data.

Using the calibrated model, we compute the welfare implications of owning the dominant asset. The steady-state welfare gain of the dominant country, relative to the other large non-dominant country, is relatively modest: 10 basis points of GDP at our benchmark calibration. This happens despite a substantial "exorbitant privilege" because the country that issues the dominant asset necessarily has a negative net foreign asset position, since the dominant

[4]Our approach is inspired by Schaal and Taschereau-Dumouchel (2015), who use a similar strategy to study the potential for two absorbing states in an economy with demand externalities.

[5]We find our benchmark model is determinate for a wide range of adjustment cost parameters, and calibrate the cost to a modest value, consistent with prior literature on international portfolio adjustment costs.

asset is widely held abroad. However, factoring in the welfare gain along the transition path from the multipolar to a coordinated steady state, which takes roughly 15 years, we find that the welfare gain of the dominant country is six times larger (60 basis points of GDP). Thus, understanding transition dynamics is crucial for assessing the benefits of owning a dominant asset.

Moreover, while the owner of the dominant asset gains, the rest-of-world actually looses when the world economy coordinates on a single medium of exchange. This happens because of congestion externalities in funding search markets: coordinated equilibria essentially waste the potential liquidity services that could otherwise have been provided by the unused, non-dominant asset. Thus, while dominant currency regimes are stable, they can easily be inefficient.

We finally consider the implications of our model for the effects of shocks to economic policy. We focus in particular on the effects of tariffs, including the situation of a single country imposing a unilateral tariff and the case of a trade war, in which all countries impose symmetric tariffs. We find that trade policy could have large effects on the status of the dominant currency and the relative asset returns across countries. This suggests that trade barriers impose additional costs beyond the usual destruction of consumer surplus. These additional costs are a consequence of changing intensity in the use of the dominant asset and, therefore, are not distributed evenly: trade barriers are costliest for the dominant country, because they reduce the effective seignorage it earns from world trade. Conversely, the dominant country has an incentive to promote free trade among the rest of the world, even when that trade does not directly involve it.

Relation to Existing Literature

Our work relates to several different strands of the literature. Motivated by some of the same empirical facts about the special international position of the US cited above, authors have sought to provide explanations focusing on the three canonical roles of money. In this paper, we provide a model where the dominant position of US assets in the international monetary system is due to their endogenous role as the main *medium of exchange*. A number of previous papers instead explore the unit of account and store of value properties of money.

Among the papers that focus on the unit of account role of money, concurrent work by Gopinath and Stein (2018) is most closely related to our own. To our knowledge, it is the only other paper that explicitly connects dollar invoicing in international trade and exorbitant privilege. The unit of account role of currency is also highlighted by Casas et al. (2016), which considers the effects of near universal dollar-denomination on shock pass-through and expenditure switching when nominal prices are sticky.

A larger portion of the related literature focuses on the capacity of countries to generate good store-of-value assets. Caballero et al. (2008) argue that the United States' superior ability — relative to developing countries — to produce store-of-value assets can explain the US experience of persistent trade deficits, falling interest rates, and rising portfolio share of US assets in developing countries. Maggiori (2017) models the endogenous emergence of a dominant reserve currency, in which dominance emerges because a higher level of financial development in the dominant country leads it to provide insurance vis-a-vis the rest of the world. Gourinchas et al. (2010) propose a different insurance framework, where the US households are assumed to have lower risk-aversion than foreign households (perhaps due to an endogenous mechanism like the one in Maggiori, 2017) and, thus, in equilibrium end up taking on most of the world's risky assets in their portfolio. Meanwhile, Farhi and Maggiori (2016) consider the different positive and welfare implications of having a single dominant reserve asset or a multipolar system; both of which can emerge endogenously in the equilibrium in our economy. He et al. (2016) also use a global-game approach to model how, in a world with two ex ante identical assets, it may be that a single safe asset emerges in equilibrium.

Two key features differentiate our work from the preceding literature cited above: (i) currency dominance emerges endogenously in equilibrium in an otherwise ex-ante symmetric world and (ii) the model features determinate equilibrium paths, with currency choice a unique function of the state variables (i.e. no sunspot multiplicity). Previous authors have developed frameworks that feature one or the other, but not the two together. For example, some existing models can deliver dominant currencies, but do so in the context of equilibrium indeterminacy. We view (i) and (ii) as a desirable combination, as in the data currency regimes occasionally change, but have historically been quite stable.

2 A Simple Model

The world consists two symmetric big countries, call them the US and the EU, and a continuum of small open economies that constitute the *rest of the world*. Economies in the rest of the world are indexed by $j \in [0, \mu_{rw}]$, where μ_{rw} measures the size of the rest of the world. We normalize $\mu_{rw} + \mu_{us} + \mu_{eu} = 1$. Within each small country j, there exists a continuum of agents $i \in [0, 1]$ — call them trading firms — who seek to engage in a transaction with partners outside of country j. If the transaction occurs, it delivers a gross profit of 2π.

Because international trade is subject to contractual frictions, firms seeking to make international transactions must acquire collateral before seeking a foreign transaction partner. In this market, two asset types serve as acceptable collateral: (safe) bonds issued by the US and (safe) bonds issued by the EU. Rest-of-world trading firms seek these assets in bond- and country-specific search and matching markets in which domestic households offer their holdings as intra-period loan to trading firms. For tractability, we assume that firms look for a fixed amount of funding, which we normalize to one, and that firms make the binary choice of either seeking dollars or euros.

The probability that a country-j trading firm seeking dollars succeeds in securing funding is given by $p_j^{\$}$. The probability of a euro-seeking firm finding collateral is $p_j^{€}$. If the firm finds collateral, it pays a fee $r > 0$ to the household for the intra-period use of the asset, and proceeds to the international transactions market.[6] If it does not find collateral, the firm exits.

Conditional on finding collateral, the trading firm seeks a trading partner across all countries j', where $j' \neq j$, with whom to transact.[7] For simplicity, we assume that trading search is undirected and that each trading firm matches with exactly one trading partner. Upon matching with the partner, trading firms transact, using their collateral to clear any payments needed, and split the resulting profits equally.

Importantly, in the event that the transacting firms' collateral is mismatched — i.e. that one side of the match arrives with dollars assets and the other side with euro assets — the firms must pay a mismatch cost of 2κ. This cost of currency mismatch captures, in reduced-form, several potential reasons that firms may wish to coordinate the currency in which they transact, including (i) the risk that collateral on one side of the exchange may not be sufficient ex post (ii) the risk of mismatched incoming and outgoing cash flows (iii) the added complication of negotiating a contract that includes a currency conversion or is conditioned on nominal exchange rates. While the existence of *some* incentive for currency coordination plays an important role in our story, we will demonstrate shortly that a small desire for coordination is all that is needed for our mechanism to work as we describe here.

[6]For simplicity, we take this fee as fixed. The fee could be made endogenous, e.g. by assuming Nash bargaining.

[7]We are assuming, for the moment, that rest of world countries do not trade with the big US and EU. We relax this assumption in our quantitative model.

Equilibrium with exogenous finding probabilities

In deciding which asset to seek as collateral, the trading firm compares the expected payoffs of searching in domestic markets for dollar or euro collateral. Since trading firms transact in decentralized markets, they must somehow share the surplus of their trade. For simplicity, we assume for now that the *ex post* surplus of trade is split evenly between the two firms, hence each side of a given transaction realizes gross profits π. Let X_j be the fraction of country j firms that choose to operate in dollars, and we can then calculate the average dollar use in the rest of the world as:

$$\bar{X} \equiv \frac{1}{\mu_{rw}} \int_0^{\mu_{rw}} X_j dj$$

This is also the probability that any given firm will be matched with a trade partner that uses dollars, and thus can be used to compute expected currency mismatch costs. With this in mind, we can calculate the expected profits of a firm in country j operating in dollars, relative to expected profits operating in euros:

$$V_j^{\$} = p_j^{\$} \left[\pi - \kappa(1 - \bar{X}) \right] - p_j^{€} \left[\pi - \kappa \bar{X} \right]. \tag{1}$$

The firm faces π gross profits regardless of what collateral it brings to the international markets, and since $1 - \bar{X}$ is the probability of matching with a firm using euros $\kappa(1 - \bar{X})$ captures the expected currency mismatch cost when operating in dollars, $\kappa\bar{X}$ is the expected mismatch cost when operating in euros. Thus, a firm in country j will choose to seek dollar funding if and only if $V_j^{\$} > 0$, and we can use that decision rule to find the equilibrium $\bar{X}$.

When $p_j^{\$}$ and $p_j^{€}$ are exogenous and constant across j, equation (1) captures the payoffs of a standard coordination game: firms find dollars more desirable whenever they expect other firms to use dollars and visa-versa for euros. This game with exogenous funding probabilities potentially has several equilibria – two corner ones where either everybody uses dollars ($\bar{X} = 1$) or euros ($\bar{X} = 0$), and a mixed strategy equilibrium in between. If the equilibrium is unique, then it is one of the corner ones. Proposition 1 summarizes the equilibria in this exogenous funding probability game.

Proposition 1. *Dollar dominance* ($\bar{X} = 1$) *is an equilibrium of the economy whenever* $\kappa \geq (1 - p^{\$}/p^{€})\pi$. *Similarly, euro dominance* ($\bar{X} = 0$) *is an equilibrium whenever* $\kappa \geq$

$(1 - p^{€}/p^{\$})\pi$. *The economy has multiple equilibria if and only if*

$$\kappa \geq (1 - p^*)\,\pi,$$

where $p^* \equiv \min\left(\frac{p^{€}}{p^{\$}}, \frac{p^{\$}}{p^{€}}\right) \leq 1$.

Proof. Proved in Appendix A. ■

Intuitively, the model has multiple equilibria so long as the probabilities of finding different types of funding are close enough. In these cases, candidate currencies have little or no "fundamental difference," so that currency choice is driven almost entirely by firms' guess about what other firms will be holding, and opens up the game to sun-spot like multiplicity.

Several strands of the literature on the endogenous emergence of money (or money-like assets) have emphasized this sort of multiplicity, which often arises in such games.[8] Proposition 1 is thus not a new result of this paper, but serves as a baseline to help understand the contribution of the asset availability mechanism we emphasize.

Equilibrium with endogenous finding probabilities

We now characterize equilibrium in which the trading firms' collateral finding probabilities, $p_j^{\$}$ and $p_j^{€}$, are endogenously determined in search and matching funding markets. On one side of these markets are households, who offer to lend their holdings of safe assets, and on the other are the firms who seek to borrow assets and use them in trade. In each country, there are two markets, one for dollar funding and one for euro funding, and domestic trading firms choose the market in which they search.

To start our analysis, in this subsection we assume that country j's holdings of the safe dollar and euro bonds are fixed at $B_j^{\$}$ and $B_j^{€}$ respectively. We assume that the number of matches that emerge in a given country-currency market is governed by the den Haan et al. (2000) matching function.[9] According to this function, the number of matches formed in a market with B units of safe assets on offer and X firms searching is

$$M^f(B, X) = \frac{BX}{(B^{\frac{1}{\varepsilon_f}} + X^{\frac{1}{\varepsilon_f}})^{\varepsilon_f}}.$$

[8]See Doepke and Schneider (2017) for one example.

[9]This functional form exhibits constant returns to scale and, unlike the Cobb-Douglas matching function, ensures matching probabilities fall between zero and one.

We call the parameter ε_f the matching function elasticity, and for tractability we set this parameter to unity for the remainder of this section. Accordingly, the probability that a firm in country-j searching for dollar assets finds them is

$$p_j^{\$} = \frac{M^f(B_j^{\$}, X_j)}{X_j} = \frac{B_j^{\$}}{B_j^{\$} + X_j},$$

while the probability of the trading firm matching in the euro market is

$$p_j^{€} = \frac{M^f(B_j^{€}, 1 - X_j)}{1 - X_j} = \frac{B_j^{€}}{B_j^{€} + 1 - X_j}.$$

Substituting the expressions for finding probabilities into equation (1) yields

$$V_j^{\$} = \frac{B_j^{\$}}{B_j^{\$} + X_j}\left[\pi - \kappa(1 - \bar{X})\right] - \frac{B_j^{€}}{B_j^{€} + 1 - X_j}.\left[\pi - \kappa\bar{X}\right] \tag{2}$$

Equation (2) captures the first key implication of the asset availability channel: the payoff of using dollars in country j is *decreasing* in the intensity of dollar use by other country-j firms. The larger the share of domestic firms looking for dollars, the higher is the competition in the domestic dollar funding market and the lower the probability any given firm finds such funding. Thus, the game in (2) captures a within-country *substitutability* in collateral choice that contrasts with the cross-country complementarity embedded in the parameter κ.

This within-country substitutability in currency choice enhances the model's ability to obtain a unique equilibrium, because it makes it harder for firms to find a particular type of funding when everyone is trying to obtain the same type of funding. Such congestion effects can preclude fully-coordinate equilibria since, as funding probabilities drop, more and more firms find it optimal to look to other markets for funding. In this way, funding availability can serve as a coordination device that precludes sunspot multiplicity.

Proposition 2 summarizes the equilibrium in the endogenous finding probability game under symmetric asset allocations, $B_j^{\$} = B^{\$}$ and $B_j^{€} = B^{€}$ for all j.

Proposition 2. *Dollar dominance ($\bar{X} = 1$) is an equilibrium of the economy whenever*

$$\kappa \geq \frac{1}{B^{\$} + 1}\pi.$$

Similarly, euro dominance ($\bar{X} = 0$) is an equilibrium whenever $\kappa \geq \frac{1}{B^{€}+1}\pi$. Moreover, the *economy has a unique (possibly mixed-strategy) equilibrium if and only if*

$$\kappa < \frac{1}{\min(B^{\$}, B^{€}) + 1}\pi.$$

Proof. Proved in Appendix A. ∎

The case of perfect symmetry is especially instructive in comparing our first and second propositions. When finding probabilities are fixed and equal ($p^{\$} = p^{€}$), Proposition 1 implies multiplicity in the currency choice for *any* positive κ. In contrast, in the corresponding perfectly symmetric world with an asset availability channel ($B^{\$} = B^{€} = \bar{B}$), the condition for uniqueness in Proposition 2 requires only that $\kappa < (\bar{B}+1)^{-1}\pi$, which is clearly a strictly weaker bound for any finite $\bar{B}$. This is the sense in which the asset availability channel, and the associated substitutability in currency choice within a country, help eliminate multiplicity. When both assets are equally available in all countries and coordination incentives are modest, coordinated equilibria cannot be sustained because of the resulting congestion in the markets for a (potentially) dominant currency.

Proposition 2 also shows that uniqueness is easier to achieve the smaller is the domestic supply of the liquidity-providing assets. This is natural, because a small stock of assets implies relatively large congestion effects in the asset markets. These congestion effects imply a stronger substitutability in currency choice, since other firms' use of an asset makes it so much harder for a given firm to find that asset. Conversely, when $B^{\$}$ and $B^{€}$ become arbitrarily large, the game in equation (2) converges to the game in equation (1), with $p_j^{\$} = p_j^{€} = 1$, i.e. the case with multiplicity for any positive κ. In this case, the availability channel essentially vanishes because firms face no scarcity.

A second key insight from Proposition 2 is that a unique dollar dominant equilibrium (an equilibrium with higher $\bar{X}$) is more likely, the higher is $B^{\$}$ relative to $B^{€}$. Intuitively, higher domestic holdings of an asset make it easier to seek collateral in that market and on the margin more firms will do so. When that asset is relatively more available broadly across countries, the asymmetry in asset availability serves as a *cross-country* coordination device, which can now reinforce the complementarity embodied in κ. As firms realize that dollars are more broadly available around the world, they correctly conjecture that on the margin foreign firms will shift towards dollars, and hence the value of seeking dollar funding increases. Firms also realize that the relatively scarce asset cannot sustain a similar level of usage, since if everyone tries to coordinate on using that asset as the medium of exchange, many firms will fail to find funding.

The black line in Figure 1 illustrates the characteristics of the equilibrium $\bar{X}$ for a particular parameterization. The figure plots the share of firms using dollars as a function of the share of dollars in the domestic portfolio. The line is upward sloping everywhere, highlighting the asset availability channel – a higher share of dollars held by domestic households makes dollar funding more easily obtainable, resulting in a (weakly) larger share of firms turning towards dollars.

Equilibrium asset holdings

Now we add the last piece of our model by endogenizing the domestic holdings of assets $B_j^{\$}$ and $B_j^{€}$. We do this by solving for optimal household portfolio allocations. As we presently, these allocations will depend on the currency choice of the firms, generating a feedback between the choices of households and firms *within* a country.

To economize on notation, we extend the range of $j \in \{[0,\mu], us, eu\}$ so it can refer either to an infinitesimal rest-of-world economy or to one of the two large economies. To keep the model tractable, we assume that the demands for intra-period dollars loans in each large country are given by exogenous values X^{us} and X^{eu} respectively. Additionally, we assume that total world stock of available assets, $\bar{B}$, is exogenous and symmetric across currencies and focus only on steady-state asset holdings.

In this context, the key asset allocation conditions are a set of country-specific Euler equations that equalize the effective rates of returns on any assets held in positive quantity. Since, bonds trade for a price Q^c and pay a liquidity premium of Δ^c, $c \in \{\$, €\}$, return equalization implies:

$$\frac{1}{\beta} = \frac{1}{Q^{\$} - \Delta_j^{\$}} = \frac{1}{Q^{€} - \Delta_j^{€}} \tag{3}$$

for all $j \in \{[0,\mu], us, eu\}$, where β is the time discount rate (i.e. required return), which we assume is the same across countries.

The liquidity premia are derived from the fees that firms pay to households when they borrow one of their assets. As such, the liquidity premium of a given type of asset in country j is equal to the probability that the household can lend out that type of an asset multiplied by the funding fee r that it would receive if successful in lending it out. Given our matching function, the premia can be expressed as

$$\Delta_j^{\$} = r \times \frac{M^f(B_j^{\$}, X_j)}{B_j^{\$}} = \frac{rX_j}{B_j^{\$} + X_j}, \tag{4}$$

$$\Delta_j^{€} = r \times \frac{M^f(B_j^{\$}, 1 - X_j)}{B_j^{\$}} = \frac{r(1 - X_j)}{B_j^{\$} + (1 - X_j)}, \tag{5}$$

for dollars and euros respectively. In the case where $\varepsilon_f = 1$, this leads to the particularly intuitive expressions that the probability of matching is equal to the ratio of searchers to the sum of available funding and searchers. The more firms that use dollars, the higher is the aggregate fee paid on dollar borrowing and thus the higher the liquidity premium all else constant. On the other hand, the higher the number of dollar bonds available in a country, the lower is the average liquidity premium each one of them earns, holding constant the dollar funding demand by firms. Note that even though the funding markets are country specific, because the markets for US and EU bonds are fully integrated, the liquidity premia must be equalized across countries:

$$\Delta_j^c = \Delta_j^c \text{ for all } j, c \in \{\$, €\}$$

Market clearing in bond holding requires

$$\bar{B} = \mu_{rw} B_{rw}^{\$} + \mu_{us} B_{us}^{\$} + \mu_{eu} B_{eu}^{\$},$$

with a similar equation for euro bonds. Combing the Euler equations with market clearing in bonds, it follows that the optimal holdings of dollar and euro bonds in country j are:

$$B_j^{\$} = \bar{B} \frac{X_j}{\mu_{rw}\bar{X} + \mu_{us}X^{us} + \mu_{eu}X^{eu}} \tag{6}$$

$$B_j^{€} = \bar{B} \frac{1 - X_j}{\mu_{rw}(1 - \bar{X}) + \mu_{us}(1 - X^{us}) + \mu_{eu}(1 - X^{eu})}. \tag{7}$$

We here note in passing that one of the nice features of the model is that it delivers determinate bond holding allocations even in the non-stochastic steady state. Portfolio indeterminacy is an important issue of standard international models. It is not a problem here, because the liquidity premia make assets (locally) not perfect substitutes, even though their rates of return are equalized in equilibrium.

Equations (6) and (7) capture equilibrium household bond holdings, taking trading firms'

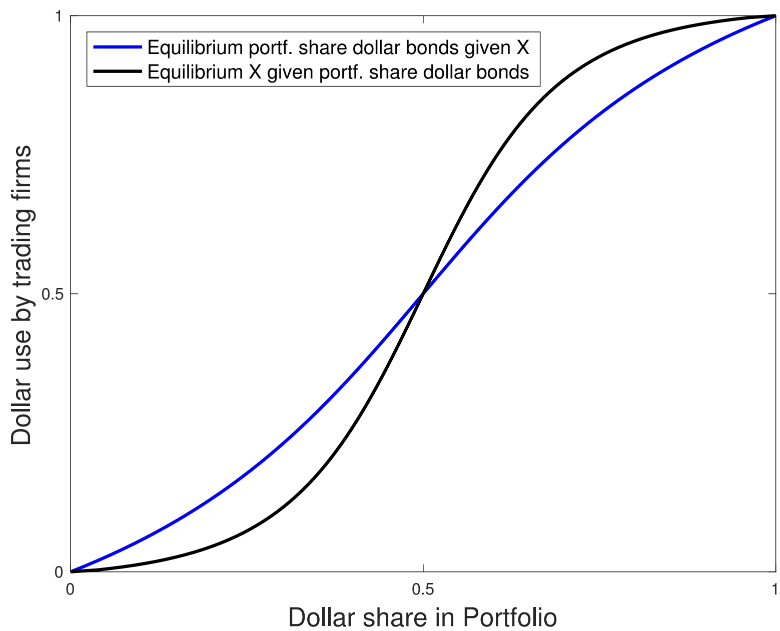

Figure 1: Equilibrium in the simple game.

collateral usage as *given*. The key insight of these equations is that household asset allocations and firm currency choice are strategic complements: higher X_j implies higher household desire the hold dollar bonds for example. Intuitively, this occurs because a bond that is widely used for liquidity will deliver higher premia to the bond holder, thereby giving domestic households an incentive to increase this type of bond holdings.

The blue line in Figure 1 illustrates the optimal portfolio choice of the households, plotting the implied share of dollar bonds (on the x-axis) as a function of firms' currency decisions (y-axis.) This line is also upward sloping everywhere, capturing the complementarity between households and firm decisions. As we prove in the next section, the lines intersect at origin and at (1,1) — corner solutions in which one type of holdings and usage become zero — and also potentially one more time somewhere in between. Interestingly, the lines have multiple crossings for any level of κ, but (except for a knife-edge case) the crossings can be no more than three.

Equilibrium at steady-state

With expressions for equilibrium collateral choices and household asset holdings, we are now ready describe the general equilibrium of the model economy in which both households and firms are making optimal decisions. We call this circumstance a *steady state equilibrium* because it is based on the steady state optimality conditions of the fully dynamic model we present below. The key feature of that model is a multiplicity of steady states, and in this section we characterize the features of the steady state equilibria analytically, while in the next section we study a calibrated version of a more complete open economy model.

For now we restrict the analysis to the case where both assets are ex-ante symmetric, hence the two big countries are of equal size, $\mu_{us} = \mu_{eu}$, and the domestic use of their assets is the same, $X_{us} = 1 - X_{eu}$. Later we will consider comparative statics in respect to these parameters. Lastly, we look for symmetric equilibria where the strategies of the ex-ante identical rest-of-world firms are the same ($X_j = X$ for all j). As suggested by Figure 1, the model has three steady state equilibria – (i) dollar dominant, (ii) euro dominant, and a symmetric multi-polar steady state where both currencies are used equally. Proposition 3 formalizes this result.

Proposition 3. *For any $\kappa \geq 0$ the symmetric economy has three steady-state equilibria: (i) a dollar dominant steady-state with $\bar{X} = 1$ and $B^{€} = 0$ (ii) a euro-dominant steady-state with $\bar{X} = 0$ and $B^{\$} = 0$, and (iii) a symmetric, multi-polar steady-state with $\bar{X} = 1/2$ and $B^{\$} = B^{€}$.*

Moreover, these are the only steady-states as long as $\kappa \neq \bar{\kappa}$, where

$$\bar{\kappa} \equiv \frac{\mu_{rw}\pi}{\bar{B} + \mu_{rw} + \mu_{us}X^{us} + \mu_{eu}X^{eu},}$$

in which case there is a continuum of equilibria with $\bar{X} \in [0, 1]$.

Proof. Proved in Appendix A. ■

In practice, a steady-state equilibrium of the economy is an intersection between the blue and black lines in Figure 1. As the figure makes clear, these lines always intersect at the origin, at (1,1), and in the center of the figure, corresponding to the three steady-states described in the theorem.

The first interesting feature of the model is that these three equilibria exist for *any* level of κ, even if it is very small. Recall that in the restricted version of the game where either the funding finding probabilities $(p^{\$}, p^{€})$ or the bond holdings $(B^{\$}, B^{€})$ were exogenously fixed there was equilibrium multiplicity *only if* κ was high enough. This was because the multiplicity came about purely from the coordination incentives in currency choice across firms. Hence, when those coordination incentives are low (κ) those games did not feature multiplicity. In contrast, the full model displays multiplicity for all levels of κ. What is going on?

The intuition is that in the model where both firms and households make optimal decisions, the key feedback loop is between households and firms in the *same* country, not between firms *across* countries. When the firms in a country increase their demand for dollars, the households find it optimal to similarly shift their portfolios towards dollars. But as the relative availability of dollars increases, this only strengthens the initial shift in the firms' currency choice towards dollars. This feedback effect can push the economy all the way out to the extremes where only dollars or euros are used, but the economy can also come to a rest right in the middle where both currencies are equally used, hence neither households nor firms have any incentive to adjust their behavior. Because of this, the value of κ is not important for the existence of the different steady states, but as will become clear shortly, it plays an important role in determining the stability properties of the different equilibria.

It is worth pointing out that even though the full model features the same types of equilibria as in the simple coordination with exogenous probabilities of funding (two coordinated ones, one multi-polar), the nature of the multiplicity is different in a way that would be important for the dynamic model. In the simple coordination game, the multiplicity arises purely due to coordination in beliefs across countries – in other words, it is a sun-spot type of multiplicity. The equilibrium is free to shift any time there is a shock that shifts the beliefs of a sufficiently large mass of agents.

Once we endogenize the funding probabilities, however, any equilibrium jump also depends on a discrete shift in the bond holdings of households. In the our dynamic model, however, will include standard portfolio adjustment costs, preventing large discrete shifts in portfolio composition. Because of this, the dynamic model features unique equilibrium paths – given state variables there is only one possible equilibrium – but multiplicity of steady states. Rather than leading to sunspot equilibria, availability feedback effects operate in the long-run, and the model can come to rest at the different steady states we describe here. These features of the dynamic economy make it well-suited for analyzing real world data, where we see that the currency regimes are persistent states of the international monetary system.

Although the model so far is very stylized, the dollar-dominant equilibrium ($\bar{X} = 1$) already captures several realistic features of the real world. First, dollar assets are far more broadly held than the competing source of liquidity. In the model, this broader availability of dollars servers as a coordination device that supports the equilibrium coordinated on dollars.

One implication is that this leads the US to have much larger foreign liabilities than the EU and, hence, and the US will display more negative Net Foreign Assets (NFA) position than its counterpart.

Second, from equations (4) and (5), we can see that the dollar liquidity premium is larger than that of the competing currency:

$$\Delta^{\$} = \frac{r\mu_{\$}}{\bar{B} + \mu_{\$}} > \frac{r\mu_{€}}{\bar{B} + \mu_{€}} = \Delta^{€},$$

where $\mu_{\$} \equiv \mu_{rw} + \mu_{us}X^{us} + \mu_{eu}X^{eu}$ and $\mu_{€} \equiv \mu_{us}(1 - X^{us}) + \mu_{eu}(1 - X^{eu})$ are, respectively, the measures of dollar and euro demand in the world economy.[10] This follows from the fact that the heavy use of dollars in rest-of-the-world leads to higher overall use of dollars in the world as a whole: $\mu_{\$} > \mu_{€}$.

Because of this difference in liquidity premia, the model displays an unconditional failure of Uncovered Interest Parity (UIP):

$$\frac{i^{€} - i^{\$}}{(1 + i^{\$})(1 + i^{€})} = \Delta^{\$} - \Delta^{€} > 0$$

where we define $1 + i^c = \frac{1}{Q^c}$ as the interest rate on the bond of country c. In the dollar dominant steady state the dollar liquidity premium is higher than that of the euro, and as a result the equilibrium interest rate on euros is higher than that on dollars. In other words, the euro asset is compensated with excess returns for the fact that it serves an inferior liquidity provision role.[11]

The unconditional failure of the UIP is interesting also for understanding the so called US "exorbitant privilege", which is the observation that in the data, US foreign assets tend to earn higher returns than the US foreign liabilities, which could potentially help support a trade deficit even with a negative NFA position. Such a force is present in our model as well. Note that the US trade balance can be expressed as

$$TB_{us} = \underbrace{(\mu_{eu}B^{\$}_{eu} + \mu_{rw}B^{\$}_{j} - \mu_{us}B^{€}_{us})}_{-NFA} i^{€} - \underbrace{(\mu_{eu}B^{\$}_{eu} + \mu_{rw}B^{\$}_{j})(i^{€} - i^{\$})}_{\text{exorbitant privilege}}$$

and thus to the extent to which the US has (i) large amount of *gross* foreign liabilities and

[10]Notice that in our stylized economy the expressions for $\Delta^{€}$ only apply in the US and EU, where the bonds are held in positive supply.

[11]While the UIP is not a focus of this paper, we note that this is a mechanism for endogenizing the convenience yield mechanism proposed as a solution to the UIP puzzle in Valchev (2017).

(ii) the foreign asset earns a higher return ($i^{€} > i^{€}$), the US could have a much smaller trade balance (possibly a trade deficit), even when carrying a large negative net position.

Stability of steady-states

A natural question is which (if any) of the steady-states described above are stable? It turns out that this depends on the level of κ, which controls the coordination incentive *across* countries. When the currency coordination incentives are relatively high, then the two coordinated steady states are stable and the symmetric steady state is not. The converse holds when currency coordination incentives are low. This result is derived in Proposition 4

Proposition 4. *For $\kappa < \bar{\kappa}$ only the symmetric steady state is locally stable, for for $\kappa > \bar{\kappa}$ only the two coordinated, dollar and euro-dominant steady states, are locally stable.*

Proof. Proved in Appendix A. ■

The key mechanism behind this proposition is that when κ is sufficiently high, the optimal currency choice of firms (X_j) around either of the coordinated steady states is relatively insensitive to off-equilibrium shifts in bond holdings. At the dollar-dominant steady state, for example, the combination of a high κ and high $\bar{X}$ gives firms a strong incentive to coordinate their actions together, and pay less attention to the funding availability channel governed by bond holdings. An off-equilibrium decrease in dollar bonds will have little effect on the use of dollars (X_j), and as a result the liquidity premia on dollars will increase since only the denominator in equation (4) will change. The rise in the dollar premium gives households an incentive to increase their dollar holdings, and thus unravel the initial shift in portfolios.

Conversely, if κ is low, then an off-equilibrium shift in bond holdings will have a large effect on X_j because firms will be primarily motivated by the change in the availability channel. In this case, a decrease in $B_j^{\$}$ will also cause a decrease in X_j as firms respond to the decreased probability of obtaining dollar funding. When κ is sufficiently low, this decrease in X_j is more than enough to offset the effect of the decrease $B_j^{\$}$ in equation (4) and thus the dollar premium will fall, leading households to lower dollar holdings even further, and unraveling the coordinated equilibrium. Similar, but opposite, intuition explains why the stability of the symmetric steady state behaves in the mirror fashion, where the multi-polar equilibrium is stable only when κ is low and the coordinated steady states are not stable, and vice versa.

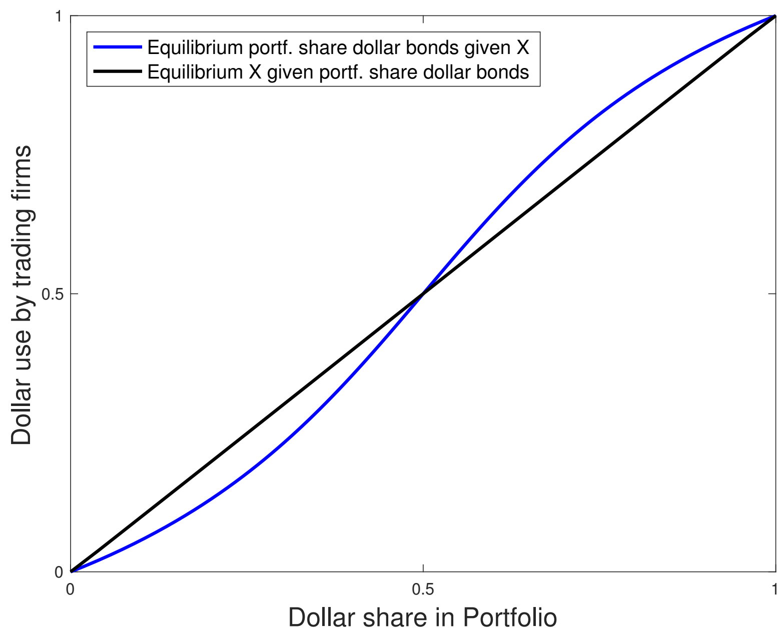

Figure 2: Equilibrium in the simple game, $\kappa < \bar{\kappa}$.

This intuition can also be seen graphically in Figure 1, which depicts a case where $\kappa > \bar{\kappa}$, and therefore the asymmetric equilibria are the stable ones. Intuitively, conjecture that the economy starts with an initial portfolio allocation on the blue line, with a dollar share just under one half. At that point, equilibrium dollar usage is found by traveling vertically downwards towards the black line. From this point, however, the best response of household is to further reduce their dollar holdings. And the process repeats until we arrive at the euro-dominant equilibrium. For comparison, Figure 2 plots a case where $\kappa < \bar{\kappa}$ and the symmetric steady state is the stable one.

Proposition 4 refers to local-stability, but it cannot rule out non-local shifts in equilibria: i.e. cases where asset allocations and asset usage both shift instantaneously and entirely from dollars to euros. In practice, such dramatic shifts can be ruled out by making either bond allocations or currency choices inertial. We find the former far more natural, and proceed this way in the full dynamic model that we present in the following section.

3 A Fully Dynamic Model

In this section, we expand the simple model described above to a fully-dynamic two country plus rest-of-world context. In doing so, we allow firms to make endogenous entry and exist decisions, include a notion of real exchange rates, and explicitly account for the evolution of bond holdings as state variables. To keep the model as tractable as possible, we focus on a version of the economy in which output arrives via an endowment process and currency usage in the large US and EU are exogenously fixed. In ongoing companion work, Chahrour and Valchev (2018) relax these assumptions and focus squarely on the implication of the model for the currency choice and welfare in the large countries. Since countries are symmetric except for their size, we let the index j refer to both small countries in the continuum of rest-of-world countries and to the large countries US and EU and point out differences between big and small countries when relevant.

Household sector

The household sector in country j consists of a representative consumer-worker, who seeks to maximize the present discounted value of utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_{jt}^{1-\sigma}}{1-\sigma}$$

where the consumption basket C_{jt} is a CES aggregator of home and foreign goods. In the two big countries, the foreign imports consist of the good of the other big country and an aggregate of rest-of-the world goods. Hence, their consumption aggregator is given by

$$C_{jt} = \left(a_h^{\frac{1}{\eta}} C_{jt}^{j\,\frac{\eta-1}{\eta}} + (1-a_h)^{\frac{1}{\eta}} \left[(\frac{\mu_{j'}}{\mu_{j'}+\mu_{rw}})^{\frac{1}{\eta}} C_{jt}^{j'\,\frac{\eta-1}{\eta}} + (\frac{\mu_{rw}}{\mu_{j'}+\mu_{rw}})^{\frac{1}{\eta}} C_{jt}^{rw\,\frac{\eta-1}{\eta}} \right] \right)^{\frac{\eta}{\eta-1}}, \tag{8}$$

where $j \in \{us, eu\}$ and we define j' as the complement of j. $C_{jt}^{j'}$ is the consumption in country j of the good of country j', and $C_{jt}^{rw} = \int (C_{jt}^{i})^{\frac{\eta-1}{\eta}} di$ the consumption of rest-of-world goods. The corresponding price index is

$$P_{jt} = \left(a_h P_{jt}^{j\,1-\eta} + (1-a_h) \left[\frac{\mu_{j'}}{\mu_{j'}+\mu_{rw}} P_{jt}^{j'\,1-\eta} + \frac{\mu_{rw}}{\mu_{j'}+\mu_{rw}} P_{jt}^{rw\,1-\eta} \right] \right)^{\frac{1}{1-\eta}}. \tag{9}$$

For one of the small countries, their consumption basket includes imports from both big countries and all the other rest-of-world small countries:

$$C_{jt} = \left(a_h^{\frac{1}{\eta}} C_{jt}^{j\,\frac{\eta-1}{\eta}} + (1-a_h)^{\frac{1}{\eta}} \left[(\frac{\mu_{us}}{\mu_{us}+\mu_{eu}+\mu_{rw}})^{\frac{1}{\eta}} C_{jt}^{us\,\frac{\eta-1}{\eta}} + (\frac{\mu_{eu}}{\mu_{us}+\mu_{eu}+\mu_{rw}})^{\frac{1}{\eta}} C_{jt}^{eu\,\frac{\eta-1}{\eta}} + (\frac{\mu_{rw}}{\mu_{us}+\mu_{eu}+\mu_{rw}})^{\frac{1}{\eta}} C_{jt}^{rw\,\frac{\eta-1}{\eta}} \right] \right)^{\frac{\eta}{\eta-1}} \tag{10}$$

with the price index

$$P_{jt} = \left(a_h P_{jt}^{j\,1-\eta} + (1-a_h) \left[(\frac{\mu_{us}}{\mu_{us}+\mu_{eu}+\mu_{rw}}) P_{jt}^{us 1-\eta} + (\frac{\mu_{eu}}{\mu_{us}+\mu_{eu}+\mu_{rw}}) P_{jt}^{eu 1-\eta} + (\frac{\mu_{rw}}{\mu_{us}+\mu_{eu}+\mu_{rw}}) P_{jt}^{rw 1-\eta} \right] \right)^{\frac{1}{1-\eta}}. \tag{11}$$

In addition, the household chooses how to allocate its savings among US and EU bonds, which are risk-free in the units of their denomination. Since trade in the differentiated US, EU, and RW products happen in decentralized markets, as described below, the bond payments are settled in terms of the international numeraire — the bonds do not promise delivery of real goods, but rather enough resources to purchase the promised consumption good. Thus, payments on the US bond are indexed to the price of US consumption — one unit of US bonds purchased at time $t-1$ yields a payment of $P_{us,t}$ — and the EU bond similarly returns $P_{eu,t}$.

The household in country j faces the following budget constraint,

$$\begin{aligned} P_{jt}C_{jt} + (1-\Delta_{jt}^{\$})P_{us,t}Q_t^{\$}B_{jt}^{\$} + (1-\Delta_{jt}^{€})P_{eu,t}Q_t^{€}B_{jt}^{€} + P_{us,t}Q_t^{\$}\frac{\tau_j^{\$}}{2}(B_{jt}^{\$}-B_{jt-1}^{\$})^2 + P_{eu,t}Q_t^{€}\frac{\tau_j^{€}}{2}(B_{jt}^{€}-B_{jt-1}^{€})^2 \\ = P_{us,t}B_{jt-1}^{\$} + P_{eu,t}B_{jt-1}^{€} + P_{jt}^{j}Y_{jt} + \Pi_{jt} + T_{jt}, \end{aligned} \tag{12}$$

where Y_{jt} is the value of the household's endowment of the domestic good, Π_{jt} is the total profit of country j's trading sector, which is returned lump-sum to the representative household, $Q_t^{\$}$ and $Q_t^{€}$ are the prices of the US and the EU bonds respectively, and $\Delta_{jt}^{\$}$ and $\Delta_{jt}^{€}$ represent the endogenous liquidity premia (i.e. lending fees) earned by the bonds lent within the period to trading firms, and T_{jt} are lump-sum taxes.

The budget constraint in (12) also includes quadratic portfolio adjustment costs parameterized by τ. Adjustment costs are parameterized to be zero in steady-state, and serve to prevent excess volatility of capital flows, without affecting the average level of bond holdings.

The first order conditions of the household yield the Euler equations:

$$1 = \beta E_t \left[\left(\frac{C_{jt+1}}{C_{jt}} \right)^{-\sigma} \frac{P_{jt}}{P_{jt+1}} \frac{P_{us,t+1}}{P_{us,t}} \frac{1+Q_{t+1}^{\$}\tau_j^{\$}(B_{j,t+1}^{\$}-B_{jt}^{\$})}{Q_t^{\$}(1-\Delta_{jt}^{\$}+\tau_j^{\$}(B_{jt}^{\$}-B_{j,t-1}^{\$}))} \right] \tag{13}$$

$$1 = \beta E_t \left[\left(\frac{C_{jt+1}}{C_{jt}} \right)^{-\sigma} \frac{P_{jt}}{P_{jt+1}} \frac{P_{eu,t+1}}{P_{eu,t}} \frac{1+Q_{t+1}^{€}\tau_j^{€}(B_{j,t+1}^{€}-B_{jt}^{€})}{Q_t^{€}(1-\Delta_{jt}^{€}+\tau_j^{€}(B_{j,t+1}^{€}-B_{jt}^{€}))} \right] \tag{14}$$

Government

The government faces expenditures fixed in proportion to domestic production:

$$G_{jt} = \phi_{jg} Y_{jt}. \tag{15}$$

The governments of the small RW countries do not issue debt and set $T_{jt} = G_{jt}$. The governments of the two big countries also issue bonds in fixed supply $\bar{B} = B^{\$} = B^{€}$, and set the level of lump-sum taxes so as to keep their stock of debt constant at $\bar{B}$.

$$G_{jt} + \bar{B} = T_{jt} + Q_t^j \bar{B}$$

The Import-Export Sector

The model of trade described here enriches our baseline model from Section 2 with endogenous firm entry, a more rigorous description of the matching process in which firms find trading partners, and additional details to ensure consistency with full general equilibrium in the world economy.

International trade in goods happens in decentralized search and matching markets, with exporting firms from one country looking to match with importers from another. Given a successful match, the exporting firm buys goods from the local producers, and sells them to the foreign importer, who then sells the good to the foreign household. The exporter-importer match then splits the resulting surplus via Nash bargaining. The equilibrium surplus is positive due to endogenous markups that are determined by zero profit conditions governing entry into the import-export sector in each country.

Transactions in these decentralized international markets are collateralized on both sides, and hence before import-export firms can trade they must first be "funded" by the domestic households. This funding takes the form of the household lending some of its safe assets, either dollar or euro bonds, to a firm so that it can use those assets as collateral guaranteeing its international transactions. In return, the trading firm pays a fee to the household for this service. At the end of the period, after the trading firm completes its transaction and receives its share of the resulting surplus, it pays the funding fee and returns the bond to the household. This funding arrangement effectively amounts to a within-period repurchasing agreement between trading firms and households.

Each period, new import-export firms are formed, operate, return profits to the local household, and are then disbanded.[12] The problem of these firms is static, but there are three stages to the life of each firm. In stage one, prospective firms choose whether or not to pay a fixed cost ϕ in domestic output units and become operational this period. In our baseline model, the firm does not know for sure whether it is going to be an importer or an exporter, or the country with which it will eventually trade. We assume that firms optimally choose these probabilities ex ante.[13] Intuitively, the firm does not know whether in the current period it will have an importing or an exporting opportunity or where (such opportunities arrive stochastically), but it can choose how hard to look for one versus the other.

In stage two, the firm looks for funding and has the choice of whether to seek funding in either dollars or euros (i.e. to seek either US or EU bonds as collateral). The firm faces a search friction in obtaining funding, hence a probability less than one of being funded. We assume that the firms look for a fixed amount of funding, which we normalize to one unit of the numeraire. If successful in obtaining funding, the firm proceeds to stage three, where it discovers whether it is an importer or an exporter, and then searches for an appropriate foreign trading partner, either an exporter or an importer respectively. If that search is successful, a trading match is formed, the importer buys goods from the exporter and sells them in its domestic market, and the resulting surplus is split between the two. If the two counter-parties to the international goods trade have chosen to operate in different currencies, the firms must pay an additional transaction cost before the trade is settled.

This framework is a tractable abstraction of the complex and varied trade financing arrangements that are the lifeblood of international trade in practice. Over 90% of international trade involves some form of credit, guarantee or insurance, but the particular arrangements differ substantially across industries and countries of destination and origin.[14] The burden for providing financing could fall on the importer, the exporter or both. All types of arrangements are prevalent in practice, and moreover most internationally active firms are both importing and exporting, and hence are rarely exclusively an importer or an exporter. Our modeling framework is designed to capture the fact that obtaining credit is important to internationally active firms, though we abstract from a formal banking sector,

[12]This market structure aligns well micro data, which shows significant amount of churn in bilateral trade relationships between firms (e.g. Eaton et al., 2016).

[13]This assumption is convenient, but not necessary for our mechanism. If we assumed that firms know whether e.g. they will import or export at the time of currency choice, we would need to analyze the game with both importer and exporter player types.

[14]For more details on the structure and importance of trade financing, see the BIS (2014) report on trade financing, and Ahn (2015), Antras and Foley (2015), and Niepmann and Schmidt-Eisenlohr (2017).

and have the firms interact directly with households, who hold all financial assets in each economy.

Stage 3: Trading Round and Profits

We solve the problem of the trading firms starting with stage three and working backwards. In the final stage, firms discover whether they are importing or exporting this period and with what country, and then search for an appropriate foreign counterpart. For each sub-market, we again assume that the total number of successful matches is given by the den Haan et al. (2000) matching function $M^t(u,v) = \frac{uv}{(u^{\frac{1}{\varepsilon_T}} + v^{\frac{1}{\varepsilon_T}})^{\varepsilon_T}}$ with elasticity parameter ε_T which may in general be different that ε_f.

Let $c = (j, j')$ be a double index, capturing an arbitrary country pair, and let $\tilde{m}^{im}_{ct}$ be the mass of *funded* importing firms in country j seeking trade with funded exporting firms in country j' at time t. Then the probability of a country j importer matching with a country j' exporter is

$$p^{ie}_{ct} = \frac{\tilde{m}^{ex}_{c',t}}{\left[(\tilde{m}^{ex}_{c',t})^{1/\xi_t} + (\tilde{m}^{im}_{ct})^{1/\xi_t}\right]^{\xi_t}},$$

where $c' \equiv (j', j)$. Using analogous definitions, the probability of a country j exporter matching with a country j' importer is

$$p^{ei}_{ct} = \frac{\tilde{m}^{im}_{c',t}}{\left[(\tilde{m}^{im}_{c',t})^{1/\xi_t} + (\tilde{m}^{ex}_{ct})^{1/\xi_t}\right]^{\xi_t}}.$$

Given a successful match, the two parties split the surplus that emerges from their trade. Here, we compute this value in the case of a successful match between a country j importer and a country j' exporter; the remaining possibilities can be computed in parallel. For each good that the two firms exchange, they earn a surplus that is equal to the difference in the price of the j' good in its origin country ($P^{j'}_{j',t}$) and its price in the destination country j ($P^{j'}_{j,t}$). If there is a currency mismatch between the two counter-parties, however, the trading surplus is reduced by an additional fraction $\kappa > 0$ of the transaction price.

We assume that the resulting surplus is split via Nash bargaining, with a weight α for the importer. The effective transaction price is thus:

$$P^{whol}_{c',t} = P^{j'}_{j',t} + (1-\alpha)(P^{j'}_{j,t} - P^{j'}_{j',t})$$

This is the wholesale price of country j' exports – the price at which the country-j importer purchases the good from the country-j' exporter. In turn, the country-j importer sells the good at its equilibrium j retail price $P^{j'}_{j,t}$. In equilibrium, $P^{j'}_{j,t} > P^{whol}_{c',t}$ and hence there is a markup and an associated positive surplus to sustain trading.

Let $\tilde{X}_{jt}$ be the fraction of funded country j firms who hold dollar collateral. Then the expected profits of country-j importer importing from j' who hold dollars is given by

$$\pi^{\$,im}_{c,t} = \frac{(1-\alpha)}{P^{whol}_{c',t}} \left[P^{j'}_{j,t} - P^{j'}_{j',t} - \kappa P^{whol}_{c',t}(1-\tilde{X}_{jt}) \right],$$

while if it hold euros, expected profits are

$$\pi^{€,im}_{c,t} = \frac{(1-\alpha)}{P^{whol}_{c',t}} \left[P^{j'}_{j,t} - P^{j'}_{j',t} - \kappa P^{whol}_{c',t}\tilde{X}_{jt} \right].$$

Similar expressions hold for exporters:

$$\pi^{\$,ex}_{c,t} = \frac{(1-\alpha)}{P^{whol}_{c,t}} \left[P^{j}_{j',t} - P^{j}_{j,t} - \kappa P^{whol}_{c,t}(1-\tilde{X}_{j',t}) \right]$$
$$\pi^{€,ex}_{c,t} = \frac{(1-\alpha)}{P^{whol}_{c,t}} \left[P^{j}_{j',t} - P^{j}_{j,t} - \kappa P^{whol}_{c,t}\tilde{X}_{j',t} \right].$$

Stage 2: Funding Stage

At this stage, the trading firms choose what type of funding to seek. We refer to funding with US safe assets as "dollar" funding, and funding with EU safe assets as "euro" funding. Firms seek their funding in matching markets, and not all firms succeed in finding funding each period. Supply in each of these markets is furnished by the domestic household, which lends its bond holdings of a particular currency. On the demand side are the domestic firms who choose to search for that currency.

In order to make their currency choice, firms compare their expected profits conditional on either being funded with dollars or euros. At this stage, they do not yet know whether they will be importers or exporters, with what country they may trade, or whether they will be able to find a successful trading matches in the next stage. Hence, they form expectations over the trading profits that they would receive, conditional on choosing one type of funding over the other.

The expected profit of a country-j trading firm funded with US assets is

$$\tilde{\Pi}_{jt}^{\$} = p_{jt}^{im} \sum_c p_{ct}^s p_{ct}^{ie} \pi_{ct}^{\$,im} + (1 - p_{jt}^{im}) \sum_c p_{ct}^s p_{ct}^{ei} \pi_{ct}^{\$,ex},$$

where p_{jt}^{im} is the probability that a US trading firm is an importer and p_{ct}^s is the probability the the firm from country j seeks a match in country j'.

The first of the two terms in the above sum equals the expected profit of being a dollar-funded importer. It equals the probability of being an importer, times the probability of then finding a successful match with a foreign exporter, times the resulting profits from that match, weighted by the relative likelihood the firm searches in country j'. The second component is the expected profit of being a dollar-funded exporter. The corresponding expected profits of a country j trading firm funded with EU assets instead is:

$$\tilde{\Pi}_{jt}^{€} = p_{jt}^{im} \sum_c p_{ct}^s p_{ct}^{ie} \pi_{ct}^{€,im} + (1 - p_{jt}^{im}) \sum_c p_{ct}^s p_{ct}^{ei} \pi_{ct}^{€,ex}.$$

We now compute the probability that firms find the funding they seek. Bonds promise payment of one unit of the issuing country's consumption good. Thus, the total value of the US bonds available for lending in country j at time t is given by $P_{us,t} B_{jt}^{\$} Q_t^{\$}$, where $B_{jt}^{\$}$ are the holdings of US bonds in the country j household's portfolio, $P_{us,t}$ is the price of the US consumption basket, and $Q_t^{\$}$ is the real price of the bond that pays off one unit of US consumption tomorrow.

Let m_{jt} be mass of trading firms operating in country j, and let X_{jt} be the fraction of country-j trading firms choosing to seek US bonds. Then, the total mass of country-j trading firms searching the domestic US bond market is $m_{jt} X_{jt}$. The total number of matches is given according to the constant returns to scale matching function, so that the probability that a country j trading firm seeking US bonds finds a suitable supplier is

$$p_{jt}^{\$} = \frac{M^f \left(m_{jt} X_{jt}, P_{us,t} B_{jt}^{\$} Q_t^{\$} \right)}{m_{jt} X_{jt}}.$$

Similarly, the probability that a country j trading firm seeking EU bonds finds a match is[15]

$$p_{jt}^{€} = \frac{M^f\left(m_{jt}(1-X_{jt}), P_{eu,t}B_{jt}^{€}Q_t^{€}\right)}{m_{jt}(1-X_{jt})}.$$

In the event that the trading firm finds the funding it seeks, it pays a fee $r_j^{\$}$ or $r_j^{€}$ (in US and EU consumption units respectively) for the funding services of dollars or euros. Thus, the expected profit of a country-j firm seeking dollar funding is given by

$$\Pi_{jt}^{\$} = p_{jt}^{\$}(\tilde{\Pi}_{jt}^{\$} - P_{us,t}r_j^{\$}), \tag{16}$$

which is simply the probability of obtaining dollar funding, $p_{jt}^{\$}$, times the expected profit net of the dollar funding costs. Similarly, we can compute the expected profit of a country-j firm seeking Euro funding:

$$\Pi_{jt}^{€} = p_{jt}^{€}(\tilde{\Pi}_{jt}^{€} - P_{eu,t}r_j^{€}). \tag{17}$$

The only equilibrium requirement for the funding fees $r_j^{\$}$ and $r_j^{€}$ are that they leave firms with a positive surplus relative to the alternative of declining funding and doing no trade. In parallel with the labor match and searching literature, these prices could be fixed exogenous parameters — so long as they fall within the surplus range of the trading firms — or they could be endogenously determined by assuming some bargaining paradigm, like Nash bargaining. For simplicity, we follow the first of these paths and fix the funding prices to a common value, $r = r_j^{\$} = r_j^{€}$.

Combining the above equations, we have the an expression for the net benefit of choosing dollars in this stage:

$$V_{jt}^{\$} = \Pi_{jt}^{\$} - \Pi_{jt}^{€} + \theta_{it},$$

where $\theta_{it} \sim N(0, \sigma_\varepsilon^2)$ and iid across firms. The shock θ_{it} captures firms' idiosyncratic incentives to prefer one currency and the other, and ensures that currency choice and asset holdings in our quantitative model are always interior, which helps numerical solutions.

Given that the expected payoff of seeking dollar funding is increasing in θ_{it}, without loss of generality we restrict our analysis to the space of monotone strategies in which trading

[15]It is now straightforward to evaluate

$$\tilde{X}_{jt} = \frac{p_{jt}^{\$}X_{jt}}{p_{jt}^{\$}X_{jt} + p_{jt}^{€}(1-X_{jt})}.$$

firms from both countries adopt the US asset so long as their private shock exceeds country-specific thresholds $\bar{\theta}_{us,t}$ and $\bar{\theta}_{eu,t}$, respectively. Given a value for these cutoffs, the resulting fraction of each country's trading firms adopting the US asset are

$$X_{jt} \equiv \int_0^1 \mathbb{1}(\theta_{it} >= \bar{\theta}_{jt})di = 1 - \Phi\left(\frac{\bar{\theta}_{jt}}{\sigma_\varepsilon}\right),$$

where $\Phi(\cdot)$ denotes the standard normal CDF.

The equilibrium cutoff $\bar{\theta}_{jt}$ is the value of the idiosyncratic preference shock that leaves the country-j importer-exporter indifferent between choosing one asset or the other, given everyone else's strategy. The equilibrium cutoff values thus solve the equation

$$\Pi^{\$}_{jt} - \Pi^{€}_{jt} + \bar{\theta}_{jt} = 0. \tag{18}$$

Stage 1: Firm Formation

At stage one, the firm optimizes over the probability of being an importer, which leads to the following optimality condition:

$$\frac{\partial \max\{\Pi^{\$}_{jt}, \Pi^{€}_{jt}\}}{\partial p^{im}_{jt}} = 0. \tag{19}$$

The solution to equation (19) above determines the share of importers and exporters in equilibrium in each country.

Given this and all of the above choices, prospective firms then decide whether or not to pay the fixed cost $\phi > 0$ in order to become operational this period. Firms enter the import-export sector until the zero-profit condition

$$\max\{\Pi^{\$}_{jt}, \Pi^{€}_{jt}\} - \phi P_{jt} = 0,$$

is satisfied, where $\Pi^{\$}_{jt}$ and $\Pi^{€}_{jt}$ are evaluated at the optimal choice of p^{im}_{jt}. The entry condition thus determine the equilibrium size, m_{jt}, of the import-export sector in each country.

Market Clearing

Market clearing in the goods market requires that domestic production is either consumed at home or exported abroad via the export sector. Since rest-of-world countries are

Table 1: Exogenously Fixed Parameters

Parameter	Concept	Value
β	Time preference	0.997
κ	Mismatch cost	0.01
$\mu_{us} = \mu_{eu}$	Big country measure	0.25
X_{us}	US dollar share	0.90
X_{eu}	EU dollar share	0.10
η	Elasticity of substitution between consumption goods	5.00
α	Exporters bargaining parameter	0.50
Y_{rw}	ROW relative income	0.90
σ	Risk Aversion	1.00
ε_T	Elasticity of trade matching function	0.25
$\phi_{us,g} = \phi_{eu,g}$	Government spending	0

symmetric, we can treat it as a single country for aggregation purposes. Then we have,

$$(1 - \phi_{jg})\mu_j Y_{jt} = \sum_{j' \in \{us,eu,rw\}} \mu_{j'} C^j_{j',t} \tag{20}$$

Here, we have assumed for simplicity that bond adjustment costs, fixed costs, and currency-mismatch costs are transferred in lump-sum back to the households located where they are incurred within each period.

Bond market clearing requires that the foreign and domestic holding of bonds combine to equal the fixed aggregate supply of government debt:

$$\bar{B}^{\$} = \sum_{j \in \{us,eu,rw\}} \mu_j B^{\$}_{j,t} \tag{21}$$

$$\bar{B}^{€} = \sum_{j \in \{us,eu,rw\}} \mu_j B^{€}_{j,t}. \tag{22}$$

Finally, note that because of the frictions in cross-border trade, the law of one price does not hold across countries. Specifically,

$$P^j_{j',t} = \Delta^j_{j',t} P^j_{j,t} \tag{23}$$

for all country pairs $j, j', j' \neq j$, so that the $\Delta^j_{j',t}$ constitute a total of six country specific wedges capturing the markups of imported goods over their production cost in the originating

Table 2: Calibration Targets

Concept	Value
ROW USD nvoice share	89%
Exorbitant privilege ($i^{€} - i^{\$}$)	1.50% annually
Gross debt	60% of GDP
ROW trade share	55% of GDP
Funding prob	99%
Import Markup	25% over production cost

country. The $\Delta^{j}_{j',t}$ are equilibrium objects pinned down by free entry of trading firms and the equilibrium of the coordination game played by trading firms.

Given processes for $\left\{A_{jt}, \Delta^{\$}_{jt}, \Delta^{€}_{jt}, \Delta^{j}_{j',t}\right\}$, an equilibrium in the domestic sector is described by the aggregate prices, $\left\{Q^{\$}_{t}, Q^{€}_{t}\right\}$, the set of country specific prices $\left\{P_{jt}, P^{us}_{jt}, P^{eu}_{jt}, P^{rw}_{jt}, W_{jt}\right\}$, and the set of country specific allocations $\left\{C_{jt}, C^{us}_{jt}, C^{eu}_{jt}, B^{\$}_{jt}, B^{€}_{jt}, G_{jt}\right\}$ that satisfy equations (10) through (23).

4 Quantifying the Mechanism

In this section, we calibrate our model in order to examine its quantitative implications for both steady-states and dynamic transitions. We fix a set of parameters to standard values, then use the remaining parameters for which we have relatively weak priors to target six steady-state moments, computed within the US-dominant steady-state. The model is able to exactly replicate our target moments. We then explore implications both for other potential (non-targeted) steady-states, as well as the potential for dynamic transitions to these alternative steady-states given unexpected shocks to the economy.

Steady-State

The set of exogenously-fixed parameters is listed in Table 1. These values are either standard in the literature, or fixed at values that favor symmetric outcomes in the economy. We set the size of each of the two big countries equal to 25% of the world economy, which is consistent with the size of the US and the EU in world GDP. We set the currency use in the big countries (X_{us} and X_{eu}) to imply 90% use of the domestic currency, to match the evidence of Gopinath (2015). We set the (monthly) time discount parameter $\beta = 0.997$ to imply annual risk-free rate of 3.5%., and assume log preferences ($\sigma = 1$). We pick an

Table 3: Calibrated Parameters

Parameter	Concept	Value
$\bar{B}$	Big country asset supply	1.663
ϕ	Fixed cost of entry	0.068
r	Funding fee	0.009
ε^f	Funding market match. elas.	0.404
a_j^h	Preference for dom. goods	0.564
σ_ϵ^2	Funding preference shock	6.5e-07

elasticity of substitution between domestic and foreign goods equal to 5, in line with micro level estimates in the trade literature. We pick the elasticity of the trade matching function, $\varepsilon_T = 0.25$, to ensure that roughly 90% of funded firms find a trading partner. Finally, we set the currency mismatch cost $\kappa = 0.01$ so that it is just 1% of the transaction surplus, and assume that the exporters and importers have the same bargaining power implying $\alpha = 0.5$.

Table 2 lays out the set of moments that we target using the remaining six parameters. These moments are selected to capture the following features of the data. (1) ROW dollar invoicing share of 89% (Gopinath, 2015); (2) return differential on US and EU assets of 1.5% a year, a value that lies in the middle of the range cited earlier; (3) government debt of 60% of GDP, consistent with US data before the crisis (4) rest-of-world trade share ($\frac{\text{Imports + Exports}}{\text{GDP}}$) of 55%, consistent with trade data from World Bank; (5) import markups of 25%, consistent with trade literature estimates; and (6) probability of finding funding of over 99%, consistent with the observation that while trade finance is big in absolute terms, it is still small compared to the overall size of the financial system.

We target these six moments with the six remaining free parameters. Listed in Table 3, these parameters are (1) the variance of the idiosyncratic currency preference shock σ_ϵ^2, which speaks directly to X_{rw}; (2) the funding fee r which helps match the return differential; (3) the supply of government debt $\bar{B}$; (4) the home bias parameter in consumption preferences (a_h) which determines the trade share; (5) the fixed cost of entry in the trading sector (ϕ) which helps determine the steady state markups and (6) the elasticity of the matching function in funding markets (ε_f), which controls the funding probability.

Using these parameters, we confirm that there are indeed three steady states – a dollar dominant one, an euro dominant one and a symmetric one. We also confirm that at our calibration, the two coordinated steady states are dynamically stable and the symmetric one

Table 4: Steady State Moments

	USD Coord.			Symmetric			EUR Coord.		
Moments	US	EU	RW	US	EU	RW	US	EU	RW
Dollar Share (X_j)	0.90	0.10	0.89	0.90	0.10	0.50	0.90	0.10	0.11
$400 \times (r^{€} - r^{\$})$	1.50	1.50	1.50	0.00	0.00	0.00	-1.50	-1.50	-1.50
100×Implied revenue/GDP	1.01	0.13	-	0.57	0.57	-	0.13	1.01	-
100×Trade balance/GDP	0.32	0.40	-0.38	0.56	0.56	-0.59	0.40	0.32	-0.38
Gross debt/GDP	0.60	0.60	0.00	0.60	0.60	0.00	0.60	0.60	0.00
NFA/GDP	-0.37	-0.15	0.27	-0.31	-0.31	0.33	-0.15	-0.37	0.27
Portfolio Home Bias	0.58	0.91	0.00	0.80	0.80	0.00	0.91	0.58	0.00
Consumption	0.962	0.961	0.896	0.959	0.959	0.898	0.961	0.962	0.896
(Im. + Ex)/GDP	0.427	0.426	0.550	0.427	0.427	0.549	0.426	0.427	0.550

is not.[16] Table 4 lists some of the key moments of the different steady states (for the US, EU and ROW respectively).

The quantitative characteristics of the dollar-dominant steady state match a number of empirical regularities. First, since the ROW primarily uses dollars for funding its trading firms in equilibrium, foreigners hold a large portion of the dollar safe assets. This generates a sizable negative NFA position for the US, which at 37% is very close to the NFA position of the US in the data.[17] Still, US bonds are not the only safe assets used in international exchange, and thus both US and ROW households also hold significant amounts of EU assets, leading the EU to have a mildly negative NFA position as well.

Still, despite the fact that the US has a much more negative NFA position, its steady state trade surplus is smaller than that of the EU (32bp vs 40bp). This is a consequence of the "exorbitant privilege" of the country issuing the dominant medium of exchange – as we can see from the second row, the annual dollar interest rate is 1.5% lower than the euro interest rate. This helps the US earn a significant premium on its foreign assets, compared to its foreign liabilities, and thus fund a portion of its massive NFA liabilities. The third line of the table (Implied Revenue) computes the interest savings the US incurs from its

[16]We discuss the stability of the steady-states in detail in our analysis of the dynamic properties of the model in the next section.

[17]When we calculate NFA in the data, we only consider debt assets as those are the only ones in the model, but the US's overall NFA is also similar.

Table 5: Consumption Equivalent gain relative to symmetric SS

	US	EU	RW
Dollar Steady State	0.31%	0.21%	-0.22%
Including Transition	0.34%	-0.24%	-0.03%

lower interest rate, which amounts to 1.01% of GDP per year. This is a function of both the exorbitant privilege, but perhaps more importantly, also of the fact that the gross liabilities of the US are quite substantial (as they are in the data). The larger is the gross stock of assets to which the return differential is applied, the bigger is the aggregate gain from the dominant currency.

Another interesting feature of the model is that the US and EU portfolios both display significant home bias, and more importantly the home bias is lower in the US than in the EU, as is also true in the data.[18] The reason for the home bias is fairly straightforward – both the US and the EU primarily use their own currency in trade financing, hence they naturally have higher demand for their home assets. The fact that the home bias in the US is lower, even though domestic currency use in the US and the EU is the same, is because as the dominant medium of international exchange, US safe assets are in high demand in the ROW. As a result, in equilibrium a large portion of the supply of dollar assets is held abroad, leaving the Americans with a more diversified portfolio. On the other hand, there is relatively little demand for EU assets abroad, and hence EU households are happy to hold the great majority of them.

The last two rows of the table show steady state consumption and trade shares. As we would expect to find, US consumption is the highest in the dollar dominant steady state (and similarly EU consumption is the highest in the euro dominant steady state). This is despite the fact that in the dollar dominant steady state the US households are significantly less wealthy than the EU households (significantly negative NFA and thus lower overall gross asset holdings). In terms of welfare, the US's dominance is worth only about 10 basis points of steady state consumption, relative to the non-dominant EU's position. Lastly, it

[18]To measure the portfolio home bias we use the usual index

$$\text{Home Bias} = 1 - \frac{\text{Share of Foreign Bonds in HH Portfolio}}{\text{Share of Foreign Bonds in World Supply}}.$$

Table 6: Trade War Scenario: 20% increase in import tariffs

	USD Coord.			Symmetric			EUR Coord.		
Moments	US	EU	RW	US	EU	RW	US	EU	RW
Dollar Share	0.90	0.10	0.98	0.90	0.10	0.50	0.90	0.10	0.02
$400 \times (r^{\$} - r^{€})$	1.26	1.26	1.26	0.00	0.00	0.00	-1.26	-1.26	-1.26
100×Implied revenue/GDP	0.76	0.01	-	0.38	0.38	-	0.01	0.76	-
100×Trade balance/GDP	0.60	0.21	-0.43	0.77	0.77	-0.80	0.21	0.60	-0.43
Gross debt/GDP	0.61	0.61	0.00	0.61	0.61	0.00	0.61	0.61	0.00
NFA/GDP	-0.38	-0.06	0.23	-0.32	-0.32	0.33	-0.06	-0.38	0.23
Home Bias	0.49	0.93	0.00	0.80	0.80	0.00	0.93	0.49	0.00
Consumption	0.942	0.946	0.878	0.940	0.940	0.882	0.946	0.942	0.878
(Im. + Ex)/GDP	0.287	0.285	0.371	0.287	0.287	0.368	0.285	0.287	0.371

is reassuring that the model matches the empirical trade shares of the US and the EU (both around 40 to 45%), even though those were not targeted. It turns out, that as it is true in the data, the big countries trade a lot less than the small countries.

While we focused most of the discussion on the dollar dominant steady state, the euro-dominant steady state is its mirror image in all respects. Finally, the middle portion of the table presents the same numbers for the symmetric steady state. In that steady state, both assets are used equally as a medium of exchange, and thus earn the same liquidity premium, and are equally distributed around the world. Thus, in all respects the implications of this steady state for the US and the EU are equivalent.

4.1 Policy Experiment: Raising Trade Barriers

In this subsection, we analyze the effect of an increase in import tariffs. Our mechanism operates through trade flows and relationships, hence restrictions on trade could affect the currency regime, the potential dominance of a single currency and the implied size of the resulting exorbitant privilege. As such, this raises the potential that trade barriers have higher costs than typically estimated, since previous work has not considered this channel. Moreover, the costs are likely to be distributed asymmetrically if the economy starts from a dominant currency steady state.

Since recent policy discussions have highlighted the heightened interest in protectionism

Table 7: Consumption Equivalent loss

	US	EU	RW
New vs old steady state	-2.08%	-1.56%	-2.01%
Including Transition	-2.09%	-1.87%	-1.71%

policies all around the world, we consider a "Trade War" scenario where all tariffs increase by 20%. We find that even with this restricted level of trade, the model still has three steady states, and the resulting moments of each are listed below in Table 6.

As we would expect, the level of trade falls precipitously – by almost 20% of GDP in the rest of the world, and close to 15% of GDP in the big countries. This clearly lowers the steady state consumption of all three countries significantly, but interestingly the country that is hurt the most is the dominant currency country (in the symmetric steady state the losses are symmetric). The reason is that in a dominant currency steady state, the dominant currency country loses not only from reduced access to foreign goods, but also because there is a lower demand for the international medium of exchange.

The exorbitant privilege that the dominant currency earns depends on the liquidity premia earned from a large base of international trade transactions. The more trading that occurs, the larger is the demand for the international medium of exchange, and the higher is the overall gain derived by the country that issues the dominant safe asset. As a result, in a potential trade war scenario the dominant country loses an additional 50bp of steady state consumption, as compared to the other big country.

Our results highlight that protecting the broad use of the dollar in international trade should be an important consideration to US trade policy, as the dominant country is the most vulnerable one in a trade war.

4.2 Dynamics

A major advantage of our approach to modeling the emergence of a dominant currency is the fact the model allows to solve for the (determinant) transition path of the economy from any initial set of states, as well as in response to unanticipated changes in policy that might affect current choice.

The first question of interest is which, if any, of the three steady states are dynamically stable. To evaluate this, in Figure 3 we plot the regions of attraction for the steady states of

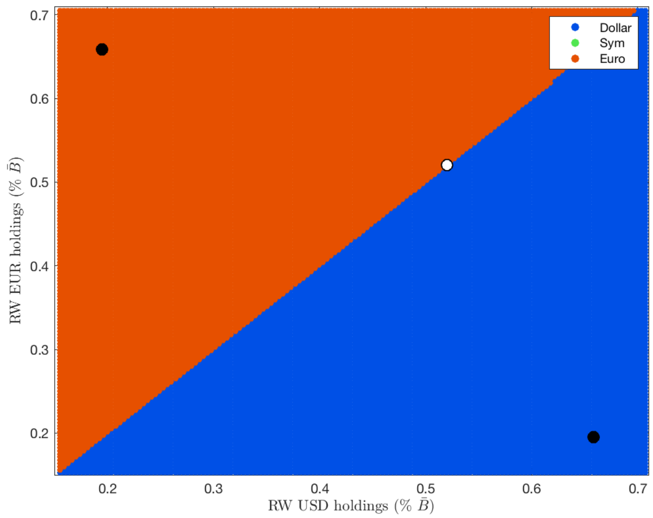

Figure 3: Attraction regions in the dynamic model.

our economy – for all possible combinations of initial asset holdings (which span the x- and y- axis) we run the model forward and see where that equilibrium path converges to. For example, the blue region in the picture encompasses all initial states that eventually converge to the dollar dominant steady state (no matter the speed of this convergence.) The figure shows that whenever the rest of world households begin with more dollars than euros, the economy converges towards the dollar dominant steady-state and, conversely, initial point with relatively more euros converge to the euro dominant steady-state. Thus, the symmetric steady state is dynamically unstable – the economy will remain there if it starts *exactly* at that point, but no other initial conditions converge to it.

While the regions of attraction tell us where the economy will *eventually* converge to, it is not necessarily true that all blue dots correspond to heavy dollar use and all red dots to heavy euro use. As the economy transitions, the equilibrium currency use can actually be much closer to symmetric. To quantify that, Figure 4 depicts a heat map of the intensity of dollar and euro usage in the economy at different points in the state space. Darker shades of blue indicate an equilibrium currency usage that is more slanted towards dollars, and similarly darker shades of orange indicate an equilibrium currency use slanted towards euros. The region of purple near to the diagonal of the figure indicate a relatively even usage of both currencies. Thus, while the symmetric steady state is unstable, and hence the economy will never converge to the point where both currencies are used equally, there is a substantial

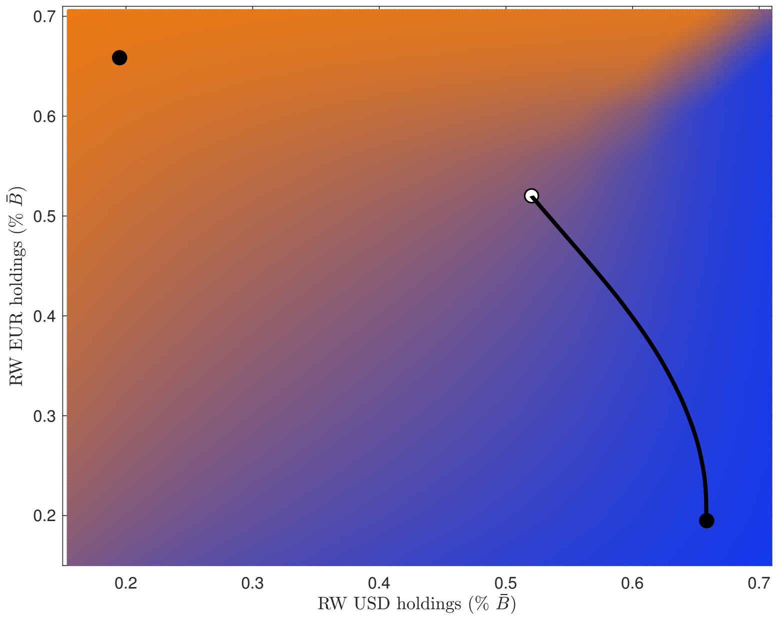

Figure 4: Currency usage in the dynamic economy. Blue indicates dollars are more intensively used, while orange indicates euros are more intensively used. black line indicates transition path from near symmetric equilibrium to dollar dominant steady-state.

portion of the state space where both currencies see a lot of use at the same time. This is because the economy does not transition out of the "mixed" currency regime immediately, but does so rather gradually.

Figure 4 offers an additional insight: the "mixed" region becomes more narrow in the upper right portion of the picture, where the initial holdings of both dollar and euro safe assets are higher. This results make sense in light of our earlier observation that, when more assets are available, agent's choices depend more heavily on their coordination motive and less on the relative availability of assets. This can lead to more rapid switches in the equilibrium currency use.

Figure 4 also depicts the transition path of the economy starting from a point that lies almost, but not quite, on top of the symmetric steady-state. Figure 5 depicts the values of several endogenous variables along this transition path. The top-left panel shows that asset holdings move slowly at first, and then accelerate their change as the dollar dominance begins to take hold. During this transition period US consumption initially rises — reflecting the increased revenue it earns from liquidity premia – and then falls as the net foreign asset position of the US deteriorates. As one can see, US consumption is substantially higher in transition than in the new steady state. This is a result of the *increasing* foreign demand for

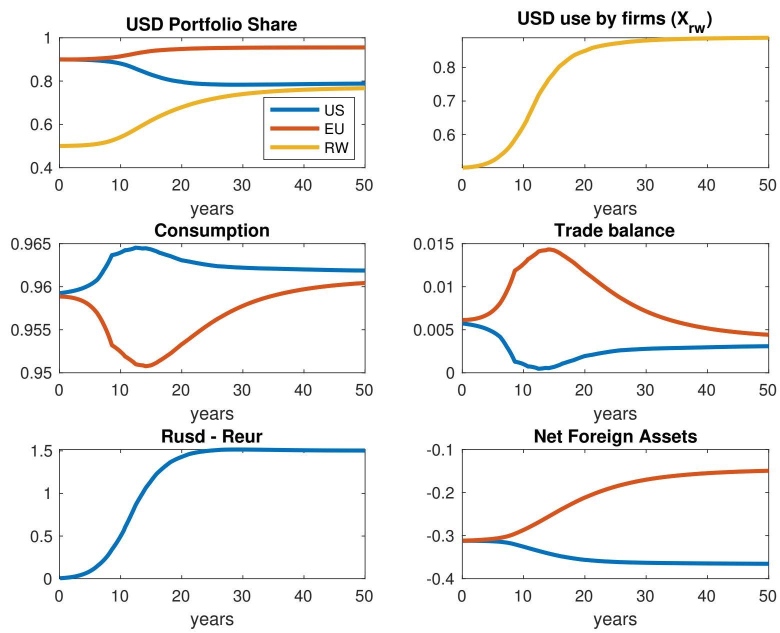

Figure 5: Transition paths from symmetric equilibrium to dollar dominant steady-state.

US assets and their concomitant liquidity services, which allows the US to steadily increase its borrowing from the rest of the world. This increased borrowing finances temporarily high consumption, but eventually consumption comes down as the US net foreign asset comes close to its new steady state value of almost 40% of GDP.

Table 5 compares the welfare of the three countries in the dollar steady state (where the economy converges to at the end of this transition) relative to the symmetric steady state (its starting point). Interestingly, both large countries enjoy a higher long run level of consumption under the dollar dominant steady state, relative to what they would have consumed in the symmetric steady-state, with the dominant country (the US) enjoying a somewhat larger benefit. The benefit to the US is relatively straight forward and is derived from the higher liquidity services offered by the USD safe asset, which essentially allows the US to earn some seignorage from its foreign liabilities. But the EU also enjoys a substantially higher consumption at the steady state, and this is because it has a significantly better Net Foreign Asset position. The EU NFA improves by over 15% of GDP as compared to the symmetric steady state, because as the foreign demand for EUR assets dries up, those assets are repatriated back in the EU, where they are in relatively higher demand.

Moreover, including the utility effects of the actual transition path that the economy takes to go to the eventual dollar steady state, reverses the welfare implications for the non-dominant large country (EU in this case), while further increasing the benefits for the

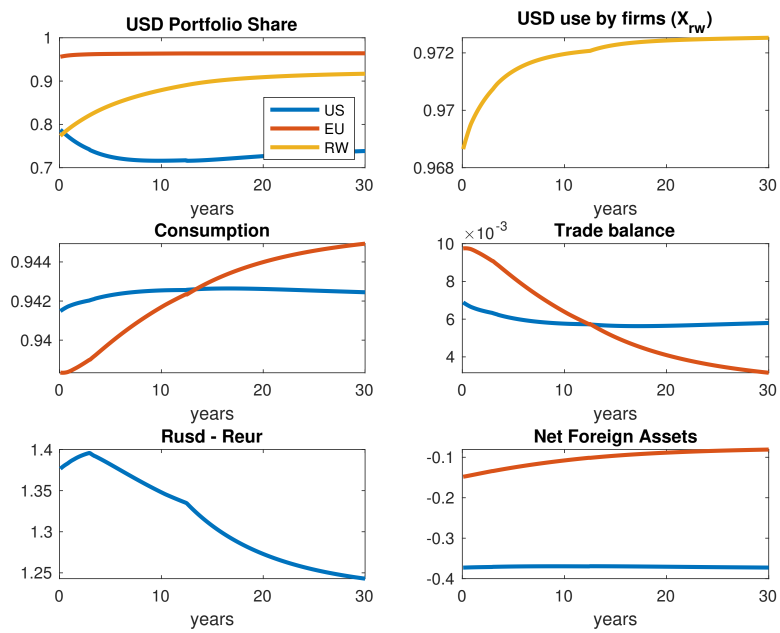

Figure 6: Transition in response to a "trade war" in which all country increase tariffs by 10%.

dominant country. Why? Because the change in asset positions required to support the dominant steady-state imply a substantial rise in the dominant country's consumption as it decumulates assets, and a substantial short fall in the consumption of the non-dominant country as is accumulates assets. More generally, the table demonstrates that including the transitions periods is important for assessing the welfare implications of the dominant currency paradigm.

4.3 Trade-war Scenario

Figure 6 depicts the transitions for several endogenous variables in after the emergence of the trade war. Consumption of the large countries falls on impact, indeed falling more for the EU than the US, as the dominance of the dollar increases in the rest of world. This happens because overall trade levels are lower, implying a fall in the scarcity of funding and increase in the importance of coordination motives on the part of trading firms.

Over time, the foreign asset position of the EU improves furthers, with virtually no effect of the NFA of the US, since demand for dollar assets is rising as dollar dominance strengthens. This leads to higher long run EU consumption. Table 7 describes the welfare implications of the trade war, showing that the dominant country the rest of the world lose the most, while the non-dominant country experiences a somewhat smaller fall in consumption. Including transitions changes the ordering of the losses of the EU and the rest of the world, but does not change change the implication that US, owner of the dominant currency, experiences the largest losses from the trade war.

5 Conclusions and Future Work

This paper has presented a new model of currency coordination that is both quantitatively realistic, and tractable enough to use for standard macroeconomic analysis. In doing so, we have abstracted from risk, both the potential for short run shocks that perturb the economy around a particular steady-state and from longer run stochastic transitions between currency regimes. We conclude by noting that both of these extensions are rather straightforward given the model we have proposed here. Business-cycle analysis can be conducted using linearized policy functions around a given steady-state. Linearized policies are already generated as part of the solution procedure we employed above. Global analysis with risk requires global solution techniques, but the policy function approach, advocated for example by Richter et al. (2013), can easily be applied to our model as well.

A Proof of Propositions

Proof of Proposition 1. When $\bar{X} = 1$, we have

$$\begin{aligned} V^{\$} &= p^{\$}(\pi - 0 \times \kappa) - p^{€}(\pi - 1 \times \kappa) \\ &= (p^{\$} - p^{€})\pi + p^{€}\kappa, \end{aligned}$$

which is weakly positive whenever $\kappa \geq (1 - p^{\$}/p^{€})\pi$. The corresponding condition for $\bar{X} = 0$ is straightforward.

The above establishes that the economy has multiple corner equilibria whenever $\kappa \geq (1 - p^*)\pi$. To see that the economy has a unique equilibrium otherwise, note that $V_j^{\$}$ is linear and increasing in $\bar{X}$. Hence an interior solution (i.e. a point where $V_j^{\$}(\bar{X}) = 0$) can exist only when $V_j^{\$}(0) \leq 0$ and $V_j^{\$}(1) \geq 0$. But we have already shown that $\kappa \geq (1 - p^*)\pi$ is both necessary and sufficient for this. Hence an an interior (mixed-strategy) solution of the form

$$X^{mix} = \frac{p^{\$}}{p^{\$} + p^{€}} + \frac{p^{€} - p^{\$}}{\kappa(p^{\$} + p^{€})}\pi.$$

exists if and only if both corner equilibria are also equilibria of the game.

■

Proof of Proposition 2. When $\bar{X} = 1$, we have

$$\begin{aligned} V^{\$} &= \frac{B^{\$}}{B^{\$} + 1}(\pi - 0 \times \kappa) - \frac{B^{€}}{B^{€} + 0}(\pi - 1 \times \kappa) \\ &= \left(\frac{B^{\$}}{B^{\$} + 1} - 1\right)\pi + \kappa, \end{aligned}$$

which is weakly positive whenever $\kappa \geq (B^{\$} + 1)^{-1}\pi$. The corresponding condition for $\bar{X} = 0$ is straightforward.

To prove the uniqueness condition, we start by noting that for $\kappa \geq \frac{1}{\min\{B^{\$}, B^{€}\}}\pi$ both corner solutions are equilibria, implying there is equilibrium multiplicity in this range. Below, we show that for $\kappa \in [\frac{1}{\max\{B^{\$}, B^{€}\}}\pi, \frac{1}{\min\{B^{\$}, B^{€}\}}\pi)$ the unique equilibrium is the corner solution that exists in that case. Then, we show that for $\kappa < \frac{1}{\max\{B^{\$}, B^{€}\}}\pi$ there is a unique interior equilibrium (and no corner equilibria).

First, we handle the case $B^{\$} \neq B^{€}$ and revisit the symmetric case below. For $\kappa \in [\frac{1}{\max\{B^{\$}, B^{€}\}}\pi, \frac{1}{\min\{B^{\$}, B^{€}\}}\pi)$ only one of the corner equilibria exists, but we need to show that there is also no interior equilibrium. Without loss of generality, assume that $B^{\$} > B^{€}$ (the proof of the mirror case proceeds in the same way).

An interior equilibrium equalizes the payoffs to dollar and euro funding and hence any interior symmetric equilibrium solves the equation:

$$V^{\$} = \frac{B^{\$}}{B^{\$} + \bar{X}}(\pi - \kappa(1 - \bar{X})) - \frac{B^{€}}{B^{€} + (1 - \bar{X})}(\pi - \kappa\bar{X}) = 0.$$

Re-arranging and cross-multiplying we get

$$B^{\$}(B^{€} + 1 - \bar{X})(\pi - \kappa(1 - \bar{X})) = B^{€}(B^{\$} + \bar{X})(\pi - \kappa\bar{X}),$$

which we can further simplify down to a quadratic equation of $\bar{X}$, that we call $P(\bar{X})$:

$$P(\bar{X}) = (B^{\$} - B^{€})\kappa\bar{X}^2 + \left[(B^{\$} + B^{€})\pi - 2B^{\$}(B^{€} + 1)\kappa\right]\bar{X} + B^{\$}(B^{€} + 1)\kappa - B^{\$}\pi$$

In particular, $P(\bar{X})$ is a convex quadratic polynomial (since $B^{\$} > B^{€}$). Hence, to show that there are no solutions for $\bar{X} \in (0, 1)$ it is enough to show that the value of the polynomial at both 0 and 1 is negative, which is the case:

$$P(0) = B^{\$}(B^{€} + 1)\kappa - B^{\$}\pi < 0$$

since $\kappa < \frac{1}{B^{€}+1}\pi$, and

$$\begin{aligned} P(1) &= (B^{\$} - B^{€})\kappa + \left[(B^{\$} + B^{€})\pi - 2B^{\$}(B^{€} + 1)\kappa\right] + B^{\$}(B^{€} + 1)\kappa - B^{\$}\pi \\ &= B^{€}(\pi - (B^{\$} + 1)\kappa) \leq 0 \end{aligned}$$

since $\kappa \geq \frac{1}{B^{\$}+1}\pi$, with equality only when $\kappa = \frac{1}{B^{\$}+1}\pi$, in which case the interior solution corresponds with the corner solution $\bar{X} = 1$. Thus, for $\kappa \in [\frac{1}{\max\{B^{\$},B^{€}\}}\pi, \frac{1}{\min\{B^{\$},B^{€}\}}\pi)$ there is a unique, corner equilibrium (equal to $\bar{X} = 1$ when $B^{\$} > B^{€}$, and $\bar{X} = 0$ when $B^{\$} < B^{€}$).

In case $\kappa < \frac{1}{\max\{B^{\$},B^{€}\}}\pi$ corner outcomes cannot be equilibria, and thus we only need to show that there exists a unique interior equilibrium. To show uniqueness, define $\phi^{\$}(\bar{X}) \equiv \frac{B^{\$}}{B^{\$}+\bar{X}}\left[\pi - \kappa(1 - \bar{X})\right]$ and $\phi^{€}(\bar{X}) \equiv \frac{B^{€}}{B^{€}+1-\bar{X}}\left[\pi - \kappa\bar{X}\right]$, and note that the we can write the condition the interior equilibrium must satisfy as:

$$V^{\$} = \phi^{\$}(\bar{X}) - \phi^{€}(\bar{X})$$

Taking derivatives,

$$\frac{\partial V^{\$}}{\partial\bar{X}} = \frac{\partial(\phi^{\$}(\bar{X}))}{\partial\bar{X}} - \frac{\partial(\phi^{€}(\bar{X}))}{\partial\bar{X}}$$

Now notice that

$$\frac{\partial(\phi^{\$}(\bar{X}))}{\partial\bar{X}} = \frac{\kappa B^{\$}}{B^{\$} + \bar{X}} - \left[\pi - \kappa(1 - \bar{X})\right]\frac{B^{\$}}{(B^{\$} + \bar{X})^2} < 0$$

if $\kappa < (B^{\$} + 1)^{-1}\pi$ and

$$\frac{\partial(\phi^{€}(\bar{X}))}{\partial \bar{X}} = -\frac{\kappa B^{€}}{B^{€} + 1 - \bar{X}} + \left[\pi - \kappa \bar{X}\right] \frac{B^{€}}{(B^{€} + 1 - \bar{X})^2} > 0$$

if $\kappa < (B^{€} + 1)^{-1}\pi$. Thus, for $\kappa < \frac{1}{\max\{B^{\$}, B^{€}\}}\pi$ we know that $\phi^{\$}(\bar{X})$ is downward sloping and $\phi^{€}(\bar{X})$ is upward sloping, hence $V^{\$}(\bar{X})$ is itself downward sloping. As such, $V^{\$}(\bar{X})$ can cross zero at most once.

To prove that a crossing does occur, notice that when $\kappa < (\max(B^{\$}, B^{€}) + 1)^{-1}\pi$ holds,

$$V^{\$}(0) = \phi^{\$}(0) - \phi^{€}(0) = \frac{1}{B^{€} + 1}\pi - \kappa > 0.$$

Conversely,

$$V^{\$}(1) = \phi^{\$}(1) - \phi^{€}(1) = \kappa - \frac{1}{B^{\$} + 1}\pi < 0.$$

Thus, we have proven the uniqueness condition for $B^{\$} \neq B^{€}$. In the case when $B^{\$} = B^{€} = B$ then we only need to establish that the interior solution is unique whenever $\kappa < \frac{1}{B+1}\pi$. Notice that in this case the polynomial $P(\bar{X})$ is linear, and it has the unique solution

$$X = \frac{B\pi - B(B+1)\kappa}{2B\pi - 2B(B+1)\kappa} = \frac{1}{2}.$$

■

Proof of Proposition 3. Restricting attention to symmetric equilibria, suppose that $B^{€} = 0$. Then, our matching function implies that $p_j^{€} = 0$. In this case

$$V^{\$} = \frac{\bar{B}}{\bar{B} + \mu_{rw}\bar{X} + \mu_{us}X^{us} + \mu_{eu}X^{eu}}\pi - 0 > 0,$$

for all $\bar{X} \in [0, 1]$. Hence, $\bar{X} = 1$ is optimal from the trading firms' perspective. Moreover, since $\mu_{eu}(1 - X^{eu}) > 0$, return equalization cannot be achieved for any positive holdings of $B^{€}$, hence $B^{€} = 0$ is sustained. A similar argument establishes that same for the euro dominant equilibrium.

When both $B^{\$}$ and $B^{€}$ are positive, we have

$$\begin{aligned} V^{\$} = & \frac{\bar{B}}{\bar{B} + \mu_{rw}\bar{X} + \mu_{us}X^{us} + \mu_{eu}X^{eu}}\left[\pi - \kappa(1 - \bar{X})\right] - \\ & \frac{\bar{B}}{\bar{B} + \mu_{rw}(1 - \bar{X}) + \mu_{us}(1 - X^{us}) + \mu_{eu}(1 - X^{eu})}\left[\pi - \kappa\bar{X}\right]. \end{aligned} \tag{24}$$

Since $\mu_{us}X^{us} = \mu_{eu}(1 - X^{eu})$ and $\mu_{eu}X_{eu} = \mu_{us}(1 - X^{us})$ by assumption, clearly $V^{\$} = 0$ when $\bar{X} = 1/2$. This establishes the existence of the steady-states equilibria stated in the theorem.

To see that these are the only steady-state equilibria (except in the case that $\kappa = \frac{\mu_{rw}\pi}{\bar{B}+\mu_{rw}+\mu_{us}X^{us}+\mu_{eu}X^{eu}}$), redefine

$$\phi^{\$}(\bar{X}) = \frac{\bar{B}}{\bar{B}+\mu_{rw}\bar{X}+\mu_{us}X^{us}+\mu_{eu}X^{eu}}\left[\pi-\kappa(1-\bar{X})\right]$$
$$\phi^{€}(\bar{X}) = \frac{\bar{B}}{\bar{B}+\mu_{rw}(1-\bar{X})+\mu_{us}(1-X^{us})+\mu_{eu}(1-X^{eu})}\left[\pi-\kappa\bar{X}\right]$$

so that $V^{\$} = \phi^{\$}(\bar{X}) - \phi^{€}(\bar{X})$. Moreover, notice that $\frac{\partial\phi^{\$}(\bar{X})}{\bar{X}}$ is negative so long as κ falls below the threshold $\frac{\mu_{rw}\pi}{\bar{B}+\mu_{rw}+\mu_{us}X^{us}+\mu_{eu}X^{eu}}$, is zero when κ equals this value, and is positive otherwise.

Conversely, $\frac{\partial\phi^{€}(\bar{X})}{\bar{X}}$ is positive when κ is below this threshold, zero at it, and negative otherwise. Hence, except in the knife-edge case, the two functions can cross only once, which we have already shown that they do. In the special case that $\kappa = \frac{\mu_{rw}\pi}{\bar{B}+\mu_{rw}+\mu_{us}X^{us}+\mu_{eu}X^{eu}}$, $\phi^{\$}(\bar{X}) = \phi^{€}(\bar{X})$, and the economy exhibits a continuum of equilibria. ■

Proof of Proposition 4. Define the total mass the two large countries, $\mu^* \equiv \mu_{us}+\mu_{eu}$, and let

$$X^* \equiv \frac{\mu_{us}X_{us}+\mu_{eu}X_{eu}}{\mu_{us}+\mu_{eu}}$$

be the average rate of dollar use among them. Thus, $\mu^*X^* = \mu_{us}X_{us}+\mu_{eu}X_{eu}$ is the total dollar use among the big countries. Given our symmetry assumptions $X^* = \frac{1}{2}$ (but can relax later).

To prove local stability of the different steady states, we want to show that best-response functions define a contraction in the neighborhood of a given steady-state. To this end, define the vector of best response function of trading firms and households in country j, given the actions of all other firms, $\bar{X}$, and households in the rest of the world $B^{\$}$ and $B^{€}$:

$$\varphi_X(\bar{X}, B^{\$}, B^{€}) = \frac{B^{\$}(\pi-\kappa(B^{€}+1)+\bar{X}\kappa(2B^{€}+1))}{(B^{\$}+B^{€})\pi+\kappa(\bar{X}(B^{\$}-B^{€})-B^{\$})}$$
$$\varphi_{B^{\$}}(\bar{X}, B^{\$}, B^{€}) = \bar{B}\frac{\bar{X}}{\mu_{rw}\bar{X}+\mu^*X^*}$$
$$\varphi_{B^{€}}(\bar{X}, B^{\$}, B^{€}) = \bar{B}\frac{1-\bar{X}}{\mu_{rw}(1-\bar{X})+\mu^*(1-X^*)}$$

Stacking the best responses in a single vector valued function $\Phi = [\varphi_X, \varphi_{B^{\$}}, \varphi_{B^{€}}]$ we have that a given steady state is locally stable if the function Φ is a local contraction map, which is the case whenever the eigenvalues of the Jacobian $\nabla\Phi$ lie inside the unit circle.

The Jacobian has the form

$$\nabla\Phi = \begin{bmatrix} \frac{\partial\varphi_X}{\partial\bar{X}} & \frac{\partial\varphi_X}{\partial B^{\$}} & \frac{\partial\varphi_X}{\partial B^{€}} \\ \frac{\partial\varphi_{B^{\$}}}{\partial\bar{X}} & 0 & 0 \\ \frac{\partial\varphi_{B^{€}}}{\partial\bar{X}} & 0 & 0 \end{bmatrix}$$

hence its eigenvalues are given by the roots of the characteristic polynomial

$$\lambda\left(\lambda^2 - \lambda\frac{\partial\varphi_X}{\partial\bar{X}} - \frac{\partial\varphi_{B^{\$}}}{\partial\bar{X}}\frac{\partial\varphi_X}{\partial B^{\$}} - \frac{\partial\varphi_{B^{€}}}{\partial\bar{X}}\frac{\partial\varphi_X}{\partial B^{€}}\right) = 0$$

Clearly, one of the solutions is $\lambda = 0$, so we just need to ensure that the roots of the quadratic expression in the parenthesis are inside the unit circle We will handle this condition for the neighborhood of each of the different steady states separately.

Case I: Symmetric Steady State

At the symmetric steady state we have that

$$\frac{\partial\varphi_X}{\partial B^{\$}} = -\frac{\partial\varphi_X}{\partial B^{€}}$$

hence the relevant condition for the eigenvalues reduces to

$$\lambda^2 - \lambda\frac{\partial\varphi_X}{\partial\bar{X}} - \frac{\partial\varphi_X}{\partial B^{\$}}\left(\frac{\partial\varphi_{B^{\$}}}{\partial\bar{X}} - \frac{\partial\varphi_{B^{€}}}{\partial\bar{X}}\right) = 0$$

with roots

$$\lambda^* = \frac{1}{2}\left(\frac{\partial\varphi_X}{\partial\bar{X}} \pm \sqrt{\left(\frac{\partial\varphi_X}{\partial\bar{X}}\right)^2 + 4\frac{\partial\varphi_X}{\partial B^{\$}}\left(\frac{\partial\varphi_{B^{\$}}}{\partial\bar{X}} - \frac{\partial\varphi_{B^{€}}}{\partial\bar{X}}\right)}\right).$$

At the symmetric steady state.

$$\frac{\partial\varphi_X}{\partial\bar{X}} = \frac{\kappa\left(4X^*\mu^*(1-X^*) + \mu_{rw}(\mu_{rw}+2\mu^*) + 2\bar{B}(\mu_{rw}+\mu^*)\right)}{(\mu_{rw}+\mu^*)^2(2\pi-\kappa)} > 0$$

since $\kappa < 2\pi$ (κ is not bigger than the gross surplus of transactions). Hence, the bigger (in absolute value) root is

$$\lambda^* = \frac{1}{2}\left(\frac{\partial\varphi_X}{\partial\bar{X}} + \sqrt{(\frac{\partial\varphi_X}{\partial\bar{X}})^2 + 4\frac{\partial\varphi_{B^{\$}}}{\partial\bar{X}}\left(\frac{\partial\varphi_X}{\partial B^{\$}} - \frac{\partial\varphi_{B^{€}}}{\partial\bar{X}}\right)}\right).$$

Lastly, since we also have that

$$\frac{\partial\frac{\partial\varphi_{B^{\$}}}{\partial\bar{X}}}{\partial\kappa} = \frac{\partial\frac{\partial\varphi_X}{\partial B^{\$}}}{\partial\kappa} = \frac{\partial\frac{\partial\varphi_{B^{€}}}{\partial\bar{X}}}{\partial\kappa} = 0$$

then the root is growing in κ. We can then find the critical threshold $\bar{\kappa}$ that ensures the roots are within the unit circle by solving

$$1-\frac{\partial\varphi_X}{\partial\bar{X}}-\frac{\partial\varphi_{B^\$}}{\partial\bar{X}}\left(\frac{\partial\varphi_X}{\partial B^\$}-\frac{\partial\varphi_{B^€}}{\partial\bar{X}}\right)=0$$

for κ, which yields

$$\bar{\kappa}=\frac{\pi(\mu_{rw}(\mu_{rw}+\mu^*)+(\mu^*)^2(1-4X^*(1-X^*)))}{(\mu_{rw}+\mu^*)(\bar{B}+\mu_{rw}+\mu^*\frac{1}{2})}$$

and with $X^*=\frac{1}{2}$ it reduces to

$$\bar{\kappa}=\frac{\pi\mu_{rw}}{\bar{B}+\mu_{rw}+\mu^*\frac{1}{2}}$$

The roots of the characteristic polynomial are then inside the unit circle (in the neighborhood of the symmetric steady state) as long as $\kappa<\bar{\kappa}$.

Case II: Dollar Dominant Steady State

At the dollar dominant steady state $\bar{X}=1$ and

$$\frac{\partial\varphi_X}{\partial\bar{X}}=\frac{\partial\varphi_X}{\partial B^\$}=0$$

and we need to ensure that

$$\lambda^2=\frac{\partial\varphi_{B^€}}{\partial\bar{X}}\frac{\partial\varphi_X}{\partial B^€}<1$$

where

$$\frac{\partial\varphi_{B^€}}{\partial\bar{X}}\frac{\partial\varphi_X}{\partial B^€}=\frac{\bar{B}}{\mu^*(1-X^*)}\frac{\pi(\mu_{rw}+\mu^*)-\kappa(\mu_{rw}+\mu^*X^*+\bar{B})}{\bar{B}\pi}$$

if $\kappa<\frac{\pi(\mu_{rw}+X^*\mu^*)}{\mu_{rw}+X^*\mu^*+\bar{B})}$ then the above expression is positive, and hence $|\lambda|<1$ if and only if the above expression is less than one, which is true if

$$\kappa>\frac{\pi(\mu_{rw}+(2X^*-1)\mu^*)}{\bar{B}+\mu_{rw}+\mu^*X^*}$$

which is equal to $\bar{\kappa}$ when $X^*=\frac{1}{2}$.

On the other hand, if $\kappa>\frac{\pi(\mu_{rw}+X^*\mu^*)}{\mu_{rw}+X^*\mu^*+\bar{B})}$ then $\phi_X=1$ because it hits the upper bound of $X\leq 1$, as a result $\frac{\partial\varphi_X}{\partial B^€}=0$ and thus

$$\lambda^2=0$$

Since all eigenvalues of $\nabla\Phi$ are zero, the system is stable. Thus, the coordinated steady state is stable for any

$$\kappa > \bar{\kappa}.$$

Case III: Euro Dominant Steady State we can proceed in similar steps to Case II. ■

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