Rising inequality and trends in leisure*

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Abstract

This paper develops a model that generates rising average leisure time and increasing leisure inequality along a path of balanced growth. Households derive utility from three sources: market goods, home goods and leisure. Home production and leisure are both activities that require time and capital. Households allocate time and capital to these non-market activities, work and rent capital out to the market place. The dynamics are driven by activity-specific TFP growth and a spread in the distribution of household-specific labor market efficiencies. When the spread is set to match the increase in wage inequality across education groups, the model can account for the observed average time series and cross-sectional dynamics of leisure time in the U.S. over the last five decades.

Keywords: leisure, labor supply, inequality, home-production, balanced growth path.

JEL classification: O41, J22, E24.

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1 Introduction

The distribution in income, consumption and wealth has received a lot of attention in economics. An important reason for this that these economic variables are key indicators of economic welfare, which is the main object of interest. Income, consumption, and wealth, however, are all related to an individual's market activity and ignore additional dimensions of heterogeneity outside the market place. In the U.S. there have been significant and systematic changes in the allocation of time between individuals of different educational groups. Overall leisure time has been increasing over time but this aggregate trend masks heterogeneity across skill groups. Whereas leisure time was relatively equally distributed across educational groups half a century ago, low skilled individuals nowadays enjoy systematically more than average leisure time. This increase in leisure inequality is mirrored in an increase in inequality of hours worked across skill groups, where high skilled individuals' hours worked decrease slower than for low skilled individuals.

The main contribution of this paper is to provide a simple growth model that is consistent with both a rise in aggregate leisure and an increase in leisure inequality along a path of balance growth in the aggregate. In our model individuals derive utility from market produced goods, home produced goods as well as leisure. Furthermore, there is rising wage inequality across skill groups due to exogenous technical change that is driving the differential trends in hours.[1] As the relative implicit price of leisure increases over time due to technical change there is a general upward trend in leisure time. This is the case since leisure and the composite of market and home goods enters the utility function as gross complements such that the income effect of a wage change dominates its substitution effect. Hence, in the time series as (average) wages increase, leisure rises too. But how do we square this with

[1]See also Elsby and Shapiro (2012) who explain the rising inequality in employment rates between high and low skilled men through returns to experience.

the cross-sectional finding that households who experienced a faster wage growth, i.e., households with higher education, experienced a slower increase in leisure time? The key aspect here in our model is intertemporal substitution of labor supplied to the market à la Lucas and Rapping (1969). This intertemporal substitution is responsible that households who face a faster wage growth decide to raise leisure time *slower*, whereas in the aggregate time series households take more and more time off as the market productivity (and average wages) increase. In contrast, a model without intertemporal substitution of labor cannot square the average time series and cross-sectional observation. The overall increase in leisure time suggests that the income effect of hours worked dominates the substitution effect. If intertemporal substitution is ignored such a formulation in turn would imply that high skilled household with a faster wage growth should increase leisure time faster (and not slower).

The model economy consists of heterogeneous households with household-specific labor market efficiencies (per unit of time) and different levels of initial wealth. The distribution of labor market efficiencies can be interpreted as mapping into the distribution of educational qualifications. The household derives utility from the consumption of market goods, home goods and leisure goods. The key assumption is that skill is more useful in the production of market goods as oppose to home and leisure goods. Market and home goods are gross substitutes with an elasticity of substitution higher than one (e.g., cooking at home versus buying a take-away), but both are poor substitutes to leisure goods, with an elasticity of substitution less than one (as they are different type of goods, watching TV versus having a haircut). Production of all three activities requires time and capital as input, and each activity has its own specific (exogenous) TFP growth rate. Traditionally, leisure time is modeled as generating directly utility. However, the majority of leisure time, such as watching TV, involves the usage of some capital (see table VII of Aguiar and

Hurst 2007a). As shown in the representative agent model in Ngai and Pissarides (2008), this generalization allows for a trend in leisure along a balanced growth path.[2]

The optimal time allocation is driven by the relative opportunity costs of the three activities as well as the intertemporal substitution of hours worked in the market. Both the activity-specific TFP growth common to all households and the household-specific change in market efficiency of time affects the relative implicit prices of the different activities. Faster TFP growth in the market leads to a rising relative implicit prices of leisure. Given that leisure and non-leisure goods are gross complements, the relative price effect shifts time allocation for all households from non-leisure toward leisure. On the other hand, the household-specific change in the market efficiency of time, i.e., wages, induces different patterns of intertemporal substitution. Increasing wage inequality implies that the more-educated tend to work (relatively) longer hours in later years whereas less-educated household tend to rather frontload their working hours. This tends to less of an increase in leisure after 1985 for the more-educated whereas leisure increases over this period even more for the less-educated. Putting together, the model can account for both the time series and cross-section facts on time allocation. Over time, the rise in leisure is due to the increase in the relative implicit price of leisure reflected in growing (average) wages. When the wage gains are similar across households, there is no change along the intertemporal substitution margin and all households increase their time allocated to leisure production. However, an increase in the wage dispersion induces in the cross-section that household with different market efficiencies exploit the intertemporal substitution margin in opposite directions, resulting in an increase

[2]See Boppart and Krusell (2016) for a theory that squares balanced growth with trends in leisure in a representative agent economy with a traditional preference formulation over leisure and consumption.

in leisure inequality.

To quantitatively assess the mechanism, we calibrate the parameters of the model to match perfectly the fractions of time allocated to the three activities of four education groups in the U.S. in 1965. We then feed in the time series of observed wages together with an estimate of the overall productivity growth to make predictions for the dynamics of the time allocation. The model successfully captures the parallel rise and the subsequent divergence in leisure shares across the four education groups. Overall, it does also a good job in accounting for the aggregate trend in leisure and the rise in leisure inequality. It accounts for all the rise in aggregate leisure and slightly over-predicts the rise in leisure inequality. We conclude from this quantification that a simple model with intertemporal labor substitution can account for the observed pattern in the data.

Consistent with the empirical work by Attanasio, Hurst and Pistaferri (2015), our theory suggests that the rising leisure inequality needs to be taken into account together with the rising inequality in market outcomes in order to make welfare statements. Our paper contributes to this issue by developing a simple model to illustrate how increasing wage inequality itself can generate a rise in leisure inequality and partially mitigates the effect of wage inequality on welfare.

In our theory, leisure production play an important role in squaring a trend in leisure time with an aggregate balanced growth path. This aspect is similar to Ngai and Pissarides (2008), which however abstracts from the cross-section facts.[3] Leisure as an activity that not only involves time but also capital, plays a key role in Vandenbroucke (2009), Kopecky (2011) and Bridgman (2016b) too.[4] These papers

[3]Our objective to develop a growth model that allows for dynamics of cross-section facts with an aggregated balanced growth path is similar to Caselli and Ventura (2000) who, however, do not study the allocation of time.

[4]See also the recent paper by Aguiar et al. (2017) that focuses on leisure "luxuries" and how innovation in video gaming and other recreational computer activities have induced young men to

study also both time trends and cross-section facts, but their main mechanism is the falling relative price of leisure capital whereas we emphasize the higher productivity growth for market production. Vandenbroucke (2009) is motivated by the differential decline in market hours across different wage-group during the period 1900–1950 and consequently treats all non-market hours as leisure and abstracts from home production. Kopecky (2011) is motivated by the trend in retirement and focuses on time use across different age groups. In contrast to our paper, Bridgman (2016b) focuses on the quantitative role played by different capital intensities across market, home and leisure production where the main objective is to account for changes in the labor market wedge.

Finally, while most of macroeconomics models feature in the long-run constant hours worked and leisure time (see, e.g., Cooley and Prescott, 1995), Boppart and Krusell (2016) propose a general class of utility functions defined over consumption and leisure to obtain trends in aggregate market hours and aggregate leisure along a balance growth path.[5] In contrast, we obtain these trends by explicitly modeling leisure and home production. Furthermore, our explicit goal in this paper is to replicate the cross-section facts on leisure and market hours.

The paper is organized as follows. The next section document the empirical facts in U.S. data that motivate the paper. In Section 2 we present a growth model with heterogeneous households and derives its balanced growth path. Section 4 shows that the balanced growth path of the model is consistent with a rise in leisure in the aggregate time series together with a rise in leisure inequality in the cross-section. Section 5 presents the quantitative results. Finally, Section 6 concludes.

shift their time allocation from market to leisure activities since 2004.

[5]See also Boppart, Krusell, and Olsson (2017) that looks at the intensive and extensive margin of labor supply separately.

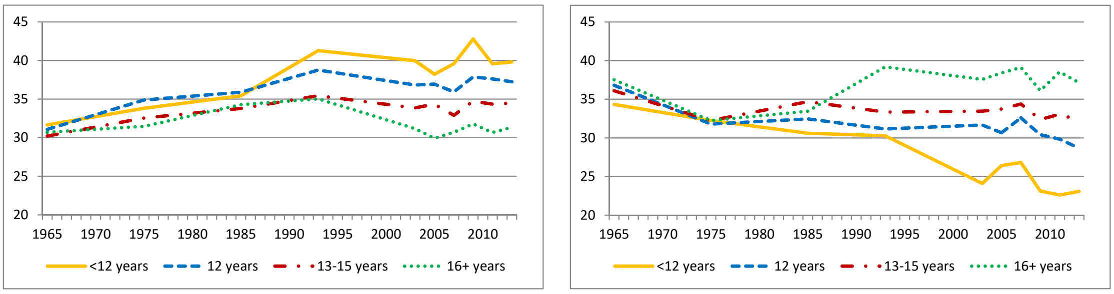
2 Empirical facts

Aguiar and Hurst (2007a) document a growing inequality in leisure that mirrors the rising inequality in wages and expenditures between 1965 to 2003. Figure 1(a) report the "rise in leisure inequality" across educational groups as documented in Aguiar and Hurst (2007a), where we updated the data to 2013.[6] Individuals with less than 12 years of education experienced a rise in leisure time over the past half century of slightly more than 8 hours per week while for college-graduates the increase in leisure is less than 1 hour per week.[7] As leisure is an important determinant of welfare, Figure 1(a) suggests that welfare calculations that are solely based on earnings and expenditure may be incomplete. Aguiar and Hurst (2009) show that the increase in leisure inequality is particularly strong for men. Since there has been a relative decline in the employment rate of less educated men, one natural question is whether the decline in market hours is not involuntary. Aguiar and Hurst's answer is no, as they find that trends in employment status explain less than half of the increase in the leisure gap between less-educated (those with 12 years of education or less) and more-educated men (those with more than 12 years of education). They conclude that most of the increase in leisure gap is voluntary and not due to either an increase in involuntary unemployment or disability.

Aguiar and Hurst (2007a) also report that overall there has been an upward trend in leisure time. As shown in Figure 1(a), over the period 1965–2013 weekly

[6] The time use data is constructed according to the methodology in Aguiar and Hurst (2007a) and the numbers are reported in Table B.1 in the Appendix B.1. Throughout the paper leisure refers to Aguiar and Hurst's "Leisure Measure 1" which includes time spent on socializing, in passive leisure, in active leisure, volunteering, in pet care and gardening.

[7] A similar rise in leisure inequality has also been documented for seven other OECD countries using Multinational Time Use Study for the period 1970s to 2000s by Gimenez-Nadal and Sevilla (2012). See Ramey and Francis (2009) for patterns of leisure in the U.S. prior to 1965.



(a) Leisure hours (b) Market hours

Figure 1: **Leisure and market hours by education group**

Notes: The figure plots leisure time and market hours 1965–2013 for four education groups.
Source: 1965–1966 America's Use of Time; 1975–1976 Time Use in Economics and Social Accounts; 1985 Americans' Use of Time; 1992–1994 National Human Activity Pattern Survey; and 2003–2009 American Time Use Surveys. The data is adjusted for changes in demographic composition: age, education, sex and presence of child, following Aguiar and Hurst's (2007a) methodology. Leisure refers to Leisure Measure 1 in Aguiar and Hurst (2007a), which includes leisure activities such as socializing, watching TV, reading etc.

leisure time increased on average by 4.5 hours.[8] This is a substantial increase, especially when viewed in the context of average time work in the market of 33–37 hours per week over the same period.[9] In a representative agent framework with a Cobb-Douglas production function and a capital share of 1/3, a simple back-of-the-envelope calculation suggests that if the increase in leisure time were instead used to increase labor input, output would be boosted by 8.5 percent (4.5/35 multiplied by 2/3).[10]

[8]It is important to note that these findings are adjusted for changes in demographic compositions in education, age, sex and presence of child. By fixing the demographic weights, the findings reflect how time spent in a given activities change over time instead of changes in demographic composition within a specific group.

[9]See also Winston (1966) and Bick et al. (2018) who document a similar negative relationship between hours worked and the level of development across countries.

[10]Note that this number only captures the static effect, that holds the capital stock constant, whereas the dynamic effect would be even larger.

Home production will be important for the model to account for the overall dynamics of time allocation given the time use evidence presented in Figure 1.[11] The decline in home hours is needed to fuel the increase in leisure for the less-educated and the increase in market hours for the more-educated post 1985. The falling trend in home hours in the model is due to the lower productivity growth for home goods relative to market goods, a process of marketization (Freeman and Schettkat, 2005).[12]

Ramey (2007) shows that the sharp increase in average leisure time in the time used survey is somewhat sensitive to the categorization—but that the rise in leisure "inequality" is robust. What about hours worked? Do other dataset than the time use survey show the same empirical pattern in hours worked as we documented here? Figure B.1 in Appendix B.1 shows the pattern in hours worked for the four skill groups in the CPS data. Overall the (average) decline in hourse worked is less pronounced but the main point of this paper—the divergence in hours worked—is clearly visible too, although the timing is slightly different. Unlike, e.g., the Census data (see Michelacci and Josep Pijoan-Mas (2016) and Wolcott (2017) for men) the CPS data suggests the diverging trend stopped in the early 90s. In this paper we focus on the time used survey data (and Aguiar and Hurst's (2007a) definition) mainly because it allows us to split non-working time further up into leisure and home production. But the main empirical motivation of rising inequality in leisure time and hours worked is indeed a robust finding.

[11]Following Aguiar and Hurst (2007a), child care is excluded. Total childcare time has been stable over time and similar across education groups in the time-use surveys between 1965–1993 at around 3 hours per week but it experienced a substantial rise of 2–3 hours during the 2000s. Ramey and Ramey (2010) argue that this rise is due to increased competition for college admission.

[12]Using personal consumption expenditure data, Bridgman (2016a) shows a substantial rise in purchased service as a share of total (home plus market) services. See also Mazzolari and Ragusa (2013) who document how a rise in the skill premium can affect the demand for unskilled services through marketization.

Figure 1(b) shows that the rise in leisure inequality is indeed accompanied by a rising inequality in market work. Figure 2, reports the wage of each education group relative to the average wage.[13] Together the two figures show suggestive evidence in favor of our channel of intertemporal substitution of leisure and hours worked. In line with our theory, most of the growing inequality in leisure and hours worked took place after 1985 (see Figure 1(a) and 1(b)), which is precisely the period of the biggest rise in wage inequality.[14] Until 1985, for which wage inequality stagnated, leisure time grew for the different education groups in parallel too. In contrast, there was a systematic raise in leisure and wage inequality since 1985. As we will show in the next section, a model with activity-specific technical change common to all households, household specific changes in labor market efficiencies and intertemporal substitution of labor can replicate both the parallel rise in leisure time prior to 1985 and its subsequent divergence.

[13]As in Figure 1, we follow Aguiar and Hurst's (2007a) methodology to control for changes in demographic composition. Due to data constraint, we cannot control for whether a child is presence, thus we have 40 demographic cells instead of 72 demographic cells.

[14]Using the same time use data, Fang and Zhu (2017) also documented a positive correlation between wage rates and market hours and the negative correlation between wage rates and home hours and leisure in the cross section. Using data for 1890s, 1973 and 1991, Costa (2000) documented a similar trend that market hours for low-wage workers have declined relative to high-wage workers.

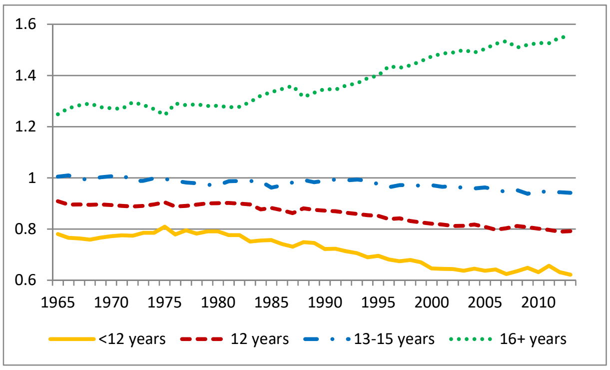


Figure 2: Wage relative to average wage by education group

Notes: The figure plots wage relative average wage 1965–2013 for four education groups.
Source: CPS/March samples. Non-farm working individuals aged 21–65 who are not student. Adjusted for changes in demographic compositions: age, education and sex, following the methodology of Aguair and Hurst (2007a)

3 Theory

3.1 Household side

3.1.1 Preferences, skill and budget constraint

There is a unit interval of heterogeneous households $i \in [0,1]$ with the following preferences

$$\mathcal{U}_i(0) = \sum_{t=0}^{\infty} \beta^t u\left(c_{m,i}(t), c_{h,i}(t), c_{z,i}(t)\right), \tag{1}$$

where $u(\cdot)$ is an instantaneous utility function defined over three different components, a market good, $c_{m,i}$, a home produced good, $c_{h,i}$, and leisure, $c_{z,i}$. $\beta < 1$ denotes the discount factor. The instantaneous utility function is assumed to take the following nested CES form

$$u\left(\cdot\right) = \frac{\varepsilon}{\varepsilon - 1} \log \left[\omega_i \left[\psi c_{m,i}(t)^{\frac{\sigma-1}{\sigma}} + (1-\psi)\, c_{h,i}(t)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma(\varepsilon-1)}{(\sigma-1)\varepsilon}} + (1-\omega_i)\, c_{z,i}(t)^{\frac{\varepsilon-1}{\varepsilon}}\right].$$

The parameter $\sigma > 0$ controls the elasticity of substitution between market goods and home production. The elasticity of substitution between leisure and the CES consumption bundle consisting of market goods and home production is given by $\varepsilon > 0$. ψ is the weight on market goods within the consumption bundle, while $1 - \omega_i \in (0, 1)$ is the weight on leisure which we allow to be household specific.[15] Household i is endowed with $a_i(0)$ units of initial capital which she can either rent out (and get a market rental rate $R(t)$) or use in home production, $k_{h,i}$, or in leisure production, $k_{z,i}$. Each household has $\bar{l}$ units of time that can be either supplied to the labor market, $l_{m,i}$, or allocated to home production, $l_{h,i}$, or leisure $l_{z,i}$. The time constraint thus reads

$$\bar{l} = l_{m,i}(t) + l_{h,i}(t) + l_{z,i}(t), \ \forall i, t. \tag{2}$$

Each unit of time supplied to the labor market is rewarded by a household specific wage rate $w_i(t)$. Differences in $w_i(t)$ across households depend on the household-specific market efficiency per unit of time, $e_i(t) > 0$. The efficiency $e_i(t)$ follows an exogenous process satisfying the following two assumptions.

Assumption 1. $\int_0^1 e_i(t)di = \bar{e}$, $\forall t$.

Assumption 2. $\lim_{t\to\infty} e_i(t)di = \hat{e}_i$, $\forall i$.

Assumption 1 states that the mean of $e_i(t)$ is constant over time and Assumption 2 makes sure that the $e_i(t)$ terms converges to a stationary distribution.[16] In the market place, the efficiency-adjusted labor input of household i that supplies $l_{m,i}(t)$ time units to the labor market is given by $e_i(t)l_{m,i}(t)$. We denote the aggregate available efficiency-adjusted labor input by L, i.e.,

$$L \equiv \int_0^1 e_i(t) \left[l_{m,i}(t) + l_{h,i}(t) + l_{z,i}(t)\right] di = \bar{l}\bar{e}, \ \forall t.$$

[15] Note that the heterogeneity in this weight allows us to match in our quantitative section the initial time allocation of all the groups perfectly.

[16] Assumption 1 excludes changes in average skill over time. However, this could be generalized and we could allow for exogenous growth in average skill at a constant rate. But given the Cobb-Douglas technologies we will impose later on, a growing average skill is mathematically identical to a change in the rate of technological change in the market place.

Since $e_i(t)$ augments the hourly labor input, $l_{m,i}(t)$, proportionally, perfect competitive labor markets allow us to write the household-specific (hourly) wage rate as

$$w_i(t) = \bar{w}(t)e_i(t), \tag{3}$$

where $\bar{w}(t) = \int_0^1 w_i(t)di$ is average wage rate per efficiency unit or the wage rate of a household with average skill, i.e., $e_i(t) = \bar{e}$. Equation (3) highlights that individual wage rates can change over time for two different reasons: (i) through $\bar{w}(t)$ changes due to aggregate dynamics common to all households like technological change or capital deepening, or (ii) through changes in the household-specific efficiency term $e_i(t)$. One interpretation of $e_i(t)$ dynamics is that a household i has an intrinsic ability to achieve certain education level. Then, the attained education is considered as fixed but the return to an education level $e_i(t)$ is changing over time, which allow us to match empirically observed increases in the wage dispersion as shown in Figure 2. Under this interpretation, and the fact that $\bar{w}(t)$ is both the average wage and the wage of a household with $\bar{e}$, the data shown in Figure 2 implies that the household with $\bar{e}$ can be interpreted as the group with 13–15 years of education as its wage follows almost exactly the average rate. As will be shown later, the theory predicts a monotonic rise in leisure for this group, which is consistent with the data presented in Figure 1(a).

There is a single market good that can be consumed or invested. The price of the market good is normalized to one in all points in time. Finally, we assume a constant depreciation rate, δ. Then, household i faces the following budget constraint

$$a_i(t+1) = R(t)\left[a_i(t) - k_{h,i}(t) - k_{z,i}(t)\right] + a_i(t)\left[1-\delta\right] + \left[\bar{l} - l_{h,i}(t) - l_{z,i}(t)\right] w_i(t) - c_{m,i}(t). \tag{4}$$

Households are heterogeneous because they differ in their initial wealth $a_i(0)$ as well as in their (return to) skill $\{e_i(t)\}_{t=0}^{\infty}$.

3.1.2 Leisure and home production

Both time, $l_{z,i}(t)$, and capital (i.e., leisure durables), $k_{z,i}(t)$, is required to generate a leisure output that enters utility $c_{z,i}(t)$. We assume that this leisure output is given by the following Cobb-Douglas aggregator

$$c_{z,i}(t) = k_{z,i}(t)^{\alpha} l_{z,i}(t)^{1-\alpha}. \tag{5}$$

Home production takes the following functional form

$$c_{h,i}(t) = k_{h,i}(t)^{\alpha} \left[A_h(t) l_{h,i}(t)\right]^{1-\alpha}, \tag{6}$$

where $k_{h,i}(t)$ is used capital (i.e., home durables), $l_{h,i}(t)$ is time used for home production, and $A_h(t) = A_h(0)\gamma_h^t$ is a Harrod-neutral technology term in home production with a gross rate of technological progress $\gamma_h > 1$.

3.1.3 Households' problem

Each household $i \in [0,1]$ maximizes (1) with respect to

$$\{a_i(t+1), c_{m,i}(t), c_{h,i}(t), c_{z,i}(t), k_{h,i}(t), k_{z,i}(t), l_{h,i}(t), l_{z,i}(t)\}_{t=0}^{\infty}$$

subject to (4), (5) and (6) as well as a standard no Ponzi game condition that can be expressed as

$$\lim_{T\to\infty} \left[a_i(T+1) \prod_{s=1}^{T} \frac{1}{1+R(s)-\delta} \right] \geq 0. \tag{7}$$

The initial wealth, $a_i(0)$, and $\{e_i(t)\}_{t=0}^{\infty}$ are exogenously given.

3.2 Production side

3.2.1 Technology

The market output good is produced under perfect competition by a representative firm according to the following technology

$$Y(t) = K_m(t)^{\alpha} \left[A_m(t) L_m(t)\right]^{1-\alpha}, \tag{8}$$

where $Y(t)$ is aggregate market output, $K_m(t)$ is the aggregate capital stock used in the market economy and $L_m(t)$ is the total skill-adjusted labor input in the market economy. The term $A_m(t) = A_m(0)\gamma_m^t$, with $\gamma_m > 1$, captures exogenous Harrod-neutral technical progress in the market place.

3.2.2 Firm's problem

The representative firm minimizes production cost of a given output level, $Y(t)$, where the firm takes the rental rate, $R(t)$, and the wage per skill-adjusted labor input, $\bar{w}(t)$, as given.

3.3 Market clearing

Market clearing on the capital and labor market requires

$$\int_0^1 [a_i(t) - k_{h,i}(t) - k_{z,i}(t)]\, di = K_m(t), \tag{9}$$

and

$$\int_0^1 l_{m,i}(t)e_i(t)di = L_m(t). \tag{10}$$

The resource constraint is given by

$$Y(t) = \int_0^1 c_{m,i}(t)di + \int_0^1 a_i(t+1) - (1-\delta)a_i(t)di, \tag{11}$$

where the left-hand side is total output and the right-hand side is total market good consumption plus total (gross) investment.

3.4 Equilibrium definition

In this economy a dynamic equilibrium is defined as follows.

Definition 1. *A dynamic equilibrium is a sequence of time and capital allocation*

$$\{l_{m,i}(t), l_{h,i}(t), l_{z,i}(t), k_{m,i}(t), k_{h,i}(t), k_{z,i}(t)\}_{t=0}^{\infty}, \ \forall i,$$

a sequence of wealth and market, home production and leisure consumption and

$$\{a_i(t), c_{m,i}(t), c_{h,i}(t), c_{z,i}(t)\}_{t=0}^{\infty}, \ \forall i,$$

a sequence of the aggregate capital stock and skill-adjusted labor used in the market economy $\{K_m(t), L_m(t)\}_{t=0}^{\infty}$*, and a sequence of rental and wage rates* $\{R(t), \bar{w}(t), w_i(t)\}_{t=0}^{\infty}$*, $\forall i$, that is jointly solving the households' problem (as specified in Section 3.1.3), the firm's problem (as specified in Section 3.2.2) and is as well consistent with the market clearing conditions* (9)–(11).

3.5 Equilibrium path

A formal characterization of the households' and firm's problem and the derivation of the first-order conditions can be found in Appendix A.1. In the following, we present the equilibrium in two steps: First, we characterize the equilibrium time and capital allocation across market, home and leisure. Second, we present the dynamic equilibrium conditions and show the existence of a balanced growth path, where the return to capital, R, is constant.

3.5.1 Intratemporal equilibrium

Define the aggregate wealth/capital stock in the economy (including household and leisure durables) as $K(t) \equiv \int_0^1 a_i(t)di$. The potential market income of household i (by renting out all the capital, and supplying all her time to the labor market) is given by

$$y_i(t) \equiv R(t)a_i(t) + \bar{l}w_i(t). \tag{12}$$

We denote the difference between this potential market income and (gross) savings as

$$c_i(t) \equiv y_i(t) - [a_i(t+1) - (1-\delta)a_i(t)]\,. \tag{13}$$

The variables $y_i(t)$ and $c_i(t)$ do not have a directly observable empirical counterpart. Nevertheless, it is helpful to introduce them to illustrates how they relate to the dynamics in the standard neoclassical growth model. In the following we show that, given the path of $c_i(t)$, the static equilibrium can be fully characterized. Describing the equilibrium dynamics of $c_i(t)$ will then be the subject of the next section.

The first-order conditions of the households' and firm's problem imply the following equilibrium conditions.

Lemma 1. *Optimal capital intensities of the households and the representative firm require*

$$\frac{k_{h,i}(t)}{e_i(t)l_{h,i}(t)} = \frac{k_{z,i}(t)}{e_i(t)l_{z,i}(t)} = \frac{K_m(t)}{L_m(t)} = \frac{K(t)}{L}. \tag{14}$$

The first-order conditions of the firm's problem combined with the market clearing conditions yield

$$\bar{w}(t) = (1-\alpha)A_m(t)\left[\frac{K(t)}{A_m(t)L}\right]^{\alpha}, \tag{15}$$

and

$$R(t) = \alpha\left[\frac{K(t)}{A_m(t)L}\right]^{\alpha-1}. \tag{16}$$

See Appendix A.1 for a proof.

The results of Lemma 1 are due to free mobility of time and physical capital, which equalize the marginal rate of technical substitution in the production of market output, home and leisure. Together with the Cobb-Douglas technologies with identical output elasticities of labor, this implies identical capital intensities across all three activities for any given household. However, note that the capital intensity will differ across households because the labor efficiency is not identical. More explicitly, household with higher market efficiency, e_i, use more capital per unit of time for home and leisure production relative to the household with lower market efficiency. Finally, the equalization of capital intensities across activities allows us to express the marginal return to labor and capital as a function of the *aggregate* capital per efficiency units of labor (see (15) and (16)).

In order to gain an intuition for a household's optimal allocation of time across market, home, and leisure it is useful to introduce implicit prices (i.e., implicit marginal cost) for $c_{h,i}$ and $c_{z,i}$

$$p_{h,i}(t) \equiv \left[\frac{w_i(t)}{(1-\alpha)A_h(t)}\right]^{1-\alpha}\left[\frac{R(t)}{\alpha}\right]^{\alpha}, \tag{17}$$

and

$$p_{z,i}(t) \equiv \left[\frac{w_i(t)}{1-\alpha}\right]^{1-\alpha}\left[\frac{R(t)}{\alpha}\right]^{\alpha}. \tag{18}$$

Given our choice of the market price as a numéraire, we have

$$p_m(t) = 1 = \left[\frac{\bar{w}(t)}{(1-\alpha)A_m(t)}\right]^{1-\alpha}\left[\frac{R(t)}{\alpha}\right]^{\alpha}. \tag{19}$$

Because the opportunity cost of time differs across households with different skills the implicit price of home production and leisure is *household specific*. For the relative implicit prices we get the following lemma.

Lemma 2. *In equilibrium, the implicit prices (relative to the market price) are given by*

$$p_{h,i}(t) = \left(\frac{A_m(t)e_i(t)}{A_h(t)}\right)^{1-\alpha}, \tag{20}$$

and

$$p_{z,i}(t) = \left[A_m(t)e_i(t)\right]^{1-\alpha}. \tag{21}$$

The relative implicit price between leisure and home is given by

$$\frac{p_{z,i}(t)}{p_{h,i}(t)} = A_h(t)^{1-\alpha}. \tag{22}$$

Proof. The expression for the relative prices follow immediately from (17)–(19) and (3). ■

Given the identical Cobb-Douglas production functions (apart from the technology terms), all the relative implicit costs can be written independently of the factor prices. Lemma 2 highlights that because lower-skilled households have a comparative advantage in home and leisure those implicit relative prices are lower for households with a lower $e_i(t)$. However, the relative prices of home to leisure are the same across households.[17] Relative prices not only vary in the cross-section but also over time, due to the differences in the pace of technological progress across activities. Together with the definitions in equation (12) and (13), we obtain the next lemma.

Lemma 3. *In equilibrium, we have*

$$y_i(t) = (1-\alpha)A_m(t)\left[\frac{K(t)}{A_m(t)L}\right]^{\alpha} e_i(t)\bar{l} + \alpha\left[\frac{K(t)}{A_m(t)L}\right]^{\alpha-1} a_i(t), \tag{23}$$

and

$$c_i(t) = c_{m,i}(t) + c_{h,i}(t)p_{h,i}(t) + c_{z,i}(t)p_{z,i}(t). \tag{24}$$

Proof. Equation (23) follows immediately from combining (3), (12), (15), and (16). Equation (24) follows from (3), (4), (14), (15), and (16) as well as the definition in (17). ∎

Lemma 3 states that the potential income $y_i(t)$ is higher for household with higher labor efficiency units $e_i(t)$ and higher wealth $a_i(t)$. The variable $c_i\,(t)$ can be expressed as the total implicit consumption expenditure of household i for market, home, and leisure goods.

In the following we define $\tilde{p}_{mh,i}(t) \equiv \left[\psi^{\sigma} + (1-\psi)^{\sigma}\, p_{h,i}(t)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$ as the implicit composite price for non-leisure goods. Note that this implicit price is household

[17]This can be generalized by allowing higher skilled household to have a relative comparative advantage in home production compared to leisure. Such a generalization however would not change the main results in this paper.

specific since the labor market efficiency unit e_i differs in the cross-section. Moreover, we define the implicit expenditure shares of leisure and home as $x_{j,i}(t) \equiv \frac{c_{j,i}(t)p_{j,i}(t)}{c_i(t)}$, $j = z, h$. Lemma 2 and 3 together imply the following lemma.

Lemma 4. *In equilibrium, the implicit expenditure shares of leisure and home are*

$$x_{z,i}(t) = \frac{1}{1 + \left(\frac{\omega_i}{1-\omega_i}\right)^{\varepsilon} \left(\frac{\tilde{p}_{mh,i}(t)}{p_{z,i}(t)}\right)^{1-\varepsilon}}, \tag{25}$$

and

$$x_{h,i}\left(t\right) = \frac{1 - x_{z,i}\left(t\right)}{1 + \left(\frac{\psi}{1-\psi}\right)^{\sigma} p_{h,i}^{\sigma-1}\left(t\right)}, \tag{26}$$

See Appendix A.1 for a proof.

For a given path of $c_i(t)$ and implicit prices $p_{h,i}(t)$ and $p_{z,i}(t)$, Lemma 4 contains closed form solutions for the equilibrium quantities of $c_{z,i}(t)$, $c_{m,i}(t)$, and $c_{h,i}(t)$.

Note that because of the (homothetic) CES structure these implicit expenditure shares are only functions of relative implicit prices, which are given in terms of exogenous technology terms in Lemma 2. Hence, together with the expressions for the relative implicit prices closed from solutions for the consumed quantities of market goods, home production, and leisure are obtained for any given $c_i(t)$. Combining the quantities in Lemma 4 with the production functions (5) and (6) and the optimality condition in production (14) allows us to express the allocation of production factors to the different activities in the following proposition.

Proposition 1. *Leisure and home production time is given by*

$$l_{j,i}(t) = x_{j,i}(t)\frac{c_i(t)}{A_m(t)e_i(t)\left[\frac{K(t)}{A_m(t)L}\right]^{\alpha}}, \;\; j = z, h. \tag{27}$$

Capital used in leisure and home production is given by

$$k_{j,i}(t) = x_{j,i}(t)c_i(t)\left[\frac{K(t)}{A_m(t)L}\right]^{1-\alpha}, \;\; j = z, h. \tag{28}$$

Proof. The allocation of the different production factors are simply obtained by combining the quantities in Lemma 4 with the production functions (5) and (6) and the optimality condition in production (14). ■

The remaining variables then follow immediately as for instance $l_{m,i}(t) = \bar{l} - l_{h,i}(t) - l_{z,i}(t)$. This illustrates that for a given distribution of $c_i(t)$, $\forall i$ and a given aggregate capital stock $K(t)$ we obtain closed form solution for all equilibrium variables. To fully solve the model we analyze the equilibrium path of $K(t)$ and $c_i(t)$ in the next section. To prepare the analysis in the cross-section and over time it is helpful to express the time of leisure relative to home production in equilibrium. This is done in the next lemma.

Lemma 5. *The relative time (and capital) used for leisure relative to home production is given by*

$$\frac{l_{z,i}(t)}{l_{h,i}(t)} = \frac{k_{z,i}(t)}{k_{h,i}(t)} = (1-\psi)^{\frac{\sigma(1-\varepsilon)}{\sigma-1}} \left(\frac{1-\omega_i}{\omega_i}\right)^{\varepsilon} \left(\frac{p_{z,i}(t)}{p_{h,i}(t)}\right)^{1-\varepsilon} \left(1 + \left(\frac{\psi}{1-\psi}\right)^{\sigma} p_{h,i}(t)^{\sigma-1}\right)^{\frac{\sigma-\varepsilon}{\sigma-1}}. \tag{29}$$

Proof. First, note that according to (27) we have $\frac{l_{z,i}(t)}{l_{h,i}(t)} = \frac{x_{z,i}(t)}{x_{h,i}(t)}$. Substituting in the values of Lemma 4 (see (25) and (26)) gives

$$\frac{l_{z,i}(t)}{l_{h,i}(t)} = \left(\frac{1-\omega_i}{\omega_i}\right)^{\varepsilon} \left(\frac{\tilde{p}_{mh,i}(t)}{p_{z,i}(t)}\right)^{\varepsilon-1} \left(1 + \left(\frac{\psi}{1-\psi}\right)^{\sigma} p_{h,i}(t)^{\sigma-1}\right). \tag{30}$$

Using the definition of $\tilde{p}_{mh,i}(t)$ allows us to rewrite this expression as (29). Finally, note that (14) implies that the capital intensities equalize between the activities home production and leisure. ■

It is important to emphasize again that—as it can be seen in Lemma 2—the relative implicit cost of a unit of home service/goods as well as leisure depends on both the technologies $A_m(t)$ and $A_h(t)$ as well as $e_i(t)$ and the heterogeneous preference weights. Consequently, the relative implicit costs vary over time and across households (with different $e_i(t)$ and preferences). Hence, the composition of consumed market goods, home production and leisure differs in the cross-section. For the same reason the allocation of time changes over time and differs in the cross-section too.

3.5.2 Intertemporal equilibrium

This section describes the equilibrium dynamics of the household wealth and consumption, $a_i(t)$ and $c_i(t)$. The dynamics of household wealth will then determine the aggregate wealth, $K(t)$, and factor prices $w(t)$ and $R(t)$ (see Lemma 1). To describe the optimal wealth accumulation, the next lemma characterizes optimal household saving behavior in the intertemporal household maximization problem.

Lemma 6. *The first-order conditions of the household problem imply*

$$a_i(t+1) = a_i(t)\left[1 + R(t) - \delta\right] + \bar{l}e_i(t)\bar{w}(t) - c_i(t), \ \forall i, \tag{31}$$

and

$$c_i(t+1) = \beta\left[R(t+1) + 1 - \delta\right] c_i(t), \ \forall i. \tag{32}$$

Hence, for a given path of factor prices, (31) and (32) characterize a system of difference equations in $a_i(t)$ and $c_i(t)$, where $a_i(0)$ is exogenously given and the transversality condition

$$\lim_{T\to\infty}\left[a_i(T+1)\prod_{s=1}^{T}\frac{1}{1+R(s)-\delta}\right] = 0, \tag{33}$$

constitutes a terminal condition. The path of the factor prices and $y_i(t)$ then follow from the aggregate capital stock (see (15), (16) and (23)).

3.6 Balanced growth path

Definition 2. *A balanced growth path is defined as an equilibrium path along which aggregate wealth/capital, $K(t)$, and the wage rate, $\bar{w}(t)$, grow at a constant rate and the rental and real interest rate are constant.*

The detrended capital stock is denoted as $\tilde{k}(t) \equiv \frac{K(t)}{A_m(t)L}$. The following proposition holds.

Proposition 2. *There exists a unique globally saddle path stable balanced growth path with* $\tilde{k}^\star = \left[\frac{\alpha}{\gamma_m/\beta - 1 + \delta}\right]^{\frac{1}{1-\alpha}}$.

See Appendix A.1 for a proof.

Along the balanced growth path, i.e., with $\tilde{k}(0) = \tilde{k}^\star$, consumption $c_i(t)$ for all households and the aggregate capital stock grow a constant rate γ_m, or formally

$$\frac{c_i(t+1)}{c_i(t)} = \frac{\int_0^1 c_i(t+1)di}{\int_0^1 c_i(t)di} = \frac{\int_0^1 a_i(t+1)di}{\int_0^1 a_i(t)di} = \frac{K(t+1)}{K(t)} = \gamma_m, \ \forall i. \tag{34}$$

The wage rate is given by

$$\bar{w}(t) = \bar{w}(t)^\star = (1-\alpha)A_m(t)\left(\tilde{k}^\star\right)^\alpha, \tag{35}$$

and grows at the same rate γ_m. The rental rate is constant and given by

$$R(t) = R^\star = \alpha\left(\tilde{k}^\star\right)^{\alpha-1}. \tag{36}$$

Given the growth rate of implicit consumption expenditure, $\frac{c_i(t+1)}{c_i(t)}$, and the path of aggregate capital and all factor prices, $c_i(0)$ is pinned down by the transversality condition, as shown in the following lemma.

Lemma 7. *Along the balanced growth path, the initial implicit consumption expenditure level is given by*

$$c_i(0) = [1 + R^\star - \delta - \gamma_m]\, a_i(0) + \sum_{t=0}^{\infty} \frac{\bar{w}(0)^\star e_i(t)\bar{l}\,[1 + R^\star - \delta - \gamma_m]}{1 + R^\star - \delta}\left(\frac{\gamma_m}{1 + R^\star - \delta}\right)^t. \tag{37}$$

See Appendix A.1 for a proof.

This lemma shows that only the permanent income pines down the initial consumption level $c_i(0)$. Thus, the entire consumption path, $c_i(t)$, of all household is know. Other equilibrium variables such as the households' time allocation and capital allocation follow directly from the intratemporal optimality conditions (see Section 3.5.1).

Equation (34)–(36) described the equilibrium dynamics along the balanced growth path, i.e., for an economy that starts with $\tilde{k}(0) = \tilde{k}^{\star}$. To complete the equilibrium analysis, note that the transitional dynamics of this economy in terms of $K(t)$, $R(t)$, $w(t)$ and $c_i(t)$ are identical to the standard neoclassical growth model.[18]

The dynamics of $K(t)$, $\bar{w}(t)$ and $R(t)$ along the balanced growth path are standard. The model predicts that the real output per hour $\frac{Y(t)}{\int_0^1 l_{m,i}(t)di}$ and the real capital stock (including the stock of consumption durables) $K(t)$ both grow at constant gross rate γ_m. The former follows from the assumption that $\int_0^1 e_i(t)di$ is constant over time (see Assumption 1). Figure B.2 in Appendix B.1 shows the real per-capita capital stock in the U.S. (where the capital stock includes the stock of consumer durables). On a logarithmic scale this series is indeed very well approximated by a linear fit. Other implications of balanced growth are a constant growth rate of average wage rates and a constant rental and interest rate.

As in Ngai and Pissarides (2007), the existence of a balanced growth path relies on the assumption of unitary intertemporal elasticity of substitution and identical output elasticities of labor across the three activities. An important aspect of this is that we explicitly model home production and leisure production as an activity that also requires physical capital (i.e., household durables or leisure goods).

[18] In general, no closed form solution for $c_i(t)$ exists along the transition. However, with $\delta = 1$ we would obtain the well-known case where a closed form solution for $c_i(t)$ exists even along the transition.

It is important to note that although the model allows for aggregation and the existence of an aggregate balance growth path, there does not exist a representative agent in this model. To put it differently, even though the wage of the group with 13–15 years of education is the same as the average wage (so they are the group with average skill $\bar{e}$), it does not imply nor require that their time allocation should be the same as the average time allocation in the economy, which is confirmed by data reported in Table B.1 in Appendix B.1. This result follows directly from the equilibrium time allocation in (27), which is non-linear in $e_i(t)$.

Finally, as $e_i(t)$ might vary over time, the growth rate of $y_i(t)$ is time varying even along the balanced growth path. Consequently, also the growth rate of wealth $a_i(t)$ changes over time accordingly. Moreover, both time allocation and consumption structure exhibits interesting dynamics across households even along the balanced growth path. Changes in the consumption structure and time allocation are driven by changes in the household-specific relative implicit prices (see Lemma 4). These relative implicit prices change due to differences in the TFP growth rates across activities γ_m and γ_h and due to changes in the labor efficiency $e_i(t)$. How changes in the relative implicit prices affect the consumption structure depends on the elasticity of substitution parameters ε and σ. Since the dynamics of time allocation and the consumption structure crucially depend on the (relative) rate of technical progress and the elasticities of substitution we will next make specific assumptions about the parameters γ_m, γ_h, ε and σ. This allows us to focus on the empirically relevant dynamics.

4 Time allocation and consumption structure along the balance growth path

In the following we focus on an economy that is along its balanced growth path, i.e., $\tilde{k}(0) = \tilde{k}^{\star}$. Furthermore, we make the following assumption with respect to the parameters γ_m, γ_h, ε and σ.

Assumption 3. *$\gamma_m > \gamma_h > 1$ and $\sigma > 1 > \varepsilon$.*

The elasticity between home and market goods being larger than one is supported by empirical findings (see the survey article by Aguiar, Hurst and Karabarbounis, 2012). Among others, Blundell and Walker (1982) and Ham and Reilly (2002) present evidence for the non-separable utility and complementarity between consumption goods and leisure.

In the following we discuss the joint dynamics of time use and the allocation of capital under Assumption 3 both in the cross-section as well as over time. This is done in three steps. First, we discuss the dynamics under the assumption that the $e_i(t)$ distribution is held fixed. These theoretical results will then be compared with the data of the period 1965–1985, that was characterized by little changes in the wage inequality across the educational groups. Second, we allow for changes in the market efficiencies and finally we analyze the asymptotic equilibrium as time goes to infinity.

4.1 Equilibrium dynamics with constant efficiency terms e_i

Analyzing the time spent for leisure relative to home production under the assumption of stationary e_i terms gives the following lemma.

Lemma 8. *For constant efficiency terms $\{e_i\}_{i=0}^{1}$, the leisure hours relative to home production hours, $\frac{l_{z,i}(t)}{l_{h,i}(t)}$, are monotonically increasing over time for all household i.*

Proof. The equilibrium expression for $\frac{l_{z,i}(t)}{l_{h,i}(t)}$ is given in Lemma 5. For a constant e_i and with $\gamma_m > \gamma_h$ we see that $p_{h,i}(t)$ and $\frac{p_{z,i}(t)}{p_{h,i}(t)}$ are monotonically increasing over time (see (20) and (22)). Hence, with $\sigma > 1 > \varepsilon$, $\frac{l_{z,i}(t)}{l_{h,i}(t)}$ is monotonically increasing over time for all i. ■

Lemma 8 implies that leisure hours relative to home production hours increase monotonically for the household with constant $\bar{e}$. This prediction is confirmed by relative time allocation reported in Figure 4 for the groups with 13-15 years of education (which is the empirical counterpart of the household with $\bar{e}$). The rise in the relative time in leisure is due to rising leisure hours and falling home hours for this group, see Table B.1. The intuition provided by the model is the following: Since the labor intensities are identical across different activities, and the labor market efficiency terms, e_i, are held constant, the changes in time spent for the different activities are determined by the changes in the implicit expenditure shares, $x_{j,i}(t)$. Because households have (nested) CES preferences the dynamics in the implicit expenditure shares are determined by changes in the implicit prices. With the elasticity of substitution between leisure and the market-home composite being smaller then one and with $\gamma_m > \gamma_h$ the (implicit) price of leisure increases relative to home production or the market good (and consequently also relative to the market-home composite). Together this implies that the implicit expenditure share of leisure increases whereas the implicit expenditure share of home production decreases.

Lemma 8 also suggests that average stock of leisure capital relative to household durables should monotonically increase. Figure 3 shows that the relative stock of leisure capital indeed grew at a remarkably fast rate and considerably faster than the stock of household durables.[19] Finally, over the period 1965–1985 when wages grew pari passu (see Figure 2), we expect $\frac{l_{z,i}(t)}{l_{h,i}(t)}$ to rise for all households, which is

[19]Figure B.3 in Appendix B.1 show a very similar trend in the ratio of leisure durables relative to household durables in terms of investments.

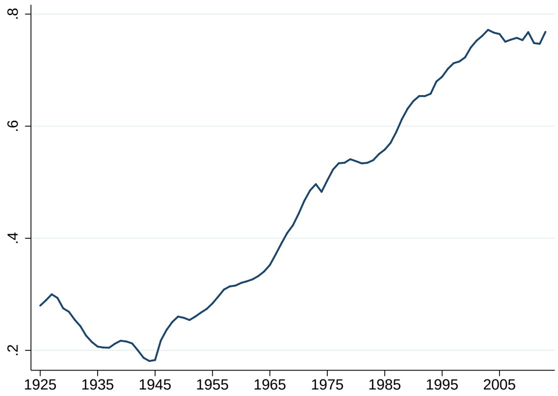


Figure 3: **Stock of leisure durables relative to household durables**

Notes: The figure plots aggregate "recreational" durable goods relative to aggregate "furnishing and household durables" corresponding to $\frac{\int_0^1 k_{z,i}(t)di}{\int_0^1 k_{h,i}(t)di}$ in the model. Source: BEA table 8.1.

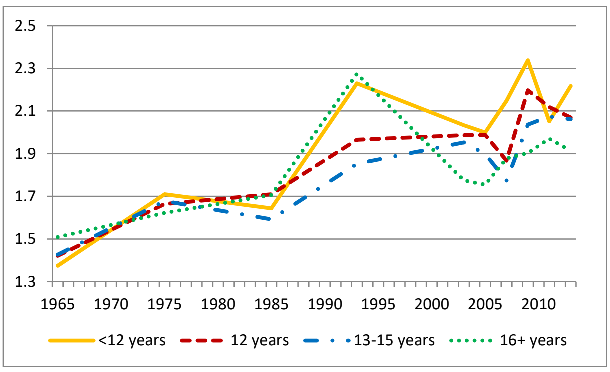


Figure 4: **Leisure hours relative to home production hours**

Notes: Source: Time use surveys. Following Aguiar and Hurst (2007a) methodology, individuals aged 21–65 who are not student nor retired. Childcare is excluded from home production and leisure refers to Leisure Measure 1 in Aguair and Hurst (2007a) confirmed by Figure 4.

The dynamics for the level of leisure hours are derived in the following lemma.

Lemma 9. *For constant efficiency terms* $\{e_i\}_{i=0}^{1}$*, we have along the balanced growth path*

$$\frac{l_{z,i}(t+1)}{l_{z,i}(t)} = \frac{x_{z,i}(t+1)}{x_{z,i}(t)} > 1, \ \forall i. \tag{38}$$

Hence, leisure hours are growing monotonically over time for all households.

Proof. With a constant e_i, the equality in (38) follows immediately from (27) and the fact that $c_i(t)$ grows at gross rate γ_m and that $\frac{K(t)}{A_m(t)L} = k^\star$ along the balanced growth path. Now $x_{z,i}(t)$ is given by (25) and is an decreasing function of $\frac{\tilde{p}_{mh,i}(t)}{p_{z,i}(t)}$ since $\varepsilon < 1$. The relative price is given by

$$\frac{\tilde{p}_{mh,i}(t)}{p_{z,i}(t)} = \left[\psi^\sigma \left[A_m(t)\, e_i(t)\right]^{-(1-\alpha)(1-\sigma)} + (1-\psi)^\sigma A_h(t)^{-(1-\alpha)(1-\sigma)}\right]^{\frac{1}{1-\sigma}}. \tag{39}$$

With a stationary e_i distribution, the gross growth rate of $\frac{\tilde{p}_{mh,i}(t)}{p_{z,i}(t)}$ is a weighted geometric mean of the gross growth factors $\gamma_m^{-(1-\alpha)} < 1$ and $\gamma_h^{-(1-\alpha)} < 1$. Hence, we have

$$\gamma_{\tilde{p}_{mh,i}}(t)/\gamma_{p_{z,i}}(t) \equiv \frac{\tilde{p}_{mh,i}(t+1)}{\tilde{p}_{mh,i}(t)} \frac{p_{z,i}(t)}{p_{z,i}(t+1)} < 1, \ \forall i. \tag{40}$$

Consequently, $x_{z,i}(t)$ and $l_{z,i}(t)$ are monotonically increasing over time for all i. ∎

Lemma 9 predicts that in periods of stationary wage distribution we should see monotonically increasing leisure hours for all educational groups. The period 1965–1985 reflects no clear trend in wage inequality (see Figure 2) and as Figure 1 shows leisure hours were indeed monotonically increasing for all educational groups over this period. The intuition of the theoretical result is again as above and hinges on $\gamma_m > \gamma_h > 1$ and the assumption that leisure and the home-market composite are gross complements (see Assumption 3).

What we have shown so far is that under the assumption of stationary e_i terms our theory suggests that leisure time should monotonically increase for all educational groups (in absolute terms and relative to home production). This is empirically the case for the period 1965–1985 for which the wage inequality was rather stable. Now, since 1985, the striking fact is that there has been an increase in leisure inequality since highly educated households worked longer market hours over time whereas less educated household increased their leisure hours further. Clearly, with a stable wage distribution (and stationary e_i terms) our theory will not generate this. It is the goal of the next section to show that the model can account for this rising leisure inequality once an (empirically observed) increase in the wage inequality is introduced.

4.2 Equilibrium dynamics with changing efficiency terms e_i

In this section we introduce systematic changes in the e_i terms into the model. The purpose of these shifts in the e_i terms is to generate the steep increase in wage inequality since the 80s. As it can be seen from (27) changes in the efficiency term e_i will affect the time allocation. Let the gross growth factor of e_i for household i be denoted as $\gamma_{e_i}(t) \equiv \frac{e_i(t+1)}{e_i(t)}$. We then obtain the following proposition.

Proposition 3. *Along the balanced growth path, the gross growth rate of leisure* $\frac{l_{z,i}(t+1)}{l_{z,i}(t)}$ *is a decreasing function of* $\gamma_{e_i}(t)$.

Proof. With a changing efficiency term $e_i(t)$, given $c_i(t)$ grows at gross rate γ_m and capital per efficiency units of labor is constant along the balanced growth path, (27) implies

$$\frac{l_{z,i}(t+1)}{l_{z,i}(t)} = \frac{x_{z,i}(t+1)}{x_{z,i}(t)}\gamma_{e_i}(t)^{-1}, \ \forall i. \tag{41}$$

The term $x_{z,i}(t)$ is given by (25) and we can express

$$\frac{x_{z,i}(t)}{x_{z,i}(t+1)} = \frac{1+\left(\frac{\omega_i}{1-\omega_i}\right)^{\varepsilon}\left(\frac{\tilde{p}_{mh,i}(t)}{p_{z,i}(t)}\right)^{1-\varepsilon}\left[\gamma_{\tilde{p}_{mh,i}}(t)/\gamma_{p_{z,i}}(t)\right]^{1-\varepsilon}}{1+\left(\frac{\omega_i}{1-\omega_i}\right)^{\varepsilon}\left(\frac{\tilde{p}_{mh,i}(t)}{p_{z,i}(t)}\right)^{1-\varepsilon}}, \tag{42}$$

which can be rewritten as

$$\frac{x_{z,i}(t)}{x_{z,i}(t+1)} = x_{z,i}\left(t\right) + \left(1 - x_{z,i}\left(t\right)\right)\left[\gamma_{\tilde{p}_{mh,i}}(t)/\gamma_{p_{z,i}}(t)\right]^{1-\varepsilon}. \tag{43}$$

Combining (41) with (43) gives

$$\frac{l_{z,i}(t)}{l_{z,i}(t+1)} = x_{z,i}\left(t\right)\gamma_{e_i}(t) + \left(1 - x_{z,i}\left(t\right)\right)\left[\gamma_{\tilde{p}_{mh,i}}(t)/\gamma_{p_{z,i}}(t)\right]^{1-\varepsilon}\gamma_{e_i}(t). \tag{44}$$

The term $\frac{\tilde{p}_{mh,i}(t)}{p_{z,i}(t)}$ is given in (39). Consequently, the gowth factor of this relative price is a weighted geometric average of $\left[\gamma_m\gamma_{e_i}(t)\right]^{-(1-\alpha)}$ and $\gamma_h^{-(1-\alpha)} < 1$, or formally

$$\gamma_{\tilde{p}_{mh,i}}(t)/\gamma_{p_{z,i}}(t) = \left[\xi\left[e_i\left(t\right)\right]\left[\gamma_m\gamma_{e_i}\left(t\right)\right]^{-(1-\alpha)(1-\sigma)} + \left\{1-\xi\left[e_i\left(t\right)\right]\right\}\gamma_h^{-(1-\alpha)(1-\sigma)}\right]^{\frac{1}{1-\sigma}}, \tag{45}$$

where the weight $\xi\left[e_i\left(t\right)\right]$, which depends on the level of $e_i(t)$ but is independent of its growth rate, is given by

$$\xi\left[e_i\left(t\right)\right] \equiv \frac{\psi_i^{\sigma}\left[A_m\left(t\right)e_i\left(t\right)\right]^{(1-\alpha)(\sigma-1)}}{\psi^{\sigma}\left[A_m\left(t\right)e_i\left(t\right)\right]^{(1-\alpha)(\sigma-1)} + \left(1-\psi\right)^{\sigma}A_h\left(t\right)^{(1-\alpha)(\sigma-1)}}. \tag{46}$$

Combining (44) and (45) gives

$$\frac{l_{z,i}(t)}{l_{z,i}(t+1)} = x_{z,i}\left(t\right)\gamma_{e_i}(t) + \left(1 - x_{z,i}\left(t\right)\right)\phi\left[e_i(t), \gamma_{e_i}(t)\right], \tag{47}$$

with

$$\phi\left[\cdot\right] = \left[\xi\left[\cdot\right]\gamma_{e_i}^{(1-\sigma)\left[\frac{\varepsilon}{1-\varepsilon}+\alpha\right]}\gamma_m\left(t\right)^{-(1-\alpha)(1-\sigma)} + \left\{1-\xi\left[\cdot\right]\right\}\gamma_{e_i}\left(t\right)^{\frac{1-\sigma}{1-\varepsilon}}\gamma_h^{-(1-\alpha)(1-\sigma)}\right]^{\frac{1-\varepsilon}{1-\sigma}}. \tag{48}$$

It follows that $\phi\left[e_i(t), \gamma_{e_i}(t)\right]$ is strictly increasing in γ_{e_i} since we have $\sigma > 1 > \varepsilon$. Consequently, in view of (47), $\frac{l_{z,i}(t)}{l_{z,i}(t+1)}$ is monotonically increasing and $\frac{l_{z,i}(t+1)}{l_{z,i}(t)}$ decreasing in γ_{e_i}. ■

The intuition behind this result is the following: along the balanced growth path, the growth of $c_i(t)$ is identical across households and independent of the changes in $e_i(t)$. Hence, what matters for the dynamics of l_z is how the implicit share spent on leisure changes $x_{z,i}(t)$ over time. The dynamics of the implicit expenditure shares depend on the changes in the implicit prices and the elasticities of substitution. The implicit relative prices change firstly due to the differences in technological change across activities. This effect is the same for all households and as the analysis of Lemma 9 shows leads to a monotonic increase in the time spent for leisure. Additionally, however, the changes in the e_i terms affect the relative implicit prices in a household specific way. For households with an increasing e_i the relative price of leisure increases at a faster rate which increases the implicit share spent on leisure even further. In a static model this would capture the whole effect. Since then a household with an increasing $e_i(t)$ term would have an additional incentive to increase leisure time squaring rising leisure inequality and generally rising leisure time would be impossible in a static framework. However there is a direct effect of changes in $e_i(t)$ (see (27)) due to intertemporal labor substitution. A household that experiences a (steeper) growth in $e_i(t)$ will react by increasing the labor supply to the market and reduce leisure and this effect will dominate. Hence, the model does replicate that households that experience a steeper wage growth will increase leisure by less (or even decrease it). As highlighted in the introduction this is exactly what we observed in the U.S. after 1985.

Proposition 4. *For educational groups with $1 \geq \gamma_{e_i}(t)$, hours of leisure are increasing over time. For educational groups with $\gamma_{e_i}(t) > 1$ leisure hours can be falling over time.*

Proof. We already showed that the growth rate of $l_{z,i}(t)$ is strictly falling in $\gamma_{e_i}(t)$ (see Proposition 3) and that the leisure growth is positive for $\gamma_{e_i} = 1$ (see Lemma 9. Hence, leisure growth must be positive for $1 \geq \gamma_{e_i}(t)$. For the second statement, note that $\gamma_{\tilde{p}_{mh,i}}(t)/\gamma_{p_{z,i}}(t)$ is monotonically decreasing in γ_{e_i} (see 45) and for $\gamma_{e_i}\gamma_m \geq \gamma_h$ we can even have $\gamma_{\tilde{p}_{mh,i}}(t)/\gamma_{p_{z,i}}(t) \leq 1$. This shows that even $x_{z,i}(t)$ can be falling household that experience a sharp increase in e_i and consequently l_z is decreasing.

■

4.3 Asymptotic equilibrium

Asymptotically, as time goes to infinity, the e_i will be stationary as stated in Assumption 2. Hence, the asymptotic dynamics of leisure time and time of home production are already discussed in Lemma 8 and 9. In addition, however, we get the following statement about the asymptotic level of time spent for leisure. Define the asymptotic detrended asset and consumption level as $\tilde{a}_i^\star \equiv \lim_{t\to\infty} a_i(t)\gamma_m^{-t}$ and $\tilde{c}_i^\star \equiv \lim_{t\to\infty} c_i(t)\gamma_m^{-t}$.

Proposition 5. *Asymptotically we have*

$$l_{z,i} = \frac{1 + R^\star - \delta - \gamma_m}{\hat{e}_i \left(\tilde{k}^\star\right)^\alpha} \tilde{a}_i^\star + (1-\alpha)\bar{l}. \tag{49}$$

Proof. As time goes to infinity, $x_{z,i}$ converges to 1 for all i, see (25) and note that $\frac{\tilde{p}_{mh,i}(t)}{p_{z,i}(t)}$ grows asymptotically at rate $\gamma_m^{-(1-\alpha)} < 1$. Then, it follows immediately from (27) that asymptotically

$$l_{z,i} = \frac{\tilde{c}_i^\star}{\hat{e}_i \left(\tilde{k}^\star\right)^\alpha}. \tag{50}$$

Finally, $\tilde{c}_i^\star$ is given by (37) where $e_i(t) = \hat{e}_i$, $\forall t$ which implies

$$\tilde{c}_i^\star = \left[1 + R^\star - \delta - \gamma_m\right] \tilde{a}_i^\star + (1-\alpha)\hat{e}_i \left(\tilde{k}^\star\right)^\alpha \bar{l}. \tag{51}$$

■

This proposition shows that leisure hours converge asymptotically to a constant. Remarkably, however, leisure hours differ—even asympotically. As it can be seen from Proposition 5 the household specific level depends on how the asymptotic (detrended) wealth level relates to the asymptotic labor efficiency $\hat{e}_i$. Only if asymptotic wealth is proportional to the asymptotic efficiency unit the terms cancel out and the asymptotic hours of leisure are identical. This is unlikely to be true given the asymptotic wealth $\tilde{a}_i$ is an equilibrium object and depends on the entire distribution of the household-specific market efficiency $e_i(t)$ and household-specific preference parameter $1 - \omega_i$ for leisure.

5 Rising leisure inequality in the U.S., 1965–2013

To quantitatively assess the role of the increase in wage dispersion in generating the rising leisure inequality, the parameters of the model are calibrated to match time allocation in 1965 in the U.S. for the four education groups. The time series of wages in Figure 2 are then used to predict the dynamics of time allocation.

5.1 Calibration

The objects of interest regarding time allocation are the hour shares $l_{j,i}/\bar{l}$, for $j = m, h, z$, which sum up to one by definition. The parameters needed to predict hour shares include elasticity parameters $\{\varepsilon, \sigma\}$, preference parameters $\{\beta, \psi, \omega_i\}$, technology parameters $\{\alpha, \delta, A_m(0), A_h(0), \gamma_m, \gamma_h\}$ and the household-specific market efficiency $e_i(t)$ for each of the four education groups.

The initial productivity $A_m(0)$ and $A_h(0)$ are normalized to one where $\{\beta, \delta, \alpha\}$ are set to the standard values in the macro literature. More specifically, the discount factor β is set to 0.97, the depreciation rate δ to 0.05, and the capital share α is set to a value of 0.3. There is an extensive literature providing estimates for the elasticity of substitution between home and market consumption, summarized by Aguiar, Hurst and Karabarbounis (2012), ranging from 1.5 to 2.5.[20] The model assumes all market goods are gross substitutes to home goods, thus the lower limit of these estimates is used and σ is set to 1.5. Regarding the elasticity of substitution across consumption and leisure ε, there is no readily available estimate. However, Blundell and Walker (1982) and Ham and Reilly (2002) present evidence for complementarity between consumption and leisure, we therefore set ε to 0.1 as the baseline parameter. Finally, the labor-augmenting productivity growth for market production, γ_m is set to 1.02 which corresponds to a 2 percent growth in per-capita terms; while γ_h for home production is set to 1.01 which is in line with the estimate of Bridgman (2016a).[21]

The remaining parameters $\{\psi, \omega_i, e_i(0)\}$ are model-specific. They are set to match the hours shares in 1965 for each of the four education group. The hour shares predicted by the model are derived in Proposition 1, and reported in (27). Along the balanced growth path, both $c_i(t)$ and $K(t)$ grow at the gross rate γ_m, and the hour shares allocated to home and leisure satisfy

$$l_{j,i}(t) = x_{j,i}(t)\left(\frac{\kappa_i}{e_i(t)}\right); \quad j = h, z; \quad \kappa_i \equiv \frac{c_i(0)}{A_m(0)\left(\tilde{k}^*\right)^{\alpha}}, \tag{52}$$

where κ_i is a constant, $\tilde{k}^*$ is derived in Proposition 2 and the initial consumption $c_i(0)$ is derived in (37). The value of $\tilde{k}^*$ is known given the parameters $\{\alpha, \beta, \delta, \gamma_m\}$, however, the value of $c_i(0)$ depends on the initial wealth and future wages. Instead of making assumption on the entire initial wealth distribution and the asymptotic distribution of market efficiency $\hat{e}_i$, there is a simpler way to calibrate κ_i directly. More specifically, since we are considering a mean-preserving spread in the labor

[20] See for example, Rupert et al. (1995), Aguiar and Hurst (2007b), Gelber and Mitchell (2012), and Fang and Zhu (2017).

[21] To be precise, Bridgman(2016a) finds that the average labor-augmenting productivity growth for home production is about 1 percent for the period 1948–2010 with a CES production function. The BEA working paper version reports that the productivity growth rate is very similar under the Cobb-Douglas production function used in this paper, see figure 5 of Bridgman (2013).

market efficiencies where the average market efficiency $\bar{e}$ is constant over time, the value of $\kappa_{\bar{e}}$ can be derived. Substituting the constant $\bar{e}$ into (37) gives

$$c_{\bar{e}}(0) = (1 + R^{\star} - \delta - \gamma_m)\, a_{\bar{e}}(0) + \bar{e}\bar{w}(0)\bar{l}. \tag{53}$$

Together with R^* and $\bar{w}\,(0)$ from (35) and (36), the value of $\kappa_{\bar{e}}$ for a household with $\bar{e}$ satisfies

$$\kappa_{\bar{e}} = \frac{\gamma_m \left(\frac{1-\beta}{\beta}\right) a_{\bar{e}}(0)}{A_m\,(0)\,\tilde{k}^*} \left(\frac{\alpha}{\gamma_m/\beta - 1 + \delta}\right) + (1-\alpha)\,\bar{e}, \tag{54}$$

where the last equality follows from the values of $\tilde{k}^*$ derived in Proposition 2. Finally, using the definition of $\tilde{k}^*$ and the market clearing conditions,

$$\tilde{k}^* \equiv \frac{K\,(0)}{A_m\,(0)\,L} = \frac{\int_i a_i\,(0)\,di}{A_m\,(0)\,\bar{l}\bar{e}}; \tag{55}$$

thus

$$\frac{\kappa_{\bar{e}}}{\bar{e}} = \alpha\chi \frac{\gamma_m\,(1-\beta)}{\gamma_m - \beta\,(1-\delta)} + (1-\alpha)\,; \quad \chi \equiv \frac{a_{\bar{e}}\,(0)}{\int_i a_i\,(0)\,di}, \tag{56}$$

where χ measures the initial wealth of the household with $e_i = \bar{e}$ relative to the average wealth of all households. We assume χ to be 1 in the baseline. This together with the calibrated values $\{\alpha, \beta, \delta, \gamma_m\}$, imply $\kappa_{\bar{e}}$ is equal to 0.79.[22]

Given the value of $\kappa_{\bar{e}}$, the preference parameters ψ can be set to match the relative time allocation across home and leisure of the $\bar{e}$ household. As explained previously, given the wage of the group with 13-15 years of education is almost exactly the same as the average, see Figure 2, this group is taken as the $\bar{e}$ household. The market efficiency $e_i\,(t)$ for the other three groups are derived from (3) using their relative wages from Figure 2.

The preference parameter ψ is then set to match the relative time allocation across home and leisure for the group with 13–15 years of education. Substituting

[22] Note that we have $\bar{e} = 1$ by the definition of average wage in (3).

the implicit expenditure share derived in (26) into (27) implies that the relative time allocation satisfies

$$\frac{l_{h,i}(0)}{l_{z,i}(0)} = \left[\frac{1 - x_{z,i}(0)}{x_{z,i}(0)}\right] \frac{1}{\left(\frac{\psi}{1-\psi}\right)^{\sigma} p_{h,i}^{\sigma-1}(0) + 1}, \quad \forall i. \tag{57}$$

For the $\bar{e}$ household, $p_{h,\bar{e}} = 1$ from (20) and $x_{z,\bar{e}}(0)$ is a function of the time share in leisure and $\kappa_{\bar{e}}$ (see (27)). Rearranging, ψ is obtained as

$$\psi = 1 - \left[\left(\frac{l_{m,\bar{e}}(0) + \kappa_{\bar{e}} - 1}{l_{h,\bar{e}}(0)}\right)^{1/\sigma} + 1\right]^{-1}, \tag{58}$$

which is equal to 0.47 given $\kappa_{\bar{e}}$ and the observed hour shares for the group with 13–15 years of education.

Given ψ and $e_i(0)$, the household-specific implicit prices, $p_{h,i}(0)$ can be computed for all education groups using (20). Equation (57) can then be used to derive $x_{z,i}(0)$ that matches relative time allocation for all education groups in 1965. Thus, the value of κ_i can be set to match the leisure share for each education group using (52) as follows

$$\kappa_i = \frac{e_i(0)}{x_{z,i}(0)} l_{z,i}(0). \tag{59}$$

The implied values for κ_i are (0.63, 0.71, 0.79, 0.98). Finally, $p_{z,i}(0)$ is computed from (21), which delivers together with $p_{h,i}(0)$ the implicit price $\tilde{p}_{mh,i}(0) \equiv \left[\psi^{\sigma} + (1-\psi)^{\sigma} p_{h,i}(0)^{1-\sigma}\right]^{1/(1-\sigma)}$. Thus using the expression for $x_{z,i}(0)$ in (25), the value of ω_i is derived as

$$\omega_i = \left[1 + \left[\left(\frac{p_{mh,i}(0)}{p_{z,i}(0)}\right)^{(1-\varepsilon)} \left(\frac{x_{z,i}(0)}{1 - x_{z,i}(0)}\right)\right]^{1/\varepsilon}\right]^{-1}. \tag{60}$$

The implied values are (0.013, 0.018, 0.026, 0.04). Table 1 summarizes once again the parameter values calibrated to match the different targets and those calibrated directly.

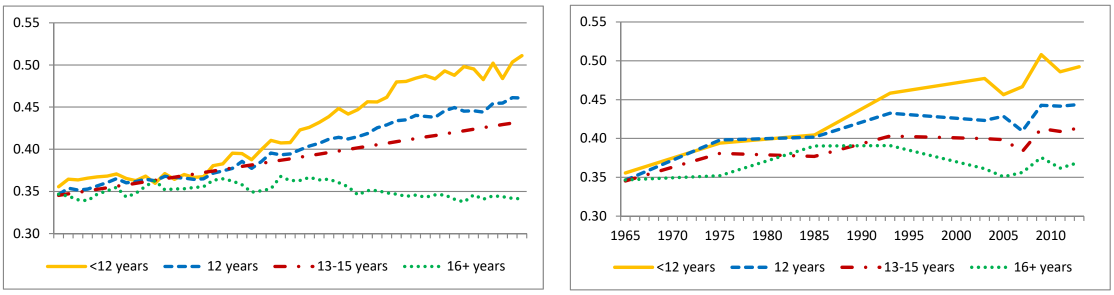
Table 1: Calibrated Parameters

Parameters	Values	Directly calibrated parameters
β	0.97	Discount factor
δ	0.05	Depreciation rate of capital
α	0.3	Capital share in production
σ	1.5	Elasticity of substitution across market and home goods
ε	0.1	Elasticity of substitution across leisure and market-home composite goods
γ_m	1.02	2 percent economic growth rate
γ_h	1.01	Labour augmenting productivity for home production, 1965–2013
Normalization, $\bar{l} = A_m(0) = A_h(0) = 1$		
Parameters matching initial time allocation		
$\psi = 0.47$		
$\kappa_i = 0.63, 0.71, 0.79, 0.98$		
$\omega_i = 0.013, 0.018, 0.026, 0.04$		

Note that the calibration only targets cross-sectional time allocation in 1965. Given the calibrated parameters, the dynamics of the time allocation for each education groups are implied by the time path of household-specific market efficiency, $e_i(t)$, which are pinned down by relative wages using equation (3).

5.2 Quantitative Results

Figure 5 show the predicted hour shares in leisure against the data. The predicted leisure share for the group 13-15 is very smooth as this group is the household with constant $\bar{e}$. Thus, as shown in Lemma 9, the dynamics of time allocation for this group follows the dynamics of $x_{z,i}(t)$, and only affected by the activity-specific productivity growth $\{\gamma_m, \gamma_h\}$. The rise in leisure share is driven by the low substitutability between leisure and consumption ($\varepsilon < 1$) and the faster productivity growth of the market-home composite ($\min\{\gamma_m, \gamma_h\} > 1$); whereas the fall in home share is driven by the high substitutability between market and home consumption



(a) Model Leisure share

(b) Data Leisure share

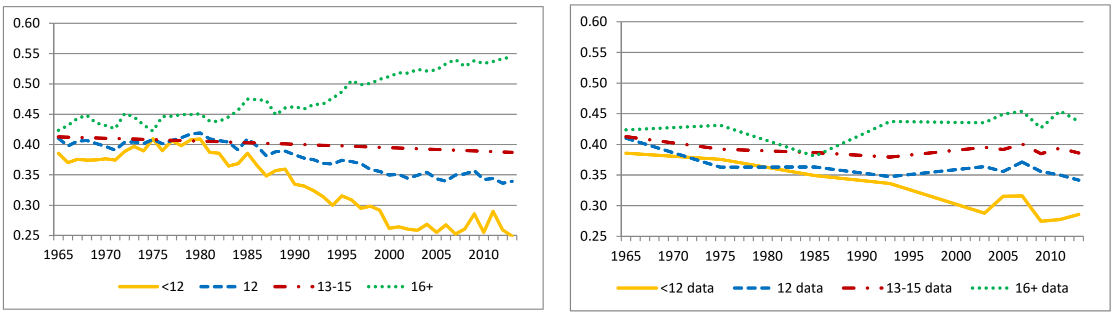
Figure 5: **Leisure share by education group, model and data**

Notes: The figure plots leisure shares predicted by the model and in the data 1965–2013 for four education groups.

($\sigma > 1$) and the faster productivity growth of market production ($\gamma_m > \gamma_h$). Quantitatively, the model does a very good job in predicting the time allocation for this group. It predicts almost perfectly the rise in leisure share from 0.35 to 0.38 in 1985 and to 0.43 in 2013; whereas in the data it increases to 0.38 and 0.41 respectively. The model also predicts quite well the decline of market share from 0.41 to 0.40 then to 0.39; whereas in the data it decreases to 0.39 in 1985 and stays around the same level throughout. The model's prediction for home share is slightly worse as it predicts a monotonic decline from 0.24 to 0.18 for the entire period whereas in the data it was flat and only started to decline after 1985 to around 0.20 in 2013. Still, these predictions are very good given the only drivers are the constant sector-specific productivity growth rates.

Turning now to the overall predictions for all four education groups, Figure 5 shows that the model captures the parallel rise and the subsequent divergence in leisure time remarkably well. It also matches time series for the first three groups extremely well. It slightly overpredicts the increase of leisure of the other three groups relative to the group with 16+ years of education. Put in numbers, the model predicts that the <12 years of education group increases the share of time allocated to leisure from 0.36 to 0.39 in 1985 and to 0.51 in 2013 while in the data it increases to 0.4 in 1985 and to 0.49 in 2013. So the model does a very good job for this group. However, relative to the 16+ group, the model predicts the leisure share for the <12 group rises from being 2% higher in 1965 to 11% higher in 1985 and 50% higher by 2013; whereas in the data it only rises to 4% higher in 1985 and 33% in 2013. The model also slightly misses the timing of the increase in leisure inequality as it predicts the increases started in 1980 whereas in the data it started around 1985.

Figure 6 and 7 report the predictions on market and home shares. Similar to the results on leisure, the model does a good job in predicting time allocation for the first three groups. It, however, over-predicts the rise in market hours and fall in home hours for the 16+ group. These results are not sensitive to the choices of the elasticity parameters σ and ε.

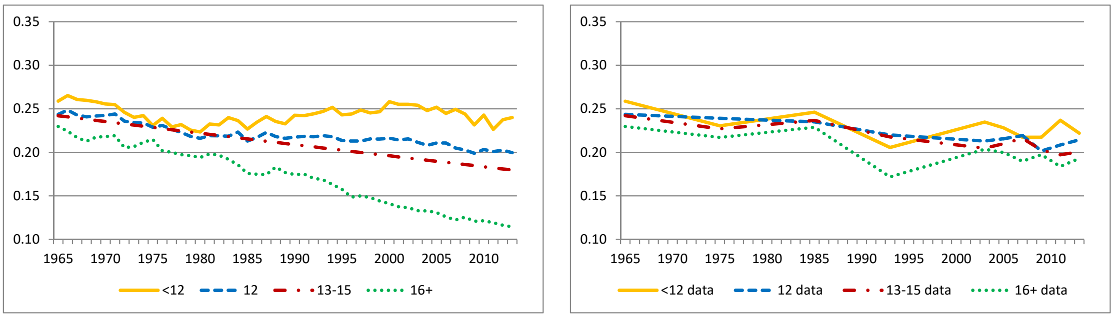


(a) Model Market share

(b) Data Market share

Figure 6: **Market share by education group, model and data**

Notes: The figure plots market shares predicted by the model and in the data 1965–2013 for four education groups.



(a) Model Home share (b) Data Home share

Figure 7: **Home share by education groups, model and data**

Notes: The figure plots home shares predicted by the model and in the data 1965–2013 for four education groups.

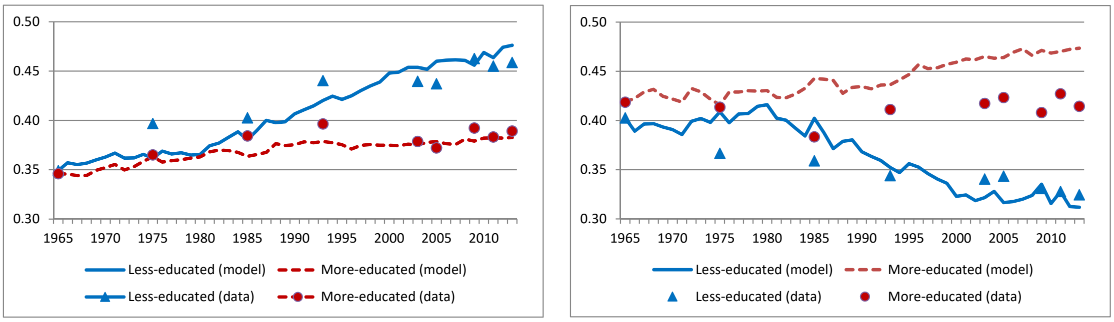
The predictions for the four education groups are aggregated using their weights in the time use data to generate the trend for aggregate leisure, market and home.[23] The model predicts very well the trend for aggregate leisure. It increases from 0.35 to 0.37 in 1985 and to 0.43 by 2013; whereas in the data it increases to 0.39 in 1985 and to 0.42 in 2013. It predicts the overall decline in aggregate market share during 1965–2013 from 0.41 to 0.39, whereas in the data it declines to 0.37. However, it misses the timing as the decline started after 1985 whereas in the data the decline was earlier. Finally, the model predicts a monotonic decline in aggregate home share during the entire period whereas in the data it has flatten out since the 1990s.

5.3 Discussion

To summarize the quantitative results in a simpler form, we aggregate the four groups into two groups using the average weights in time use surveys: the less-

[23]The average weights of the four groups are (0.15, 0.34, 0.23, 0.28) in the sample of time use surveys and (0.16, 0.36, 0.22, 0.26) in the CPS sample.

educated (those with less than or equal to 12 years of education) and the more-educated (those with 13 or more years of education).[24] The results on leisure and market share are reported in Figure 8. Their leisure share was about the same in 1965, the model predicts the leisure of the less-educated relative becomes 5 percent higher in 1985 (which exactly matches the data) then rises to 24 percent higher by 2013, whereas in the data it is only 18 percent higher. Turning to market share, the less-educated work 4 percent less in the market in 1965, the model predicts it drops to being 9 percent lower in 1985 (which was 6 percent lower in the data), then jumps to 32 percent lower by 2013 whereas in the data it is only 22 percent. The over-prediction in both cases are due to the fact that the model over-predicts the rise in market hours for the group with 16 or more years of education, which was driven by the substantial rise in their relative wages.



(a) Leisure shares

(b) Market shares

Figure 8: **Leisure and Market shares for more-educated and less-educated**

Notes: The figure plots leisure and market shares predicted by the model and in the data 1965–2013 for two education groups. Less-educated include those with 12 or less years of education and more-educated include those with 13 or more years of education.

Overall the model does a good job in accounting for the rising leisure inequality

[24]The average weights are 0.49 (0.15+0.34) for the less-educated and 0.51 (0.23+0.28) for the more-educated group.

and aggregate trend in leisure. It performs less well in disentangling the trend in non-leisure hours into market and home; and for the time allocation of 16+ group. There are two observations to make. First, Bridgman (2016a) documents a significant decline in the labor-augmenting productivity growth for the home sector from about 2.5% before 1980 to zero growth afterward. Intuitively, this helps to lower the predicted market share prior to 1980 bringing it closer to the data and it may also help to fix the timing of the increase in leisure inequality predicted by the model. Quantitatively, the effects are small, as shown in Appendix Figure B.4. The other predictions on market and home shares are also similar. Second, regarding the poor predictions for the 16+ group, recall that the model assumes full anticipation of the fast relative wage growth post-1985. Intuitively, if the rise in college-premium was not fully anticipated, this could reconcile why the rise in market hours and fall in leisure hours observed in the data are less than the model's predictions. Modeling expectation, however, is beyond the scope of our paper.

6 Conclusion

Market efficiency, initial capital and time are the primitives that ultimate constrain the behavior of households. While the former two are most likely subject to some form of exogenous distributions, time constraint is the same for all individual. Thus, being able to freely allocate one's time is an important tool for the "less-privileged" household to partly "reverse" the welfare inequality induced by the two exogenous inequalities in market efficiency and initial capital. This is indeed what is observed in the data where the less-educated allocating more time to leisure while more-educated allocating more time to market hours and obtain higher market income. Consistent with the empirical findings of Aguiar and Hurst (2007a) and Attanasio, Hurst and Pistaferri (2015), the increase in leisure inequality has partly offset the welfare effects of the rising income and consumption inequality. This is done through both the direct channel of higher leisure time for the less-educated (low market efficiency individuals) and the equilibrium channel where the more-educated (high market efficiency individuals) work more in the market which increases the aggregate market production. One interesting application for policy analysis would be to use the model to evaluate the welfare loss and increase in welfare inequality for regulations on working hours in all sectors of the economy.

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A.1 Appendix A

A.1.1 Solving the household and firm problem

Replacing $c_{z,i}$ and $c_{h,i}$ and in the utility function by (5) and (6) allows us to write the Lagrangian of the household problem as

$$\begin{aligned}\mathcal{L}_i \quad = \quad & \sum_{t=0}^{\infty} \beta^t u\left(c_{m,i}(t), k_{h,i}(t)^\alpha \left[A_h(t) l_{h,i}(t)\right]^{1-\alpha}, k_{z,i}(t)^\alpha l_{z,i}(t)^{1-\alpha}\right) + \\ & \sum_{t=0}^{\infty} \beta^t \lambda_i(t) \left[R(t)\left[a_i(t) - k_{h,i}(t) - k_{z,i}(t)\right] + a_i(t)\left[1-\delta\right] + \left[1 - l_{h,i}(t) - l_{z,i}(t)\right] w_i(t) - c_{m,i}(t)\right]\end{aligned}$$

The first-order conditions are then given by

$$\lambda_i(t) = \beta \lambda_i(t+1)\left[1 + R(t+1) - \delta\right], \tag{A.1}$$

$$\frac{\omega_i \psi \left[\psi c_{m,i}(t)^{\frac{\sigma-1}{\sigma}} + (1-\psi)\, c_{h,i}(t)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma(\varepsilon-1)}{(\sigma-1)\varepsilon}-1} c_{m,i}(t)^{\frac{\sigma-1}{\sigma}}}{\omega_i \left[\psi c_{m,i}(t)^{\frac{\sigma-1}{\sigma}} + (1-\psi)\, c_{h,i}(t)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma(\varepsilon-1)}{(\sigma-1)\varepsilon}} + (1-\omega_i)\, c_{z,i}(t)^{\frac{\varepsilon-1}{\varepsilon}}} = \lambda_i(t) c_{m,i}(t), \tag{A.2}$$

$$\frac{\alpha \omega_i (1-\psi) \left[\psi c_{m,i}(t)^{\frac{\sigma-1}{\sigma}} + (1-\psi)\, c_{h,i}(t)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma(\varepsilon-1)}{(\sigma-1)\varepsilon}-1} c_{h,i}(t)^{\frac{\sigma-1}{\sigma}}}{\omega_i \left[\psi c_{m,i}(t)^{\frac{\sigma-1}{\sigma}} + (1-\psi)\, c_{h,i}(t)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma(\varepsilon-1)}{(\sigma-1)\varepsilon}} + (1-\omega_i)\, c_{z,i}(t)^{\frac{\varepsilon-1}{\varepsilon}}} = \lambda_i(t) R(t) k_{h,i}(t), \tag{A.3}$$

$$\frac{(1-\alpha)\omega_i (1-\psi) \left[\psi c_{m,i}(t)^{\frac{\sigma-1}{\sigma}} + (1-\psi)\, c_{h,i}(t)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma(\varepsilon-1)}{(\sigma-1)\varepsilon}-1} c_{h,i}(t)^{\frac{\sigma-1}{\sigma}}}{\omega_i \left[\psi c_{m,i}(t)^{\frac{\sigma-1}{\sigma}} + (1-\psi)\, c_{h,i}(t)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma(\varepsilon-1)}{(\sigma-1)\varepsilon}} + (1-\omega_i)\, c_{z,i}(t)^{\frac{\varepsilon-1}{\varepsilon}}} = \lambda_i(t) w_i(t) l_{h,i}(t), \tag{A.4}$$

$$\frac{\alpha\,(1-\omega_i)\, c_{z,i}(t)^{\frac{\varepsilon-1}{\varepsilon}}}{\omega_i \left[\psi c_{m,i}(t)^{\frac{\sigma-1}{\sigma}} + (1-\psi)\, c_{h,i}(t)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma(\varepsilon-1)}{(\sigma-1)\varepsilon}} + (1-\omega_i)\, c_{z,i}(t)^{\frac{\varepsilon-1}{\varepsilon}}} = \lambda_i(t) R(t) k_{z,i}(t), \tag{A.5}$$

$$\frac{(1-\alpha)\,(1-\omega_i)\, c_{z,i}(t)^{\frac{\varepsilon-1}{\varepsilon}}}{\omega_i \left[\psi c_{m,i}(t)^{\frac{\sigma-1}{\sigma}} + (1-\psi)\, c_{h,i}(t)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma(\varepsilon-1)}{(\sigma-1)\varepsilon}} + (1-\omega_i)\, c_{z,i}(t)^{\frac{\varepsilon-1}{\varepsilon}}} = \lambda_i(t) w_i(t) l_{z,i}(t). \tag{A.6}$$

The representative firm solves

$$\max_{K_m(t), L_m(t)} K_m(t)^\alpha \left[A_m(t) L_m(t)\right]^{1-\alpha} - R(t) K_m(t) - \bar{w}(t) L_m(t). \tag{A.7}$$

The first-order conditions are

$$\alpha \left[\frac{K_m(t)}{A_m(t)L_m(t)} \right]^{\alpha-1} = R(t), \tag{A.8}$$

$$(1-\alpha)A_m(t) \left[\frac{K_m(t)}{A_m(t)L_m(t)} \right]^{\alpha} = \bar{w}(t). \tag{A.9}$$

A.1.2 Proof of Lemma 1

Proof. Combining the first-order conditions (A.3) and (A.4) as well as (A.5) and (A.6) gives

$$\frac{k_{h,i}(t)}{l_{h,i}(t)} = \frac{k_{z,i}(t)}{l_{z,i}(t)} = \frac{e_i(t)\bar{w}(t)}{R(t)} \frac{\alpha}{1-\alpha}, \ \forall i. \tag{A.10}$$

Using this together with the first-order conditions of the firm's problem, (A.9) and (A.8), and the market clearing conditions (9) and (10) (as well as (3)) gives

$$\frac{K_m(t)}{L_m(t)} = \frac{\bar{w}(t)}{R(t)} \frac{\alpha}{1-\alpha} = \frac{K(t)}{L}. \tag{A.11}$$

Using this fact in (A.9) and (A.8) gives (15) and (16). Finally, combining (A.10) and (A.11) gives (14). ■

A.1.3 Proof of Lemma 4

Proof. By combining (A.2)-(A.4), (6) and (17) gives

$$\frac{1-\psi}{\psi} \left(\frac{c_{m,i}(t)}{c_{h,i}(t)} \right)^{1/\sigma} = p_{h,i}(t). \tag{A.12}$$

This equation has the interpretation that under optimal behavior the marginal rate of substitution between m and h has to equalize their implicit relative price. Let $c_{mh,i}(t) \equiv \left[\psi c_{m,i}(t)^{\frac{\sigma-1}{\sigma}} + (1-\psi)\, c_{h,i}(t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$ be the consumption of the composite non-leisure good. Given the definition of implicit price for non-leisure good, $\tilde{p}_{mh,i}(t) = \left[\psi^{\sigma} + (1-\psi)^{\sigma} p_{h,i}(t)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$, and (A.12) we have

$$\tilde{p}_{mh,i}(t) c_{mh,i}(t) = c_{m,i}(t) + p_{h,i}(t) c_{h,i}(t). \tag{A.13}$$

Note that we also have

$$\frac{p_{h,i}(t)c_{h,i}(t)}{\tilde{p}_{mh,i}(t)c_{mh,i}(t)} = \frac{1}{1+\left(\frac{\psi}{1-\psi}\right)^{\sigma} p_{h,i}(t)^{\sigma-1}}, \tag{A.14}$$

and

$$\frac{c_{m,i}(t)}{\tilde{p}_{mh,i}(t)c_{mh,i}(t)} = \frac{1}{1+\left(\frac{\psi}{1-\psi}\right)^{-\sigma} p_{h,i}(t)^{1-\sigma}}. \tag{A.15}$$

Equating the marginal rate of substitution across $c_{mh,i}(i)$ and $c_{z,i}(i)$, ob by combining (A.13) with (A.5) and (A.6), we obtain the relative expenditure

$$\frac{\tilde{p}_{mh,i}(t)c_{mh,i}(t)}{p_{z,i}(t)c_{z,i}(t)} = \left(\frac{\omega_i}{1-\omega_i}\right)^{\varepsilon}\left(\frac{p_{z,i}(t)}{\tilde{p}_{mh,i}(t)}\right)^{\varepsilon-1}. \tag{A.16}$$

Combining (A.13) with (24) and (A.16) and

$$\frac{\tilde{p}_{mh,i}(t)c_{mh,i}(t)}{c_i(t)} = \frac{1}{1+\left(\frac{\omega_i}{1-\omega_i}\right)^{-\varepsilon}\left(\frac{p_{z,i}(t)}{\tilde{p}_{mh,i}(t)}\right)^{1-\varepsilon}}. \tag{A.17}$$

Finally, (A.17) together with (A.14) and (A.15) yield (25) and (26). ■

A.1.4 Proof of Lemma 6

Proof. Using the definition of $c_i(t)$ it the budged constraint (4) gives (31). By summing the first-order conditions (A.2)–(A.6) we obtain

$$c_i(t) = \frac{1}{\lambda_i(t)}. \tag{A.18}$$

Combining this with the first-order condition (A.1) gives (32). ■

A.1.5 Proof of Proposition 2

Proof. With $\tilde{k}(t) = \tilde{k}^{\star}$ we have $\frac{\int_0^1 c_i(t+1)di}{\int_0^1 c_i(t)di} = \frac{\int_0^1 a_i(t+1)di}{\int_0^1 a_i(t)di} = \gamma_m$, $\forall i$ and the transversality conditions are fulfilled. To see global saddle path stability, note that the system (31), (32), (15) and (16) is identical to the one of a one sector neoclassical growth model with Cobb-Douglas production and logarithmic instantaneous utility over $c_i(t)$. ■

A.1.6 Proof of Lemma 7

Proof. The budget constraint (4) can be written as (see (12) and (13))

$$a_i(t+1) = [1 + R(t) - \delta]\, a_i(t) + \bar{w}(t)e_i(t)\bar{l} - c_i(t). \tag{A.19}$$

Consolidating these budget constraints over time and using the transversality condition gives

$$\begin{aligned} c_i(0) + \sum_{t=1}^{\infty} c_i(t) \prod_{s=1}^{t} \frac{1}{1 + R(s) - \delta} &= (1 + R(0) - \delta)a_i(0) \\ &+ \bar{w}(0)e_i(0)\bar{l} + \sum_{t=1}^{\infty} \bar{w}e_i(t)\bar{l} \prod_{s=1}^{t} \frac{1}{1 + R(s) - \delta}. \end{aligned}$$

Substituting in the factor prices along the balanced growth path gives (37). ∎

B.1 Appendix B: Additional tables and figures

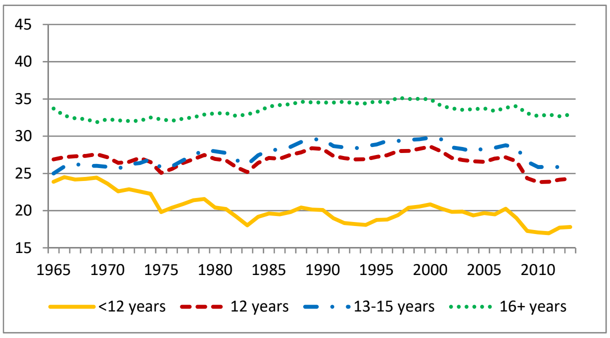


Figure B.1: Market hours by education group: CPS data

Notes: The figure plots market hours 1965–2013 for four education groups using CPS data.
Source: CPS. Non-farm working individuals aged 21–65 who are not student. Adjusted for changes in demographic compositions as done by Aguair and Hurst (2007a) for the time used data.

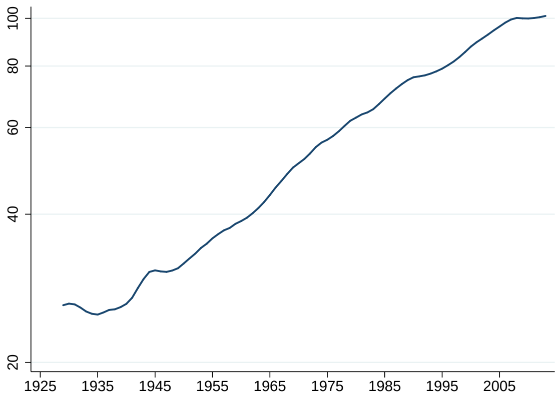


Figure B.2: **Real capital per capita**

Notes: The figure plots real fixed assets plus consumer durables per capita (corresponding to $K(t)$ in the model) on a logarithmic scale. The series is normalized to 100 in the year 2009. Source: BEA table 1.2 and 7.1 for the population data.

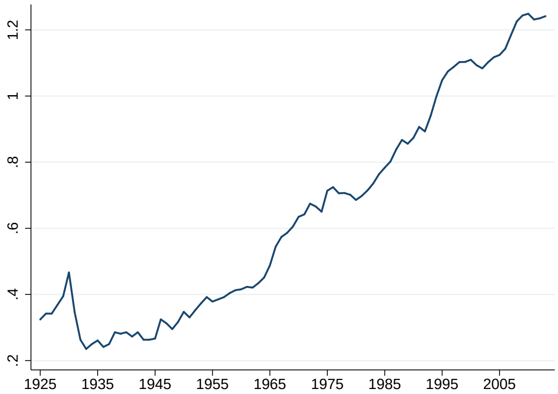
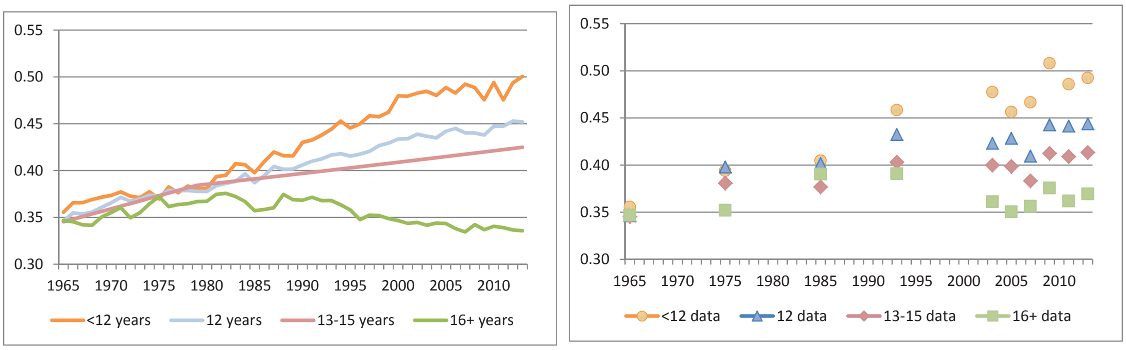


Figure B.3: **Expenditures of leisure durables relative to household durables**

Notes: The figure plots aggregate personal consumption expenditure of "recreational" durable goods relative to aggregate personal consumption expenditure of "furnishing and household durables" corresponding to $\frac{\int_0^1 \dot{k}_{z,i}(t)+\delta k_{z,i}(t)di}{\int_0^1 \dot{k}_{h,i}(t)+\delta k_{h,i}(t)di}$ in the model. Source: BEA table 8.7.



(a) Model Market share (b) Data Market share

Figure B.4: **Leisure share by education group, predicted when there is a slowdown in home productivity since 1980**

Notes: The figure plots leisure share predicted by the model when there is a slowdown in home productivity growth as found by Bridgman (2016a). γ_h is set to 1.025 before 1980 and 1 after 1980.

Table B.1: Time allocation (hours per week) for different education groups

Panel 1: Leisure					
Years of education	All	<12	12	13–15	16+
1965	30.9	31.7	31.1	30.2	30.7
1975	33.2	33.8	34.9	32.5	31.5
1985	34.9	35.4	35.9	33.8	34.3
1993	37.3	41.3	38.8	35.4	35.0
2003	35.1	40.0	36.8	33.9	31.2
2013	35.4	39.8	37.2	34.5	31.3
1965-1985	4.0	3.8	4.8	3.6	3.6
1985-2013	0.5	4.4	1.3	0.7	-2.9
Panel 2: Total Market Work					
Years of education	All	<12	12	13–15	16+
1965	36.5	34.3	36.8	36.1	37.5
1975	34.1	32.2	31.8	33.5	38.5
1985	33.0	30.6	32.4	34.7	33.4
1993	33.7	30.3	31.2	33.3	39.2
2003	32.6	24.1	31.7	33.5	37.6
2013	31.0	23.1	28.7	32.2	37.1
1965-1985	-3.5	-3.7	-4.4	-1.4	-4.1
1985-2013	-2.0	-7.5	-3.8	-2.5	3.7
Panel 3: Total Non-Market Work					
Years of education	All	<12	12	13–15	16+
1965	21.5	23.0	21.9	21.2	20.3
1975	20.0	19.8	21.0	19.4	19.4
1985	20.9	21.6	21.0	21.2	20.1
1993	18.2	18.5	19.7	19.1	15.4
2003	18.2	19.7	18.5	17.3	17.6
2013	17.3	18.0	18.0	16.7	16.4
1965-1985	-0.6	-1.5	-0.9	0.0	-0.3
1985-2013	-3.6	-3.6	-3.0	-4.5	-3.7

Notes: Source: 1965–1966 America's Use of Time; 1975–1976 Time Use in Economics and Social Accounts; 1985 Americans' Use of Time; 1992–1994 National Human Activity Pattern Survey; and 2003–2009 American Time Use Surveys, following Aguiar and Hurst (2007a) Methodology. Non-retired, non-student individuals between the ages of 21–65. The column "All" is comparable to Aguiar and Hurst (2007a) table II. "Leisure" is Aguiar and Hurst (2007a) "Leisure Measure 1". Childcare is excluded.