Intergenerational Elasticity of Life-cycle Earnings Growth/Risk

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[PRELIMINARY]

Motivation and Literature

How important is family background in determining a child's earning prospects?

- Measurement error attenuates estimates. Especially tough when shocks are persistent!
  - Solon (1992, 1999), Black & Devereux (2011), Mazumdar (2005, 2016)
- Variation in earnings over life-cycle further complicate the matter
  - Haider & Solon (2006), Bohlmark & Lindquist (2006), Nilsen, et al (2008)
- Mechanisms: Education, Neighborhoods, etc.
  - Chetty, et al (2014); Chetty, et al (2018); Chetty & Hendren (2018a,b)
- Literature tends to focus on elasticity of permanent income *level* between parents/children

What explains cross-sectional variance of earnings over life-cycle?

- Cross-sectional variance of earnings rises over life-cycle – heterogeneous profiles vs persistent shocks
  - Baker (1997), Mazumdar (2001), Haider (2001), Guvenen (2009), Sabelhaus & Song (2009, 2010), Huggett, Ventura, Yaron (2011), Altonji, Smith, Vidangos (2013), Guvenen, et al (2016)
- Timing of earnings and knowledge about earnings potential matter for spending:
  - High growth with little financial buffer → inability to smooth consumption over lifecycle
  - Lack of knowledge about future earnings growth → consumption reflects updating beliefs about earnings
- Differences in initial conditions at age 23 determine most of earnings variance (Huggett, Ventura, Yaron, 2011)
  - This view may not be robust to age-varying heteroskedasticity in shocks (Sabelhaus & Song, 2009, 2010)

Overview

Standard IGE estimates using “permanent income” are a combination of level and growth elasticities

Goal: Estimate how earnings *profiles* of children are related to parents.

Model/estimate a parametric empirical process of earnings

- Explicitly control for measurement error and shocks to earnings when worker is observed (standard)
- Allows for intergenerational elasticity in both level and growth of earnings profile (new)
- In progress: Expand to allow for intergenerational transfer of household-specific risk

Results suggest heterogeneity in earnings *growth* determined by dad’s earnings

- “True” IGE may vary over lifecycle because earnings early and late in life are driven more by parents
- Supportive evidence that some variation life-cycle earnings growth is known ex ante

An Empirical Process of Earnings

Let log real earnings for individual i at age h, time t be given by:

$$y_{ht}^{i} = \mu_{ht} + (a_i + b_i h) + z_{ht}^{i} + \varepsilon_{ht}^{i}$$

- Common, time-varying, age-specific profile: μ_{ht}
  - Cohort-specific life-cycle profiles allows changes in return to experience (Katz & Autor, 1999)
  - Alternative specs also allowing for changes in return to education (Katz & Murphy, 1992)
- Ex ante heterogeneous income profiles over life-cycle: $a_i + b_i h$,
  - $E[a_i, b_i] = [0,0],\ Var(a_i, b_i) = \left[\sigma_\alpha^2, \sigma_{\alpha\beta}; \sigma_{\alpha\beta}, \sigma_\beta^2\right]$
  - a_i is level shift from cohort average at start of career
  - b_i is how earnings moves away from cohort average as worker ages
- AR(1) shock and IID shock:
  - AR(1): $z_{ht}^{i} = \rho z_{h-1,t-1}^{i} + \eta_{ht}^{i}$, $Var\left(\eta_{ht}^{i}\right) = \phi_t \sigma_\eta^2$
  - IID shock: $\varepsilon_{ht}^{i}, Var\left(\varepsilon_{ht}^{i}\right) = \pi_t \sigma_\varepsilon^2$
  - ϕ_t and $\pi_t$allows variance of shock to change over *time*
  - includes persistent/iid measurement error

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- Heterogeneous income profiles over life-cycle: $a_i + b_i h$
- AR(1) shock and IID shock (including measurement error)

Individuals are related to parents via (a_i, b_i)

$$a_{son} = R_a a_{father} + u^a_{son}$$
$$b_{son} = R_b b_{father} + u^b_{son}$$

- Note: Other channels may be possible – ie, parents determine initial shock z_{0t} which fades

What does naïve OLS yield?

Common way of estimating IGE is to regress son's earnings on father, controlling for age/time of observation:

$$y_{ht}^{son} = R y_{h't'}^{father} + \gamma[\mu_{ht}, \mu_{h't'}] + v$$

Probability limit of $\hat{R}$ is given by:

$$\hat{R} \rightarrow_p \frac{R_a var(\alpha) + R_b var(\beta) h^2 + cov(\alpha,\beta)(R_a + R_b)h}{var(\alpha) + var(\beta){h'}^2 + 2cov(\alpha,\beta)h' + var(z_{h'}) + var(\varepsilon)}$$

Estimate is attenuated by standard errors-in-variables issue (Solon, 1992 and others)

- Not simply solved by time-aggregation if errors are persistent (Mazumdar, 2001)
- Jointly estimate parameters of income process to account for bias

Even adjusting for attenuation yields weighted average of level and growth rate when $(var(\beta) > 0)$

- Evidence for HIP in data – Baker (1997), Guvenen (2009), Guvenen et al (2017)
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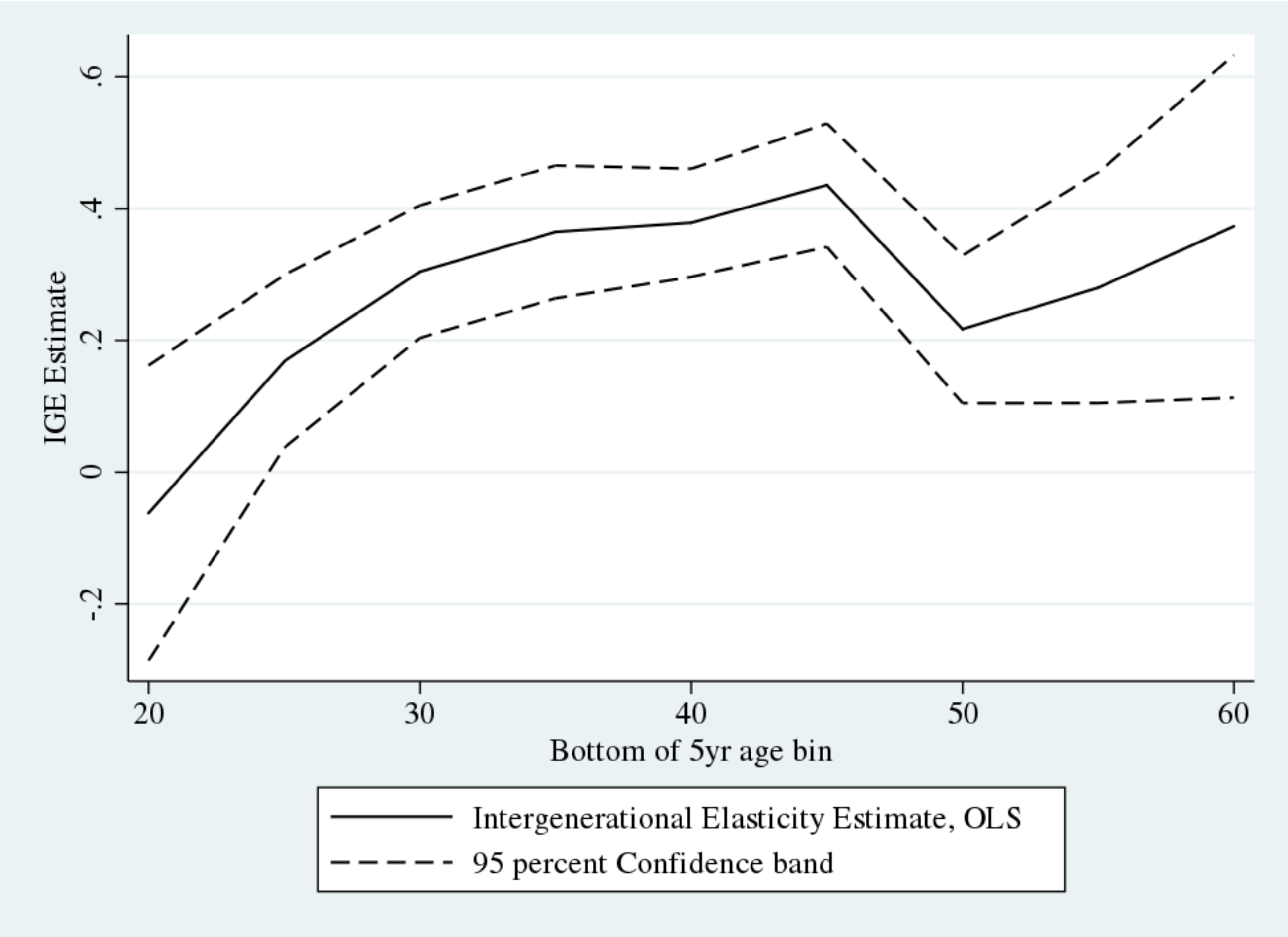
➢ OLS by Age

Why not run OLS age by age?



➢ Back

A More Structural Approach

Goal: Address biases and decompose R into pieces R_α and R_β

- To what extend do parents transfer initial earnings level α
- To what extent do parents transfer learning ability, education, etc in the form of β?

Doing this requires estimating variances of $\alpha_i, \beta_i, z_{ht}^i, \varepsilon_{ht}^i$ jointly

Two-step approach following Abowd & Card (1989) also used by Mazumdar (2001), Guvenen (2009):

- Remove common component with age/time effects
- Minimum Distance Estimator using covariance structure of data and empirical model

Citations above focus on earning process alone

- Contribution here is estimating link to parents

PSID Data

Identification requirements:

- Parent-child match
- Long labor histories: β_i, z_{ht}^i, and ε_{ht}^i have differing effects at long lags

Panel Study of Income Dynamics

- Sampling frame explicitly related to family structure - children of respondents also followed
- Longitudinal panel from 1968 – 2015 (almost 50 years)

Would like to focus on workers with some labor force attachment

- Need long labor history to differentiate between AR1 shocks and profiles
- Not explicitly modeling labor supply decision, so focus on strong labor force attachment

Sample Selection

Sample selection criteria:

- Male household heads ages 20 to 64 from main SRC sample
- “Attached” to labor force for 10/20 years (based on hours, hourly wages, and earnings)
- Valid observations with no labor earnings are coded to $1
- Individuals without validly matched parents are retained to estimate income process

10yr attachment sample: 4,661 workers and 1,386 matched father-son pairs

20yr attachment sample: 2,230 workers and 431 matched father-son pairs

Estimating Structural Model

Let the common time-age profile be characterized by Mincer-type regression:

$$\mu_{ht} = m_0^t + m_1^t h + m_2^t h^2 + m_3^t h^3$$

- Allow returns to experience to vary over time (Katz & Autor, 1999)
- Returns to education absorbed by β_i, since education groups are pooled in baseline

Estimates yield residual log real earnings given by:

$$\tilde{y}_{ht}^i = y_{ht}^i - \mu_{ht} = (a_i + b_i h) + z_{ht}^i + \varepsilon_{ht}^i$$

- Estimation error from first stage will be absorbed into measurement error in z and ε

Model Covariance Structure

Given the parametric model for $\tilde{y}_{ht}^i$:

$$\tilde{y}_{ht}^i = (a_i + b_i h) + z_{ht}^i + \varepsilon_{ht}^i$$

The auto-covariance of log earnings is:

$$E\left[\tilde{y}_{ht}^i * \tilde{y}_{h-\ell,t-\ell}^i\right]$$
$$= \underbrace{\sigma_\alpha^2 + \sigma_{\alpha\beta}(2h-\ell) + \sigma_\beta^2 h(h-\ell)}_{heterog.\,profiles} + \underbrace{\rho^\ell var\left(z_{h-\ell,t-\ell}^i\right)}_{AR1\ shock} + \underbrace{1(\ell=0)\pi_t^2\sigma_\varepsilon^2}_{idio.\,shock}$$

where

$$var\left(z_{ht}^i\right) = \rho^2 var\left(z_{h-1,t-1}^i\right) + \phi_t^2\sigma_\eta^2$$
$$var\left(z_{0t}^i\right) = \phi_0^2\sigma_\eta^2$$
$$var\left(z_{h0}^i\right) = \phi_0^2\sigma_\eta^2\sum_{j=0}^{h-1}\rho^{2j}$$

Model Covariance Structure

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where

$$var\left(z^i_{ht}\right) = \rho^2 var\left(z^i_{h-1,t-1}\right) + \phi^2_t\sigma^2_\eta$$

No AR shock inherited $\longrightarrow$

$$var\left(z^i_{0t}\right) = \phi^2_0\sigma^2_\eta$$

Variance was stable before 1968 $\longrightarrow$

$$var\left(z^i_{h0}\right) = \phi^2_0\sigma^2_\eta\sum_{j=0}^{h-1}\rho^{2j}$$

Model Covariance Structure

Model also implies intergenerational covariance.

Using intergenerational link:

$$a_i = R_a a_{father(i)} + u_i^a$$
$$b_i = R_b b_{father(i)} + u_i^b$$

Covariance of son of age h at time t with his father at age h' at time t' is:

$$E\left[\tilde{y}_{ht}^{i} * \tilde{y}_{h',t'}^{father(i)}\right] = R_\alpha\left(\sigma_\alpha^2 + \sigma_{\alpha\beta}h'\right) + R_\beta\left(\sigma_\beta^2 hh' + \sigma_{\alpha\beta}h\right)$$

Identification of R_α and R_β relies on how intergenerational covariance depends on father and son's ages h' and h

Minimum Distance Estimator

Sample corollaries of covariances are used to construct moments:

$$\frac{1}{N_1}\sum_i \tilde{y}^i_{ht} * \tilde{y}^i_{h-\ell,t-\ell} - g_1(\theta; h, t, l) = 0$$

$$\frac{1}{N_2}\sum_i \tilde{y}^i_{ht} * \tilde{y}^{father(i)}_{h't'} - g_2(\theta; h, h') = 0$$

- $g_1(\cdot)$ and $g_2(\cdot)$ denote auto-covariance and intergenerational covariance.
- θ is vector of $8 + 2T$ parameters: $[\sigma_a^2, \sigma_b^2, \sigma_{ab}, \rho, \sigma_\eta^2, \sigma_\varepsilon^2, \phi_t, \pi_t, R_a, R_b]$

One-step (equally-weighted) GMM estimate of parameter vector θ

- Sample size is likely too small for optimally-weighted GMM to perform well.

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	(0.14396)		(0.38173)		(0.1879)		(0.39814)	
R_b	0.36703	***	0.55691	***	0.34652	**	0.59534	***
	(0.07975)		(0.14728)		(0.16414)		(0.17845)	
sigma2_a	0.121	**	0.08		0.05659		0.06974	
	(0.0573)		(0.062)		(0.03804)		(0.06419)	
sigma2_b	0.00019	***	0.00022	***	0.00001		0.0002	***
	(0.00006)		(0.00006)		(0.00004)		(0.00006)	
sigma_ab	-0.0012		-0.0014	**	0.00186	**	-0.00124	**
	(0.00091)		(0.00062)		(0.00088)		(0.00061)	
rho	0.80536	***	0.86491	***	0.81869	***	0.87261	***
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sigma2_eta	0.03168		0.01863		0.01311		0.01791	
	(0.02869)		(0.02233)		(0.0229)		(0.02167)	
sigma2_eps	0.00897		0.00547		0.04107		0.04719	
	(0.08779)		(0.11727)		(0.09414)		(0.13228)	
Sample	10yr LF Attachment		20yr LF Attachment		10yr LF Attachment		20yr LF Attachment	
Common Component	Age x Time		Age x Time		Age x Time x Educ		Age x Time x Educ	
Total Individuals	4661		2230		4661		2230	
Matched Father Pairs	1386		421		1386		421	

➢ Model Fit

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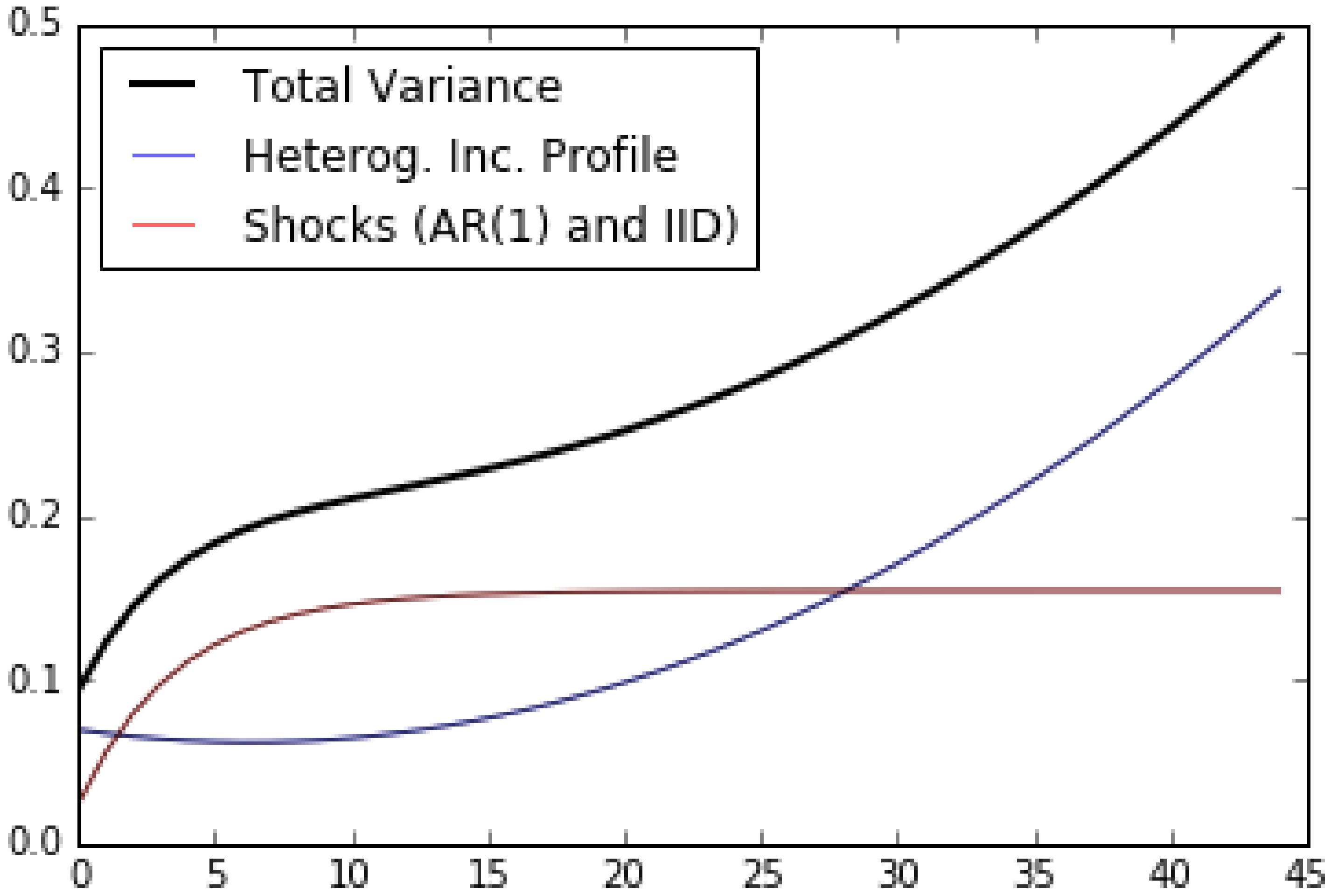

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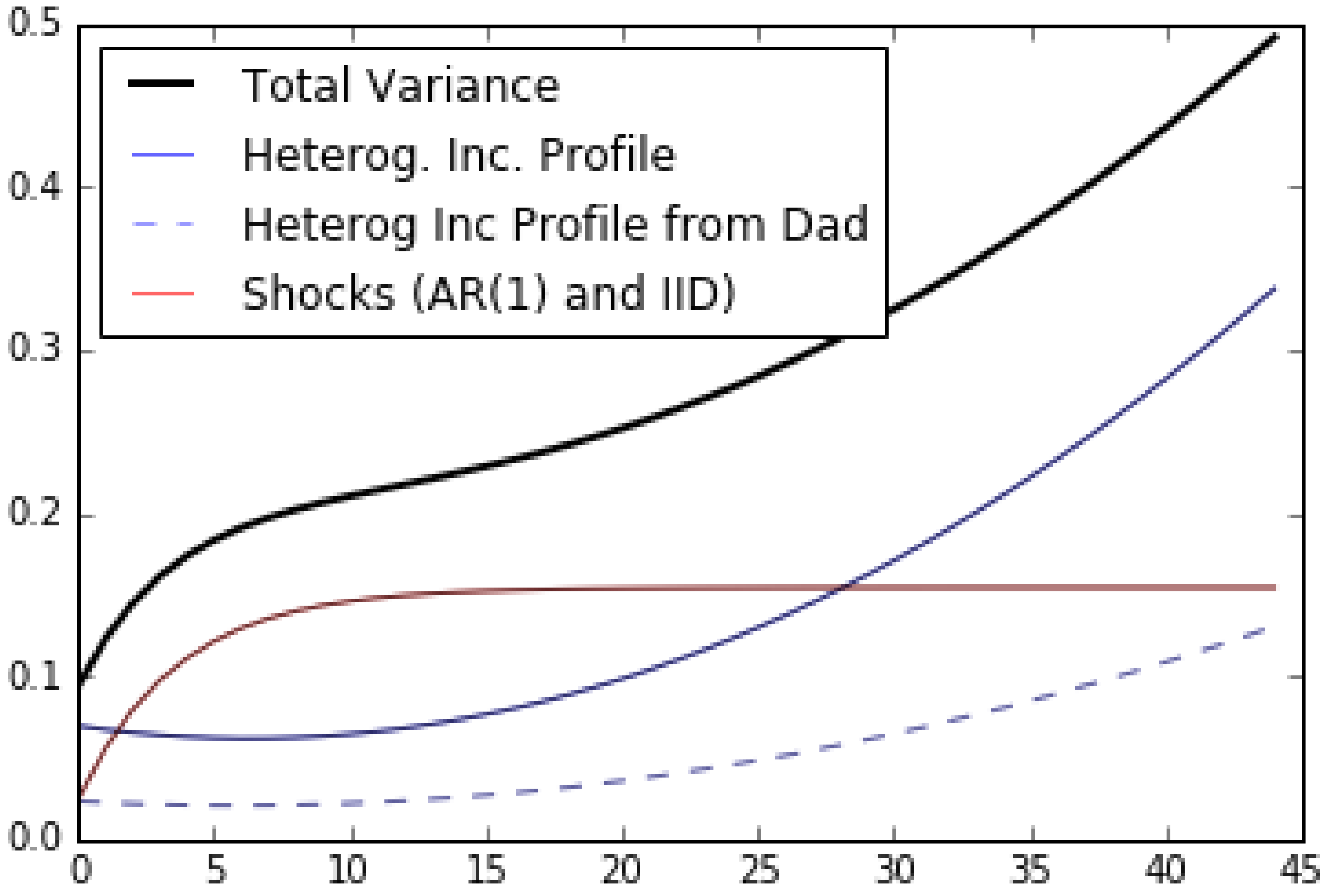
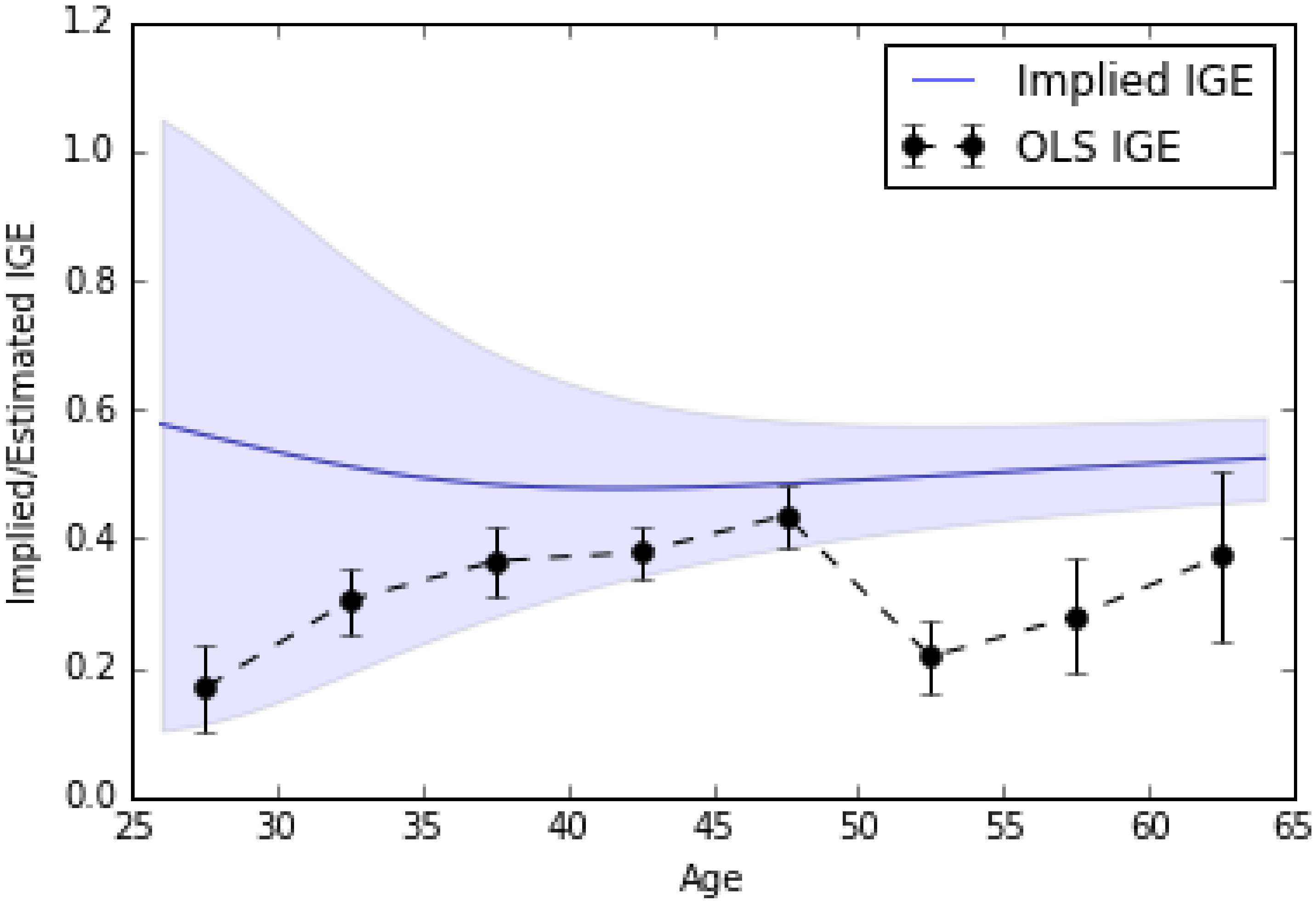


Implied IGE of Income Level by Age



What might this tell us about earnings?

OLS estimates of the IGE are in the 0.2-0.4 ballpark whereas GMM estimates are closer to 0.35-0.6.

Estimates also imply parents contribute to *growth* in earnings more than levels

- Possibly suggests the mechanism works through learning ability or human capital with returns later in life

What does this tell us about consumption?

- Some amount of lifetime earnings is knowable ex ante (evidence for “HIP” earnings process)
- Quantitatively matches “indirect” approach to backing out priors as in Guvenen (2007)
- Suggests covariance between consumption and income over life-cycle is more driven by constraints rather than information

Conclusions

Overall IGE can be decomposed into level and growth

Estimate structural model using the PSID
- Accounts for measurement error and transitory shocks that may lead to attenuation bias
- Uses covariance structure to separately recover level and growth components of IGE

Preliminary results suggest
- There is heterogeneity in life-cycle profiles
- Individual-specific earnings growth is tied to parent, with less link in starting level
- Implies higher standard IGE later in life