Global Ownership and Hierarchies of Firms. 'What is essential is invisible to the eye'.

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Abstract

To date, little is known about the complexity of the global ownership of firms. In this contribution, we introduce a basic network framework to study the separation of cash rights from control rights, therefore assessing the extent of a firm boundary when coordinated management decisions have to be transmitted along alternative and often overlapping ownership paths. After we apply our framework to a dataset of 53.5 million of companies operating in 206 countries in 2015, we provide some useful insights into the hidden heterogeneity of corporate hierarchies, made by parents and subsidiaries. We detect a strong concentration of corporate power, usually unobserved, as less than 1% of parent companies control more than 50% of global sales. Therefore, we document policy-relevant cases of indirect control, when the nationality of the investor may be not immediately apparent, in multiple-passports (19.1%), indirectly foreign (24.5%), and round-tripping subsidiaries (1.33%). Finally, we test that indirect control and pyramidal corporate structures preferably run through intermediate jurisdictions with good financial institutions, even after controlling for the critical role of offshore financial centers.

Keywords: ownership, corporate control, multinational enterprises, financial networks, financial institutions, offshore

JEL codes: G32; G34; F23; F36; C63; C71; L14

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1 Introduction

The ownership of firms has become a global and concentrated activity. In 2017, FDI stocks registered a record high ever of 39% of world GDP, and in the same year global cross-border mergers and takeovers amounted to 694 billion dollars (UNCTAD, 2018). When it comes to companies, we know that the top global 500 come from only 33 countries, they generate $30 trillion in revenues and employ about 67.7 million people worldwide (Fortune, 2018).

However, to date, little more is known of the complexity of modern corporations, when interlocking shareholding activity stretches within and between countries. For example, according to rough estimates by UNCTAD (2016), 55% of foreign affiliates around the world may not be controlled directly by headquarters but through ownership chains that run across several national borders, making it difficult for the policymakers the identification of the investors' nationality.

Against this background, we aim to unravel the complexity of global ownership structures through a network approach that provides some useful insights on how corporate control emerges from often convoluted ownership structures. The main intuition is that coordination between a parent company and its subsidiaries requires a concentration of voting rights in interlocking assemblies of shareholders. In this case, management decisions shall be passed along *ownership paths,* where possibly consolidated majorities of votes allow approval of proposals at the shareholding assemblies of subsidiaries in a sequence.

In graph theory, the problem amounts to spot spanning subgraphs from otherwise bigger ownership networks on equity markets, where investors decide both portfolio and direct investment operations. Often the difference between direct investment and portfolio operations is subtle and, in this regard, we provide a probabilistic measure of corporate control.

Therefore, once the full extent of the corporate boundary is revealed, we show how concentrated is global output under a relatively small number of corporations, and how multinational enterprises extend control paths across national borders through chains of subsidiaries. Eventually, we positively test that the emergence of indirect control, hence corporate pyramidal structures, is associated with the institutional environment of the countries that are crossed along the control paths. Countries with better financial institutions are likely chosen as intermediate jurisdictions to coordinate economic activities at a distance, even after controlling for the presence of offshore financial jurisdictions. *Ceteris paribus*, parent countries with poorer financial institutions more likely choose indirect control and pyramidal structures.

The issue is certainly relevant for the taxman because murky internal structures can be exploited for shifting profits where fiscal jurisdictions are all too benevolent (Torslov et al., 2018). Well-known is the hearing before the US Senate (2012), when Microsoft and Hewlett Packard testified as case studies of U.S multinational companies accused of taking advantage of their internal ownership structures to organize transfer pricing among subsidiaries scattered around the world. Eventually, more than 115 countries recently acknowledged the relevance of the issue while supporting the initiative by the OECD and the G20 countries for the implementation of a Base Erosion and Profit Shifting (BEPS) Package (OECD, 2017), which aims at reducing the gaps in national tax rules that allow for profit shifting.

Besides the taxman, complex ownership structures challenge international investment law because, when ownership becomes globally intertwined, attempts by investment agreements to discriminate by investor nationality are frustrated. The issue is also relevant for competition authorities, because screenings of market concentration may be hindered by long ownership chains, when seemingly independent competitors end up being coordinated by the same headquarters under horizontal or vertical integration.

From a practical viewpoint, we show how the problem of unravelling complex ownership structures essentially reduces to a computational problem[1] for the identification of firms' boundaries that

[1] For a discussion on the avenues recently opened to scholars after the integration of computer science with economics, see Blume et al., (2015). Complex network analysis, computational games, applied market design and machine learn-

are plunged into interlocking shareholding activity, to separate what managers and shareholders can actually coordinate from what they cannot. While the simplest boundary is just made of a plant specialized on a single economic activity, modern corporations may collect many divisions, branches and subsidiaries that are coordinated by central headquarters. Different from divisions and branches, subsidiaries typically maintain a legal autonomy, i.e., they are controlled by a parent company through equity stakes that bestow voting rights at a shareholding assembly. When the subsidiaries start to acquire equity stakes in other subsidiaries, then complex ownership structures emerge.

Naturally, the issue touches upon themes that have been studied in various strands of research, although from different perspectives, both theoretically and empirically.

On the one hand, the organization of the firm has been studied without regard to the emergence of complex ownership structures. On the other hand, complex ownership structures have been studied without regard to the organization of a firm's boundary.

The determinants of a firm's boundary have been extensively introduced in relation to property rights theory and transaction cost economics, based on the seminal works by Williamson (1971; 1975; 1985), Grossman and Hart (1986), and Hart and Moore (1990). The crucial questions are still how, why, where and when a firm integrates economic activities that could be otherwise performed among independent firms that exchange goods and services on the market. For some surveys, see Holmstrom and Tirole (1989), Whinston (2001), Joskow (2005) and Aghion and Holden (2011). Additionally, the boundaries of the multinational enterprises have been extensively studied in relation to their geographic outreach: in how many countries they operate and why. For a review, see Antràs and Yeaple (2014), Antràs and Rossi-Hansberg (2009), Helpman et al. (2008), Helpman (2006), Antràs and Helpman (2004), Antràs (2003). More recently, the decision to shape firms' boundaries on fragmented global value chains has been studied in Antràs and Chor (2013), Alfaro et al. (2017), Del Prete and Rungi (2017), Antràs and de Gortari (2017), Fally and Hillberry (2018).

The study of complex ownership structures naturally belongs to the field of corporate finance. La Porta et al. (1999) crucially show how one or more shareholders may have voting rights that are significantly in excess of dividends rights, thanks to an indirect control on companies organized over pyramidal structures. More than often, scholars in this field are concerned with the institutional environments of countries where a group of insiders can expropriate resources from minority shareholders, after the adoption of complex ownership structures (Mathews, 2007; Almeida and Wolfenzon, 2006; Lemmon and Vins, 2003; Faccio and Lang, 2002; Claessens et al., 2000; La Porta et al., 1999; Fama and Jensen, 1983). For example, majority controllers could tunnel and prop financial resources along ownership chains (Riyanto and Toolsema, 2008; Almeida and Wolfenzon, 2006). More recently, Levy and Szafarz (2016) find that cross-ownership linkages may allow managers (not shareholders) to seize indirect voting rights, therefore shielding firms from outside shareholders, in yet another attempt of the agents (in this case, the managers) to escape the control of the principal (in this case, the shareholders). On a similar line of research, Morck et al. (2005) find that indirect voting rights gained through ownership chains often pave the way for a concentration of economic power by a few, who do not make a commensurate capital investment, hence endangering innovation and an optimal allocation of financial resources.

Apparently, the studies on firm's boundaries and the ones on corporate structures do not talk to each other. Indeed, a complete review of the issues at stake would be interesting but beyond the scope of this paper. Neither we are interested in advancing a theoretical framework that jointly explains the emergence of complex ownership structures in relation to firms' boundaries. In fact, ownership chains have been around since the invention of the share-capital companies and are here to stay[2], because

ing are successful stories of contamination among these fields. See also the recent efforts in algorithmic game theory (Roughgarden, 2016).

[2] Eventually, any governance that ends up with a concentration of voting rights contradicts the ideal model of a *modern corporation* sketched by Means (1930; 1931) and in the seminal book by Berle and Means (1932), whose capital shares were supposed to be widely held by small shareholders while relying on accountable professional managers. Instead, corporate personhood allows the single company to enter into property rights contracts and hold stakes in other companies, hence establishing a form of indirect control on economic activities.

they emerge every time a company invests in the share capital of another company. Yet we do not know much about how they look like nowadays. This is the reason why in this contribution we step aside from theory to assess the outreach of global ownership and corporate control through the lens of a network topology, hopefully providing a first and non-exhaustive description of the dimensions along which the complexity emerges in ownership structures.

The basic network topology that we propose is based on a definition of an *ownership space* made of companies and their shareholders, where we can track loops and pyramidal structures along *ownership paths* made of direct and indirect shareholders. Therefore, following a concentration of voting rights over equity linkages, we can assess the outreach of a firm's boundary, i.e., how far a decision taken by the managers of a parent company can go along possibly multiple and overlapping ownership paths.

In its essence, ours is just an equity accounting framework that takes stock of four modes through which corporate control can be exerted in fragmented ownership networks:

i) direct control, when a shareholder holds the absolute majority of equity stakes in another company;

ii) indirect control by transitivity, when a shareholder has direct control of a company that in turn has direct control over another company, in a sequence;

iii) indirect control by a consolidation of equity stakes, when a shareholder is able to control a company by summing up all the minority stakes she holds in her portfolio or in the portfolio of other companies she controls;

iv) dominant shareholding, when a shareholder is able to control a company with a minority stake because other shareholders are too much fragmented to form an opposing coalition.

As we will observe hereinafter, when ownership structures are much fragmented, an additional element of uncertainty is introduced that may not allow the managers and the shareholders to coordinate the economic activities within the full extent of the firm's boundary. In this case, we make use of a probabilistic measure of corporate control, estimated through the Banzhaf (1965) index, which can encompass a variety of governance structures in prominent real-world corporations.

In the end, our network topology usefully provides an algorithm that builds upon previous works, whose original scope was to study the corporate power of agents within already assigned control structures (Levy and Szafarz, 2016; Levy, 2011; Dorofeenko et al., 2008; Chapelle and Szafarz, 2007; Crama and Leruth, 2013; Crama and Leruth. 2007). Contrary to the previous literature, we assume that the boundary of any corporation is unknown, plunged into an *ownership space*, and we want precisely to cut it out.

Therefore, we can finally represent the boundary of a modern corporation as a peculiar graph, a *hierarchy of firms*, made of a parent company and its subsidiaries, all ordered on *hierarchical layers*. The position on the hierarchy is given by the *control distance* between the parent company and each subsidiary, which we define as the minimum number of intermediate *middlemen subsidiaries* that link them through. In other words, the *control distance* is the minimum ownership path that could transmit the management decisions through interlocking assemblies of shareholders.

When we apply our network framework to a dataset of about 53.5 million firms active in 208 countries in the year 2015, we can finally provide a systematic assessment of the heterogeneity in the design of ownership structures and firms' boundaries around the world. As far as we know, ours is the first attempt to apply a big data algorithm for the detection of ownership structures.

First, we detect an intense concentration of economic power by few corporations, because less than 1% coordinate more than 100 subsidiaries, but these are responsible for more than 50% of global sales in our data.Interestingly, we find that indirect corporate control is more common than we expected because *subsidiaries of subsidiaries* represent about 15% of domestic and 54% of foreign companies. More specifically, peculiar cases emerge of foreign affiliates, whose ultimate nationality can be ambiguous:

a) when a company has *multiple passports*, because indirect control paths cross various countries, in 19.1% of our sample of foreign affiliates;

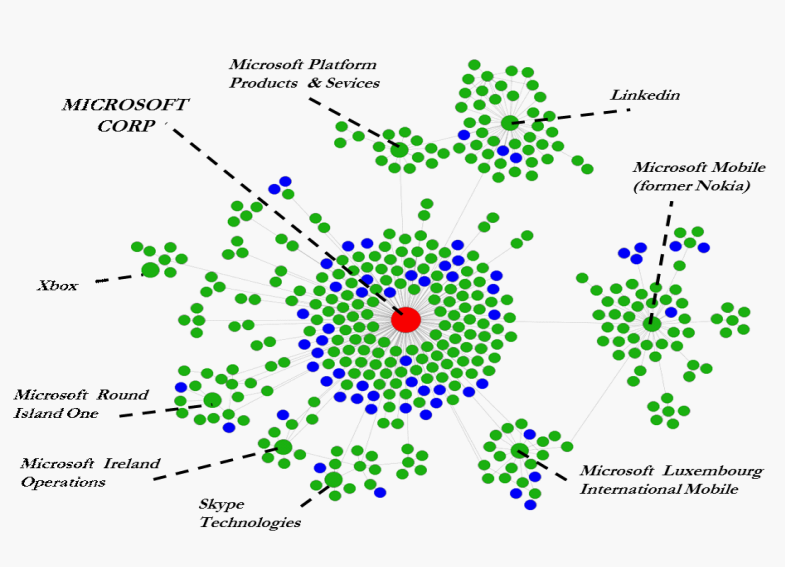
b) when a company is apparently domestic, because we look at the immediate shareholders, but it proves to be *indirectly foreign* once we check for upper layers of ownership, in 24.5% of our sample

of foreign affiliates;

c) when a domestic investor exploits a foreign jurisdiction to reinvest back in her country of origin, starting a so-called *round-tripping* activity, in a non-negligible 1.33% of our sample of foreign affiliates.

Finally, we relate the emergence of indirect *vis à vis* direct control by a parent company to indicators of geography, contractual and financial frictions at the country level. We find that parent companies less likely establish pyramidal structures when they originate in countries with better financial institutions since the latter probably foster more transparent forms of corporate governance. However, *ceteris paribus*, better financial institutions in both the countries of the *middlemen* and of the *final* subsidiaries increase the probability that a parent company establishes indirect control. The latter finding is coherent with the idea that, once vertical structures are established, their operability requires lower frictions along control paths so that management decisions can be enforced from the top of headquarters. These findings are robust after controlling for the role of offshore financial centers, which are also preferred jurisdictions for the establishment of *middlemen* subsidiaries thanks to a lower taxation and a lack of transparency in financial disclosure.

Figure 1: The corporate control network of Microsoft

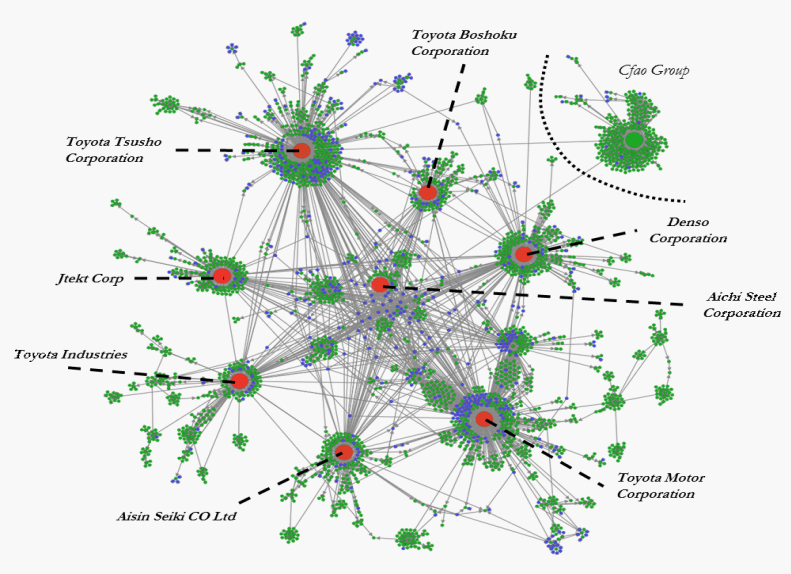


Corporate network of Microsoft Corp visualized with a ForceAtlas2 layout (Jacomi et al., 2014) using Gephi software. A red node indicates a parent company, while green nodes represent majority-owned subsidiaries and blue nodes are affiliates controlled with a dominant stake.

To have an idea of how complex some ownership structures can be, we show two examples extracted from our elaborations in Figure 1 and Figure 2. Microsoft Corp and the Toyota Group are among the top 0.1% *hierarchies of firms* that we detect, in terms of both numbers of subsidiaries and consolidated sales in 2015, and they also rank among the Fortune top global 500 corporations. Nonetheless, their drawings on paper show different corporate structures. Each node of the graphs represents a legally autonomous firm that participates or is participated by other companies. Therefore, the participations to share capital are depicted as directed edges from one firm to another.

Microsoft Corp is a leading IT company providing computer software and consumer electronics since 1975. According to our elaborations, it collects 389 subsidiaries operating in 79 countries in 2015. The second is one of the biggest manufacturers of motor vehicles historically organized as a Japanese

Figure 2: The corporate control network of the Toyota Group



Corporate network of the Toyota Group visualized with a ForceAtlas2 (Jacomi et al., 2014) layout using Gephi software. Red nodes indicate (interlocked) parent companies, while green nodes represent majority-owned subsidiaries and blue nodes are affiliates controlled with a dominant stake.

'keiretsu', i.e., as a conglomerate structure with cross-shareholding exchanges among constituent firms (Berglof and Perotti, 1994). It was born in the 1930s and, according to our elaborations, it collects 2,239 subsidiaries active in 125 countries. Subsidiaries are organized from the top by eight different parent companies.

In the case of Microsoft, most of the *ownership linkages* run in sequence from the parent company to subsidiaries, i.e., the parent directly controls a majority of subsidiaries, which in turn can control other subsidiaries. In some cases, subsidiaries of Microsoft Corp are organized in clusters, for example around Linkedin, Microsoft Mobile (former Nokia) and Skype Technologies. These clusters are actually some former parent companies and their subsidiaries that were acquired time ago by Microsoft, therefore their ownership structure has just been embedded within the hierarchy of the acquirer. Other clusters can be detected around Microsoft Ireland Operations, Microsoft Round Island One, Microsoft Luxembourg International Mobile, Xbox and Microsoft Platform Products and Services, which instead are all born inside Microsoft as financial or technological holdings.

In Figure 2, the Toyota Group has a more sophisticated internal structure with patterns of ownership often crossing each other, as we can expect in a Japanese 'keiretsu', where cross-holdings are a signal of long-term reciprocal commitment, while managers can sit at interlocking assemblies. Exceptionally, the Toyota Group has eight different parent companies on top of its corporate structure, each of them holds a minority albeit dominant stake in the other seven, in a range of about 20-25% of total equity. The case of Toyota seems to be an exception from our elaborations, as it is the most sophisticated structure we can find. A separate cluster of firms is detected around CFAO, which indeed was an autonomous group acquired by Toyota only in 2012 and specialized in the distribution of cars, consumer goods and pharmaceutical products in African countries.

What both these corporate structures have in common? They have to coordinate their economic activity on multiple and overlapping ownership paths, on which a distribution of voting rights can be

detected. Only through a concentration of voting rights they can enforce management proposals at each shareholders' assembly. In fact, the headquarters have to find a way to transmit management decisions through those complex ownership structures, where paths seem often to overlap.

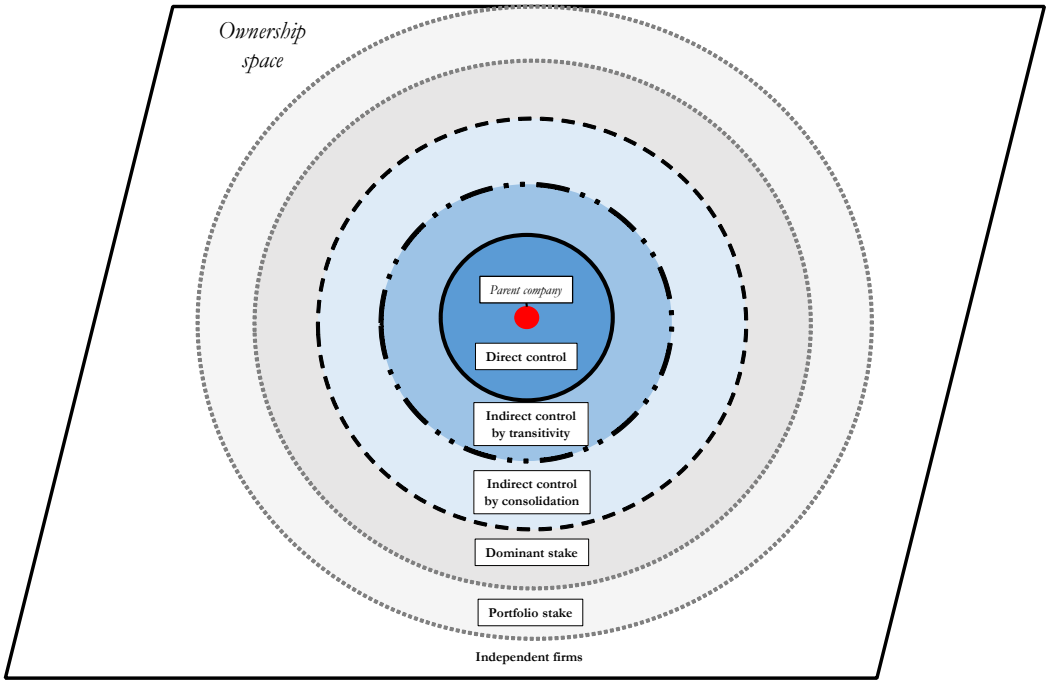
In Section 2 we sketch our basic network framework, then we apply it to real data in Section 3, where we also provide insights on the ownership space and the organization of firms' hierarchies. In Section 4, we show how indirect control relates to the quality of financial institutions and other geographic characteristics. In Section 5 we conclude and point to future lines of research.

2 Firms, ownership and corporate control

We aim at explaining complex ownership structures by introducing a basic network topology, which eventually drives the outreach of a parent company in an *ownership network*, when management decisions have to be transmitted along multiple and often overlapping *ownership paths*.

Before introducing any formal notation, we visualize here our general idea on a fictional map of a firm's boundary, when companies are plunged into webs of equity stakes.

Figure 3: Modularity of corporate control



First, we introduce the notion of an *ownership space* as a common playfield for all equity investors, in which *property rights* in companies are distributed to shareholders. Since *indirect ownership* arises any time a company becomes a (corporate) shareholder of another company, the basic observational unit of our interest is any *ownership path* that connects two nodes in an *ownership network*.

Hence, when ownership structures become much complex, a non-trivial coordination effort is required to agree on a management proposal across interlocking assemblies of shareholders.

In this case, we propose a recursive backward induction solution based on a simple accounting of equity stakes at the company level, which detects a concentration of voting rights over the mapping of all *ownership paths* in an *ownership network.* The result is an identification of a *hierarchy of firms*, made of a parent company that is able to coordinate a set of subsidiaries through an architecture of ownership linkages. Following graph theory, we can say that the *hierarchy of firms* is a spanning

subgraph[3] of the larger *ownership network*, including both parents and subsidiaries together with all the ownership edges that allow for a concentration of voting rights.

Figure 3 illustrates the modularity that we imagine. A first ring of directly controlled subsidiaries is followed by a second ring where the parent company exerts control by transitivity, i.e., through *subsidiaries of subsidiaries*. Then, it is possible that a portfolio of minority stakes, distributed across subsidiaries, can be consolidated by the parent company to obtain a majority of voting rights in another subsidiary. This is what we plot in a third ring in Figure 3.

Finally, we introduce a probabilistic notion of corporate control when no absolute majority can be reached at a corporate assembly. In this case, we are able to plot a fourth and a fifth ring around a fictional parent company. The parent becomes a *dominant shareholder* for companies in a fourth ring, when she or one of her subsidiaries are able to win the consensus on a management proposal in more than half of the possible coalitions among direct shareholders. The parent or any of its subsidiaries have just portfolio stakes in firms included in the fifth ring of Figure 3, at the edge of the *hierarchy of firms*, because minority shareholdings still allow a minimal albeit non-dominant influence in the company's management.

2.1 An ownership space

We can define an *ownership network* as made of companies and shareholders connected through equity linkages. Trivially, networks emerge because companies can in turn become (corporate) shareholders after acquiring stakes in other companies. Thus, formally, a directed network can be sketched by a graph:

$$\mathcal{G}\left(N_{\mathcal{G}}, V_{\mathcal{G}}, K_{\mathcal{G}}, W_{\mathcal{G}}\right), \tag{1}$$

which is made of:

i) a set of nodes, $N_{\mathcal{G}}$, collecting companies and their (individual or corporate) shareholders;
ii) a set of attributes of the nodes, $V_{\mathcal{G}}$, i.e, the equity values of companies[4];
iii) a set of edges, $K_{\mathcal{G}}$, i.e., the ownership linkages running from shareholders to companies, resulting from investment in the equity;
iv) a set of attributes of the edges, $W_{\mathcal{G}}$, i.e., the *property rights* granted to shareholders to vote and collect profits (or losses), in general proportional to the investment in the equity[5].

Around the world, there may be several *ownership networks* disconnected one from the other. The connectivity of the ownership space can be already some useful information on the organization of global ownership. Therefore, we can think of the *ownership space* as a common playfield for all equity investors that participate to the risk of the enterprise. Formally, the *ownership space*[6] is the union of all the *ownership networks*, $\bigcup_{g} \mathcal{G} = \Omega$, where g is the number of separate networks such that:

$$\mathcal{G}\left(N_{\mathcal{G}}, V_{\mathcal{G}}, K_{\mathcal{G}}, W_{\mathcal{G}}\right) \in \Omega\left(N, V, K, W\right) \tag{2}$$

[3] For more details on the notion of spanning subgraphs and its relevance in modern graph theory, see Bollobas (1998).

[4] Alternatively, we can substitute the equity values with any other firm-level indicator, e.g., size, productivity, or financial constraints, according to the peculiar scope of the analysis. In this case, keeping the equity values by company just allows us to better frame the *ownership networks* as the result of decisions by investors on the market for equity.

[5] Please note how special categories of stakes may exist, which can balance voting rights and cash rights in various ways (e.g., privileged capital shares, golden shares, etc.). For the purpose of this paper, a full account of these categories implies only a rescaling of the voting rights among shareholders of a company, without loss of generality.

[6] Going one step further, we can say that the *ownership space*, Ω , is a fully-fledged topological space (Munkres, 2000), because given the collection of open subsets, T, we have: 1) the empty set $\varnothing$ is in T; 2) Ω is in T; 3) The intersection of a finite number of sets in T is also in T; 4) The union of an arbitrary number of sets in T is also in T.

and the sets N, V, K and W are the total nodes, equity values, edges and weights present on a global scale.

In each *ownership network*, the set of nodes, $N_{\mathcal{G}}$, includes the subset of companies ($I_{\mathcal{G}} \subset N_{\mathcal{G}}$) and the subset of their shareholders ($S_{\mathcal{G}} \subseteq N_{\mathcal{G}}$). Since a company can in turn become a (corporate) shareholder, we generally have $I_{\mathcal{G}} \cap S_{\mathcal{G}} \neq \varnothing$. Any single shareholder, $s_i \in S_{\mathcal{G}}$, who has invested in a company, $i \in I_{\mathcal{G}}$, can be either an individual investor, another company, a public authority (e.g., governmental agencies , regional agencies, etc.) or some non-profit organizations (e.g., foundations, trust funds, etc.).

By nature and by human institutions, individual shareholders, public authorities and non-profit organizations cannot be owned by anybody else, i.e. they are the orginal sources of any *ownership network.* More specifically, we call them the *ultimate owners*, $U_{\mathcal{G}} \subseteq S_{\mathcal{G}}$, because they ultimately benefit from the productive activities of all the companies in the network, directly or indirectly, after the distribution of profits (or losses)[7]. In the end, we an *ownership network*, $\mathcal{G}$, cannot exist without *ultimate owners*, such that $U_{\mathcal{G}} \neq \varnothing$.

Across all *ownership networks*, single equity values, $v_i \in V_{\mathcal{G}}$, sum up to the amount of equity capital, E, scattered around the globe, such that $\sum_i v_i = E$. For sake of completeness, we can assume that *ultimate owners,* $U_{\mathcal{G}}$, have null equity values.

In each *ownership network,* the *ownership linkages,* $K_{\mathcal{G}}$, entitle to *property rights,* $W_{\mathcal{G}}$, representing the investment by each shareholder s_i in the ith company. *Property rights* include both the rights to vote at the shareholders' assembly and the right to cash the dividends by each shareholder, $w_{s_i i} \in W_{\mathcal{G}}$, expressed as percentage, $w_{s_i i} \in (0, 1]$.

Eventually, the *ownership space*, Ω, is a perfect partition of all *ownership networks*, such that $\bigcup_{\mathcal{G}} N_{\mathcal{G}} = N$, and $\bigcup_{\mathcal{G}} K_{\mathcal{G}} = K$, but also $\bigcap_{\mathcal{G}} N_{\mathcal{G}} = \varnothing$ and $\bigcap_{G} K_G = \varnothing$.

In a world where ownership is fragmented, a company does not participate in more than one network, and a shareholder does not own stakes outside of it. Obviously, we cannot exclude the possibility that the global *ownership space* can become a unique huge *ownership network* when all companies and shareholders can become seamlessly connected. In the following analyses, after looking at ownership data, we will see that this is not the case, although a huge network component emerges at a global level. More in general, framing the distribution of *property rights* within a companies' *ownership space* allows us systematically studying a cross-country heterogeneity in corporate governance, a granularity of firm-level networks and the connectivity of modern corporations.

2.2 An ownership relationship

In the *ownership space,* we can formally introduce an *ownership (or shareholding) relationship* between companies and shareholders.

After a shareholder acquires a stake in the equity of a company, a general relationship is established, $\mathcal{S} : S \rightarrow I$, such that a direct *ownership linkage* ($k_{s_i i} \in K$) runs from any shareholder s_i to the company i, and the amount of stake a shareholder holds attributes *property rights* in a percent range, $w_{s_i i} \in W = (0, 1]$. Each company, $i \in I$, has at least one *direct shareholder*, s_i, and the sum of *property rights* is equal to $\sum_i w_{s_i i} = 1$. The *ownership in-degree* is $|S_i|$, and it equals the numerosity of the shareholders' assembly.

[7] Please note how our notion of *ultimate owner* can compare to relatively recent legal efforts to identify *ultimate beneficial ownership*. For example, the Financial Action Task Force (FATF) combats money laundering and states that the expressions *"[...] 'ultimately owns or controls' and 'ultimate effective control' refer to situations in which ownership/control is exercised through a chain of ownership or by means of control other than direct control"*. More recently, the EU's Fourth Anti-Money Laundering Directive implemented in 2017 has substantially addressed the issue asking for a continuous update of registries at the country level with information on ownership chains and indirect control. Interestingly, Moulton and van de Ven (2018) systematically review the issue of ultimate ownership in national accounts in times of economic globalization.

For the scope of our analysis, we can define the *ownership relationship* defined on *ownership paths*. On an *ownership path* we may find *successors* and *predecessors*. Any direct shareholder s_i in a company i is a (direct) *predecessor* of that company, and any receiving company is a (direct) *successor* in ownership. When there is more than one *predecessor* for a company i, the shareholder s_{i+m} becomes an indirect shareholder, and her *property rights* can be exerted through a number of *middlemen* (corporate) shareholders.

Formally, we can define the *ownership path* as a finite sequence linking two nodes in the *ownership network*, such that:

$$P_{s_j,i}^{(m)} = (s_{i+m}, s_{i+m-1}..., , s_i, i) \tag{3}$$

where $s_{i+m} \in N$ is a generic *predecessor* of company i. It can be either an *ultimate owner* or a corporate shareholder. The *length* of any *ownership path* is given by m, which is equivalent to the number of *predecessors* in the path. In case of a direct shareholder, the length of the path is simply $m = 1$, thus any *ownership path* of length m can be decomposed exactly in m ownership paths of length 1.

Please note how more complex ownership structures may include many and alternative *ownership paths* linking two arbitrary nodes. In fact, in some cases *ownership cycles* can be found, whose presence is a peculiarity of *ownership networks* that makes the identification of *ultimate ownership* more difficult (Levy and Szafarz, 2016).

We can say that an *ownership network* is *cyclic* when it contains at least one *ownership path* that departs from a (corporate) shareholder and ends back to it. In other words, each company on an *ownership cycle* is at the same time a *predecessor* and a *successor* of itself.

A peculiar cycle is the one when a company buys on the market some of her stakes, usually allowed within some limits and for specific objectives, e.g. to distribute stock options. In this case, we have cases of *self-ownership* with an elementary *ownership path* of length 1 that starts and ends in the same company.

At this point, we can introduce a notion of *ownership distance* between any two nodes[8] as trivially given by the shortest path that links them through:

$$d_{ji} = \min_m \left[P_{s_j,i}^{(m)} \right] \tag{4}$$

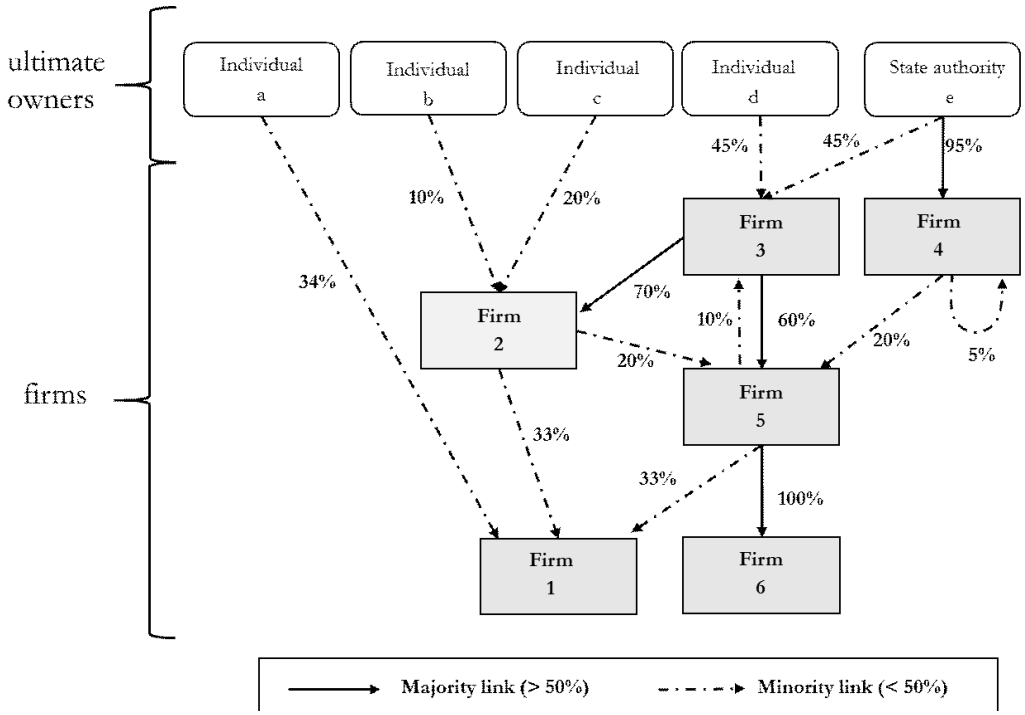
In Figure 4, we illustrate a fictional case to show the reader all the hurdles we may encounter in an *ownership network*, and the utility of the definitions we have introduced until now. Our fictional network is made of six companies, four individuals and a state authority, for a total of eleven nodes. Overall, fifteen *ownership linkages* are generated by investment in the equity of companies, each with *property rights* expressed as percentage. The *ultimate owners* of the graphs are the ultimate beneficiaries of the economic activities throughout the network. We can think of our fictional graph as extracted from the bigger *ownership space*, where other *ownership networks* are present.

Let us consider the *ownership relationship* between shareholders and companies.

Company [1] has two corporate shareholders, [2] and [5], and an individual shareholder, [*a*]. No shareholder in [1] has an outright majority, i.e. each arc has a weight below 50% of voting rights. Company [1] has no successors in ownership and it is at the bottom of any ownership path. Company

[8] Going one step further, defining a distance in an *ownership network* makes the latter a metric space and introduces a network topology. The distance on the single graph, $d : N_{\mathcal{G}} \times N_{\mathcal{G}} \to [0, +\infty)$, satisfies conditions of: 1) non-negativity, $d(s_j, i) > 0$; 2) identity of indiscernibles, $d(s_j, i) = 0 \iff s_j = i$, 3) simmetry, $d(s_j, i) = d(i, s_j)$; 4) triangle inequality, $d(s_j, i) \leq d(s_j, s_i) + d(s_i, i)$.

Figure 4: A stylized ownership network with two corporate boundaries



[2] participates directly in the equity of two different companies, [1] and [5], and it has three distinct shareholders,[*b*], [*c*], and [3], among which company [3] has *direct control*. Company [3] has two individual shareholders, [*d*] and [*e*], who sit in the assembly together with managers appointed by company [5]. Company [4] is directly controlled by a state authority, [*e*], but it also holds some of its own shares in portfolio, producing a case of *self-ownership*, which is reported as an arc that both starts and ends in the same company. Company [5] has three corporate shareholders ([2],[3], [4]). Among them, company [3] holds an absolute majority of voting rights. Companies [5] and [3] engage in cross-holdings. Company [6] has only one direct corporate shareholder [5].

We can now consider a mapping of the *ownership relationship* in Figure 4. There are about 54 possible *ownership paths* connecting any two nodes in the network. For example, company [1] is found at the end of 19 different paths and company [6] at the end of 15 different paths. They may have variable length, for example $P_{c1}^{(2)} = (c, 2, 1)$ and $P_{c1}^{(3)} = (c, 2, 5, 1)$ both connect company [1] with individual shareholder [*c*]. There are four paths with length equal to seven, among them for example $P_{d6}^{(7)} = (d, 3, 2, 5, 3, 2, 5, 6)$. Direct *ownership paths* have always length equal to 1, for example $P_{21}^{(1)} = (2, 1)$. Note how longer paths also contain the *ownership cycles*, as for example company [2] and [5] appear as *predecessors* of themselves in $P_{d6}^{(7)}$. A case of *self-ownership* implies that the company is always considered as a *predecessor* of itself, as for example in $P_{e6}^{(4)} = (e, 4, 4, 5, 6)$. Please note that all *ultimate owners* of the network can in principle claim a direct or indirect ownership in company [1]. The same is not true, for example, for company [6], whose *ultimate owners* are in the subset $\{b, c, d, e\}$.

At this stage, we can already anticipate our intuition of corporate control as from the *ownership network* in Figure 4, which includes a *hierarchy of firms*, made of a parent company [3] and four subsidiaries: [2], [5], [1] and [6]. In other words, parent company [3] concentrates enough voting rights in the network to coordinate the economic activities of a set of subsidiaries. Control is direct on companies [2] and [5]. Interestingly, company [6] is controlled only indirectly, through a *middleman*

[5], i.e. it is a *subsdiary of a subsidiary* once we assume a *transitivity* in corporate control. Instead, company [1] is controlled by [3] only after a *consolidation* of voting rights from first-level subsidiaries [2] and [5], which together sum up to an absolute majority of control (66%) in [6].

In the next sections, we will systematically sketch a *corporate control relationship* in which *transitivity* and *consolidation* of voting rights are two possible forms of indirect control, whose role is peculiar in modern corporations.

2.3 Hierarchies of firms

We can define a *hierarchy of firms* as a corporate structure organized by a parent company that coordinates the economic activity of a set of subsidiaries through the transmission of decisions along *ownership paths*.

From our network perspective, the *hierarchy* is a subgraph plunged into an *ownership network*, where a concentration of voting rights is detected[9]. More formally, we can introduce the *hierarchy of firms*, $\mathcal{C}$, as an element of a *corporate control space*, Λ, which is the union set that includes all the *hierarchies*:

$$\mathcal{C}(N_{\mathcal{C}}, V_{\mathcal{C}}, K_{\mathcal{C}}, W_{\mathcal{C}}) \in \Lambda\left(N_{\mathcal{C}}, V_{\mathcal{C}}, K_{\mathcal{C}}, W_{\mathcal{C}}\right) \tag{5}$$

In a *hierarchy of firms*, we have a set of nodes, $N_{\mathcal{C}}$, including a parent company, $h_o \in N_{\mathcal{C}}$, and its subsidiaries, which are connected through a set of *ownership linkages*, $K_{\mathcal{C}}$. Please note that *ultimate owners* are by definition excluded from the hierarchy.

Obviously, the set of attributes of firms falling under *hierarchies*, $v_i \in V_{\mathcal{C}}$, collects the same invariant equity values of the firms that were present in the *ownership network*, although this time the global equity value may be lower than the total distributed in *ownership networks*, $\sum_{i\in\Lambda} v_i \leqslant \sum_{i\in\Omega} v_i$, because we must exclude independent firms, which are neither subsidiaries nor parents.

Eventually, a *hierarchy of firms,* $\mathcal{C}$, can be considered a *spanning subgraph*[10] of an *ownership network*, $\mathcal{G}$, because it includes a set of its nodes, $N_{\mathcal{C}} \subseteq N_{\mathcal{G}}$, and all the *ownership linkages,* $K_{\mathcal{C}} \subseteq K_{\mathcal{G}}$, that join its constituent nodes, $N_{\mathcal{C}}$.

From another perspective, we can say that a *hierarchy* is characterized by an authority structure in the sense introduced by Gilles (2010). In fact, adapting to the case of firms' ownership, we must have that:

i) a unique parent company, $h_o \in N_{\mathcal{C}}$, controls directly or indirectly a set of subsidiaries;

ii) the parent company has no *superior* in corporate control, i.e. there is no other company that controls the parent;

iii) there is no control cycle in the hierarchy, i.e., no firm is a *superior* of itself.

Briefly, in an authority structure coordination flows always from top to down, in our case from the parent to its subsidiaries. In this sense we use the notions of *superior* and *inferior*, when looking at (corporate) shareholders in an ownership sequence. An authority structure is obviously more hierarchical than a general *dominance structure* (Gilles, 2010) established within a (directed) *ownership network*, where instead we can find *ownership cycles* and a firm can be a *predecessor* (or *successor*) of itself.

[9] In this contribution, we rule out the possibility that corporate control is established on contractual terms, for example between a buyer and a supplier, given special contract terms, or by using privileged equity shares when control rights exceed dividend rights. Of course, having information on these peculiar cases, we could easily extend later applications and give a numerical value to control. For example, privileged voting rights could be weighted for their real power in the assembly of a company.

[10] For more details on the notion of spanning subgraph, its properties and other notions of graph theory here introduced, see Bollobas (1998).

More formally, a *corporate control relationship*, $\mathfrak{h}$, can be defined such that authority flows from the parent company to its subsidiaries, extracting the *hierarchy* from the whole network, $\mathcal{G}$, in which it is plunged:

$$\mathfrak{h} : \mathcal{G}(N_{\mathcal{G}}, K_{\mathcal{G}}, V_{\mathcal{G}}, W_{\mathcal{G}}) \rightarrow \mathcal{C}(M_{\mathcal{C}}, K_{\mathcal{C}}, V_{\mathcal{C}}) \tag{6}$$

Please note that, in principle, there can be more than one *hierarchy of firms* in an *ownership network* because more than one parent company is able to exert control on some subsidiaries. On the other hand, it is possible that no *hierarchy* at all is established in an *ownership network*, because all linkages just represent minority shareholdings that do not allow for a clear coordination. In the next paragraph, we will generalize and extend our framework by introducing a probabilistic notion of corporate control that measures also the impact of minority shareholders. For the moment, we assume that a control relationship is just binary as it uniquely decides whether the parent company is able to implement control on a subsidiary or not.

Before introducing how a *control relationship* works, we need a final step to define any *hierarchy of firms*, $\mathcal{C}$, as a mapping of *ownership paths,* $\mathfrak{h}^{+}$, on which control is actually exerted, in the form:

$$\mathfrak{h}^{+}(h_0) = \left\{ i \in M_{\mathcal{C}} \mid \exists\ P^{(l)}_{h_o,i} = \{h_o, h_{i+l-1}..., , h_i, i\} \text{ in } \mathcal{C} \right\} \tag{7}$$

where h_0 is the parent company and a path has a generic length l. Any generic element h_{i+l-n}, with $n > l$, is an intermediate controller of company i, which we can call *middleman subsidiary.*

The sign of the mapping, $\mathfrak{h}^{+}$, defines the orientation of the relation from *superior* nodes to *inferior* nodes, as we assume that headquarters by parent company is where coordination of management starts. That is, we assume that once a decision is made at the headquarters level, it is then passed along *ownership paths.*

Since there can be more than one *ownership path,* $P^{(l)}_{h_o,i}$, running from a parent to each of its subsidiaries, with variable lengths, we can define the *hierarchical distance* of a subsidiary from the parent company as the minimum length of connecting paths, in the form[11]:

$$d_{h_o i} = \min_{m} \left[P^{(m)}_{h_o,i} \right] \tag{8}$$

where h_o is the parent and i the representative subsidiary.

In other words, a parent company can exploit interlocking shareholders' assemblies once they are hierarchically aligned according to a relationship that we can consider of *partial order*[12], such that $h_0 \succeq h_{i+m-1} \succeq ... \succeq i$. Since there is not such a clear alignment in presence of multiple overlapping paths and cycles, the parent company has to find its way through it and manage to reach any subsidiary taking the shortest route, e.g. passing decisions by phone calls to managers on the shortest *ownership path.* Our definition allows to assume that companies aim at minimizing the communication costs in

[11] Going one step further, please note how the introduction of a *hierarchical distance* makes the *control space*, Λ, a metric space similarly as in the case of the *ownership space*, Ω, which supports it.

[12] According to properties of partial order: i) any company can be a superior of itself (reflexivity: $h_m \succeq h_m$); ii) if a company controls another and the latter control the first, they are on the same hierarchical level (antisimmetry: if $h_m \succeq h_{m+1}$, and $h_{m+1} \succeq h_m$, then $h_{m+1} \sim h_m$); iii) if a company controls another, and the latter in turn has a subsidiary, then the company on top control both (transitivity: if $h_m \succeq h_{m+1}$, and $h_{m+1} \succeq h_{m+2}$, then $h_m \succeq h_{m+2}$). Please note how in this case framing a *partial order* relationship is more useful than framing a *strict order* relationship, because it allows keeping situations when more than one *ownership path* of the same length runs from the parent company down to a single subsidiary. In this case, we can assume that the parent company can choose through which shortest path she can transmit decisions.

passing decisions downstream from the headquarters. Ideally, in a coordinated sequence, a decision starts from the parent company and is enforced first by its immediate subsidiary, which in turn pass it to *subsidiaries of subsidiaries*. If there is any crossroad on the ownership network, the parent company will prefer to take a shortcut and reduce information frictions. The way to reach coordination along control paths is the object of the next paragraphs.

2.4 The corporate control relationship

We have now all the elements to derive the outreach of a parent company in an *ownership network*, i.e. to assess how far a management decision implemented by the headquarters can navigate in interlocking assemblies of shareholders.

From the perspective of the managers of a parent, it is the solution of a coordination problem to transmit management proposals along more or less complex ownership structures. Operationally, it implies winning voting sessions held in sequences at the shareholders' assemblies of all the firms involved along all *ownership paths*[13]. The coordination becomes complicated in presence of multiple and often overlapping *ownership paths*, when stakes are dispersed throughout a network.

For sake of simplicity, we can assume that all shareholders always apply a simple *voting rule* such that a winning proposal requires an absolute majority on a binary choice: *yes* or *no.* If the proposal is approved, then the decision can be transmitted to the next *successor*, if any, along the *ownership path.* The easiest way to exert control is holding an absolute majority of voting rights at the shareholders' assembly. However, the possibility that companies invest in the equity of other companies paves the way for forms of *indirect control* in two ways:

i) by transitivity, when a subsidiary acquires the majority of voting rights in another company;
ii) by consolidation of voting rights, when a majority of voting rights is reached after summing up the stakes that are held by more than one subsidiary, or by the parent company and one or more subsidiaries.

We can start by assuming that there is a perfect information on the global distribution of equity stakes among shareholders, given an ownership matrix, $A^{(0)}$, where each entry corresponds to the equity stake $w_{s_i i}$ held by a direct shareholder, s_i, in the ith firm. Crucially, the managers of a parent company can check the extent of their corporate control by a backward solution, starting from the distributions of equity stakes in each company within the *ownership network* where they operate, excluding on the way to the top all the minority shareholders who do not have an impact on the management decisions[14]. In Appendix Figure B1, we report an exemplification of the corporate control function applied to the fictional network of Figure 4.

2.4.1 Direct control

When a shareholder s_i has an absolute majority in the ith company and $w_{s_i i} > 0.5$, the matrix $A^{(0)}$ transforms[15] in a *direct control matrix* $A^{(1)}$, whose single element is:

[13] For the sake of simplification, we are ruling out agency problems between managers and shareholders, or delegation of authority. That is, we are exclusively interested in the *formal control* exerted through voting rights at the shareholders' assembly. Considering agency problems implies introducing a friction at the meetings of the shareholders' assembly. See for example Levy and Szafarz (2016) for the study of cross-holdings in this context.

[14] Here we follow the spirit of Chapelle and Szafarz (2007), who assess the distribution of control power within already assigned parents and subsidiaries. However, in our case, the extent of the full corporate structure is unknown. Moreover, in our framework a Banzhaf (1965) index is applied after (not before) any consolidation of control, in a more conservative way.

[15] Please note that the superscripts on following matrices, $A^{(m)}$, $T^{(m)}$, $C^{(m)}$, always indicate the length of the *ownership paths* considered before transformations. This notation is also useful to track until which *hierarchical distance* the company is found from a parent company. See eq. 8.

$$a_{s_i i}^{(1)} = \begin{cases} 1 \text{ if } \exists\ s_i : P_{s_i i}^{(1)} = (s_i, i) \text{ et } w_{s_i i} > 0.5 \\ 0 \text{ if } \exists\ s_k \neq s_i : P_{s_k i}^{(1)} = (s_k, i) \text{ et } w_{s_i i} > 0.5 \\ w_{s_i i} \text{ otherwise} \end{cases} \tag{9}$$

Briefly, the matrix $A^{(1)}$ identifies all the *direct controllers* (if any) among direct shareholders who reach absolute majority in a company. A minority shareholder has few chances to have an impact at the shareholders' assembly when another has absolute majority, therefore we can substitute with 0s all the minority stakes when direct control is detected. On the contrary, the weight of an ownership linkage is left unchanged in $A^{(1)}$ when no *direct controller* is spotted.

2.4.2 Indirect control by transitivity

At this point, we can go up one level in the *ownership paths* and check whether a *direct controller* is in turn immediately controlled by another (corporate) shareholder. In this case, we transform the *direct control matrix*, $A^{(1)}$, in a *transitivity matrix*, $T^{(2)}$, whose element is:

$$t_{s_j i}^{(2)} = \begin{cases} 1 \text{ if } \exists\ s_j : P_{s_j i}^{(2)} = (s_j, s_i, i), \text{ when } a_{s_i i}^{(1)} = 1 \text{ et } w_{s_j s_i} > 0.5 \\ 0 \text{ if } \exists\ s_k \neq s_j : P_{s_k i}^{(2)} = (s_k, s_i, i), \text{ when } a_{s_i i}^{(1)} = 1 \text{ et } w_{s_k s_i} > 0.5 \\ a_{s_i i}^{(1)} \text{ otherwise} \end{cases} \tag{10}$$

where s_j and s_k are to be found among all *direct shareholders of direct shareholders*, i.e. considering all the generic paths of length two, $P^{(2)}$, reaching a company i. Please note that the superscripts on matrices, $A^{(m)}$, $T^{(m)}$, $C^{(m)}$, indicate the length of the *ownership paths* considered before transformations.

2.4.3 Indirect control by consolidation of voting rights

At this stage, it is possible that a (corporate) shareholder can consolidate majority stakes exploiting the portfolio of equity held by itself and one or more subsidiaries.

From the perspective of the owned company, it is possible that capital shares are apparently dispersed among direct shareholders, but at an upper level the voting rights are actually coordinate by the same (corporate) shareholder.

Hence, we can build a consolidation matrix, $C^{(2)}$, on the basis of the transitivity matrix, $T^{(2)}$, such that each of its elements is:

$$c_{s_j i}^{(2)} = \begin{cases} 1 \text{ if } t_{s_j i}^{(2)} + \sum_{q:\ t_{s_j s_q}^{(1)} = 1} t_{s_q i}^{(2)} > 0.5 \\ 0 \text{ if } \exists\ s_l \neq s_j : t_{s_l i}^{(2)} + \sum_{l:\ t_{s_j s_l}^{(1)} = 1} t_{s_l i}^{(2)} > 0.5 \\ t_{s_j i}^{(2)} \text{ otherwise} \end{cases} \tag{11}$$

where s_q and s_l are *successors* of s_j and also *predecessors* of i on paths of length one, $P^{(1)}$, i.e. they are both direct shareholders of a firm i.

After a first round when we detected direct control, transitivity and consolidation, we can start iterating consecutively, bottom-up from the ownership of single companies, after setting $C^{(2)} = A^{(2)}$. In this way, we will always look at a generic upper mth level of shareholders to detect a *transitivity*

matrix $T^{(m-1)}$, and then we will look immediately downstream at a generic level $(m-1)$ to detect consolidation of voting rights among *successors* in a *consolidation matrix* $C^{(m)}$.

We finally come up to the point when no further direct control, transitivity and consolidation can be detected, such that $A^{(n)} = A^{(n+1)}$. At the end of the process, the managers of a parent company have full information on the *hierarchy of firms*, $\mathcal{C}$, within which any subsidiary locates at a hierarchical distance, m, given by the shortest *ownership paths,* $P^{(m)}_{h_o i} = \{h_o, h_{i+m-1}..., , h_i, i\}$, which convey management decisions.

2.4.4 A probabilistic measure of corporate control

So far we have not considered minority stakes assuming that they do not have any impact on the management decisions. Yet we may find companies whose ownership is fragmented and no (individual or corporate) shareholder has an outright absolute majority.

Still, it is possible that a *dominant* shareholder can emerge[16], because other shareholders are not equally able to win consensus at the corporate assembly, whereas one shareholder may be able to pool together the necessary voting rights in a winning coalition. This is for example the case of companies quoted at the stock exchange, when an ocean of small minority shareholders is not able to build a coalition, and some core shareholders may have full control.

For all these cases, we propose to extend the notion of *corporate control* adopting a probabilistic measure comprised in a range $[0, 1]$, whose limit value 1 is just the previous case of a unique majority shareholder with a stake above 50% of direct or indirect control, including consolidation of voting rights. Instead, a probability value between 0 and 1 can indicate a variety of distributions of stakes among shareholders, where corporate power is shared.

In fact, as soon as no absolute majority is held by a single shareholder, the control power of any shareholder is a function of not only the amount of the equity stakes held by the same shareholder, but it is also a function of the distribution of equity stakes among all other shareholders.

Let us continue to assume, as before, that an assembly of shareholders, S_i, of company i adopts a majority voting rule to reach consensus on each management proposal.

In this case, the binary outcome of each possible vote in a company i is $r(B_i) \in \{0, 1\}$, where $B_i \subseteq S_i$ is a winning coalition, and the *'yes'* and *'no'* are respectively proxied by 1s and 0s. Conventionally, we can assume that a coalition wins when its proposal is approved (*'yes'*) by the absolute majority of voting rights ($> 50\%$). Therefore, the generic winning coalition B_i emerges if:

$$\sum_{s_i \in B_i} w_{s_i i} \in (0.5, 1] \tag{12}$$

Eventually, the final outcome of voting sessions depends on the overall distribution of weights, $w_{s_i i} \in W_i$, among shareholders. For sake of completeness, we include among possible coalitions also the empty set, when no agreement is reached ($B_i = \{\varnothing\}$), and the *grand coalition* ($B_i = S_i$), when all shareholders agree on a proposal.

In the end, we can measure the *control probability* by each shareholder, $\pi(s_i)$, after the computation of a Banzhaf (1965) index[17] in the form:

[16] Until now we followed international standards for concentration of voting rights above 50% by a unique shareholder, with or without consolidation and transitivity (OECD, 2005). For an example of a more flexible definition in international law, we may refer to the Article 3(2) of the EU Council Regulation (2004), where control is defined as the possibility of exercising *decisive influence* on an undertaking, which can be acquired through purchase of securities or assets or by rights, contracts, or any other means.

[17] The index is sometimes also called the Penrose-Banzhaf-Coleman power index. For previous uses, see the review by Crama and Leruth (2013). Two main power indices are originally designed for horizontal voting models. The alternative is the Shapley value (1953) index, which calculates the fraction of the possible voting *sequences,* in which a shareholder is decisive for the approval. Our preference falls on the Banzhaf (1965) index as it is valid whatever the voting sequence. See also Dubey and Shapley (1979) for an introduction to its mathematical properties. For a previous example of shareholders' power, but using the Shapley value in (non-interlocking) assemblies of shareholders, see Nenova (2003).

$$\pi(s_i) = \frac{1}{2^{|S_i|-1}} \sum_{B_i \subseteq S_i \setminus \{s_i\}} r\left(B_i \cup \{s_i\}\right) - r\left(B_i\right) \tag{13}$$

where $|S_i|$ is the cardinality of the corporate assembly S_i. The *control probability*, $\pi(s_i)$, is in the range $(0, 1]$ and represents the probability that the shareholder s_i exerts the decisive vote after considering all the possible winning coalitions with her, $B_i \cup \{s_i\}$, and without her, B_i.

When a single shareholder holds above 50% of equity stakes, she has *control probability* equal to 1 and we just fall in the case of binary control introduced in previous sections. In this case, any other minority shareholder has a control probability equal to 0. When no shareholder has absolute majority, different distributions of control power can be detected within the company, between 0 and 1, which may be not linear in the distribution of equity stakes[18]. The more fragmented is the ownership among other shareholders, the higher the control probability by one shareholder.

Then, what is the threshold of voting rights after which a stake in a company can be considered influential? In the probabilistic framework like the one given by eq. 13 there is no single answer, although we can reasonably assume that a shareholder is *dominant* when the control probability is higher than 0.5, i.e. her voting rights are determinant in more than half of the possible coalitions at the assembly of shareholders.

Finally, we can consider the notion of *dominant (corporate) shareholder* as just another addition to the corporate control relationship that already includes the notions of direct control, indirect control by transitivity, and indirect control by consolidation of voting rights. In this case, when a parent company and/or one of her subsidiaries is *dominant* at the shareholders' assembly, then that company belongs to the same *hierarchy of firms* but with a probabilistic value in a range $(0.5, 1]$.

Please note how, after including a notion of *control probability*, the number of firms falling in the *corporate control space,* Λ, could increase. Eventually, only independent firms owned by *ultimate owners* or joint ventures[19] are left in the set difference, $N_{\mathcal{G}} - N_{\mathcal{C}}$.

3 Data on global ownership and hierarchies of firms

We source data on global ownership and financial accounts from Orbis, a firm-level database compiled by Bureau Van Dijk that collects original information from a variety of national and international registries, regulatory bodies, companies' annual reports, websites and specialized press[20]. For our purpose, we extract information on shareholding activity for companies active in 206 countries in year 2015.

We end up with a dataset made of 10,294,391 firms that participate in ownership networks out of about 53.5 million companies for which ownership information is present in Orbis. In Figure 5 we report an immediate visualization of the stratification of our sample, before and after the application of the algorithm from eqs. (9) - (11).

We do find that a majority of companies do not participate in any *ownership network* and consists of single traders/proprietorships or other unlimited liability companies, which do not usually have

[18] The literature on voting power indices is full of practical examples that show the non-linearity in the distribution of the decision power when voting rights are fragmented. Here we just show the example of three shareholders with equity stakes 49%, 2% and 49%. At the margin, all shareholders have 1/3 of probability to pass their proposal at the shareholders' assembly.

[19] In the case of joint ventures with equal participation by the shareholders, by design no party is dominant on the other and any power index would also find an equal probability of control.

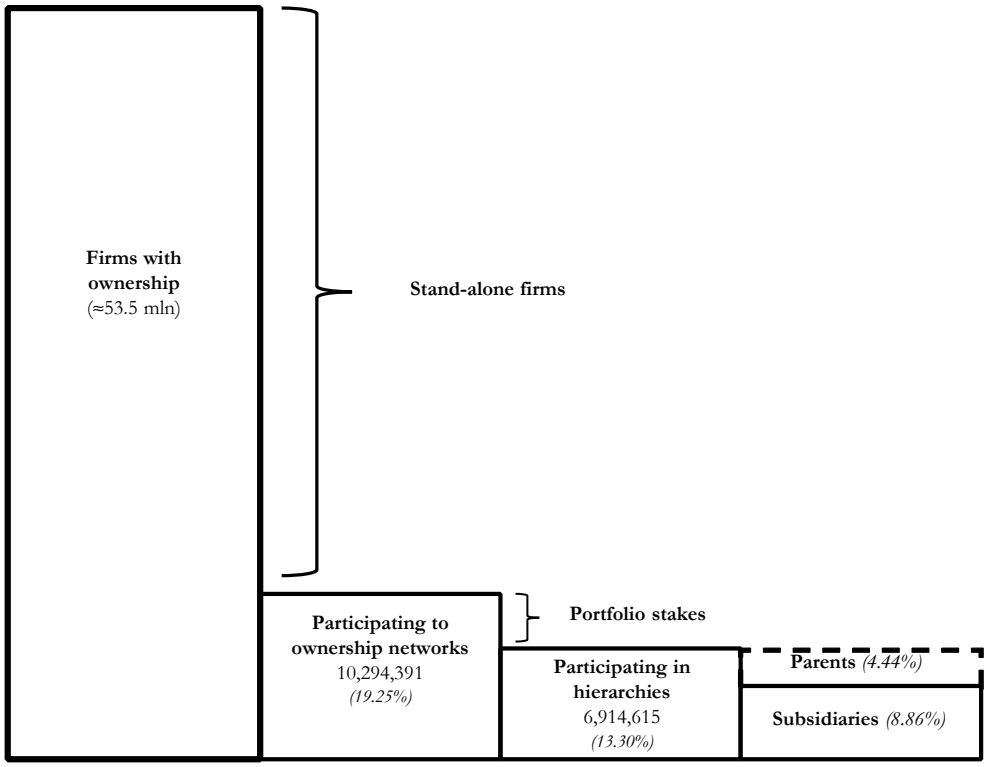
[20] The ownership data present in Orbis are increasingly used in academia for the study of multinational enterprises. For a useful introduction and a discussion of pitfalls for researchers, see Kalemli-Ozcan et al. (2015) and Ribeiro et al. (2010). The OECD currently maintains an AMNE (Activities of Multinational Enterprises) database that is based on ownership from Orbis (see a description by Cadestin et al., 2018). For previous publications using the same data, see Cravino and Levchenko (2017), Alviarez et al. (2017), Del Prete and Rungi (2017).

corporate shareholders and did not invest in the equity of any company. A share of about 38% consists of unlimited liability companies whose ownership structure is so simple that they just have ultimate owners on top and no participation in the equity of other companies.

Indeed, in *ownership networks*, there are only companies that have at least one *ownership linkage* with another company, i.e. participating in an ownership structure where there is at least a corporate shareholder, whatever the amount of the equity stake.

Only after the identification of a concentration of voting rights, as from the previous section, we can spot both parent companies and subsidiaries included in what we define *hierarchies of firms*, after separating about 6% of companies that exchange only portfolio stakes.

Figure 5: From global ownership to hierarchies of firms in our sample.



Please note that our elementary unit of observation is any linkage made of a company and a (corporate or individual) shareholder, therefore in Table 1 we also have a brief look at the various categories of shareholders. On top of any network, we have *ultimate owners*[21], an overwhelming majority of which is made of individuals or families that share the risk of the enterprises. Other *ultimate owners* include non-profit organizations (foundations, trust funds, etc.) and public authorities (government, regional authorities, etc.). If we look at corporate shareholders, we can further separate between industrial and financial companies based on the activity declared by companies and originally present in the data. We find that more than 1.6 million of industrial companies are engaged as investors in the equity of other companies, whereas only 2.57% of shareholders can be classified as professional investors in financial activities, including equity. In other words, the construction of

[21] The ownership data in Orbis come with an internal application that assigns a so-called global ultimate owner (GUO) to companies (see also the discussion by Kalemli-Ozcan et al., 2015). Unfortunately, the application does not exploit the full network information present in the original data, and it just looks at direct shareholding tracks upwards. It excludes *a priori* the presence of multiple ownership paths, on which voting rights can consolidate. According to our framework, a GUO in Orbis can actually be either a parent company, an *ultimate owner* or just another intermediate (corporate) shareholder, depending on the hidden complexity of the company's ownership structure. For this reason, we start from the original shareholding information to reconstruct the entire *ownership space*.

complex ownership structure is not at all a phenomenon generated by financial intermediaries or companies' holdings. Industrial companies around the world systematically hold property rights in other companies.

In Table 2, we report the coverage of our *ownership space* by main geographic areas. Global ownership spans across 206 countries[22], but as expected the European Union and the United States are the most covered regions, with about 3.9 million firms and 3.7 million firms, respectively. Indeed, together with Japan, these are the countries where equity markets are much sophisticated. However, all world continents are well represented, including Africa, Asia, South America and Oceania. Interestingly, firms of any size are engaged in shareholding activities[23], and smaller firms represent 86.5% of the entire dataset.

Unfortunately, there is no official source against which we could validate the coverage of our sample, because by now no other institution has been able to standardize ownership information on a global scale. As a matter of fact, information on shareholders seems to be the most compulsory at the firm level, because dividends are an important fiscal base of national taxations, whereas more details on financial accounts are often on a voluntary basis when firms fall below some size thresholds, to alleviate red tape burdens. We find that only about one-third of the companies reported in Table 2 also reports detailed balance sheet information, potentially leading to some sample selection bias in analyses concerning financial accounts. In fact, developed countries seem much more able to set up reliable national registries. On a case-by-case basis, we will make use of robustness checks against a possible geographic or firm-size bias.

Table 1: Companies participating to corporate control networks. Geographic coverage and multinational status.

Shareholder category	**Frequency**	*%*
Ultimate owners		
Individuals/families	11,023,436	*83.61*
Non-profit organizations	133,216	*1.01*
Public authority	17,321	*0.13*
Corporate shareholders		
Industrial companies	1,671,415	*12.68*
Financial companies	339,058	*2.57*
Total	**13,184,446**	*100*

In Table 3, we report the geographic coverage of our sample after we identify *hierarchies of firms*, hence excluding minority shareholdings.

The practical computation of eqs. (9) - (11) on original ownership data sourced from Orbis is made by implementing a C++ code, following basic principles of object-oriented programming.

[22] Please note that the database also includes many 'depedencies' separately reported, e.g. Hong Kong, Puerto Rico and British Virgin Islands, which have some degree of autonomy from another sovereign country, China, the United States and the United Kindgom. Following membership to the United Nations, only 193 are fully-fledged sovereign states.

[23] A combination of criteria classifies companies by size in Orbis: A) Large or very large companies report more than 10 million EUR revenues, or more than 20 million EUR total assets, or more than 150 employees, or over 0.5 million EUR capitalization, or they are listed at a stock exchange; B) Medium-sized companies register more than 1 million EUR revenues, or more than 2 million EUR total assets, or more than 15 employees, or over 50 thousand EUR capitalization; C) Small companies includes the residual, i.e. they are not in the medium or in the large and very large categories.

Table 2: Companies participating to ownership networks. Geographic coverage and firm size.

Economy	Small	Medium	Large	Total
European Union	2,920,807	713,176	323,924	**3,957,907**
of which:				
Germany	*536,578*	*138,180*	*50,170*	*724,928*
France	*101,064*	*76,576*	*38,558*	*216,198*
United Kingdom	*437,778*	*77,485*	*68,046*	*583,309*
Italy	*203,179*	*96,872*	*32,245*	*332,296*
Spain	*133,862*	*48,305*	*24,021*	*206,188*
United States	3,620,543	45,067	62,494	**3,728,104**
Russia	437,180	105,822	34,367	**577,369**
Other Europe	228,161	64,132	20,802	**313,095**
Asia	553,644	116,435	131,578	**801,657**
of which:				
Japan	*118,195*	*27,931*	*30,774*	*176,900*
China	*99,942*	*10,874*	*34,595*	*145,411*
India	*17,032*	*4,738*	*8,990*	*30,760*
Africa	39,750	2,151	4,475	**46,376**
Central and South America	121,580	20,159	17,752	**159,491**
of which:				
Brazil	*8,631*	*9,534*	*5,365*	*23,530*
Argentina	*5,358*	*1,174*	*1,432*	*7,964*
Mexico	*12,994*	*1,754*	*2,627*	*17,375*
the Caribbean countries	*41,662*	*141*	*1,676*	*43,479*
Australia	381,878	48,311	10,211	**440,400**
Rest of the world	606,339	21,300	17,473	**645,112**
TOTAL	**8,909,882**	**1,136,553**	**623,076**	**10,294,391**

In the end, numbers by country/area slightly change with respect to Table 2. Now we have 2,369,892 parent companies and 4,740,352 subsidiaries in Table 3, where both parents and affiliates are classified according to the hosting economy, i.e. the country where they operate. Therefore, parent companies are reported first overall and then considering only multinational enterprises (MNEs) originated in the country/area, in the first half of Table 3. In turn, subsidiaries are reported first overall and then considering only foreign status, in the second half of Table 3. We consider a parent company as leading a multinational enterprise if it controls at least one foreign subsidiary that is located in a country different from the origin country.

In our sample, the United States host by far the highest numbers of both parent companies (60.56%) and subsidiaries (45.10%). The United Kingdom and Germany are respectively the second and third country hosting the highest number of both parents (2.45% and 2.49%) and subsidiaries (4.14% and 6.02%), at great distance from US numbers. Asia as a whole is the third area where *hierarchies of firms* operate, although hosting only 4.45% of total parents and 6.67% of subsidiaries.

When we look at MNEs only, we further separate multinational enterprises among *hierarchies of firms*. As expected, in the European Union we detect a higher share of multinational enterprises (55.41%) than in US, essentially for an accounting issue, because there is a high level of integration at

the European continental level, after companies can locate activities intra-EU thanks to a relatively high freedom of capital movements. The EU has developed a quasi-internal market among albeit sovereign countries, whereas we certainly do not consider investing from one US state to another as a multinational operations.

Eventually. we detect 752,355 foreign subsidiaries around the world, i.e. about 15.8% of the total. More than half of them are located in the European Union. Asia is the second continent with the highest number of foreign affiliates (99,624), 36% of which is hosted by China. Among Latin American countries, a noteworthy 2.4% of world share of foreign affiliates is attracted by Caribbean countries, especially in British Virgin Islands, Bahamas and Curacao, well in excess of their economic weight. As these are also often considered offshore countries chosen for lack of ownership and financial disclosure, they will be the object of further investigations in the next sections.

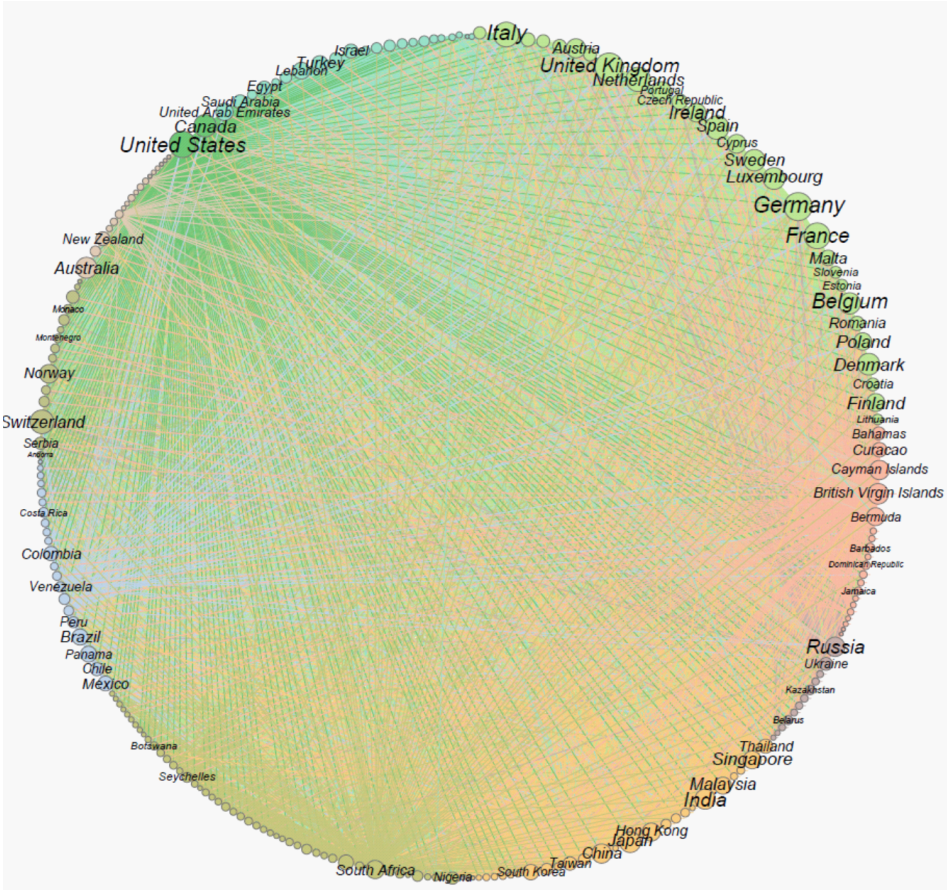
Table 3: Companies participating to hierarchies of firms. Geographic coverage and multinational status.

	Parent companies				Subsidiaries			
Hosting economy	**All**	**%**	**Multinational**	**%**	**All**	**%**	**Foreign**	**%**
European Union	600,829	25.35%	111,522	55.41%	1,625,508	34.29%	387,006	51.44%
of which:								
Germany	*58,969*	*2.49%*	*11,261*	*5.59%*	*196,426*	*4.14%*	*64,405*	*8.56%*
France	*31,389*	*1.32%*	*6,650*	*3.30%*	*96,749*	*2.04%*	*26,631*	*3.54%*
United Kingdom	*58,138*	*2.45%*	*12,361*	*6.14%*	*285,286*	*6.02%*	*62,818*	*8.35%*
Italy	*40,555*	*1.71%*	*8,680*	*4.31%*	*88,091*	*1.86%*	*19,357*	*2.57%*
Spain	*34,600*	*1.46%*	*5,530*	*2.75%*	*103,454*	*2.18%*	*21,823*	*2.90%*
United States	1,435,218	60.56%	22,511	11.18%	2,138,025	45.10%	63,220	8.40%
Russia	29,741	1.25%	974	0.48%	110,232	2.33%	50,541	6.72%
Other Europe	36,073	1.52%	14,089	7.00%	84,045	1.77%	22,441	2.98%
Asia	105,449	4.45%	19,142	9.51%	316,014	6.67%	99,624	13.24%
of which:								
Japan	32,526	1.37%	3,259	1.62%	82,316	1.74%	5,214	0.69%
China	11,048	0.47%	2,995	1.49%	83,311	1.76%	35,983	4.78%
India	3,516	0.15%	1,501	0.75%	14,971	0.32%	6,775	0.90%
Africa	5,102	0.22%	4,169	2.07%	30,346	0.64%	17,088	2.27%
Latin America	30,058	1.27%	18,247	9.07%	83,227	1.76%	51,693	6.87%
of which:								
Brazil	2,905	0.12%	342	0.17%	15,960	0.34%	9,443	1.26%
Argentina	600	0.03%	126	0.06%	3,907	0.08%	3,059	0.41%
Mexico	1,356	0.06%	322	0.16%	12,728	0.27%	8,132	1.08%
the Caribbean countries	13,023	0.55%	12,662	6.29%	21,684	0.46%	18,049	2.40%
Australia	58,788	2.48%	2,771	1.38%	136,189	2.87%	14,750	1.96%
Rest of the world	68,634	2.90%	7,847	3.90%	216,766	4.57%	45,992	6.11%
TOTAL	**2,369,892**	**100.00%**	**201,272**	**100.00%**	**4,740,352**	**100.00%**	**752,355**	**100.00%**

3.1 Preliminary evidence on global ownership

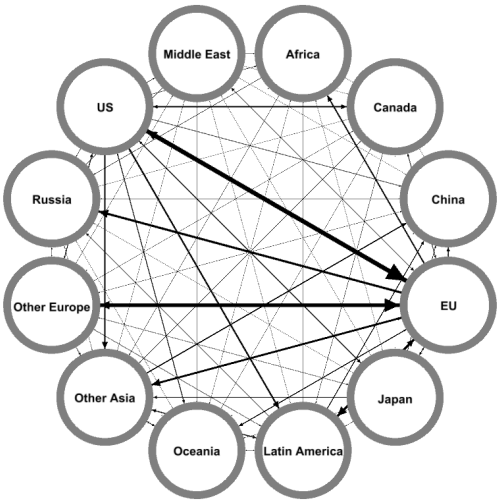
In Figure 6, we report a visualization of global ownership on a ring layout, after aggregating firms within countries. Bigger nodes indicate a higher weighted outdegree, i.e. a higher number of ownership linkages with the rest of the world as weighted by the percentage of property rights they entitle. Therefore, linkages are colored according to the continent to which origin countries belong and directed through the *ownership space* to reach other countries ordered on the ring. As expected, major country players in global ownership are the United States and European countries. Weighted outdegree values for top country players are reported in Table 4.

Figure 6: Country-level ownership space



The active role of shareholders from the United States and the European countries is evident and largely expected. Members of the European Union are also considerably integrated through ownership linkages. Less intuitive is the weight that the Caribbean have on equity markets. British Virgin Islands, Cayman Islands, Curaçao and Bermuda are overseas territories and dependencies of European countries that offer minimal tax liabilities and poor external disclosure to foreign investors. Their role in global equityis certainly disproportionate in relation to their economic size, in part because investors use these countries to control economic activities elsewhere. Similar observations can be valid for Taiwan, Hong Kong and Singapore in Asia, or for Luxembourg and Cyprus in Europe. Only when we look at the concentration of voting rights within ownership networks, as from the next paragraph, we are finally able to track down who is actually controlling economic activities through *middlemen* legal entities in these countries.

Figure 7: Main global regions and the ownership space.



In Figure 7, we further aggregate firm-level ownership linkages within main world regions to observe intercontinental integration, where the size of the arcs and their arrows indicate the magnitude of the weighted outdegrees. The relationship between the United States and the European Union strikes for its relative strength, although the direct investment from the US is dominant in the bilateral exchanges. In general, shareholders from the United States have important equity stakes in all main world regions, while countries in Africa and South America are the less connected regions. Switzerland, Norway and the Balkan countries are the main origin and destination of linkages between the European Union and the rest of Europe.

In the first column of Table 4, we are able to quantify the role of top 25 countries across the *ownership space.* We make use of a measure of eigenvector centrality (Bonacich, 1986) at the country level, which assesses the influence of a node based on the centrality of its neighbors. It has become a quite common tool for application to network data, including economic and financial networks (see Battiston et al., 2012, or Rungi and Fattorini, 2018). Unexpectedly, Germany emerges as a hub in the *ownership space*, given also the country position at the center stage of the European economic integration. Other European Union members are included on top of the list of Table 4, besides the well-known financial hubs of the United States and the United Kingdom. In the Appendix Table XXX, we repeat the exercise including only financial companies before aggregation at the country level, and we find again most of the offshore financial centers (British Virgin Islands, Cayman Islands, Bermuda) among the top 25 countries in the world.

3.2 Preliminary evidence on hierarchies of firms

The main advantage of adopting our network perspective is the possibility to observe full ownership paths often running through several firms located in different countries. In the end, a parent company can be linked with each of its subsidiaries in many ways. As far as we know, this information has never been explored before. The simple fact that *middlemen* subsidiaries are used to exert corporate control on other more downstream subsidiaries has been neglected until now.

In particular, we want to provide some first insights on the characteristics of control paths through which parent companies decide to invest to control a final subsidiary. Obviously, we rely on our framework, and specifically on eq. (8), to attribute a *hierarchical distance* between a parent and each of its subsidiaries, i.e. the length of the shortest path within the *hierarchy of firms.*

Therefore, in Table 5, we plot *hierarchical distances* also separating domestic and foreign subsidiaries, i.e., controlling for the origin country of the parent company. An overwhelming majority of

Table 4: Centrality of countries in ownership - all firms.

Country	Eigenvector centrality	Rank	Weighted degree	Weighted indegree	Weighted outdegree
Germany	1.000	1	3520070	1778547	1741523
United Kingdom	0.928	2	1133540	558974	574566
Italy	0.894	3	1811298	895715	915583
United States	0.891	4	5020248	2457440	2562808
Russian Federation	0.869	5	1483438	768504	714934
Australia	0.835	6	1519408	769871	749537
France	0.810	7	452903	209647	243256
Cyprus	0.797	8	99440	40084	59356
Netherlands	0.793	9	736932	355217	381715
Romania	0.792	10	204113	120511	83602
Luxembourg	0.761	11	91009	43758	47251
Czech Republic	0.754	12	159879	88137	71742
Austria	0.752	13	348713	172049	176664
Spain	0.741	14	476102	234363	241739
Switzerland	0.739	15	152359	52662	99697
Poland	0.722	16	175255	98303	76952
Canada	0.720	17	229778	115721	114057
Ukraine	0.713	18	138490	72371	66119
Denmark	0.700	19	352103	167873	184230
China	0.698	20	250885	147345	103540
Portugal	0.668	21	153750	79946	73804
British Virgin Islands	0.661	22	39440	14922	24518
Bulgaria	0.653	23	367441	185931	181510
India	0.645	24	85325	47922	37403
Arab Emirates	0.634	25	20585	10938	9647

subsidiaries (78.6%) are located at just distance one from the parent company. Of course, most of the simplest hierarchies made of only one parent and one subsidiary fall in this category. However, *middlemen* subsidiaries located on indirect control paths represent a non-negligible 21.5% of the total.

Interestingly, indirect control is much more likely to emerge in the case of foreign subsidiaries (53.7%), when a *hierarchy of firms* becomes a multinational enterprise. That is, the majority of foreign subsidiaries is controlled by at least one other *middleman* subsidiary located upstream and finally controlled by the parent company. The chain of control can become longer until more than 10 hierarchical layers inserted between a parent and the manager of a subsidiary for a bunch of about 2,132 companies in our data[24].

The main issue that we consider relevant in this paper is the possibility that *middlemen* subsidiaries are located in different countries, when the control paths extend along different financial and contractual jurisidictions[25].

In Figure 8, we start by assessing the relevance of peculiar cases of foreign subsidiaries, when the identification of the ultimate nationality may not be trivial. The problem is well known by international agencies promoting standards for foreign direct investment (OECD, 2005; UNCTAD, 2009). Such peculiar cases may have important implications at the moment of drafting investment policies that try to discriminate companies based on the investor nationality (UNCTAD, 2016).

[24] As an example of extremely long control path, we report the round-tripping case of Europarks U.K. Limited, which is located at hierarchical distance twenty from its parent company at the end of 2015. In fact, its control path crosses the border twice, in Luxembourg and in UK, where we also found its parent company Bottom-up. In between, we have: Europarks U.K Limited - Europarks Limited - National Car Parks Limited - National Parking Corporation Limited - NCP Holdings Limited - National Car Parks Group Limited - Pointspec Limited - Statusaward Limited - Trendcycle Limited - Primepanel Limited - PIHL (2003) Limited - Oval (2041) Limited - Oval (2042) Limited - Parking International Holdings Limited - Parking Holdings Limited - MEIF II CP Holdings 3 Limited - MEIF II CP Holdings 2 Limited - MEIF II CP Holdings 1 Limited - MEIF II CP Holdings SARL - Macquarie European Infrastructure Fund II.

[25] In a recent document, OECD (2015) suggests reporting separately the case of special purpose entities (SPEs), which channel investments (i.e. as *middlemen*) through several countries. Hence, SPEs are defined as entities that have little or no employment, physical presence, or operations in a country, while holding assets and liabilities or raising capital for the multinational enterprise. Actually, a cursory glance to our data suggests that only few *middlemen* subsidiaries may be alleged SPEs, while most have non-negligible employment and may be actively engaged in economic activities in their hosting countries.

Table 5: Hierarchical distance of domestic and foreign subsidiaries from parent company

Hierarchical distance	Domestic subsidiaries	%	Foreign subsidiaries	%	All companies	%
1	3,223,182	84.991%	347,959	46.249%	3,571,141	78.578%
2	376,511	9.928%	186,062	24.731%	562,573	12.379%
3	123,021	3.244%	104,063	13.832%	227,084	4.997%
4	43,079	1.136%	55,414	7.365%	98,493	2.167%
5	15,354	0.405%	28,135	3.740%	43,489	0.957%
6	5,934	0.156%	14,182	1.885%	20,116	0.443%
7	2,518	0.066%	8,132	1.081%	10,650	0.234%
8	1,321	0.035%	3,765	0.500%	5,086	0.112%
9	600	0.016%	2,104	0.280%	2,704	0.059%
10	268	0.007%	987	0.131%	1,255	0.028%
> 10	580	0.015%	1552	0.206%	2132	0.047%
Total	3,792,368	100.000%	752,355	100.000%	4,544,723	100.000%

As from our elaborations, we find that 24.51% of foreign subsidiaries are only *indirect foreign*. That is, they belong to an *ultimate* parent company abroad, but indirect control is exerted through at least one domestic *middleman* subsidiary. We report the estimates of this case from our data in the first panel of Figure 8.

On the second panel of the same figure, we sketch the case of companies with *multiple passports*[26], i.e. subsidiaries whose control path involves more than one country before ending in the country of origin of the parent. They account for 19.16% of the total of foreign affiliates in our data.

A third peculiar case is reported in panel c) of Figure 8. A *round-tripping* investment occurs when an investor brings capital abroad in an incorporated company, and then she exploits that foreign company to control a company located in her country of origin. From our data, only 1.33% of foreign subsidiaries can be considered *round-tripping* operations, although it is possible that we underestimate these cases from our data, because we exclude the possibility that on top of the control chain can be an individual investor, who is more difficult to track and whose identity is less likely disclosed in official registries. Main factors explaining round-tripping may include the possibility to exploit offshore advantages in tax heavens, as well as scant protection of property rights in the origin country, which requires shielding capital abroad. On the latter line of research, see Ledyaeva et al. (2015), who study the case of Russian investors.

3.3 Corporate control and economic entrenchment

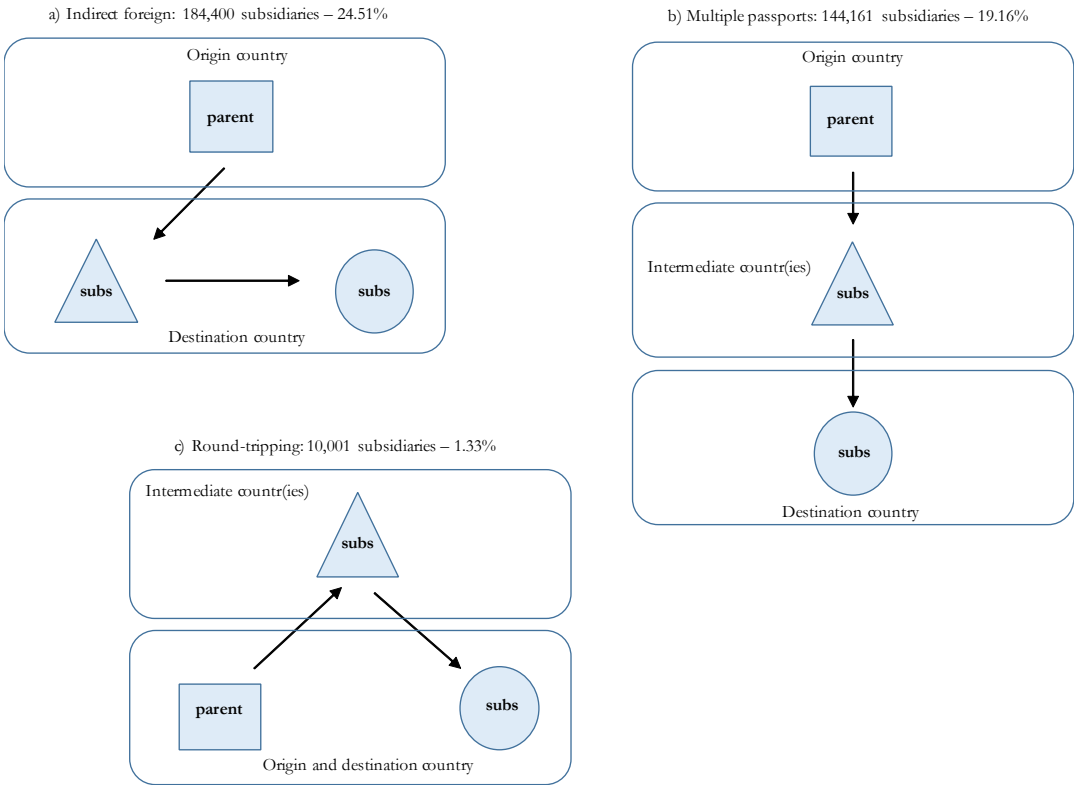
After we explored some very preliminary evidence on the organization of the *ownership space* and about the characteristics of the control paths, in this section we provide an elementary insight on how much concentrated is global output under *hierarchies of firms.*

In this regard, we argue that the full extent of the corporate boundary is a dimension that is too often missing in previous studies, mainly for lack of viable data, although its disclosure can reveal otherwise unobserved market concentration and market power. The statistical unit that is usually observed is the single firm unplugged from its ownership network. At best, a single subsidiary is often observed as in connection with its parent company, in a binary relationship, without regard to the full extent of the set of co-subsidiaries within the same *hierarchy of firms.*

Scant literature has observed, as Baker et al. (2002), that network-like forms of firms' organizations are relevant as they can simulate market-like environments, but with some important differences. For example, subsidiaries maintain formal property rights on their production assets, but they can

[26] Extracting randomly from our data, consider the case of Safilo Optical Sdn Bhd in Myanmar. It is finally controlled by the Safilo Group SpA in Italy, although we find *middlemen* first in Singapore and then in Hong Kong.

Figure 8: Peculiar cases of foreign affiliates: indirect foreign, multiple passports, and round-tripping investment



exchange inputs at prices that are coordinated by the parent company, as in the case of transfer pricing, which optimizes tax base. On the other hand, a parent and its subsidiaries can internally relocate financial resources at more favorable conditions than external financing, via the development of so-called internal capital markets.

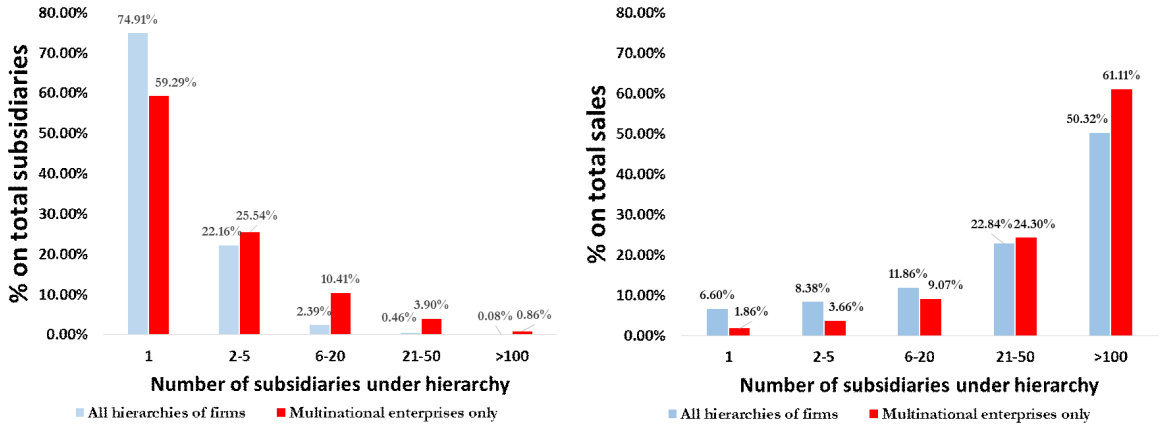
In Figure 9, we start assessing the size distribution of our *hierarchies of firms*, measured in terms of the number of subsidiaries, and check how much output they can command, given their size. Looking at the first panel on the left, we observe that the majority of these objects has very simple structures, since we count 74.91% of *hierarchies* made of only two firms, one parent and a single subsidiary (59.29% in the case of multinationals). Actually, only about 0.08% of the parent companies control more than 100 of foreign or domestic subsidiaries. Multinational enterprises have on average more subsidiaries.

However, once we look at the right panel of Figure 9, we find that the skewness of the distribution is reversed when we consider aggregations of firm-level output. In the end, there are just about 2,000 *hierarchies of firms* that command more than half of the sales on a global scale[27], and the degree of concentration is relatively higher within the sample of multinational enterprises (61%). On the contrary, smaller hierarchies with one subsidiary collect just 6.6% of the total.

In the end, we can provide accurate estimates of the overdispersion in the distribution of *hierarchies of firms*, as plotted in Figure 10, assuming that it resembles a negative binomial distribution on count

[27] Sensitivity analysis shows that half of the global output ranges between 0.08% and 1% of sample hierarchies. Such variation is due to missing values on turnover, which is present in less than half of the subsidiaries in the sample. We may assume that smaller firms, in smaller networks, can affect our findings on concentration. Nonetheless, our descriptive finding is robust to the exclusion of all corporate networks where information on turnover is not complete. We further try to reconstruct the size distribution on the left panel of Figure 9 using only companies with non-missing value on turnover. The magnitude of output concentration falls in the same range of values.

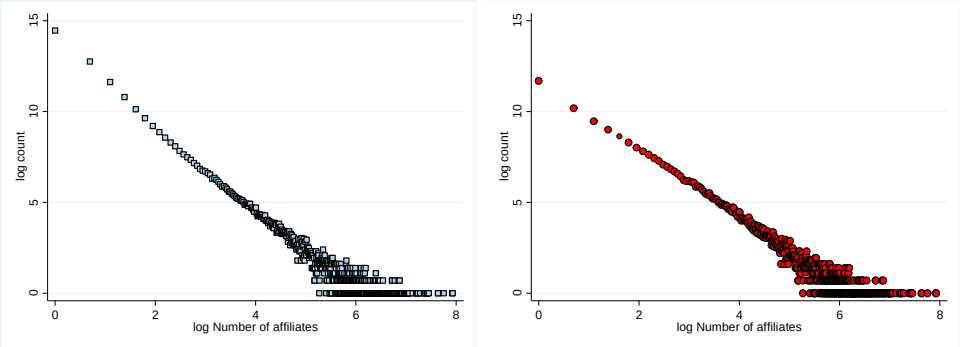
Figure 9: Distributions of subsidiaries and global output under hierarchies of firms.



data. In particular, we may be interested in the dispersion that such a distribution entails. For mroe details, see Cameron and Trivedi (1986).

On the x-axis, we report the (log of the) number of its subsidiaries, while the (log of the) count of how many hierarchies have that same size is reported on the y-axis. If we assume that Figure 10 underlies a negative binomial distribution, we estimate that (over)dispersion is equal to 1.58 for all hierarchies in the first panel, and it is 2.89 for multinationals in the second panel. In either case, we have significant thick right tails that point to the existence of a scale-free network (Choromanski et al., 2013), whose outdegree distribution follows a power law, at least asymptotically.

Figure 10: Overdispersion in distributions of subsidiaries under hierarchies.



a) All hierarchies of firms (2,369,892). We estimate dispersion (1+alpha) parameter 1.58 assuming a negative binomial distribution.

b) Multinational corporate networks (201,272). We estimate dispersion (1+alpha) parameter 2.89 assuming a negative binomial distribution.

4 Financial institutions and indirect control

Ultimately, the emergence of indirect control and pyramidal structures has been studied in relation to the financial environment in which companies originate (e.g., Mathews, 2007; Almeida and Wolfenzon, 2006; Lemmon and Vins, 2003; Faccio and Lang, 2002; Claessens et al., 2000; La Porta et al., 1999; Fama and Jensen, 1983).

As far as we know, ours is the first study that systematically investigates also the role of intermediate jurisdictions through which pyramidal structures eventually run.

We organize econometric investigations as follows. First, we test the emergence of indirect *vis à vis* direct control on a subsidiary, as the first choice is the way parent companies start organizing pyramidal structures. In particular, we want to assess whether a parent's choice is driven by institutional environment and geography.

Second, once pyramidal structures emerge, we test whether institutions and geography matter for running control paths between a specific couple of countries of origin and final destination. Finally, we test the choice of intermediate countries, as jurisdictions where *middlemen* subsidiaries are located.

4.1 Emergence of indirect control

We assume that a parent company can choose either to establish direct control over a subsidiary or to coordinate the latter through one or more *middlemen* subsidiaries, i.e. through one or more *predecessors* in corporate control, following eq. 8. Eventually, if indirect control is chosen, a pyramidal structure emerges on which subsidiaries are coordinated by the parent company. Accordingly, we test the following probit model:

$$Indirect_{h_0(c_o)i(c_i)} = \beta_0 + \beta_1 X_{c_o} + \beta_2 X_{c_i} + \beta_3 Z_{c_o c_i} + \beta_4 F_{h_0} + \beta_5 F_i + \varepsilon_{h_0(c_o)i(c_i)} \tag{14}$$

where the dependent variable is binary and equal to one if the parent company chooses indirect control, and zero otherwise. The sets of regressors, X_{c_o} and X_{c_i}, include characteristics of the country of origin of the parent and of the subsidiary, respectively, with a focus on financial and contractual institutions. Among them, both metrics of financial development and contract enforcement are sourced from the World Bank's Development Indicators, as relative to year 2015. The first is the value of domestic credit provided to the private sector as a percentage of GDP. The second is the cost required to to enforce a contract through the courts, as a percentage of the claim. Both measures have been used frequently in other works to assess the quality of financial institutions (see, for example, Acemoglu et al., 2009, and Rajan and Zingales, 1998). Specifically, we control for the entry cost required to start a business in the country of a subsidiary, as a percentage of income per capita. Further, we include tax rates on commercial profits and levels of GDP for both countries of parents and subsidiaries. Additionally, the set $Z_{c_o c_i}$ collects bilateral gravity controls sourced from Head et al. (2010) and Head and Mayer (2013), updated to year 2015, with a specific focus on geography, commercial agreements and common institutions between a parent's and each subsidiary's locations. Firm size categories[28] (small, medium, large, very large) are included in F_{h_0} and F_i. Based on the activity of the parent, we further separate industrial and financial groups, the latter including banks, insurance companies, private equity, mutual and pension funds. Standard errors are clustered by parent company. We report nested results in Table 6.

[28] See note 17 above for details on how categories are originally drawn in the Orbis database.

Table 6: Emergence of indirect control

Dependent variable:	**(1)**	**(2)**	**(3)**	**(4)**
Indirectly controlled$_{h0(c0)i(ci)}$	**All**	**All**	**Industrial group**	**Financial group**
Financial development$_{c0}$	.0006**	-.0008***	-.0007***	-.0001
	(.0002)	(.0002)	(.0002)	(.0006)
Financial development$_{ci}$	.0031***	.0026***	.0030***	.0005*
	(.0001)	(.0001)	(.0001)	(.0003)
Entry cost$_{ci}$	.0020**	.0012	.0017**	-.0031**
	(.0007)	.0007	(.0009)	(.0015)
Contract enforcement$_{c0}$	-.0027**	-.0020	-.0042***	.0008
	(.0012	(.0012)	(.0014)	(.0030)
Contract enforcement$_{ci}$	.0054***	.0057***	.0059***	.0046***
	(.0004)	(.0004)	(.0005)	(.0010)
Profit tax$_{c0}$	-.0026**	-.0029**	-.0044***	.0052*
	(.0011	(.0011)	(.0012)	(.0024)
Profit tax$_{ci}$	.0018***	.0014***	.0014***	.0012
	(.0004)	(.0004)	(0004)	(.0008)
(log of) GDP$_{c0}$	.0565***	.0642***	.0600***	.0437**
	(.0065)	(.0064)	(.0067)	(.0192)
(log of) GDP$_{ci}$	.0264***	.0381***	.0346***	.0650***
	(.0038)	(.0040)	(.0045)	(.0097)
(log of) geographic distance$_{c0ci}$		-.0185*	-.0193*	.0412*
		(.0098)	(.0102)	(.0243)
Contiguous borders$_{c0ci}$		-.2443***	-.2700***	-.1533***
		(.0190)	(.0213)	(.0346)
Common language$_{c0ci}$		.0411**	.0610***	-.1533***
		(.0175)	(.0200)	(.0346)
Colonial relationship$_{c0ci}$		.0823***	.0824***	.0967*
		(.0225)	(.0254)	(.0513)
Common currency$_{c0ci}$		.1818***	.1728***	.1054***
		(.0190)	(.0205)	(.0437)
Common legal origin$_{c0ci}$		-.0985***	-.1054***	-.0679**
		(.0142)	(.0154)	(.0323)
WTO members$_{c0ci}$		-.0594	-.0216	-.2447
		(.1032)	(.0999)	(.2230)
Regional trade agreement$_{c0ci}$		.0559***	.0252	.0439
		(.0178)	(.0190)	(.0473)
Constant	-2.4947***	-2.7379***	-2.5817***	-2.9370***
	(.1565)	(.1808)	(.1846)	(.5245)
Pseudo R squared	.0251	.0293	.0329	.0273
Log pseudolikelihood	-406,283.88	-404,562.31	-320,718.85	-74,199.48
Erros clustered by parent	Yes	Yes	Yes	Yes
Firm-level controls	Yes	Yes	Yes	Yes
N. of observations	604,785	604,785	478,432	126,353

Clustered standard errors by parent company in parenthesis *** p<0.01, ** p<0.05, * p<0.1

We find that it is more likely a parent chooses direct control over indirect control, when it is located in a country with good institutions, specifically in the case of industrial groups. On the contrary, lower financial and contractual frictions in the location of subsidiaries more likely foster choices of indirect control, possibly because in these countries parents are more able to coordinate from remote the economic activities of subsidiaries. In this context, the financial institutions seem to act as a *push & pull* factor. Depending on which side of the control paths we look at. From top to down, financial frictions encourage direct control paths from the parent's perspective, when we look at the origin countries. From bottom up, poor quality of both financial and contractual institutions encourage the establishment of indirect control paths.

In Appendix Table B3, we further run a robustness check limiting our analysis only to domestic subsidiaries, for which the host country is the same of the parent company. Indeed, we find that the correlation between indirect control and financial frictions in the origin country is statistically significant only in the case of domestic companies owned by MNEs, whereas no significance is detected in the case of exclusively domestic hierarchies of firms. made of parent companies and subsidiaries located in the same country. Briefly, the *push & pull* effect of financial frictions manifest itself only when companies can choose among different institutional environments, at home and abroad, along complex ownership paths.

Among additional controls, as expected, lower tax rates in the parent country discourage the establishment of indirect control. On the contrary, higher tax rates in the country of subsidiaries incentive indirect control. In fact, we could reasonably suspect that one of the main motivations behind pyramidal structures is the possibility to shift and hide profits where taxation is lower. Moreover, geographic contiguity and longer geographic distances correlate with direct control. On the other hand, bigger economies, sharing a common language or a common currency, and a colonial common past positively correlate with indirect control.

In the last column of Table 6, when we separate financial groups, many of previous results seem not robust and significant on institutions and taxation, possibly because the corporate governance of these hierarchies is mainly based on the possibility to exploit financial gains, rather than on the necessity to coordinate productive activities.

4.2 Origin and destination of indirect control paths

In this section, we restrict our analyses only to cases of indirect control in multinational enterprises (MNEs), to test which combinations of countries of origin and destination prevails after looking at institutional environments, physical and political geography.

In the following equation 15, the dependent variable is an integer counting of how many times a parent company h_0 located in c_0 has made the choice to locate subsidiaries in a country c_i. It measures the within-choice of a parent company to pick a destination, given its origin country.

$$N_{h_0(c_o c_i)} = \beta_0 + \beta_1 X_{c_o} + \beta_2 X_{c_i} + \beta_3 Z_{c_o c_i} + \beta_4 F_{h_0} + \varepsilon_{h_0(c_o c_i)} \tag{15}$$

Based on the nature of the dependent variable, and coherently with what we assume about the data generating process of these objects in Section 3.3, we implement a negative binomial regression model. On the right-hand side of the equation, we test the same determinants as in eq. 14, now excluding only the firm size of the subsidiary because the information is aggregated at the parent level. Standard errors are clustered by parent companies. Nested results are reported in Table 7.

We find that, after the choice of indirect control is made, what matters is the quality of financial institutions in the country of the subsidiaries: lower financial frictions in a country correlate with a higher number of indirectly controlled subsidiaries coordinated by a parent company. Here, as well, our intuitition is that a parent establishes indirect control preferably where it is easier to coordinate management decisions from remote. Note that in this case contract enforcement is not statistically

significant. Finally, we observe that the choice of a negative binomial model for the data generating process is corroborated by the estimates of the dispersion parameter, which is significantly above one.

Table 7: Origin and destination of indirect control paths, geography and institutions

Dependent variable:	**(1)**	**(2)**	**(3)**	**(4)**
Number of indirect control paths$_{h0(c0\ ci)}$	**All**	**All**	**Industrial group**	**Financial group**
Financial development$_{c0}$	.0001	-.0001	.0016	-.0025
	(.0013)	(.0014)	(.0012)	(.0023)
Financial development$_{ci}$	.0049**	.0033***	.0049***	.0002
	(.0012)	(.0011)	(.0010)	(.0016)
Entry cost$_{ci}$	-.0090	-.0088	-.0099*	-.0115*
	(.0064)	(.0059)	(.0058)	(.0062)
Contract enforcement$_{c0}$	-.0019	-.0037	.0003	-.0093
	(.0069)	(.0074)	(.0054)	(.0101)
Contract enforcement$_{ci}$	.0013	.0015	.0057	-.0036
	(.0043)	(.0047)	(.0044)	(.0049)
Profit tax$_{c0}$	.0002	-.0025	.0010	-.0060
	(.0008)	(.0077)	(.0061)	(.0117)
Profit tax$_{ci}$	-.0080*	-.0100***	-.0054	-.0124***
	(.0041)	(.0033)	(.0037)	(.0036)
(log of) GDP$_{c0}$	-.0969**	-.0685*	-.0784**	-.0373
	(.0421)	(.0385)	(..3042)	(.0521)
(log of) GDP$_{ci}$	.2861***	.3204***	.3042***	.2782***
	(.0312)	(.0303)	(.0345)	(.0329)
(log of) geographic distance$_{c0ci}$		.0749	.0349	.1949**
		(.0490)	(.0663)	(.0771)
Contiguous borders$_{c0ci}$		-.1176	.0557	-.2954*
		(.1644)	(.1988)	(.1689)
Common language$_{c0ci}$		.0650	.0173	.2240
		(.1330)	(.1632)	(.1403)
Colonial relationship$_{c0ci}$		.3650**	.1804	.8756***
		(.1765)	(.1942)	(.2247)
Common currency$_{c0ci}$		.3615*	.1713	.6251**
		(.2005)	(.1597)	(.2447)
Common legal origin$_{c0ci}$		-.0312	.1968	-.5262**
		(.1370)	(.1360)	(.2054)
WTO members$_{c0ci}$		-.8252*	-.0109	-1.1419**
		(.4689)	(..3540)	(.5301)
Regional trade agreement$_{c0ci}$		-1954	-.0109	.6129***
		(.1340)	(.1573)	(.1804)
Constant	-2.173**	-3.4750***	-4.0661***	-2.6452*
	(1.0445)	(.9367)	(1.1201)	(1.4690)
Pseudo R squared	.0288	.0312	.0384	.0281
Log pseudolikelihood	-1,373,088.7	1,372,756.5	-986,059.9	-380,433.11
Estimated dispersion parameter	1.7819***	1.7547***	1.7004***	1.7128***
Erros clustered by parent	Yes	Yes	Yes	Yes
Firm-level controls	Yes	Yes	Yes	Yes
N. of observations	329,586	329,586	240,096	89,400

Clustered standard errors by parent company in parenthesis *** p<0.01, ** p<0.05, * p<0.1

Table 8: Intermediate jurisdictions along control paths

Dependent variable:	**(1)**	**(2)**	**(3)**	**(4)**
Number of indirect control paths $_{(c0\text{-}cm\text{-}ci)}$	**All**	**All**	**Industrial group**	**Financial group**
Financial development$_{cm}$	.0061***	.0049***	.0028***	.0060***
	(.0020)	(.0014)	(.0010)	(.0019)
Offshore financial country$_{cm}$	.2666*	.2975**	.3924***	.2554**
	(.1383)	(.1126)	(.1395)	(.1356)
Entry cost$_{cm}$	-.0134	-.0069	-.0314***	.0059
	(.0165)	(.0164)	(.0091)	(.0228)
Contract enforcement$_{cm}$	.0028	.0034	.0135**	-.0102
	(.0080)	(.0069)	(.0058)	(.0097)
Profit tax$_{cm}$	-.0030	-.0054	-.0012	-.0106*
	(.0061)	(.0044)	(.0048	(.0064)
(log of) GDP$_{cm}$	.2298***	.2538***	.2847***	.2137***
	(.0416)	(.0339)	(.0362)	(.0472)
(log of) geographic distance$_{c0\text{-}cm\text{-}ci}$		.2943***	.1806***	.2137***
		(.0460)	(.0448)	(.0472)
Contiguous borders$_{c0\text{-}cm\text{-}ci}$		.5415***	.4036*	.7644***
		(.1735)	(.2199)	(.1556***
Common language$_{c0\text{-}cm\text{-}ci}$		.1599	.3024	-.2496
		(.2008)	(.2320)	(.1603)
Colonial relationship$_{c0\text{-}cm\text{-}ci}$		.4185*	.3365	.6831***
		(.2427)	(.2778)	(.2184)
Common currency$_{c0\text{-}cm\text{-}ci}$		.4086**	.3310**	.5198**
		(.1645)	(.1315)	(.2272)
Common legal origin$_{c0\text{-}cm\text{-}ci}$		-.1361	-.0168	-.2899**
		(.1014)	(.1078)	(.1359)
WTO members$_{c0\text{-}cm\text{-}ci}$		-.2803*	-.3230*	-.3371*
		(.1595)	(.1734)	(.1838)
Regional trade agreement$_{cm\text{-}co\text{-}ci}$		.4256***	.1420	.8589***
		(.1234)	(.1099)	(.1871)
Constant		-6.5708***	-6.6205***	-6.5552***
		(.8440)	(.9475)	(1.2572)
Pseudo R squared	.0202	.0292	.0348	.0302
Log pseudolikelihood	-1,667,385	-1,625,354.3	-1,167,943.1	-449,763.44
Estimated dispersion parameter	1.8469***	1.7477***	1.7011***	1.6815***
Erros clustered by parent	Yes	Yes	Yes	Yes
Origin-destination fixed effects	Yes	Yes	Yes	Yes
Firm level controls	Yes	Yes	Yes	Yes
N. of observations	397,645	390,883	285,406	126,353

Clustered standard errors by parent company in parenthesis *** p<0.01, ** p<0.05, * p<0.1

4.3 Indirect control and the role of intermediate countries

Only looking at the full *ownership network*, as we do, it is possible to track through which intermediate jurisdictions an *ownership path* runs. In fact, in Section 2.3, we defined the hierarchical distance as the minimum path connecting a parent and its subsidiary. Therefore, at this stage, we can finally test which features, among institutions and geography, drive the parent's choice to choose an intermediate jurisdiction along indirect control paths.

We estimate the following equation, assuming a data generating process of the dependent variable resembling a negative binomial distribution:

$$\mathrm{E}\left[N_{h_0(c_o c_m c_i)} | X_{c_m}, Z_{c_o c_i}, F_{h_0}\right] \tag{16}$$

$$\Pr[inflate_{h_0(c_o c_i)} \mid X_{c_o/c_1}, Z_{c_o c_i}, F_{h_0}] \tag{17}$$

where $N_{h_0(c_o c_m c_i)}$ is an integer indicating the number of subsidiaries indirectly controlled by a parent company, h_0, originated in the country c_o, which are organized on control paths crossing (one or more) intermediate countries c_m, before reaching a destination country c_i. We include a set of country indicators for intermediate jurisdictions, X_{c_m}, on the right-hand side. Then, we add a set of gravity indicators, $Z_{c_o c_m c_i}$, for the multilateral characteristics of the countries involved along the *ownership paths*. All characteristics, X_{c_m}, of intermediate jurisdictions are averaged when there is more than one country along path. As a novelty with respect to previous specifications, X_{c_m} includes also a binary variable that indicate whether there is at least one country that can be considered an *offshore financial center*, following the classification by the IMF (2014) and the OECD (2016) international assessment.

Among multilateral controls, $Z_{c_o c_m c_i}$, geographic distance is summed up to take into account the entire route running from the parent country c_o, going through the intermediate countr(ies) c_m, and finally reaching a destination c_i. The rest of multilateral controls are binary variables equal to one if (all) intermediate countr(ies) on the control path verify a common condition, respectively, of contiguity, common language, shared colonial relationship, common currency, common legal origin, WTO membership, and they are all parts of a regional agreement.

Please note that, different from eq. 15, the institutional environment in origin and destination countries is neutralized after the inclusion of two sets of fixed effects, θ_{c_o} and λ_{c_i}. Parent size is introduced under F_{h_0}. Standard errors are clustered by parent company. Nested results are reported in Table 8

We find that the level of financial development in intermediate jurisdictions is a significant driver of the parent choice along indirect control paths. This finding adds to previous results shown in Tables 6 and 7, where we also commented that managers of the parent prefer to minimize coordination costs along ownership paths.

Nonetheless, *ceteris paribus*, we also find that a parent company prefers an *offshore financial center* to locate a *middleman* subsidiary, probably due to the possibility to limit disclosure of financial information and thanks to favorable taxation. The (average) level of tax rates on profit in intermediate countries is not statistically significant. Finally, richer countries, more distant but sharing a national border with the parent and thesubsidiaries are preferably chosen for the location of *middlemen* subsidiaries.

5 Conclusions

As far as we know, our contribution is the first to study the ownership and corporate control of companies systematically, on a global scale, after adopting a basic network framework for webs of interlocking shareholding activities.

First, we propose a simple generalization of an *ownership space*, which we imagine as the common playfield for all investors in the share capital of firms. Then, we introduce a model to detect a concentration of voting rights iteratively, in the presence of pyramidal structures and cross-holding exchanges among companies. From a perspective of graph theory, the problem amounts to finding *spanning subgraphs* in wider *ownership networks*, in our case including parent companies, their subsidiaries and all the ownership edges that allow for corporate control. In this case, we show that firm boundaries can actually represent peculiar graphs, the *hierarchies of firms*, where parent companies on top can enforce management decisions in downstream subsidiaries organized on different hierarchical layers.

We argue that our basic network framework is particularly useful to understand different shareholding architectures around the world, including network dimensions that are involved in the design of firms' boundaries but often neglected in economic literature. In this context, we stress the role of indirect control through *middlemen subsidiaries*, which are crucial in the organization of pyramidal structures. These are companies that are controlled by a parent on top but in turn control *subsidiaries of subsidiaries* on downstream layers of the hierarchy.

After we exploit a unique dataset of more than 53.5 million of firms with ownership information in 206 countries in year 2015, we provide some first insights on the heterogeneous and complex distributions of these objects both within and across countries. Among others, we document policy-relevant cases of multiple-passports, indirectly foreign and round-tripping subsidiaries, which amount to about 54% of foreign affiliates from our data.

As the emergence of complex ownership structures has been usually associated to the quality of the financial institutions of the parent country, we can extend on findings from existing literature. Exploiting information on jurisdictions encountered along the ownership paths, we test that indirect control and pyramidal structures preferably run through countries with good financial institutions, even after controlling for the critical role of offshore financial centers. Moreover, we uncover a *push & pull* effect started by institutional environments in MNEs structures. Less financial and contractual frictions in the country of a parent drive to more transparent forms of corporate governance, whereas less financial frictions in the countries of subsidiaries, permit a stretching of pyramidal structures and the establishment of indirect control, possibly because a parent can better afford to coordinate management decisions from remote.

We believe that our work paves the way to a variety of follow-ups. Once the network nature of modern corporations is acknowledged, we can study for example how the intra-firm trade of goods and services develops, probably at prices commanded by the headquarters that are different from market prices, possibly initiating profit-shifting operations. We could also study if and how internal capital markets develop to switch financial resources across countries and industries on unsynchronized business cycles, as an alternative to external financial markets. More interesting, it seems to us the possibility to understand if and how the concentration of corporate control observed here on a global scale also implies an increasing concentration of market power, raising an issue of lack of competition that can be detrimental to overall welfare.

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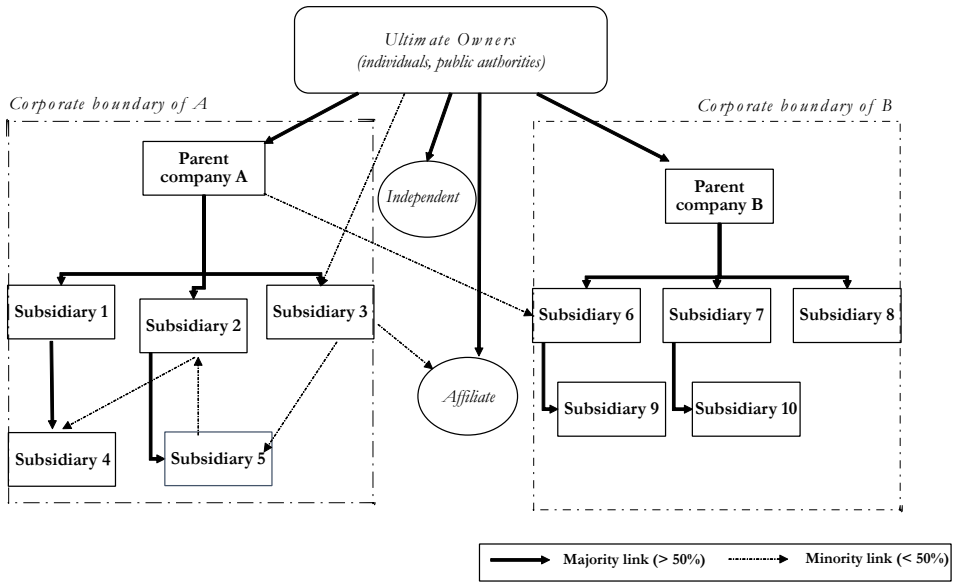
Appendix A: Definitions

In Figure A1 we sketch a fictional ownership network where we can spot two different *hierarchies of firms*. Assuming we are able to separate a boundary A from a boundary B, we summarize here the definitions we use of ultimate owners, parent company, subsidiary company, affiliated company, and independent company, which we use throughout our contribution.

Figure A1: A fictional ownership network with two hierarchies of firms.



Ultimate owners (UOs) are the shareholders (individuals, families, public authorities) that are on top of any ownership path, because they cannot be owned by any other shareholder. Equivalently, we can say that they are the ultimate sources of the *ownership space* because they do not have any *predecessor* in ownership. In fact, they are the starters of any ownership network as from initial decisions to invest in companies' capital shares. As depicted in Figure A1, we consider UOs as on top but outside of corporate boundaries. See also Figure 4.

Our loose notion of UOs provided here does differ from a more specific legal definition of Ultimate Beneficial Owner (UBOs), which identifies subjects that are for their actions ultimately liable in front of the law. For example, the EU's Fourth Anti-Money Laundering Directive (MLD4) identifies an UBO as a subject that holds more than 25% of voting rights in a legal entity. On the other hand, the Financial Action Task Force on Money Laundering (FATF) is an intergovernmental body participated by 37 member countries, which defines an UBO more flexibly as the natural person who ultimately own or control a legal entity and/or the natural person on whose behalf a business is being conducted.

We argue that keeping our definition of UOs more inclusive allows us to catch better the variety of wedges that can be inserted between ownership and control of a company, given also the variety of corporate structures we may encounter (see also Vermeulen et al., 2013).

A **parent company** is a company that coordinates the activities of one or more subsidiaries in a *hierarchy of firms*. It is not controlled by any other corporate shareholder, although one or more corporate shareholders can sit at its assembly, with either dominant stakes or portfolio stakes.

A **subsidiary company** is a company that is directly or indirectly controlled by a parent company through an absolute majority of capital shares or a dominant stake.

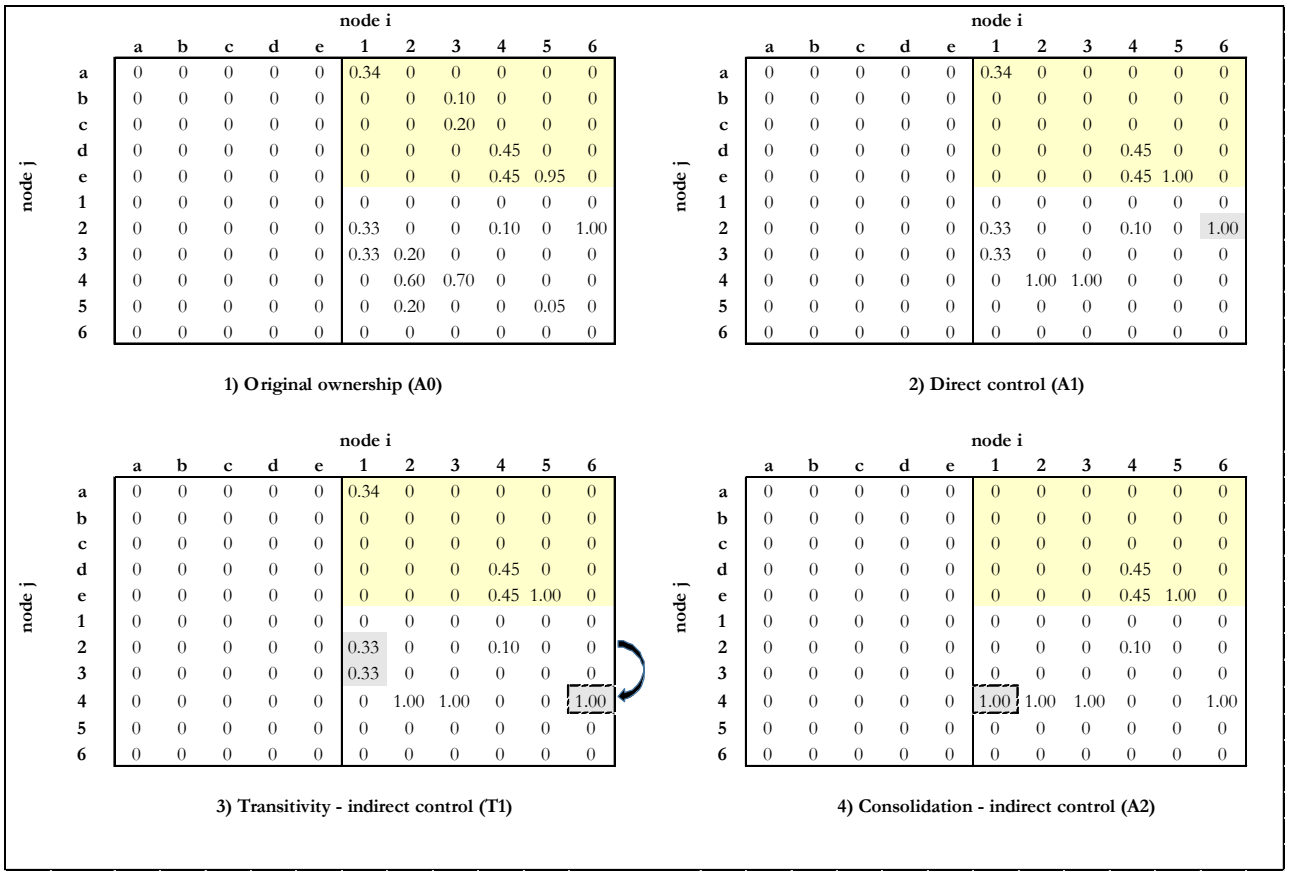
An **affiliated company** is a company in which another company holds a minority portfolio stake. It can be an **independent company,** because it is neither a parent nor a subsidiary. It can be a

subsidiary of another parent company because the latter has outright absolute majority or a dominant stake at the shareholders' assembly.

More than often, **corporate control** is a matter of degree when ownership is fragmented. When we identify a **binary notion of control**, in Section 2.4, we follow international standards by UNCTAD (2009) and OECD (2005) that identify a threshold of voting rights above 50% + 1 stake. When we extend our framework including a **probabilistic notion** of corporate control, we can relate to more recent attempts to identify *decisive influence* in fragmented ownership. See also the Article 3(2) of the EU Merger Regulation.

APPENDIX B: TABLES AND GRAPHS

Figure B1: From the ownership matrix to the control matrix: an application



Starting from the original ownership adjacency matrix (A0) of the illustrative network of Figure 4, any jth node may have a stake in an ith node. The coloured area is the sub-matrix of ultimate ownership, which does not change after application of the corporate control transformation. Following matrices A1, T1 and A2 represent applications of eqs. 9, 10 and 11, respectively. Grey-colored cells show substitutions. No iteration is needed in this case: matrix A2 represents corporate control in the given ownership network.

Figure B2: Banzhaf (1965) Index, capital share and number of shareholders for minority-owned companies

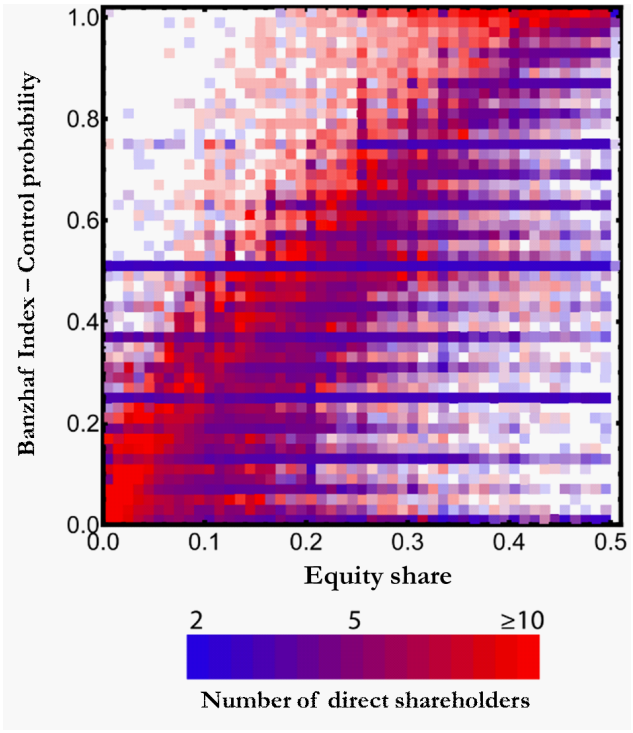


Table B1: Companies participating to ownership networks: geographic coverage and firm size

Economy	Small	Medium	Large	Total
European Union	2,920,807	713,176	323,924	**3,957,907**
of which:				
Germany	*536,578*	*138,180*	*50,170*	*724,928*
France	*101,064*	*76,576*	*38,558*	*216,198*
United Kingdom	*437,778*	*77,485*	*68,046*	*583,309*
Italy	*203,179*	*96,872*	*32,245*	*332,296*
Spain	*133,862*	*48,305*	*24,021*	*206,188*
United States	3,620,543	45,067	62,494	**3,728,104**
Russia	437,180	105,822	34,367	**577,369**
Other Europe	228,161	64,132	20,802	**313,095**
Asia	553,644	116,435	131,578	**801,657**
of which:				
Japan	*118,195*	*27,931*	*30,774*	*176,900*
China	*99,942*	*10,874*	*34,595*	*145,411*
India	*17,032*	*4,738*	*8,990*	*30,760*
Africa	39,750	2,151	4,475	**46,376**
Central and South America	121,580	20,159	17,752	**159,491**
of which:				
Brazil	*8,631*	*9,534*	*5,365*	*23,530*
Argentina	*5,358*	*1,174*	*1,432*	*7,964*
Mexico	*12,994*	*1,754*	*2,627*	*17,375*
the Caribbean countries	*41,662*	*141*	*1,676*	*43,479*
Australia	381,878	48,311	10,211	**440,400**
Rest of the world	606,339	21,300	17,473	**645,112**
TOTAL	**8,909,882**	**1,136,553**	**623,076**	**10,294,391**

Table B2: Top 20 countries where firms establish reciprocal cross-holdings

Top 20 countries	N. firms	Avg share	st. dev.	Min	Max
Germany	4956	92.69	23.17	0.03	100
United States	1556	50.09	8.59	0.10	100
Japan	1493	14.09	13.24	0.01	100
Russia	1427	36.97	36.89	0.01	100
Spain	1123	49.77	37.26	0.01	100
United Kingdom	1016	72.29	39.19	0.01	100
Australia	960	58.94	38.85	0.01	100
Belgium	787	43.06	41.16	0.01	100
Italy	648	29.73	33.44	0.01	100
Israel	630	43.91	43.49	0.01	100
Portugal	587	33.99	34.19	0.02	100
Ireland	293	49.36	39.43	0.01	100
France	207	28.59	35.79	0.01	100
Turkey	162	26.01	31.09	0.02	100
Norway	146	42.11	39.60	0.15	100
Bulgaria	128	50.51	40.75	0.02	100
Romania	116	34.12	37.00	0.02	100
Taiwan	109	14.92	25.54	0.03	100
Netherlands	107	74.02	33.15	0.01	100
Poland	106	58.13	36.04	0.01	100
Other countries	*1437*	*39.45*	*28.70*	*0.01*	*100*

Table B3: Ownership centralities at the country level - financial companies only

Country	Eigenvector centrality	Rank	Weighted degree	Weighted indegree	Weighted outdegree
United Kingdom	1.000	1	246931	119135	127796
Germany	0.876	2	200353	103247	97106
United States	0.844	3	684670	323181	361489
Netherlands	0.819	4	372170	177178	194992
Luxembourg	0.779	5	16879	9951	6928
British Virgin Islands	0.779	6	7433	4218	3215
Italy	0.773	7	149883	73888	75995
Spain	0.759	8	84858	42947	41911
Cayman Islands	0.759	9	15167	9488	5679
Cyprus	0.745	10	9536	4655	4881
Russian Federation	0.736	11	96139	49886	46253
Switzerland	0.721	12	58414	18055	40359
France	0.717	13	119708	56233	63475
Canada	0.693	14	49906	23232	26674
Hong Kong	0.685	15	17310	8989	8321
Bermuda	0.671	16	14864	6230	8634
Ireland	0.659	17	21861	12630	9231
Australia	0.651	18	232845	117346	115499
Poland	0.650	19	22075	13732	8343
India	0.642	20	29376	17913	11463
Arab Emirates	0.637	21	4273	2267	2006
China	0.635	22	60655	34440	26215
Romania	0.635	23	12422	8197	4225
Singapore	0.631	24	17003	8902	8101
Belgium	0.630	25	46409	21278	25131

Table B4: Indirect vs direct corporate control in domestic subsidiaries

Dependent variable: **Indirectly controlled$_{h0(c0)i(c0)}$**	**All domestic subs**	**Domestic subs of MNEs**	**Domestic subs of domestic groups**
Financial development$_{c0}$	-.0006*	-.0020***	.0004
	(.0002)	(.0004)	(.0003)
Entry cost$_{c0}$	-.0120***	-.0327***	-.0014**
	(.0021)	(.0033)	(.0005)
Contract enforcement$_{c0}$	.0022*	.0134***	-.0005
	(.0009)	(.0015)	(.0009)
Profit tax$_{c0}$	.0037***	.0005	.0052***
	(.0009)	(.0002)	(.0009)
(log of) GDP$_{c0}$	-.1505***	-.0114	-.1785***
	(.0058)	(.0121)	(.0057)
Constant	2.2776***	-.6311**	2.870***
	(.1307)	(.2761)	(.1422)
Pseudo R squared	0.3746	0.1013	0.2981
Log pseudolikelihood	-964,026.55	-290,510.97	-649,983.6
Erros clustered by parent	Yes	Yes	Yes
Firm-level controls	Yes	Yes	Yes
N. of observations	3,665, 804	474,897	3,190, 804

Clustered standard errors by parent company in parenthesis. *** p<0.01, ** p<0.05, * p<0.1