(Un)expected Monetary Policy Shocks and Term Premia

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The term structure of interest rates is crucial for the transmission of monetary policy to financial markets and the macroeconomy. Disentangling the impact of monetary policy on the components of interest rates, expected short rates and term premia, is essential to understanding this channel. To accomplish this, we provide a quantitative structural model with endogenous, time-varying term premia that are consistent with empirical findings. News about future policy, in contrast to unexpected policy shocks, has quantitatively significant effects on term premia along the entire term structure. This provides a plausible explanation for partly contradictory estimates in the empirical literature.

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1. Introduction

Gauging how monetary policy tools affect the entire term structure of interest rates is important for understanding the monetary transmission mechanism. The term structure comprises expected future short term interest rates and term premia.

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Thus, understanding the effect of monetary policy shocks on the term structure requires us to disentangle the effects on both components. There is, however, no consensus so far on the impact of monetary policy shocks on these premia (for a discussion see, for example, Nakamura and Steinsson, 2017). The empirical literature faces the challenge of identifying monetary policy shocks when multiple instruments at the same time, say, unexpected changes in the policy rate and news about future policy, are used concurrently. This challenge has increased in relevance since the 1990s as the Federal Reserve has increasing relied on communication in transmitting monetary policy (see, for example, Gürkaynak, Sack, and Swanson, 2005a; Campbell, Evans, Fisher, and Justiniano, 2012). While a structural model in general can help to disentangle these instruments, the existing literature has trouble providing a tractable framework that enables an empirical analysis of endogenous, time-varying term premia.

This paper makes two important contributions to the literature. First, we provide a medium scale macro-finance model that is estimated with U.S. macroeconomic and Treasury bond time series using Bayesian likelihood methods. The estimated model implies historical time series of term premia that match those found in reduced form empirical estimates without sacrificing the macroeconomic fit. We therefore provide a structural framework for the analysis of endogenous, time-varying term premia. Second, we then use this model to study the impact of monetary policy shocks on the term structure. We specifically differentiate between unexpected changes of the policy rate and news about future monetary policy. We find that this distinction is crucial with unexpected changes in the policy rate having limited effects while news about future policy has strong effects on term premia of all maturities. This finding is in line with the recent empirical literature. Furthermore, this difference can help understand partly contradictory estimates of the effects of monetary policy shocks on the term structure in the empirical literature.

Many studies have found that nominal term premia are sizable, volatile, and have been on the decline since the beginning of the 1980s (see Rudebusch, Sack, and Swanson, 2007, for a summary). The literature has also emphasized the role of real term premia relative to inflation risk premia, (see, e.g., Gürkaynak, Sack, and Wright, 2010; Chernov and Mueller, 2012) manifesting itself in an upward sloping real yield curve. This sets the yardstick for our structural model. To this end, a joint model of the interaction between the macroeconomy and the term structure of interest rates that allows for time-varying risk premia is needed. This is beyond the reach of standard structural modelling approaches – like the linear New Keynesian models commonly used in policy analysis, as its linearization renders the model certainty equivalent, shutting down risk premia altogether. While the macro-finance literature has offered punctual solutions for selective empirical facts,[1] the investigation of a comprehensive model is hampered by the computationally burdensome solution and estimation methods. Consequentially, the literature has focused either on matching selected moments (Rudebusch and Swanson, 2012; Andreasen, Fernández-Villaverde, and Rubio-Ramírez, 2017) or on highly stylized models (van Binsbergen, Fernández-Villaverde, Koijen, and Rubio-Ramírez, 2012). This tension is poignantly noted by Gürkaynak and Wright (2012, p. 354): "A general problem with a structural model . . . is that it is challenging to maintain computational tractability and yet obtain time-variation in term premia."

We are the first to our knowledge to provide a quantitatively meaningful, joint model of the macroeconomy and the term structure of interest rates. Our medium scale New Keynesian macro-finance DSGE model is fitted to U.S. macroeconomic and Treasury bond time series from 1983:Q1 to 2007:Q4 using Bayesian likelihood methods. To do so, we apply a novel procedure that captures both constant and time-varying risk premia while maintaining linearity in states and shocks (Meyer-Gohde, 2016). The closest contributions to ours are Andreasen (2011) and Dew-Becker (2014). While the former is silent about model predictions of stylized macroeconomic facts and other financial facts besides the nominal term structure, the latter, in addition, predicts historical time-varying term premia which are at odds with the empirical literature. In contrast, our estimated structural model implies a histori-

[1]E.g., Piazzesi and Schneider (2007) highlight the role of recursive preferences and supply shocks for an upward sloping nominal yield curve and Wachter (2006) points to the role of habit formation for real yields.

cal 10-year term premium comparable in level, pattern, and volatility with recent reduced-form empirical estimates, see Figure 1.[2] The model implies both an upward

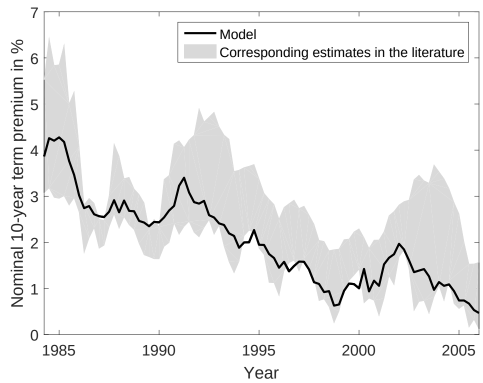


FIGURE 1. MODEL IMPLIED 10-YEAR NOMINAL TERM PREMIUM.

Note: The figure compares the model implied term premium (black line) with a range of corresponding estimates in the literature (gray area). In particular, the gray area presents the range (maximum and minimum) of the estimates from models developed by Kim and Wright (2005), Rudebusch and Wu (2008), Bernanke, Reinhart, and Sack (2004), Adrian, Crump, and Moench (2013), and Bauer (2018). The first three measures were calculated by Rudebusch et al. (2007) and Rudebusch, Swanson, and Wu (2006). A description of the estimates can be found there.

sloping nominal yield curve in line with the data and an upward sloping real yield curve in line with empirical estimates (see, for example, Gürkaynak et al., 2010; Chernov and Mueller, 2012), but in contrast to many DSGE models (see, for example, van Binsbergen et al., 2012; Andreasen, 2012; Swanson, 2016) that imply flat or downward sloping real yield curves. Additionally, our results suggest that 2/3 of the average slope of the nominal term structure is related to real rather than to inflation risk and an upward sloping inflation risk premium – both consistent with recent estimates in the literature (see, for example, Abrahams, Adrian, Crump, Moench, and Yu, 2016).

While the general impact of monetary policy on the term structure of interest rates is largely agreed upon (Piazzesi, 2005), there exists considerable disagreement on the effects of monetary policy shocks on term premia. Hanson and Stein (2015) argue for quantitatively strong effects of monetary policy shocks on real risk premia,

[2]We are very grateful to Eric T. Swanson and Michael Bauer for sharing their estimates with us.

while Nakamura and Steinsson (2017) find rather small effects despite sizable effects on nominal term premia overall. More strikingly, the literature disagrees even on the qualitative effects: Gertler and Karadi (2015) and Abrahams et al. (2016) find a positive, while Crump, Eusepi, and Moench (2016) and Nakamura and Steinsson (2017) find a negative correlation between the policy rate and nominal term premia. There are many potential reasons for this disagreement; we take our cue from Ramey (2016) and focus on disentangling the effects of structural shocks arising from different monetary policy tools on term premia.

We find that an unexpected tightening of monetary policy via a simple innovation to the Taylor rule reduces risk premia especially at longer maturities, in line with the empirical work by Crump et al. (2016) but in contrast to e.g. Gertler and Karadi (2015). Yet overall such a shock has quantitatively limited effects, in line with findings from other structural models (see, for example, Rudebusch and Swanson, 2012). In contrast, a shock to the inflation target or unconditional forward guidance[3] reveals news about future paths of macroeconomic variables, affecting households' precautionary motives and thereby their demand for risk premia. In particular, a change of the inflation target might be interpreted as a change in the systematic component of monetary policy (see Cogley, Primiceri, and Sargent, 2010) as it affects agents' perception of the macroeconomy in the longer run. Similarly, forward guidance communicates the expected path of future short rates and is likewise informative as to the central bank's commitment to allow higher inflation in the future. The recent empirical literature (Gürkaynak et al., 2005a; Nakamura and Steinsson, 2017) argues that news revealed by monetary policy about its future path has strong effects on the term structure of interest rates. In line with this, we find that a shock to the inflation target has strong effects on the term structure of interest rates and term premia across all maturities. As laid out by Rudebusch and Swanson (2012), a change to the inflation target introduces long-run (nominal) risk

[3]For a discussion of different forms of forward guidance see Campbell et al. (2012) and Akkaya, Gürkaynak, Kısacıkoğlu, and Wright (2015). Particularly such a distinction is a significant challenge in many empirical approaches (see, for example, the discussion in Nakamura and Steinsson, 2017; Campbell, Fisher, Justiniano, and Melosi, 2016).

that strongly affects households' expectation formation and precautionary savings motives. Unconditional forward guidance likewise affects term premia substantially, causing real term premia and inflation risk premia to rise as agents expect more volatile inflation and output in the future in line with the empirical findings of Akkaya et al. (2015).

Our analysis of the effects of monetary policy shocks on term premia suggests that the quantitatively large effects found in the empirical literature seem primarily driven by monetary policy news about its medium- or long-term stance rather than changes in the policy rate. Beyond identification, Monte-Carlo analysis of small samples with common empirical models demonstrates that estimation uncertainty results in a wide range of quantitatively and qualitatively different point estimates. Thus, our structural model can help to rationalize empirical findings. Finally, given the model's tractability it can be easily applied and extended to serve as a building block for future research.

The remainder of the paper reads as follows: Section 2 presents the model. Then, section 3 describes the solution method, the data, and the Bayesian estimation approach in greater detail. Section 4 presents the estimation results and discusses the model fit. The effects of unexpected and expected monetary policy on the term structure and puts them in comparison with empirical estimates are presented in section 5. Section 6 concludes the paper.

2. Model

In the following section, we present our dynamic stochastic general equilibrium (DSGE) model, a standard New Keynesian model but with recursive preferences (Epstein and Zin, 1989, 1991; Weil, 1989) and both real and nominal long-run risk (Bansal and Yaron, 2004; Gürkaynak, Sack, and Swanson, 2005b), highlighted in the literature for the explanation of many financial moments in consumption-based asset pricing.

2.1. Firms

A perfectly competitive representative firm produces the final good y_t, which is aggregated from a continuum of intermediate goods $y_{j,t}$ by $y_t = \left(\int_0^1 y_{j,t}^{(\theta_p-1)/\theta_p} dj\right)^{\theta_p/(1-\theta_p)}$ where $\theta_p > 1$ the elasticity of substitution. Cost-minimization yields the demand function for the intermediate good $y_{j,t} = (P_{j,t}/P_t)^{-\theta_p} y_t$ and the aggregate price level is then $P_t = \left(\int_0^1 P_{j,t}^{1-\theta_p} dj\right)^{\frac{1}{1-\theta_p}}$.

The intermediate good j is produced by a monopolistic competitive firm with

$$y_{j,t} = \exp\{a_t\} k_{j,t-1}^{\alpha} (z_t l_{j,t})^{1-\alpha} - z_t^+ \Omega_t \tag{1}$$

where $k_{j,t}$ and $l_{j,t}$ denote capital and labor inputs used for production by the jth intermediate good producer. The capital share is α and Ω_t the fixed costs of production. Short-run risk is present via the stationary technology shock a_t that follows

$$a_t = \rho_a a_{t-1} + \sigma_a \epsilon_{a,t}, \text{ with } \epsilon_{a,t} \overset{iid}{\sim} N(0,1) \tag{2}$$

and long-run risk via the stochastic aggregate productivity trend z_t

$$\mu_{z,t} \doteq \ln\{z_t/z_{t-1}\} = (1-\rho_z)\bar{\mu}_z + \rho_z \mu_{z,t-1} + \sigma_z \epsilon_{z,t}, \text{ with } \epsilon_{z,t} \overset{iid}{\sim} N(0,1). \tag{3}$$

Croce's (2014) specification for productivity, which mirrors Bansal and Yaron's (2004) for consumption, is captured as a special case.[4]

Alongside the stochastic trend z_t, we assume a deterministic trend in the relative price of investment Υ_t with $\exp\{\bar{\mu}_\Upsilon\} = \Upsilon_t/\Upsilon_{t-1}$. Following Altig, Christiano, Eichenbaum, and Linde (2011) we define $z_t^+ = \Upsilon_t^{\frac{\alpha}{1-\alpha}} z_t$ as an overall measure of technology with associated trend $\mu_{z^+,t} = \frac{\alpha}{1-\alpha}\bar{\mu}_\Upsilon + \mu_{z,t}$.

[4] Short-run risk (SSR) is white noise as in Croce (2014) and Bansal and Yaron (2004) if $\rho_a = 1$

$\varpi_t \doteq \exp\{a_t\} z_t^{1-\alpha} \Rightarrow \ln\{\varpi_t/\varpi_{t-1}\} = (1-\alpha)\ln\{z_t/z_{t-1}\} + a_t - a_{t-1} = \underbrace{(1-\alpha)\mu_{z,t}}_{\text{LRR}} + \underbrace{(\rho_a - 1) a_{t-1} + \sigma_a \epsilon_{a,t}}_{SRR}$

Finally, we scale Ω_t by z_t^+ to ensure the existence of a balanced growth path and let production costs be time-varying as proposed by Andreasen (2011).

$$\ln\left(\Omega_t/\bar{\Omega}\right) = \rho_\Omega \ln\left(\Omega_{t-1}/\bar{\Omega}\right) + \sigma_\Omega \epsilon_{\Omega,t}, \text{ with } \epsilon_{\Omega,t} \overset{iid}{\sim} N(0,1). \tag{4}$$

Following Calvo (1983), intermediate goods firms face staggered price setting and adjust their prices only with probability $(1-\gamma_p)$ each period. Non-adjusted prices evolve according to the indexation rule: $P_{j,t} = P_{j,t-1}\pi_{t-1}^{\xi_p}$, where $\pi_t = P_t/P_{t-1}$ is gross inflation. Firms that are able to adjust their prices, choose the same price $\tilde{p}_t = P_{j,t}$ to maximize the present value of their expected future profits, accounting for demand, indexation, and the readjustment probability. Firms are owned by the households and discount with their real stochastic discount factor M_{t+1}. The optimality conditions, where mc_t are real marginal costs, are

$$\mathcal{K}_t = y_t \tilde{p}_t^{-\theta_p} + \gamma_p E_t\left[M_{t+1}\left(\pi_t^{\xi_p}/\pi_{t+1}\right)^{1-\theta_p}\left(\tilde{p}_t/\tilde{p}_{t-1}\right)^{-\theta_p}\mathcal{K}_{t+1}\right] \text{ and} \tag{5}$$

$$\mathcal{K}_t = \frac{\theta_p}{\theta_p - 1} y_t mc_t \tilde{p}_t^{-\theta_p-1} + \gamma_p E_t\left[M_{t+1}\left(\pi_t^{\xi_p}/\pi_{t+1}\right)^{-\theta_p}\left(\tilde{p}_t/\tilde{p}_{t-1}\right)^{-\theta_p-1}\mathcal{K}_{t+1}\right]. \tag{6}$$

2.2. Households

We assume that the representative household has recursive preferences to disentangle risk aversion and the intertemporal elasticity of the substitution (IES). Following Rudebusch and Swanson (2012), the value function of the household is

$$V_t = \begin{cases} u_t + \beta\left(E_t\left[V_{t+1}^{1-\sigma_{EZ}}\right]\right)^{\frac{1}{1-\sigma_{EZ}}} & \text{if } u_t > 0 \text{ for all } t \\ u_t - \beta\left(E_t\left[\left(-V_{t+1}\right)^{1-\sigma_{EZ}}\right]\right)^{\frac{1}{1-\sigma_{EZ}}} & \text{if } u_t < 0 \text{ for all } t \end{cases} \tag{7}$$

where u_t is the period utility kernel and $\beta \in (0,1)$ the subjective discount factor.

Similarly to Andreasen et al. (2017), we assume the following utility kernel

$$u_t = \exp\{\varepsilon_{b,t}\}\left[\frac{1}{1-\gamma}\left(\left(\frac{c_t - bh_t}{z_t^+}\right)^{1-\gamma} - 1\right) + \frac{\psi_L}{1-\chi}\left(1-l_t\right)^{1-\chi}\right] \tag{8}$$

with consumption c_t, the habit h_t, hours l_t, and preference parameters γ, χ, and ψ_L. We assume an external habit in last period's aggregate consumption $h_t = C_{t-1}$, the degree of which is controlled by $b \in (0,1)$. The preference shock $\varepsilon_{b,t}$ is given by

$$\varepsilon_{b,t} = \rho_b \varepsilon_{b,t-1} + \sigma_b \epsilon_{b,t}, \text{ with } \epsilon_{b,t} \overset{iid}{\sim} N\left(0,1\right) \tag{9}$$

The household's period budget constraint equates real expenditures with income

$$c_t + I_t/\Upsilon_t + b_t + T_t = w_t l_t + r_t^k k_{t-1} + b_{t-1}\exp\left\{R_{t-1}^f\right\}/\pi_t + \int_0^1 \Pi_t\left(j\right)dj. \tag{10}$$

Expenditures comprise consumption, investment I_t, a lump-sum tax T_t, and a one-period bond b_t that accrues the risk-free nominal interest R_t^f in the following period; while income comprises labor income $w_t l_t$ with w_t the real wage, capital service income $r_t^k k_{t-1}$, the pay-off from last period's bonds b_{t-1}, and the dividends from the intermediate firms – indexed by j – owned by households $\Pi\left(j\right)$.

Households own the physical capital stock, which accumulates as

$$k_t = \left(1-\delta\right)k_{t-1} + \exp\{\varepsilon_{i,t}\}\left(1 - \frac{\nu}{2}\left(\frac{I_t}{I_{t-1}} - \exp\{\bar{\mu}_{z^+} + \bar{\mu}_\Upsilon\}\right)^2\right)I_t \tag{11}$$

δ is the depreciation rate and $\nu \geq 0$ controls the investment adjustment costs as in Christiano, Eichenbaum, and Evans (2005), which are zero along the balanced growth path via $\exp\{\bar{\mu}_{z^+} + \bar{\mu}_\Upsilon\}$. Following Justiniano, Primiceri, and Tambalotti (2010), $\varepsilon_{i,t}$ represents an investment shock that evolves as

$$\varepsilon_{i,t} = \rho_i \varepsilon_{i,t-1} + \sigma_i \epsilon_{i,t}, \text{ with } \epsilon_{i,t} \overset{iid}{\sim} N\left(0,1\right) \tag{12}$$

2.3. Monetary Policy

We follow Rudebusch and Swanson (2008, 2012) and specify monetary policy via the following interest rate rule

$$R_t^f = \rho_R R_{t-1}^f + (1-\rho_R)\left(\bar{r} + \ln \pi_t + \frac{\eta_y}{4}\ln\left(\frac{y_t}{z_t^+ \bar{y}}\right) + \frac{\eta_\pi}{4}\ln\left(\frac{\pi_t^4}{\pi_t^*}\right)\right) + \frac{\sigma_m}{4}\epsilon_{m,t} \tag{13}$$

where $\bar{r}$ is the deterministic steady-state real interest rate. The policy parameters ρ_R, η_y, and η_π characterize the degree of monetary policy's aim to smooth the interest rate, stabilize deviations in output from its balanced growth path – $\ln\left(y_t/z_t^+\bar{y}\right)$ – and those in inflation from the central bank's inflation target π_t^* – $\ln\left(\pi_t^4/\pi_t^*\right)$. Departures from these aims are captured by $\epsilon_{m,t} \overset{iid}{\sim} N(0,1)$. Following Gürkaynak et al. (2005b), the inflation target is time-varying and is governed by

$$\log \pi_t^* - 4\log\bar{\pi} = \rho_\pi\left(\log \pi_{t-1}^* - 4\log\bar{\pi}\right) + 4\zeta_\pi\left(\log \pi_{t-1} - \log\bar{\pi},\right) + \sigma_\pi\epsilon_{\pi,t} \tag{14}$$

with $\epsilon_{\pi,t} \overset{iid}{\sim} N(0,1)$ a shock to the inflation target.

2.4. Aggregation and Market Clearing

The aggregate market clearing constraint in the goods market is given by

$$p_t^+ y_t = \exp\{a_t\} k_{t-1}^\alpha \left(z_t l_t\right)^{1-\alpha} - z_t^+ \Omega_t \tag{15}$$

where $l_t = \int_0^1 l(j,t)\ dj$ and $k_t = \int_0^1 k(j,t)\ dj$ are aggregated labor and capital. Price dispersion, $p_t^+ = \int_0^1 \left(\frac{P_{j,t}}{P_t}\right)^{-\theta_p} dj$, arises from staggered price setting and evolves as

$$p_t^+ = (1-\gamma_p)\left(\tilde{p}_t\right)^{-\theta_p} + \gamma_p\left(\pi_{t-1}^{\xi_p}/\pi_t\right)^{-\theta_p} p_{t-1}^+ \tag{16}$$

The economy's aggregate resource constraint implies that

$$y_t = c_t + I_t/\Upsilon_t + g_t \tag{17}$$

where government expenditures $g_t = \bar{g}z_t^+ \exp\{\varepsilon_{g,t}\}$ grow with the economy, are financed by lump-sum taxes $g_t = T_t$, and are subject to shocks via $\varepsilon_{g,t}$ given as

$$\varepsilon_{g,t} = \rho_g \varepsilon_{g,t-1} + \sigma_g \epsilon_{g,t}, \text{ with } \epsilon_{g,t} \overset{iid}{\sim} N(0,1) \tag{18}$$

Finally, the aggregate price index is $1 = \gamma_p \left(\pi_{t-1}^{\xi_p}/\pi_t\right)^{1-\theta_p} + (1-\gamma_p)(\tilde{p}_t)^{1-\theta_p}$.

2.5. The Nominal and Real Term Structures

The nominal and real term structures follow the procedures of, e.g., Rudebusch and Swanson (2008, 2012) and Andreasen (2012) identically: assets are priced following standard no-arbitrage arguments as the sum of their stochastically discounted state-contingent payoffs in period $t+1$. For example, the price of a default free n-period zero-coupon bond that pays one unit of cash at maturity satisfies

$$P_{n,t} = E_t\left[M_{t,t+n}^{\$} 1\right] = E_t\left[M_{t,t+1}^{\$} P_{n-1,t+1}\right] \tag{19}$$

where $M_{t,t+1}^{\$}$ is the household's nominal stochastic discount factor given by

$$M_{t,t+1}^{\$} = \beta \frac{\lambda_{t+1}}{\lambda_t \pi_{t+1}} (V_{t+1})^{-\sigma_{EZ}} E_t\left[V_{t+1}^{1-\sigma_{EZ}}\right]^{\frac{\sigma_{EZ}}{1-\sigma_{EZ}}} \tag{20}$$

with λ_t the marginal utility of consumption. The continuously compounded yield to maturity on the n-period zero-coupon nominal bond is $\exp\left\{-nR_{n,t}^{\$}\right\} = P_{n,t}^{\$}$.

Following, e.g., Rudebusch and Swanson (2012), the term premium is the difference between a bond's yield and its unobserved risk-neutral equivalent yield. This risk-neutral bond, which also pays one unit of cash at maturity, is priced as

$$\hat{P}_{n,t} = \exp\left\{-R_t^f\right\} E_t\left[\hat{P}_{n-1,t+1}\right] \tag{21}$$

In contrast to eq. (19), discounting is performed using the risk-free rate and the nominal term premium on a bond with maturity n is given by

$$TP_{n,t}^{\$} = \frac{1}{n}\left(\log \hat{P}_{n,t} - \log P_{n,t}^{\$}\right) \tag{22}$$

Similarly, we can derive the yield to maturity of a real bond $R_{n,t}$ as well its risk-neutral equivalent, leading analogously to the associated real term premium $TP_{n,t}$. Finally, we follow the literature and define inflation risk premia $TP_{n,t}^{\pi}$ as

$$TP_{n,t}^{\pi} = TP_{n,t}^{\$} - TP_{n,t} \tag{23}$$

3. Model Solution and Estimation

3.1. Solution Method

We solve the model with the method of Meyer-Gohde (2016), delivering a nonlinear in risk, but linear in states approximation at the means of the endogenous variables.[5] Unlike standard higher order perturbations or affine approximation methods, this allows us to use the standard set of macroeconometric tools for estimation and analysis of linear models, without limiting the approximation to the certainty-equivalent approximation around the deterministic steady state. We adjust the points and slopes of the decision rules for risk out to the second moments of the exogenous processes to capture both constant and time-varying risk premium, as well as the effects of conditional heteroskedasticity (e.g. van Binsbergen et al., 2012). Our resulting

[5] Meyer-Gohde (2016) provides derivations for adjustments around the deterministic and stochastic steady states, along with those around the mean that we derive and apply here, accuracy checks and formal justifications for the method.

linear in states approximation is

$$y_t \simeq \tilde{y}(\sigma) + y_y(\tilde{y}(\sigma), 0, \sigma)\,(y_{t-1} - \tilde{y}(\sigma)) + y_\varepsilon(\tilde{y}(\sigma), 0, \sigma)\varepsilon_t \tag{24}$$

where y_t are endogenous and ε_t exogenous variables, $\tilde{y}(\sigma)$ the means of the endogenous variables, and $y_y(\tilde{y}(\sigma), 0, \sigma)$ and $y_\varepsilon(\tilde{y}(\sigma), 0, \sigma)$ the first derivatives of the policy function evaluated at the means $\tilde{y}(\sigma)$. The hyperparameter σ scales the distribution of the exogenous variables with $\sigma = 1$ returning the stochastic model we analyze and $\sigma = 0$ its deterministic counterpart. All of the objects in the approximation can be recovered from Taylor series in σ using the derivative information available from standard higher order perturbations at the deterministic steady state. Appendix B provides a self-contained overview of the derivations involved in this approximation.

The tension between the nonlinearity need to capture the time-varying effects of risk underlying asset prices on the one hand and the difficulties of using nonlinear estimation routines on such models on the other is highlighted by van Binsbergen et al. (2012), who model inflation as exogenous in a New Keynesian model to make the particle filter tractable. The advantage of a linear in state approximation for estimation has also been noted by, e.g., Ang and Piazzesi (2003), Hamilton and Wu (2012), Dew-Becker (2014). Our approach compromises between nonlinearity in risk and the endogenous stochastic discount factor to price financial variables consistent with the macroeconomy on the one hand, and the need for linearity in states to make the estimation of medium scale policy relevant models feasible on the other. To further reduce the computational burden, we apply the PoP method of Andreasen and Zabczyk (2015) that solves the model in a two-step fashion.

3.2. Data

We estimate the model with quarterly U.S. data from 1983:q1 to 2007:q4, covering the Great Moderation and stopping before the onset of the Great Recession. While the systematic behavior of monetary policy is an important driver of the yield curve, as pointed out, for example, by Campbell, Pflueger, and Viceira (2014), we chose a time episode which is characterized by a relatively stable monetary policy regime.First, it is widely accepted in the literature that the U.S. faced a systematic change in monetary policy after Paul Volcker became chairman of the Federal Reserve. Second, the start of the Great Recession, the financial crisis of 2008, along with the zero interest policy rates that prevailed from December 2008 onward marks another structural change in U.S. monetary policy.

Our estimation uses four macroeconomic time series, six time series from the nominal yield curve, and two time series of survey data on interest rate forecasts.[6] The macroeconomic series comprise real GDP growth, real private investment growth, real private consumption growth, and annualized GDP deflator inflation. While the last is measured in levels, the remaining variables are expressed in per capita log-differences using the civilian noninstitutional population over 16 years (CNP16OV) series from the U.S. Department of Labor, Bureau of Labor Statistics.

The nominal yield curve is measured by the 1-quarter, 1-year, 3-year, 5-year, and 10-year annualized interest rates of U.S. Treasury bonds. The data are from Gürkaynak, Sack, and Wright (2007) with the exception of the 1-quarter rate, where we use the 3-month T-Bill rate from the Board of Governors of the Federal Reserve System. To have a consistent description of the yield curve, we use this interest rate as the policy rate ($R_t^f = R_{1,t}^{\$}$) in our model instead of the effective Fed funds rate.

Among others, Kim and Orphanides (2012); Andreasen (2011) have shown that survey data on interest rate forecasts can improve the identification of term structure models. For this reason, we incorporate 1 and 4-quarter ahead expectations of the 3-month T-Bill from the Survey of Professional Forecasters into the estimation.

3.3. Bayesian Estimation

We now present our priors for the parameters we estimate and the calibration of those we do not. Given the choice of our observable variables and the characteristics

[6] See Appendix C for details on the source and a description of all data used in this paper.

of our model, for example, the highly stylized labor market, some of the model parameters can hardly be expected to be identified. These parameters are calibrated either following the literature or related to our observables and are summarized in Table 1. The remaining parameters of the model are estimated.

We calibrate the steady state growth rates, $\bar{z}^+$ and $\bar{\Psi}$ to 0.54/100 and 0.08/100 which implies growth rates of 0.54 and 0.62 percent for GDP and investment as in our sample. Moreover, we calibrate the capital depreciation rate, δ, to 10% per year and the share of capital, α, in the production function to 1/3. We also assume that in the deterministic steady state, the labor supply $\bar{l}$ and government consumption to GDP ratio $\bar{g}/\bar{y}$ are 1/3 and 0.19, respectively. The discount rate β is set equal to 0.99 and the steady state elasticity of substitution between intermediate goods θ_p to 6, implying a markup of 20%. Following Andreasen et al. (2017), we set the price indexation $\xi_p = 0$ and calibrate the Frisch elasticity of labor supply FE to 0.5. Hence, we can solve recursively for $\chi = 1/FE \cdot \left(1/\bar{l} - 1\right)$.

Description	Symbol	Value
Technology trend in percent	$\bar{z}^+$	0.54/100
Investment trend in percent	$\bar{\Psi}$	0.08/100
Capital share	α	1/3
Depreciation rate	δ	0.025
Price markup	$\theta_p/(\theta_p - 1)$	1.2
Price indexation	ξ_p	0
Discount factor	β	0.99
Frisch elasticity of labor supply	FE	0.5
Labor supply	$\bar{l}$	1/3
Ratio of government consumption to output	$\bar{g}/\bar{y}$	0.19

Table 1—Parameter calibration.

Since our focus is to jointly explain macroeconomic and asset pricing facts, we pay special attention to selected first and second moments. The practical problem boils down to having just one observation on the means, e.g., of the slope, curvature, and level of the yield curve, while there are many observations to identify parameters crucial for the model dynamics. To mitigate this imbalance, we apply an endogenous prior approach similar to Del Negro and Schorfheide (2008) and Christiano, Trabandt, and Walentin (2011) and begin with a set of initial priors, $p(\theta)$, independent across parameters. Then, we use two sets of first and second moments from a pre-sample,[7] treating them in separate blocks to capture potentially different precisions of beliefs regarding these moments.

We focus on the first moments of inflation and as well as means of level, slope, and curvature factors of the yield curve. We include the mean of inflation because the non-linearities in our model impose strong precautionary motives that push the predicted ergodic mean of inflation away from its deterministic steady state, $\bar{\pi}$, as is also discussed by Tallarini (2000) and Andreasen (2011). The second moments of interest are is a set of variances of macroeconomic variables (GDP growth, consumption growth, investment growth, inflation, and the policy rate).[8]

4. Estimation Results

We now turn to the results of our estimation. We begin with the estimated parameters, turn then to the predicted first and second moments of endogenous variables, and conclude with a comparison of the estimated components of the ten-year yield predicted by our model with those from the literature.

4.1. Parameter Estimates

As discussed in section 3, our solution method, unlike standard perturbations (e.g. Andreasen et al., 2017), maintains linearity in states and shocks which allows us to estimate the model with the standard Bayesian techniques familiar to linear DSGE analysis. We estimate the posterior mode of the distribution and employ a random walk Metropolis-Hasting algorithm to simulate the posterior distribution of the parameters, quantifying the uncertainty of our estimates. We run two chains,

[7]We follow Christiano et al. (2011) and use the actual sample as our pre-sample because of the monetary regime changes.

[8]In the supplemental appendix D, we describe the method of endogenously formed priors regarding first and second moments as well as its practical application in the paper.

each with 100,000 parameter vector draws where the first 50% have been discarded. Table 2 provides the resulting posterior mode, posterior mean and the 90% posterior credible set of the estimated parameters.[9] The results indicate that the posterior distributions of all structural parameters are well approximated and differ from the initial prior distribution.

Name	Symbol	Mode	Mean	5%	95%
Relative risk aversion	RRA	89.860	91.427	75.581	108.489
Calvo parameter	γ_p	0.853	0.855	0.843	0.866
Investment adjustment	ν	1.417	1.440	1.204	1.667
Habit formation	b	0.685	0.679	0.614	0.741
Intertemporal elas. substitution	IES	0.089	0.089	0.077	0.101
Steady state inflation	$100\,(\bar{\pi}-1)$	1.038	1.034	0.981	1.091
Interest rate AR coefficient	ρ_R	0.754	0.752	0.718	0.786
Interest rate inflation coefficient	η_π	3.124	3.164	2.839	3.491
Interest rate output coefficient	η_y	0.156	0.159	0.114	0.204
Inflation target coefficient	$100\zeta_\pi$	0.210	0.242	0.109	0.366
AR coefficient technology	ρ_a	0.366	0.356	0.304	0.412
AR coefficient preference	ρ_b	0.820	0.817	0.793	0.843
AR coefficient investment	ρ_i	0.956	0.955	0.949	0.961
AR coefficient gov. spending	ρ_g	0.910	0.909	0.880	0.937
AR coefficient inflation target	ρ_π	0.934	0.925	0.901	0.950
AR coefficient long-run growth	ρ_z	0.630	0.611	0.500	0.729
AR coefficient fixed cost	ρ_Ω	0.928	0.928	0.922	0.933
S.d. technology	$100\sigma_a$	2.333	2.460	1.929	2.985
S.d. preference	$100\sigma_b$	4.878	4.880	4.180	5.570
S.d. investment	$100\sigma_i$	2.516	2.523	2.337	2.689
S.d. monetary policy shock	$100\sigma_m$	0.561	0.572	0.494	0.653
S.d. government spending	$100\sigma_g$	2.010	2.018	1.825	2.220
S.d. inflation target	$100\sigma_\pi$	0.167	0.180	0.130	0.226
S.d. long-run growth	$100\sigma_z$	0.345	0.353	0.253	0.446
S.d. fixed cost	$100\sigma_\Omega$	9.766	9.705	9.022	10.372
ME 1-year T-Bill	$400R^{\$}_{4,t}$	0.185	0.188	0.161	0.214
ME 2-year T-Bill	$400R^{\$}_{8,t}$	0.084	0.085	0.071	0.100
ME 3-year T-Bill	$400R^{\$}_{12,t}$	0.078	0.081	0.067	0.095
ME 5-year T-Bill	$400R^{\$}_{20,t}$	0.152	0.156	0.130	0.181
ME 10-year T-Bill	$400R^{\$}_{40,t}$	0.287	0.297	0.251	0.346
ME 1Q-expected policy rate	$400E_t\left[R^f_{t,t+1}\right]$	0.456	0.464	0.408	0.522
ME 4Q-expected policy rate	$400E_t\left[R^f_{t,t+4}\right]$	0.738	0.750	0.660	0.842

Table 2—Posterior statistics. Posterior means and parameter distributions are based on a standard MCMC algorithm with two chains of 100,000 parameter vector draws each, 50% of the draws used for burn-in, and a draw acceptance rates about 1/3.

We find a low intertemporal elasticity of substitution ($IES = 0.089$) and a high

[9]Figures E1 and E2 in the appendix illustrate the posterior distribution of each parameter relative to its initial prior distribution.

relative risk aversion ($RRA \approx 90$). Our estimated IES is in line with, e.g., Hall (1988), but differs from estimates in the long-run and valuation risk literatures that argue for an IES above one. With this in mind, it is not surprising that the model needs a high relative risk aversion to fit the data. Nevertheless, our estimate is still in line with much of the existing macro-finance literature (see, for example, van Binsbergen et al., 2012; Rudebusch and Swanson, 2012; Swanson, 2016). Though a direct comparison is difficult as all of these studies use different samples, ours covers the Great Moderation, and their models differ in their specification of structural shocks. As pointed out by van Binsbergen et al. (2012), models that feature a higher volatility of shocks (higher risk) thereby increasing the volatility of the stochastic discount factor need, e.g., less risk aversion to match average bond yields. This notwithstanding, our estimate of risk aversion is higher than in endowment economy studies and in micro-studies (Barsky, Juster, Kimball, and Shapiro, 1997). Potential explanations are present in the literature, with Malloy, Moskowitz, and Vissing-Jørgensen (2009) having shown that risk aversion estimated for stockholders in the U.S. is substantially lower than a representative agent using aggregate consumption (which they find increases to 81) .and Barillas, Hansen, and Sargent (2009) argue that a small amount of model uncertainty can substitute for the large degree relative risk aversion often found in the literature.

We estimate a quarterly deterministic steady state inflation of around 1.04% which is substantially higher than the average observed inflation rate (0.64%). Due to the non-linearities in our model, the difference is related to household's precautionary motives, as also discussed by Tallarini (2000), but the approximated ergodic mean of inflation, see the subsequent subsection, is similar to the average U.S. inflation over our sample.

For the inflation target, we estimate $\rho_\pi = 0.93$ and $\zeta_\pi = 0.002$. The latter coefficient is similar to Rudebusch and Swanson (2012), while the former coefficient is slightly smaller than their calibration, implying a less persistent effect of nominal risk in our model. Moreover, we estimate a moderate size of investment adjustment costs ($\nu = 1.4$) and comparable estimates to the literature for price stickiness ($\gamma_p = 0.85$)

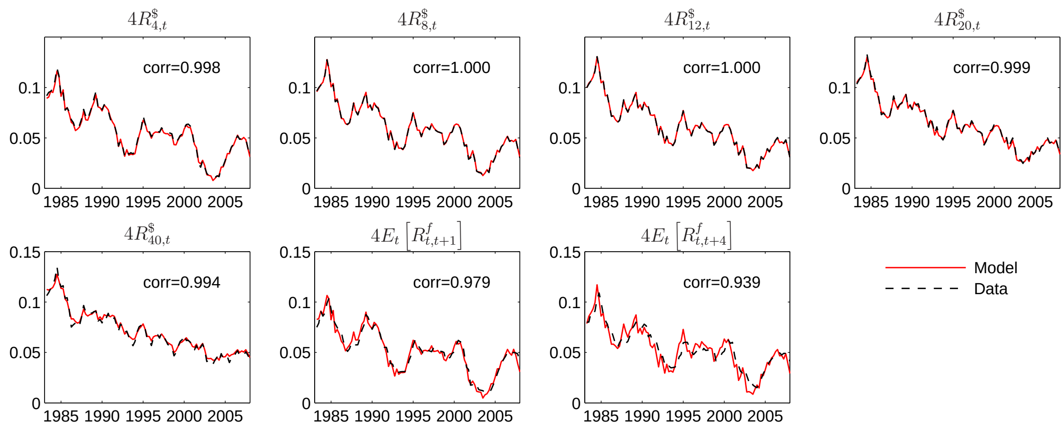


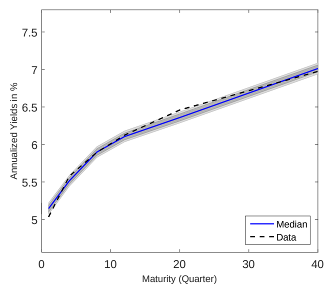
FIGURE 2. OBSERVED AND MODEL IMPLIED NOMINAL RETURNS OF TREASURY BILLS AND RETURNS OF EXPECTED SHORT RATES AT THE POSTERIOR MODE.

and external habit formation ($b = 0.67$). Finally, we find that monetary policy puts more weight on stabilizing the inflation gap ($\eta_\pi = 3.13$) than on the output gap ($\eta_y = 0.16$) and smoothes changes in the policy rate ($\rho_R = 0.75$).

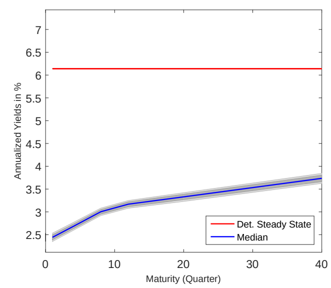
Figure 2 shows the historical time series (dash-dotted line) and the model implied smoothed time series (solid line) for the seven variables estimated with measurement error. Note that we estimate small measurement errors along the yield curve. In particular, the measurement errors range between 7 and 29 basis points, implying a correlation between the smoothed model implied yields and the data of 0.99 or higher. The measurement errors for the 1-quarter ahead and 1-year ahead expectations of the 3-month T-Bill are 45 and 74 basis points, respectively, delivering high correlations (0.94 and 0.98) of our model-based expectations with the data from the Survey of Professional Forecasters.

4.2. Predicted Moments

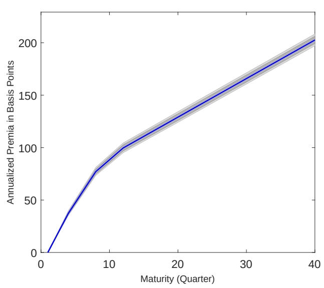
In the following subsection, we begin our posterior analysis with respect to the predicted first and second moments. Figure 3(a) shows the predicted ergodic means of the nominal yields in relation to the means of the corresponding data. The figure illustrates the success of our estimation approach, with the a priori information



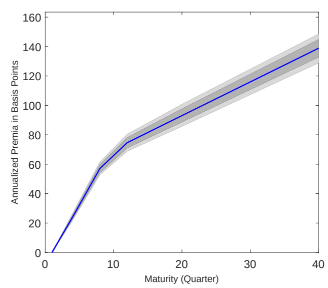
(a) Nominal yield curve



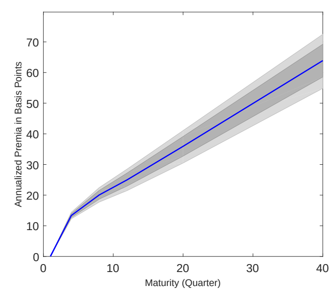
(b) Real yield curve



(c) Nominal term premium



(d) Real term premium



(e) Inflation risk premium

Figure 3. Term structure of interest rates

about the level, slope, and curvature, based on only 3-month, 2-year, and 10-year nominal yields, sufficient to estimate first moments for all maturities.

Backus, Gregory, and Zin (1989) and den Haan (1995) formulated the bond-pricing puzzle with the question as to why the yield curve is upward sloping. That is, long-term bonds should carry an insurance-like negative risk premium and therefore the yield curve should be downward sloping. However, the data for nominal yields as well as estimates for the nominal term premium suggest the opposite, as does our model (see Figure 3(c)). The mechanism can be found in, e.g., Rudebusch and Swanson (2012): supply shocks move consumption and inflation in opposite directions, imposing a negative correlation between the two. Thus, inflation reduces the real value of nominal bonds precisely in states of low consumption when agents would particularly value higher payouts, thereby generating a positive term premium. To this end, Piazzesi and Schneider (2007) show that consumption and inflation were negatively correlated in the period 1952-2004 for the U.S., which suggests that supply shocks play a relatively important role in generating the upward sloping nominal term structure in the data and in our model.

The negative correlation between consumption growth and inflation can explain the positive slope in the nominal term structure by appealing to inflation risk, but absent another mechanism cannot account for the real term structure. If it is solely inflation risk driving the upward slope of the nominal term structure, then the real term structure should be downward sloping as spells of low consumption growth will be associated with low real rates (and hence high prices for real bonds). This gives agents a higher payout precisely when they would value it highly and implying that real bonds should carry negative, insurance-like risk premia. Nevertheless, as illustrated by Figures 3(b) and 3(d), our model also predicts an upward-sloping real term structure which is in line with the literature (see, for example, Gürkaynak et al., 2010; Chernov and Mueller, 2012). The mechanism in our model follows that described in Wachter (2006) and Hördahl, Tristani, and Vestin (2008), as our households' habit formation introduces a hump-shaped response of consumption. This makes consumption growth positively autocorrelated while reducing agents' precautionary saving motive for longer maturities: households will seek to maintain their habit in the face of a slowdown in consumption, drawing down their precautionary savings and driving down real bond prices, implying that payouts on real bonds are negatively correlated with marginal utility and that real bonds demand a positive risk premium. The precautionary motive is illustrated in Figure 3(b), where the red line shows the real yield curve in absence of risk, i.e., at the deterministic steady state. When confronted with risk, agents accumulate additional capital, driving down its return. This reduction, however, is decreasing in the maturity due to the positive real risk premium, resulting in our estimated upward sloping real term structure.

Figure 3(e) shows that our model predicts an upward sloping inflation risk premium consistent with recent estimates in the literature (see, for example, Abrahams et al., 2016), with our ergodic mean term structure of inflation risk comfortably between the estimates of Buraschi and Jiltsov (2005) and Chen, Liu, and Cheng (2010).

The ergodic mean of inflation risk is approximately half the size of the real term premia for all maturities, consistent with Kim and Wright's (2005) estimates for the ten year inflation and real risk premia. Consequentially, our results suggest that most of the average slope of the nominal term structure is related to real rather than to inflation risk. Again, this finding is consistent with recent estimates for the U.S. (see, for example, Kim and Wright, 2005) and is also qualitatively comparable to the results by Hördahl and Tristani (2012) for the Euro area. So far most of the DSGE models (see, for example, van Binsbergen et al., 2012; Swanson, 2016) generally attribute a stronger insurance-like character to real bonds, that lead to flat or downward sloping real yield curves.

Name	Data		Model	
	Mean	S.d.	Mean	S.d.
GDP growth	0.540	0.573	0.540	0.790
			[0.368; 0.712]	[0.665; 0.934]
Consumption growth	.610	0.411	0.540	0.545
			[0.378; 0.702]	[0.453; 0.652]
Investment growth	0.620	2.049	0.620	2.253
			[0.297; 0.943]	[1.854; 2.720]
Annualized inflation	2.496	0.922	2.469	1.117
			[1.829; 3.108]	[0.930; 1.372]
Annualized policy rate	5.034	2.222	5.144	2.461
			[3.058; 7.235]	[1.763; 3.524]
1-year T-Bill	5.578	2.400	5.515	2.174
			[3.567; 7.464]	[1.517; 3.203]
2-year T-Bill	5.896	2.431	5.900	1.886
			[4.158; 7.642]	[1.281; 2.834]
3-year T-Bill	6.125	2.421	6.107	1.682
			[4.549; 7.665]	[1.135; 2.541]
5-year T-Bill	6.460	2.357	6.360	1.383
			[5.083; 7.641]	[0.932; 2.089]
10-year T-Bill	6.975	2.198	7.014	0.927
			[6.054; 7.974]	[0.624; 1.414]

Table 3—Simulated and empirical moments of selected macro and financial variables.

Note: The simulated moments are based on 1200 parameter vector draws from the posterior. For each draw, we simulate 1000 time series for each variable of interest. After removing a burn-in of 5000 periods for each simulation the final simulated time series have the same length (T=100) as the vector of observables. The number in brackets indicate 5% and 95% probabilities.

Table 3 compares empirical and simulated first and second moments of selected variables.[10] The results illustrate that our estimation approach delivers an ergodic mean of inflation comparable to the mean of the data and, as a result, captures households' precautionary savings motives. Moreover, the simulated second moments regarding the macroeconomic variables are in line with the data, highlighting the ability of our New Keynesian DSGE model to match financial and macroeconomic moments jointly (see also Andreasen et al., 2017).[11] Regarding treasury bonds, our model misses the high volatility for longer maturities, but matches the monotonic decrease in volatility with the maturity. This result in general equilibrium models has been described in den Haan (1995) and is related to some missing source of persistence in the model (see Hördahl et al., 2008). We do not see this, however, as a fatal shortcoming of our analysis. Firstly, the uncertainty related to these moments is quite high and, secondly, it rather illustrates the tension in the competing goals the model faces: matching highly volatile nominal treasury bonds while predicting a very smooth inflation rate.

4.3. Model Implied Historical Fit

In the following subsection, we discuss our model implied historical time series for the nominal term premium, break-even inflation rate, real rate, and inflation risk premium. It is important to stress that these measures did not enter into our estimation and, instead, are produced as estimated latent variables in our analysis. To judge the quality of our estimated model, we contrast our estimates with various estimates from the literature. Following the majority of the empirical literature, we limit our discussion to 10-year maturities.

In Figure 1, we compare our 10-year nominal term premium with several different prominent estimates from the literature. As Rudebusch et al. (2007) show, all of the estimated term premia, which they investigate, follow a similar pattern and are highly correlated. This is also true for our extented sample which includes two more

[10] Appendix E presents further statistics for the DSGE model.

[11] Additionally, figure E3 in appendix E presents the autocorrelation of hp-filtered macroeconomic variables which also illustrates the good fit.

recent estimates by Adrian et al. (2013) and Bauer (2018).[12] Table 4 presents the correlations between these five measures of the term premium and the estimate of our model. Our estimate shows also a remarkably high correlation with all measures, but especially with those of Kim and Wright (2005) and Bauer (2018) (0.94 and 0.93, respectively). Given that our model is arguably closest in structure to the model used by Rudebusch and Wu (2008), we would have expected our model to display a much higher correlation with their measure than it actually does. Also, while the model by Rudebusch and Wu (2008) predicts a smooth term premium, all other models including the model presented in this paper predict a much more volatile measure.

	Bernanke et al.	Rudebusch and Wu	Kim and Wright	Adrian et al.	Bauer	S.d.
Bernanke et al. (2004)	1.000					1.294
Rudebusch and Wu (2008)	0.763	1.000				0.336
Kim and Wright (2005)	0.976	0.811	1.000			0.981
Adrian et al. (2013)	0.817	0.941	0.891	1.000		1.033
Bauer (2018)	0.853	0.734	0.936	0.885	1.000	1.182
Model	0.904	0.800	0.940	0.868	0.932	0.943

Table 4—Six measures of the 10-year term premium.

Note: Correlations among six measures of the 10-year term premium from 1984:q1-2005:q4. The last column presents the standard deviation over the sample. Statistics related to the estimates by Bauer (2018) are based on a shorter sample starting 1990.

The reason that our model produces a large and volatile term premium is similar to explanations postulated in the recent literature (see, for example, Andreasen et al., 2017). Beside the role of supply shocks in our model that generate a sizable term premium, the presence of long-run nominal risk is important in generating a volatile term premium (see Rudebusch and Swanson, 2012). Additionally, our model captures a channel recently postulated by Andreasen et al. (2017), namely the role of steady-state inflation for the mean and volatility of risk premia. In

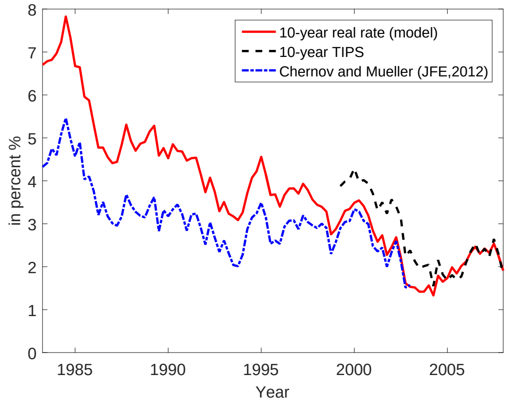
[12]The estimates by Bauer (2018) start in 1990, so all calculations using this estimate are restricted to a shorter sample.

particular, steady-state inflation generates more heteroscedasticity in the stochastic discount factor which eventually produces more volatile risk premia. This channel is present despite the fact that the shocks in our model are all homoscedastic. More specifically, the endogenously generated heteroscedasticity in the pricing kernel is a byproduct of the heteroscedasticity in price dispersion due to positive steady-state inflation.

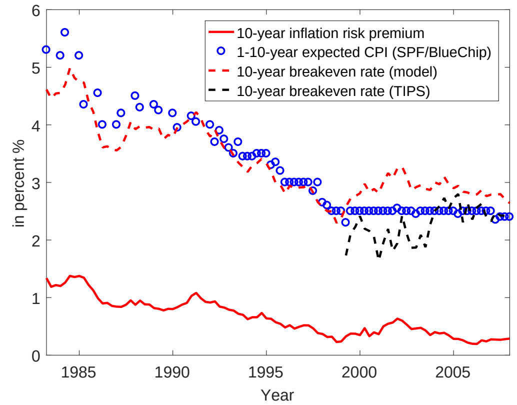
Figure 4(a) compares our 10-year real rate with the estimates provided by Gürkaynak et al. (2010) using TIPS data and those of Chernov and Mueller (2012) using survey-based forecasting data. Both measures are not fully identical with the real rate measured by our model, for example, while our real rates are based on GDP inflation the aforementioned measures are based on CPI data. Also, our model has no role for a liquidity premium component that is arguably a non-negligible component of TIPS (see, for example, Abrahams et al., 2016). Nevertheless, our estimate captures the downward trend since the 1980s found likewise in Chernov and Mueller (2012). Additionally, our estimate demonstrates a high correlation with both (0.9 with Gürkaynak et al. (2010) and 0.94 with Chernov and Mueller (2012)) of these alternative measures, derived from empirical reduced-form models.

Figure 4(b) shows the model implied 10-year break-even inflation rate. At the beginning of the sample, the breakeven inflation rate declines continuously until 1998. From 1999 onward we find a stable breakeven rate fluctuating around 3 percent. Over this period, our estimate is comparable in levels and pattern with those by Gürkaynak et al. (2010). Moreover, the continuous decline in the model's breakeven rate until 1998 is accompanied by a decreasing inflation risk premium. This pattern is commensurate with declining inflation expectations in this period.

In summary, our model-implied estimates of the components of 10-year bond yields demonstrate a considerable alignment with various empirical estimates in the literature. This alignment is all the more remarkable as these components of the yields were not used in our estimation procedure. Our results reiterates Swanson's (2016) conclusion that DSGE models with recursive preferences and nominal rigidities can jointly replicate observations on the macroeconomy and financial markets



(a) 10-year real interest rate.



(b) 10-year break-even rate and its components.

FIGURE 4. 10-YEAR REAL INTEREST RATE AND 10-YEAR BREAK-EVEN RATE.

Note: The left panel shows the model implied 10-year real rate (red solid), 10-year TIPS of Gürkaynak et al. (2010) (black dashed), and 10-year real rate of Chernov and Mueller (2012)(blue dash-dotted). The right panel shows the model implied 10-year inflation risk premium (red solid) and the 10-year break-even inflation rate (red-dashed), the 10-year break-even inflation rate of Gürkaynak et al. (2010) (black dashed), and 1 to 10-year average expected CPI inflation from SPF and BlueChip (blue circle).

and extends this result to a richer and estimated model. This provides us with a high degree of confidence in our model's ability to replicate stylized term structure facts as we now turn to the structural analysis of the effects of monetary policy on the term structure of interest rates and its components.

5. Monetary Policy Through the Lens of Our Model

5.1. Comparison of Monetary Policy Tools

In this subsection, we analyze the effects of monetary policy shocks on term premia and distinguish between three different policy actions.[13] First, a surprise shock to the policy rate via the residual of the Taylor rule. Second, a shock to the inflation target that might be interpreted as a change in the systematic component of monetary policy (see Cogley et al., 2010) as it affects agents' perception of inflation in the long run. Third, we investigate the effects of a commitment by the

[13] We again use the risk-adjusted linear method of Meyer-Gohde (2016). See the supplementary appendix E.E3 for generalized impulse responses using a third order perturbation as well as an investigation into the role of certainty nonequivalence in our risk-adjusted linear approximation.

monetary authority to a path for future short rates; i.e., forward guidance by means of a credible announcement to change the policy rate in the future while holding it constant until then.[14] The first two actions follow directly from the model, we implement the forward guidance scenario by altering the Taylor rule in eq. (13) following Laséen and Svensson (2011) and others by adding a sequence of anticipated shocks to the Taylor rule that allow the monetary authority to keep the policy rate upon announcement constant until the announced interest rate change (here a cut) is implemented as follows

$$r_t^f = R\left(r_{t-1}^f, \pi_t, y_t\right) + \sigma_m \left(\epsilon_{m,t} + \sum_{k=1}^{K} \epsilon_{m,t+k}\right), \ \epsilon_{m,t+k} \overset{iid}{\sim} N(0,1) \tag{25}$$

where $R(\cdot)$ characterizes the systematic response of monetary policy, $\epsilon_{m,t}$ is the usual contemporaneous policy shock, and $\sum_{k=1}^{K} \epsilon_{m,t+k}$ a sequence of policy shocks known to agents at time t but that affect the policy rule k periods later, i.e., at time $t+k$.

Figure 5 shows the impact responses of the nominal and real term structures.[15] The unexpected monetary policy shock (left column) shows that the response on impact of the term structure becomes more muted with the maturity, as would be expected in accordance with the expectations hypothesis and the path of the policy rate (assumed identical to the short rate). Similarly, the response on impact of the real yield curve, see the second row of Figure 5, is driven primarily by the expectations hypothesis and the Fisher equation with the response likewise becoming more muted with the maturity.

With the expectations hypothesis being the predominate driver of the impact on real and nominal rates, an unexpected monetary policy shock – a simple innovation

[14] While this may seem a narrow aspect of recent experience with unconventional monetary policy, Woodford (2012), for example, argues that even quantitative easing itself can at least partially be interpreted as forward guidance through the signalling channel, building on results by e.g. Bauer and Rudebusch (2014). Furthermore, forward guidance has been a component of standard monetary policy at major central banks even before its explicit implementation since the financial crisis (see Gürkaynak et al., 2005a).

[15] Figures E9 and E10 in Appendix E.E5 present the dynamic responses of macroeconomic variables and of 1-year and 10-year maturities, respectively.

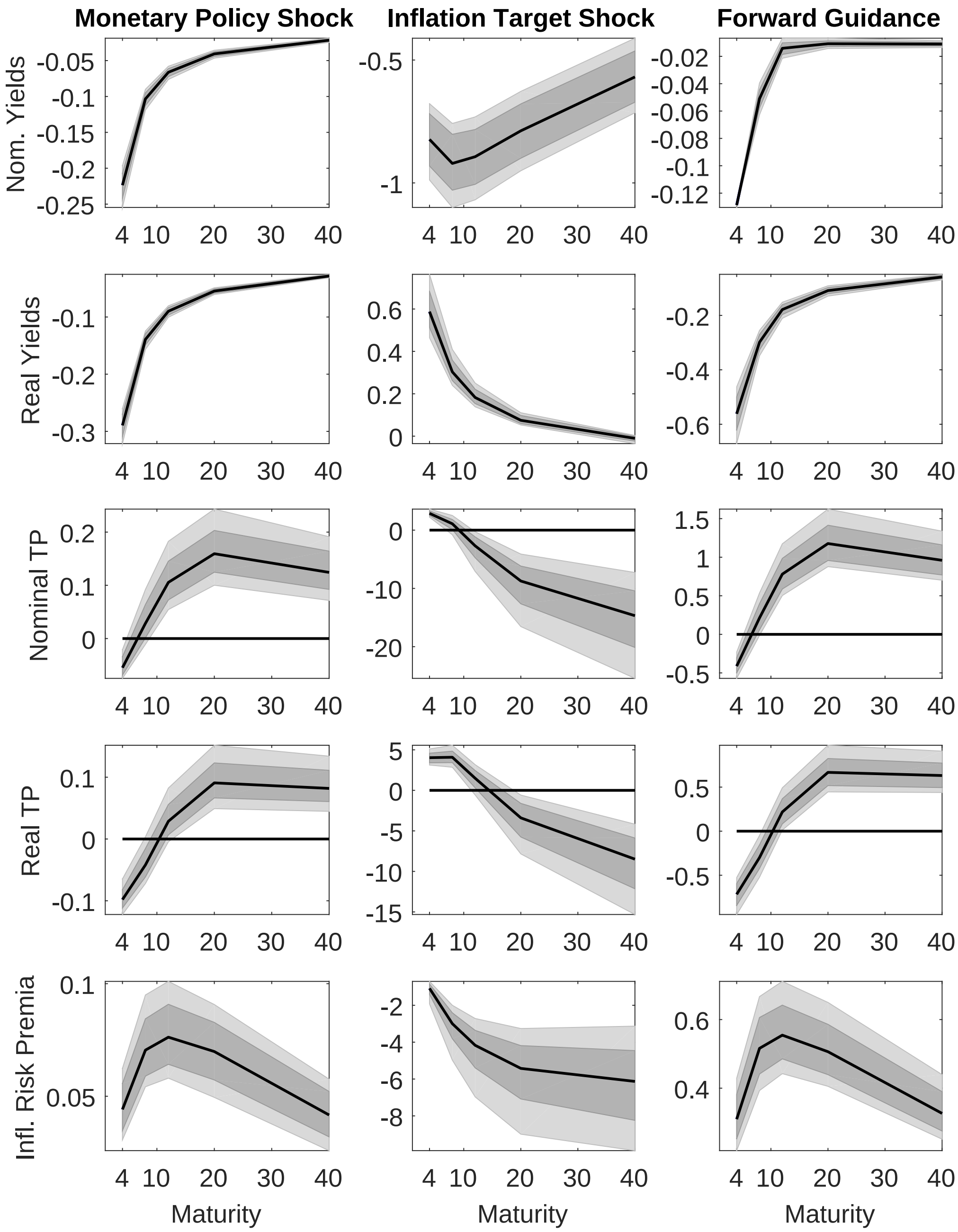


Figure 5. Impact responses of nominal and real term structures.

Note: The figure shows the impact response across all maturities to a surprise 50 basis point policy rate cut, a surprise cut in the inflation target leading to a 50 basis point policy rate cut, and forward guidance of a 50 basis point policy rate cut in 4 quarters. The deviations of yields are in percentage points while the deviations of risk premia are presented in basis points. Shaded areas represent the 90% and 68% posterior credible sets.

to the Taylor rule – has limited, though nonzero, effects on the risk premia along all maturities. This finding is in line with those of other structural models (see, for example, Rudebusch and Swanson, 2012). On impact, see the third row of Figure 5, bond holders demand higher total premia for holding nominal bonds for longer maturities and lower total premia for shorter maturities.The stimulative effects in the short run generate increased confidence in the absence of downside risks to the economy, reflecting the fall in the short run premia. The delayed contractionary effects of the loosening of monetary policy are reflected in the higher medium to long run premia demanded on impact. The effects on impact for the real term premia qualitatively mirror those of the nominal term premia, confirming that the primary driver of the nominal term premia is indeed the real economy and associated risks. On impact, the real term premia, see the fourth row of Figure 5, are shifted downward across all maturities relative to the impact response of the nominal term premia, reflecting the elevation in the inflation risk premia, see the bottom row of Figure 5, demanded by investors in response to the inflationary effects of the expansionary monetary policy. The negative initial response of real term premia associated with shorter maturities and positive response of those associated with longer maturities can be understood roughly from the comovement of the real yields and the consumption relative to its habit in the pricing kernel. Yields on real bonds at all maturities drop on impact whereas consumption relative to its habit initially rises but then falls. This generates a positive comovement between the pricing kernel and bond prices on shorter maturities that thus contain a negative, insurance-like premium. At longer maturities, this comovement becomes negative as consumption drops relative to its habit and thus real bonds of longer maturities bear a positive risk premium to induce households to hold these bonds that pay less when payoffs are more highly valued. The timing of when the ten year real term premium turns negative coincides with the onset of the contraction in the real economy. Finally, on impact, investors demand a higher premium across all maturities to compensate them for inflation risks associated with the surprise change in monetary policy.

In contrast, a surprise shock to the inflation target has a much stronger effect on the risk premia of interest rates across all maturities, see the second column of Figure 5, with the effects roughly two orders of magnitude larger. While this stronger effect on the nominal term premia can also be found in Rudebusch and Swanson (2012), our findings show that monetary policy substantially affects real term premia. This is consistent with the interpretation of the shock to the inflation target as being a shift in the systematic monetary policy: long run downside risks to the economy are reduced by the more aggressive response of monetary policy at the cost of heightened short run risks. With both the inflation target and realized inflation reduced by the more aggressive posture of monetary policy towards inflation, investors' perception of upside risks to inflation are ameliorated, leading to a reduction in the inflation risk premia that they demand at all horizons on impact, see the bottom middle panel of Figure 5. While the nominal term premia are still primarily driven by risks associated with the real economy in response to the inflation target shock, the effects of inflation risk premia are disproportionately increased in magnitude, consistent with the interpretation of this experiment being not only a change in the systematic response of monetary policy, but more specifically a more aggressive posture towards inflation.

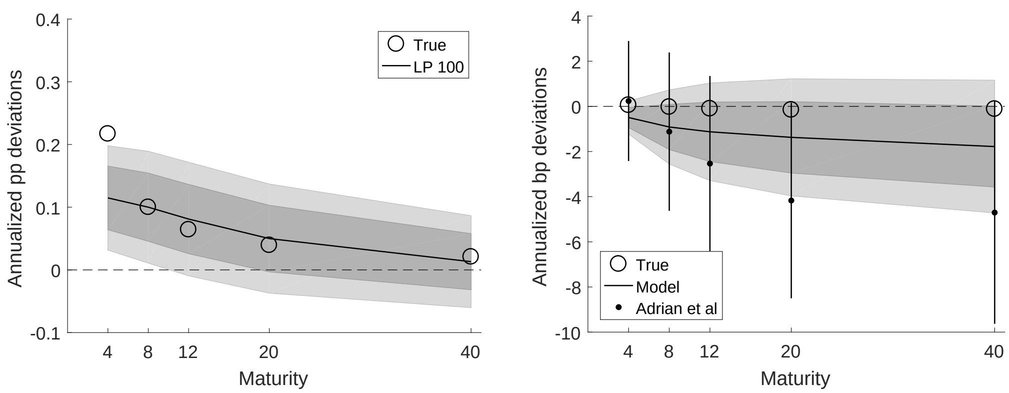
Turning to forward guidance, the dynamic responses of interest rates are driven by the countervailing effects of the expectations hypotheses and risk premia. As in standard models under the expectations hypothesis, the dynamics of interest rates with longer maturities reflect the dynamic adjustment of the risk free short rate, determined by the monetary authority's Taylor rule. The large effects on inflation and output imply that the policy rate rises quickly above its ergodic mean only few quarters after its announced fall. This explains, at least in part, why we observe only a mild drop on impact in nominal bonds with a maturity longer than 2 years (see the upper right panel of Figure 5). While the yield of a 1-quarter real bond falls by around 30 basis points on impact, the yield of a 10-year real bond falls by around 3 basis points (see the second row of the right column in Figure 5). Our findings illustrate that bondholders demand higher nominal premia on impact for all maturities from 2 years onward to compensate them for the downside risks they perceive in the nominal economy. This is in line with the empirical findings of Akkaya et al. (2015). While there is some increased short to medium term confidence in the real economy, as can be seen by the fall in the real premium demanded for two year real bonds on impact, this is outweighed by the larger increase in inflation risk perceived by the bondholders, see the bottom two rows of the right column in Figure 5. This overall increase in risk premia prevents nominal and real long rates from falling as strongly as the expectations hypothesis would predict and therefore dampens the expansionary effects of the announced cut in the policy rate. Finally, the increase in inflation risk premia follows what theory would predict. While forward guidance does communicate the expected path of future short rate, it is just as informative about the central bank's commitment to allow higher inflation in the future. This commitment drives households' demand for higher inflation risk premia.

In sum, our findings show the importance to distinguish between different policy tools when assessing the effects of monetary policy shocks on term premia.[16] Unexpected monetary policy shocks die out quite quickly, limiting their effects on business cycle frequencies and, consequentially, on risk premia. But news about monetary policy, which reveal information about macroeconomic variables in the future have quantitatively much stronger effects. To this end, our structural model confirms the empirical finding from Gürkaynak et al. (2005a) but highlights that monetary policy communications transmits on long-term bonds especially through risk premia. Most of the empirical literature, which investigates the effects of monetary policy shocks on term premia, focuses on samples starting in the early 1990s. At this time, the Federal reserve has increasingly used communication as a policy tool. To this end, our findings suggest that the quantitative strong effects on term premia found by the empirical literature are primarily driven by monetary policy news about their mid- or longterm stance rather then changes in the policy rate.

[16] See the supplementary appendix E.E3 for a robustness exercise using generalized impulse responses in a third order perturbation as well as an investigation into the role of certainty nonequivalence in our risk-adjusted linear approximation.

5.2. Comparison with Empirical Findings

In the following subsection, we compare the findings from our structural model with those from the empirical literature, focusing on the effects of a surprise shock to the policy rate via the residual of the Taylor-rule. We run a local linear projection following Jordà (2005) by regressing the variables of interest like treasury yields and historical estimates for nominal term premia on the monetary policy shock identified by our model.[17]



(a) Nominal yield curve (b) Nominal term premia

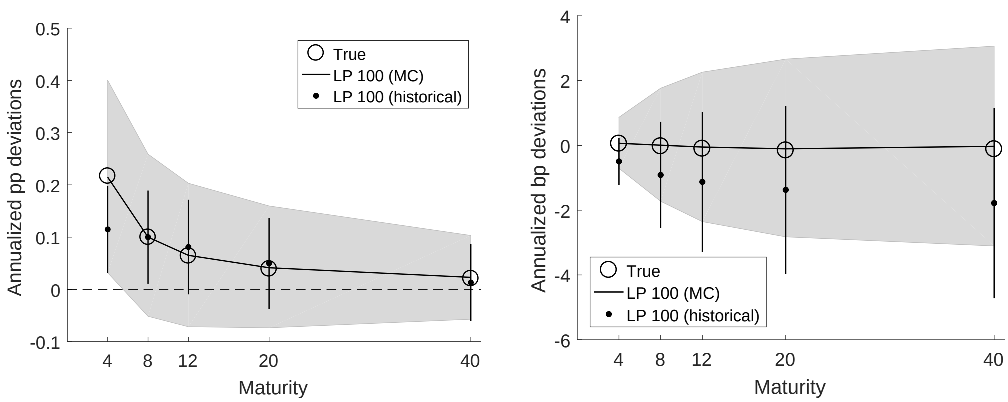
Figure 6. Impact effect of monetary policy shock on nominal yields and nominal term premia for different maturities.

Note: The solid line and shaded areas show median response, the 68%, and 90% confidence bands from the local projection with the model implied historical term premia as dependent variable, respectively. The circles indicate the theoretical, true response. Additionally, the dots and vertical lines in the right panel show median response and 90% confidence bands from the local projection with term premia estimates from Adrian et al. (2013). We use the Newey-West correction for the standard errors.

Figure 6 shows the impact effects of a surprise monetary policy shock across maturities. We scaled the median response of the 2-year treasury bond to be 0.1 annualized percentage points which ensures that the impact responses, especially at longer maturities, are comparable. As can be seen in Figure 6(a), the local projection fails to recover the true impact effect at shorter maturities, underestimating the true monetary policy surprise and imposing too strong a persistence across maturities.

[17]Appendix E.E4 provides additional details of the empirical model along with further results.

Figure 6(b) shows the effects on nominal term premia. The impact responses from the empirical model contain the true theoretical responses, but are not significantly different from zero for all maturities (on 90% confidence level). At a 68% confidence level, the impact response is negative for longer maturities as are our theoretical responses. This finding also holds when using different estimates for the nominal term premium (those of Adrian et al. (2013) are also depicted in Figure 6(b)).[18] All the local projection estimates deliver qualitatively and quantitatively similar results: A tightening of monetary policy rates reduces nominal term premia, especially for longer maturities, in line with the empirical work by Nakamura and Steinsson (2017) and Crump et al. (2016) but in contrast to, e.g., Gertler and Karadi (2015).



(a) Nominal yield curve (b) Nominal term premium

FIGURE 7. MONTE-CARLO EXERCISE

Note: Impact effect of monetary policy shock on nominal yields and nominal term premia for different maturities. The solid line and shaded areas show median response and 90% confidence bands from the local projection with sample length 100 based on 1000 simulations, respectively. The circles indicate the theoretical, true response. Additionally, the dots and vertical lines show median response and 90% confidence bands from the local projection with historical model implied term premia.

However, we also find that local projections using our model's smoothed series predict larger effects than the true, theoretical responses. For further investigation, we perform a Monte-Carlo exercise with simulated time series for nominal yields, monetary policy shocks, and nominal term premia from our model at the poste-

[18]In Appendix E.E4, we show that this also holds for an alternative measure for monetary policy shocks as well as for most of the empirical estimates of the 10-year nominal term premia in the literature.

rior mean.[19] Figure 7 shows that the empirical model on average replicates the true response of the DGP and, therefore, shows no systematic small sample bias (see discussion in Jordà, 2005). Nevertheless, the estimation uncertainty is especially pronounced in small samples, capturing a wide range of quantitatively and qualitatively different estimates in the empirical literature.

6. Conclusion

This paper provides a medium scale macro-finance model, estimated with full information on U.S. macroeconomic and Treasury bond data. We estimate historical time series of term premia that match those found in reduced form empirical estimates without sacrificing the macroeconomic fit or other financial variables. We therefore provide a structural framework for the analysis of endogenous, time-varying term premia.

Distinguishing between different monetary policy actions is important. While unexpected shocks to the policy rate have quantitatively small effects, shocks revealing information about the future of monetary policy (e.g. forward guidance) can have quantitatively much stronger effects. Hence, with this disentangling of shocks, an ongoing challenge for empirical models, our findings can provide insight on some of the seemingly contradictory findings in this literature (see, for example among others, Hanson and Stein, 2015; Nakamura and Steinsson, 2017; Gertler and Karadi, 2015).

We offer a first step toward understanding the transmission of monetary policy on the term structure of interest rates from a structural Bayesian perspective, but many salient questions need further investigation. For example, while our model features a frictionless asset trade, a model featuring market segmentation could affect the policy conclusions of our paper (see, for example, Fuerst, 2015). Additionally, a further extension would be the incorporation of the zero lower bound for interest rates, which remains a not fully resolved methodological challenge for nonlinear

[19] We run 1000 simulations with a sample length of 100 after having discarded the first 5000 observations. Afterwards, we run the same local projections as before.

DSGE models as well affine term structure models. Moreover, investigating the impact of unconventional monetary policy on risk premia or the impact of monetary policy on asset valuation more generally are natural questions of currently high interest. We acknowledge but leave these extensions for future work, providing an estimated macro-finance model in this paper able to provide a structural analysis of the impact of monetary policy on the term structure of interest rates.

References

Abrahams, M., T. Adrian, R. K. Crump, E. Moench, and R. Yu (2016): "Decomposing real and nominal yield curves," *Journal of Monetary Economics*, 84, 182–200.

Adrian, T., R. K. Crump, and E. Moench (2013): "Pricing the term structure with linear regressions," *Journal of Financial Economics*, 110, 110–138.

Akkaya, Y., R. Gürkaynak, B. Kisacikoğlu, and J. Wright (2015): "Forward guidance and asset prices," *IMES Discussion Paper Series*, E-6.

Altig, D., L. Christiano, M. Eichenbaum, and J. Linde (2011): "Firm-Specific Capital, Nominal Rigidities and the Business Cycle," *Review of Economic Dynamics*, 14, 225–247.

Andreasen, M. M. (2011): "Explaining Macroeconomic and Term Structure Dynamics Jointly in a Non-linear DSGE Model," Creates research papers, School of Economics and Management, University of Aarhus.

——— (2012): "An estimated DSGE model: Explaining variation in nominal term premia, real term premia, and inflation risk premia," *European Economic Review*, 56, 1656–1674.

Andreasen, M. M., J. Fernández-Villaverde, and J. F. Rubio-Ramírez (2017): "The Pruned State-Space System for Non-Linear DSGE Models: Theory and Empirical Applications," *Review of Economic Studies*, forthcoming.

Andreasen, M. M. and P. Zabczyk (2015): "Efficient bond price approximations in non-linear equilibrium-based term structure models," *Studies in Nonlinear Dynamics & Econometrics*, 19, 1–33.

Ang, A. and M. Piazzesi (2003): "A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables," *Journal of Monetary Economics*, 50, 745–787.

Backus, D. K., A. W. Gregory, and S. E. Zin (1989): "Risk premiums in the term structure : Evidence from artificial economies," *Journal of Monetary Economics*, 24, 371–399.

Bansal, R. and A. Yaron (2004): "Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles," *Journal of Finance*, 59, 1481–1509.

Barillas, F., L. P. Hansen, and T. J. Sargent (2009): "Doubts or variability?" *Journal of Economic Theory*, 144, 2388–2418.

Barsky, R. B., F. T. Juster, M. S. Kimball, and M. D. Shapiro (1997): "Preference Parameters and Behavioral Heterogeneity: An Experimental Approach in the Health and Retirement Study," *The Quarterly Journal of Economics*, 112, 537–579.

Bauer, M. D. (2018): "Restrictions on Risk Prices in Dynamic Term Structure Models," *Journal of Business & Economic Statistics*, 36, 196–211.

Bauer, M. D. and G. D. Rudebusch (2014): "The Signaling Channel for Federal Reserve Bond Purchases," *International Journal of Central Banking*, 10, 233–289.

Bernanke, B. S., V. R. Reinhart, and B. P. Sack (2004): "Monetary Policy Alternatives at the Zero Bound: An Empirical Assessment," *Brookings Papers on Economic Activity*, 35, 1–100.

Buraschi, A. and A. Jiltsov (2005): "Inflation risk premia and the expectations hypothesis," *Journal of Financial Economics*, 75, 429–490.

Calvo, G. A. (1983): "Staggered prices in a utility-maximizing framework," *Journal of Monetary Economics*, 12, 383–398.

Campbell, J. R., C. L. Evans, J. D. M. Fisher, and A. Justiniano (2012): "Macroeconomic Effects of FOMC Forward Guidance," *Brookings Papers on Economic Activity*, 44, 1–80.

Campbell, J. R., J. D. M. Fisher, A. Justiniano, and L. Melosi (2016): "Forward Guidance and Macroeconomic Outcomes Since the Financial Crisis," in *NBER Macroeconomics Annual 2016, Volume 31*, National Bureau of Economic Research, Inc, NBER Chapters.

Campbell, J. Y., C. Pflueger, and L. M. Viceira (2014): "Monetary Policy Drivers of Bond and Equity Risks," Working Paper 20070, National Bureau of Economic Research.

Chen, R.-R., B. Liu, and X. Cheng (2010): "Pricing the term structure of inflation risk premia: Theory and evidence from TIPS," *Journal of Empirical Finance*, 17, 702–721.

Chernov, M. and P. Mueller (2012): "The term structure of inflation expectations," *Journal of Financial Economics*, 106, 367–394.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 113, 1–45.

Christiano, L. J., M. Trabandt, and K. Walentin (2011): "Introducing financial frictions and unemployment into a small open economy model," *Journal of Economic Dynamics and Control*, 35, 1999–2041.

Cogley, T., G. E. Primiceri, and T. J. Sargent (2010): "Inflation-Gap Persistence in the US," *American Economic Journal: Macroeconomics*, 2, 43–69.

Croce, M. M. (2014): "Long-run productivity risk: A new hope for production-based asset pricing?" *Journal of Monetary Economics*, 66, 13–31.

Crump, R. K., S. Eusepi, and E. Moench (2016): "The term structure of expectations and bond yields," Staff report, FRB of NY.

Del Negro, M. and F. Schorfheide (2008): "Forming priors for DSGE models (and how it affects the assessment of nominal rigidities)," *Journal of Monetary Economics*, 55, 1191–1208.

den Haan, W. J. (1995): "The term structure of interest rates in real and monetary economies," *Journal of Economic Dynamics and Control*, 19, 909–940.

Dew-Becker, I. (2014): "Bond Pricing with a Time-Varying Price of Risk in an Estimated Medium-Scale Bayesian DSGE Model," *Journal of Money, Credit and Banking*, 46, 837–888.

Epstein, L. G. and S. E. Zin (1989): "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework," *Econometrica*, 57, 937–69.

——— (1991): "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: An Empirical Analysis," *Journal of Political Economy*, 99, 263–86.

Fuerst, T. S. (2015): "Monetary policy and the term premium," *Journal of Economic Dynamics and Control*, 52, 1–10.

Gertler, M. and P. Karadi (2015): "Monetary policy surprises, credit costs, and economic activity," *American Economic Journal: Macroeconomics*, 7, 44–76.

Gürkaynak, R. S., B. Sack, and E. Swanson (2005a): "Do Actions Speak Louder Than Words? The Response of Asset Prices to Monetary Policy Actions and Statements," *International Journal of Central Banking*, 1, 55–93.

——— (2005b): "The Sensitivity of Long-Term Interest Rates to Economic News: Evidence and Implications for Macroeconomic Models," *American Economic Review*, 95, 425–436.

Gürkaynak, R. S., B. Sack, and J. H. Wright (2007): "The U.S. Treasury yield curve: 1961 to the present," *Journal of Monetary Economics*, 54, 2291–2304.

——— (2010): "The TIPS Yield Curve and Inflation Compensation," *American Economic Journal: Macroeconomics*, 2, 70–92.

Gürkaynak, R. S. and J. H. Wright (2012): "Macroeconomics and the Term Structure," *Journal of Economic Literature*, 50, 331–367.

Hall, R. E. (1988): "Intertemporal Substitution in Consumption," *Journal of Political Economy*, 96, 339–357.

Hamilton, J. D. and J. C. Wu (2012): "Identification and estimation of Gaussian affine term structure models," *Journal of Econometrics*, 168, 315–331.

Hanson, S. G. and J. C. Stein (2015): "Monetary policy and long-term real rates," *Journal of Financial Economics*, 115, 429–448.

Hördahl, P. and O. Tristani (2012): "Inflation risk premia in the term structure of interest rates," *Journal of the European Economic Association*, 10, 634–657.

Hördahl, P., O. Tristani, and D. Vestin (2008): "The Yield Curve and Macroeconomic Dynamics," *Economic Journal*, 118, 1937–1970.

Jordà, O. (2005): "Estimation and Inference of Impulse Responses by Local Projections," *American Economic Review*, 95, 161–182.

Justiniano, A., G. E. Primiceri, and A. Tambalotti (2010): "Investment shocks and business cycles," *Journal of Monetary Economics*, 57, 132–145.

Kim, D. H. and A. Orphanides (2012): "Term Structure Estimation with Survey Data on Interest Rate Forecasts," *Journal of Financial and Quantitative Analysis*, 47, 241–272.

Kim, D. H. and J. H. Wright (2005): "An arbitrage-free three-factor term structure model and the recent behavior of long-term yields and distant-horizon forward rates," Finance and Economics Discussion Series 2005-33, Board of Governors of the Federal Reserve System (U.S.).

Laséen, S. and L. E. Svensson (2011): "Anticipated Alternative Policy Rate Paths in Policy Simulations," *International Journal of Central Banking*.

Malloy, C. J., T. J. Moskowitz, and A. Vissing-Jørgensen (2009): "Long-Run Stockholder Consumption Risk and Asset Returns," *Journal of Finance*, 64, 2427–2479.

Meyer-Gohde, A. (2016): "Risk-Sensitive Linear Approximations," mimeo, Hamburg University.

Nakamura, E. and J. Steinsson (2017): "High frequency identification of monetary non-neutrality: The information effect," *Quarterly Journal of Ecnomics*, forthcoming.

Piazzesi, M. (2005): "Bond Yields and the Federal Reserve," *Journal of Political Economy*, 113, 311–344.

Piazzesi, M. and M. Schneider (2007): "Equilibrium Yield Curves," in *NBER Macroeconomics Annual 2006, Volume 21*, National Bureau of Economic Research, Inc, NBER Chapters, 389–472.

Ramey, V. A. (2016): "Macroeconomic Shocks and Their Propagation," in *Handbook of Macroeconomics*, ed. by J. B. Taylor and H. Uhlig, Elsevier, vol. 2, chap. 2, 71–162.

Rudebusch, G. D., B. P. Sack, and E. T. Swanson (2007): "Macroeconomic implications of changes in the term premium," *Federal Reserve Bank of St. Louis Review*, 89, 241–269.

Rudebusch, G. D. and E. T. Swanson (2008): "Examining the bond premium puzzle with a DSGE model," *Journal of Monetary Economics*, 55, S111–S126.

——— (2012): "The Bond Premium in a DSGE Model with Long-Run Real and Nominal Risks," *American Economic Journal: Macroeconomics*, 4, 105–43.

Rudebusch, G. D., E. T. Swanson, and T. Wu (2006): "The Bond Yield "Conundrum" from a Macro-Finance Perspective," *Monetary and Economic Studies*, 24, 83–109.

Rudebusch, G. D. and T. Wu (2008): "A Macro-Finance Model of the Term Structure, Monetary Policy and the Economy," *Economic Journal*, 118, 906–926.

Swanson, E. T. (2016): "A Macroeconomic Model of Equities and Real, Nominal, and Defaultable Debt," mimeo, University of California at Irvine.

Tallarini, T. D. (2000): "Risk-sensitive real business cycles," *Journal of Monetary Economics*, 45, 507–532.

van Binsbergen, J. H., J. Fernández-Villaverde, R. S. Koijen, and J. Rubio-Ramírez (2012): "The term structure of interest rates in a DSGE model with recursive preferences," *Journal of Monetary Economics*, 59, 634–648.

Wachter, J. A. (2006): "A consumption-based model of the term structure of interest rates," *Journal of Financial Economics*, 79, 365–399.

Weil, P. (1989): "The equity premium puzzle and the risk-free rate puzzle," *Journal of Monetary Economics*, 24, 401–421.

Woodford, M. (2012): "Methods of policy accommodation at the interest-rate lower bound," *Proceedings - Economic Policy Symposium - Jackson Hole*, 185–288.

A. Model Solution (not for publication)

A1. Stationarized Model

Household:

$$V_t = \left[\frac{e^{\varepsilon_{b,t}}}{1-\gamma}\left(\left(c_t - \frac{bc_{t-1}}{e^{z_t^+}}\right)^{1-\gamma} - 1\right) + \frac{e^{\varepsilon_{b,t}}\psi_L\left(1-L_t\right)^{1-\chi}}{1-\chi}\right] + \beta\left(E_t\left[V_{t+1}^{1-\sigma_{EZ}}\right]\right)^{\frac{1}{1-\sigma_{EZ}}} \tag{A-1}$$

$$\lambda_t = e^{\varepsilon_{b,t}}\left(c_t - \frac{bc_{t-1}}{e^{z_t^+}}\right)^{-\gamma} \tag{A-2}$$

$$q_t = \frac{1 - E_t\left[M_{t+1}q_{t+1}\nu\left(\frac{I_{t+1}e^{z_{t+1}^+ + \Psi_{t+1}}}{I_t} - e^{\bar{z}^+ + \bar{\Psi}}\right)e^{\varepsilon_{I,t+1}}\left(\frac{I_{t+1}e^{z_{t+1}^+ + \Psi_{t+1}}}{I_t}\right)^2\right]}{\left(1 - \frac{\nu}{2}\left(\frac{I_t e^{z_t^+ + \Psi_t}}{I_{t-1}} - e^{\bar{z}^+ + \bar{\Psi}}\right)^2 - \nu\left(\frac{I_t e^{z_t^+ + \Psi_t}}{I_{t-1}} - e^{\bar{z}^+ + \bar{\Psi}}\right)\frac{I_t e^{z_t^+ + \Psi_t}}{I_{t-1}}\right)e^{\varepsilon_{I,t}}} \tag{A-3}$$

$$q_t = E_t\left[\frac{M_{t+1}}{e^{\Psi_{t+1}}}\left(r_{t+1}^k + q_{t+1}\left(1-\delta\right)\right)\right] \tag{A-4}$$

$$w_t\lambda_t = e^{\varepsilon_{b,t}}\psi_L\left(1-L_t\right)^{-\chi} \tag{A-5}$$

$$M_t = \beta e^{-z_{t+1}^+}\frac{\lambda_t}{\lambda_{t-1}}\left(V_t\right)^{-\sigma_{EZ}} E_{t-1}\left[V_t^{1-\sigma_{EZ}}\right]^{\frac{\sigma_{EZ}}{1-\sigma_{EZ}}} \tag{A-6}$$

$$1 = M_{t+1}\frac{\exp\left(R_t^f\right)}{\pi_{t+1}} \tag{A-7}$$

Price setting:

$$K_t^p = e^{\varepsilon_{p,t}}y_t\tilde{p}_t^{-\theta_p} + \gamma_p E_t\left[M_{t+1}\left(\frac{\pi_t^{\xi_p}}{\pi_{t+1}}\right)^{1-\theta_p}\left(\frac{\tilde{p}_t}{\tilde{p}_{t-1}}\right)^{-\theta_p}e^{z_t^+ + 1}K_{t+1}^p\right] \tag{A-8}$$

$$\frac{\theta_p - 1}{\theta_p}K_t^p = y_t mc_t\tilde{p}_t^{-\theta_p - 1} + \gamma_p E_t\left[M_{t+1}\left(\frac{\pi_t^{\xi_p}}{\pi_{t+1}}\right)^{-\theta_p}\left(\frac{\tilde{p}_t}{\tilde{p}_{t-1}}\right)^{-\theta_p - 1}\frac{\theta_p - 1}{\theta_p}e^{z_t^+ + 1}K_{t+1}^p\right] \tag{A-9}$$

$$1 = \gamma_p\left(\frac{\pi_{t-1}^{\xi_p}}{\pi_t}\right)^{1-\theta_p} + \left(1-\gamma_p\right)\left(\tilde{p}_t\right)^{1-\theta_p} \tag{A-10}$$

Intermediate Goods Producer:

$$p_t^+ y_t = e^{a_t}\left(\frac{k_{t-1}}{e^{z_t^+ + \Psi_t}}\right)^{\alpha}\left(L_t\right)^{1-\alpha} - \Phi_t \tag{A-11}$$

$$w_t = mc_t e^{a_t}\left(1-\alpha\right)\left(\frac{k_{t-1}}{e^{z_t^+ + \Psi_t}}\right)^{\alpha}L_t^{-\alpha} \tag{A-12}$$

$$r_t^k = mc_t e^{a_t}\alpha\left(\frac{k_{t-1}}{e^{z_t^+ + \Psi_t}}\right)^{\alpha - 1}L_t^{1-\alpha} \tag{A-13}$$

Aggregation:

$$k_t = \left(1-\delta\right)\frac{k_{t-1}}{e^{z_t^+ + \Psi_t}} + e^{\varepsilon_{I,t}}\left(1 - \frac{\nu}{2}\left(\frac{I_t e^{z_t^+ + \Psi_t}}{I_{t-1}} - e^{\bar{z}^+ + \bar{\Psi}}\right)^2\right)I_t \tag{A-14}$$

$$p_t^+ = (1-\gamma_p)\left(\tilde{p}_t\right)^{-\theta_p} + \gamma_p \left(\frac{\pi_{t-1}^{\xi_p}}{\pi_t}\right)^{-\theta_p} p_{t-1}^+ \tag{A-15}$$

$$y_t = c_t + I_t + \bar{g}e^{g_t} \tag{A-16}$$

$$z_t^+ = \frac{\alpha}{1-\alpha}\bar{\Psi} + z_t \tag{A-17}$$

Monetary Policy:

$$4R_t^f = 4\rho_R R_{t-1}^f + (1-\rho_R)\left(4\bar{r}^{real} + \eta_y \ln\left(\frac{y_t}{\bar{y}}\right) + 4\ln(\pi_t) + \eta_\pi\left[4\ln(\tilde{\pi}_t) - \ln(\pi_t^*)\right]\right) + \sigma_m \epsilon_{m,t} \tag{A-18}$$

$$\ln \pi_t^* - 4\ln\bar{\pi} = \rho_\pi\left(\ln \pi_{t-1}^* - 4\ln\bar{\pi}\right) + 4\zeta_\pi\left(\ln \pi_{t-1} - \ln\bar{\pi},\right) + \sigma_\pi \epsilon_{\pi,t} \tag{A-19}$$

Shock Processes:

$$g_t = \rho_g g_{t-1} + \sigma_g \epsilon_{g,t} \tag{A-20}$$

$$a_t = \rho_a a_{t-1} + \sigma_a \epsilon_{a,t} \tag{A-21}$$

$$\varepsilon_{I,t} = \rho_I \varepsilon_{I,t-1} + \sigma_I \epsilon_{I,t} \tag{A-22}$$

$$\varepsilon_{b,t} = \rho_b \varepsilon_{b,t-1} + \sigma_b \epsilon_{b,t} \tag{A-23}$$

$$z_t - \bar{z} = \rho_z\left(z_{t-1} - \bar{z}\right) + \sigma_z \epsilon_{z,t} \tag{A-24}$$

$$\ln\left(\Omega_t/\bar{\Omega}\right) = \rho_\Omega \ln\left(\Omega_{t-1}/\bar{\Omega}\right) + \sigma_\Omega \epsilon_{\Omega,t} \tag{A-25}$$

A2. Deterministic Steady State

Given our parameterizations for $\frac{\bar{g}}{\bar{y}}$, $\bar{\pi}$, and $\bar{L}$, the deterministic steady state is

$$\bar{\tilde{p}} = \left[\frac{1-\gamma_p \bar{\pi}^{(\xi_p-1)(1-\theta_p)}}{1-\gamma_p}\right]^{\frac{1}{1-\theta_p}} \tag{A-26}$$

$$\bar{p}^+ = \frac{(1-\gamma_p)\,\bar{\tilde{p}}^{-\theta_p}}{1-\gamma_p \bar{\pi}^{(1-\xi_p)\theta_p}} \tag{A-27}$$

$$\bar{R}^f = \ln\left(\frac{\bar{\pi}}{\beta}\right) - \left(-\gamma(1-\phi) - \phi\right)\bar{z}^+ \tag{A-28}$$

$$\bar{M} = \beta \exp\left(-\bar{z}^+\right) \tag{A-29}$$

$$\bar{r}^k = \frac{\exp\left(\bar{z}^+ + \bar{\Psi}\right)}{\beta} - (1-\delta) \tag{A-30}$$

$$\overline{mc} = \bar{\tilde{p}}\frac{\theta_p - 1}{\theta_p}\frac{\left(1-\gamma_p \beta \bar{\pi}^{(1-\xi_p)\theta_p}\right)}{\left(1-\gamma_p \beta \bar{\pi}^{(\xi_p-1)\theta_p}\right)} \tag{A-31}$$

$$\bar{k} = \bar{L}\left(\frac{\bar{r}^k}{\overline{mc}\alpha}\frac{1}{\exp\left(\bar{z}^+ + \bar{\Psi}\right)}\right)^{-\frac{1}{1-\alpha}} \tag{A-32}$$

$$\bar{w} = \overline{mc}\,(1-\alpha)\left(\exp\left(\bar{z}^+ + \bar{\Psi}\right)\right)^{-\alpha}\left(\frac{\bar{r}^k}{\overline{mc}\alpha}\frac{1}{\exp\left(\bar{z}^+ + \bar{\Psi}\right)}\right)^{\frac{\alpha}{1-\alpha}} \tag{A-33}$$

$$\bar{y} = \bar{r}^k\left(\frac{\bar{k}}{\exp\left(\bar{z}^+ + \bar{\Psi}\right)}\right) + \bar{w}\bar{L} \tag{A-34}$$

$$\Phi = \left(\frac{\bar{k}}{\exp\left(\bar{z}^{+} + \bar{\Psi}\right)}\right)^{\alpha} \bar{L}^{1-\alpha} - \bar{y}\bar{p}^{+} \tag{A-35}$$

$$\bar{I} = \left(1 - \frac{1-\delta}{\exp\left(\bar{z}^{+} + \bar{\Psi}\right)}\right)\bar{k} \tag{A-36}$$

$$\bar{g} = \left(\frac{\bar{g}}{\bar{y}}\right)\bar{y} \tag{A-37}$$

$$\bar{c} = \bar{y} - \bar{g} - \bar{I} \tag{A-38}$$

$$\bar{\lambda} = \left(\bar{c} - \frac{b\bar{c}}{\exp\left(\bar{z}^{+}\right)}\right)^{-\gamma} \tag{A-39}$$

$$\psi_L = \bar{w}\bar{\lambda}\left(1 - \bar{L}\right)^{\chi} \tag{A-40}$$

$$\bar{K}^{p} = \frac{\bar{y}\bar{\tilde{p}}^{-\theta_p}}{1 - \gamma_p \beta \bar{\pi}^{(\xi_p - 1)(1-\theta_p)}} \tag{A-41}$$

$$\bar{V} = \frac{1}{1-\beta}\left(\frac{\left(\bar{c} - \frac{b\bar{c}}{\exp\left(\bar{z}^{+}\right)}\right)^{1-\gamma} - 1}{1-\gamma} - \frac{\psi_L\left(1-\bar{L}\right)^{1-\chi}}{1-\chi}\right) \tag{A-42}$$

B. Approximation (not for publication)

B1. Risk-Adjusted Linear Approximation

The method of Meyer-Gohde (2016) differs from others in constructing an approximation centered around a risk-adjusted critical point, such as Juillard (2010), Kliem and Uhlig (2016), and Coeurdacier, Rey, and Winant (2011). First, it is direct and noniterative relying entirely on perturbation methods to construct the approximation. Second, it enables us to construct the approximation around (an approximation of) the ergodic mean of the true policy function instead of its stochastic or "'risky" steady state, placing the locality of our approximation in a region with a likely high (model-based) data density. The closest methods in the macro-finance term structure literature are Dew-Becker (2014) and Lopez, Lopez-Salido, and Vazquez-Grande (2015), who both approximate the nonlinear macro side of the model to obtain a linear in states approximation with adjustments for risk and then derive affine approximation of the yield curve taking this macro approximation as given. The exact meaning of these risk adjustments remains unclear, however, whereas the method by Meyer-Gohde (2016) adjusts the coefficients out to the second moments in shocks around the mean of the endogenous variables, itself approximated out to the second moments in shocks.

Thus instead of either a linear certainty-equivalent or nonlinear non-certainty-equivalent approximation, the method constructs a linear non-certainty-equivalent approximation. By using higher order derivatives of the policy function at the deterministic steady state, it approximates the ergodic mean of endogenous variables and the first derivatives of the policy function around this ergodic mean. Unlike standard higher order polynomial perturbations[20] or affine approximation methods,[21] this linear in states approximation gives us significant computational advantages.

Stacking our n_y endogenous variables into the vector y_t and our n_ε normally distributed exogenous shocks into the vector ε_t, we collect our equations into the following vector of nonlinear rational expectations difference equations

$$0 = E_t[f(y_{t+1}, y_t, y_{t-1}, \varepsilon_t)] = \hat{F}(y_{t-1}, \varepsilon_t) \tag{B-1}$$

where f is an $(n_{eq} \times 1)$ vector valued function, continuously M-times differentiable in all its arguments and with as many equations as endogenous variables ($n_{eq} = n_y$).

The solution to the functional problem in (B-1) is the policy function

$$y_t = g^0(y_{t-1}, \varepsilon_t) \tag{B-2}$$

Generally, a closed form for (B-2) is not available, so recourse to numerical approximations is necessary.

We assume that the related deterministic model

$$0 = f(y_{t+1}, y_t, y_{t-1}, 0) = \overline{F}(y_{t-1}, 0) \tag{B-3}$$

admits the calculation of a fix point, the deterministic steady state, defined as $\overline{y} \in \mathbb{R}^{n_y}$ such that $0 = \overline{F}(\overline{y}, 0)$. We are, however, interested in the stochastic version of the model and will now proceed to nest the deterministic model, for which we can recover a fix point, and the stochastic model, for which we cannot, within a larger continuum of models, following standard practice in the perturbation DSGE literature.

We introduce an auxiliary variable $\sigma \in [0,\ 1]$ to scale the stochastic elements in the model. The value $\sigma = 1$ corresponds to the "true" stochastic model and $\sigma = 0$ returns the deterministic model in (B-3). Accordingly, the stochastic model, (B-1), and the deterministic model, (B-3), can be nested inside the following continuum of models

$$0 = E_t[f(y_{t+1}, y_t, y_{t-1}, \tilde{\varepsilon}_t)] = F(\sigma, y_{t-1}, \tilde{\varepsilon}_t),\ \tilde{\varepsilon}_t \equiv \sigma\varepsilon_t \tag{B-4}$$

[20] Among others, recent third order perturbation approximations for DSGE models of the term structure include Rudebusch and Swanson (2008, 2012), van Binsbergen et al. (2012) Andreasen (2012), and Andreasen et al. (2017). While second order approximations such as Hördahl et al. (2008) provide nonzero but constant premia and De Graeve, Emiris, and Wouters (2009) is an example of a purely linear model that neglects endogenous premia. Additionally, many recent perturbations, Andreasen and Zabczyk (2015), Andreasen (2012), Andreasen et al. (2017), prune to ensure asymptotic stability.

[21] These approaches separate the macro and financial variables, generally using a (log) linear approximation of the former and an affine approximation for the yield curve following the empirical finance literature. Bonds are priced in an arbitrage free setup using either the endogenous pricing kernel implied by households' stochastic discount factors, as Dew-Becker (2014), Bekaert, Cho, and Moreno (2010), and Palomino (2012), or an estimated exogenously specified kernel, as Hördahl, Tristani, and Vestin (2006) , Hördahl and Tristani (2012), Ireland (2015), Rudebusch and Wu (2007), Rudebusch and Wu (2008).

with the associated policy function

$$y_t = g(y_{t-1}, \tilde{\varepsilon}_t, \sigma) \tag{B-5}$$

Notice that this reformulation allows us to express the deterministic steady state as the fix point of (B-4) for $\sigma = 0$, i.e., $\overline{y} \in \mathbb{R}^{n_y}$ such that $0 = F(0, \overline{y}, 0) = \overline{F}(\overline{y}, 0)$ and, as a consequence $\overline{y} = g(\overline{y}, 0, 0)$. We use this deterministic steady state and derivatives of the policy function in (B-5), recovered by the implicit function theorem,[22] evaluated at at $\overline{y}$ (both in the deterministic model, (B-3), and towards our stochastic model, (B-1), to construct our approximation of and around the ergodic mean.

Since y in the policy function (B-5) is a vector valued function, its derivatives form a hypercube.[23] Adopting an abbreviated notation, we write $g_{z^j\sigma^i} \in \mathbb{R}^{n_y \times n_z^j}$ as the partial derivative of the vector function g with respect to the state vector z_t j times and the perturbation parameter σ i times evaluated at the deterministic steady state.

Instead of using the partial derivatives to construct a Taylor series as is the standard procedure,[24] we would like to construct a more accurate linear approximation of the true policy function (B-2), centered at the mean of y_t. Accordingly, we will construct a linear approximation of (B-2) around the ergodic mean, which we formalize in the following.

PROPOSITION 1: *Linear Approximation around the Ergodic Mean*
Nest the means of the stochastic model ($\sigma = 1$) and of the deterministic model ($\sigma = 0$) through

$$\tilde{y}(\sigma) \equiv E\left[g(y_{t-1}, \sigma\varepsilon_t, \sigma)\right] = E\left[y_t\right] \tag{B-8}$$

Then for any $\sigma \in [0,\ 1]$, the linear approximation of the policy function, (B-2), around the mean of y_t defined in (B-8) and that of ε_t is

$$y_t \simeq \tilde{y}(\sigma) + g_y(\tilde{y}(\sigma), 0, \sigma)\left(y_{t-1} - \tilde{y}(\sigma)\right) + g_\varepsilon(\tilde{y}(\sigma), 0, \sigma)\varepsilon_t \tag{B-9}$$

[22]See Jin and Judd (2002).

[23]We use the method of Lan and Meyer-Gohde (2014) that differentiates conformably with the Kronecker product, allowing us to maintain standard linear algebraic structures to derive our results as follows: Let $A(B) : \mathbb{R}^{s\times 1} \to \mathbb{R}^{p\times q}$ be a matrix-valued function that maps an $s \times 1$ vector B into a $p \times q$ matrix $A(B)$, the derivative structure of $A(B)$ with respect to B is defined as

$$A_B \equiv \mathscr{D}_{B^T}\{A\} \equiv \left[\frac{\partial}{\partial b_1} \quad \dots \quad \frac{\partial}{\partial b_s}\right] \otimes A \tag{B-6}$$

where b_i denotes i'th row of vector B, ^{T} indicates transposition; n'th derivatives are

$$A_{B^n} \equiv \mathscr{D}_{(B^T)^n}\{A\} \equiv \left(\left[\frac{\partial}{\partial b_1} \quad \dots \quad \frac{\partial}{\partial b_s}\right]^{\otimes[n]}\right) \otimes A \tag{B-7}$$

[24]The Taylor series approximation at a deterministic steady state, assuming (B-5) is C^M with respect to all its arguments, can be written as $y_t = \sum_{j=0}^{M} \frac{1}{j!}\left[\sum_{i=0}^{M-j} \frac{1}{i!} g_{z^j\sigma^i}\sigma^i\right](z_t - \overline{z})^{\otimes[j]}$

Furthermore, the mean of y_t defined in (B-8) and the two additional unknown functions in this linear approximation

$$\tilde{y}_y(\sigma) \equiv g_y(\tilde{y}(\sigma), 0, \sigma) \tag{B-10}$$

$$\tilde{y}_\varepsilon(\sigma) \equiv g_\varepsilon(\tilde{y}(\sigma), 0, \sigma) \tag{B-11}$$

can be approximated out to second order in σ as

$$\tilde{y}(\sigma) = E[y_t] \approx \overline{y} + \frac{1}{2}\tilde{y}''(0) \tag{B-12}$$

$$g_y(\tilde{y}(\sigma), 0, \sigma) \approx g_y + \frac{1}{2}\left(g_{y^2}\left(\tilde{y}''(0) \otimes I_{n_y}\right) + g_{\sigma^2 y}\right) \tag{B-13}$$

$$g_\varepsilon(\tilde{y}(\sigma), 0, \sigma) \approx g_\varepsilon + \frac{1}{2}\left(g_{y\varepsilon}\left(\tilde{y}''(0) \otimes I_{n_\varepsilon}\right) + g_{\sigma^2 \varepsilon}\right) \tag{B-14}$$

where

$$\tilde{y}''(0) = \left(I_{n_y} - g_y\right)^{-1}\left(\left(g_{\varepsilon^2} + \left(I_{n_y} - g_y^{\otimes[2]}\right)^{-1} g_\varepsilon^{\otimes[2]}\right) E\left[\varepsilon_t^{\otimes[2]}\right] + g_{\sigma^2}\right) \tag{B-15}$$

PROOF:

See the next subsection.

B2. Proof of Proposition 1

We will recover the first order partial derivatives by applying the implicit function theorem on (B-4) and higher order partials through successive differentiation.[25]

Beginning with the unknown point of approximation, the ergodic mean, construct a Taylor series around the deterministic steady state

$$\tilde{y}(\sigma) = \tilde{y}(0) + \tilde{y}'(0)\sigma + \frac{1}{2}\tilde{y}''(0)\sigma^2 \dots \tag{B-16}$$

under the assumption of analyticity, the ergodic mean $\tilde{y}(1)$ can be approximated by

$$\tilde{y}(1) \approx \tilde{y}(0) + \tilde{y}'(0) + \frac{1}{2}\tilde{y}''(0) + \cdots + \frac{1}{n!}\tilde{y}^{(n)}(0) \tag{B-17}$$

Analogously for the two first derivatives of the policy function (B-2)

$$\tilde{y}_y(1) \approx \tilde{y}_y(0) + \tilde{y}_y{}'(0) + \frac{1}{2}\tilde{y}_y{}''(0) + \cdots + \frac{1}{(n-1)!}\tilde{y}_y{}^{(n-1)}(0) \tag{B-18}$$

[25]See Jin and Judd (2002) for a local existence theorem as well as Juillard and Kamenik (2004) for derivations with successive differentiation and Lan and Meyer-Gohde (2014) for solvability conditions for perturbations of arbitrary order.

$$\tilde{y_\varepsilon}(1) \approx \tilde{y_\varepsilon}(0) + \tilde{y_\varepsilon}'(0) + \frac{1}{2}\tilde{y_\varepsilon}''(0) + \cdots + \frac{1}{(n-1)!}\tilde{y_\varepsilon}^{(n-1)}(0) \tag{B-19}$$

Note that the approximations of $\tilde{y_\varepsilon}(1)$ and $\tilde{y_y}(1)$ are expressed up to order $n-1$, whereas the approximation of $\tilde{y}(1)$ is expressed up to order n. As the first two are derivatives of the third, terms of the order of $n-1$ in these two are actually of the order n with respect to derivatives of the underlying policy function (B-5), from which we will construct the approximations. Additionally, the assumption of analyticity, here in a domain encompassing both the deterministic steady state and ergodic mean of (B-5), while hardly innocuous, underlies standard perturbations methods that approximate the stochastic model using derivatives of the meta policy function (B-5) evaluated at the deterministic steady state.

Now we will show that the Taylor series representations of (B-8), (B-10), and (B-11) can be recovered from the derivatives of the policy function (B-5) evaluated at the deterministic steady state used in standard perturbations.

We will start with (B-8), the point of approximation,

$$\tilde{y}(1) \approx \tilde{y}(0) + \tilde{y}'(0) + \frac{1}{2}\tilde{y}''(0) \tag{B-20}$$

we need the three terms on the right hand side—$\tilde{y}(0)$, $\tilde{y}'(0)$, and $\tilde{y}''(0)$—to construct this approximation. Proceeding in increasing order of differentiation, we begin with $\tilde{y}(0)$. From (B-8),

$$\tilde{y}(0) = E\left[g(y_{t-1}, 0, 0)\right] = g(\overline{y}, 0, 0) = \overline{y} \tag{B-21}$$

the first derivative, $\tilde{y}'(\sigma)$, is

(B-22)
$$\tilde{y}'(0) = \mathscr{D}_\sigma\{E\left[y_t\right]\}\Big|_{\sigma=0} = \mathscr{D}_\sigma\{E\left[g(y_{t-1}, \sigma\varepsilon_t, \sigma)\right]\}\Big|_{\sigma=0} = E\left[\mathscr{D}_\sigma\{g(y_{t-1}, \sigma\varepsilon_t, \sigma)\}\right]\Big|_{\sigma=0}$$

where the expectation is with respect to the infinite sequence of $\{\varepsilon_{t-j}\}_{j=0}^{\infty}$ with invariant i.i.d. distributions, thus and assuming stability of y_t, gives the final equality. Taking derivatives and expectations and evaluating at the deterministic steady state

$$\mathscr{D}_\sigma\{E\left[y_t\right]\}\Big|_{\sigma=0} = g_y\mathscr{D}_\sigma\{E\left[y_{t-1}\right]\}\Big|_{\sigma=0} + g_\varepsilon E\left[\varepsilon_t\right] + g_\sigma \tag{B-23}$$

$$= g_y\mathscr{D}_\sigma\{E\left[y_{t-1}\right]\} \tag{B-24}$$

where the second line follows from the assumption of ε_t being mean zero.[26] Thus,

$$\tilde{y}'(0) = 0 \tag{B-25}$$

[26]Thus, $E\left[\varepsilon_t\right] = 0$ follows directly and g_σ consequentially, see Schmitt-Grohe and Uribe (2004), Jin and Judd (2002), or Lan and Meyer-Gohde (2014).

as g_y has all its eigenvalues inside the unit circle. The second derivative, $\tilde{y}''(\sigma)$, is

$$\tilde{y}''(0) = \mathscr{D}_{\sigma^2}\{E[y_t]\}\Big|_{\sigma=0} = E\left[\mathscr{D}_{\sigma^2}\{g(y_{t-1}, \sigma\varepsilon_t, \sigma)\}\right]\Big|_{\sigma=0} \tag{B-26}$$

Taking derivatives and expectations, evaluating at the deterministic steady state, and recalling results from the first derivative above[27]

(B-27)

$$\begin{aligned}
\mathscr{D}_{\sigma^2}\{E[y_t]\}\Big|_{\sigma=0} =& E\left[g_y\mathscr{D}_{\sigma^2}\{y_{t-1}\} + g_{y^2}\mathscr{D}_{\sigma}\{y_{t-1}\}^{\otimes[2]} + 2g_{y\varepsilon}\varepsilon_t \otimes \mathscr{D}_{\sigma}\{y_{t-1}\}\right. \\
&\left. +2g_{y\sigma}\mathscr{D}_{\sigma}\{y_{t-1}\} + 2g_{\varepsilon\sigma}\varepsilon_t + g_{\varepsilon^2}\varepsilon_t^{\otimes[2]} + g_{\sigma^2}\right]\Big|_{\sigma=0} \\
=& g_y\mathscr{D}_{\sigma^2}\{E[y_{t-1}]\}\Big|_{\sigma=0} + g_{y^2}E\left[\mathscr{D}_{\sigma}\{y_{t-1}\}^{\otimes[2]}\right]\Big|_{\sigma=0} \\
&+ g_{\varepsilon^2}E\left[\varepsilon_t^{\otimes[2]}\right] + g_{\sigma^2} \\
=& g_y\mathscr{D}_{\sigma^2}\{E[y_{t-1}]\}\Big|_{\sigma=0} + g_{y^2}\left(I_{n_y} - g_y^{\otimes[2]}\right)^{-1} g_{\varepsilon}^{\otimes[2]}E\left[\varepsilon_t^{\otimes[2]}\right] \\
&+ g_{\varepsilon^2}E\left[\varepsilon_t^{\otimes[2]}\right] + g_{\sigma^2}
\end{aligned}$$

$$\tilde{y}''(0) = \mathscr{D}_{\sigma^2}\{E[y_t]\}\Big|_{\sigma=0} = \left(I_{n_y} - g_y\right)^{-1}\left(\left(g_{\varepsilon^2} + \left(I_{n_y} - g_y^{\otimes[2]}\right)^{-1} g_{\varepsilon}^{\otimes[2]}\right)E\left[\varepsilon_t^{\otimes[2]}\right] + g_{\sigma^2}\right)$$

where the second to last equality follows[28]—taking expectations, evaluating at the deterministic steady state, and recalling results from the first derivative above—as

$$\begin{aligned}
E\left[\mathscr{D}_{\sigma}\{y_t\}^{\otimes[2]}\right]\Big|_{\sigma=0} =& E\left[\left(g_y\mathscr{D}_{\sigma}\{y_{t-1}\} + g_{\varepsilon}\varepsilon_t + g_{\sigma}\right)^{\otimes[2]}\right]\Big|_{\sigma=0} \\
=& g_y^{\otimes[2]}E\left[\mathscr{D}_{\sigma}\{y_{t-1}\}^{\otimes[2]}\right]\Big|_{\sigma=0} + g_{\varepsilon}^{\otimes[2]}E\left[\varepsilon_t^{\otimes[2]}\right]
\end{aligned} \tag{B-28}$$

Thus, $\tilde{y}''(0)$ adjusts the zeroth order mean $\tilde{y}(0)$ or deterministic steady state for the cumulative—$\left(I_{n_y} - g_y\right)^{-1}$—influence of the variance of shocks, directly through $E\left[\varepsilon_t^{\otimes[2]}\right]$ and indirectly through the influence of risk on the policy function captured by g_{σ^2}.

Moving on to the derivative of the policy function with respect to y_{t-1}, (B-10), for small deviations of y_{t-1} and ε_t from their respective means

$$\tilde{y_y}(1) \approx \tilde{y_y}(0) + \tilde{y_y}'(0) + \frac{1}{2}\tilde{y_y}''(0) \tag{B-29}$$

[27]The notation $x^{\otimes[n]}$ represents Kronecker powers, $x^{\otimes[n]}$ is the n'th fold Kronecker product of x with itself: $x \otimes x \cdots \otimes x$.

[28]The second line follows as $g_{y\sigma}$ and $g_{\varepsilon\sigma}$ are zero, see Schmitt-Grohe and Uribe (2004), Jin and Judd (2002), or Lan and Meyer-Gohde (2014).

we need the three terms on the right hand side—$\tilde{y_y}(0)$, $\tilde{y_y}'(0)$, and $\tilde{y_y}''(0)$. Starting with $\tilde{y_y}(0)$,

$$\tilde{y_y}(0) = \mathscr{D}_{y_{t-1}}\{y_t\}\Big|_{\sigma,\varepsilon_t=0} = \mathscr{D}_{y_{t-1}}\{g(\tilde{y}(\sigma),\tilde{\varepsilon}_t,\sigma)\}\Big|_{\sigma,\varepsilon_t=0} = g_y \tag{B-30}$$

Turning to $\tilde{y_y}'(0)$

$$\begin{aligned}\tilde{y_y}'(0) = \mathscr{D}_{\sigma y_{t-1}}\{y_t\}\Big|_{\sigma,\varepsilon_t=0} =&\mathscr{D}_{\sigma y_{t-1}}\{g(\tilde{y}(\sigma),\tilde{\varepsilon}_t,\sigma)\}\Big|_{\sigma,\varepsilon_t=0}\\ &= \mathscr{D}_{\sigma}\{g_y(\tilde{y}(\sigma),\tilde{\varepsilon}_t,\sigma)\}\Big|_{\sigma,\varepsilon_t=0}\\ &= g_{y^2}\mathscr{D}_{\sigma}\{\tilde{y}(\sigma)\}\Big|_{\sigma=0}\otimes I_{n_y} + g_{\sigma y}\\ &= 0 \end{aligned}\tag{B-31}$$

The first term is zero as $\mathscr{D}_{\sigma}\{\tilde{y}(\sigma)\}\Big|_{\sigma=0}$ was shown to be zero above and the second is equal to zero following standard results in the perturbation literature as discussed above. Finally, $\tilde{y_y}''(0)$

$$\begin{aligned}\tilde{y_y}''(0) = \mathscr{D}_{\sigma^2 y_{t-1}}\{y_t\}\Big|_{\sigma,\varepsilon_t=0} =&\mathscr{D}_{\sigma^2 y_{t-1}}\{g(\tilde{y}(\sigma),\tilde{\varepsilon}_t,\sigma)\}\Big|_{\sigma,\varepsilon_t=0}\\ &= \mathscr{D}_{\sigma^2}\{g_y(\tilde{y}(\sigma),\tilde{\varepsilon}_t,\sigma)\}\Big|_{\sigma=0}\\ &= g_{y^3}\mathscr{D}_{\sigma}\{\tilde{y}(\sigma)\}\Big|_{\sigma=0}^{\otimes[2]}\otimes I_{n_y} + 2g_{\sigma y^2}\mathscr{D}_{\sigma}\{\tilde{y}(\sigma)\}\Big|_{\sigma=0}\otimes I_{ny}\\ &+ g_{y^2}\mathscr{D}_{\sigma^2}\{\tilde{y}(\sigma)\}\Big|_{\sigma=0}\otimes I_{n_y} + g_{\sigma^2 y}\\ &= g_{y^2}\mathscr{D}_{\sigma^2}\{\tilde{y}(\sigma)\}\Big|_{\sigma=0}\otimes I_{n_y} + g_{\sigma^2 y}\end{aligned}\tag{B-32}$$

The final equality follows as $\mathscr{D}_{\sigma}\{\tilde{y}(\sigma)\}\Big|_{\sigma=0}$ and $g_{\sigma y^2}$ are both zero following the results and discussions above.

Finally, the derivative of the policy with respect to ε_t, (B-11), follows analogously to the derivative with respect to y_{t-1},

$$\tilde{y_\varepsilon}(1) \approx \tilde{y_\varepsilon}(0) + \tilde{y_\varepsilon}'(0) + \frac{1}{2}\tilde{y_\varepsilon}''(0) \tag{B-33}$$

Again, we need the three terms on the right hand side—$\tilde{y_\varepsilon}(0)$, $\tilde{y_\varepsilon}'(0)$, and $\tilde{y_\varepsilon}''(0)$. Starting with $\tilde{y_\varepsilon}(0)$,

$$\tilde{y_\varepsilon}(0) = \mathscr{D}_{\varepsilon_t}\{y_t\}\Big|_{\sigma,\varepsilon_t=0} = \mathscr{D}_{\varepsilon_t}\{g(\tilde{y}(\sigma),\tilde{\varepsilon}_t,\sigma)\}\Big|_{\sigma,\varepsilon_t=0} = g_\varepsilon \tag{B-34}$$

then $\tilde{y_\varepsilon}'(0)$

$$
\begin{aligned}
\tilde{y_\varepsilon}'(0) = \mathscr{D}_{\sigma\varepsilon_t}\{y_t\}\Big|_{\sigma,\varepsilon_t=0} &= \mathscr{D}_{\sigma\varepsilon_t}\{g(\tilde{y}(\sigma),\tilde{\varepsilon}_t,\sigma)\}\Big|_{\sigma,\varepsilon_t=0} \\
&= \mathscr{D}_{\sigma}\{g_\varepsilon(\tilde{y}(\sigma),\tilde{\varepsilon}_t,\sigma)\}\Big|_{\sigma,\varepsilon_t=0} \\
&= g_{y\varepsilon}\mathscr{D}_{\sigma}\{\tilde{y}(\sigma)\}\Big|_{\sigma=0} \otimes I_{n_\varepsilon} + g_{\sigma\varepsilon} \\
&= 0 \qquad \text{(B-35)}
\end{aligned}
$$

The first term is zero as $\mathscr{D}_{\sigma}\{\tilde{y}(\sigma)\}\Big|_{\sigma=0}$ was shown to be zero above and the second is equal to zero following standard results in the perturbation literature as discussed above. Finally, $\tilde{y_y}''(0)$

$$
\begin{aligned}
\tilde{y_\varepsilon}''(0) = \mathscr{D}_{\sigma^2\varepsilon_t}\{y_t\}\Big|_{\sigma,\varepsilon_t=0} &= \mathscr{D}_{\sigma^2\varepsilon_t}\{g(\tilde{y}(\sigma),\tilde{\varepsilon}_t,\sigma)\}\Big|_{\sigma,\varepsilon_t=0} \\
&= \mathscr{D}_{\sigma^2}\{g_\varepsilon(\tilde{y}(\sigma),\tilde{\varepsilon}_t,\sigma)\}\Big|_{\sigma=0} \\
&= g_{y^2\varepsilon}\mathscr{D}_{\sigma}\{\tilde{y}(\sigma)\}\Big|_{\sigma=0}^{\otimes[2]} \otimes I_{n_\varepsilon} + 2g_{\sigma y\varepsilon}\mathscr{D}_{\sigma}\{\tilde{y}(\sigma)\}\Big|_{\sigma=0} \otimes I_{n_\varepsilon} \\
&\quad + g_{y\varepsilon}\mathscr{D}_{\sigma^2}\{\tilde{y}(\sigma)\}\Big|_{\sigma=0} \otimes I_{n_\varepsilon} + g_{\sigma^2\varepsilon} \\
&= g_{y\varepsilon}\mathscr{D}_{\sigma^2}\{\tilde{y}(\sigma)\}\Big|_{\sigma=0} \otimes I_{n_\varepsilon} + g_{\sigma^2\varepsilon} \qquad \text{(B-36)}
\end{aligned}
$$

The final equality follows as $\mathscr{D}_{\sigma}\{\tilde{y}(\sigma)\}\Big|_{\sigma=0}$ and $g_{\sigma y\varepsilon}$ are both zero following the results and discussions above.

C. Data (not for publication)

Real GDP: *BEA NIPA table 1.1.6 line 1 (A191RX1).*

Nominal GDP: *BEA NIPA table 1.1.5 line 1 (A191RC1).*

Implicit GDP Deflator: the ratio of **Nominal GDP** to **Real GDP**.

Private Consumption: Real consumption expenditures for non-durables and services is the sum of *BEA NIPA table 1.1.5 line 5 (DNDGRC1)* and *BEA NIPA table 1.1.5 line 6 (DNDGRC1)* deflated by the implicit GDP deflator.

Private Investment: Total real private investment is the sum of Gross Private Investment *BEA NIPA table 1.1.5 line 7 (A006RC1)* and Personal Consumption Expenditures: Durable Goods *BEA NIPA table 1.1.5 line 4 (DDURRC1)* deflated by the implicit GDP deflator.

Civilian Population: This series is calculated from monthly data of *civilian non-institutional population over 16 years (CNP16OV)* from the U.S. Department of Labor: Bureau of Labor Statistics.

Policy Rate: 3-Month Treasury Bill: Secondary Market Rate *TB3MS* provided by Board of Governors of the Federal Reserve System. The quarterly aggregation is end of period.

Treasury Bond Yields: 1-year, 2-year, 3-year, 5-year, and 10-year zero-coupon bond yields measured end of quarter. The original series are daily figures based on the updated series by Adrian et al. (2013).
Source: https://www.newyorkfed.org/research/data_indicators/term_premia.html

Nominal Interest Rate Forecasts: 1-quarter (TBILL3) and 4-quarter (TBILL6) ahead forecasts of the 3-Month Treasury Bill. The time series are the median responses by the Survey of Professional Forecasters from the Federal Reserve Bank of Philadelphia.
Source: https://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/data-files

D. Endogenous prior (not for publication)

Following Del Negro and Schorfheide (2008), we assume $\hat{F}$ to be a vector that collects the first moments of interest from our pre-sample and $F_M(\theta)$ be a vector-valued function which relates model parameters and ergodic means

$$\hat{F} = F_M(\theta) + \eta \tag{D-1}$$

where η is a vector of measurement errors. In our application, we assume that the error terms η are independently and normally distributed. Hence, we express eq. (D-1) as a quasi-likelihood function which can be interpreted as the conditional density

$$\begin{aligned}\mathcal{L}\left(F_M(\theta)\,|\hat{F}, T^*\right) &= \exp\left\{-\frac{T^*}{2}\left(\hat{F} - F_M(\theta)\right)' \Sigma_\eta^{-1}\left(\hat{F} - F_M(\theta)\right)\right\} \\ &= p\left(\hat{F}|F_M(\theta), T^*\right)\end{aligned} \tag{D-2}$$

This quasi-likelihood is small for values of θ that lead the DSGE model to predict first moments that strongly differ from the measures of the pre-sample. The parameter T^* captures, along with the standard deviation of η, the precision of our beliefs about the first moments. In practice we set T^* to the length of the pre-sample.

For the application in this paper, we assume that the vector $\hat{F}$ contains the mean of inflation and the means of proxies for the level, slope, and curvature factors of the yield curve. We include the mean of inflation because the non-linearities in our model impose strong precautionary motives that push the predicted ergodic mean of inflation away from its deterministic steady state, $\bar{\pi}$, as is also discussed by Tallarini (2000) and Andreasen (2011). Regarding $\mathcal{L}\left(F_M(\theta)\,|\hat{F}\right)$, we assume that $E_t\left[400\pi|\theta\right]$ is normally distributed with mean 2.5 and variance 0.1.

We follow, e.g., Diebold, Rudebusch, and Aruoba (2006) and specify common proxies for the level, slope, and curvature factors of the yield curve. Specifically, the proxy for the level factor is $\left(R^{\$}_{1,t}+R^{\$}_{8,t}+R^{\$}_{40,t}\right)/3$, with all yields expressed in annualized terms and the nominal yield of the 1-quarter Treasury Bond equal to the policy rate in the model. Additionally, the proxies for the slope and curvature factors are defined as $R^{\$}_{1,t}-R^{\$}_{40,t}$ and $2R^{\$}_{8,t}-R^{\$}_{1,t}-R^{\$}_{40,t}$, respectively. Regarding $\mathcal{L}\left(F_M(\theta)\,|\hat{F}\right)$, we assume that the ergodic mean of each factor is normally distributed, with the mean equal to its empirical counterpart of the pre-sample. Moreover, we assume that the means of level, slope, and curvature have a variance of 22, 12, and 9 basis points respectively. Thus, the means and variances can be interpreted as $\hat{F}$ value and the variance of the measurement error η in eq. (D-1).

Additionally, we use the second moments of macroeconomic variables, about which we have a priori knowledge, to inform our prior distribution and apply the approach of Christiano et al. (2011). This approach uses classical large sample theory to form a large sample approximation to the likelihood of the pre-sample statistics. The approach is conceptually similar to the one proposed by Del Negro and Schorfheide (2008), but differs in some important respects. Specifically, Del Negro and Schorfheide (2008) focus on the model-implied p-th order vector autoregression, which implies that the likelihood of the second moments is known exactly conditional on the DSGE model parameters and requires no large-sample approximation in contrast to the approach by Christiano et al. (2011). Yet, the latter approach is more flexible insofar as the statistics to target are concerned. Accordingly, let S be a column vector containing the second moments of interest, then, as shown by Christiano et al. (2011) under the assumption of large sample, the estimator of S is

$$\hat{S}\sim N\left(S^0,\frac{\hat{\Sigma}_S}{T}\right) \tag{D-3}$$

with S^0 the true value of S, T the sample length, and $\hat{\Sigma}_S$ the estimate of the zero-frequency spectral density. Now, let $S_M(\theta)$ be a function which maps our DSGE model parameters θ into S. Then, for n targeted second moments and sufficiently large T, the density of $\hat{S}$ is given by

$$p\left(\hat{S}|\theta\right)=\left(\frac{T}{2\pi}\right)^{\frac{n}{2}}\left\|\hat{\Sigma}_S\right\|^{-\frac{1}{2}}\exp\left\{-\frac{T}{2}\left(\hat{S}-S_M(\theta)\right)'\hat{\Sigma}_S^{-1}\left(\hat{S}-S_M(\theta)\right)\right\} \tag{D-4}$$

In our application, S is a set of variances of macroeconomic variables (GDP growth, consumption growth, investment growth, inflation, and the policy rate). In sum, the overall endogenous prior distribution takes the following form

$$p\left(\theta|\hat{F},\hat{S},T^{*}\right)=C^{-1}p\left(\theta\right)p\left(\hat{F}|F_{M}\left(\theta\right),T^{*}\right)p\left(\hat{S}|\theta\right) \tag{D-5}$$

where $p\left(\theta\right)$ is the initial prior distribution and C a normalization constant. Two points are noteworthy. First, while the initial priors are independent across parameters, as is typical in Bayesian analysis, the endogenous prior is not independent across parameters. Second, the normalization constant C is necessary for, e.g., posterior odds calculation but not for estimating the model. Accordingly, we do not calculate this constant, which has otherwise to be approximated (see, for example, Del Negro and Schorfheide, 2008; Kliem and Uhlig, 2016). So, the posterior distribution is given by

$$p\left(\theta|X,\hat{F},\hat{S},T^{*}\right)\propto p\left(\theta|\hat{F},\hat{S},T^{*}\right)p\left(X|\theta\right) \tag{D-6}$$

with $p\left(X|\theta\right)$ the likelihood of the data conditional on DSGE model parameters θ.

Table D1 summarizes the initial prior distributions of the remaining parameters. While the prior distributions for most of the parameters are chosen following the literature, it is noteworthy to highlight some deviations. First, we do not use a prior for the preference parameters, γ and α_{EZ}, directly, but rather impose priors for the intertemporal elasticity of substitution, IES, and the coefficient relative risk aversion, RRA, and solve for the underlying parameters. The intertemporal elasticity of substitution, IES, in our model with external habit formation is

$$IES=\frac{1}{\gamma}\left[1-\frac{b}{\exp\left(\bar{z}^{+}\right)}\right] \tag{D-7}$$

We follow Swanson (2012) by using his closed-form expressions for risk aversion, RRA, which takes into account that households can vary their labor supply. Hence, our model implies

$$RRA=\frac{\gamma}{1-\frac{b}{\exp(\bar{z}^{+})}+\frac{\gamma}{\chi}\left(1-\bar{l}\right)\frac{\bar{w}}{\bar{c}}}+\alpha_{EZ}\frac{1-\gamma}{1-\frac{b}{\exp(\bar{z}^{+})}-\left(1-\frac{b}{\exp(\bar{z}^{+})}\right)^{\gamma}\bar{c}^{\gamma-1}+\frac{\bar{w}\left(1-\bar{l}\right)}{\bar{c}}\frac{1-\gamma}{1-\chi}} \tag{D-8}$$

where $\bar{l}$ is the steady state labor supply, while $\bar{c}$ and $\bar{w}$ are consumption and the real wage in the deterministic steady state, respectively. Given the wide range of different estimates for relative risk aversion in the macro- and finance literatures, we initially assume a uniform prior with support over the interval 0 to 2000; our endogenous prior approach, however, does impose an informative prior. We proceed analogously for the deterministic steady state of inflation and choose an uninformative initial prior distribution. Finally, we add measurement errors to the 1-year,

Name	Symbol	Domain	Density	Para(1)	Para(2)
Relative risk aversion	$RRA/100$	$\mathbb{R}^+$	Uniform	0	20
Calvo parameter	γ_p	$[0,1)$	Beta	0.5	0.1
Investment adjustment	ν	$\mathbb{R}^+$	Gamma	4.0	0.75
Habit formation	b	$[0,1)$	Beta	0.5	0.1
Intertemporal elas. substitution	IES	$[0,1)$	Beta	0.25	0.1
Steady state inflation	$100\,(\bar{\pi}-1)$	$\mathbb{R}^+$	Uniform	0	6
Interest rate AR coefficient	ρ_R	$[0,1)$	Beta	0.8	0.1
Interest rate inflation coefficient	η_π	$\mathbb{R}^+$	Gamma	1	0.15
Interest rate output coefficient	η_y	$\mathbb{R}^+$	Gamma	0.5	0.1
Inflation target coefficient	$100\zeta_\pi$	$[0,1)$	Beta	0.3	0.1
AR coefficient technology	ρ_a	$[0,1)$	Beta	0.75	0.1
AR coefficient preference	ρ_b	$[0,1)$	Beta	0.75	0.1
AR coefficient investment	ρ_i	$[0,1)$	Beta	0.75	0.1
AR coefficient gov. spending	ρ_g	$[0,1)$	Beta	0.75	0.1
AR coefficient inflation target	ρ_π	$[0,1)$	Beta	0.95	0.025
AR coefficient long-run growth	ρ_z	$[0,1)$	Beta	0.75	0.1
AR coefficient fixed costs	ρ_Ω	$[0,1)$	Beta	0.75	0.1
S.d. technology	$100\sigma_a$	$\mathbb{R}^+$	InvGam	0.5	2
S.d. preference	$100\sigma_b$	$\mathbb{R}^+$	InvGam	0.5	2
S.d. investment	$100\sigma_i$	$\mathbb{R}^+$	InvGam	0.5	2
S.d. monetary policy shock	$100\sigma_m$	$\mathbb{R}^+$	InvGam	0.5	2
S.d. government spending	$100\sigma_g$	$\mathbb{R}^+$	InvGam	0.5	2
S.d. inflation target	$100\sigma_\pi$	$\mathbb{R}^+$	InvGam	0.06	0.03
S.d. long-run growth	$100\sigma_z$	$\mathbb{R}^+$	InvGam	0.5	2
S.d. fixed costs	$100\sigma_\Omega$	$\mathbb{R}^+$	InvGam	0.5	2
ME 1-year T-Bill	$4R^{\$}_{4,t}$	$\mathbb{R}^+$	InvGam	0.005	∞
ME 2-year T-Bill	$4R^{\$}_{8,t}$	$\mathbb{R}^+$	InvGam	0.005	∞
ME 3-year T-Bill	$4R^{\$}_{12,t}$	$\mathbb{R}^+$	InvGam	0.005	∞
ME 5-year T-Bill	$4R^{\$}_{20,t}$	$\mathbb{R}^+$	InvGam	0.005	∞
ME 10-year T-Bill	$4R^{\$}_{40,t}$	$\mathbb{R}^+$	InvGam	0.005	∞
ME 1Q-expected policy rate	$4E_t\left[R^f_{t,t+1}\right]$	$\mathbb{R}^+$	InvGam	0.005	∞
ME 4Q-expected policy rate	$4E_t\left[R^f_{t,t+4}\right]$	$\mathbb{R}^+$	InvGam	0.005	∞

Table D1—Initial prior distribution. Para(1) and Para(2) correspond to means and standard deviations for the Beta, Gamma, Inverted Gamma, and Normal distributions and to the lower and upper bounds for the Uniform distribution.

2-year, 3-year, 5-year, and 10-year Treasury bond yields as well as to the expected policy rate expected 1 and 4-quarters ahead. By adding measurement errors along the yield curve, we are following the empirical term structure literature (see, for example, Diebold et al., 2006) and the measurement errors on the expectations of the short rate align the imperfect fit of the data with the model's rational expectation assumption.

E. Supplementary Results (not for publication)

E1. Initial Prior vs Posterior Plots

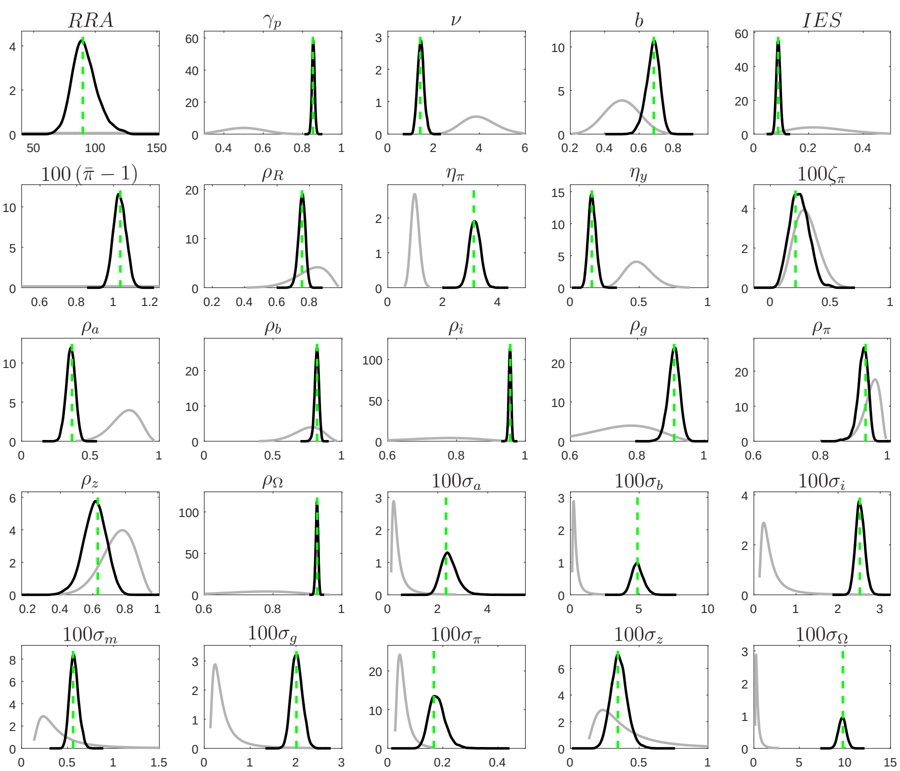


Figure E1. Prior (gray) and posterior (black) distribution of the model parameters, the green dashed line indicates the posterior mode.

E2. Predicted Moments

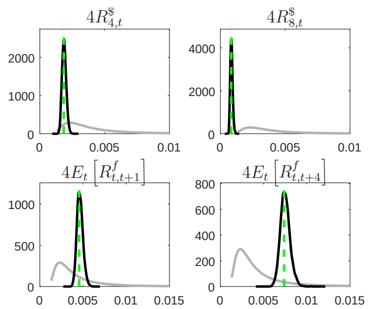


FIGURE E2. PRIOR (GRAY) AND POSTERIOR (BLACK) DISTRIBUTION OF MEASUREMENT ERRORS, THE GREEN DASHED LINE INDICATES THE POSTERIOR MODE.

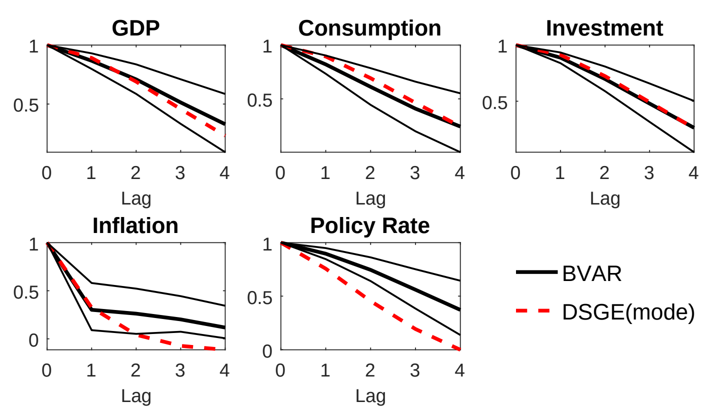


FIGURE E3. PREDICTED AUTOCORRELATION OF SELECTED HP-FILTERED MACRO VARIABLES AT THE POSTERIOR MODE AND THE CORRESPONDING POPULATION MOMENTS OF THE DATA CALCULATED BY USING A BAYESIAN VECTOR AUTOREGRESSION MODEL WITH TWO LAGS. THE THIN BLACK LINES REPRESENT THE 90% PROBABILITY BANDS.

Name	Symbol	Mean			S.d.		
		50%	5%	95%	50%	5%	95%
1-year real T-Bill	$R_{4,t}$	2.68	1.09	4.26	1.81	1.28	2.62
2-year real T-Bill	$R_{8,t}$	3.00	1.58	4.42	1.53	1.04	2.29
3-year real T-Bill	$R_{12,t}$	3.17	1.90	4.43	1.35	0.91	2.04
5-year real T-Bill	$R_{20,t}$	3.33	2.31	4.34	1.10	0.74	1.66
10-year real T-Bill	$R_{40,t}$	3.73	3.05	4.43	0.72	0.48	1.08
1-year nominal term premium	$TP^{\$}_{4,t}$	37.36	27.93	46.96	8.83	5.92	13.39
2-year nominal term premium	$TP^{\$}_{8,t}$	77.14	55.92	98.46	19.55	13.16	29.58
3-year nominal term premium	$TP^{\$}_{12,t}$	99.69	71.02	128.40	25.64	17.25	38.89
5-year nominal term premium	$TP^{\$}_{20,t}$	129.06	91.12	167.03	31.37	20.90	48.30
10-year nominal term premium	$TP^{\$}_{40,t}$	202.69	148.52	256.89	37.85	24.37	60.81
1-year real term premium	$TP_{4,t}$	23.95	18.76	29.57	4.82	3.21	7.36
2-year real term premium	$TP_{8,t}$	56.96	42.78	71.59	13.22	8.86	20.04
3-year real term premium	$TP_{12,t}$	74.54	54.60	94.85	18.10	12.17	27.40
5-year real term premium	$TP_{20,t}$	93.09	66.69	119.70	22.09	14.80	33.75
10-year real term premium	$TP_{40,t}$	138.88	101.06	176.17	26.26	17.08	41.68
1-year inflation risk premium	$TP^{\pi}_{4,t}$	13.34	8.77	18.03	4.14	2.78	6.30
2-year inflation risk premium	$TP^{\pi}_{8,t}$	20.03	12.52	27.98	6.47	4.30	9.93
3-year inflation risk premium	$TP^{\pi}_{12,t}$	24.93	15.50	35.19	7.67	5.04	11.97
5-year inflation risk premium	$TP^{\pi}_{20,t}$	35.68	22.99	49.76	9.37	6.01	15.02
10-year inflation risk premium	$TP^{\pi}_{40,t}$	63.46	44.89	84.47	11.61	7.17	19.44

Table E1—Simulated moments of further financial variables.

Note: The simulated moments are based on 1200 parameter vector draws from the posterior. For each draw, we simulate 1000 time series for each variable of interest. After removing a burn-in of 5000 periods for each simulation the final simulated time series have the same length (T=100) as the vector of observables. The number in brackets indicate 5% and 95% probabilities. All returns are measured in annualized percentage points and all risk premia are measured in annualized basis points.

E3. Risk-Adjusted Impulse Responses versus Generalized Impulse Responses

Here, we compare our impulse responses using the solution method of Meyer-Gohde (2016) with generalized impulse responses from a standard nonlinear solution method (see Koop, Pesaran, and Potter (1996); Andreasen et al. (2017)). We use our posterior mean parameters and compute a standard third order perturbation of our model. The generalized impulse response of a variable y^j_{t+s} to a shock ε^i_t is given by

$$GIRF(s, \omega, y_{t-1}) = E\left[y^j_{t+s}|y_{t-1}, \varepsilon^i_t = \omega\right] - E\left[y^j_{t+s}|y_{t-1}\right] \tag{D-1}$$

To calculate the impulse responses, we run 10,000 simulations of 5,040 periods each for the third order perturbation, where an impulse ω occurs at period 5,001.[29] We start the simulations from the deterministic steady state and then discard the first 5,000 periods so that the simulated values will likely have converged to the ergodic distribution. The average value over all the simulations, as well as the 90% and 68% coverage of the simulations can be found in Figure E4.

The figure also contains impulse responses from standard linear approximations around the deterministic steady state. Whereas both the generalized impulse response and the impulses calculated from the risk adjusted linear approximation are in deviations from the ergodic mean, the standard linear approximation returns impulses in deviations from the deterministic steady state. While it is tempting to look for the term premia to span the distance between our risk-adjusted and a standard linear approximation for bond yields, the different points of approximation that encompass covariance terms and the like preclude this.

As can clearly be seen in the figure, our risk adjusted linear approximated model is very successful in capturing the effects of monetary policy changes that a fully nonlinear approximation would predict. In contrast to the standard linear approximation, the nonlinearity in risk captured by the method we use captures the effects on term premia. Conspicuously, the forward guidance experiment from Figure 5 is missing here. Both this and the estimation of our model would be nontrivial tasks for a standard nonlinear approximation. Thus, we conclude that the gains from maintaining linearity in states by using the risk adjusted linear approximation outweigh the costs of apparently small accuracy loses.

Our approximation is noncertainty equivalent despite its linearity in states; i.e., the underlying risk in the economy affects the predicted response to any shock. To illustrate this, Figure E5 contains the impact responses of the yield curve and components to monetary policy shocks (1) at our posterior mean estimates and (2) at our posterior mean estimates with the variance of all other shocks set to zero. Ad-

[29] To maximize comparability with the main text, we ensure that the average impulse leads to a 50 basis point drop in the policy rate on impact. Due to the nonlinearity in states of the third order perturbation, we cannot simply scale the impulse responses, but must solve a fixed point problem to recover the ω that leads to this 50 basis point drop.

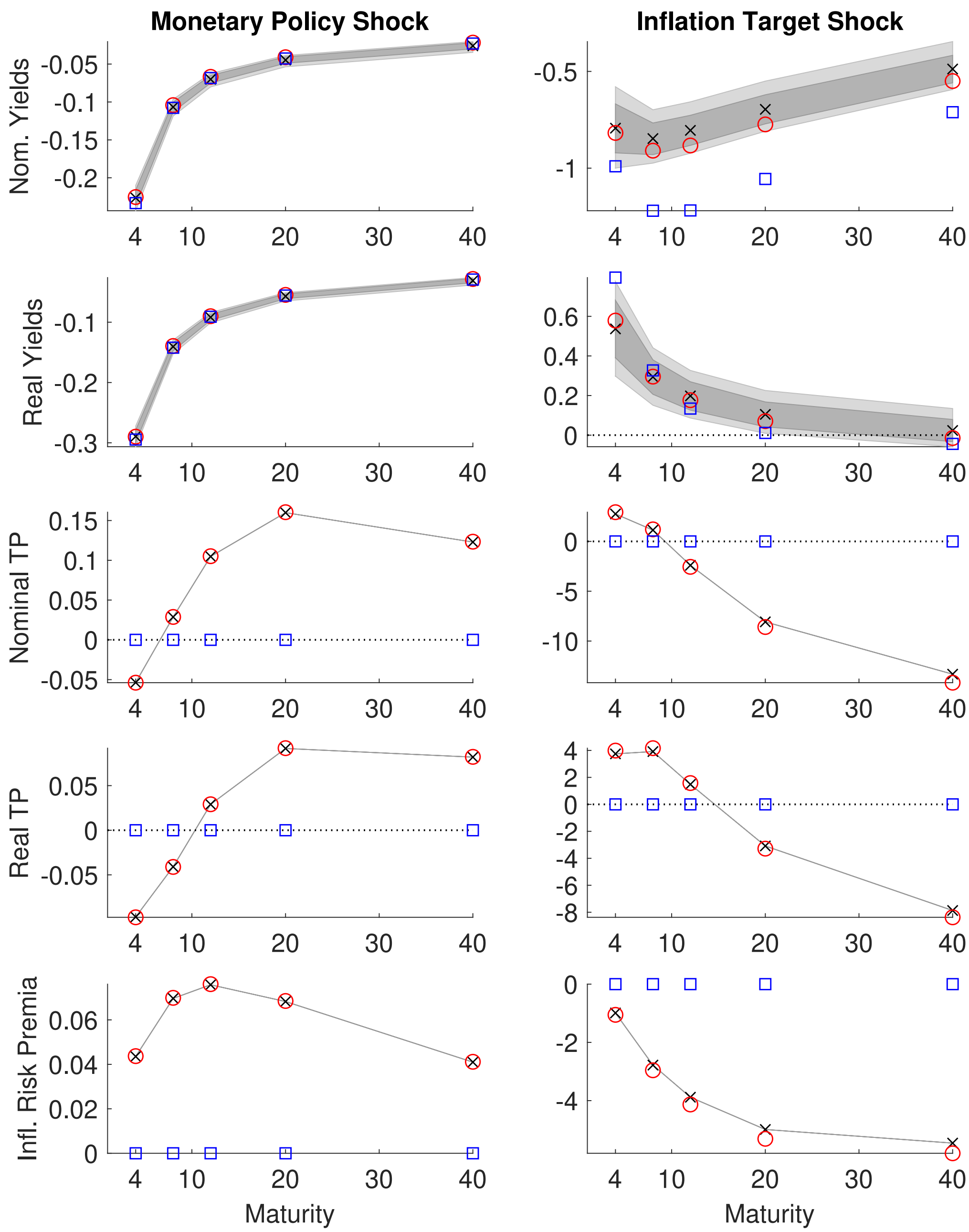


Figure E4. Solution method and impact responses of nominal and real term structures.

Note: The figure shows the impact response across all maturities to a surprise 50 basis point policy rate cut and a surprise cut in the inflation target leading to a 50 basis point policy rate cut. The deviations of yields are in percentage points while the deviations of risk premia are presented in basis points. The black crosses (median) and shaded areas (90% and 68% coverage) give generalized impulse responses calculated with a full third order perturbation at our posterior mode. The red circles give the responses from the risk-adjusted linear approximation. The blues squares give the responses from a standard linear approximation.

ditionally, the impact responses of the standard deterministic linear approximation are also plotted.

Under the standard linear approximation at the deterministic steady state, the impulse response functions are invariant to the volatility of shocks. Under the risk adjusted solution, they differ significantly due to the risk dependence of the solution. This also underlines why having a rich, estimated stochastic environment is essential even to analyses focusing on a single aspect of the macroeconomy (say, monetary policy) in the absence of certainty equivalence.

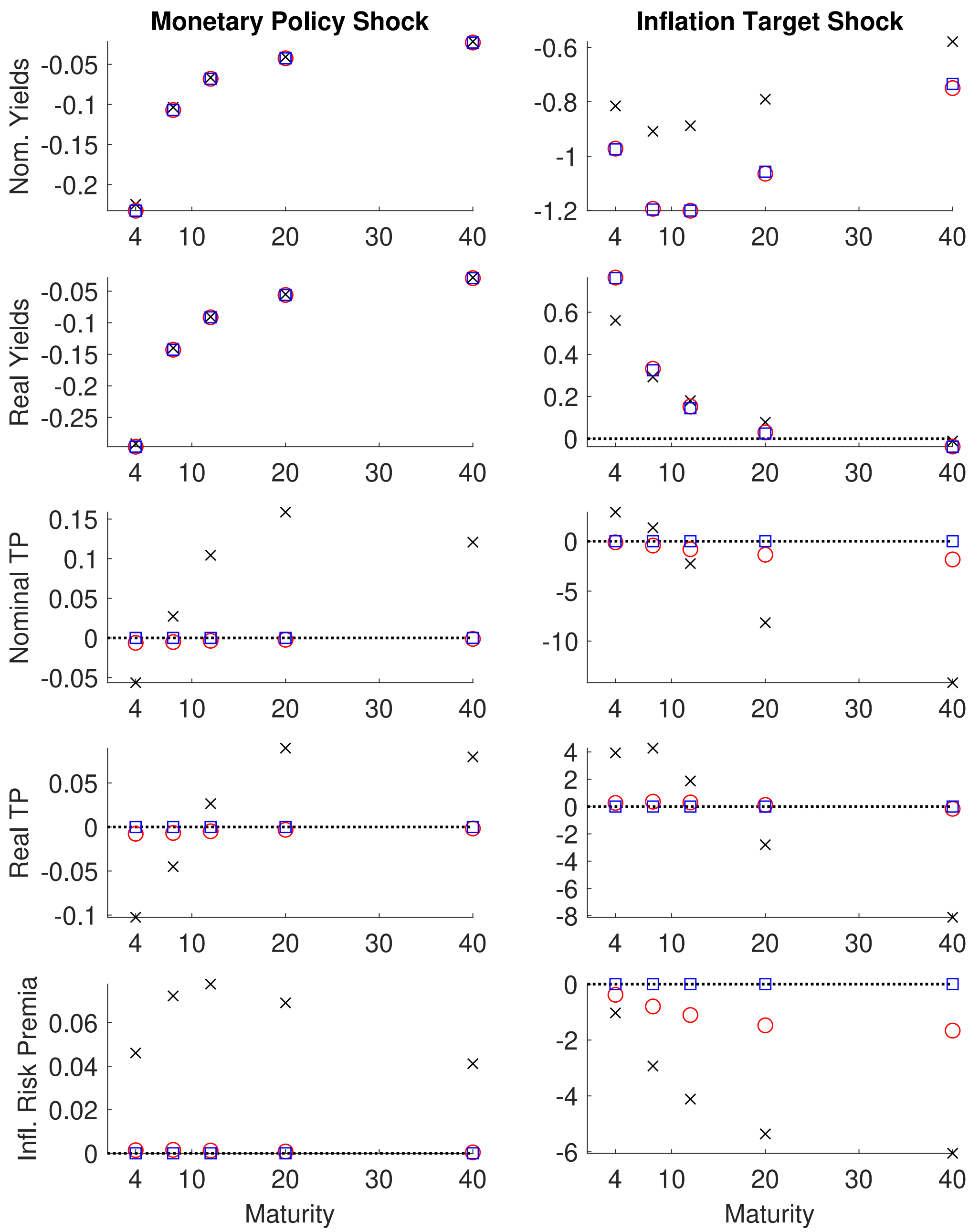


Figure E5. Risk dependence of impact responses of nominal and real term structures.

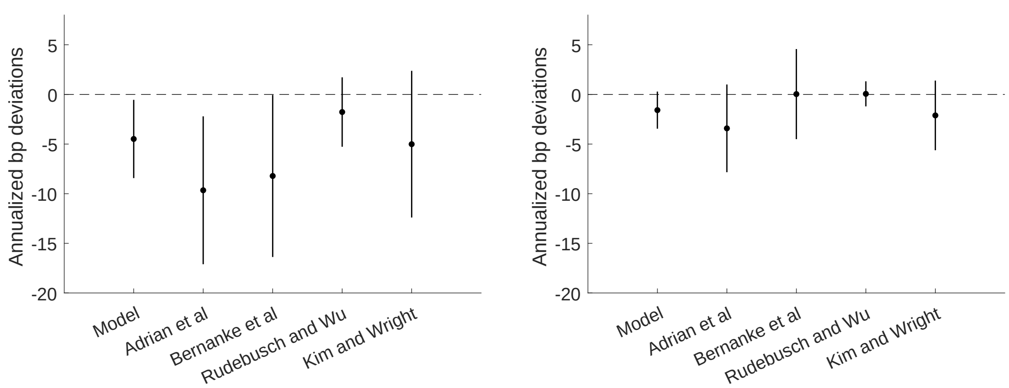
Note: The figure shows the impact response across all maturities to a surprise 50 basis point policy rate cut and a surprise cut in the inflation target leading to a 50 basis point policy rate cut. The deviations of yields are in percentage points while the deviations of risk premia are presented in basis points. The black crosses give the responses at our posterior mode from the risk-adjusted linear approximation. The red circles give the responses from the risk-adjusted linear approximation with the variances of all other shocks set to zero. The blues squares give the responses from a standard linear approximation.

E4. Empirical evidence

In this subsection, we compare the impulse responses from our structural model with those from the empirical literature in greater detail. In particular, we apply a linear local projection following Jordà (2005). Our model setup is very flexible and encompasses the commonly used linear projections in the empirical literature (e.g. Hanson and Stein, 2015; Nakamura and Steinsson, 2017; Crump et al., 2016). The linear model is given as follows

$$x_{t+h} = \alpha_h + \psi_h(L)\, z_{t-1} + \beta_h shock_t + \varepsilon_{t+h} \quad \text{for} \quad h = 0, 1, 2 \ldots, \tag{F-1}$$

where x is the variable of interest, z a vector of control variables, $\psi_h(L)$ a polynomial in the lag operator, and *shock* the identified monetary policy shock. In our applications, $\psi_h(L)$ is a polynomial of order 2, the vector of controls z comprise GDP growth and inflation along with the variable of interest and the identified shock (see, for example, Stock and Watson, 2018). Finally, the variables of interest x are nominal yields and nominal term premia with a maturity between 4 and 40 quarters. Figure 6 in the main text presents the results for $h = 0$ of this local linear projection. For comparison, we scaled all results so that the median response of the 2-year bond is equal to 0.1 annualized percentage points. The results are similar for the model implied historical term premia as well as for the estimates from Adrian et al. (2013). The left panel of Figure E6 extends this result using alternative measures of the 10-year nominal term premium from the literature. As the available sample differs in length among the estimates, Figure E6 shows the results for 1984:q1-2005:q4, while Figure 6 is based on our full sample 1983:q1-2007:q4.

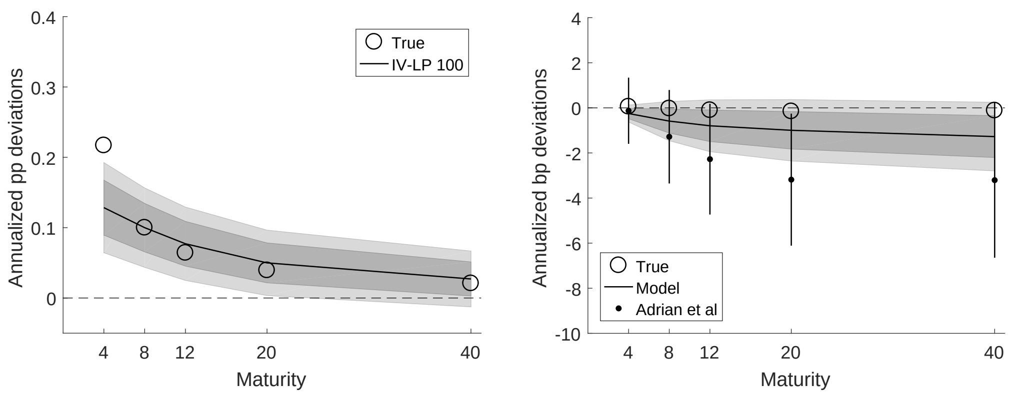


(a) monetary policy measure: Model (b) monetary policy measure: Romer and Romer

Figure E6. Impact effect of monetary policy shock on 10-year nominal term premia.

Note: The dots and vertical lines show median response and 95% confidence bands from the local projection for different historical 10-year nominal term premia as dependent variable, respectively. We use the Newey-West correction for the standard errors.

In the following, we investigate the effects from an alternative measure for monetary policy. We use the measure by Romer and Romer (2004) as updated by Wieland and Yang (2016). This measure is based on a structural interpretation of a monetary policy rule and, therefore, has a close relation to an innovation in the Taylor-rule as in our model. However, this measure is at its best a proxy for such a innovation. Accordingly, we use an instrumental variable local projection (IV-LP) as proposed by Stock and Watson (2018). Figure E7 shows the corresponding results. The IV-LP with Romer and Romer (2004)-shocks as instruments gives qualitatively similar but quantitatively smaller and often insignificant results for the impact response of nominal term premia. This continues to hold under alternative estimates for the 10-year term premia as variable of interest x (see right panel in Figure E6)

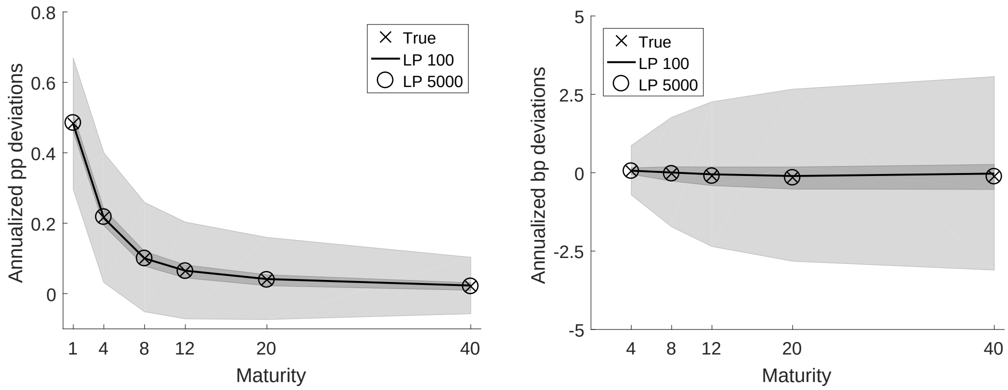


(a) Nominal yield curve (b) Nominal term premium

FIGURE E7. IMPACT EFFECT OF ROMER AND ROMER MONETARY POLICY SHOCK ON NOMINAL YIELDS AND NOMINAL TERM PREMIA FOR DIFFERENT MATURITIES.

Note: The solid line and shaded areas show median response, the 68%, and 90% confidence bands from the IV-LP with the model implied historical variables as dependent variable. The circles indicate the theoretical, true response. Additionally, the dots and vertical lines in the right panel show median response and 90% confidence bands from the IV-LP with term premia estimates from Adrian et al. (2013). We use the Newey-West correction for the standard errors.

In the following, we perform a Monte-Carlo exercise to evaluate the small sample properties of the linear projection estimator. At the posterior mean, we simulate 1,0000 time series with a length of 10,000 for all variables of interest, control variables, and monetary policy shocks from the model. After discarding the first 5,000 observations, we run two sets of local linear projections with a sample length of 100 and 5,000 respectively. Figure E8 presents the results. On average, both linear projections deliver estimates close to the true, theoretical response and, therefore, show no systematic small sample bias (Jordà, 2005). However, the Monte-Carlo exercise shows a high estimation uncertainty in small samples, consistent with the wide range of quantitatively and qualitatively different estimates in the empirical literature.



(a) Nominal yield curve (b) Nominal term premium

Figure E8. Monte-carlo experiment

Note: Impact effect of monetary policy shock on nominal yields and nominal term premia for different maturities. The dark gray area and light gray area represent the 95% confidence interval from the local projection with sample length 5000 and 100, respectively. The circles and solid line are the corresponding median responses while the crosses present theoretical response. All results are based on 1000 simulations at the posterior mean.

E5. Impulse response functions

The three columns in Figure E9 contain the IRFs of macroeconomic variables to a surprise shock to the policy rate (left column), to a surprise inflation target shock (middle column), and to a four-quarter ahead forward guidance shock (right column). All shocks are normalized to yield a median lowering of the policy rate by 50 basis points on impact (or in four quarters for the forward guidance shock).

The responses of the macroeconomy to the surprise policy rate shock are contained in first column of Figure E9. As is standard in the literature, the expansionary policy due to surprise policy rate cut (left column of Figure E9) leads to an increase in aggregate demand and its components as well as inflation. As the policy rate begins to return to its mean level with inflation still elevated, the resulting increase in expected real rates reverses the expansion, depressing aggregate demand and its components, before the macroeconomy then settles back to its mean position after around 10 quarters.

The middle column of Figure E9 shows the impulse responses to a surprise inflation target shock. The reduction in the inflation target is accompanied with a nearly two annualized percentage point reduction in inflation, roughly the same magnitude as the reduction of the target, which corresponds to a substantial change in the systematic behavior of monetary policy. The lowering of the policy rate is hump shaped with the maximal decrease of about 110 annualized basis points occurring about a year after the lowering of the inflation target. This lowering of the policy rate is not sufficient to overcome the initial contractionary effects of the lowered inflation target and associated disinflation as can be seen by the negative responses

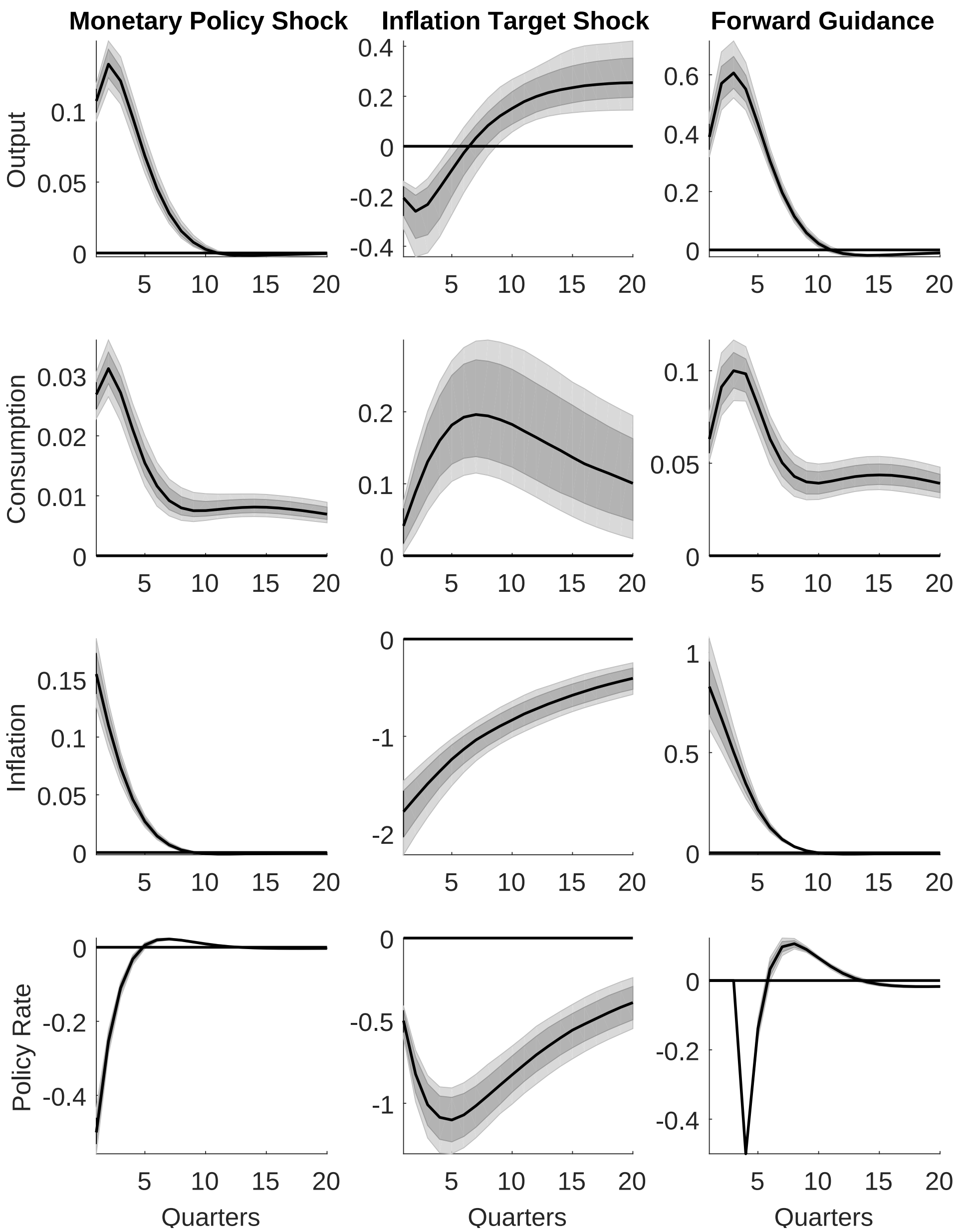


Figure E9. Posterior impulse responses of macro variables

Note: The figure shows a surprise 50 basis point policy rate cut, a surprise cut in the inflation target leading to a 50 basis point policy rate cut, and forward guidance of a 50 basis point policy rate cut in 4 quarters. The deviations of yields are in percentage points while the deviations of risk premia are presented in basis points. Shaded areas represent the 90% and 68% posterior credible sets.

on aggregate demand. Moreover, our results illustrate that a shock to the inflation target is much more long lasting and therefore has stronger effects on business cycle and lower frequencies, in contrast to a simple innovation to the Taylor-rule which quickly dissipates. This confirms the interpretation of Rudebusch and Swanson (2012) that a change in the inflation target, or more generally a change in the systematic behavior of monetary policy, introduces long-run nominal risk into the economy.

The right column in Figure E9 shows the evolution of macroeconomic variables following the forward guidance experiment. Similarly to most studies, we find that forward guidance increases macroeconomic activity and substantially increases inflation. Output and inflation both increase on impact with output reaching its peak after 3 quarters and falling slightly below its mean value after 12 quarters. The response to the announcement is driven by expectations of lower nominal short term interest rates and of future inflation. Expected higher inflation leads to a rise in current inflation through forward looking price setting, with a consequential fall in current and expected real interest rates and associated increase in economic activity on impact. Therefore, comparable to a change in the inflation target, forward guidance communicates the central bank's commitment to allow higher inflation in the future, which has more stronger and more long lasting effects on households' expectation and so on their precautionary savings motives.

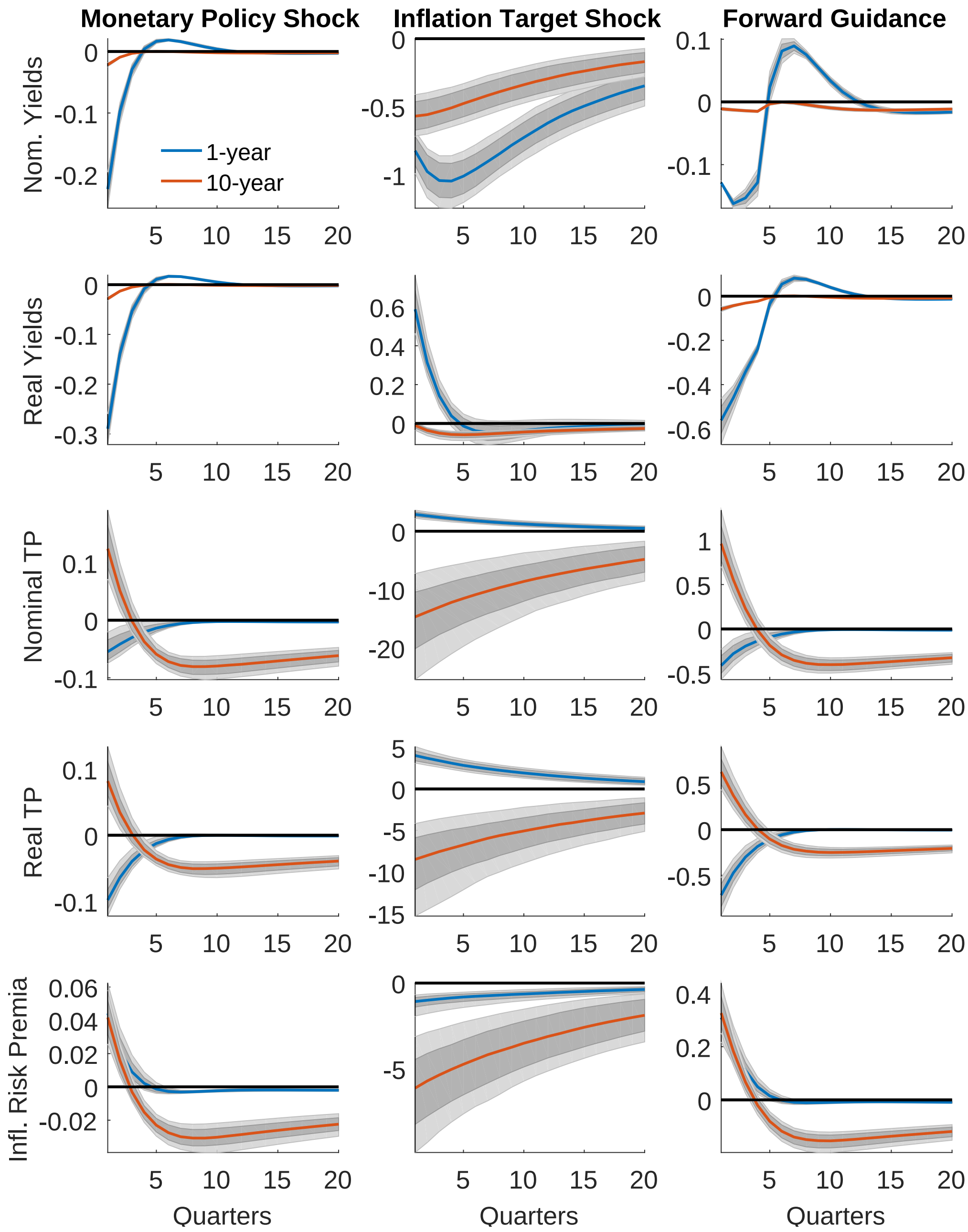


Figure E10. Posterior impulse responses of nominal and real term structure at the short and long end.

Note: The figure shows a surprise 50 basis point policy rate cut, a surprise cut in the inflation target leading to a 50 basis point policy rate cut, and forward guidance of a 50 basis point policy rate cut in 4 quarters. The deviations of yields are in percentage points while the deviations of risk premia are presented in basis points. Shaded areas represent the 90% and 68% posterior credible sets.

References Appendix

Adrian, T., R. K. Crump, and E. Moench (2013): "Pricing the term structure with linear regressions," *Journal of Financial Economics*, 110, 110–138.

Andreasen, M. M. (2011): "Explaining Macroeconomic and Term Structure Dynamics Jointly in a Non-linear DSGE Model," Creates research papers, School of Economics and Management, University of Aarhus.

——— (2012): "An estimated DSGE model: Explaining variation in nominal term premia, real term premia, and inflation risk premia," *European Economic Review*, 56, 1656–1674.

Andreasen, M. M., J. Fernández-Villaverde, and J. F. Rubio-Ramírez (2017): "The Pruned State-Space System for Non-Linear DSGE Models: Theory and Empirical Applications," *Review of Economic Studies*, forthcoming.

Andreasen, M. M. and P. Zabczyk (2015): "Efficient bond price approximations in non-linear equilibrium-based term structure models," *Studies in Nonlinear Dynamics & Econometrics*, 19, 1–33.

Bekaert, G., S. Cho, and A. Moreno (2010): "New Keynesian Macroeconomics and the Term Structure," *Journal of Money, Credit and Banking*, 42, 33–62.

Christiano, L. J., M. Trabandt, and K. Walentin (2011): "Introducing financial frictions and unemployment into a small open economy model," *Journal of Economic Dynamics and Control*, 35, 1999–2041.

Coeurdacier, N., H. Rey, and P. Winant (2011): "The Risky Steady State," *American Economic Review*, 101, 398–401.

Crump, R. K., S. Eusepi, and E. Moench (2016): "The term structure of expectations and bond yields," Staff report, FRB of NY.

De Graeve, F., M. Emiris, and R. Wouters (2009): "A structural decomposition of the US yield curve," *Journal of Monetary Economics*, 56, 545–559.

Del Negro, M. and F. Schorfheide (2008): "Forming priors for DSGE models (and how it affects the assessment of nominal rigidities)," *Journal of Monetary Economics*, 55, 1191–1208.

Dew-Becker, I. (2014): "Bond Pricing with a Time-Varying Price of Risk in an Estimated Medium-Scale Bayesian DSGE Model," *Journal of Money, Credit and Banking*, 46, 837–888.

Diebold, F. X., G. D. Rudebusch, and S. B. Aruoba (2006): "The macroeconomy and the yield curve: a dynamic latent factor approach," *Journal of Econometrics*, 131, 309–338.

Hanson, S. G. and J. C. Stein (2015): "Monetary policy and long-term real rates," *Journal of Financial Economics*, 115, 429–448.

Hördahl, P. and O. Tristani (2012): "Inflation risk premia in the term structure of interest rates," *Journal of the European Economic Association*, 10, 634–657.

Hördahl, P., O. Tristani, and D. Vestin (2006): "A joint econometric model of macroeconomic and term-structure dynamics," *Journal of Econometrics*, 131, 405–444.

——— (2008): "The Yield Curve and Macroeconomic Dynamics," *Economic Journal*, 118, 1937–1970.

Ireland, P. N. (2015): "Monetary policy, bond risk premia, and the economy," *Journal of Monetary Economics*, 76, 124–140.

Jin, H.-H. and K. L. Judd (2002): "Pertubation Methods for General Dynamic Stochastic Models," Mimeo, Stanford University.

Jordà, O. (2005): "Estimation and Inference of Impulse Responses by Local Projections," *American Economic Review*, 95, 161–182.

Juillard, M. (2010): "Local approximation of DGSE models around the risky steady state," mimeo, Bank of France.

Juillard, M. and O. Kamenik (2004): "Solving Stochastic Dynamic Equilibrium Models: a k-Order Perturbation Approach," Mimeo, Banque de France.

Kliem, M. and H. Uhlig (2016): "Bayesian estimation of a dynamic stochastic general equilibrium model with asset prices," *Quantitative Economics*, 7, 257–287.

Koop, G., M. H. Pesaran, and S. M. Potter (1996): "Impulse Response Analysis in Nonlinear Multivariate Models," *Journal of Econometrics*, 74, 119–147.

Lan, H. and A. Meyer-Gohde (2014): "Solvability of Perturbation Solutions in DSGE Models," *Journal of Economic Dynamics and Control*, 45, 366–388.

Lopez, P., D. Lopez-Salido, and F. Vazquez-Grande (2015): "Exponential-Affine Approximations of Macro-Finance Models," mimeo, Banque de France.

Meyer-Gohde, A. (2016): "Risk-Sensitive Linear Approximations," mimeo, Hamburg University.

Nakamura, E. and J. Steinsson (2017): "High frequency identification of monetary non-neutrality: The information effect," *Quarterly Journal of Ecnomics*, forthcoming.

Palomino, F. (2012): "Bond Risk Premiums and Optimal Monetary Policy," *Review of Economic Dynamics*, 15, 19–40.

Romer, C. D. and D. H. Romer (2004): "A New Measure of Monetary Shocks: Derivation and Implications," *American Economic Review*, 94, 1055–1084.

Rudebusch, G. D. and E. T. Swanson (2008): "Examining the bond premium puzzle with a DSGE model," *Journal of Monetary Economics*, 55, S111–S126.

——— (2012): "The Bond Premium in a DSGE Model with Long-Run Real and Nominal Risks," *American Economic Journal: Macroeconomics*, 4, 105–43.

Rudebusch, G. D. and T. Wu (2007): "Accounting for a Shift in Term Structure Behavior with No-Arbitrage and Macro-Finance Models," *Journal of Money, Credit and Banking*, 39, 395–422.

——— (2008): "A Macro-Finance Model of the Term Structure, Monetary Policy and the Economy," *Economic Journal*, 118, 906–926.

Schmitt-Grohe, S. and M. Uribe (2004): "Solving dynamic general equilibrium models using a second-order approximation to the policy function," *Journal of Economic Dynamics and Control*, 28, 755–775.

Stock, J. H. and M. W. Watson (2018): "Identification and Estimation of Dynamic Causal Effects in Macroeconomics Using External Instruments," Working Paper 24216, National Bureau of Economic Research.

Swanson, E. T. (2012): "Risk Aversion and the Labor Margin in Dynamic Equilibrium Models," *American Economic Review*, 102, 1663–91.

Tallarini, T. D. (2000): "Risk-sensitive real business cycles," *Journal of Monetary Economics*, 45, 507–532.

van Binsbergen, J. H., J. Fernández-Villaverde, R. S. Koijen, and J. Rubio-Ramírez (2012): "The term structure of interest rates in a DSGE model with recursive preferences," *Journal of Monetary Economics*, 59, 634–648.

Wieland, J. F. and M.-J. Yang (2016): "Financial Dampening," NBER Working Papers 22141, National Bureau of Economic Research, Inc.