Investing in Children's Skills: Equilibrium Analysis of Social Interactions and Parental Investments

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Motivation

- **How do social interactions affect the dynamics of skill formation?**
  - Peer effects (effect of friends' achievements on a child's outcome)
  - Parental investments respond to the child's social interactions

- **Many policies have lasting effects on peers' composition**
  - Examples: school busing policies, re-drawing school's district boundaries, etc
  - Size of the policy matters (no. of children)
    - School composition is changed
    - Children make new friends
    - Parental investments endogenously respond to changes in peers

This Paper

- **Dynamic equilibrium model of child development and social interactions**
  - Children grow up in different *environments*

    (peers composition , neighborhood quality , school quality)
  - Endogenous peer network formation and parental investments
  - Technology of skill formation
  - Equilibrium effects within each environment:
    - Individual return of investing is affected by other parents' investments (through children's social interactions)
    - Explain part of developmental gaps between different environments

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- **Preview of Results:** Moving many children to better environment:
  - Important dynamic equilibrium effects
    - Receiving children: up to -10% SD skills at age 16
  - Heterogeneous effects due to endogenous formation of new peers

Data and Empirical Evidence

Data

- The National Longitudinal Study of Adolescent Health (Add Health)
- Representative for US schools in 94-95
  - 144 public and private schools
  - In-school survey: 90,118 adolescents in grades 7-12
  - In-home survey: 20,745 subsample of In-school survey
  - Contextual information about Census Tract (e.g.: median household income)
- Friendships nomination
  - Friendship network within school roster
- Measures for adolescents achievements (skills)
  - Peabody Picture Vocabulary Test (PPVT)
  - Math, Science, English and History Grades
- Measures for parental investments (In-home survey)

Summary of Empirical Evidence

1. Homophily-bias in friendship formation
   - Race Race
   - Skills (New Fact) Skills

2. Parental investments respond to peer compositions (New Fact)
   - Empirical challenge: peer groups are formed endogenously
   - I exploit within-school/across-cohorts variation in peer composition (see Hoxby, 2000)
     - Shifts in the *choice set* from which children can select their friends

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- What's the effect on child development of changes in peer composition?
  - To answer this question, I need a model with:
  1. Endogenous formation of new peer groups
  2. Parents respond to peer changes
     - Equilibrium effects of other parents' investments on a child development

The Model

The Model

- Children will be between 13 and 16 years old
- Different environments $e \in \{1, \ldots, E\}$:
  - Populated by N_e families
  - Neighborhood quality d
  - School quality s
- Families are formed by one parent and one child
  - Heterogeneous in many dimensions: skills, race, income

1. **Children**:
   - Select their peers based on their observed characteristics and skills
2. **Parents**:
   - Take children's decision as given
   - Invest their time to foster their children's skills

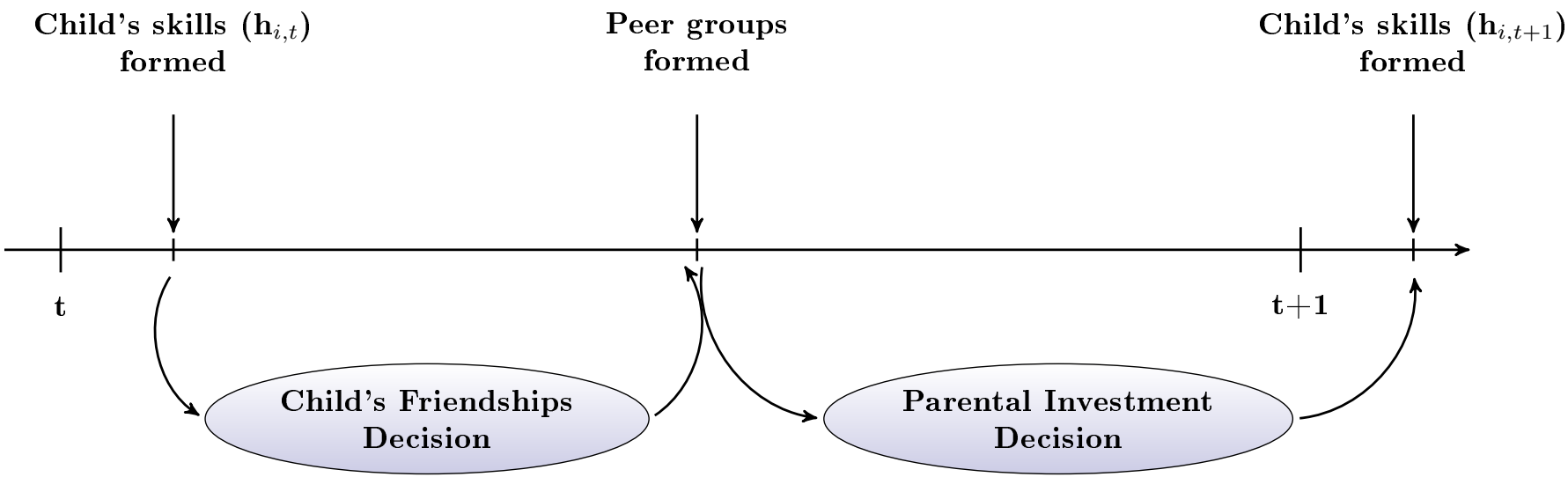
- Equilibrium: Parental investments have to be consistent with each other (Equilibrium concept: Markovian Equilibrium)

Technology of Skill Formation

$$h_{i,t+1} = h_{i,t}^{\alpha_1} \left[\alpha_2 \left(I_{i,t}\right)^{\alpha_3} + (1-\alpha_2)\left(\overline{H}_{i,t}\right)^{\alpha_3}\right]^{\frac{\alpha_4}{\alpha_3}} \cdot e^{A_d + A_s + A_t + \eta_{i,t+1}}$$

- $h_{i,t+1}$: Next-period stock of skills
- $h_{i,t}$: Current stock of skills
- $I_{i,t}$: Parental investments
- $L_{i,j,t}$: Indicator of friendship ($= 1$ if i and j are friends)
- Peer effects: $\overline{H}_{i,t} = \frac{1}{\sum_{j=1,j\neq i}^{H} L_{i,j,t}} \sum_{j=1,j\neq i}^{H} L_{i,j,t}\, h_{j,t}$
- A_d neighborhood effect
- A_s school effect
- A_t trend
- $\eta_{i,t+1}$ skills shock
- Age of children: $t \in \{13, \ldots, 16\}$

Timeline


Child's skills ($h_{i,t}$) formed
Peer groups formed
Child's skills ($h_{i,t+1}$) formed
t
t+1
Child's Friendships Decision
Parental Investment Decision

Estimation

Sample Statistics

	Mean (1)	Standard Deviation (2)
Child's Age	15.65	1.74
Fraction black	0.16	0.37
Fraction hispanic	0.17	0.38
Fraction white	0.67	0.47
N of reported friends (In-School)	6.98	3.28
Schools characteristics:		
School size	1,042	629
Cohort size	261	156
Measures for skills:		
PPVT	64.26	11.14
English	2.83	0.98
Math	2.72	1.03
History	2.86	1.01
Science	2.82	1.01
Family's characteristics:		
Income ($ 1994)	42,844	27,724
Mother's education	13.13	2.35
No of Obs		
In-School Survey	90,118	
In-Home Survey	14,267	

Structural Estimation

- Estimator: Simulated Method of Moments (SMM)

- Dynamic latent factor model (skills and investments are unobserved)
  - Cunha et al. (2010), Agostinelli and Wiswall (2016)

- Moments selection and identification:
  - Indirect Inference:
    - Elasticities of parental investments w.r.t. peers' skills
    - Autocorrelation in skill formation and parental investments
  - Distribution of skills by age between environments Skills
  - Moments on homophily-bias in friendship formation Homophily
  - School and neighborhood valued added

Indirect Infecence: Auxiliary Model

- I want to identify the peer effects on parental investments
- 2SLS estimator (both in data and simulated data):

$$\text{(Second Stage)} \quad \Delta_s I_{i,t} = \gamma_1 \, \Delta_s \ln h_{i,t} + \gamma_2 \Delta_s \ln \overline{H}_{i,t} + \Delta_s X_i' \gamma_3 + \Delta_s \gamma_t + \Delta_s \epsilon_{i,t}$$

$$\text{(First Stage)} \quad \Delta_s \ln \overline{H}_{i,t} = \beta_1 \, \Delta_s \ln h_{i,t} + \beta_2 \Delta_s Z_{i,t} + \Delta_s X_i' \beta_3 + \Delta_s \beta_t + \Delta_s u_{i,t}$$

- Δ_s: within-school transformation
- β_2 identifies degree of homophily in friendships formation
- γ_2 identifies parents-peers complementarities in skill formation
- $\Delta_s Z_{i,t}$: within-school/between-cohorts variation in % same-race children
  - Common IV in peer effects literature (see Hoxby, 2000)
  - I allow interaction with child's skills to account for homophily in skills

Sample Fit: Auxiliary Regressions Coefficients

	Dependent Variable Fraction (%) of Invested Parental Time	
	Instrumental Variables (IV)	Instrumental Variables (IV)
	Data	Model
Peer's Skills (Log)	0.720 (0.354) [0.026,1.414]	0.895
	First Stage	First Stage
$Z_{1,i,t}$ (Minorities Children)	-0.104 (0.052) [-0.206,-0.002]	-0.127
$Z_{2,i,t}$ (White Children)	0.082 (0.037) [0.009,0.155]	0.105
F-Stat Excl. Instruments	11.78	
P-value	0.000	

Each regression includes age and school fixed effects and controls for family characteristics. Standard errors are clustered at school level.

Structural Estimates

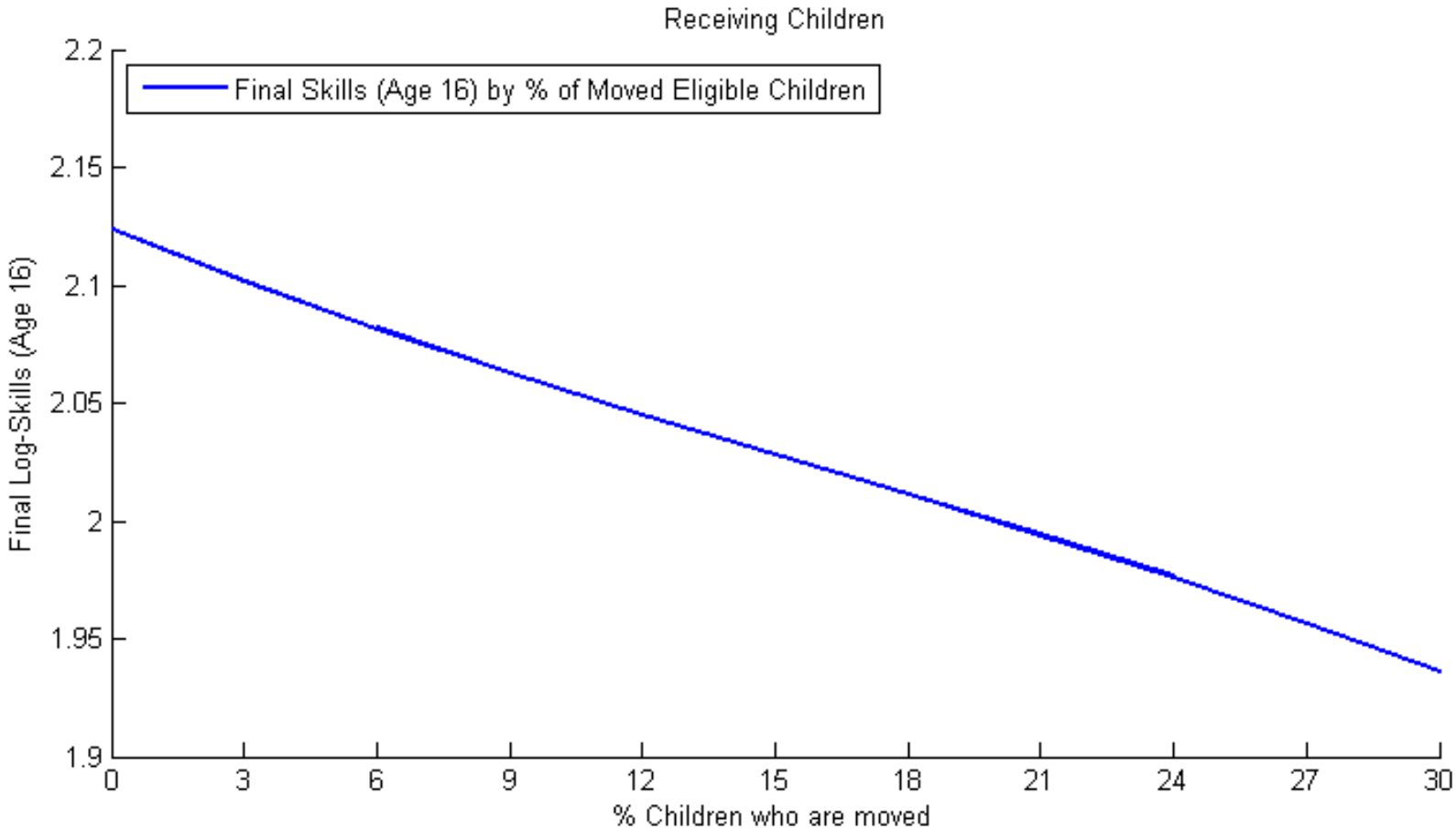
- **Technology:**
  - CES complementarity parameter $= 0.944$ (s.e. 0.087)
    - Almost perfect substitute
  - Self-Productivity $= 0.744$ (s.e. 0.068)
    - $\uparrow$ 1% current skills $\Rightarrow \uparrow$ 0.74% next period skills (elasticity)
- **Peer-Network Formation:**
  - A white child with low-skills (first quintile skills distribution)
    - 2.5 times more likely to befriend a same-race child
    - 2 times more likely to befriend a same-skill child
  - A black child with low-skills (first quintile skills distribution)
    - 4 times more likely to befriend a same-race child
    - 2 times more likely to befriend a same-skill child

All estimates technology | Estimates Initial Conditions | Peer Formation Parameters | Other parameters

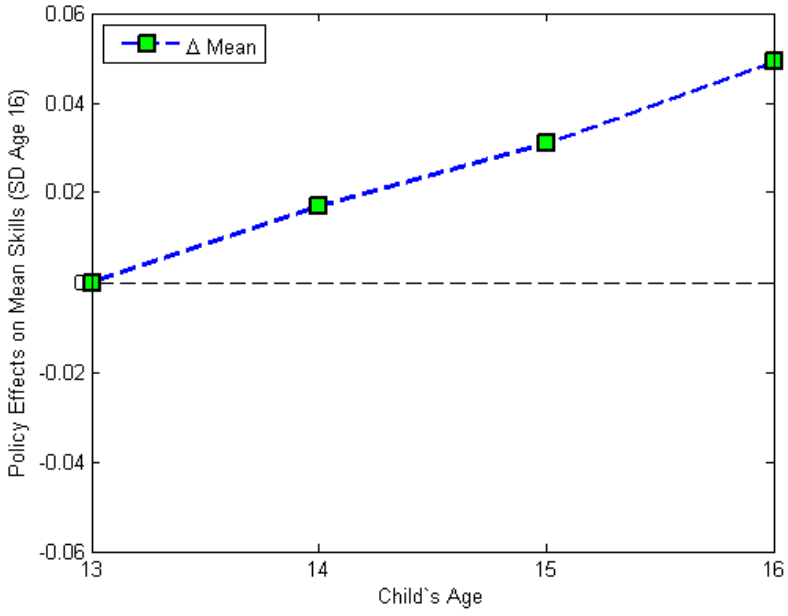
Larger-scale policy

- Moving children at age 13 from *low-income* environment
  - First quartile of skill distribution
  - From 1% to 30% of population of the receiving neighborhood
  - Median family income $\approx$ 25k (in 2017 dollars)
  - Racial composition: 10 % white, 43% hispanic, 47% black
- Receiving *high-income* environment
  - Median family income $\approx$ 100k (in 2017 dollars)
  - Racial composition: 84 % white, 10% hispanic, 6% black
- Caveats:
  - No endogenous response of changing environment
  - Neighborhood and School quality are policy invariant

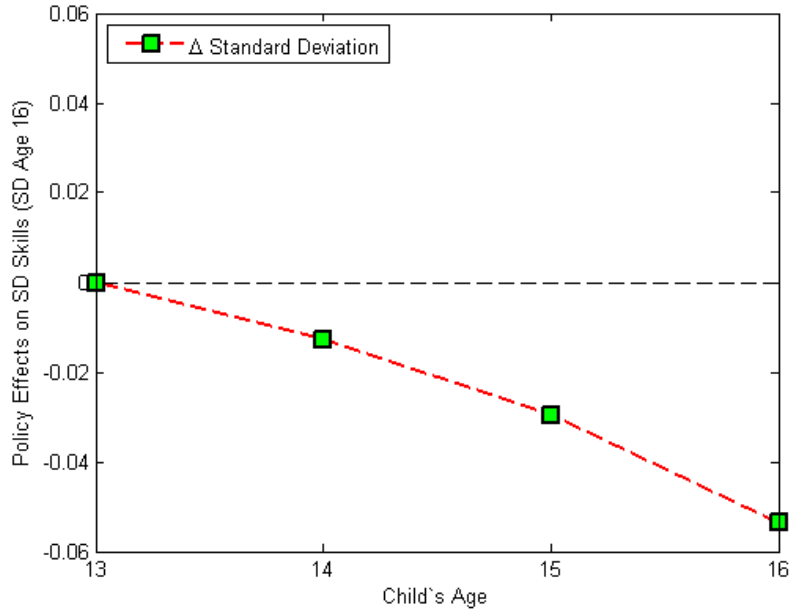
Treatment Effect by Fraction of Moved Eligible Children



Aggregate Effects on Skill Distribution



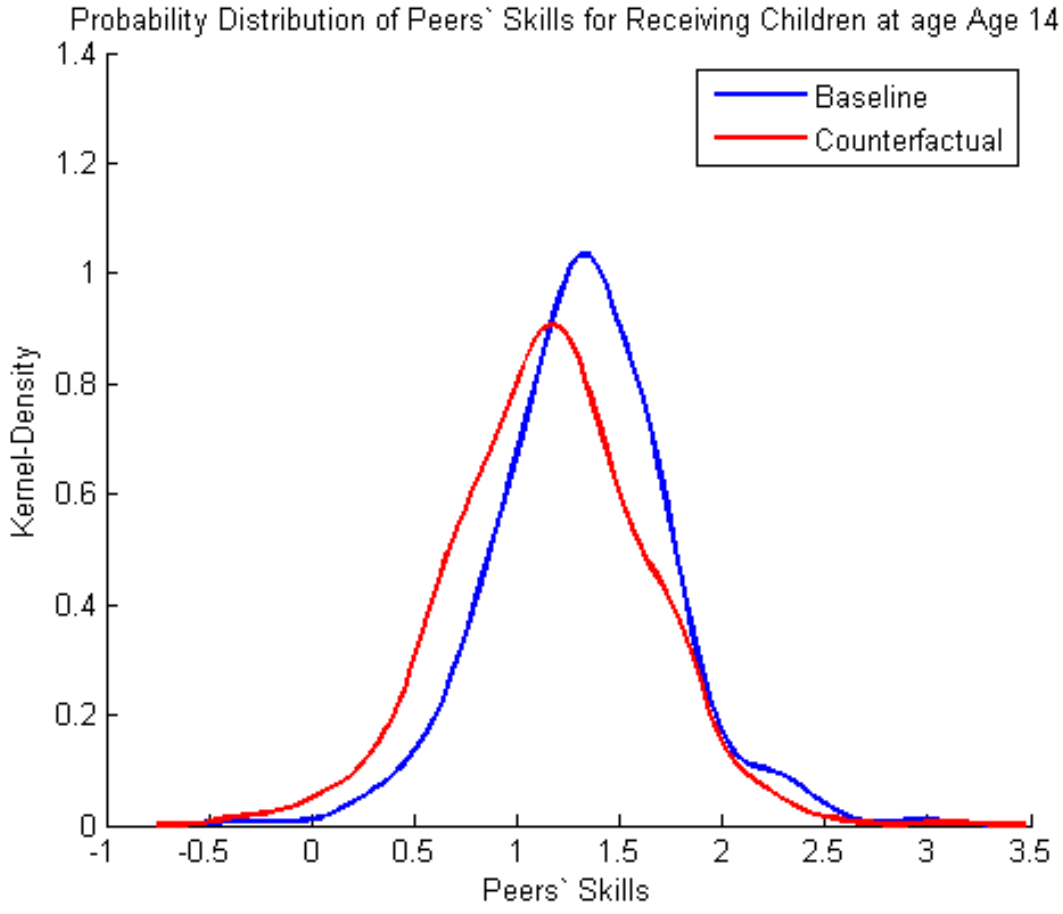
Change in Aggregate Mean Skills



Change in Aggregate SD Skills

Why are receiving children negatively affected?

Expected Peers for Receiving Children (10% Policy)



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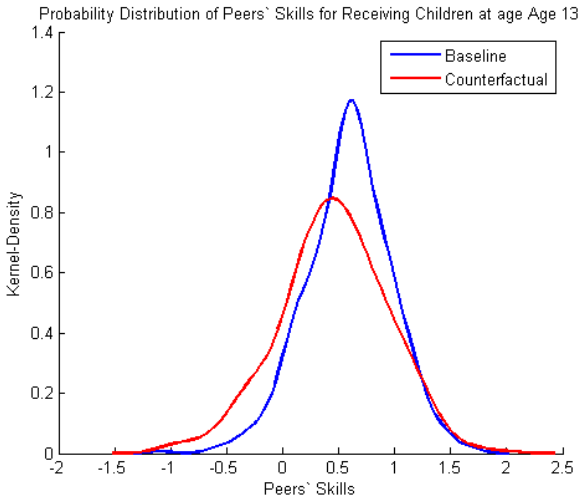
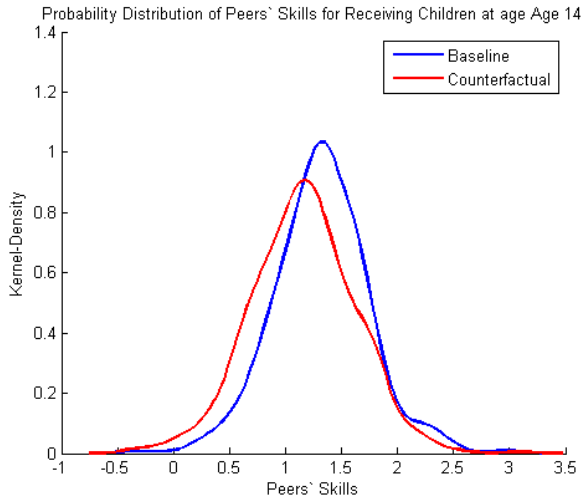
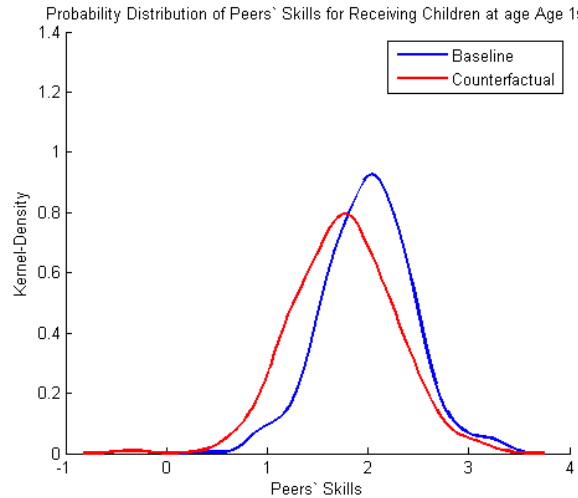






Are effects on receiving children heterogeneous?

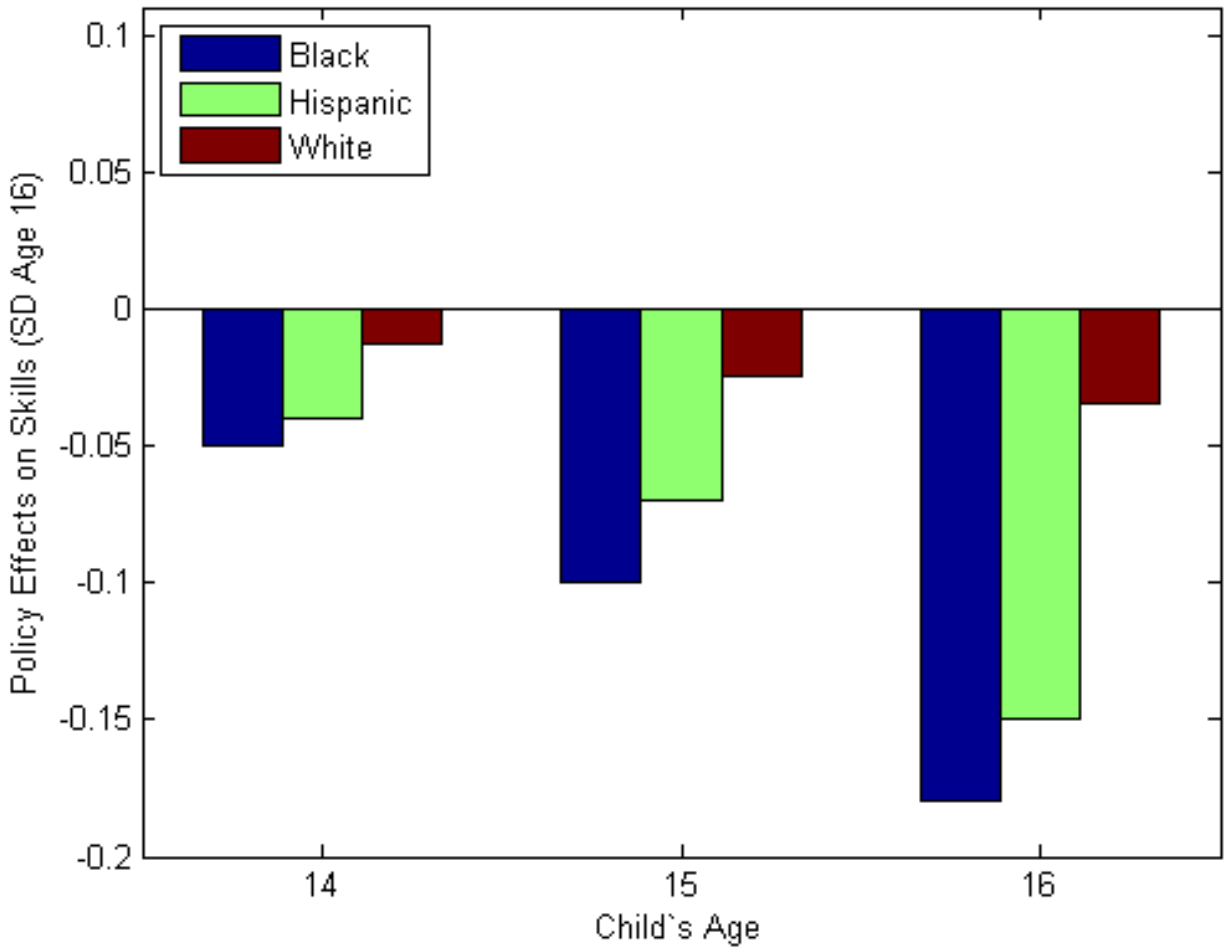
- Evidences on differential (stronger) peer effects on **minorities**

  (Hoxby (2000); Angrist and Lang (2004); Imberman, Kugler and Sacerdote (2012))

- Is it a story of endogenous social interactions?

Skills

Effects on Receiving Children by Race



- Stronger policy effects for minorities

Conclusions

- I built and estimated a model of child development and social interactions
- Estimated model replicates previous findings on childhood exposure effects
  - Treatment effects are not informative for large-scale policies
- Large-scale policies
  - Dynamic-equilibrium effects are key for policy predictions
  - Heterogeneous effects based on endogenous formation of new peer groups

Moved Children

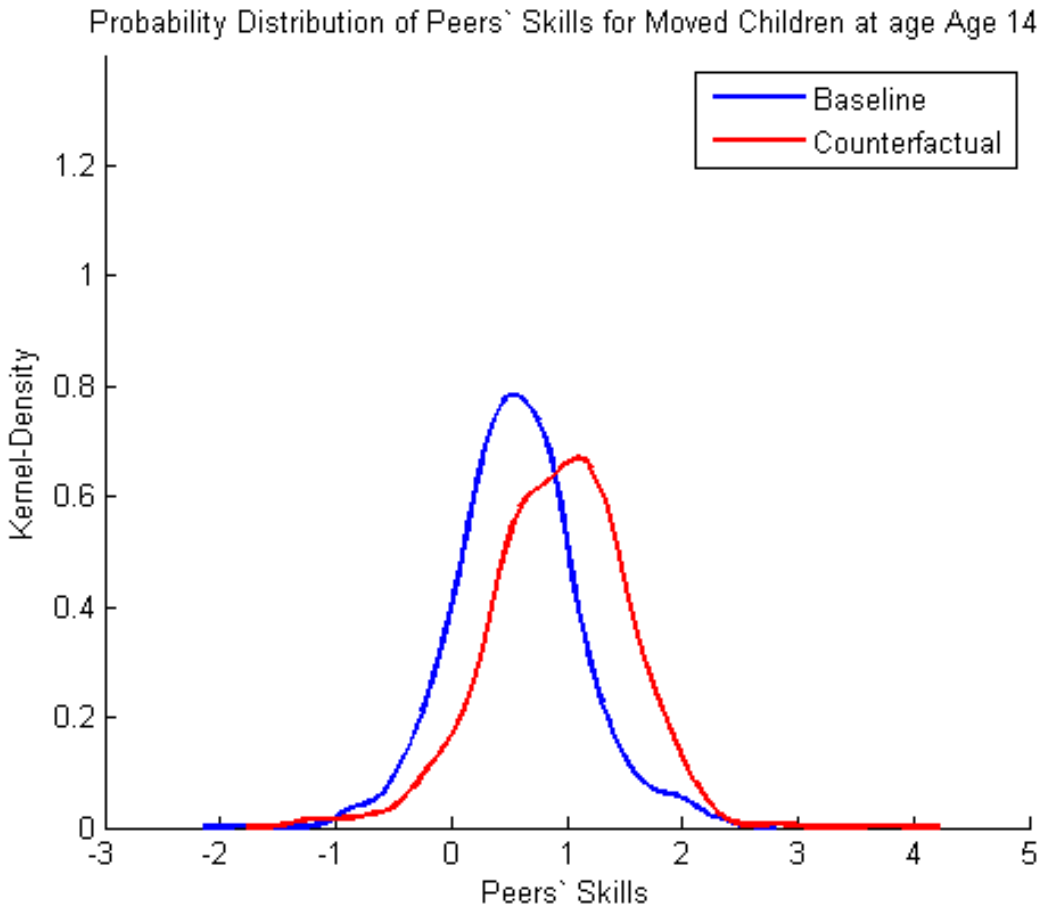
Panel A: Effects on Children's Log-Skills (Mean)

	Counterfactual (Equilibrium)	Counterfactual (No Equilibrium)
Age 13	0.00	0.00
Age 14	+0.09	+0.04
Age 15	+0.16	+0.10
Age 16	+0.31	+0.26

Panel B: Effects on Parent's Investment Decision (Mean)

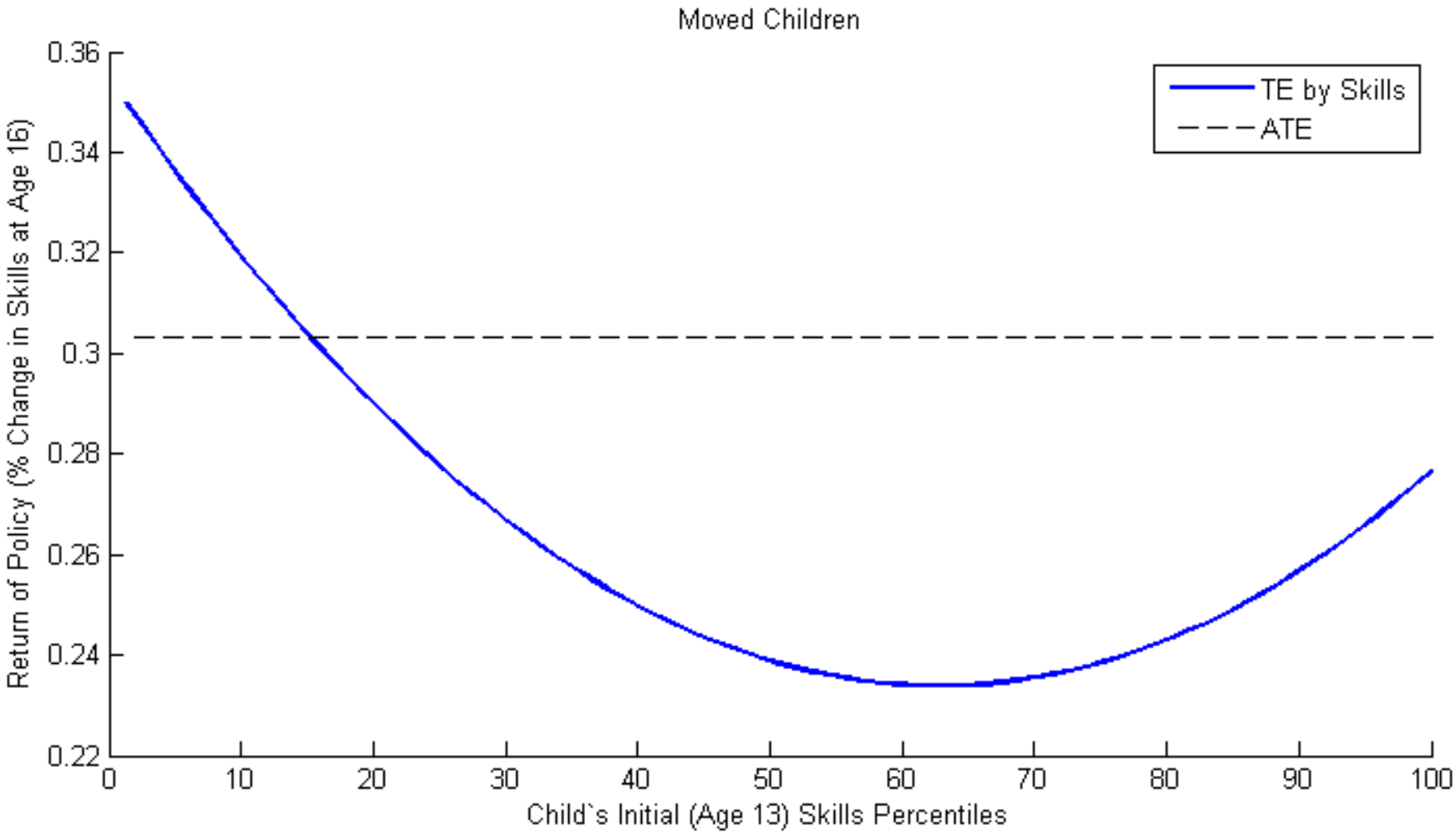
	Counterfactual (Equilibrium)	Counterfactual (No Equilibrium)
Age 13	+1.63	-0.03
Age 14	+0.62	-0.85
Age 15	-0.42	-0.79

Expected Peers for Moved Children: Baseline vs Counterfactual



Are effects heterogeneous by intial skill endowment?

Heterogeneous Effects in Moved Children by Skills



Latent Factor Model for Skills

- Measures for skills I use:
  - PPVT
  - Math Grades
  - Science Grades
  - English Grades
  - History Grades
- Latent factor model for some measure/proxy m :

$$\underbrace{Z_{i,t,m}}_{\text{Observed proxy}} = \mu_{t,m} + \lambda_{t,m} \underbrace{\ln h_{i,t}}_{\text{Latent skills}} + \underbrace{\epsilon_{i,t,m}}_{\text{Measurement error}}$$

  - t = age of child
  - $\mu_{t,m}$ = location of measure
  - $\lambda_{t,m}$ = factor loading/scale of measure

Return

Latent Factor Model for Investments

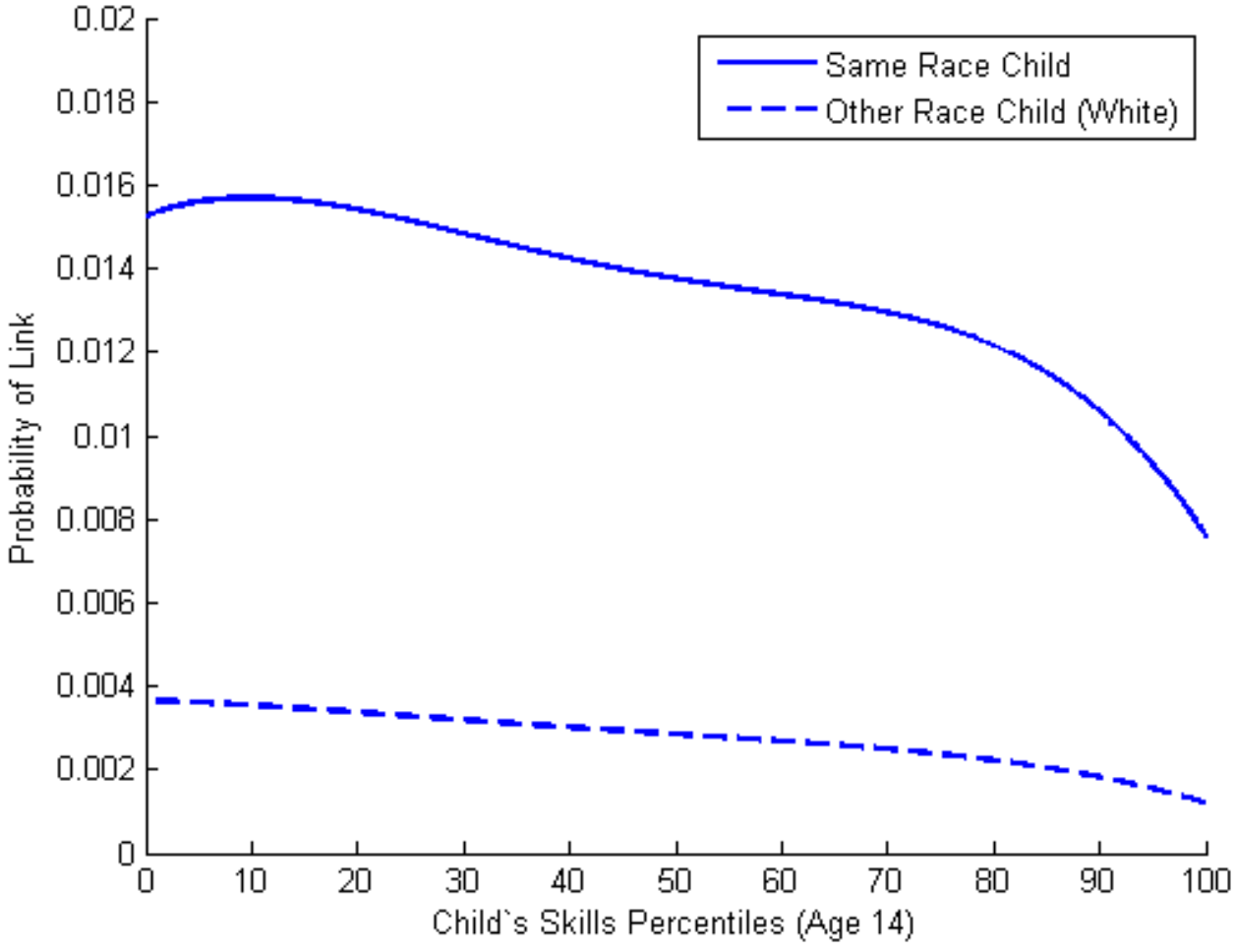
- $Z_{i,k,t} \in \{0, 1\}$ Observed measure of investments
- $p(I_{i,t})$: probability $Z_{i,k,t} = 1$ function of latent investment
- Assumptions:

  1. $p(I_{i,t}) \sim \text{Beta}(\ \alpha + Z_{i,k,t},\ 1 + \beta - Z_{i,k,t}\ )$
  2. $p(I_{i,t}) = (\frac{I_{i,t}}{\tau})^{\lambda_{k,t}}$ where $\frac{I_{i,t}}{\tau}$ is fraction of invested time

- $\{I_{i,t}\}_i$, α, β, $\{\lambda_{k,t}\}_k$ are identified up to normalization (scale and location)
- Look at ATUS to identify mean and variance of fraction of time invested (location and scale for latent investments)

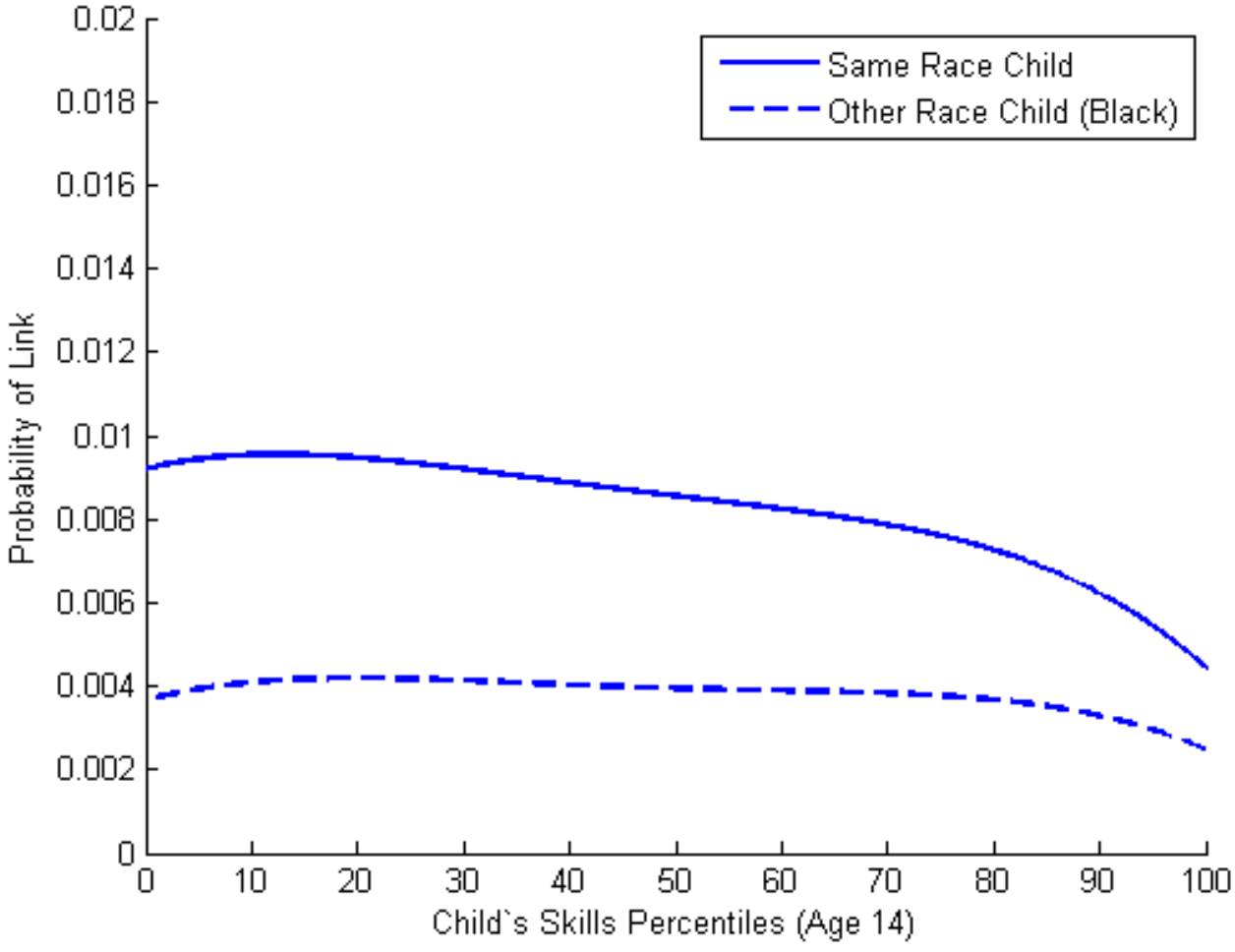
Return

Peer Group Formation: Black - Low Skills - Child



Return

Peer Group Formation: White - Low Skills - Child



Return

Estimates for Initial Conditions

Panel A: Mean Initial Child's and Mother's Skills

	Neighborhood 1		Neighborhood 2		Neighborhood 3	
	Child	Mother	Child	Mother	Child	Mother
Black	-0.47	-0.07	-0.40	0.36	-0.30	0.44
	(0.08)	(0.15)	(0.27)	(0.25)	(0.29)	(0.20)
Hispanic	-0.49	-0.93	-0.48	-0.77	-0.34	-0.36
	(0.11)	(0.19)	(0.26)	(0.19)	(0.25)	(0.19)
White	0.00	0.00	0.02	0.26	0.22	0.58
	(-)	(-)	(0.24)	(0.18)	(0.24)	(0.19)

Panel B: Variance-Covariance Initial Child's and Mother's Skills

	Neighborhood 1		Neighborhood 2		Neighborhood 3	
	Child	Mother	Child	Mother	Child	Mother
	0.65		0.87		0.89	
	(0.05)		(0.08)		(0.15)	
Black	0.20	0.61	0.31	0.67	0.30	0.64
	(0.08)	(0.14)	(0.09)	(0.17)	(0.16)	(0.14)
	0.84		1.10		0.78	
	(0.09)		(0.10)		(0.12)	
Hispanic	0.22	1.59	0.26	1.58	0.28	1.33
	(0.08)	(0.32)	(0.08)	(0.35)	(0.10)	(0.34)
	1.00		1.09		0.99	
	(-)		(0.09)		(0.13)	
White	0.48	1.00	0.37	0.74	0.36	0.78
	(0.07)	(-)	(0.04)	(0.19)	(0.06)	(0.17)

Return

Estimates: Technology

Parameter	Estimate	S.E.
Child's Skills (α_1)	0.744	0.0682
Investments (Yearly Hours, α_2)	0.009	0.0014
Elasticity Investment vs Peers (α_3)	0.944	0.0270
Return to Scale (α_4)	0.767	0.0283
Std of Shocks (σ_ξ)	0.700	0.0461
Panel B: Neighborhood TFP		
Constant ($\gamma_{0,tfp}$)	-1.329	0.1256
Neighborhood Quality ($\gamma_{1,tfp}$)	0.008	0.0003
Age Trend ($\gamma_{2,tfp}$)	0.030	0.0008
Panel C: School-Quality Effects		
Low Income Neighborhood		
Mean ($\eta_{s,1}$)	-0.033	0.0350
Standard Deviation ($\sigma_{s,1}$)	0.262	0.0264
Medium Income Neighborhood		
Mean ($\eta_{s,2}$)	0.006	0.0277
Standard Deviation ($\sigma_{s,2}$)	0.244	0.0278
High Income Neighborhood		
Mean ($\eta_{s,3}$)	0.041	0.0318
Standard Deviation ($\sigma_{s,3}$)	0.188	0.0249

Return

Estimate of Preferences and Wage/Income Process

Parameter	Estimate	S.E.
Panel A: Preferences Parameters		
Curvature on consumption (γ_1)	0.786	0.0046
Weight on Child's Skills (γ_2)	0.901	0.0030
Weight on Final Child's Skills (γ_4)	2.475	0.2455
Curvature on Child's Skills (γ_3)	0.562	0.0256
Curvature on Final Child's Skills (γ_5)	0.465	0.0011
Panel B: Parameters of Labor and Non-Labor Income		
Constant (Wage, $\kappa_{1,0}$)	2.750	0.0067
Mother's Skills (Wage, $\kappa_{1,1}$)	0.438	0.0048
Constant (Non-Labor Income, $\kappa_{2,0}$)	9.992	0.0174
Mother's Skills (Non-Labor Income, $\kappa_{2,1}$)	1.033	0.0113

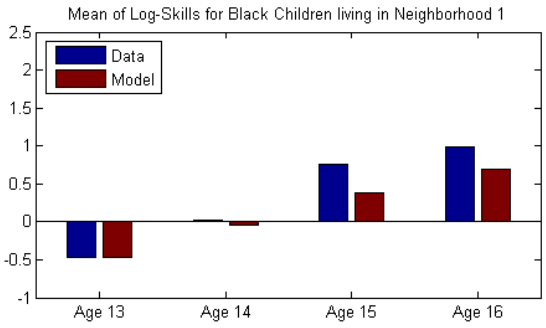
Return

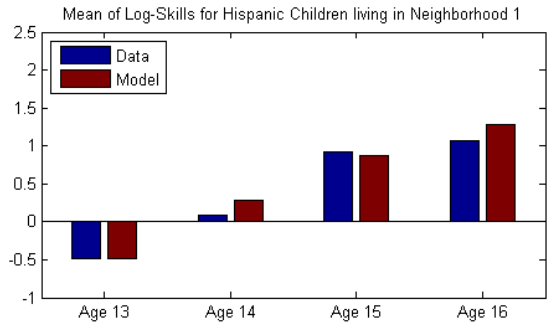
Estimate: Child's Utility

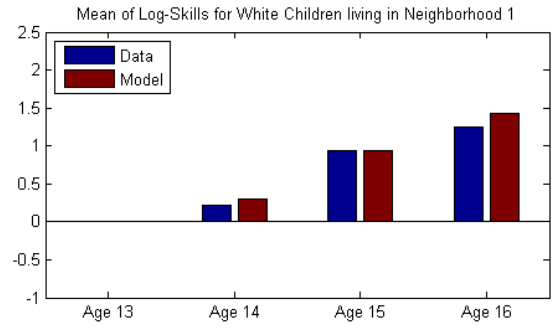
Parameter	Estimate	S.E.
Constant (δ_1)	-0.246	0.0172
Child's Log-Skills (δ_2)	0.088	0.0048
Black ($\delta_{3,1}$)	0.075	0.0023
Hispanic ($\delta_{3,2}$)	-0.005	0.0001
Both Black ($\delta_{4,1}$)	0.763	0.0317
Both Hispanic ($\delta_{4,2}$)	0.701	0.0298
Both White ($\delta_{4,3}$)	0.559	0.0475
Distance in Children's Skills (δ_5)	-0.038	0.0014
N of Children (Hundreds, $\delta_{6,1}$)	-0.890	0.0003
N of Children Squared (Hundreds, $\delta_{6,2}$)	0.001	0.0000
Distance in Children's Skills · %White ($\delta_{6,3}$)	- 0.063	0.0032
Distance in Children's Skills · %Black ($\delta_{6,4}$)	0.042	0.0025
Age (δ_7)	-0.050	0.0010
Additional Unobserved Heterogeneity ($\zeta_{i,j,t}$)		
Correlation with Skill Shocks	-0.404	0.0212
Standard Deviation	0.110	0.0095

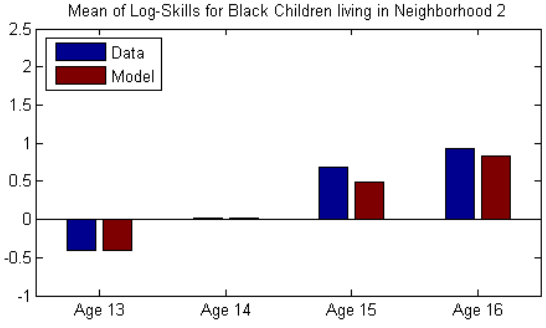
Return

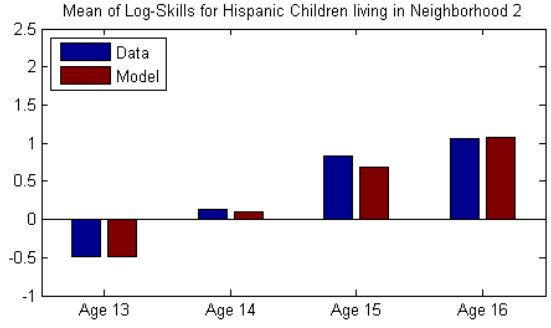
Dynamics of Mean Children's Skills by Race and Neighborhood (Return)

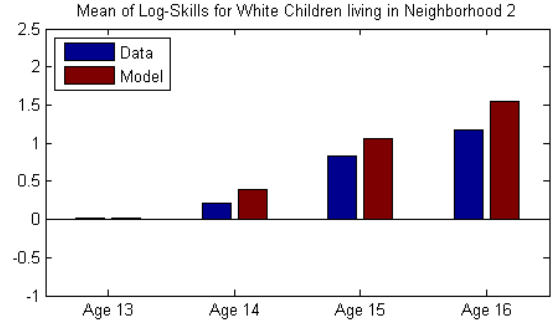


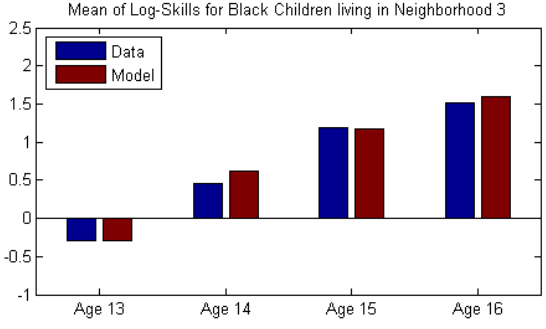


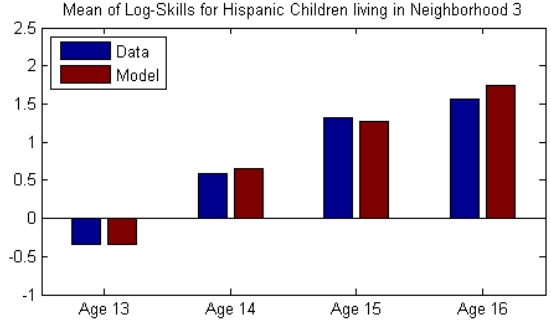


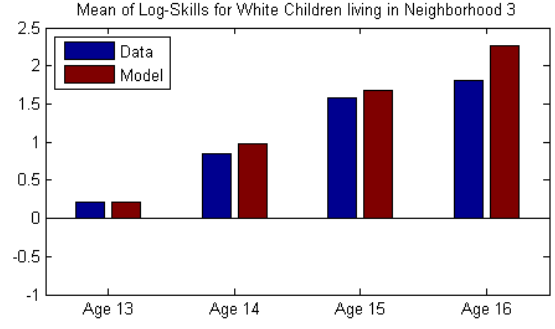




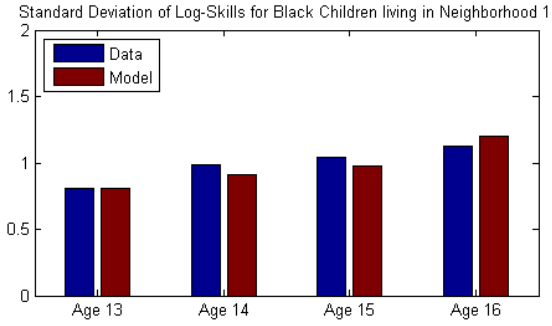


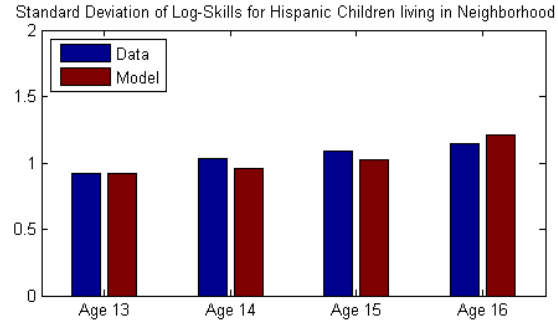


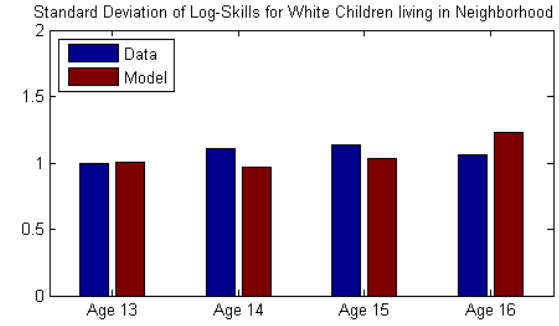


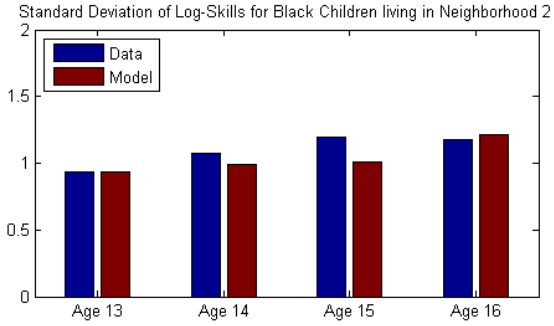


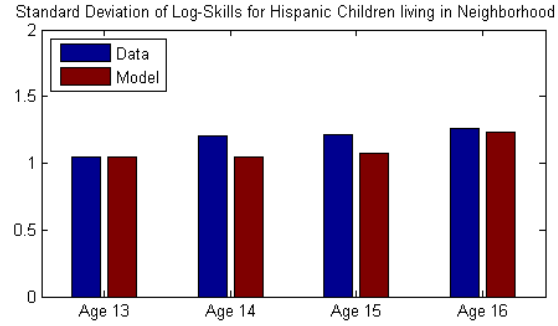
Dynamics of Std Children's Skills by Race and Neighborhood (Return)

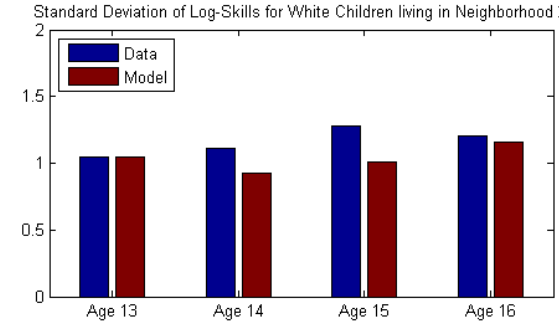


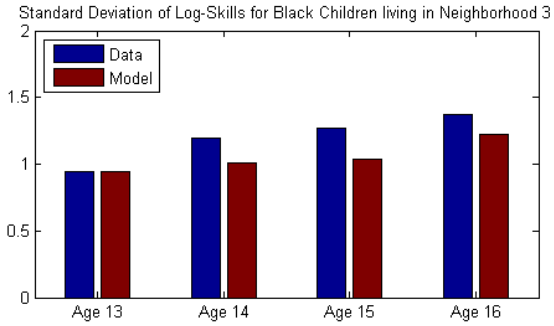


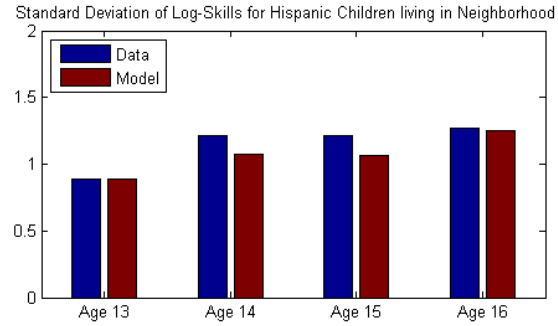


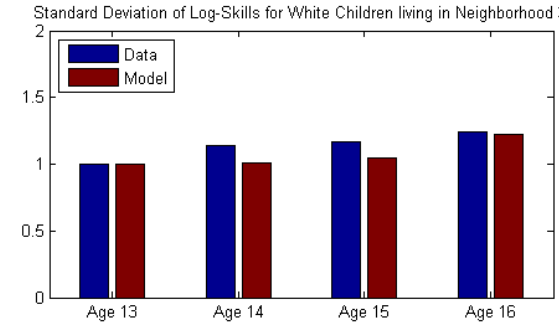




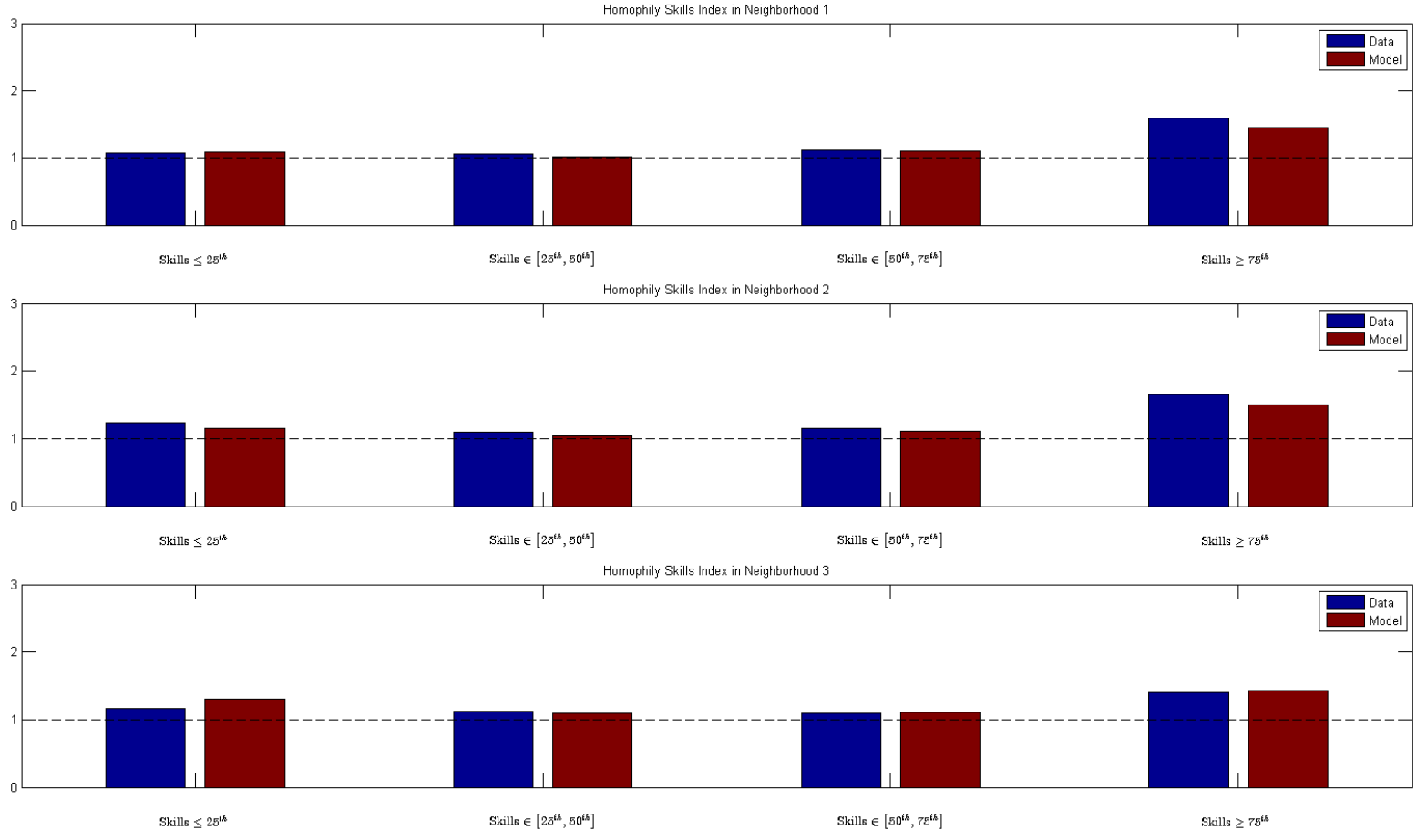






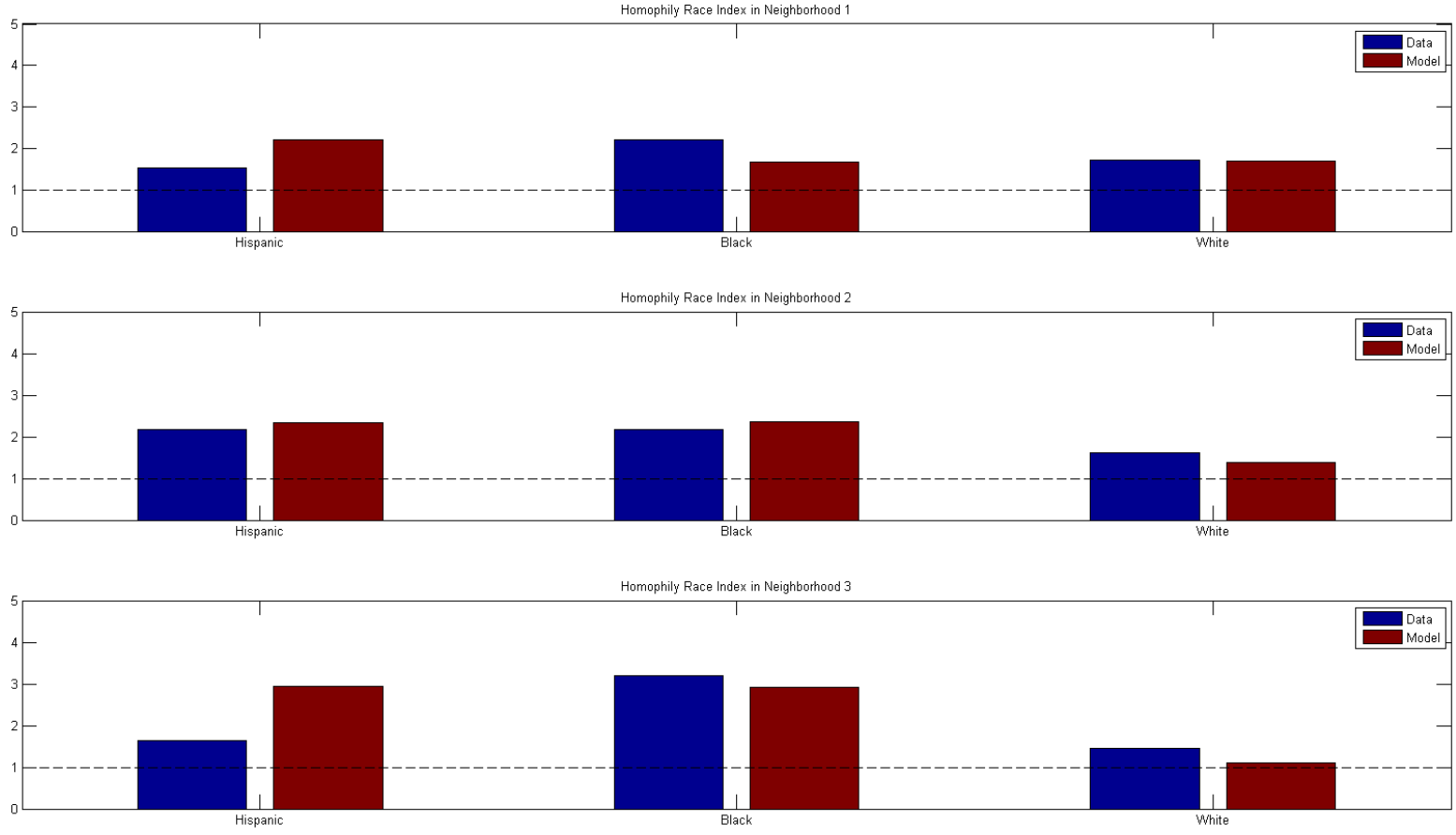


Homophily Skill Index by Skills and Neighborhood



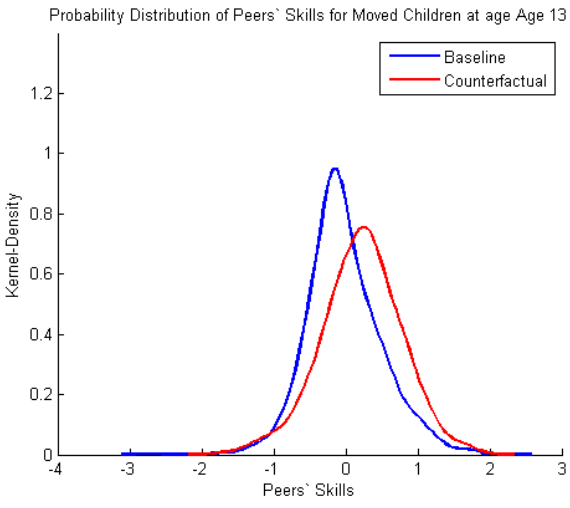
Return

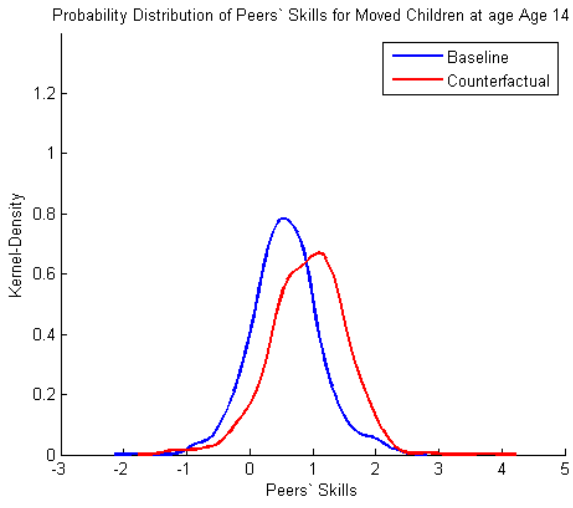
Homophily Race Index by Race and Neighborhood

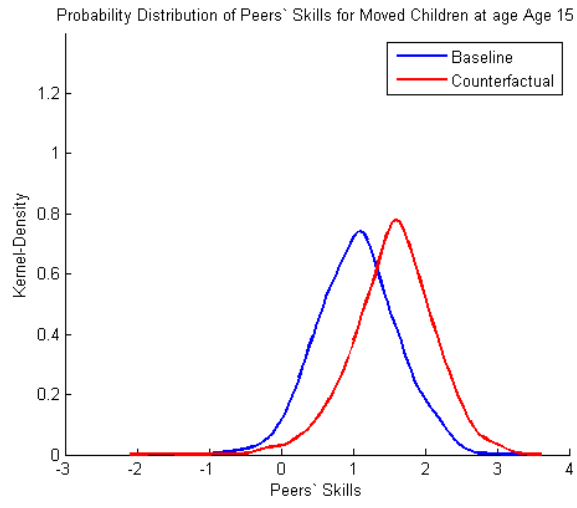


Return

Expected Peers for Moved Children: Baseline vs Counterfactual







Return

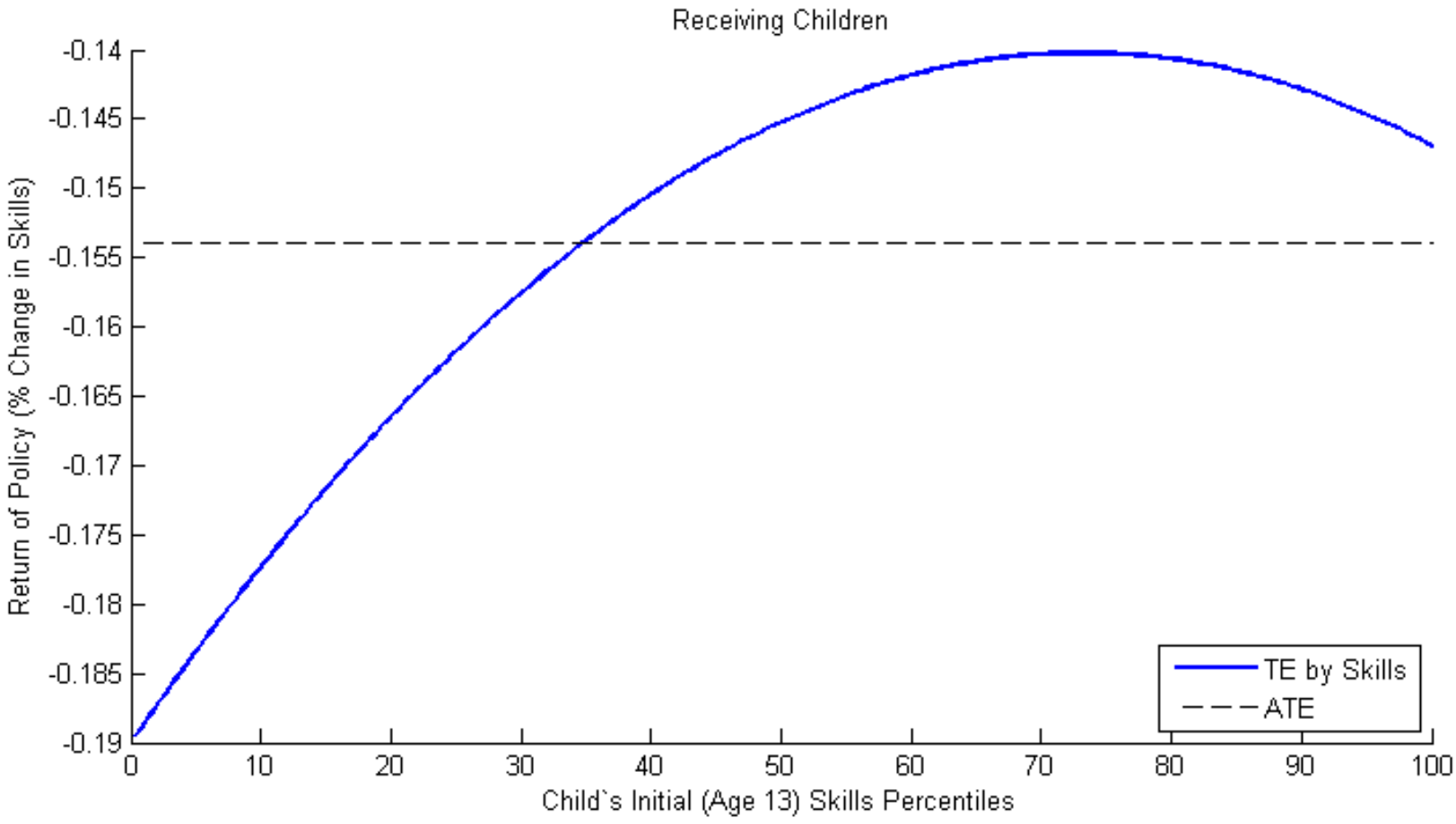
Technology of Skill Formation

$$h_{i,t+1} = h_{i,t}^{\alpha_1} \left[\alpha_2 \left(I_{i,t}\right)^{\alpha_3} + \alpha_4 \left(\overline{H}_{i,t}\right)^{\alpha_3} + \alpha_5 \left(A_s\right)^{\alpha_3}\right]^{\frac{\alpha_6}{\alpha_3}} \cdot A_{d,t} \cdot e^{\eta_{i,t+1}}$$

- $\overline{H}_{i,t}$ and $\eta_{i,t+1}$ correlated via unobserved heterogeneity in peer groups formation
- Peer effects: $\overline{H}_{i,t} = \frac{1}{\sum_{j=1,j\neq i}^{H} L_{i,j,t}} \sum_{j=1,j\neq i}^{H} L_{i,j,t}\, h_{j,t}$
- $A_{d,t}$ neighborhood quality
- A_s school quality
- $\eta_{i,t+1}$ skills shock (it is realized end of each period)

Return

Heterogenous Treatment Effect by Skills Receiving Children



Return

Latent Parental Investments and Skills

- Dynamic latent factor model (as in Del Boca et al., 2014, Cunha et al., 2010, Agostinelli and Wiswall, 2016)

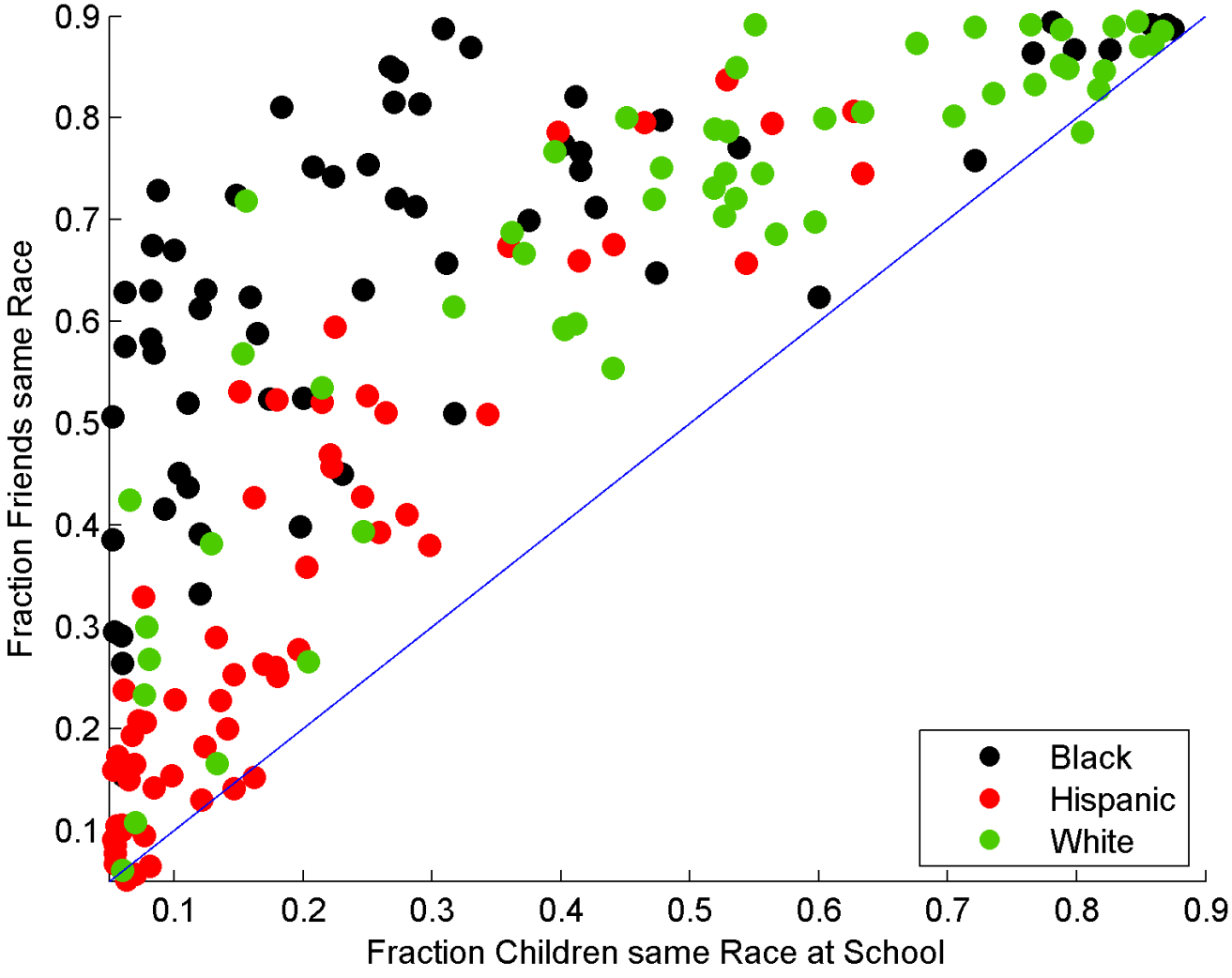
1. Investments ($I_{i,t}$):
   - Gone shopping
   - Played a sport
   - Gone to a religious service
   - Gone to a movie, play, museum, concert, or sports event
   - Had a talk about a personal problem
   - Had a serious argument about your behavior
   - Talked about your school work or grades
   - Worked on a project for school
   - Talked about other things you are doing in school

1. Child's skills ($\theta_{i,t}$):
   - Peabody Picture Vocabulary Test (PPVT)
   - Math, Science, English and History Grades

Latent Factor Model Investments Latent Factor Model Skills

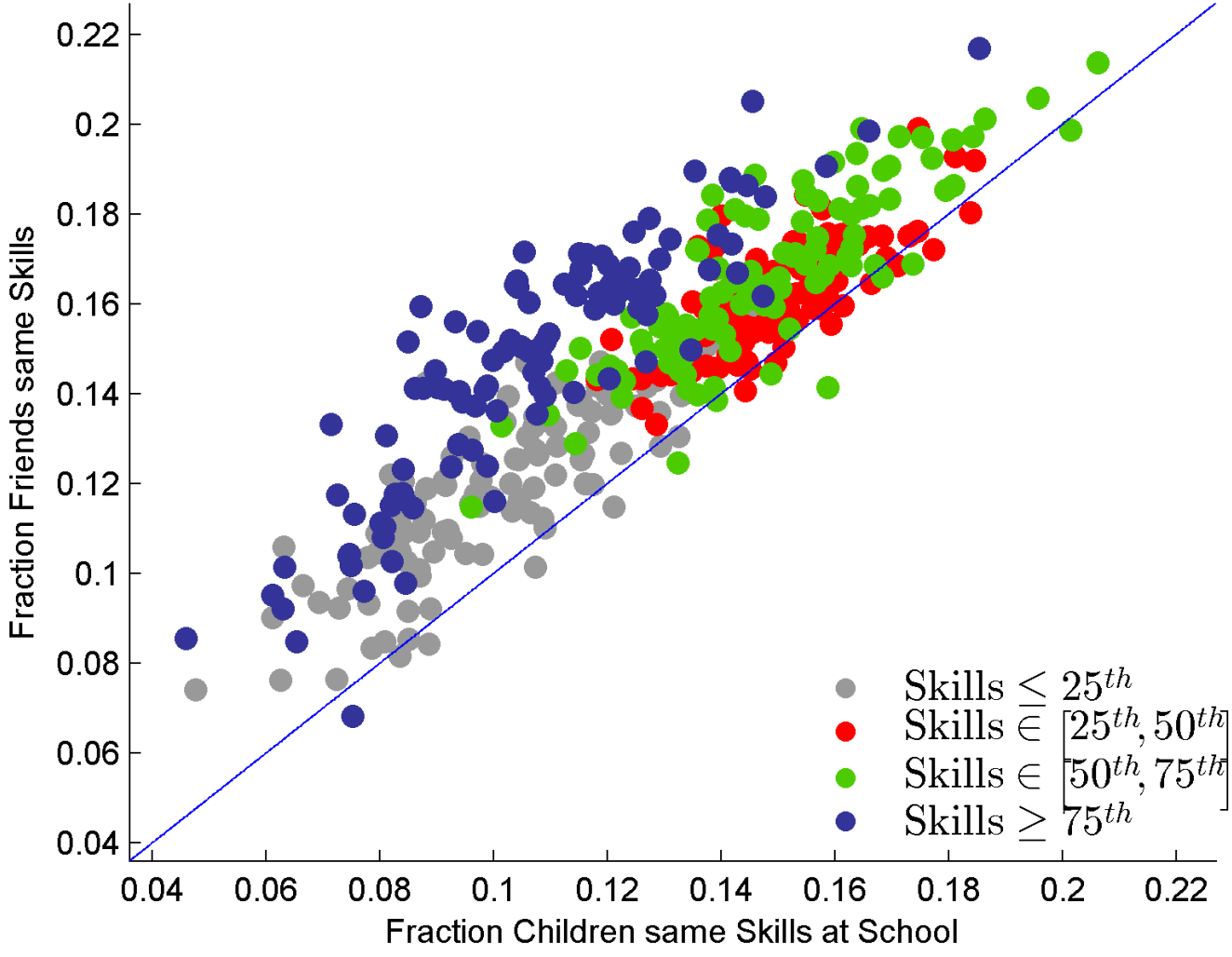
Return

Endogenous Peer Groups Formation: Race



Return

Endogenous Peer Groups Formation: Skills



Graphs with other boundaries Return

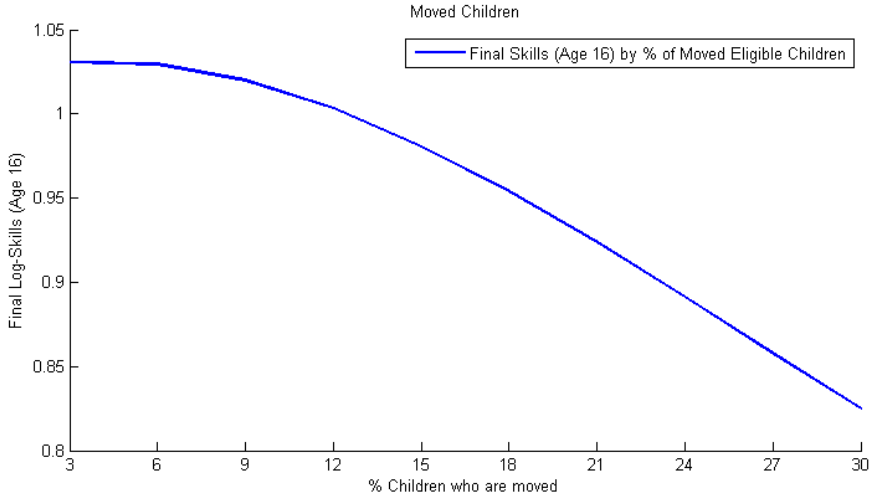
If peers' skills double:

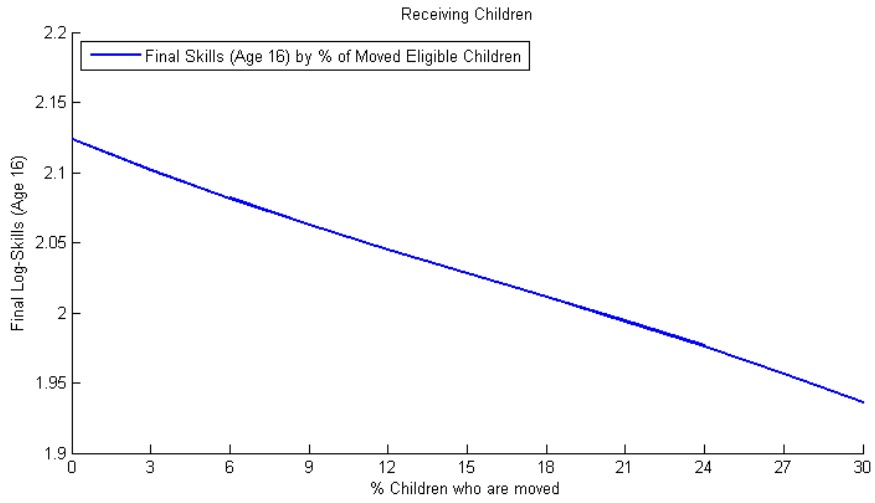
$$16 * (-0.01441) * 7 * 52 = -84 \text{ hours per year}$$

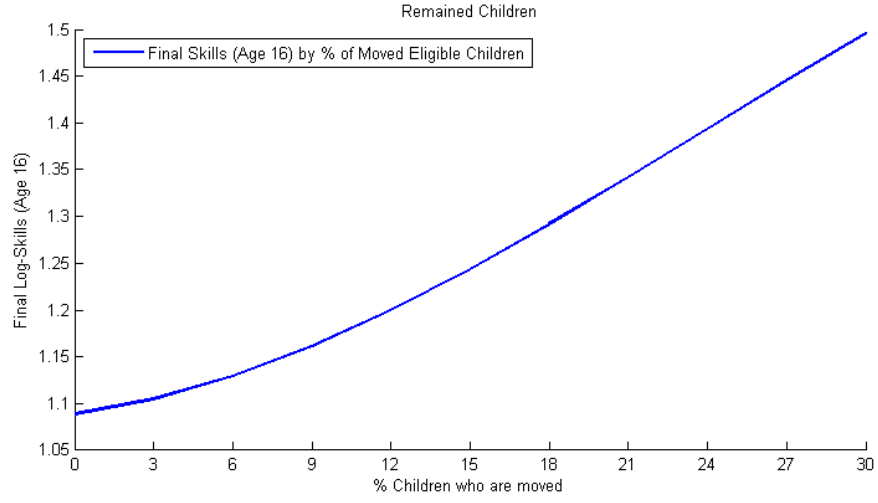
$$84 * 15 = 1258 \$ \, per \, year$$

Return

Treatment Effect by Fraction of Moved Eligible Children







Return

Existence of Equilibrium

- The existence proof follows the lattice programming argument (Topkis, 1998)
- The goal is preserving supermodularity in the value function

  (Datta, Mirman and Reffett, 2002; Datta, Mirman, Morand and Reffett, 2002; Mirman, Morand and Reffett, 2008; Datta, Reffett and Wozny, 2017)
- The supermodularity here is preserved because of the technology:

$$h_{i,t+1} = h_{i,t}^{\alpha_1} \left[\alpha_2 \left(I_{i,t}\right)^{\alpha_3} + (1-\alpha_2) \left(\overline{H}_{i,t}\right)^{\alpha_3}\right]^{\frac{\alpha_4}{\alpha_3}} \cdot e^{A}$$

  - Technology is supermodalar in $I_{i,t}$ and $\overline{H}_{i,t}$
  - Technology is supermodalar in $h_{i,t}$ and $\overline{H}_{i,t}$