

# Financial Intermediation, Capital Accumulation and Crisis Recovery

Hans Gersbach (ETH Zurich)  
Jean-Charles Rochet (Univ. Zurich)  
Martin Scheffel (KIT)

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# Outline

- 1 Introduction
- 2 Model Setup
- 3 Intra-temporal Equilibrium
- 4 Analysis of Steady States and Transition Dynamics
- 5 Short-run Dynamics and Sensitivity of Bank Leverage
- 6 Speeding up Recovery
- 7 Quantitative Analysis
- 8 Extensions and Conclusion

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# Motivation/Contribution

- Conceptual: Integration of banks into two-sector neoclassical growth model  
→ existence and type of steady states?
- Issues to be investigated:
  - 1 Role of bank leverage as amplifier and automatic stabilizer.
  - 2 Optimal crisis recovery with bank recapitalization and dividend payout restrictions.
  - 3 Explaining typical business cycle patterns such as procyclical leverage, bank lending and countercyclical bond issuance.
  - 4 Quantitative analysis of Great Recession.

## Relation to the Literature (1)

- The classics: Bernanke and Gertler (1989), Bernanke, Gertler and Gilchrist (1996), Kiyotaki and Moore (1997).
- Recent papers integrating financial intermediation into neoclassical growth model:  
Gertler and Kiyotaki (2010), Quadrini (2014), Brunnermeier and Sannikov (2015), Rampini and Viswanathan (2017).
- Difference:
  - Dual role of bank leverage and quantitative analysis.
  - Set-up with two sectors (bank and bond finance) and smooth consumption / savings decisions.
  - Coupled accumulation rules for household capital and bank capital (like Rampini and Viswanathan).

## Relation to the Literature (2)

- New DSGE models with an explicit banking sector examine the impact of financial frictions on
  - Efficiency of monetary policy:  
Gertler and Kiyotaki (2010), Gertler and Karadi (2011),
  - Role of bank capital in propagating shocks:  
Meh and Moran (2010), Angeloni and Faia (2013), Rampini and Viswanathan (2014),
  - Bank leverage cycles and crises:  
Adrian and Boyarchenko (2012), Brunnermeier and Sannikov (2014).

## Relation to the Literature (3)

- Policy: Dividend payout restrictions on banks (Acharya et al. (2013), Shin (2016), excessive payouts during financial crises).
- Stylized facts: During recessions and banking crises,
  - volume of loans decreases but volume of bonds increases (Kashyap, Stein and Wilcox (1993), De Fiore and Uhlig (2012)),
  - bank leverage is pro-cyclical (Adrian and Shin (2014)).

Both bank loans and bonds are qualitatively important in the financing of firms.

# Outline

Simplest two-sector accumulation model combined with the micro-founded form of banking based on Hart and Moore (1994), Holmström and Tirole (1997) or Gertler and Kiyotaki (2011).

We proceed as follows:

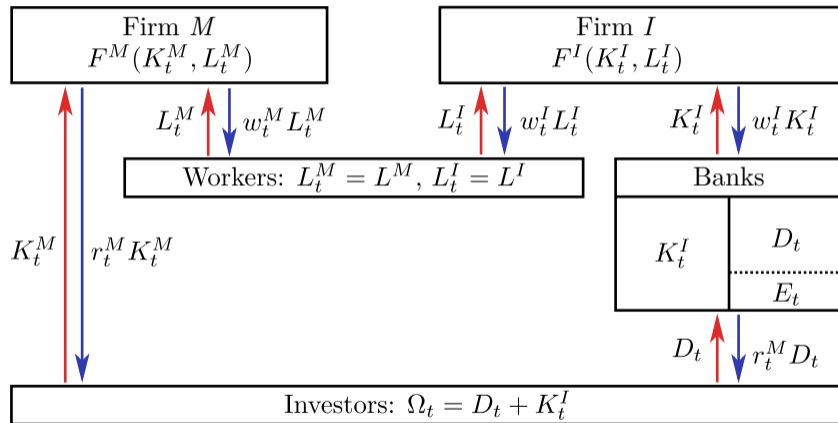
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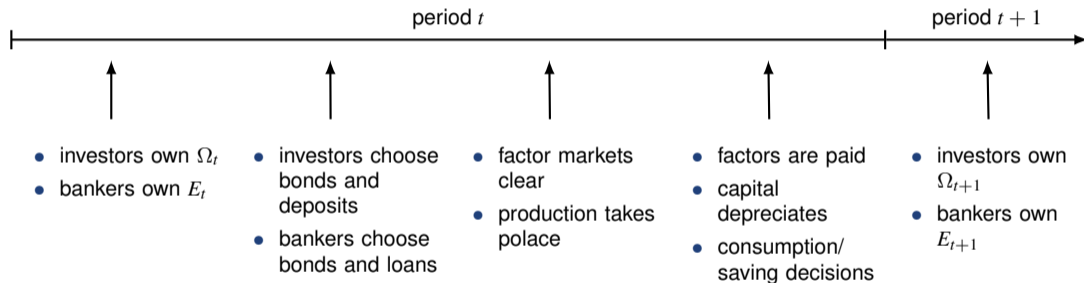
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# Model Set-up



# Sequence of Events



# Bankers and Leverage

- Incentive compatibility condition for deposit contracts:

$$(1 + r_t^M)(K_t^I - E_t) \leq K_t^I(1 + r_t^I - \theta).$$

- As bankers maximize  $\theta K_t^I$ , this condition will always be binding in equilibrium when  $E_t$  is not too large.
- $\Rightarrow$  Bank leverage:

$$\lambda_t = \frac{K_t^F}{E_t} = \frac{1 + r_t^M}{r_t^M - r_t^I + \theta}.$$

- Remark: As  $r_t^I > r_t^M$  in equilibrium when financial frictions matter, bankers are always better off by leveraging.

# Equilibrium Definition

## Definition

*A sequential markets equilibrium is a sequence of factor prices and allocations*

*$\{w_t^M, w_t^I, r_t^M, r_t^I, \Omega_t, E_t, K_t^M, K_t^I, C_t^H, C_t^B\}_{t=0}^\infty$  such that*

- 1 given  $\Omega_0$  and  $\{r_t^M\}_{t=0}^\infty$ , the allocation  $\{C_t^H, \Omega_t\}_{t=0}^\infty$  solves the investor's problem (1),*
- 2 given  $E_0$  and  $\{r_t^M, r_t^I\}_{t=0}^\infty$ , the allocation  $\{C_t^B, E_t\}_{t=0}^\infty$  solves the banker's problem (2),*
- 3 for each  $t \geq 0$ , given  $(w_t^M, w_t^I, r_t^M, r_t^I)$ , the firm allocation  $(K_t^M, K_t^I, L_t^M, L_t^I)$  solves the firms' problems,*
- 4 factor and output markets clear,*
- 5 leverage constraint is binding ( if financial frictions matter) or non-binding (with  $r_t^I = r_t^M$ ).*

# Three Cases

(A): Immobile labor.

(B): Flexible labor ( $w_t^I = w_t^M$ ).

(C): Some labor mobile and some labor immobile.

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# Comparative Statics

Shocks	Bank leverage	Loans	Bonds	Output
TFP ↓	-	-	+	-
$\Omega$ ↓	-	-	-	-
E ↓	+	-	+	-



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# Laws of Motion for Log-utilities

- Log-utilities imply

$$\Omega_{t+1} = \beta_H(1 + r_t^M - \delta)\Omega_t$$

$$E_{t+1} = \beta_B R_t^B E_t,$$

where  $R_t^B$  is the (net) return on equity factor in period  $t$  given by

$$R_t^B := \begin{cases} \theta\lambda(r_t^M, r_t^I) - \delta & \text{if } E_t < \bar{E}(K_t), \\ 1 + r_t^M - \delta & \text{if } E_t \geq \bar{E}(K_t). \end{cases}$$

- Bankers benefit from capital return differences between sector I and M and from leverage:

$$R_t^B = 1 + r_t^M - \delta + \lambda(r_t^M, r_t^I)(r_t^I - r_t^M).$$

- Assumption:  $\beta_B < \beta_H$  ( $\Leftrightarrow \rho_B > \rho_H$ ).

# Existence of Steady States

## Proposition

*Suppose  $\rho_B > \rho_H$ .*

*Then, the system has a unique and globally stable state  $(\hat{E}, \hat{\Omega})$ . Financial frictions always bind in the long run.*

Remarks:

- Steady state can be explicitly (iteratively) calculated for log utilities.
- Interesting consequence of permanent shock:  
An increase of  $\theta$  increases the banker's utility if  $\rho_B$  is close to  $\rho_H$ .

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# Amplification, Persistence and Stabilization

- Temporary negative shock to TFP:  $(K^M \uparrow, K^I \downarrow), Y \downarrow$   
but  $\lambda \downarrow, Y \downarrow\downarrow$ : amplification and persistence.
- Temporary negative shock to  $E$ :  $(K^M \uparrow, K^I \downarrow\downarrow), Y \downarrow\downarrow\downarrow$   
but automatic stabilization,  $\lambda \uparrow, Y \uparrow$ .

# Bond and Loan Financing over the Business Cycle

Empirical literature: De Fiore and Uhlig (2012), Contessi et al. (2013)

- Bank lending is procyclical,
- Bond issuing reacts little to booms and busts, and may even be countercyclical.

This feature can be derived when a downturn is associated with

- a temporary negative aggregate productivity shock,
- a negative shock to bank equity,
- a negative trust shock,

or any combination of these shocks.

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# Workout of Banking Slump

## Proposition (Dividend Payout Restrictions and Capital Injections)

*Suppose there is a shock that leads to a temporary decline in bank equity capital in period 0, with  $1 - \delta_1^E > \beta_H(1 - \delta)$ . Then, there exists a feasible sequence of transfer payments from investors to banks,  $\{Tr_t\}_{t=0}^\infty$ , and an associated sequence of dividend payout restrictions,  $\{d_t\}_{t=0}^\infty$  with the following properties:*

- (i) Total capital  $K_t$  and total output  $Y_t$  exceed their respective laissez-faire values in all periods.*
- (ii) Lifetime-utility of bankers is constant by construction and lifetime-utility of workers increases. The impact on lifetime-utility of investors is ambiguous.*



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# Quantitative Analysis

- Calibration to US economy (1991 Q1 – 2017 Q4) with shock process involving  $A_t, \delta_t^E, \theta_t$  captured by VAR(1) process.
- First step: Time-invariant parameters to match steady state to long-run stylized facts.
- Second step: Estimation of joint stochastic process.
- Data: FED, PWT, Call Report Data, De Fiore and Uhlig (2011).

# Parameters and Calibration Targets

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PARAMETERS				
$\alpha$	$z^M$	$z^I$	$\delta^H$	$\delta^B$
0.3484	1.0000	1.0168	0.0146	0.0146
$\beta^H$	$\beta^B$	$\theta$	$L$	$l$
0.9871	0.9731	0.0967	1.0000	0.5885

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CALIBRATION TARGETS				
$\bar{s}$	$\overline{K/Y}$	$\bar{\lambda}$	$\bar{r}^B$	$\overline{K^I/K^M}$
0.1801	12.3763	10.7808	0.0276	0.6667

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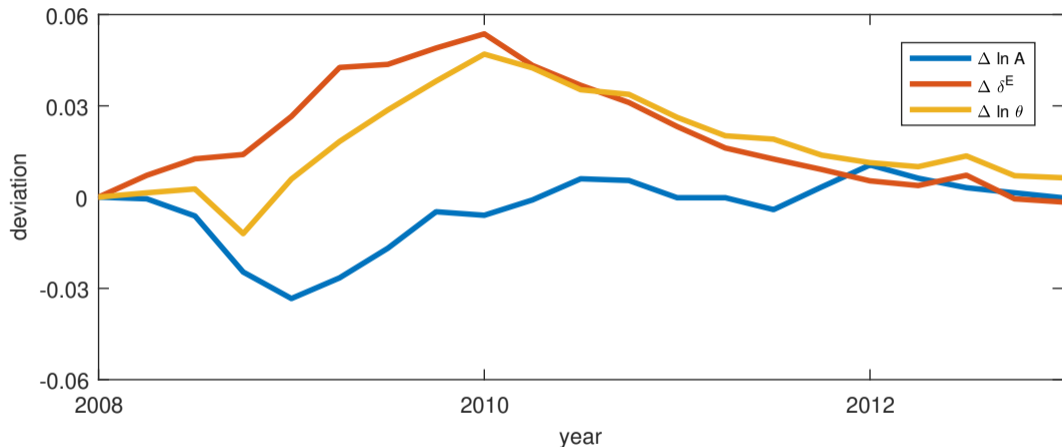
# Correlation of Shocks and Leverage

	$\Delta \ln(A)$	$\Delta \delta^E$	$\Delta \ln(\theta)$	$\Delta \ln(Y)$	$\Delta \ln(\lambda)$	$\Delta \ln(K^I)$
$\Delta \ln(A)$	+1.0000	-0.2746 (0.0042)	+0.0295 (0.7630)	+0.6591 (0.0000)	-0.0939 (0.3360)	+0.1688 (0.0822)
$\Delta \delta^E$		+1.0000	+0.7946 (0.0000)	-0.4947 (0.0000)	+0.0436 (0.6556)	+0.2797 (0.0035)
$\Delta \ln(\theta)$			+1.0000	-0.2642 (0.0060)	-0.2491 (0.0097)	-0.1121 (0.2503)
$\Delta \ln(Y)$				+1.0000	+0.2073 (0.0321)	+0.4888 (0.0000)
$\Delta \ln(\lambda)$					+1.0000	+0.3943 (0.0000)

*Note:  $\Delta x$  refers to the deviation of  $x$  from its HP-trend with smoothing parameter 1600.*

*The numbers are temporary cross-correlations and the associated p-values are in parentheses.*

# Great Recession – Shock Sequences



*Note:  $\Delta x$  refers to the deviation of  $x$  from its HP-trend with smoothing parameter 1600. The deviations from trend are further normalized by their respective 2008Q1 value.*

# Welfare and Output Costs of the Great Recession

shocks to ...		output cost	welfare cost		
			investor	worker	banker
$\lambda^{reg} = \infty$	$(A, \delta^E, \theta)$ -shock	+0.5323	+0.5640	+0.3408	+3.3963
	$(A, \delta^E)$ -shock	+0.5235	+0.5408	+0.3349	+3.9666
	$(A)$ -shock	+0.2678	+0.0976	+0.1434	+0.1402
$\lambda^{reg} = 1.01\hat{\lambda}$	$(A, \delta^E, \theta)$ -shock	+0.6257	+0.7367	+0.4152	+4.4756
	$(A, \delta^E)$ -shock	+0.9119	+1.2565	+0.6530	+8.361
	$(A)$ -shock	+0.2678	+0.0976	+0.1434	+0.1402

*Note: Simulation results for  $(A, \delta^E, \theta)$ -shocks – Great Recession –  $(A, \delta^E)$ -shocks, and  $(A)$ -shocks for different regulatory regimes: laissez faire refers to  $\lambda^{reg} = \infty$ , weak regulation refers to  $\lambda^{reg} = 1.05\hat{\lambda}$ , and strong regulation refers to  $\lambda^{reg} = 1.01\hat{\lambda}$ . Output costs are denominated in percent of the present discounted value of output. Welfare costs are denominated in percent of consumption equivalent units.*

# Accelerating Recoveries

## Welfare and Output Costs of the Great Recession: *Balanced Bailout vs Laissez-Faire*

shocks to ...		output cost	welfare cost		
			investor	worker	banker
$\lambda = 10.7808$	<i>laissez faire</i> ( $\zeta = 0.00$ )	+0.5323	+0.5640	+0.3408	+3.3963
	<i>balanced bailout</i> ( $\zeta = 0.33$ )	+0.5254	+0.6059	+0.3384	+3.3963
	<i>balanced bailout</i> ( $\zeta = 0.66$ )	+0.5220	+0.6492	+0.3382	+3.3963

*Note: Simulation results for  $(A, \delta^E, \theta)$ -shocks – Great Recession – for different policy regimes. The policy regimes are convex combinations between the laissez-faire path of bank equity capital and the steady state value of bank equity capital, where parameter  $\zeta$  is the weight given to laissez-faire. Output costs are denominated in percent of the present discounted value of output. Welfare costs are denominated in percent of consumption-equivalent units.*

# Policy Implications

- Automatic stabilization of leverage is quantitatively important  
→ countercyclical capital requirements are important.
- Balanced bailout speeds up recovery.
- Unbalanced bailout strongly accelerates recovery.  
→ debt-financed bank recapitalization and dividend payment restriction



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# Extensions

- Extensions:
  - Anticipated bank equity shocks
  - Costs of intermediation
  - Saving workers
  - General utility and production function
- Application: Resilience of economies relying more heavily on bank loans (Eurozone and much of Asia) compared to the ones relying more on corporate bonds (USA).
- Challenge: Model with completely flexible labor.

# Conclusion

- Parsimonious model of capital accumulation and growth in which both bank credit and bonds play an essential role.
- Useful for dual role of bank leverage, explaining facts and designing policy for crisis management and prevention.
- Many possible avenues for further research

# Backup

# Macroeconomic Environment

- Time  $t \in \{0, 1, 2, \dots\}$
- Four types of competitive agents (represented by continua in  $[0, 1]$ ):
  - Workers (each one supplies one unit of labor)
  - Entrepreneurs (manage non-financial firms)
  - Investors (own capital  $\Omega_t$ )
  - Bankers (manage banks, own capital  $E_t$ )
- Competitive markets
  - ⇒ Representative agents acting competitively
- Two goods: physical good and labor
- Physical good
  - produced by capital  $K_t$  and labor  $L_t$
  - consumed or invested in future periods
- Capital depreciates at rate  $\delta$ .

# Production Technologies

- At the end of each period, agents decide how much to consume and how much to save.
- Total capital  $K_t = E_t + \Omega_t$  allocated between two sectors:  
 $j = M$  (firms obtaining **market** finance) and  
 $j = I$  (firms needing **intermediated** finance)
- Cobb-Douglas technologies:

$$Y_t^j = z^j A (K_t^j)^\alpha (L_t^j)^{1-\alpha}$$

- $z_j$  specific productivity in each sector: allows to calibrate the relative size of two sectors  
 $K_t = K_t^M + K_t^I$

# Financial Frictions (1)

- Sector M (large/mature firms)
  - Uninformed lending through financial markets
  - $K_t^M$  supplied by households only
- Sector I (small/young firms)
  - Moral hazard problem of entrepreneurs
  - Monitoring technology of banks (in basic version: costless)
  - $K_t^I$  denotes bank capital supplied by bankers and households

⇒ Access to capital markets only through informed bank lending

## Financial Frictions (2)

- Banking technology
  - Moral hazard at bank managers' level
  - Bankers cannot pledge a fraction  $\theta$  of their banks' assets
  - Non-pledgeable part is thus  $\theta K_t^I$
  - Can be explained by
    - moral hazard à la Holmström and Tirole (1997)
    - asset diversion (Gertler and Karadi (2011))
    - non-alienability of human capital (Hart and Moore (1994), Diamond and Rajan (2000))



# Labor and Capital

- Competitive firms maximize profits, given interest rates  $r_t^j$  and wages  $w_t^j$ .
- Segmented labor markets, fixed labor supply ( $L_t^M = 1, L_t^I = 1$ )
- Segmented capital markets:
  - $I$ -firms only financed by banks (loan rate  $r_t^I$ );
  - $M$ -firms financed by markets (interest rate  $r_t^M$ ).
- In equilibrium:
  - positive spread between loan and bond rates  $r_t^I > r_t^M$

# Preferences

- Bankers and investors (households) choose their saving and consumption levels to maximize

$$\sum_{t=0}^{\infty} (\beta^k)^t \ln(C_t^k), k = B, H. \quad \beta^B \equiv \frac{1}{1 + \rho^B} < \beta^H \equiv \frac{1}{1 + \rho^H}.$$

s.t. **Budget Constraints.**

- Investors are indifferent between bonds and deposits.
- Banks issue deposits to leverage their equity.
- Workers supply labor and own no assets. For implicit solutions: focus on case in which they consume all of their income.
- Entrepreneurs are competitive and make zero profit.

# Intertemporal Budget Constraints

- Bankers

$$C_t^B + E_{t+1} = \theta K_t^I - \delta E_t = \left( \theta \frac{1 + r_t^M}{r_t^M - r_t^I + \theta} - \delta \right) E_t$$
$$C_t^B, E_{t+1} \geq 0, \quad E_0 \text{ is given}$$

- Households

$$C_t^H + \Omega_{t+1} = r_t^M K_t^M + r_t^D D_t + (1 - \delta)\Omega_t = r_t^M \Omega_t + (1 - \delta)\Omega_t$$
$$K_t^M + D_t = \Omega_t$$
$$C_t^H, D_t, K_t^M, \Omega_{t+1} \geq 0, \quad \Omega_0 \text{ is given}$$

# Investors

$$\begin{aligned} \max_{\{C_t^H, \Omega_{t+1}\}_{t=0}^{\infty}} & \left\{ \sum_{t=0}^{\infty} \beta_H^t \ln(C_t^H) \right\} & (1) \\ \text{s.t.} & C_t^H + \Omega_{t+1} = r_t^M K_t^M + r_t^D D_t + (1 - \delta)\Omega_t \\ & K_t^M + D_t = \Omega_t \\ & C_t^H, D_t, K_t^M, \Omega_{t+1} \geq 0 \\ & \Omega_0 \text{ given} \end{aligned}$$

- $C_t^H$  denotes investors' consumption.
- $D_t$  denotes the (aggregate) amount of deposits.
- $\beta_H = \frac{1}{1+\rho_H}$  ( $0 < \beta_H < 1$ ) denotes the discount factor and  $\rho_H$  the discount rate.

# Bankers

$$\begin{aligned} \max_{\{C_t^B, E_{t+1}\}_{t=0}^{\infty}} & \left\{ \sum_{t=0}^{\infty} \beta_B^t \ln(C_t^B) \right\} \\ \text{s.t. } & C_t^B + E_{t+1} = \theta K_t^I - \delta E_t = \left( \theta \frac{1 + r_t^M}{r_t^M - r_t^I + \theta} - \delta \right) E_t \\ & C_t^B, E_{t+1} \geq 0 \\ & E_0 \text{ given} \end{aligned} \tag{2}$$

- $C_t^B$  denotes the bankers' consumption.
- $\beta_B = \frac{1}{1+\rho_B}$  denotes the bankers' discount factor and  $\rho_B$  the discount rate.

# Intra-temporal Equilibrium

- Profit-maximization of firms yields

$$r_t^j = \alpha A z^j (K_t^j)^{\alpha-1}, \quad j \in \{M, I\} \quad (3)$$

$$w_t^j = (1 - \alpha) A z^j (K_t^j)^\alpha, \quad j \in \{M, I\} \quad (4)$$

# Irrelevant Financial Frictions

- No funds channeled from households to the banking technology

$$K_t^M = (E_t - K_t^I) + \Omega_t$$

- $r_t^I = r_t^M$
- Equilibrium values

$$K_t^M = \frac{K_t}{1+z}, \quad K_t^I = \frac{zK_t}{1+z}$$

- Net earnings of bankers amount to  $K_t^I(1+r_t^I) + (E_t - K_t^I)(1+r_t^M) = E_t(1+r_t^M)$
- Incentive compatibility constraint requires that  $E_t(1+r_t^M) \geq \theta K_t^I$   
or equivalently

$$E_t \geq \frac{\theta z}{\left(1 + \alpha z^M \left(\frac{K_t}{1+z}\right)^{\alpha-1}\right)(1+z)} K_t \equiv \bar{E}(K)$$

# Binding Financial Frictions

- Allocation

$$K_t^I = \lambda_t E_t$$

$$K_t^M = K_t - \lambda_t E_t = \Omega_t + E_t - \lambda_t E_t$$

$$\lambda_t = \frac{1 + \alpha z^M (\Omega_t + E_t - \lambda_t E_t)^{\alpha-1}}{\alpha z^M (\Omega_t + E_t - \lambda_t E_t)^{\alpha-1} - \alpha z^I (\lambda_t E_t)^{\alpha-1} + \theta}$$

- Equilibrium leverage  $\lambda$  satisfies

$$\varphi(\lambda) = \alpha z^M (\Omega + E - \lambda E)^{\alpha-1} \left(1 - \frac{1}{\lambda}\right) - \frac{1}{\lambda} + \theta - \alpha z^I (\lambda E)^{\alpha-1} = 0 \quad (5)$$

- If  $E < \bar{E}(K)$ , the intermediate value theorem and strict monotonicity of  $\varphi(\lambda)$  delivers the existence and uniqueness of  $\lambda_t^*$  that solves (5).
- $r_t^I > r_t^M$



# Existence and Uniqueness

## Proposition (Intra-temporal Equilibrium)

*For all pairs  $(E_t, K_t)$  with  $0 < E_t < K_t$ , there exists a unique equilibrium.*

- (i) If  $E_t \geq \bar{E}(K_t)$ , we obtain  $\left(K_t^M = \frac{K_t}{1+z}, K_t^I = z \frac{K_t}{1+z}\right)$  and financial frictions do not matter.*
- (ii) If  $E_t < \bar{E}(K_t)$ , financial constraints bind and leverage  $\lambda_t$  is determined by  $\theta \lambda_t = 1 + r_t^M(\lambda_t) + \lambda_t(r_t^I(\lambda_t) - r_t^M(\lambda_t))$ .*

# Comparative Statics (1/2)

## Corollary

Suppose that financial frictions matter, i.e.  $E_t < \bar{E}(K_t)$ . Then,

- (i)  $\lambda_t$  increases in  $z^I$ ,  $\Omega_t$ ,
  - (ii)  $\lambda_t$  decreases in  $z^M$ ,  $E_t$  and  $\theta$ .
- 
- Suppose both total factor productivity parameters  $z^M$  and  $z^I$  are affected by the same relative shock

$$\epsilon := \frac{\Delta z^M}{z^M} = \frac{\Delta z^I}{z^I}.$$

- Then, the effect on leverage is as follows:

## Corollary

Suppose financial frictions are binding. Then  $\frac{\partial \lambda}{\partial \epsilon} > 0$  where  $\epsilon$  is a proportional change of  $z^M$  and  $z^I$ .

## Comparative Statics (2/2)

- Impact of higher  $E_t$  and thereby higher  $K_t$

### Corollary

*Suppose that financial frictions matter. Then, an increase in bank equity  $E_t$  (and a corresponding increase of  $K_t$ ) raises  $K_t^I$ .*

- Impact of higher  $\Omega_t$  and thereby higher  $K_t$

### Corollary

*Suppose that financial frictions matter. Then, an increase in household wealth  $\Omega_t$  (and a corresponding increase of  $K_t$ ) raises  $K_t^M$ .*

# Intuition why Bank Leverage is Pro-cyclical

- In Adrian and Shin (2008) and Adrian and Boyarchenko (2013), banks are confronted with VaR constraints: the higher the risk the lower the leverage. Then leverage is pro-cyclical because risk is anti-cyclical.
- In our model, leverage is given by the "skin in the game" constraint for bankers:

$$\lambda = \frac{1+ar^M}{\theta-a(r^I-r^M)} \text{ increases in TFP } a.$$

# Pro-cyclicality of Bank Lending

Figure I: Procyclicality of Intermediary Financial Assets

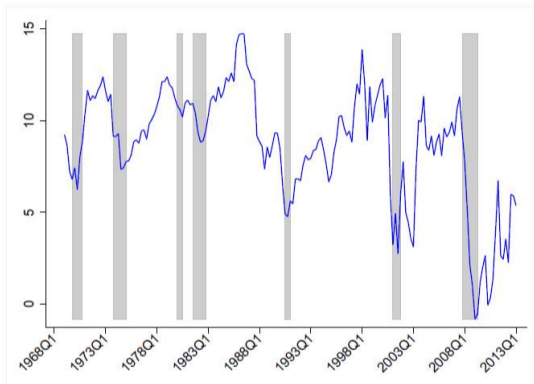


Figure: Total growth of US banks' assets.

Source: Adrian and Boyarchenko (2013). NBER recessions in grey.

# Comparative Statics

- First row (recession): bank leverage and bank assets decrease, bond issuance increases. Conform with empirical evidence: Adrian-Shin (2008), Adrian-Colla-Shin (2013).
- Second row (financial crisis): both bank loans and bond issuance decreases.
- Third row (banking crisis without capital injections): bank leverage increases, bank credit decreases, bond issuance increases.

# Laws of Motion

## Lemma

*The necessary conditions for the solution of the investor's problem imply*

$$\begin{aligned}C_t^H &= (1 - \beta_H)(1 + r_t^M - \delta)\Omega_t, \\ \Omega_{t+1} &= \beta_H(1 + r_t^M - \delta)\Omega_t.\end{aligned}$$

- We make the following assumption:

## Assumption

*Bankers are more impatient than investors, i.e.  $\beta_B < \beta_H$  or  $\rho_B > \rho_H$ .*

# Existence of Steady States

- There is no steady state when financial frictions are irrelevant.
- Otherwise, if  $\hat{E} > 0$ , the laws of motion would imply

$$\frac{\hat{E}}{\hat{\Omega}} = \frac{\beta_B}{\beta_H} \frac{\hat{E}}{\hat{\Omega}} < \frac{\hat{E}}{\hat{\Omega}}.$$

- Note that the case  $\hat{E} = 0$  will be excluded, based on the analysis of the transitional dynamics.
- Therefore, we obtain

## Proposition

*Suppose  $\rho_B > \rho_H$ . Then, the system has a unique and globally stable state  $(\hat{E}, \hat{\Omega})$  described by equations (11) to (16). Financial frictions always bind in the long run.*



## Phase Diagram (1/4)

- Suppose first that financial frictions matter.
- The laws of motion reads

$$E_{t+1} = \beta_B E_t [\theta \lambda(E_t, \Omega_t) - \delta], \quad (6)$$

$$\Omega_{t+1} = \beta_H \Omega_t [1 - \delta + \alpha z^M (E_t + \Omega_t - \lambda(E_t, \Omega_t) E_t)^{\alpha-1}]. \quad (7)$$

- We define  $\Omega^1(E)$  and  $\Omega^2(E)$  such that

$$E_{t+1} = E_t \Leftrightarrow \Omega^1(E_t) = \Omega_t,$$

$$\Omega_{t+1} = \Omega_t \Leftrightarrow \Omega^2(E_t) = \Omega_t.$$

- We obtain

### Lemma

$$\Omega^2(E_t) > \Omega^1(E_t) \Leftrightarrow E_t < \hat{E}_t$$

## Phase Diagram (2/4)

- We also obtain

### Corollary

*When financial frictions are binding,*

- (i)  $\Omega^1(E_t) < \Omega_t \Leftrightarrow E_t < E_{t+1}$ ,
- (ii)  $\Omega_t < \Omega^2(E_t) \Leftrightarrow \Omega_{t+1} > \Omega_t$ .

- We define  $E^i$  for  $i = 1, 2$  implicitly by

$$E^i = \bar{E}(E^i + \Omega^i(E^i)).$$

- By continuity,
  - at  $(E^1, \Omega^1(E^1))$ ,  $r^{M^1} = \hat{r}^M = \delta + \rho_B$ ,
  - at  $(E^2, \Omega^2(E^2))$ ,  $r^{M^2} = \delta + \rho_H$ .
- With obvious notations, we obtain  $K^1 < \hat{K} < K^2$ .

## Phase Diagram (3/4)

- When financial frictions do not matter,

$$r_t^M = r_t^I = \alpha z^M \left( \frac{K_t}{1+z} \right)^{\alpha-1} = \alpha z^I \left( \frac{zK_t}{1+z} \right)^{\alpha-1}, \quad (8)$$

$$E_{t+1} = \beta_B \left[ 1 + \alpha z^M \left( \frac{K_t}{1+z} \right)^{\alpha-1} - \delta \right] E_t, \quad (9)$$

$$\Omega_{t+1} = \beta_H \left[ 1 + \alpha z^M \left( \frac{K_t}{1+z} \right)^{\alpha-1} - \delta \right] \Omega_t. \quad (10)$$

- We can easily derive the following:

### Corollary

*When financial frictions do not matter,*

- (i)  $K_t < K^1 \Leftrightarrow E_{t+1} > E_t$ ,
- (ii)  $K_t < K^2 \Leftrightarrow \Omega_{t+1} > \Omega_t$ .

- From these considerations, we can draw the phase diagram and derive convergence towards the steady state.

# Existence of Steady States

$$\hat{r}^M = \delta + \rho_H \quad (11)$$

$$\hat{r}^I = \hat{r}^M + \frac{\theta(\rho_B - \rho_H)}{1 + \delta + \rho_B} \quad (12)$$

$$\hat{K}^M = \left( \frac{\alpha z^M}{\hat{r}^M} \right)^{\frac{1}{1-\alpha}} \quad (13)$$

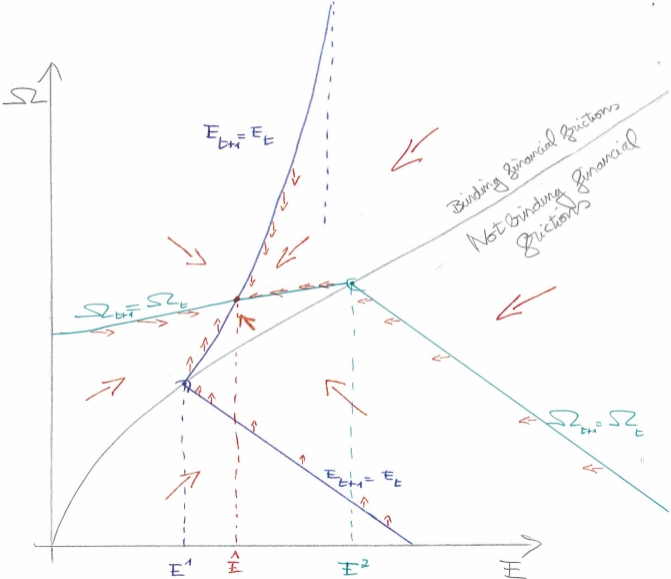
$$\hat{K}^I = \left( \frac{\alpha z^I}{\hat{r}^I} \right)^{\frac{1}{1-\alpha}} \quad (14)$$

$$\hat{E} = \left( \frac{\alpha z^I}{\hat{r}^I} \right)^{\frac{1}{1-\alpha}} \frac{\theta}{1 + \delta + \rho_B} \quad (15)$$

$$\hat{\Omega} = \hat{K} - \hat{E} \quad (16)$$

Remark: Frictionless case:  $\bar{r}^M = \bar{r}^I = \delta + \rho_H$

# Phase Diagram



# Impact of Financial Frictions

- They reduce the steady state capital stock in the intermediated sector (but not in the market sector).
- Spread between loan rates and bonds rate persists in the limit, due to the combination of financial frictions and the bankers' impatience.
- Frictions reduce the speed of convergence towards steady state.

# Permanent Shocks to Financial Frictions (1)

- A negative shock to financial frictions ( $\theta \rightarrow \theta'$  with  $\theta < \theta'$ ) may result from
  - worsening moral hazard in banking,
  - lowered trust in bankers.

## Corollary

*An increase of the intensity of financial frictions, i.e. an increase of  $\theta$ ,*

- (i) lowers the steady state value  $\hat{K}$ ,*
- (ii) increases bank equity  $\hat{E}$  if bankers are not too impatient.*

## Permanent Shocks to Financial Frictions (2)

Moreover,

### Proposition

*Suppose that  $\rho_B$  is sufficiently close to  $\rho_H$  and that the economy is hit by a negative permanent shock to financial frictions ( $\theta \rightarrow \theta'$ ). Then, the bankers' intertemporal utility after the shock is higher than in the steady state associated with  $\theta$ .*



# Impact of Technological Progress

- Exogenous technological progress leaves structure of economy (e.g. share of banking) unchanged.

# Temporary Shocks to Financial Frictions

- Shock:  $\theta \rightarrow \theta'$  where  $\theta < \theta'$
- Lowers output but boosts bank equity accumulation
- Shock ends:  $\theta' \rightarrow \theta$
- Higher levels of bank equity may allow temporary higher investment in sector I, thereby boosting output.

## Hypothesis

*A temporary shock  $\theta \rightarrow \theta'$  ( $\theta < \theta'$ ) may cause a bust/boom cycle, i.e. aggregate output first declines, then turns into a boom before it returns to the steady state.*

Remark: The same may occur when an negative shock to household wealth occurs (in particular when labor markets are not segmented).

# Temporary Shocks to Productivity

**Hitting both sectors:**  $\epsilon = \frac{\Delta z^M}{z^M} = \frac{\Delta z^I}{z^I} < 0$  (only at  $t = 0$ )

- The borrowing constraint on bankers is tightened;
- leverage decreases (cf. Corollary 1);
- at period 1, bank equity will decline;
- as a consequence of lower bank equity and leverage, more capital will be employed in sector M, meaning that  $r^M$  will decline;
- at period 1, households' wealth will decline;
- then, recovery occurs with capital starting its build-up.

**Hitting sector M only:**  $\Delta z^M < 0$  (only at  $t = 0$ )

- Leverage increases (cf. Corollary 1);
- therefore,  $K_0^I > \hat{K}^I$  and  $r_0^I < \hat{r}^I$ ;
- lower returns in sector I implies lower returns in sector M:  $r_0^M < \hat{r}^M$ ;
- therefore,  $E_1 > \hat{E}$  and  $\Omega_1 < \hat{\Omega}$ ;
- shock hurts households, but benefits bankers;
- recovery is qualitatively different than for aggregate productivity shock.