

# Intertemporal Elasticity of Substitution with Leisure Margin

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## Abstract

This paper investigates whether the estimation of the intertemporal elasticity of substitution of consumption (IES) would be affected when leisure time is allowed to vary. To this end, we adopt a utility specification that allows interactions between consumption and leisure and estimate IES using a pair of Euler equations. We find that the IES estimates that allow leisure to respond to the market interest rate are consistently lower than the IES estimates using the conventional method that keeps leisure constant. We show that time spent on home production explains majority of the difference between the two IES estimates due to the higher substitutability of home production time, particularly the childcare component, compared with other leisure time. When we exclude home production from nonmarket time, we find the IES estimates become larger. Our findings demonstrate the importance of time allocation when individuals make decisions on consumption and saving.

## Motivation

**Why leisure margin?** Existing studies typically estimate IES while holding leisure constant or ignore the role of leisure by assuming additively separable utility function. Heckman (1974) argues that *nonzero* cross-partial of marginal utility for consumption and leisure  $u_{c,l}$  is the key to understanding the hump-shaped life-cycle profile of consumption.

- Joint decision on consumption and leisure:** people adjust both consumption and leisure time in response to the interest rate. Allowing adjustment at the leisure margin may effectively lower the response of consumption. IES with leisure held constant would not capture this margin (→ Fig. 1a).
- Leisure is not the same as nonmarket time:** leisure time is heterogeneous in nature and the strength of substitution between consumption and leisure time is also different, with childcare and housework showing the strongest substitution and exercise showing the weakest (→ Fig. 1b).

## Main Result

- Leisure-varying IES  $\theta^{cl}$  is lower than leisure-held-constant IES  $\theta$  (→ Tab. 1a). Robust to different controls (→ Tab. 1b).
- Leisure-varying IES estimated using “nonwork time” (nonmarket time *less home production*) is larger than the IES estimated using nonmarket time. We find childcare time is highly substitutable with consumption (→ Tab. 1c, Fig. 2).

Tab. 1a. Comparing  $\theta^{cl}$  with  $\theta$

Leisure-varying IES $\theta^{cl}$	Leisure-constant IES $\theta$
0.115***	0.327***

Tab. 1c. Estimates of  $\theta^{cl}$  with different leisure measures

Benchmark using nonmkt time	Using nonmkt less housework	Using nonmkt less childcare	Using nonwork time
0.115***	0.133***	0.190***	0.240***

Tab. 1b. Result with Different Controls

	(a)	(b)	(c)	(d)	(e)
$\theta^{cl}$	0.091***	0.115***	0.113***	0.115***	0.120***
$\theta$	0.000	0.000	0.000	0.000	0.000
$\beta$	0.304***	0.322***	0.323***	0.324***	0.343***
$\alpha$	0.000	0.000	0.000	0.000	0.000
$\Delta \ln(\text{adult})$	0.241***	0.035	0.031	0.001	0.000
$\Delta \ln(\text{children})$	3.558***	2.726***	2.734***	2.708***	2.824***
$\Delta \ln(\text{single})$	0.000	0.000	0.000	0.000	0.000
$\Delta \text{ spouse fulltime}$	0.000	0.000	0.000	0.000	0.000
$\Delta \text{ spouse nonmkt}$	0.000	-0.072***	-0.023	-0.030	-0.024**
$\Delta \text{ other CU salary}$	0.000	0.524***	0.375***	0.429***	0.000
Sargan criterion	61.107	60.166	60.867	60.290	62.011
Consave utility? C and L are substitutes?	Yes	Yes	Yes	Yes	Yes

Note:  $\theta^{cl}$  is constructed based on Equation (9) in the main text and for the null hypotheses  $H_0: \theta^{cl} = \theta$ , we use Wald-type of tests and the delta method to estimate the standard errors. The number in the parentheses represents the p-value for the test. \*\*\*, \*\*, and \* represent statistical significance at the 1%, 5%, and 10% level, respectively. The instruments in (a)-(c) include the second, third, and fourth lags of consumption growth, leisure growth, nominal interest rate, inflation, and labor income growth, and the second and third lag of the number of adults, children, and elderly (those older than 64), number of earners, single status, whether the spouse works full-time, spouse's nonmarket time, average age, age squared, and three seasonal dummies. The instruments in (d)-(e) further include the second, third, and fourth lags of spouse salary and salary of other CU members, respectively. In addition to the variables presented in the table, three seasonal dummies are also included in estimation.

Fig. 1a. Life Cycle Profiles of Key Variables

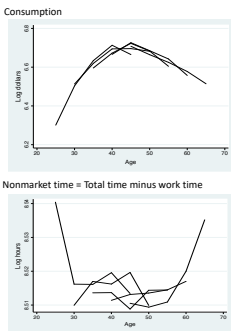
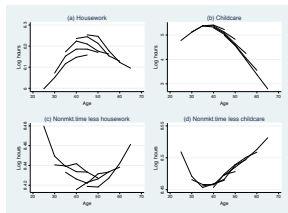
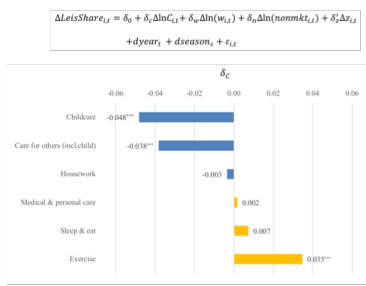


Fig. 1b. Life Cycle Profiles of Leisure Measures



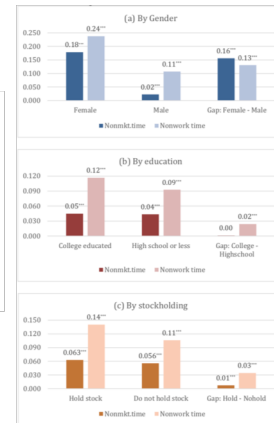
Note: The panels plot the average cohort leisure by age. Each line segment represents one cohort and the sample period of 1996-2014 in the CEX. All leisure measures are predicted using data from the ATUS and the CEX and apply for employed persons.

Fig. 2. Relationship between Consumption and Specific Time Use



Note: The numbers reported are the coefficient of nondurable consumption ( $\delta_c$ ) in Equation (17) in the main text. Standard errors clustered at the individual level are included in the parentheses. \*\*\* indicates statistical significance at the 1% level.

Fig. 3. Subsample Analysis



Note: The columns represent the IES estimates for subsamples using nonmarket time (dark color) and nonwork time (light color), respectively. The last column shows the difference between the two subsamples presented in the first and second column. For a given pair of subsamples, four equations (two times consumption and leisure Euler) are jointly estimated. The control variables include the number of adults, the number of children, and seasonal dummies. A formal test with regard to the significance of the gaps are provided in the Online Appendix Table O.5-O.7. \*\*\* indicates statistical significance at the 1% level.

## Methods

- Allow nonseparability of consumption  $C$  and leisure  $L$ 
  - Assume utility function of King-Plosser-Rebelo form
$$u(C_t, L_t) = (1 - \gamma)^{-1} C_t^{1-\gamma} L_t^{\gamma(1-\gamma)}$$
  - Allow leisure to respond to interest rate while wage held constant (cf. Swanson, 2012). IES becomes
$$\theta^{cl} \equiv [\gamma - \chi(1 - \gamma)]^{-1}$$
- Estimate IES combining consumption and *estimated* leisure using individual data
  - Consumer Expenditure Survey (CEX, main data): Sample 1996-2014
  - Combine synthetic cohort approach and General Method of Moments *a la* Attanasio and Weber (1995)
  - Sample: individuals who are working, age 21-45 in 1996, quarterly frequency
  - Estimating two Euler equations for consumption and leisure jointly
- Check sensitivity of IES using three different leisure measures
  - Three Leisure measures: nonmarket time less housework, nonmarket time less childcare, nonwork time (=nonmarket time less housework and childcare)
  - For these additional leisure measures, we used American Time Use Survey (ATUS) to predict the *fraction* of leisure time over nonmarket time

## Additional Findings

- Subsample analysis:** We confirm that the main findings also apply to the subsamples distinguished by gender, education, and stock-holding status (→ Fig. 3).
- Gender difference:** The IES for men (0.02) is lower than IES for women (0.18). For men, the IES falls sharply from 0.11 in the case when nonwork time is used as leisure measure to 0.02 when nonmarket time is used instead. This suggest that men actively substitute consumption and home production. This is not the case for women (→ Fig. 3, panel (a)).
- Spouse's leisure:** When joint leisure is considered, the leisure-varying IES falls from 0.115 to 0.006 for nonmarket time and from 0.240 to 0.147 for nonwork time, suggesting spouse's leisure serves as an additional channel of consumption smoothing (results shown in the working paper).

## Summary

- We endogenized leisure in estimating IES, which has not been done by previous studies. Our results show that adjustment at the leisure margin is highly relevant in estimating the IES (0.115, as opposed to 0.3 or higher). We show that substitutability of home production time (in particular childcare) matters in the IES estimates.

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