

# Paths of Ideological Conflict: Closing the Gap Between Gamson's Law and Theory

Julia Belau

TU Dortmund University

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 technische universität  
dortmund

# Portfolio Allocation and Gamson's Law

- Literature on government formation typically concentrates on governments in “minority legislatures”
- Central issue: assignment of ministerial portfolios to parties  
cf. [Laver(1998)] and [Laver and Schofield(1998)] for literature review
- Most prominent landmark: **Gamson's Law**  
→ portfolio payoffs are proportional to **relative seat share within the coalition** cf. [Gamson(1961)]
- Strong empirical evidence, but poor theoretical foundation and conflict with bargaining theory → **outside options**  
cf. [Browne and Franklin(1973)], [Warwick and Druckman(2006)], [Snyder et al.(2005)Snyder, Ting, and Ansolabehere], [Carroll and Cox(2007)] among others

# Political Science, Power Indices and Factual Content

- Economic theory suggests the use of **power-indices** as the Banzhaf-Power-Index or the Coalitional Bargaining Solution  
cf. [Banzhaf(1952)]/[Coleman(1971)]/[Penrose(1946)] and [Compte and Jehiel(2010)]
- Approaches based on power indices and bargaining theory **stay behind** Gamson's Law w.r.t. explanatory power  
cf. [Linhart et al.(2008)Linhart, Pappi, and Schmitt]
- "[The Power-index approach] should not (even) be considered as part of political science. Viewed as a scientific theory, it is a branch of probability theory and can safely be ignored by political scientists. [...] It has **no factual content** and can therefore not be used for purposes of prediction or explanation."  
cf. [Albert(2003)]

# Example: 2016 State Parliament Election Rhineland-Palatinate

## Table : 2016 State Parliament Election Rhineland-Palatinate

party	CDU	SPD	FDP	Grüne	AfD
seats	35%	39%	7%	6%	14%

## Table : Ministerial Positions Cabinet Dreyer II

party	SPD	FDP	Grüne
# ministers	5 (56%)	2 (22%)	2 (22%)

Gamson's Law: 75% , 13% , 12% → 7 , 1 , 1

Banzhaf Index: 71% , 14% , 14% → 6-7 , 1 , 1

both ignore ideological closeness/conflict potential

Our Approach: 51-54% , 23-25% , 23-24% → 5 , 2 , 2

## Towards Factual Content

- We suggest portfolio allocation due to **relative weakness proportionality** (cf. interpretation of Coalitional Bargaining Solution)
- In contrast to CBS, we do not derive weakness by unblocked coalitions, but by **election specific ideological closeness** → factual content
- More precisely, we interpret ideological closeness of a coalition as **proportional to its materialization probability**
- This yields a measure of **weakness for non-member parties**
- Finally, we suggest portfolio allocation to be proportional to relative weakness

## Relative Weakness Proportionality

- Let  $N = \{1, \dots, n\}$  denote the parties in a parliament
- Let  $\{\mu_S\}_{S \subseteq N}$  be a measure of coalitional strength which satisfies  $\mu_S \in [0, 1]$  for all coalitions  $S \subseteq N$  and  $\mu_S = 0$  for all non-winning coalitions
- Then,  $m_i^\mu = \sum_{S \subseteq N \setminus \{i\}} \mu_S$  denotes a party  $i$ 's weakness
- We define bargaining power  $x_i$  to be proportional to  $i$ 's relative weakness: for all parties  $i, j$  we have

$$x_i = \frac{m_j^\mu}{m_i^\mu} x_j \quad \text{where } \tilde{m}_i^\mu = \begin{cases} 1 + m_i^\mu & \exists \text{ pivotal party} \\ m_i^\mu & \nexists \text{ pivotal party} \end{cases}$$

→ if  $i$  is weaker than  $j$  (i.e.  $m_i^\mu > m_j^\mu$ ), we have  $x_i < x_j$   
 the weaker  $i$  compared to  $j$ , the lower  $x_i$  compared to  $x_j$

## Relative Weakness Index

- Normalizing bargaining power by  $\sum_{i \in N} x_i = 1$  (index on the unit interval) yields

$$x_i = \frac{1}{\tilde{m}_i^\mu} \cdot \left( \sum_{l=1}^{|\mathcal{N}|} \frac{1}{\tilde{m}_l^\mu} \right)^{-1}$$

by solving the corresponding system of equations

- Portfolio allocation can be calculated by relative bargaining power within the government coalition

How to measure coalitional strength, i.e.  $\{\mu_S\}_{S \subseteq N}$ ?

→ ideological closeness via VAA data

# Factual Content and VAAs

- Voting Advice Applications (VAA) are a commonly used tool in Europe and “ slowly but surely are gaining ground in other parts of the world”

[Garzia and Marschall(2012)], [Marschall and Garzia(2014)],

[Van Camp et al.(2014)Van Camp, Lefevere, and Walgrave]

- VAAs provides yes/no/neutral positions for the “most important” election statements of potential parliament parties
- We use data from the German “Wahl-o-Mat”
- Equivalent use with “StemWijzer” (Netherlands), “Smartvote” (Switzerland), “Vote Compass” (Canada, USA, Australia, New Zealand), ...



# Measuring Consensus via VAA data

- For each statement  $s = 1, \dots, S$ , the parties  $i = 1, \dots, N$  **self-position** by choosing “agree”/“not agree”/“neutral”
- For each two parties  $i, j$  and each statement  $s$ , we define the **consensus value**  $c_{ij}^s$  according to

	agree	neutral	not agree
agree	2	1	0
neutral	1	2	1
not agree	0	1	2

$$\Rightarrow c_{ij}^s \in \{0, 1, 2\}$$

# Bilateral Closeness and Example Revisited

Definition (Bilateral Closeness between parties  $i$  and  $j$ )

$$bilclos_{ij} := \left( \sum_{s=1}^S c_{ij}^s \right) \frac{1}{2 \cdot S} \in [0, 1]$$

Table : Closeness Matrix RP 2016

	CDU	SPD	FDP	Grüne
CDU	1	0.645	0.697	0.395
SPD	0.645	1	0.579	0.750
FDP	0.697	0.579	1	0.408
Grüne	0.395	0.750	0.408	1

Closeness of SPD and FDP: 57.9%  
 Closeness of SPD and Green: 75%  
 Closeness of FDP and Green: 40.8%

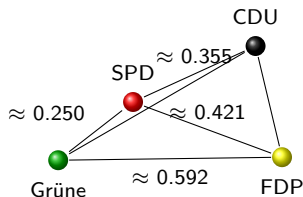
}  $\Rightarrow$  Closeness of  
 Traffic-Light  
**Coalition?**

# Distance Network

Instead of consensus, the statement specific **conflict value** can be measured by  $2 - c_{ij}^s$ . This yields **distance** between each two parties:

$$d_{ij} := \left( \sum_{s=1}^S 2 - c_{ij}^s \right) \frac{1}{2 \cdot S} = 1 - bilclos_{ij}$$

Figure : Distance Network RP 2016



## Coalitional Closeness: Possible vs Actual Conflict

- Centrality approach: Closeness by **inverting the length of paths of least distance** (cf. [Freeman(1978)])
- Problems regarding **scales, relative differences** and since  $\text{distance} = 1 - \text{bilclos}$  is **cumulated** across statements
- We use **differences to possible conflict** on negotiation paths w.r.t. **statement-specific consensus/conflict**

### Possible vs. Actual Conflict

We interpret closeness as **the difference between possible and actual conflict** on a conflict path.

## Coalitional Closeness: Sequential Negotiation on Paths

- Let  $K \subseteq N$  be a coalition of parties
- Let  $p^K := \{ij \mid i, j \in K, i \neq j\}$  be the link set of the complete graph with node-set  $K$ 
  - set of all (bilateral) negotiation possibilities within  $K$
- 1. Average conflict across complete negotiation graph
  - coincides with average closeness of coalition
- 2. Overall conflict across complete negotiation graph
  - conflict potential of coalition
- 3. Least possible conflict across connecting negotiation path
  - path of least conflict in coalition

## Possible vs. Actual Conflict: Average Conflict

## Definition (Average Conflict)

Average conflict closeness of  $K$  is given by

$$\text{AVCclos}_K := \sum_{s=1}^S \left( 2 - \sum_{\substack{i,j \in K: \\ i \neq j}} (2 - c_{ij}^s) \binom{|K|}{2}^{-1} \right) (2S)^{-1}$$

where we normalize by maximal average conflict across statements (index on unit interval).

**Lemma:** we have  $\text{AVCclos}_K = \sum_{\substack{i,j \in K: \\ i \neq j}} \text{bilclos}_{ij} \binom{|K|}{2}^{-1}$ .

## Possible vs. Actual Conflict: Conflict Potential

**Theorem:** For each statement  $s = 1, \dots, S$ , the maximal overall conflict within  $K \subseteq N$  is  $\left\lfloor \frac{|K|^2}{2} \right\rfloor$ .

Definition (Conflict Potential Closeness)

Conflict Potential closeness of  $K$  is given by

$$\text{CP}_{\text{clos}_K} := \sum_{s=1}^S \left( \left\lfloor \frac{|K|^2}{2} \right\rfloor - \sum_{\substack{i,j \in K \\ i \neq j}} (2 - c_{ij}^s) \right) \left( \left\lfloor \frac{|K|^2}{2} \right\rfloor \cdot S \right)^{-1}$$

where we normalize by the maximal conflict value across statements to obtain an index on a normalized scale.

## Possible vs. Actual Conflict: Path of Least Conflict

Interpret coalitional negotiation as a *minimal* sequential process of bilateral negotiations. The conflict value on a **path of least conflict (PLC)** w.r.t. statement  $s$  is given by

$$plc_K^s := \min \left\{ \sum_{ij \in p} (2 - c_{ij}^s) \mid p \text{ connects } K \right\}$$

### Definition (Path of Least Conflict Closeness)

*Path of Least Conflict closeness* of  $K$  is given by

$$PLC_{\text{clos}_K} := \sum_{s=1}^S \left( 2(|K| - 1) - plc_K^s \right) \left( 2(|K| - 1) \cdot S \right)^{-1}$$

where we normalize by the maximal **bilateral** conflict on a minimal connecting path across statements.



# Comparison of Closeness Measures

Table : Coalitional Closeness RP 2016 (normalized to 100 %)

Coalition	CDU & SPD	SPD, FDP & Grüne
AVC-Closeness	64.47	57.89
CP-Closeness	64.47	36.84
PLC-Closeness	64.47	68.42

Party 1	Party 2	Party 3	CP	AVC	PLC	SMC
agree	agree	agree	2	2	2	2
agree	agree	neutral	1	1.33	1.5	1
agree	agree	not agree	0	0.67	1	0
agree	neutral	not agree	0	0.67	1	0.5

CP rules out certain consensus in a group of 3 parties: at least 2 parties have equal or not strongly opposite positions!

## Application: Parliament Elections Germany

- Always adjust for incompatibilities!
- Analysis on ministerial positions (w/o prime minister)
- 31 Parliament Elections, comparison to Gamson's Law
  - Baden-Wuerttemberg 06 ✓, 11 ✓, 16 ✓
  - Bavaria 18 ✓
  - Berlin 06 ✓, 11 ✓, 16 ✓
  - Bremen 07 ✓, 11 ✓, 15 ✓
  - Federal Parliament 05 ✓, 09 ✓, 13 ✓, 17 ✓
  - Hamburg 08 ✓, 15 ✓
  - Lower Saxony 08 ✓, 13 ✓
  - North-Rhine-Westphalia 05 ✓, 10 ✓, 12 ✓, 17 ✓
  - Rhineland-Palatinate 12 ✓, 16 ✓
  - Saarland 12 ⚡, 17 ✓
  - Schleswig-Holstein 12 ✓, 17 ✓
  - Saxony 14 ✓
  - Saxony-Anhalt 16 ⚡
  - Thuringia ✓
- ✓: confirm (20), ✓: not confirm but better (9)
- confirmation rate for Gamson's Law:  
91% (20 out of 31-9=22) / 93.5% (20+9=29/31)

# Application: Parliament Elections Germany

Table : Proxy-Performance for Portfolio Allocation

	Correct Proxy	Best Proxy	Gamson's Law
Gamson	18	20	
Banzhaf	4	7	8
NoWeight	4	8	9
AVCclos	15	18	14 (22)
CPclos	19	21	15 (23)
PLCclos	16	20	12 (22)
Merged	25	29	20 (29)

# Conclusion

- We suggest portfolio allocation due to relative weakness proportionality: Closeness interpreted as materialization probability → weakness for non-member parties
- Ideological closeness is derived by conflict path analysis / consensus and conflict from VAA data
- Analysis of 31 elections in Germany → 91% / 93.5% confirmation rate
- Further research
  - More data ( “StemWijzer”/“Smartvote”/“Vote Compass”)
  - Hybrid between CP and PLC → SMC
  - Centrality/Distance analysis

**Thank you for your attention**

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