Intertemporal Substitution, Precautionary Saving, and Currency Risk Premium

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Introduction

- exchange rate S_t, home currency price per unit of foreign currency
- home short rate r_t and foreign short rate r_t^*
- log exchange rate, $s_t \equiv \ln S_t$
- currency excess return is: $\rho_{t+1} = s_{t+1} - s_t + r_t^* - r_t$
- currency risk premium is: $E_t(\rho_{t+1}) = E_t(s_{t+1} - s_t) + r_t^* - r_t$

A Paradox in FX Market

Engel's paradox (AER, 2016), two empirical regularities

- Forward Premium Puzzle (Short Premium Puzzle)
  $cov(E_t[\rho_{t+1}], r_t^* - r_t) > 0$

- Excess Co-movement Puzzle or Level Puzzle
  $cov(\sum_{j=0}^{\infty} E_t[\rho_{t+j+1}], r_t^* - r_t) < 0$

  $\Rightarrow$ (Long Premium Puzzle)
  $cov(E_t[\rho_{t+j+1}], r_t^* - r_t) < 0$ for large j

- Engel: these 2 empirical regularities constitute a paradox

Economics of the Forward Premium Puzzle

- forward premium puzzle: $\mathrm{cov}(\mathrm{E}_t(\rho_{t+1}), r_t^* - r_t) > 0$
- when foreign interest rate is high, the average excess currency return is high
- this justifies the carry-trade strategy: borrowing low interest currency to invest in high interest currency
- one explanation for high average excess return is compensation for risk (Backus, Foresi and Telmer (2001), Brennan and Xia (2006))

Economics of the Level Puzzle

- The second empirical regularity is linked to level of exchange rate by telescoping: $\rho_{t+j+1} = s_{t+j+1} - s_{t+j} + r^*_{t+j} - r_{t+j}$

$$s_t - E_t[s_{t+k}] \quad = \quad \underbrace{\sum_{j=0}^{k} E_t[r^*_{t+j} - r_{t+j}]}_{interest\ \ rate\ \ parity\ \ term} \quad - \quad \underbrace{\sum_{j=0}^{k} E_t[\rho_{t+j+1}]}_{cumulative\ \ risk\ \ premium}$$

- when s_t is mean reverting, $E_t[s_{t+k}]$ becomes a constant in the limit
- $cov(\sum_{j=0}^{\infty} E_t[\rho_{t+j+1}], r^*_t - r_t) < 0$ leads to excessive covariance with s_t
- implies more excessive over-shooting than the classical Dornbusch model and Mundell-Fleming model which assumes UIP

Our Contribution

We show that a fairly standard model with time varying risk premium can resolve Engel's paradox

- Intertemporal substitution plays an important role
- Mean consumption growth depends on both consumption volatility and variance
- Risk can account both forward premium puzzle and the excess co-movement puzzle
- Did not use recursive utility, long run risk, and bounded rationality

- Existing models can not account these two puzzles simultaneously

Exchange Rate and No Arbitrage

Exchange Rate

- two-agents (country) model
- each country (home and foreign) - a representative agent
- $C_t^i, \ i \in \{h, f\}$, could be interpreted as a quantity index of multiple goods
- Lucas (1982), Cole and Obstfeld (1991), Backus and Smith (2013), Colacito and Croce (2013)

Exchange Rate and No Arbitrage

Exchange Rate

- by no-arbitrage, Backus et. al. (JF, 2001)

$$s_{t+1} - s_t = \ln \pi^*_{t+1} - \ln \pi_{t+1}$$

  where π^*_{t+1} and π_{t+1} are foreign and home country pricing kernels
- we only need to model pricing kernels for each country
- each country has a representative agent, same parameters, independent and identical consumption processes

Expected Utility

- representative agent with expected CRRA utility

$$\sum_{t=0}^{\infty} \mathrm{E}_0 \left[e^{-\beta t} \frac{C_t^{1-\gamma}}{1-\gamma} \right].$$

- β is the subjective discount coefficient
- γ is the risk-aversion coefficient
- home country pricing kernel π_{t+1}

$$\pi_{t+1} = e^{-\beta} e^{-\gamma(c_{t+1} - c_t)}$$

- foreign country π_{t+1}^*

$$\pi_{t+1}^* = e^{-\beta} e^{-\gamma(c_{t+1}^* - c_t^*)}$$

Interest Rate

- log consumption growth

$$c_{t+1} - c_t = \mu_{ct} + \sigma_{ct}\varepsilon_{t+1}$$

- interest rate

$$r_t = \beta + \gamma\mu_{ct} - \frac{1}{2}\gamma^2\sigma_{ct}^2$$

- intertemporal substitution component : $i.s. = \gamma\mu_{ct}$
- precautionary saving component: $p.s. = -\frac{1}{2}\gamma^2\sigma_{ct}^2$

Currency Premium

- currency premium of the simple return, $\frac{S_{t+1}}{S_t} e^{r_t^f}$, is

$$E_t \left(\frac{S_{t+1}}{S_t} e^{r_t^f - r_t^h} \right) = e^{\gamma^2 \sigma_{ct}^{h2}} \quad ,$$

- depends on home consumption variance $\gamma^2 \sigma_{ct}^{h2}$ but not on foreign consumption variance.
- only home risk is priced.

Currency Premium

- currency premium for the log return is

$$E_t[\rho_{t+1}] = E_t\left(\ln\left[\frac{S_{t+1}}{S_t} - r_t^h + r_t^f\right]\right)$$

$$= \underbrace{\gamma^2\sigma_{ct}^{h2}}_{compensation\ \ for\ \ risk} \quad - \underbrace{\frac{\gamma^2}{2}(\sigma_{ct}^{h2} + \sigma_{ct}^{f2})}_{Jensen's\ \ effect}$$

- can be written as the differential of home and foreign country premiums:

$$E_t[\rho_{t+1}] = \frac{\gamma^2}{2}(\sigma_{ct}^{h2} - \sigma_{ct}^{f2}) \quad .$$

- log return makes it symmetric
- from now on, we will focus on one country.

Precautionary Saving and Risk Premium

- risk premium: $\nu_t^h = \frac{\gamma^2}{2}\sigma_{ct}^2$
- interest rate: $r_t = i.s. + p.s.$
  - intertemporal substitution component : $i.s. = \gamma\mu_{ct}$
  - precautionary saving component: $p.s. = -\frac{1}{2}\gamma^2\sigma_{ct}^2$
    negatively proportional to risk premium
- positive correlation between the risk premium and interest rates has to come from μ_{ct}, this channel is ignored in existing literature

Our Model

- Our Model

Consumption Process

- log consumption growth

$$c_{t+1} - c_t = \underbrace{\lambda\sigma_{ct} + (h - 1/2)\sigma_{ct}^2}_{\mu_{ct}} \; + \; \sigma_{ct} \; \varepsilon_{t+1}^c$$

- conditional volatility

$$\sigma_{ct} = x_t + \theta$$

$$x_{t+1} = \varphi x_t + \sigma\varepsilon_{t+1}^x$$

  OU process, Stein and Stein (1991) and Constantinides (1992)
- conditional mean

$$\mu_{ct} = \lambda\sigma_{ct} + (h - 1/2)\sigma_{ct}^2$$

  depends on both consumption volatility and variance
- empirically documented in Bekaert and Liu (2004) (new to literature)

Expected Future Risk Premium

- the expected future risk premium $\nu_t = \frac{\gamma^2}{2}\sigma_{ct}^2$

$$\mathrm{E}_t[\sigma_{ct+j}^2] = \mathrm{E}_t[(x_{t+j} + \theta)^2] = 2\theta x_t \varphi^j + x_t^2 \varphi^{2j} + \cdots$$

- the expected future risk premium depends on both consumption variance x_t^2 and consumption volatility x_t

- the term with consumption volatility dominates the term with consumption variance when j is large

- existing (Affine) models only have consumption variance, thus only 1 decay mode

Interest Rate

- the interest rate is

$$r_t = \beta + \underbrace{\gamma(\lambda(x_t+\theta) + (h-1/2)(x_t+\theta)^2)}_{intertemporal\ \ substitution} \underbrace{-\frac{1}{2}\gamma^2(x_t+\theta)^2}_{precautionary\ \ saving}$$

$$= \beta + \gamma\lambda(x_t+\theta) - \gamma\Big(\frac{1+\gamma}{2} - h\Big)(x_t+\theta)^2$$

- positively depends on the consumption volatility $x_t + \theta$ through the intertemporal substitution effect
- negatively depends on the consumption variance $(x_t+\theta)^2$ through the precautionary saving effect

Interest Rate

- when $x_t + \theta$ is large, $(x_t + \theta)^2$ dominates, precautionary effect dominates
- interest rate decreases with consumption variance

- when $x_t + \theta$ is small, $x_t + \theta$ dominates, intertemporal substitution effect dominates
- interest rate increases with consumption volatility

Interest Rate

- the competing mechanism of these two effects makes the interest rate a nonmonotonic function of $x_t + \theta$
- key to resolving Engel's paradox
- existing models only have conditional variance

- Resolution of Engel's Paradox

Covariance Between Interest Rate and Risk Premium

- interest rate

$$r_t = \beta + \gamma\lambda(x_t + \theta) - \gamma\left(\frac{1+\gamma}{2} - h\right)(x_t + \theta)^2$$

- expected future premium

$$\nu_t^h = \frac{\gamma^2}{2}\mathrm{E}_t[\sigma_{t+j}^2] = \frac{\gamma^2}{2}(2\theta x_t \varphi^j + x_t^2 \varphi^{2j} + \cdots)$$

- the covariance

$$cov[\nu_t^h, -r_t] = \gamma^3\left[2\left(2\theta^2\left(\frac{1+\gamma}{2} - h\right) - \lambda\theta\right)\ \varphi^j Var[x_t] + \left(\frac{1+\gamma}{2} - h\right)\ \varphi^{2j} Var[{x_t}^2]\ \right]$$

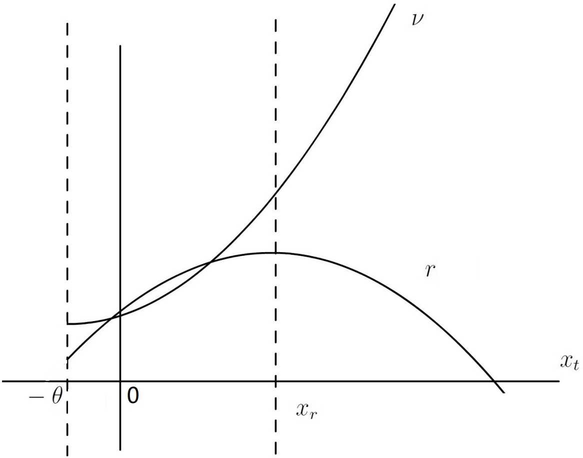
The Covariance

- the covariance

$$cov[\nu_t^h, -r_t] = \gamma^3 \Big[2 \left(2\theta^2 \left(\frac{1+\gamma}{2} - h \right) - \lambda\theta \right) \varphi^j Var[x_t] + \left(\frac{1+\gamma}{2} - h \right) \varphi^{2j} Var[{x_t}^2] \Big]$$

- first term due to conditional volatility x_t and second term due to conditional variance
- when j is large, the first term dominates

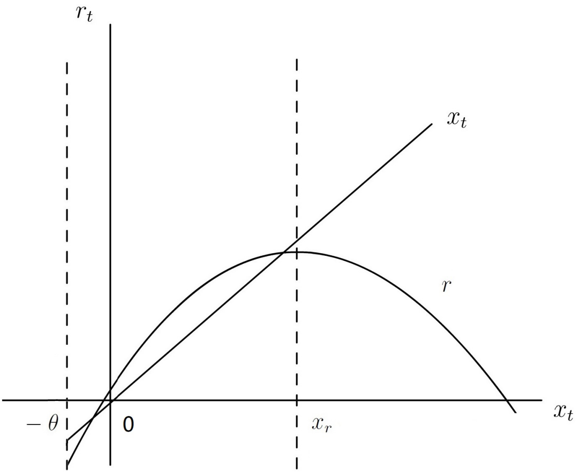
Short Premium Puzzle



Short Premium Puzzle

- $r(x_t)$ and $\nu(x_t)$ are both non-monotone in x_t
- for $x_t \in (-\theta, x_r)$, $x_r = -\theta + \frac{\lambda}{\gamma+1-2h}$, both $r(x_t)$ and $\nu(x_t)$ increase with x_t and thus increase with each other
- for $x_t > x_r$ or $x_t < -\theta$, $r(x_t)$ decrease with $\nu(x_t)$
- the unconditional covariance between the two is negative if λ is small enough (place an upper bound on λ)
- Note that the region $x_t < -\theta$ has negligible probability mass if $\theta \gg 0$

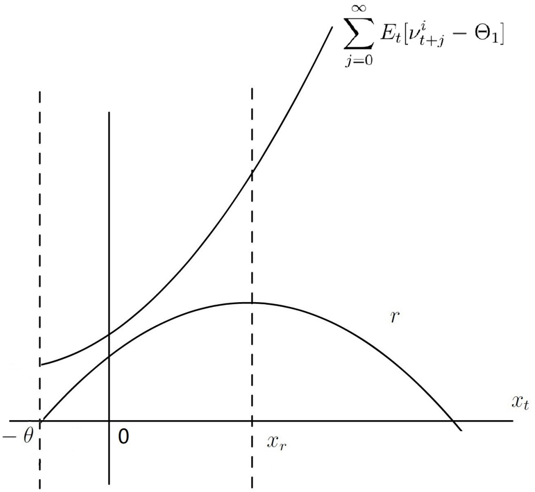
Long Premium Puzzle



Long Premium Puzzle

- long country premium is proportional to x_t
- $r(x_t)$ is non-monotone in x_t
- $r(x_t)$ increases with x_t for $x_t \leq x_r$ and decrease for $x_t > x_r$, $x_r = -\theta + \frac{\lambda}{\gamma+1-2h}$
- over all correlation between $r(x_t)$ and x_t is positive if $x_r > 0$ (place an lower bound on λ)

Cumulative Premium Puzzle



Cumulative Premium Puzzle

- $r(x_t)$ and $\sum_{j=0}^{\infty} E_t[\nu_{t+j}](x_t)$ are both non-monotone in x_t
- both $r(x_t)$ and $\sum_{j=0}^{\infty} E_t[\nu_{t+j}](x_t)$ increase with x_t, and thus, increase with each other if $-\theta(1+\varphi) < x_t < x_r$, $x_r = -\theta + \frac{\lambda}{\gamma+1-2h}$
- for $x_t > x_r$ or $x_t < -\theta(1+\varphi)$, $r(x_t)$ decreases with $\sum_{j=0}^{\infty} E_t[\nu_{t+j}](x_t)$
- the unconditional covariance between the two is positive if λ is large enough.

Resolving the Paradox

- for the short premium, the precautionary saving effect (higher consumption variance causes investors to save more) dominates on average

- places an upper bound on λ

- for the long premium, the intertemporal substitution effect (higher consumption volatility implies higher consumption growth) dominates on average

- provides a lower bound on λ

- there is a range for λ such that both bounds are satisfied, thus resolving Engel's paradox.

Term structure of $\mathrm{Cov}(E_t[\rho_{t+j+1}], r_t^* - r_t)$

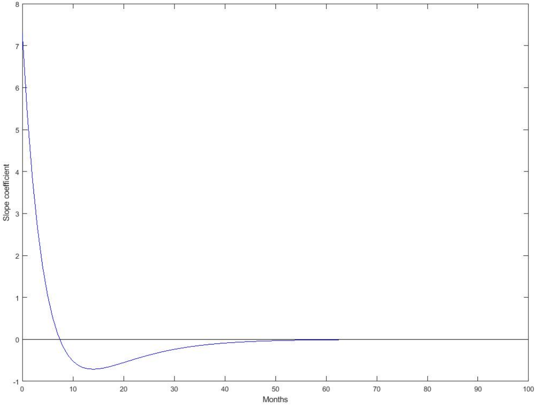


Figure: $\theta = 0.05$, $\sigma = 0.04$, $\varphi = 0.9$, $\gamma = 15$, $\lambda = 5$ and $h = -20$

Extension

- recursive utility
- stationary currency level
- both in closed form

Conclusion

- We provide a risk based rational model to resolve Engel's paradox
- Our model is parsimonious model with stochastic volatility and variance in mean consumption growth
- The intertemporal substitution account for excess co-movement puzzle while the precautionary saving account for the forward premium puzzle
- Our results point to new features for asset pricing models