Belief Update and Mispricing

Munenori Nakasato (*Aoyama Gakuin Univ., Tokyo, nakasato@gsim.aoyama.ac.jp*)
Tomoki Kitamura (*Tohoku Gakuin Univ., Miyagi, PXL03406@nifty.ne.jp*)
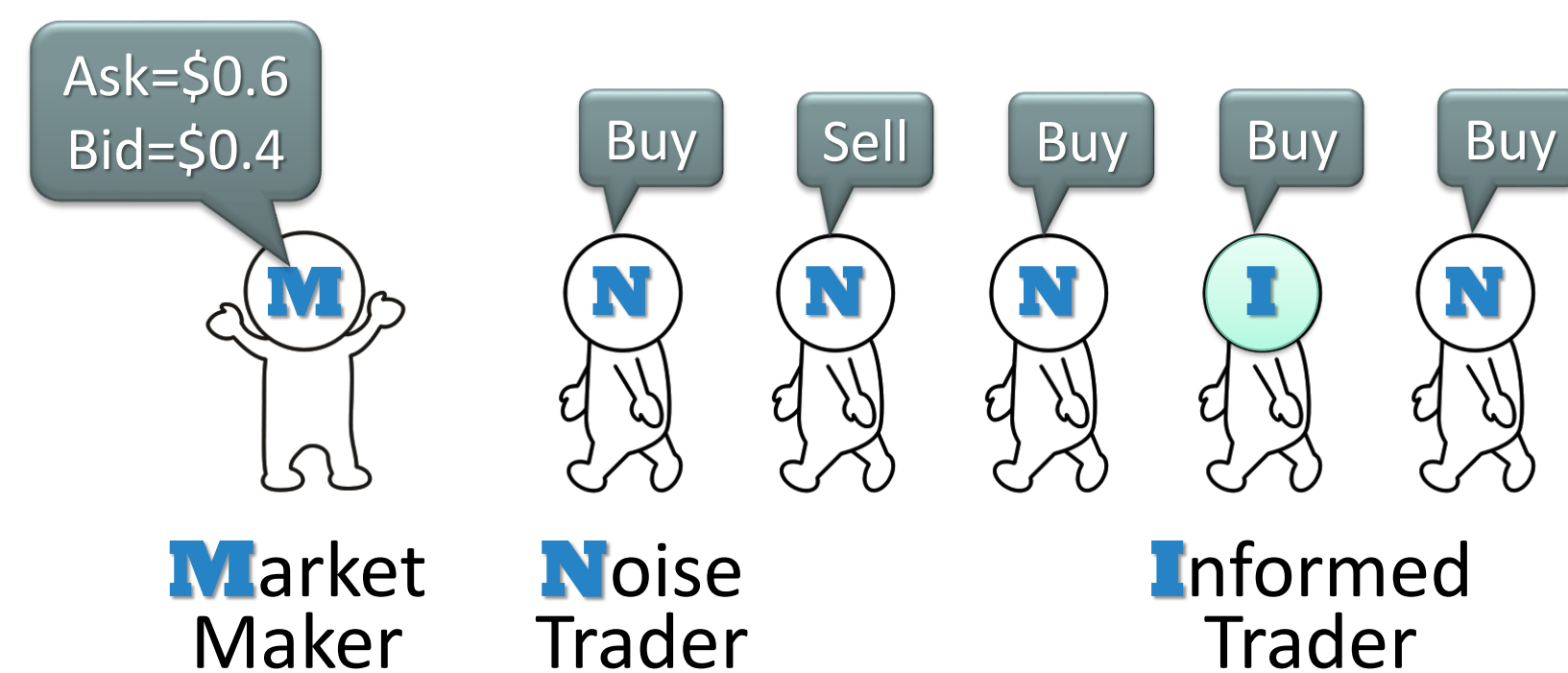Hirotaka Fushiya (*Aoyama Gakuin Univ., Tokyo, fushiya@gmail.com*)

Abstract

- We assume an **asymmetric information** environment.
- There are two uncertainties: **uncertainty of the asset value** and **uncertainty of the existence of informed traders**.
- When a market maker does not know whether informed traders exist or not and informed traders actually do not exist, the asset price systematically deviates from the fair value, causing **asset mispricing**.
- This situation is close to the **information mirage** that Camerer and Weigelt (1991) found in their asset market experiments.
- After the market maker sufficiently updates his belief, he adequately finds the non-existence of informed traders. Then asset mispricing shrinks, and the market becomes **efficient**.
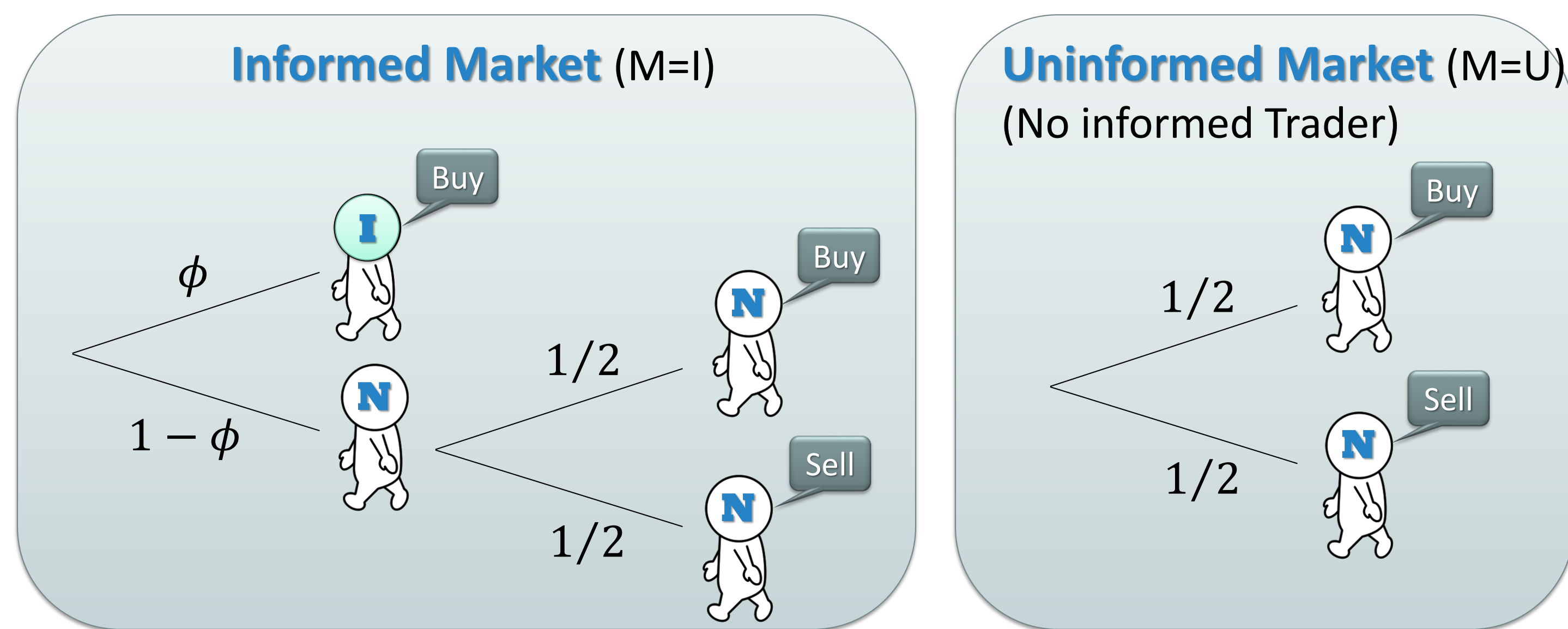
Market

1) Risky Asset

Asset value : $\theta = \begin{cases} 1 & (prob. = 1/2) \\ 0 & (prob. = 1/2) \end{cases}$

2) Sequential Trading

Ask=$0.6
Bid=$0.4

Buy Sell Buy Buy Buy

Market Maker   **N**oise Trader   **I**nformed Trader

3) Market Type (Given $\theta = 1$)

Informed Market (M=I)

ϕ
$1 - \phi$
Buy
$1/2$
$1/2$
Buy
Sell

Uninformed Market (M=U)
(No informed Trader)

$1/2$
$1/2$
Buy
Sell

Belief Update

1) Market Maker's belief

$\mu_n = \text{Prob}(\theta = 1 \mid M = I, \mathcal{H}_n)$ $\mathcal{H}_n = \omega_1 \omega_2 \cdots \omega_n$: *Buy/Sell history*
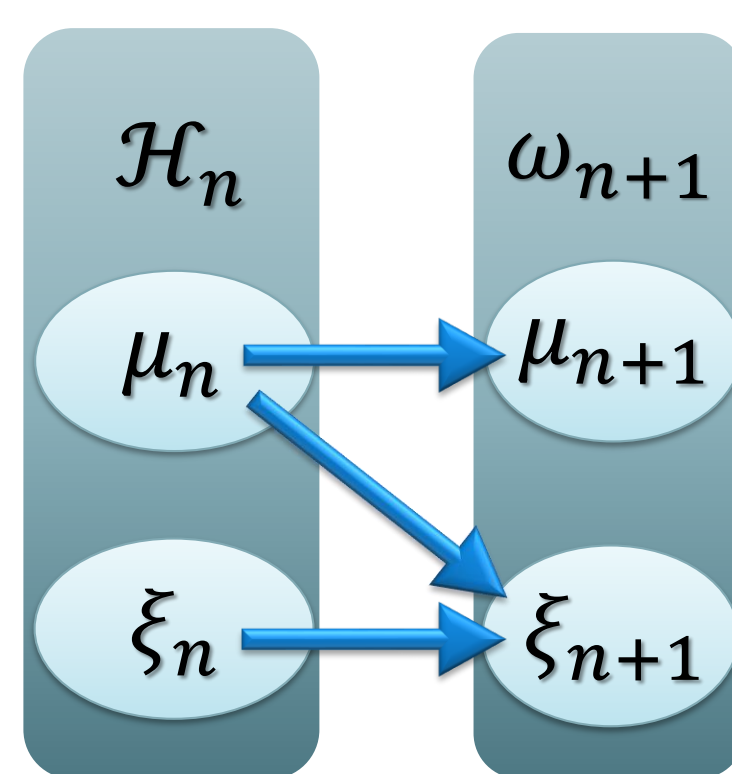
Example $\mathcal{H}_n$ =BSBBB

$\xi_n = \text{Prob}(M = I \mid \mathcal{H}_n)$ *Suspicion of informed traders*
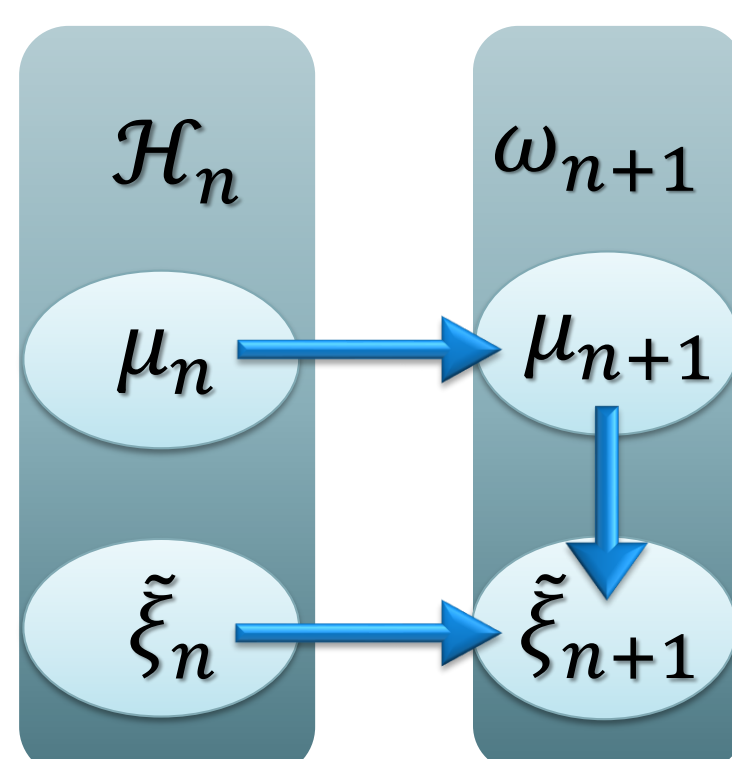
2) Bayesian Update

$\mu_{n+1} = \begin{cases} \dfrac{(1+\phi)\mu_n}{1-\phi+2\phi\mu_n} & (\omega_{n+1} = B) \\[2em] \dfrac{(1-\phi)\mu_n}{1+\phi-2\phi\mu_n} & (\omega_{n+1} = S) \end{cases}$

$\xi_{n+1} = \begin{cases} \dfrac{0.5 + \phi(\mu_n - 0.5)}{0.5 + \phi(\mu_n - 0.5)\xi_n} \cdot \xi_n & (\omega_{n+1} = B) \\[2em] \dfrac{0.5 - \phi(\mu_n - 0.5)}{0.5 - \phi(\mu_n - 0.5)\xi_n} \cdot \xi_n & (\omega_{n+1} = S) \end{cases}$

$\mathcal{H}_n \rightarrow \omega_{n+1}$
$\mu_n \rightarrow \mu_{n+1}$
$\xi_n \rightarrow \xi_{n+1}$

3) Quasi-Bayesian Update

$\tilde{\xi}_{n+1} = \begin{cases} \dfrac{0.5 + \phi(\mu_{n+1} - 0.5)}{0.5 + \phi(\mu_{n+1} - 0.5)\tilde{\xi}_n} \cdot \tilde{\xi}_n & (\omega_{n+1} = B) \\[2em] \dfrac{0.5 - \phi(\mu_{n+1} - 0.5)}{0.5 - \phi(\mu_{n+1} - 0.5)\tilde{\xi}_n} \cdot \tilde{\xi}_n & (\omega_{n+1} = S) \end{cases}$

$\mathcal{H}_n \rightarrow \omega_{n+1}$
$\mu_n \rightarrow \mu_{n+1}$
$\tilde{\xi}_n \rightarrow \tilde{\xi}_{n+1}$

Theorem

1) $\xi_n \xrightarrow{a.s.} True\ Value$ *Bayesian markets become efficient.*

2) $\tilde{\xi}_n \xrightarrow{a.s.} True\ Value$ *Quasi-Bayesian markets become efficient.*

3) $\xi_n < \tilde{\xi}_n$ **for all** n $(and\ all\ \mathcal{H}_n)$

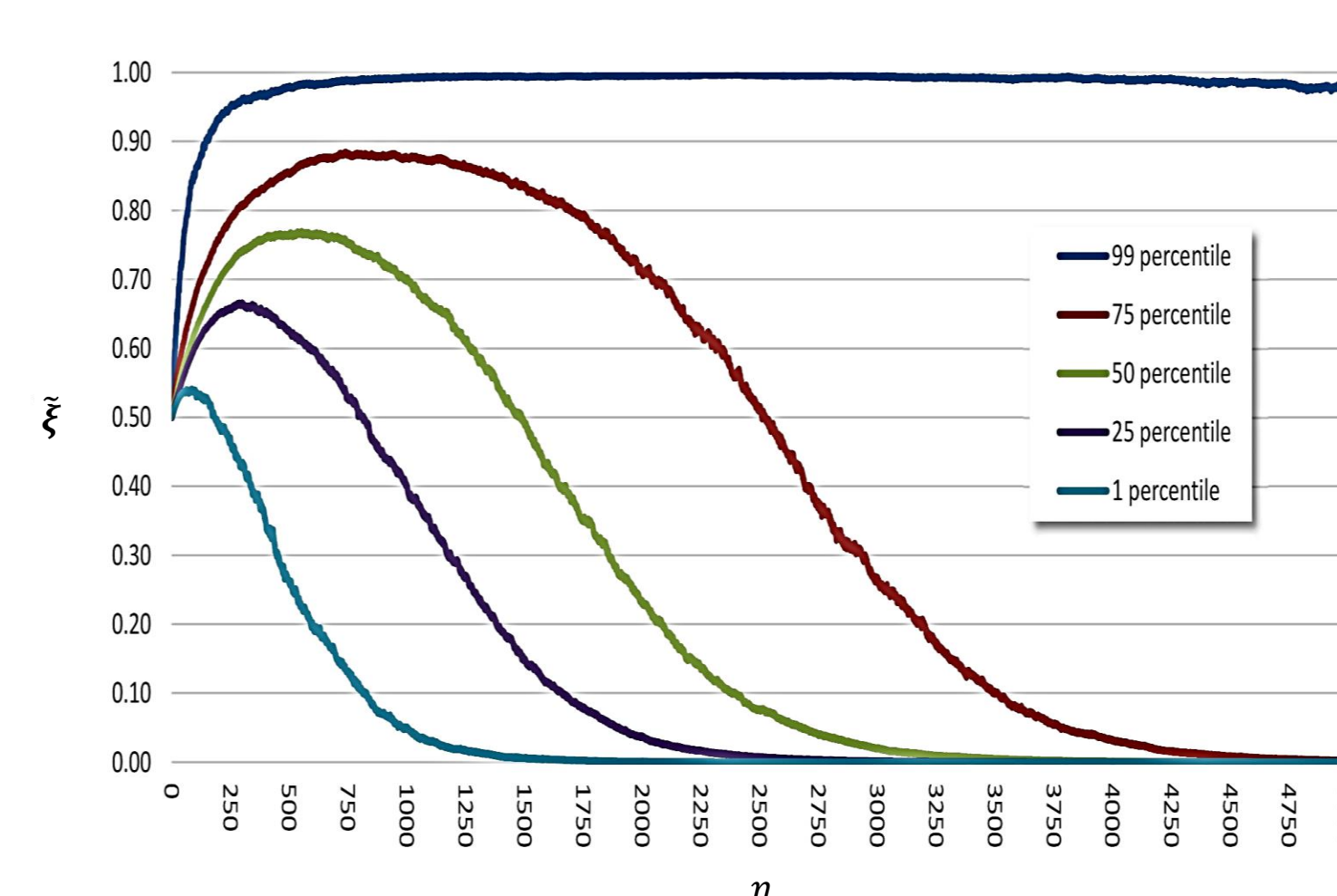4) $E[\tilde{\xi}_n]$ has *Local Maximum*   *Temporal Miss price (Bubble) occurs.*
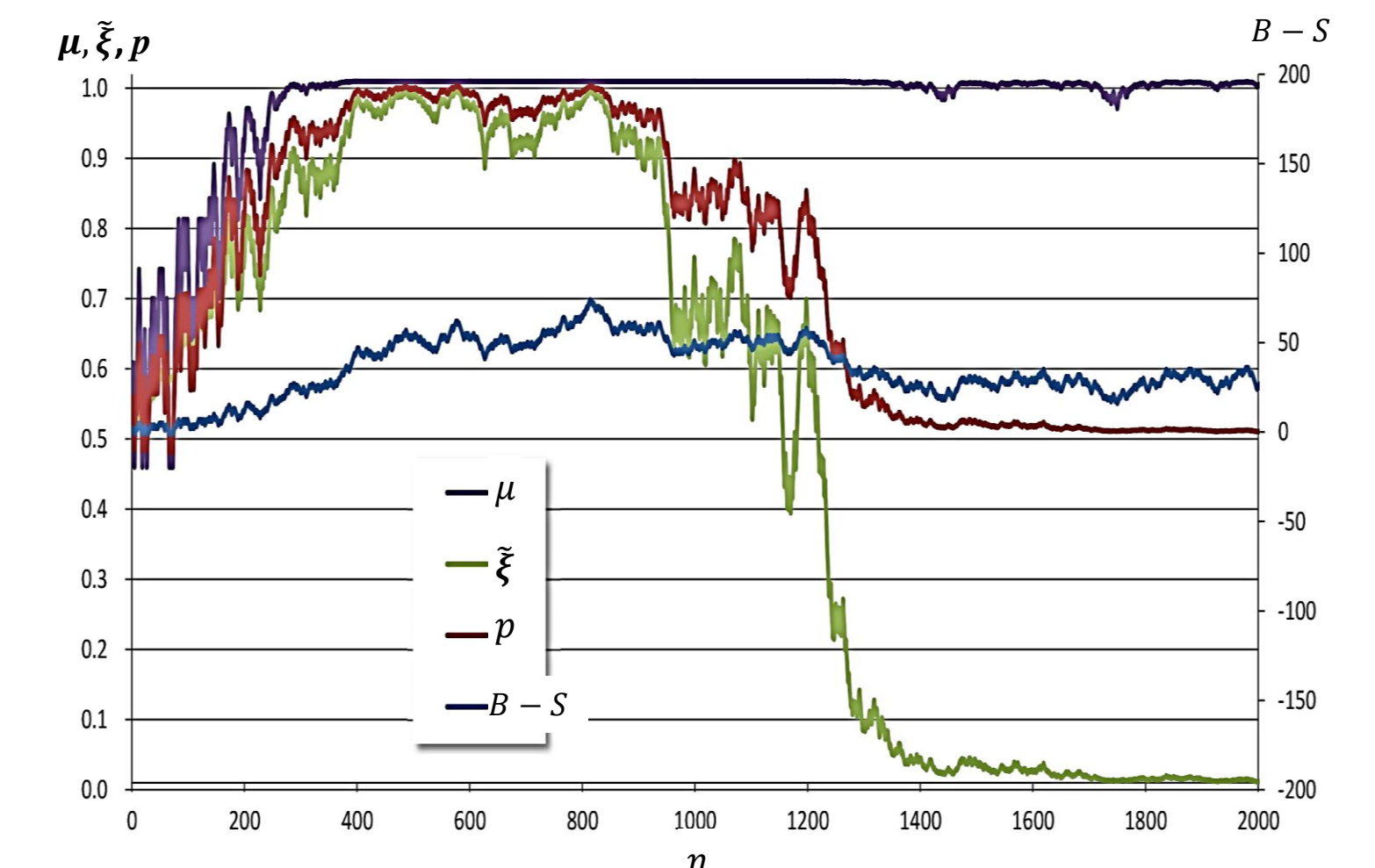
Figure 1. Transition of $\tilde{\xi}$.

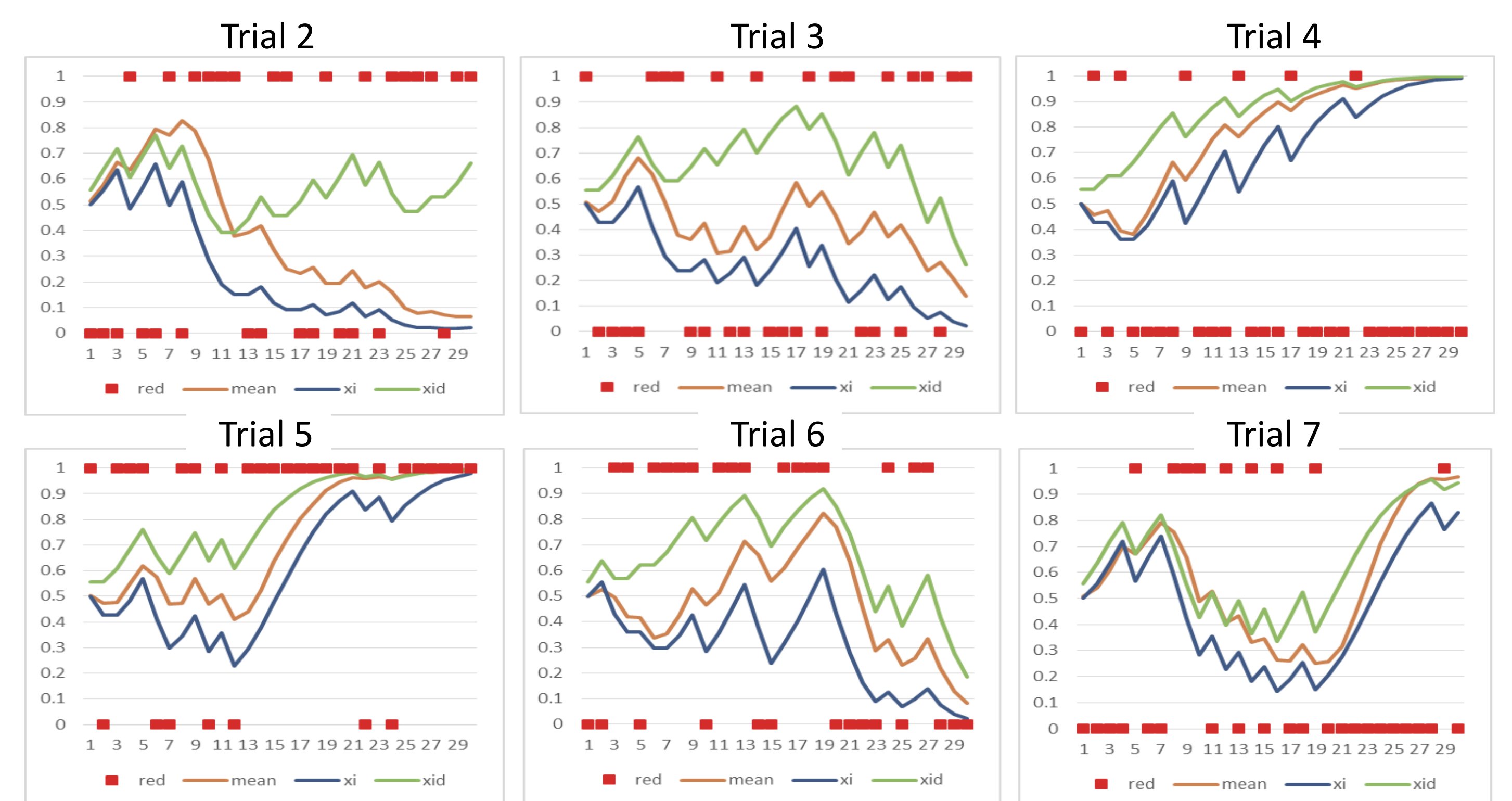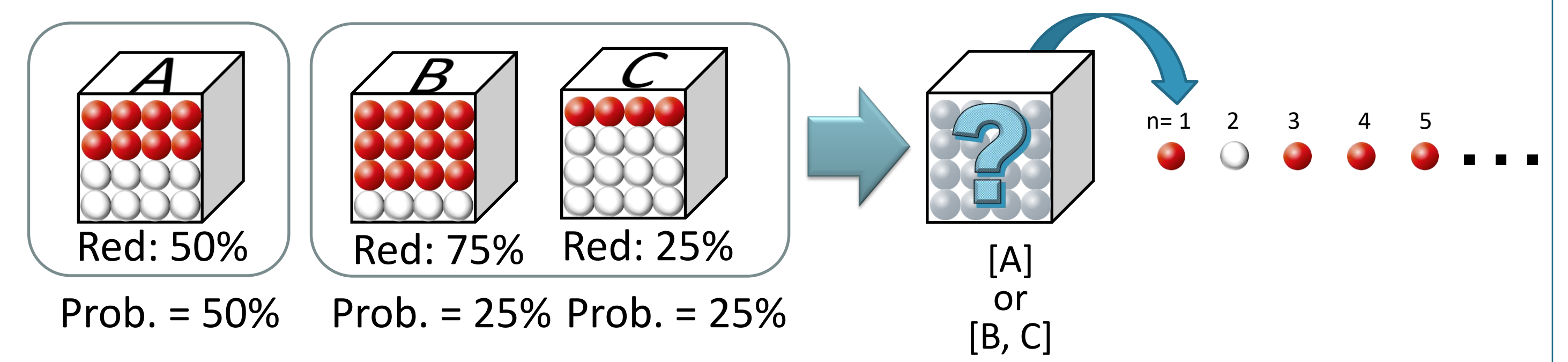**Figure 2.** A typical pass of miss pricing.

Experiment

A Red: 50% Prob. = 50%
B Red: 75% Prob. = 25%
C Red: 25% Prob. = 25%

[A] or [B, C]

n=1 2 3 4 5 . . .

Trial 2   Trial 3   Trial 4
Trial 5   Trial 6   Trial 7

red — mean — xi — xid

Figure 3. Paths of ξ and $\tilde{\xi}$.

Trial 2   Trial 3   Trial 4
Trial 5   Trial 6   Trial 7

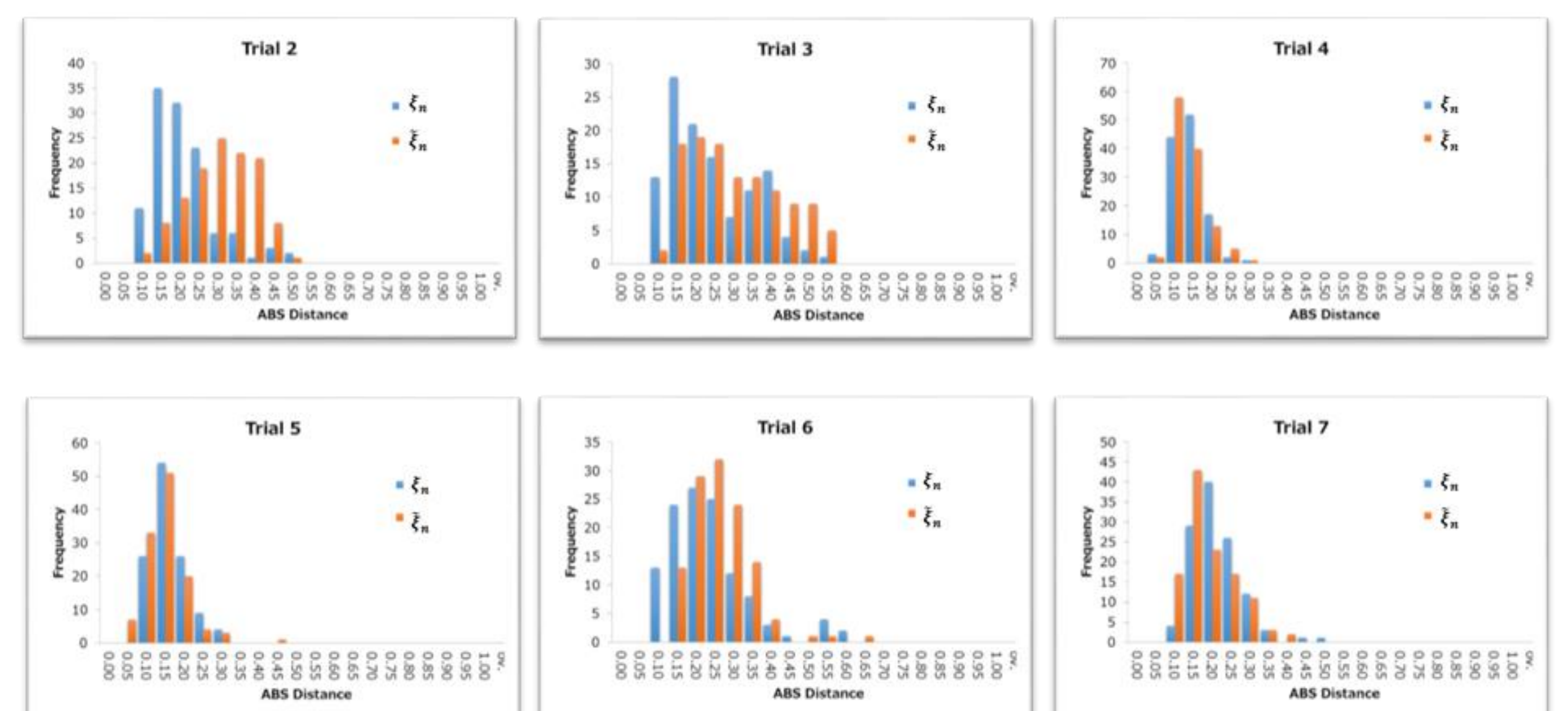ξ_n $\tilde{\xi}_n$

ABS Distance

Figure 4. $D_i = \sum_{n=1}^{30} \left| X_{i,T,n} - \xi_{T,n} \right|$

Conclusion

Our results suggest that mispricing may occur when investors believe that private information exists in stock markets and trade on the basis of their own belief even if private information does not exist. For stock markets to be efficient, controlling the information flow is important for policy makers and regulators.