Lying to Speak the Truth: Selective Manipulation and Improved Information Transmission[*]

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Abstract

We show that firms may benefit from allowing some earnings management, because it can make noisy signals more informative. We model a firm that cannot observe a manager's cost of effort, her effort choice, and whether she manipulated a publicly observable signal. An optimal contract links compensation to both the eventually realized firm value and the (possibly manipulated) signal, since both are noisy measures of effort provision. It may be optimal to allow for manipulation of the signal by a manager who exerted a high effort level: Doing so can convert a falsely unfavorable signal into a favorable signal, thereby strengthening the link between effort and compensation.

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1 Introduction

Financial reporting allows investors to monitor the performance of firms in which they invest. However, financial reporting is noisy, which adds frictions to the design of incentive compensation and may cause suboptimal decisions. Some have argued that investors may benefit from allowing executives some discretion in "managing" financial reports, if this reduces the noise in their reporting (e.g., Subramanyam 1996). But such discretion can be abused by managers if their compensation depends on the perceived performance of their firms, and a large literature focusing on agency problems (discussed in more detail below) views earnings management as undesirable.[1]

We show that the two views are not necessarily in conflict. We analyze an optimal contracting model in which financial reports are noisy and managers can manipulate their firm's reports, at a cost. Importantly, we assume that a firm can make earnings manipulation prohibitively costly for its managers, at no cost to itself. We find that firms can benefit from allowing "selective manipulation": it can be optimal to allow manipulation by managers who (i) expect their firm to perform well and (ii) expect an intermediate financial report to be unfavorable. We obtain this result for the case in which a manager's effort is moderately productive; in contrast, if effort is sufficiently productive, the firm finds it optimal to always prevent manipulation.

Inducing selective manipulation makes noisy financial reports less noisy, because "false negatives" are corrected endogenously. The possibility to "fix" a false unfavorable financial report makes incentive contracts more powerful, but this comes at a cost. If a manager who exerts high effort has to incur manipulation costs if she expects an unfavorable report, this causes a deadweight loss (and potentially lower profits), and it reduces the power of the incentive compensation.

Our results are consistent with the attitudes of financial executives. The survey results in Graham, Harvey, and Rajgopal (2005) and De Jong et al. (2014) show that it is common

[1] Various terms are used in the literature to describe various extents of manipulation, for example, fraud, irregularities, misconduct, misreporting, or misrepresentation; see Amiram et al. (2018) for an overview.

for CFOs to manage earnings — often through reductions in discretionary spending, thereby reducing the long-term value of their firms. Importantly, it seems that CFOs regard such earnings management as being in their firms' best interest, possibly because failing to meet or beat analyst forecasts could be interpreted as a sign that there are underlying problems so severe that the earnings could not be artificially raised. The view that earnings management is benign is also evident from an episode described in Jack Welch's memoir (Welch and Byrne 2003), in which he complains about the managers of one division of GE who were unwilling to "pitch in" to make up for an unexpected earnings shortfall.[2] It is also consistent with the common use of non-GAAP measures in financial reporting.

Our results are also consistent with empirical evidence in Fang and Fu (2018), who analyze equity undervaluation around the time of fire sales caused by market shocks. Firms can meet or beat analyst forecasts using either accounting manipulation (e.g., using accruals) or real manipulation (e.g., delaying or cutting R&D). Fang and Fu (2018) find that firms focusing on cutting R&D underperform after fire sales, compared with firms focusing on earnings manipulation, suggesting that it can be optimal to allow managers some ability to manage financial reports. Note that there is no choice between two manipulation methods in our model; but the findings suggest that, consistent with our model, investors benefit if firms can at times manipulate reports that would otherwise convey false unfavorable information.

In our model, a manager must exert costly effort, but investors do not know how costly it is for the manager to exert effort (that is the manager's private information). Whether effort is exerted is also unobservable to investors. Inducing effort is beneficial since it increases the chances of the firm earning a high terminal cash flow. It also increases the chances that a good intermediate financial report is realized. Both this financial report and the eventual terminal cash flow are verifiable and can be used to incentivize effort (with an appropriate incentive compensation contract). However, the manager can manipulate the financial report

[2] After a negative earnings surprise of $350m was discovered, Welch was pleased by the GE division managers' offers to "pitch in": *"The response of our business leaders to the crisis was typical of the GE culture. [...] many immediately offered to pitch in [...]. Some said they could find an extra $10 million, $20 million, and even $30 million from their businesses to offset the surprise. [...] their willingness to help was a dramatic contrast to the excuses I had been hearing from the Kidder people."* (Welch and Byrne 2003, ch. 15).

before it is realized: At a personal cost, she can convert a bad report into a good report. The optimal incentive scheme determines whether a manager exerts effort (depending on her realized cost of effort) and whether a manager manipulates the report if it is unfavorable. Asymmetric information about the cost of effort causes an adverse selection problem and therefore information rents for the manager. Allowing manipulation of the financial report complicates the firm's optimization problem.

A critical assumption in our model is that the firm can freely choose how costly it is for the manager to manipulate the financial report. The board of directors can influence the quality of the financial reporting, or more generally the strength of corporate governance, thus making it either easy or difficult to manipulate financial reports. We assume that this choice does not cause any costs to the firm: It is equally costly to have relaxed financial reporting standards as it is to have strict standards that prevent manipulation. Arguably, stricter standards are likely to be more costly to implement, which could make it optimal to allow for some limited extent of manipulation. But this would be an uninteresting, mechanical explanation for the potential optimality of allowing for some earnings management at the margin. As we show, even if manipulation could be prevented *at no cost* to the firm, it may be optimal to induce some degree of manipulation.

The novel results of this paper are on the surface counter-intuitive: Firms may choose to allow some extent of manipulation even if they could prevent it completely at no cost; and manipulation can make financial reports less noisy and more informative. When the optimal contract allows for some manipulation, it may induce the manager to manipulate financial reports when she exerted high effort but not when she exerted low effort. In doing that, it makes the reports more informative about the manager's effort choice, and therefore about the terminal cash flow. The manager's compensation is increasing in both the reported performance and the realized cash flow, so that the manager's expected marginal benefit from manipulating the report increases in the effort level she has chosen. In situations where managers must be incentivized to exert costly effort, performance manipulation may therefore not only be unavoidable, as the literature has argued, but it can actually be desirable: allowing the manager to overstate firm performance enables the principal to design a more efficient compensation scheme.

This result implies that misreporting can be used to smooth over problems that are temporary and not indicative of fundamental problems. Our result thus confirms the intuition often applied by practitioners when arguing that frequent reporting requirements cause CEOs to become short-termist: by allowing CEOs to inflate short-term reports, they can more effectively focus on long-term value creation. For example, in diversified conglomerates, the skill of a CEO may be to gather and manage a portfolio of unrelated operations, focusing on the efficiency of each of the operations. Allowing such a CEO to hide a limited set of bad news may be beneficial, because it allows her to focus on making the best use of her skills.

An important distinction of our results from earlier contributions is that in our model, a manager who exerted high effort may be induced (by the optimal contract) to manipulate the information that investors observe, while it is always suboptimal to induce manipulation from a manager who exerted low effort. Earlier work either found that less productive managers manipulate financial reports, or that there is a signal jamming equilibrium in which both productive and less productive managers manipulate financial reports (see our literature discussion below).

However, allowing manipulation is not always optimal. Specifically, it is optimal only when the incremental productivity of exerting high effort is not too high. If it is sufficiently high, then it is optimal not to allow any manipulation of the financial reports. Thus, managers who argue that their firms benefit from earnings management inadvertently reveal that their ability to add value (by making good decisions, exerting effort, etc.) is not "above-average."

A variety of explanations for the presence of earnings management have been offered in the literature. First, numerous authors argue that misreporting is an unavoidable feature of large, widely held firms, that it is too costly to completely prevent it, and therefore misreporting can only be managed, not avoided (e.g., Stein 1989; Demski, Frimor, and Sappington 2004; Goldman and Slezak 2006; Crocker and Slemrod 2007; Beyer, Guttman, and Marinovic 2014; Marinovic and Povel 2017; Bertomeu, Darrough, and Xue 2017). That approach is different from our model, since we assume that the firm can completely eliminate all misreporting, at no cost to itself, yet it may be optimal to allow some misreporting.

Second, if there are limits to communication, contractibility, or commitment, then it may be optimal to let an agent manipulate information (Dye 1988; Arya, Glover, and Sunder 1998; Demski 1998). Our results do not rely on such constraints, and the results are driven by asymmetric information.

Third, some authors assume that managers may enjoy the ability to misreport as a perquisite, and tolerating some extent of misreporting can reduce the size of the compensation package a manager expects, thus increasing the firm's profit (Acharya and Volpin 2010; Dicks 2012). There is no such assumption in our model: The manager benefits from manipulation only if it increases her net payoff.

Fourth, the current shareholders in a firm may benefit from earnings manipulation if it allows the firm to raise funds from third parties at favorable rates (e.g., Bar-Gill and Bebchuk 2003; Povel, Singh, and Winton 2007; Strobl 2013). This is different from our model, since there is no second period in which funds need to be raised.

Fifth, market imperfections can make it suboptimal to have perfect disclosure (e.g., Hirshleifer 1971; Morris and Shin 2002). Such imperfections do not play a role in our model.

Sixth, firms may rely on information generated by investors (and revealed through market prices) when making decisions, and it can then be optimal to allow for some manipulation if it strengthens the incentive to generate such information (e.g., Gao and Liang 2013). There is no such effect in our model.

2 The Model

We study an agency model with two risk-neutral parties, a board of directors and a manager, that takes place over times 0, 1, 2, and 3. At time 0, the board (the principal) chooses the firm's governance system (explained below) and hires a manager (the agent) to run the firm. The board represents the interests of shareholders and offers the manager a contract that maximizes the value of the firm, net of the cost of managerial compensation. At time 1, the manager exerts an unobservable effort to enhance the value of the firm. At time 2, the firm's accounting system produces a public report concerning the manager's performance. A key feature of our model is that this performance report can be manipulated by the manager. At time 3, the firm's terminal cash flow v is realized and paid out to shareholders.

The firm's cash flow is either high ($v = v_h$) or low ($v = v_\ell < v_h$). The distribution of v depends on the manager's effort choice $e \in \{0, 1\}$. If the manager exerts high effort ($e = 1$), v is equal to v_h with probability one; if she exerts low effort ($e = 0$), v is equal to v_h with probability $\lambda < 1$ and equal to v_ℓ with probability $1 - \lambda$. The manager's private utility cost of exerting high effort, denoted by c, is drawn from a uniform distribution over the interval $[0, \bar{c}]$; the cost of low effort is normalized to zero. The manager's effort choice e and effort cost c are her private information and hence cannot be used for contracting purposes. The following assumption ensures that inducing high managerial effort is not optimal for sufficiently high realizations of the effort cost c.

Assumption 1. $\bar{c} > (1 - \lambda)(v_h - v_\ell)$.

Prior to the realization of the cash flow v, the firm's accounting system provides a (noisy) signal r to the market concerning the manager's effort choice (and thus the value of the firm). This signal, which we refer to as an earnings report, can take on one of two values, r_h or r_ℓ. Absent any managerial intervention, the report is correlated with the manager's effort choice as follows:

$$\text{prob}\,[r = r_h | e = 1] = \text{prob}\,[r = r_\ell | e = 0] = \delta, \tag{1}$$

where $\delta \in (\frac{1}{2}, 1)$. The parameter δ measures the quality of the firm's accounting system. It represents various accounting standards and conventions in the economy as well as firm- and auditor-specific factors such as the transparency of the firm's operations and the auditor's experience in the industry.

Although the report is produced by the firm's accounting system, the manager can influence its outcome—for example, by exploiting any leeway in accounting rules or by hiding information from the auditor. Specifically, we assume that, by incurring a utility cost g, the manager can turn an unfavorable report r_ℓ into a favorable report r_h with probability ϕ. We denote the manager's decision whether or not to take such an action by $m \in \{0, 1\}$, where $m = 1$ (respectively, $m = 0$) denotes the case where the manager engages (respectively, does not engage) in earnings manipulation.

The manipulation cost g may reflect the time spent coming up with creative ways to manage the firm's earnings or the effort involved in convincing the auditor to accept a biased report.[3] This cost is related to the legal environment in the economy, but also depends on firm-specific factors such as the firm's internal control system that affect the manager's opportunities to misreport the firm's performance. We interpret g as an observable characteristic that represents the quality of the firm's governance and reporting arrangements. For example, a higher g may be the result of the board's decision to implement a more elaborate internal control system or to appoint more financial experts to the audit committee. To stack the deck against finding equilibria with weak governance, we assume that the board of directors can improve the firm's governance—and hence increase the manager's manipulation cost—at no cost to the firm's shareholders. That is, at time 0 the board can choose any $g \geq 0$, without having to spend any resources.

The board represents the interests of shareholders and chooses the firm's governance system and the manager's contract to maximize the value of the firm, net of the cost of managerial compensation. A contract specifies the manager's compensation as a function of the earnings report r and the terminal cash flow v. The manager is risk neutral, has no wealth, and is protected by limited liability so that all payments must be nonnegative. Her reservation level of utility is normalized to zero.

The contractual frictions in our model are created by asymmetric information about the cost of effort, the manager's effort choice, the realization of the signal, and the manager's manipulation choice. The firm could achieve the first-best outcome if the cost of effort, c, and the chosen effort level, e, were verifiable: The signal would then have no information

[3]By diverting resources from more productive uses, these activities may also negatively impact the firm's performance. In the baseline model, we abstract from such a cost to the firm's shareholders and assume that the manager's choice of m does not affect the firm's cash flow v.

value, and the manager would have no incentive to manipulate it. The board would find it optimal to elicit high effort if and only if $v_h - c \geq \lambda v_h + (1-\lambda)v_\ell$, and so the first-best effort level is given by

$$e_{FB} = \begin{cases} 1 & \text{if } c \leq (1-\lambda)(v_h - v_\ell), \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

3 Analysis

This is a model of costly state falsification. Our specification of the set of available contracts is without loss of generality in the sense that it is fully consistent with the revelation principle. Thus, we can restrict attention to truthful direct revelation mechanisms. It is important to note that this does not imply that shareholders will induce the manager to abstain from manipulating the firm's earnings report: the manager's decision to manipulate the report is an action and not a message. Instead, it implies that any allocation that can be achieved through a contract that is contingent on the report and the firm's cash flow can also be achieved through a truthful direct mechanism.

In the ensuing analysis, let $w(r, v|c)$ denote the compensation scheme under the direct mechanism. The fact that the manager has no wealth means that all compensation payments must be nonnegative. Note that this also implies that the manager's participation constraint is trivially satisfied: by choosing to exert zero effort and to not manipulate the report, the manager can always achieve a nonnegative payoff.

3.1 Preliminary Results

We first show that, for a given compensation scheme, the manager's manipulation strategy depends on her cost of effort only through its effect on the effort choice e, that is, $m(e, c) = m(e)$, for all $c \in [0, \bar{c}]$. The manager's manipulation strategy can thus be fully characterized by the vector $(m_0, m_1) \in \{0, 1\}^2$.

Lemma 1. *Suppose that $w(r, v|c) = w(r, v)$ for all $c \in C \subseteq [0, \bar{c}]$, $r \in \{r_h, r_\ell\}$, and $v \in \{v_h, v_\ell\}$. Then, the cost of effort, c, influences the manager's manipulation choice, m, only* *through its effect on her effort choice, e. That is, $m(e,c) = m(e,c')$, for all $e \in \{0,1\}$ and $c, c' \in C$.*

Lemma 2. *There exists a threshold $\hat{c} \in [0, \bar{c}]$ such that the optimal menu of contracts induces high managerial effort (i.e., $e = 1$) for all $c < \hat{c}$ and low managerial effort (i.e., $e = 0$) for all $c > \hat{c}$.*

We next show that the allocation resulting from an optimal direct mechanism can be implemented through a menu of contracts that pools all managers of type $c < \hat{c}$ and of type $c > \hat{c}$. Thus, without loss of generality, we can set $w(r, v|c) = w_1(r, v)$ for all $c \in [0, \hat{c})$ and $w(r, v|c) = w_0(r, v)$ for all $c \in (\hat{c}, \bar{c}]$.

Lemma 3. *The optimal mechanism can be implemented by offering the manager a menu of contracts that pools all types $c \in [0, \hat{c})$ and all types $c \in (\hat{c}, \bar{c}]$.*

The optimal compensation scheme can therefore be characterized by the menu $\{\boldsymbol{w}_0, \boldsymbol{w}_1\}$, where $\boldsymbol{w}_e = (w_e(r_h, v_h), w_e(r_\ell, v_h), w_e(r_h, v_\ell), w_e(r_\ell, v_\ell))$, $e \in \{0, 1\}$.

3.2 The Optimization Problem

To simplify notation, let $\pi_{e,m_e}(r, v)$ denote the probability that a report $r \in \{r_h, r_\ell\}$ and a cash flow $v \in \{v_h, v_\ell\}$ is observed if the manager chooses effort level $e \in \{0, 1\}$ and follows the manipulation strategy $m_e \in \{0, 1\}$. That is,

$$\pi_{1,m_1}(r_h, v_h) = \delta + (1 - \delta)\, \phi\, m_1, \tag{3}$$

$$\pi_{0,m_0}(r_h, v_h) = \lambda(1 - \delta + \delta\, \phi\, m_0), \tag{4}$$

$$\pi_{1,m_1}(r_\ell, v_h) = (1 - \delta)(1 - \phi\, m_1), \tag{5}$$

$$\pi_{0,m_0}(r_\ell, v_h) = \lambda\, \delta(1 - \phi\, m_0), \tag{6}$$

$$\pi_{1,m_1}(r_h, v_\ell) = 0, \tag{7}$$

$$\pi_{0,m_0}(r_h, v_\ell) = (1 - \lambda)(1 - \delta + \delta\, \phi\, m_0), \tag{8}$$

$$\pi_{1,m_1}(r_\ell, v_\ell) = 0, \tag{9}$$

$$\pi_{0,m_0}(r_\ell, v_\ell) = (1 - \lambda)\delta(1 - \phi\, m_0). \tag{10}$$

Also, define $\Delta\pi_{m_0,m_1}(r, v) = \pi_{1,m_1}(r, v) - \pi_{0,m_0}(r, v)$.

Based on the results stated in Lemmas 1 to 3, we can then express the conditions that characterize the optimal contract as follows. First, to induce the manager to follow the desired manipulation strategy (m_0, m_1), the compensation scheme $\{w_0(r, v), w_1(r, v)\}$ that pools manager types as outlined in Lemma 3 has to satisfy the incentive compatibility constraints

$$(2m_1 - 1)\left[\phi\left(w_1(r_h, v_h) - w_1(r_\ell, v_h)\right) - g\right] \geq 0, \tag{11}$$

$$(2m_0 - 1)\left[\phi\left(\lambda\left(w_0(r_h, v_h) - w_0(r_\ell, v_h)\right) + (1 - \lambda)\left(w_0(r_h, v_\ell) - w_0(r_\ell, v_\ell)\right)\right) - g\right] \geq 0. \tag{12}$$

Second, for managers of type $c < \hat{c}$ to exert high effort and for managers of type $c > \hat{c}$ to exert low effort, we must have

$$\sum_{r,v} \pi_{1,m_1}(r, v)\, w_1(r, v) - \hat{c} - (1 - \delta)gm_1 \geq \max_{m \in \{0,1\}} \sum_{r,v} \pi_{0,m}(r, v)\, w_1(r, v) - \delta gm, \tag{13}$$

$$\sum_{r,v} \pi_{0,m_0}(r, v)\, w_0(r, v) - \delta gm_0 \geq \max_{m \in \{0,1\}} \sum_{r,v} \pi_{1,m}(r, v)\, w_0(r, v) - \hat{c} - (1 - \delta)gm. \tag{14}$$

Finally, to ensure that the manager truthfully reports her type c, it must be that

$$\sum_{r,v} \pi_{1,m_1}(r,v)\, w_1(r,v) - c - (1-\delta) g m_1 \geq$$
$$\max_{e,m\in\{0,1\}} \sum_{r,v} \pi_{e,m}(r,v)\, w_0(r,v) - e\big(c + (1-\delta)gm\big) - (1-e)\delta g m, \quad \forall c \in [0,\hat{c}), \quad (15)$$

$$\sum_{r,v} \pi_{0,m_0}(r,v)\, w_0(r,v) - \delta g m_0 \geq$$
$$\max_{e,m\in\{0,1\}} \sum_{r,v} \pi_{e,m}(r,v)\, w_1(r,v) - e\big(c + (1-\delta)gm\big) - (1-e)\delta g m, \quad \forall c \in (\hat{c},\bar{c}]. \quad (16)$$

For a given threshold $\hat{c}$ and manipulation strategy (m_0, m_1), the optimal contract $\mathcal{C} = (\boldsymbol{w}_0, \boldsymbol{w}_1, g)$ minimizes the expected payment to the manager, that is, it solves the problem

$$\min_{\boldsymbol{w}_0,\boldsymbol{w}_1,g} \left(\frac{\hat{c}}{\bar{c}}\right) \sum_{r,v} \pi_{1,m_1}(r,v)\, w_1(r,v) + \left(1 - \frac{\hat{c}}{\bar{c}}\right) \sum_{r,v} \pi_{0,m_0}(r,v)\, w_0(r,v), \quad (17)$$

subject to the constraints in (11)–(16) and the nonnegativity constraints

$$g \geq 0, w_0(r,v) \geq 0, w_1(r,v) \geq 0, \quad \forall r \in \{r_h, r_\ell\}, v \in \{v_h, v_\ell\}. \quad (18)$$

Setting $e = 0$ and $m = m_0$ on the right-hand side of (15) yields

$$\sum_{r,v} \pi_{1,m_1}(r,v)\, w_1(r,v) \geq \sum_{r,v} \pi_{0,m_0}(r,v)\, w_0(r,v) + \hat{c} + G(m_0,m_1), \quad (19)$$

where $G(m_0, m_1)$ denotes the difference in the manager's expected manipulation cost when she exerts high rather than low effort, that is, $G(m_0, m_1) = [(1-\delta)m_1 - \delta m_0]\, g$. Similarly, setting $e = 1$ and $m = m_1$ on the right-hand side of (16), we have

$$\sum_{r,v} \pi_{0,m_0}(r,v)\, w_0(r,v) \geq \sum_{r,v} \pi_{1,m_1}(r,v)\, w_1(r,v) - \hat{c} - G(m_0,m_1). \quad (20)$$

An inspection of (19) and (20) shows that both constraints must be binding, and the principal's objective function can therefore be written as

$$\min_{\boldsymbol{w}_0,\boldsymbol{w}_1,g} \sum_{r,v} \pi_{1,m_1}(r,v)\, w_1(r,v) - \left(1 - \frac{\hat{c}}{\bar{c}}\right) \left(\hat{c} + G(m_0,m_1)\right). \tag{21}$$

The firm's optimization problem is thus to minimize the manager's expected compensation in (21), subject to the constraints in (11)–(16) and (18).

3.3 Optimal Contracts

We solve for the optimal contract in three steps. First, for a given cost threshold $\hat{c}$ and manipulation strategy (m_0, m_1), we characterize the compensation scheme and manipulation cost that induces the manager to exert high effort if and only if $c \leq \hat{c}$ and to implement the manipulation strategy (m_0, m_1) at minimum cost to the firm. Second, for a given cost threshold $\hat{c}$, we compare the firm's profit across different manipulation strategies. We show that it is never optimal to incentivize the manager to manipulate a low report when she exerted low effort, which allows us to restrict our attention to contracts that may or may not induce manipulation from a manager who exerted high effort. Finally, we solve for the cost threshold $\hat{c}$ that maximizes the firm's expected profit. This allows us to compare the feasible firm values generated by the contracts, and to determine which contract is optimal for a given set of parameters.

The following proposition characterizes the optimal no-manipulation contract, that is, the optimal contract that never induces the manager to manipulate the report, irrespective of her chosen effort level.

Proposition 1. *For any cost threshold $\hat{c} \in [0, \bar{c}]$, the optimal no-manipulation contract $\mathcal{C}^n = (\boldsymbol{w}_0^n, \boldsymbol{w}_1^n, g^n)$ consists of a compensation scheme*

$$w_0^n(r,v) = w_1^n(r,v) = \begin{cases} \frac{\hat{c}}{\delta - \lambda(1-\delta)} & \textit{if } r = r_h \textit{ and } v = v_h, \\ 0 & \textit{otherwise}, \end{cases} \tag{22}$$

and a manipulation cost

$$g^n \geq \frac{\phi\,\hat{c}}{\delta - \lambda(1-\delta)}. \tag{23}$$

This contract induces the manager to exert high effort if $c \leq \hat{c}$ and low effort if $c > \hat{c}$, and to follow the manipulation strategy $(m_0, m_1) = (0, 0)$ at minimum cost.

In order to induce effort, a positive compensation should be offered only if the realized outcome is most likely generated by a manager who exerted high effort, that is, if $r = r_h$ and $v = v_h$. The amount offered in that case determines which effort-cost types do exert effort. And the cost of manipulation g^n is chosen such that manipulation is never optimal.

We next turn to the optimal contract that prompts the manager to implement the manipulation strategy $(m_0, m_1) = (0, 1)$, that is, that induces the manager to manipulate a low report if she exerted high effort, but not if she exerted low effort. We refer to such a contract as a partial-manipulation contract.

Proposition 2. *For any cost threshold $\hat{c} \in [0, \bar{c}]$, the optimal partial-manipulation contract $\mathcal{C}^p = (\boldsymbol{w}_0^p, \boldsymbol{w}_1^p, g^p)$ consists of a compensation scheme*

$$w_0^p(r,v) = w_1^p(r,v) = \begin{cases} \frac{\hat{c}}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)} & \text{if } r = r_h \text{ and } v = v_h, \\ 0 & \text{otherwise.} \end{cases} \tag{24}$$

and a manipulation cost

$$g^p = \frac{\lambda\phi\,\hat{c}}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)}. \tag{25}$$

This contract induces the manager to exert high effort if $c \leq \hat{c}$ and low effort if $c > \hat{c}$, and to follow the manipulation strategy $(m_0, m_1) = (0, 1)$ at minimum cost.

As in the no-manipulation case, a positive compensation is earned only when the outcome is most likely generated by a manager who exerted high effort. The cost of manipulation g^p is chosen such that only a manager who exerted high effort will manipulate if the realized report is r_ℓ. The amount of compensation is set such that, incorporating both the cost of effort and the expected cost of manipulation, high effort is exerted if and only if the manager's cost of effort is no higher than the threshold $\hat{c}$.

Our next result shows that it is never optimal for the firm to incentivize the manager to manipulate a low report if she exerted low effort, that is, to choose a manipulation strategy $(m_0, m_1) = (1, 0)$ or $(m_0, m_1) = (1, 1)$.

Proposition 3. *Any contract that induces manipulation after low effort ($m_0 = 1$) is strictly suboptimal.*

Manipulation by a manager who exerted low effort does not benefit the firm: it makes it more difficult to infer the manager's effort choice from the report, which increases the expected compensation payment. This means that, for any desired cost threshold $\hat{c}$, the optimal contract is either contract $\mathcal{C}^n$ defined in Proposition 1) that prevents manipulation entirely or contract $\mathcal{C}^p$ defined in Proposition 2) that permits manipulation only after high effort. The following proposition compares the manager's expected compensation under these two contracts (taking the threshold $\hat{c}$ as given).

Proposition 4. *Let* $\kappa = \frac{(1-\delta)(1-\lambda)}{\delta-\lambda(1-\delta)} \in (0, 1)$. *Then,*

(i) for any cost threshold $\hat{c} \in (0, \kappa\bar{c})$, the expected compensation under the no-manipulation contract $\mathcal{C}^n = (\boldsymbol{w}_0^n, \boldsymbol{w}_1^n, g^n)$ defined in Proposition 1 is strictly higher than the expected compensation under the partial-manipulation contract $\mathcal{C}^p = (\boldsymbol{w}_0^p, \boldsymbol{w}_1^p, g^p)$ defined in Proposition 2;

(ii) for any cost threshold $\hat{c} \in (\kappa\bar{c}, \bar{c}]$, the expected compensation under the no-manipulation contract $\mathcal{C}^n = (\boldsymbol{w}_0^n, \boldsymbol{w}_1^n, g^n)$ defined in Proposition 1 is strictly lower than the expected compensation under the partial-manipulation contract $\mathcal{C}^p = (\boldsymbol{w}_0^p, \boldsymbol{w}_1^p, g^p)$ defined in Proposition 2.

Proposition 4 shows that, for a low cost threshold $\hat{c}$, the firm prefers to offer a partial-manipulation contract, whereas for a high cost threshold, it prefers a no-manipulation contract. In other words, if effort is moderately productive and thus incentivized only if its cost is very low, the firm prefers to allow for some misreporting, but if effort is more productive and incentivized even at a higher cost, it is better to prevent all misreporting.

To understand this result, we need to analyze the net expected payoff of the manager. For a manager with $c > \hat{c}$, who does not exert effort, the probability of being paid is the same under both contracts, $\lambda(1-\delta)$, but the payment is larger under the no-manipulation contract because $w^n(r_h, v_h) > w^p(r_h, v_h)$. So a high-cost manager strictly prefers contract $\mathcal{C}^n$ to contract $\mathcal{C}^p$.

For a manager with $c < \hat{c}$, the expected payment is larger with partial manipulation, because

$$(\delta + (1-\delta)\phi)\left(\frac{\hat{c}}{\delta - \lambda(1-\delta) + (1-\delta)(1-\lambda)\phi}\right) > \delta\left(\frac{\hat{c}}{\delta - \lambda(1-\delta)}\right) \tag{26}$$

(the expressions on the left- and right-hand side are equal if $\phi = 0$, and the expression on the left-hand side is increasing in ϕ). However, this ignores the expected cost of manipulation, $(1-\delta)\, g^p$, and the cost of exerting effort, c. Under the partial-manipulation contract $\mathcal{C}^p$, a manager with $c < \hat{c}$ expects a net payoff of

$$-c - (1-\delta)g^p + \frac{(\delta + (1-\delta)\phi)\,\hat{c}}{\delta - \lambda(1-\delta) + (1-\delta)(1-\lambda)\phi} = -c + \frac{(\delta + (1-\delta)(1-\lambda)\phi)\,\hat{c}}{\delta - \lambda(1-\delta) + (1-\delta)(1-\lambda)\phi}, \tag{27}$$

which is smaller than the expected net payoff under the no-manipulation contract $\mathcal{C}^n$, given by $-c + \frac{\delta\hat{c}}{\delta - \lambda(1-\delta)}$ (the two payoffs are the same for $\phi = 0$, and the net payoff under contract $\mathcal{C}^p$ is decreasing in ϕ). So a manager with $c < \hat{c}$ also strictly prefers the no-manipulation contract $\mathcal{C}^n$.

By allowing for manipulation, the contract $\mathcal{C}^p$ reduces the manager's information rent, because the report becomes more informative about the manager's effort choice and, hence, whether the manager's cost of effort is above or below the threshold $\hat{c}$. For a high-cost manager, this reduction in the information rent is easily verified: the expected compensation is lower than under contract $\mathcal{C}^p$. For a low-cost manager, the expected compensation is higher under contract $\mathcal{C}^p$, but the manager may have to incur manipulation costs, which are not fully offset by the higher expected compensation. For sufficiently low $\hat{c}$, reductions in the information rent are key to the firm's preference for the partial-manipulation contract $\mathcal{C}^p$.

However, this holds only if $\hat{c}$ is sufficiently low. As $\hat{c}$ increases, the variables $w^n(r_h, v_h)$, $w^p(r_h, v_h)$ and g^p increase linearly in $\hat{c}$, so their proportions remain unchanged, and the focus must be on their respective probabilities. Importantly, the probability of manipulation is linear in $\hat{c}$, so the expected cost of manipulation is quadratic in $\hat{c}$. So for very small $\hat{c}$, the expected cost of manipulation is of a smaller order of magnitude than the expected payments to the manager, which are most likely unproductive payments to the low-effort manager (with $c > \hat{c}$). The payment to a high-effort manager (with $c < \hat{c}$) is higher under $\mathcal{C}^p$, but the firm is unlikely to actually face this type of manager with a very low $\hat{c}$. As $\hat{c}$ increases, the likelihood of beneficial savings (if $c > \hat{c}$) decreases, and the likelihood of larger payments to high-effort managers increases. In the limit as $\hat{c} = \bar{c}$, offering the contract $\mathcal{C}^p$ has no benefits, only drawbacks. Hence, the firm prefers the contract $\mathcal{C}^n$ for some $\hat{c} \in (0, \bar{c})$.

This intuition, that partial manipulation reduces information rents, is consistent with some comparative statics for the cut-off $\kappa\bar{c}$, which is decreasing in both δ and λ. If the pre-manipulation accounting information is more informative (higher δ), the partial-manipulation contract $\mathcal{C}^p$ becomes relatively less attractive, because the manager's informational advantage is smaller. Similarly, if λ is larger, the manager's informational advantage is less important, since a high-cost manager is more likely to generate a high firm value v_h even without exerting effort.

The intuition is also consistent with the firm's choice of optimal contract, which we analyze next. We find that when effort adds little value, the firm chooses a low threshold $\hat{c}$ and implements it using a partial-manipulation contract. However, when effort adds sufficient value, the firm implements a higher threshold $\hat{c}$ using a no-manipulation contract. We first determine the optimal value of the threshold $\hat{c}$ for each type of contract (Proposition 5), and we then provide conditions for the optimality of each type (Proposition 6).

Proposition 5. *Under the no-manipulation contract $\mathcal{C}^n = (\boldsymbol{w}_0^n, \boldsymbol{w}_1^n, g^n)$ defined in Proposition 1, firm value is maximized at a cost threshold of*

$$\hat{c}_n = \max\left\{\frac{1}{2}\left((1-\lambda)(v_h - v_\ell) - \frac{\lambda(1-\delta)\,\bar{c}}{\delta - \lambda(1-\delta)}\right), 0\right\}. \tag{28}$$

In contrast, under the partial-manipulation contract $\mathcal{C}^p = (\boldsymbol{w}_0^p, \boldsymbol{w}_1^p, g^p)$ defined in Proposition 2, firm value is maximized at a cost threshold of

$$\hat{c}_p = \max\left\{\frac{1}{2}\left(\frac{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}{\delta-(1-\delta)(\lambda-\phi)}\right)\left((1-\lambda)(v_h-v_\ell)-\frac{\lambda(1-\delta)\,\bar{c}}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}\right),0\right\}. \tag{29}$$

Under both types of contract, the firm may or may not provide incentives for the manager to exert effort. High effort is induced (i.e., $\hat{c} > 0$) only if the expected value-added of high effort, $(1-\lambda)(v_h - v_\ell)$, is sufficiently large. For small values of $(1-\lambda)(v_h - v_\ell)$, the firm optimally sets the threshold $\hat{c}$ to zero (which is implemented by setting all compensation payments to zero). An inspection of (28) and (29) reveals that $\hat{c}_n = 0$ if $\hat{c}_p = 0$, but not vice versa. This means that, under certain conditions, high effort can only be induced with a partial-manipulation contract. The expression for $\hat{c}_p$ in (29) immediately implies the following result.

Corollary 1. *The optimal contract implements a cost threshold $\hat{c} > 0$ (i.e., incentivizes the manager to exert high effort with a strictly positive probability) if and only if*

$$\frac{v_h - v_\ell}{\bar{c}} > \frac{\lambda(1-\delta)}{(1-\lambda)\left[\delta-(1-\delta)(\lambda-\phi+\lambda\phi)\right]}. \tag{30}$$

Having determined the optimal cost threshold $\hat{c}$ under the no-manipulation and partial-manipulation contract, we can now solve for the optimal contract by analyzing which of these two contracts generates a higher firm value when the cost threshold is chosen optimally (i.e., when $\hat{c}$ is set to $\hat{c}_n$ under the no-manipulation contract and to $\hat{c}_p$ under the partial-manipulation contract).

Proposition 6. *If the condition*

$$\frac{v_h - v_\ell}{\bar{c}} \leq \frac{1-\delta}{\delta-\lambda(1-\delta)}\left(\frac{1}{1-\lambda}+\sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}\right) \tag{31}$$

is satisfied, then the optimal contract is the partial-manipulation contract $\mathcal{C}^p = (\boldsymbol{w}_0^p, \boldsymbol{w}_1^p, g^p)$

defined in Proposition 2. If the above condition is not satisfied, then the optimal contract is the no-manipulation contract $\mathcal{C}^n = (\boldsymbol{w}_0^n, \boldsymbol{w}_1^n, g^n)$ *defined in Proposition 1.*

The optimal contract offered by the firm depends on the value-added by high managerial effort, $v_h - v_\ell$. There are three distinct regions. If $v_h - v_\ell$ is sufficiently small (relative to the maximum cost of effort, $\bar{c}$) so that condition (30) is not satisfied (and, hence, $\hat{c}_p = \hat{c}_n = 0$ according to Corollary 1), it is not optimal for the firm to induce high effort for any effort cost $c > 0$. In this case, the manager's manipulation strategy is irrelevant and both the no-manipulation and the partial-manipulation contract lead to the same firm value of $\lambda v_h + (1-\lambda)v_\ell$ (and zero compensation for the manager). For higher values of $v_h - v_\ell$ such that both conditions (30) and (31) are satisfied, it is optimal to induce high effort if $c < \hat{c}_p$ and to incentivize manipulation in case the manager exerted high effort. For even higher values of $v_h - v_\ell$ such that condition (30) is satisfied but condition (31) is violated, it is optimal to induce high effort if $c < \hat{c}_n$ and to prevent all manipulation.

The results in Proposition 6 are consistent with the intuition we provided after Proposition 4, comparing the costs of implementing a given threshold $\hat{c}$ using either the no-manipulation contract $\mathcal{C}^n$ or the partial-manipulation contract $\mathcal{C}^p$. As we discussed, the implementation cost is lower (higher) with a partial-manipulation contract if $\hat{c}$ is low (high). The reason for this result goes back to the trade-off between the costs of incentivizing manipulation (the manipulation cost g) and the benefits of being able to contract on a more precise signal r (thanks to selective manipulation). For low values of $\hat{c}$, the expected deadweight loss due to manipulation is small (because high effort provision and hence manipulation is unlikely), but it grows as $\hat{c}$ increases, and eventually it outweighs the informational benefits. Hence, for sufficiently high values of $v_h - v_\ell$ such that firms find it optimal to provide strong incentives to exert effort (i.e., to implement a high threshold $\hat{c}$), it is optimal to use the no-manipulation contract $\mathcal{C}^n$.

Having identified conditions for the optimality of the partial-manipulation and the no-manipulation contract, we can now compare the compensation payments that the manager earns under each contract, given that such a contract is offered in equilibrium (to implement an optimal cost threshold of either $\hat{c}_p$ or $\hat{c}_n$).

Proposition 7. *The compensation payment $w^n(r_h, v_h)$ under any no-manipulation contract offered in equilibrium exceeds the compensation payment $w^p(r_h, v_h)$ under any partial-manipulation contract offered in equilibrium.*

Proposition 7 shows that we can rank the promised payments under the two potentially optimal contracts: A partial-manipulation contract never offers more compensation (contingent on $r = r_h$ and $v = v_h$) than any no-manipulation contract. The compensation schedule has therefore a higher slope under a no-manipulation contract, which suggests that a no-manipulation contract (when it is optimal) generates stronger effort incentives because it is more high-powered.

4 Empirical Predictions

A key result from our analysis is that firms may find it optimal to incentivize managers to manipulate the signal r if they exerted a high effort level, but not if they exerted a low effort level. Our model therefore predicts that manipulation is positively related to effort provision. Furthermore, since managers are induced to exert high effort only if the cost c is sufficiently low, manipulation is negatively related to the cost of effort.

Prediction 1. *Earnings manipulation is expected to occur more frequently when managers exerted high effort and when managers have low costs of effort.*

Effort and manipulation are positively correlated in our model under certain conditions (i.e., when conditions (30) and (31) are satisfied). However, this does not imply that more high-powered incentive contracts go along with more manipulation. The incentive to manipulate also depends on the cost of manipulation g, which is set higher when condition (31) is violated (to prevent all manipulation) than when it is satisfied (to induce a high-effort manager to manipulate if $r = r_\ell$). The power of incentive contracts depends on the payments that a manager can potentially realize. Proposition 7 shows that for any no-manipulation and partial-manipulation contract that firms may offer their managers, we have $w^n(r_h, v_h) > w^p(r_h, v_h)$. The payments $w^n(r_h, v_h)$ and $w^p(r_h, v_h)$ measure the *maximum* compensation that can be achieved by a manager, or (equivalently) the difference between the highest and lowest possible compensation level. (Note that this is different from the manager's *expected* compensation.) These amounts can thus be used as measures of how high-powered the incentive compensation is under the two contracts. Recalling that manipulation is incentivized only under a partial-manipulation contract, we obtain the following prediction.

Prediction 2. *Earnings manipulation is expected to occur more frequently if incentive compensation is low-powered.*

We now analyze how some key parameters affect the possible optimality of manipulation by studying how these parameters affect the boundary between the no-manipulation region and the partial-manipulation region (Proposition 6). The key parameters of interest are λ, δ, and ϕ. Letting Γ denote the term on the right-hand side of the inequality in (31), our focus is on the signs of $\frac{d\Gamma}{d\lambda}$, $\frac{d\Gamma}{d\delta}$, and $\frac{d\Gamma}{d\phi}$.

In our model, managerial effort is more productive when λ is small because the expected increase in firm value due to high effort is $(1-\lambda)(v_h - v_\ell)$. Thus, exerting high effort adds less value when λ is large. An increase in λ increases the set of parameter values for which the partial-manipulation contract is optimal. In fact, it is straightforward to show that $\frac{d\Gamma}{d\lambda} > 0$, which leads to our next prediction.

Prediction 3. *Earnings manipulation is expected to occur more frequently in firms with less productive managerial effort.*

Our model predicts that if top executives are not the key value drivers in a firm or industry, then it is more likely that (selective) earnings manipulation is optimal. In contrast, when CEO talent and focus on creating value are essential to a firm's success, it is more likely that firms make manipulation prohibitively costly. This is consistent with Prediction 2: If an optimally designed compensation scheme offers higher-powered incentives because effort is crucial, then it is more likely that manipulation is sub-optimal. But in firms in which incentives are optimally low-powered, it may be optimal to allow for some manipulation.

A second parameter of interest is δ, which measures the quality of the firm's accounting system in the absence of manipulation: a higher δ makes the unmanipulated report more informative. From the inequality in (31), it immediately follows that $\frac{d\Gamma}{d\delta} < 0$. Thus, a decrease in δ increases the region of parameter values for which the partial-manipulation contract optimal, which leads to the following prediction.

Prediction 4. *Earnings manipulation is expected to occur more frequently in firms with less informative accounting systems (in the absence of manipulation).*

This prediction, which distinguishes our model from most of the existing literature (e.g., Strobl 2013), is driven by the fact that (selective) manipulation is more valuable when the accounting system is less informative because it can offset the noise inherent in the accounting system. A key difference between our model and other models of manipulation is that, in our model, only a manager who exerted *high* effort may have an incentive to manipulate financial reports. One should therefore not interpret evidence of manipulation as evidence of poor performance or of severe agency problems in the firm that are not addressed.

In our model, manipulation is more effective if ϕ is high: a higher ϕ means that a manipulation attempt by the manager is more likely to succeed and thus to produce a favorable report. It is therefore not surprising that an increase in ϕ increases the set of parameter values for which the partial-manipulation contract is optimal. In fact, from the inequality in (31), we have $\frac{d\Gamma}{d\phi} > 0$.

Prediction 5. *Earnings manipulation is expected to occur more frequently in firms in which it has a stronger effect on reported earnings.*

Our model predicts that firms are more likely to tolerate manipulation when manipulation techniques are more effective. It is important to keep in mind that only "good" managers who exerted high effort have an incentive to manipulate financial reports in our model, and that allowing this is optimal for the firm. In our model, the firm can choose to avoid any manipulation by the manager at no cost. However, empirically, the prediction is also consistent with the traditional view in the literature, that manipulation is largely unavoidable and can at best be managed; hence, if it is more effective, it is more likely to be used.

Predictions 3–5 are based on how Γ, the threshold of $\frac{v_h - v_\ell}{\bar{c}}$ that separates the partial-manipulation region from the no-manipulation region (i.e., the right-hand side of the inequality in (31)), responds to changes in one parameter of the model. However, such a change also affects the set of values of $\frac{v_h - v_\ell}{\bar{c}}$ for which the firm offers the manager an incentive contract: If $\frac{v_h - v_\ell}{\bar{c}}$ is sufficiently low so that the condition in (30) is not satisfied, the manager receives a flat (i.e., outcome-independent) compensation (of zero) and thus has no incentive to exert high effort. We want to emphasize though that taking this effect into account does not change our comparative statics results qualitatively. In fact, letting Λ denote the term on the right-hand side of the inequality in (30), it is straightforward to show that $0 < \frac{d\Lambda}{d\lambda} < \frac{d\Gamma}{d\lambda}$, $\frac{d\Gamma}{d\delta} < \frac{d\Lambda}{d\delta} < 0$, and $\frac{d\Lambda}{d\phi} < 0 < \frac{d\Gamma}{d\phi}$. An increase in λ causes both thresholds Γ and Λ to increase, but Γ increases by more than Λ so that the region of $\frac{v_h - v_\ell}{\bar{c}}$ for which the partial-manipulation contract is optimal becomes larger, consistent with Prediction 3. Similarly, an increase in δ reduces both thresholds, but the reduction is larger for Γ than for Λ so that the partial-manipulation region shrinks in size, consistent with Prediction 4. Finally, an increase in ϕ moves the two thresholds in opposite directions and pushes them further apart, thereby increasing the size of the partial-manipulation region, consistent with Prediction 5.

5 Conclusion

Practitioners have long argued that manipulation is helpful if it eliminates some of the noise that is inherent in financial reporting, in particular unfavorable reports that shed a wrong (negative) light on how a firm is performing. However, an obvious drawback of allowing manipulation is the possibility of opportunistic behavior by managers, who may manipulate financial reports to increase their compensation. The literature has analyzed several possible explanations for the presence of earnings manipulation (as discussed in the Introduction), but none of them directly analyze this trade-off: Whether firms may *choose* to allow some manipulation even when faced with possible opportunistic behavior by managers, because the informational benefits outweigh the costs.

We have analyzed a simple model that incorporates both of these features: Allowing manipulation can improve the information content of a noisy performance measure, but managers can use manipulation to improve their expected compensation. When using optimally designed incentive compensation contracts and accounting or governance regimes, firms benefit from allowing managers who exerted high effort (and who are expecting the firm to perform well) to "correct" a false unfavorable financial report. Allowing this type of manipulation makes the report more informative, since false unfavorable reports are corrected, while valid unfavorable reports are not changed; and this makes it more likely that a manager who exerted high effort will eventually earn a high payoff, so it increases the power of incentive contracts. But manipulation is costly, and this makes allowing manipulation unattractive in some cases.

Importantly, our model suggests that only managers who exerted high effort may be induced to manipulate; managers who exerted low effort should not be allowed to manipulate. This is not immediately intuitive, since standard models of earnings manipulation find that *less* productive managers manipulating performance measures (in order to increase their compensation), or that all types of manager should manipulate (in models with signal jamming).

However, this does not imply that the "best" managers should be allowed to manipulate. The model suggests that manipulation should only be induced when managerial effort adds moderate value to their firms — if choosing high effort adds significant value, then manipulation is not desirable and should be completely prevented.

Appendix: Proofs

Proof of Lemma 1. This result follows immediately from the fact that the manager's cost of effort is not contractible. Hence, if manipulating (respectively, not manipulating) an unfavorable report r_ℓ for a given effort choice e maximizes the manager's expected compensation payment when the effort cost is c, manipulating (respectively, not manipulating) the report must also maximize the manager's expected compensation payment when the effort cost is $c' \neq c$, as long as the compensation scheme is the same. ■

Proof of Lemma 2. We prove this result by contradiction. Suppose the result does not hold. Then, there must exist a cost $c_0 > 0$ that induces effort choice $e = 0$ and a cost $c_1 > c_0$ that induces effort choice $e = 1$. Thus, letting $U(e, m, c)$ denote the manager's expected utility if she chooses effort e and manipulation strategy m when facing a cost of effort c (that she reports truthfully), we must have

$$U(0, m_0, c_0) \geq U(1, m_1, c_0), \tag{32}$$

$$U(1, m_1, c_1) \geq U(0, m_0, c_1), \tag{33}$$

where m_e denotes the manager's optimal manipulation choice for a given effort choice e. Furthermore, let $\hat{U}(e, m, c, c')$ denote a type-c manager's expected utility from choosing e and m when she mimics the behavior of a type-c' manager (i.e., claims to be of type c' and chooses e and m accordingly). Since a type-c_0 manager prefers not to mimic the behavior of a type-c_1 manager, we have

$$U(0, m_0, c_0) \geq \hat{U}(1, m_1, c_0, c_1) > U(1, m_1, c_1), \tag{34}$$

where the last inequality follows from the fact that $c_1 > c_0$. Similarly, since a type-c_1 manager prefers not to mimic the behavior of a type-c_0 manager, we have

$$U(1, m_1, c_1) \geq \hat{U}(0, m_0, c_1, c_0) = U(0, m_0, c_0), \tag{35}$$

where the equality follows from the fact that the effort cost does not directly affect the manager's expected utility if she chooses low effort $e = 0$. Clearly, the two inequalities in (34) and (35) are inconsistent with each other, proving that such a case cannot exist. The result must therefore be true. ■

Proof of Lemma 3. From Lemma 2, it follows that all manager types $c \in [0, \hat{c})$ choose the same effort $e = 1$ and hence the same manipulation strategy m_1 (Lemma 1). Thus, these types face the same probability of generating outcome (r, v), for all $r \in \{r_h, r_\ell\}$ and

$v \in \{v_h, v_\ell\}$. This means that, under an incentive-compatible mechanism, these types must all receive the same expected compensation. Otherwise, they would all report to be of the type that generates the highest expected compensation. Without loss of generality, we can therefore set $w(r, v|c) = w_1(r, v)$, for all $c \in [0, \hat{c})$. An analogous argument holds for all manager types $c \in (\hat{c}, \bar{c}]$, so that, without loss of generality, we can set $w(r, v|c) = w_0(r, v)$, for all $c \in (\hat{c}, \bar{c}]$. ■

Proof of Proposition 1. We derive the optimal no-manipulation contract by first considering a simplified optimization problem and then showing that the solution to this simplified problem is also a solution to the full optimization problem in (11)–(16), (18), and (21). In particular, we first solve for the optimal compensation scheme $\boldsymbol{w}_1$ that implements an effort choice characterized by the threshold $\hat{c} \in (0, \bar{c}]$ for a given manipulation strategy $m_0 = m_1 = 0$ and (temporarily) ignore the contracting variables $\boldsymbol{w}_0$ and g, the effort-choice constraint in (14) (for the case when $c > \hat{c}$), and the truth-telling constraints in (15) and (16). Since $G(m_0, m_1) = 0$ when $m_0 = m_1 = 0$, the simplified problem is thus given by

$$\min_{\boldsymbol{w}_1} \sum_{r,v} \pi_{1,0}(r, v)\, w_1(r, v) - \left(1 - \frac{\hat{c}}{\bar{c}}\right) \hat{c} \tag{36}$$

$$\text{s.t.} \sum_{r,v} \Delta\pi_{0,0}(r, v)\, w_1(r, v) \geq \hat{c} \tag{37}$$

$$w_1(r, v) \geq 0, \quad \forall r \in \{r_h, r_\ell\}, v \in \{v_h, v_\ell\} \tag{38}$$

Denoting the Lagrangian multiplier of the constraint in (37) by ν and the respective multipliers of the limited liability constraints in (38) by $\xi_{r,v}$, we derive the first order condition of the above optimization problem with respect to $w_1(r, v)$ as

$$\pi_{1,0}(r, v) - \nu\, \Delta\pi_{0,0}(r, v) - \xi_{r,v} = 0, \tag{39}$$

with the complementary slackness condition $\xi_{r,v} w_1(r, v) = 0$. We first show that the IC constraint in (37) must be binding. For the constraint to be satisfied for any $\hat{c} > 0$, the payment $w_1(r_h, v_h)$ or $w_1(r_\ell, v_h)$ must be strictly positive because $\Delta\pi_{0,0}(r_h, v_\ell) < 0$ and $\Delta\pi_{0,0}(r_\ell, v_\ell) < 0$. (Note that $\Delta\pi_{0,0}(r_\ell, v_h)$ may be positive or negative, whereas $\Delta\pi_{0,0}(r_h, v_h)$ is always positive.) If the constraint in (37) were not binding for any $\hat{c} > 0$, the expected compensation in (36) could therefore be reduced by lowering one of these positive payments without violating any constraints. Optimality thus requires that the IC constraint in (37) be binding and that $\nu > 0$. Since $\pi_{1,0}(r, v) = 0$ and $\Delta\pi_{0,0}(r, v) < 0$ for the two outcomes (r_h, v_ℓ) and (r_ℓ, v_ℓ) and since $\nu > 0$, the first order condition in (39) implies that $\xi_{r_h, v_\ell} > 0$ and $\xi_{r_\ell, v_\ell} > 0$. Thus, complementary slackness requires that $w_1(r_h, v_\ell) = w_1(r_\ell, v_\ell) = 0$.

Furthermore, for the IC constraint in (37) to hold for $\hat{c} > 0$, at least one of the two remaining payments, $w_1(r_h, v_h)$ and $w_1(r_\ell, v_h)$, must be positive. However, they cannot both be positive: if $\xi_{r_h,v_h} = \xi_{r_\ell,v_h} = 0$, the first order condition in (39) would require that

$$\frac{\delta}{\delta - \lambda(1-\delta)} = \frac{\pi_{1,0}(r_h, v_h)}{\Delta\pi_{0,0}(r_h, v_h)} = \nu = \frac{\pi_{1,0}(r_\ell, v_h)}{\Delta\pi_{0,0}(r_\ell, v_h)} = \frac{1-\delta}{1-\delta-\lambda\delta}, \tag{40}$$

which cannot hold since $\delta > \frac{1}{2}$ and $\lambda > 0$. Consequently, the IC constraint in (37) implies that either

$$w_1(r_h, v_h) = \frac{\hat{c}}{\Delta\pi_{0,0}(r_h, v_h)} = \frac{\hat{c}}{\delta - \lambda(1-\delta)} \quad \text{and} \quad w_1(r_\ell, v_h) = 0 \tag{41}$$

or

$$w_1(r_h, v_h) = 0 \quad \text{and} \quad w_1(r_\ell, v_h) = \frac{\hat{c}}{\Delta\pi_{0,0}(r_\ell, v_h)} = \frac{\hat{c}}{1-\delta-\lambda\delta}. \tag{42}$$

The latter case is only feasible if $1-\delta-\lambda\delta > 0$, since the payment $w_1(r_\ell, v_h)$ would otherwise be negative and hence violate the limited liability constraint in (38). However, even if the payment scheme $w_1(r_h, v_h) = 0$ and $w_1(r_\ell, v_h) > 0$ is feasible, it is never optimal. To see this, consider an increase in $w_1(r_h, v_h)$ to $\varepsilon_1 > 0$ and a decrease in $w_1(r_\ell, v_h)$ by $\varepsilon_2 > 0$ such that the IC constraint in (37) remains binding, that is,

$$\varepsilon_2 = \frac{\Delta\pi_{0,0}(r_h, v_h)}{\Delta\pi_{0,0}(r_\ell, v_h)}\,\varepsilon_1 = \frac{\delta - \lambda(1-\delta)}{1-\delta-\lambda\delta}\,\varepsilon_1. \tag{43}$$

Such a change in payments would change the manager's expected compensation by

$$\pi_{1,0}(r_h, v_h)\,\varepsilon_1 - \pi_{1,0}(r_\ell, v_h)\,\varepsilon_2 = \delta\,\varepsilon_1 - (1-\delta)\,\frac{\delta-\lambda(1-\delta)}{1-\delta-\lambda\delta}\,\varepsilon_1 = -\frac{\lambda(2\delta-1)}{1-\delta-\lambda\delta}\,\varepsilon_1, \tag{44}$$

which is negative since $\delta > \frac{1}{2}$ and $1-\delta-\lambda\delta > 0$. A positive payment $w_1(r_\ell, v_h)$ can therefore not be optimal. The optimal compensation scheme is hence given by $w_1(r_h, v_h) = \frac{\hat{c}}{\delta-\lambda(1-\delta)}$ and $w_1(r_\ell, v_h) = w_1(r_h, v_\ell) = w_1(r_\ell, v_\ell) = 0$. This is intuitive: The expected compensation in (36) is minimized if the manager receives a positive payment only in the state of nature with the highest likelihood ratio $\frac{\pi_{1,0}(r,v)}{\pi_{0,0}(r,v)}$, which is state (r_h, v_h) in which both the earnings report and the terminal cash flow signal high managerial effort. Now consider the "no-manipulation" contract $\mathcal{C}^n = (\boldsymbol{w}_0^n, \boldsymbol{w}_1^n, g^n)$ with $w_1^n(r_h, v_h) = \frac{\hat{c}}{\delta-\lambda(1-\delta)}$ and $w_1^n(r_\ell, v_h) = w_1^n(r_h, v_\ell) = w_1^n(r_\ell, v_\ell) = 0$ as above, $w_0^n(r, v) = w_1^n(r, v)$ for all $r \in \{r_h, r_\ell\}$ and $v \in \{v_h, v_\ell\}$, and $g^n \geq \phi\, w_1^n(r_h, v_h)$. Since $\boldsymbol{w}_0$ and g are not part of the simplified problem, this contract is clearly a solution to the simplified problem in (36)–(38). Furthermore, since the objective functions in (21) and (36) are identical when $m_0 = m_1 = 0$ and since the constraints in (37) and (38) are implied by the constraints in (13) and (18), the contract $\mathcal{C}^n$

is also a solution to the full optimization problem characterized in Section 3.2 if it satisfies the additional constraints in (11)–(16) and (18). The contract $\mathcal{C}^n = (\boldsymbol{w}_0^n, \boldsymbol{w}_1^n, g^n)$ clearly satisfies the nonnegativity constraints in (18). Furthermore, any $g^n \geq \phi\, w_1^n(r_h, v_h)$ satisfies the manipulation incentive constraints in (11) and (12) when $m_0 = m_1 = 0$. Since $g^n \geq \phi\, w_1^n(r_h, v_h)$, the right-hand side of (13) is maximized by setting $m = 0$: the expected gain from manipulating, $\lambda\delta\phi\, w_1^n(r_h, v_h)$, is lower than the expected cost, δg^n. The constraint in (13) then becomes identical to the constraint in (37) and is binding. The right-hand side of (14) is also maximized by setting $m = 0$: the expected gain from manipulating, $(1-\delta)\phi\, w_0^n(r_h, v_h)$, cannot exceed the expected cost, $(1-\delta)g^n$, when $g^n \geq \phi\, w_1^n(r_h, v_h)$. Since $\boldsymbol{w}_0^n = \boldsymbol{w}_1^n$, this means that the expression on the right-hand side of (14) is identical to the expression on the left-hand side of (13) when $m_1 = 0$. Furthermore, the expression on the left-hand side of (14) is identical to the expression on the right-hand side of (13) when $m_0 = 0$ because the right-hand side of (13) is maximized by setting $m = 0$, as demonstrated above. Thus, the result that (13) is binding implies that (14) is also binding. The truth-telling constraint in (15) is implied by the constraint in (13) when $e = 0$ on the right-hand side of (15). To see this, note that, for $c = \hat{c}$, (13) is identical to (15) when $e = 0$ because $\boldsymbol{w}_0^n = \boldsymbol{w}_1^n$. Thus, (15) must be satisfied for all $c \leq \hat{c}$ when $e = 0$. When $e = 1$, the constraint in (15) is (weakly) more restrictive when $m = 0$ on the right-hand side: the expected gain from manipulating is $(1-\delta)\phi\, w_0^n(r_h, v_h)$ and hence cannot exceed the expected cost of $(1-\delta)g^n$ since $g^n \geq \phi\, w_1^n(r_h, v_h)$. This means that the constraint is trivially satisfied when $e = 1$ because, for $m = 0$ (and $m_1 = 0$), the expression on the left-hand side equals the expression on the right-hand side. Similarly, the truth-telling constraint in (16) is implied by the constraint in (14) when $e = 1$ on the right-hand side of (16). To see this, note that, for $c = \hat{c}$, (14) is identical to (16) when $e = 1$ because $\boldsymbol{w}_0^n = \boldsymbol{w}_1^n$. Thus, (16) must be satisfied for all $c \geq \hat{c}$ when $e = 1$. When $e = 0$, the constraint in (16) is (weakly) more restrictive when $m = 0$ on the right-hand side: the expected gain from manipulating is $\lambda\delta\phi\, w_1^n(r_h, v_h)$ and hence is lower than the expected cost of δg^n since $g^n \geq \phi\, w_1^n(r_h, v_h)$. This means that the constraint is trivially satisfied when $e = 0$ because, for $m = 0$ (and $m_0 = 0$), the expression on the left-hand side equals the expression on the right-hand side. ■

Proof of Proposition 2. The derivation of the optimal contract that induces manipulation by the manager when she exerted high effort but not when she exerted low effort (i.e., when $c < \hat{c}$) is similar to that of the optimal no-manipulation contract. We again first consider a simplified optimization problem that minimizes the cost of implementing an effort choice characterized by the threshold $\hat{c}$ for a given manipulation strategy $m_0 = 0$ and $m_1 = 1$ and then show that its solution is also a solution to the full optimization problem in (11)–(16), (18), and (21). The simplified problem consists of the objective function in (21) (ignoring the contracting variable $\boldsymbol{w}_0$), the effort-choice constraint in (13) for the case when $c < \hat{c}$ (both for $m = 0$ and $m = 1$ on the right-hand side), and the nonnegativity constraint for $\boldsymbol{w}_1$ in (18). Since $G(m_0, m_1) = (1-\delta)g \geq 0$ when $m_0 = 0$ and $m_1 = 1$, the simplified problem is thus given by

$$\min_{\boldsymbol{w_1},g} \sum_{r,v} \pi_{1,1}(r,v)\, w_1(r,v) - \left(1 - \frac{\hat{c}}{\bar{c}}\right)\left(\hat{c} + (1-\delta)g\right) \tag{45}$$

$$\text{s.t.} \sum_{r,v} \Delta\pi_{0,1}(r,v)\, w_1(r,v) \geq \hat{c} + (1-\delta)g \tag{46}$$

$$\sum_{r,v} \Delta\pi_{1,1}(r,v)\, w_1(r,v) \geq \hat{c} + (1-2\delta)g \tag{47}$$

$$w_1(r,v) \geq 0, \quad \forall r \in \{r_h, r_\ell\}, v \in \{v_h, v_\ell\} \tag{48}$$

Denoting the Lagrangian multiplier of the constraint in (46) by ν, the multiplier of the constraint in (47) by μ, and the respective multipliers of the limited liability constraints in (48) by $\xi_{r,v}$, we derive the first order condition of the above optimization problem with respect to $w_1(r,v)$ as

$$\pi_{1,1}(r,v) - \nu\, \Delta\pi_{0,1}(r,v) - \mu\, \Delta\pi_{1,1}(r,v) - \xi_{r,v} = 0, \tag{49}$$

with the complementary slackness condition $\xi_{r,v} w_1(r,v) = 0$, and the first order condition with respect to g as

$$-\left(1 - \frac{\hat{c}}{\bar{c}}\right)(1-\delta) + \nu(1-\delta) + \mu(1-2\delta) = 0. \tag{50}$$

We first show that it is optimal to set $w_1(r_h, v_\ell) = w_1(r_\ell, v_\ell) = 0$. Suppose this is not the case. If $w_1(r, v_\ell) > 0$, complementary slackness requires that $\xi_{r,v_\ell} = 0$. But since $\pi_{1,1}(r,v) = 0$, $\Delta\pi_{0,1}(r,v) < 0$, and $\Delta\pi_{1,1}(r,v) < 0$ for the two outcomes (r_h, v_ℓ) and (r_ℓ, v_ℓ), this implies that the first order condition in (49) can only be satisfied if $\nu = \mu = 0$ (the multipliers have to be nonnegative), which means that the IC constraints in (46) and (47) are not binding. This, in turn, implies that it is uniquely optimal to set $w_1(r_h, v_h) = w_1(r_\ell, v_h) = 0$ because $\pi_{1,1}(r_h, v_h) > 0$ and $\pi_{1,1}(r_h, v_h) > 0$. But this makes it impossible to elicit high effort for any nonzero $\hat{c}$: since $\Delta\pi_{0,1}(r_h, v_\ell) < 0$ and $\Delta\pi_{0,1}(r_\ell, v_\ell) < 0$, (46) is violated if $w_1(r_h, v_h) = w_1(r_\ell, v_h) = 0$. Thus, we must have that $w_1(r_h, v_\ell) = w_1(r_\ell, v_\ell) = 0$. We next argue that the IC constraints in (46) and (47) must both be binding. Suppose this is not the case. If the constraint in (46) is slack, we must have $\nu = 0$. The first order condition in (50) then implies that $\mu < 0$ (since $\delta > \frac{1}{2}$). But this violates the condition that the multiplier μ has to be nonnegative at the optimum. Thus, the constraint in (46) must be binding. Similarly, if the constraint in (47) is slack, we must have $\mu = 0$. Since a payment $w_1(r,v)$ can only be strictly positive if $\xi_{r,v} = 0$, the first order condition in (49) then implies that $\nu = \frac{\pi_{1,1}(r,v)}{\Delta\pi_{0,1}(r,v)} = \frac{\pi_{1,1}(r,v)}{\pi_{1,1}(r,v)-\pi_{0,0}(r,v)}$. However, this expression either exceeds one (if $\pi_{1,1}(r,v) > \pi_{0,0}(r,v) > 0$) or it is nonpositive (if $\pi_{1,1}(r,v) < \pi_{0,0}(r,v)$). In both cases, it violates the first order condition in (50) when $\mu = 0$, which requires that $\nu = 1 - \frac{\hat{c}}{\bar{c}} \in (0,1]$ for any nonzero $\hat{c}$. Thus, the constraint in (47) must be binding. Since both IC constraints in (46) and (47) must be binding at the optimum, we can combine them to obtain g (using the fact that $w_1(r_h, v_\ell) = w_1(r_\ell, v_\ell) = 0$):

$$g = \frac{1}{\delta}\sum_{r,v}\left(\Delta\pi_{0,1}(r,v) - \Delta\pi_{1,1}(r,v)\right)\, w_1(r,v) \tag{51}$$

$$= \frac{1}{\delta}\sum_{r,v}\left(\pi_{0,1}(r,v) - \pi_{0,0}(r,v)\right)\, w_1(r,v) \tag{52}$$

$$= \lambda\phi\left(w_1(r_h, v_h) - w_1(r_\ell, v_h)\right). \tag{53}$$

Note that, with this choice of g, the two IC constraints in (46) and (47) become identical. We can therefore drop one of the constraints. Substituting g into the objective function in (45) and the constraint in (46), we can rewrite the optimization problem as

$$\min_{\boldsymbol{w}_1} \sum_{r,v}\pi_{1,1}(r,v)\, w_1(r,v) - \left(1 - \frac{\hat{c}}{\bar{c}}\right)\left[\hat{c} + (1-\delta)\lambda\phi\left(w_1(r_h,v_h) - w_1(r_\ell,v_h)\right)\right] \tag{54}$$

$$\text{s.t. } \sum_{r,v}\Delta\pi_{0,1}(r,v)\, w_1(r,v) = \hat{c} + (1-\delta)\lambda\phi\left(w_1(r_h,v_h) - w_1(r_\ell,v_h)\right) \tag{55}$$

$$w_1(r_h,v_h) \geq 0, w_1(r_\ell,v_h) \geq 0, w_1(r_h,v_\ell) = 0, w_1(r_\ell,v_\ell) = 0 \tag{56}$$

As before, denote the Lagrangian multiplier of the constraint in (55) by ν and the multipliers of the limited liability constraints by ξ_{r_h,v_h} and ξ_{r_ℓ,v_h}. The first order conditions with respect to $w_1(r_h,v_h)$ and $w_1(r_\ell,v_h)$ are then

$$\delta + (1-\delta)\phi - \left(1 - \frac{\hat{c}}{\bar{c}}\right)(1-\delta)\lambda\phi - \nu\left[\delta - (1-\delta)(\lambda - \phi + \lambda\phi)\right] - \xi_{r_h,v_h} = 0, \tag{57}$$

$$(1-\delta)(1-\phi) + \left(1 - \frac{\hat{c}}{\bar{c}}\right)(1-\delta)\lambda\phi - \nu\left[(1-\delta)(1-\phi+\lambda\phi) - \lambda\delta\right] - \xi_{r_\ell,v_h} = 0, \tag{58}$$

where we have substituted in the expressions for $\pi_{0,0}(r,v)$ and $\pi_{1,1}(r,v)$ from (3)–(6). For the IC constraint in (55) to hold for $\hat{c} > 0$, at least one of the payments $w_1(r_h,v_h)$ and $w_1(r_\ell,v_h)$ must be positive. However, they cannot both be positive. If they were, complementary slackness would require that $\xi_{r_h,v_h} = \xi_{r_\ell,v_h} = 0$. But then the first order conditions in (57) and (58) would imply that

$$\frac{\delta + (1-\delta)\phi - \left(1-\frac{\hat{c}}{\bar{c}}\right)(1-\delta)\lambda\phi}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)} = \frac{(1-\delta)(1-\phi) + \left(1-\frac{\hat{c}}{\bar{c}}\right)(1-\delta)\lambda\phi}{(1-\delta)(1-\phi+\lambda\phi) - \lambda\delta}, \tag{59}$$

or, equivalently, that

$$\frac{\hat{c}}{\bar{c}} = 1 + \frac{2\delta - 1}{(1-\delta)(1-\lambda)\phi}, \tag{60}$$

which cannot be the case because $\delta > \frac{1}{2}$ and $\hat{c} \leq \bar{c}$. Consequently, the IC constraint in (55) implies that either

$$w_1(r_h, v_h) = \frac{\hat{c}}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)} \quad \text{and} \quad w_1(r_\ell, v_h) = 0 \tag{61}$$

or

$$w_1(r_h, v_h) = 0 \quad \text{and} \quad w_1(r_\ell, v_h) = \frac{\hat{c}}{(1-\delta)(1-\phi+\lambda\phi) - \lambda\delta}. \tag{62}$$

In the former case, the payment $w_1(r_h, v_h)$ is positive because $\delta > \frac{1}{2}$. In the latter case, the payment $w_1(r_\ell, v_h)$ is positive only if $(1-\delta)(1-\phi+\lambda\phi) - \lambda\delta > 0$. Thus, the latter payment scheme may not be feasible because it may violate the limited liability constraint in (56). However, even if it is feasible, this payment scheme is never optimal. To see this, consider an increase in $w_1(r_h, v_h)$ to $\varepsilon_1 > 0$ and a decrease in $w_1(r_\ell, v_h)$ by $\varepsilon_2 > 0$ such that the IC constraint in (55) remains binding, that is,

$$\varepsilon_2 = \frac{\Delta\pi_{0,1}(r_h, v_h) - (1-\delta)\lambda\phi}{\Delta\pi_{0,1}(r_\ell, v_h) + (1-\delta)\lambda\phi}\,\varepsilon_1 = \frac{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)}{(1-\delta)(1-\phi+\lambda\phi) - \lambda\delta}\,\varepsilon_1. \tag{63}$$

Such a change in payments would change the manager's expected compensation by

$$\left[\pi_{1,1}(r_h, v_h) - \left(1-\frac{\hat{c}}{\bar{c}}\right)(1-\delta)\lambda\phi\right]\varepsilon_1 - \left[\pi_{1,1}(r_\ell, v_h) + \left(1-\frac{\hat{c}}{\bar{c}}\right)(1-\delta)\lambda\phi\right]\varepsilon_2 \tag{64}$$

$$= \left[\delta + (1-\delta)\left(\phi - \left(1-\frac{\hat{c}}{\bar{c}}\right)\lambda\phi\right)\right]\varepsilon_1 - \left[(1-\delta)\left(1-\phi+\left(1-\frac{\hat{c}}{\bar{c}}\right)\lambda\phi\right)\right]\varepsilon_2 \tag{65}$$

$$= -\frac{\lambda\left[2\delta - 1 + \left(1-\frac{\hat{c}}{\bar{c}}\right)(1-\delta)(1-\lambda)\phi\right]}{(1-\delta)(1-\phi+\lambda\phi) - \lambda\delta}\,\varepsilon_1, \tag{66}$$

which is negative since $\delta > \frac{1}{2}$ and $(1-\delta)(1-\phi+\lambda\phi) - \lambda\delta > 0$. A positive payment $w_1(r_\ell, v_h)$ can therefore not be optimal. The optimal solution to the problem in (45)–(48) is thus given by the compensation scheme $w_1(r_h, v_h) = \frac{\hat{c}}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}$, $w_1(r_\ell, v_h) = w_1(r_h, v_\ell) = w_1(r_\ell, v_\ell) = 0$, and the manipulation cost $g = \lambda\phi\, w_1(r_h, v_h)$. Now consider the "partial-manipulation" contract $\mathcal{C}^p = (\boldsymbol{w}_0^p, \boldsymbol{w}_1^p, g^p)$ with $w_1^p(r_h, v_h) = \frac{\hat{c}}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}$, $w_1^p(r_\ell, v_h) =$

$w_1^p(r_h, v_\ell) = w_1^p(r_\ell, v_\ell) = 0$, $g^p = \lambda\phi\, w_1^p(r_h, v_h)$ as above, and $w_0^p(r, v) = w_1^p(r, v)$ for all $r \in \{r_h, r_\ell\}$ and $v \in \{v_h, v_\ell\}$. Since $\boldsymbol{w}_0$ is not part of the simplified problem, this contract is clearly a solution to the simplified problem in (45)–(48). Furthermore, since the objective functions in (21) and (45) are identical when $m_0 = 0$ and $m_1 = 1$ and since the constraints in (46), (47), and (48) are implied by the constraints in (13) and (18), the contract $\mathcal{C}^p$ is also a solution to the full optimization problem characterized in Section 3.2 if it satisfies the additional constraints in (11)–(16) and (18). The contract $\mathcal{C}^p = (\boldsymbol{w}_0^p, \boldsymbol{w}_1^p, g^p)$ clearly satisfies the nonnegativity constraints in (18). Furthermore, $g^p = \lambda\phi\, w_1^p(r_h, v_h)$ satisfies the manipulation incentive constraints in (11) and (12) when $m_0 = 0$ and $m_1 = 1$ ((11) is slack and (12) is binding). Since $g^p = \lambda\phi\, w_1^p(r_h, v_h)$, the right-hand side of (13) is the same for $m = 0$ and $m = 1$: the expected gain from manipulating, $\lambda\delta\phi\, w_1^p(r_h, v_h)$, is equal to the expected cost, δg^p. The constraint in (13) then becomes identical to the constraint in (46) and is binding. The right-hand side of (14) is maximized by setting $m = 1$: the expected gain from manipulating, $(1-\delta)\phi\, w_0^p(r_h, v_h)$, exceeds the expected cost, $(1-\delta)g^p = (1-\delta)\lambda\phi\, w_1^p(r_h, v_h)$. Since $\boldsymbol{w}_0^p = \boldsymbol{w}_1^p$, this means that the expression on the right-hand side of (14) is identical to the expression on the left-hand side of (13) when $m_1 = 1$. Furthermore, the expression on the left-hand side of (14) is identical to the expression on the right-hand side of (13) when $m_0 = 0$ because the right-hand side of (13) is maximized by setting $m = 0$, as demonstrated above. Thus, the result that (13) is binding implies that (14) is also binding. The truth-telling constraint in (15) is implied by the constraint in (13) when $e = 0$ on the right-hand side of (15). To see this, note that, for $c = \hat{c}$, (13) is identical to (15) when $e = 0$ because $\boldsymbol{w}_0^p = \boldsymbol{w}_1^p$. Thus, (15) must be satisfied for all $c \leq \hat{c}$ when $e = 0$. When $e = 1$, the constraint in (15) is more restrictive when $m = 1$ on the right-hand side: the expected gain from manipulating is $(1-\delta)\phi\, w_0^p(r_h, v_h)$ and hence exceeds the expected cost of $(1-\delta)g^p$ since $g^p = \lambda\phi\, w_1^p(r_h, v_h)$. This means that the constraint is trivially satisfied when $e = 1$ because, for $m = 1$ (and $m_1 = 1$), the expression on the left-hand side equals the expression on the right-hand side. Similarly, the truth-telling constraint in (16) is implied by the constraint in (14) when $e = 1$ on the right-hand side of (16). To see this, note that, for $c = \hat{c}$, (14) is identical to (16) when $e = 1$ because $\boldsymbol{w}_0^p = \boldsymbol{w}_1^p$. Thus, (16) must be satisfied for all $c \geq \hat{c}$ when $e = 1$. When $e = 0$, the constraint in (16) is (weakly) more restrictive when $m = 0$ on the right-hand side: the expected gain from manipulating is $\lambda\delta\phi\, w_1^p(r_h, v_h)$ and hence equals the expected cost of δg^p since $g^p = \lambda\phi\, w_1^p(r_h, v_h)$. This means that the constraint is trivially satisfied when $e = 0$ because, for $m = 0$ (and $m_0 = 0$), the expression on the left-hand side equals the expression on the right-hand side. ■

Proof of Proposition 3. We prove this result by showing that any contract that induces the manager to choose the manipulation strategy $(m_0, m_1) = (1, 0)$ is dominated by the no-manipulation contract $\mathcal{C}^n$ derived in Proposition 1, and any contract that induces the manager to choose the manipulation strategy $(m_0, m_1) = (1, 1)$ is dominated by the partial-manipulation contract $\mathcal{C}^p$ derived in Proposition 2. If $m_0 = 1$, the manager's expected compensation in (21) cannot be lower than

$$\sum_{r,v} \pi_{1,m_1}(r, v)\, w_1(r, v) - \left(1 - \frac{\hat{c}}{\bar{c}}\right) \left(\hat{c} + (1 - \delta)gm_1\right), \tag{67}$$

the expected compensation if $m_0 = 0$, because $G(1, m_1) = [(1 - \delta)m_1 - \delta]\, g \leq (1 - \delta)gm_1 = G(0, m_1)$, with a strict inequality if $g > 0$. Consider the case where $m_0 = 1$ and $m_1 = 0$. The IC constraint in (13) then requires that

$$\sum_{r,v} \Delta\pi_{0,0}(r, v)\, w_1(r, v) \geq \hat{c}. \tag{68}$$

This constraint is identical to the IC constraint in (37) of the simplified problem analyzed in the proof of Proposition 1. Furthermore, the objective function of that problem in (36) is identical to (67) if $m_1 = 0$. The optimal no-manipulation contract $\mathcal{C}^n$ thus minimizes the expected compensation in (67) (with $m_1 = 0$) subject to the IC constraint in (68) and the limited liability constraints $w_1(r, v) \geq 0$. But these constraints also have to be satisfied by any contract that implements the manipulation strategy $(m_0, m_1) = (1, 0)$. Furthermore, the additional constraints in (11)–(16) cannot reduce the manager's expected compensation. Hence, there cannot exist a contract that implements the threshold $\hat{c}$ and the manipulation strategy $(m_0, m_1) = (1, 0)$ at a lower cost than the contract $\mathcal{C}^n$. Next, consider the case where $m_0 = m_1 = 1$. The IC constraint in (13) then requires that

$$\sum_{r,v} \Delta\pi_{0,1}(r, v)\, w_1(r, v) \geq \hat{c} + (1 - \delta)g, \tag{69}$$

and that

$$\sum_{r,v} \Delta\pi_{1,1}(r, v)\, w_1(r, v) \geq \hat{c} + (1 - 2\delta)g. \tag{70}$$

These constraints are identical to the IC constraints in (46) and (47) of the simplified problem analyzed in the proof of Proposition 2. Furthermore, the objective function of that problem in (45) is identical to (67) if $m_1 = 1$. The optimal partial-manipulation contract $\mathcal{C}^p$ thus minimizes the expected compensation in (67) (with $m_1 = 1$) subject to the IC constraints in (69) and (70) and the limited liability constraints $w_1(r, v) \geq 0$. But these constraints also have to be satisfied by any contract that implements the manipulation strategy $(m_0, m_1) = (1, 1)$. Furthermore, the additional constraints in (11)–(16) cannot reduce the manager's expected compensation. Hence, there cannot exist a contract that implements the threshold $\hat{c}$ and the manipulation strategy $(m_0, m_1) = (1, 1)$ at a lower cost than the contract $\mathcal{C}^p$. ■

Proof of Proposition 4. From the objective function in (21) and the compensation scheme in Proposition 1, it follows that, for any cost threshold $\hat{c} \in [0, \bar{c}]$, the expected compensation required to induce the manager to exert high effort if and only if $c \leq \hat{c}$ and to follow a no-manipulation strategy (i.e., to choose the strategy $m_0 = m_1 = 0$) is given by

$$\mathbb{E}w^n(\hat{c}) = \pi_{1,0}(r_h, v_h)\, w_1^n(r_h, v_h) - \left(1 - \frac{\hat{c}}{\bar{c}}\right)\hat{c} = \left(\frac{\lambda(1-\delta)}{\delta - \lambda(1-\delta)} + \frac{\hat{c}}{\bar{c}}\right)\hat{c}. \tag{71}$$

Similarly, from (21) and Proposition 2, it follows that the expected compensation necessary to induce the manager to exert high effort if and only if $c \leq \hat{c}$ and to follow the partial-manipulation strategy $m_0 = 0$ and $m_1 = 1$ is given by

$$\mathbb{E}w^p(\hat{c}) = \pi_{1,1}(r_h, v_h)\, w_1^p(r_h, v_h) - \left(1 - \frac{\hat{c}}{\bar{c}}\right)\big(\hat{c} + (1-\delta)g^p\big) \tag{72}$$

$$= \left(\frac{\delta + (1-\delta)\phi - \left(1 - \frac{\hat{c}}{\bar{c}}\right)(1-\delta)\lambda\phi}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)} - 1 + \frac{\hat{c}}{\bar{c}}\right)\hat{c} \tag{73}$$

$$= \left(\frac{\left[1 + \left(\frac{\hat{c}}{\bar{c}}\right)\phi\right](1-\delta)\lambda}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)} + \frac{\hat{c}}{\bar{c}}\right)\hat{c}. \tag{74}$$

For any cost threshold $\hat{c} > 0$, the expressions in (71) and (74) imply that $\mathbb{E}w^p(\hat{c}) \underset{(>)}{<} \mathbb{E}w^n(\hat{c})$ if and only if

$$\frac{\left[1 + \left(\frac{\hat{c}}{\bar{c}}\right)\phi\right](1-\delta)\lambda}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)} \underset{(>)}{<} \frac{\lambda(1-\delta)}{\delta - \lambda(1-\delta)}, \tag{75}$$

or, equivalently, if and only if

$$\frac{\hat{c}}{\bar{c}} \underset{(>)}{<} \frac{(1-\delta)(1-\lambda)}{\delta - \lambda(1-\delta)}. \tag{76}$$

■

Proof of Proposition 5. For a given cost threshold $\hat{c} \in [0, \bar{c}]$, the value of the firm (net of the cost of managerial compensation) under the optimal no-manipulation contract $\mathcal{C}^n = (\boldsymbol{w}_0^n, \boldsymbol{w}_1^n, g^n)$ specified in Proposition 1 is given by

$$V_n(\hat{c}) = \left(\frac{\hat{c}}{\bar{c}}\right) v_h + \left(1 - \frac{\hat{c}}{\bar{c}}\right)\big(\lambda\, v_h + (1-\lambda)\, v_\ell\big) - \mathbb{E}w^n(\hat{c}), \tag{77}$$

where the expected compensation $\mathbb{E}w^n(\hat{c})$ is given by (71) in the proof of Proposition 4. Substituting the expression in (71) into the above equation yields

$$V_n(\hat{c}) = V_0 + (1-\lambda)(v_h - v_\ell)\left(\frac{\hat{c}}{\bar{c}}\right) - \left(\frac{\lambda(1-\delta)}{\delta - \lambda(1-\delta)} + \frac{\hat{c}}{\bar{c}}\right)\hat{c}, \tag{78}$$

where $V_0 = \lambda v_h + (1-\lambda)v_\ell$. Note that V_n is a strictly concave function of $\hat{c}$ with $V_n'(\bar{c}) < (1-\lambda)(v_h - v_\ell)/\bar{c} - 2 < 0$ because, under Assumption 1, $(1-\lambda)(v_h - v_\ell) < \bar{c}$. Thus, if $V_n'(0) \geq 0$, the optimal cost threshold that maximizes V_n is uniquely determined by the first order condition

$$\hat{c}_n = \frac{1}{2}\left((1-\lambda)(v_h - v_\ell) - \frac{\lambda(1-\delta)\,\bar{c}}{\delta - \lambda(1-\delta)}\right). \tag{79}$$

If $V_n'(0) < 0$, the above expression is negative and the optimal cost threshold is zero. Similarly, the value of the firm under the optimal partial-manipulation contract $\mathcal{C}^p = (\boldsymbol{w}_0^p, \boldsymbol{w}_1^p, g^p)$ specified in Proposition 2 is given by

$$V_p(\hat{c}) = \left(\frac{\hat{c}}{\bar{c}}\right) v_h + \left(1 - \frac{\hat{c}}{\bar{c}}\right)\left(\lambda\, v_h + (1-\lambda)\, v_\ell\right) - \mathbb{E}w^p(\hat{c}), \tag{80}$$

where the expected compensation $\mathbb{E}w^p(\hat{c})$ is given by (74). Substituting the expression in (74) into the above equation yields

$$V_p(\hat{c}) = V_0 + (1-\lambda)(v_h - v_\ell)\left(\frac{\hat{c}}{\bar{c}}\right) - \left(\frac{\left[1 + \left(\frac{\hat{c}}{\bar{c}}\right)\phi\right]\lambda(1-\delta)}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)} + \frac{\hat{c}}{\bar{c}}\right)\hat{c}, \tag{81}$$

where, as before, $V_0 = \lambda v_h + (1-\lambda)v_\ell$. Similarly to V_n, V_p is a strictly concave function of $\hat{c}$ with $V_p'(\bar{c}) < (1-\lambda)(v_h - v_\ell)/\bar{c} - 2 < 0$. Thus, if $V_p'(0) \geq 0$, the optimal cost threshold that maximizes V_p is uniquely determined by the first order condition

$$\hat{c}_p = \frac{1}{2}\left(\frac{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)}{\delta - (1-\delta)(\lambda - \phi)}\right)\left((1-\lambda)(v_h - v_\ell) - \frac{\lambda(1-\delta)\,\bar{c}}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)}\right). \tag{82}$$

If $V_p'(0) < 0$, the above expression is negative and the optimal cost threshold is zero. ■

Proof of Corollary 1. From Proposition 3, we know that, for any cost threshold $\hat{c}$, the optimal contract is either the no-manipulation contract $\mathcal{C}^n = (\boldsymbol{w}_0^n, \boldsymbol{w}_1^n, g^n)$ defined in Proposition 1 or the partial-manipulation contract $\mathcal{C}^p = (\boldsymbol{w}_0^p, \boldsymbol{w}_1^p, g^p)$ defined in Proposition 2. Proposition 5 shows that the optimal cost threshold under the no-manipulation contract, $\hat{c}_n$, is zero whenever the optimal cost threshold under the partial-manipulation contract, $\hat{c}_p$, is zero. Thus, a necessary and sufficient condition for the optimal contract to induce high effort is that $\hat{c}_p > 0$, which is equivalent to the condition in (30). ■

Proof of Proposition 6. From Proposition 3, we know that, for any cost threshold $\hat{c}$, the optimal contract is either the no-manipulation contract $\mathcal{C}^n = (\boldsymbol{w}_0^n, \boldsymbol{w}_1^n, g^n)$ defined in Proposition 1 or the partial-manipulation contract $\mathcal{C}^p = (\boldsymbol{w}_0^p, \boldsymbol{w}_1^p, g^p)$ defined in Proposition 2. Furthermore, Proposition 5 shows that firm value under the no-manipulation contract (respectively, the partial-manipulation contract) is maximized at a cost threshold of $\hat{c}_n$ (respectively, $\hat{c}_p$). Thus, to prove the result it suffices to show that $V_p(\hat{c}_p) \geq V_n(\hat{c}_n)$ if and only if (31) is satisfied, where, as in the proof of Proposition 5, $V_n(\hat{c})$ denotes firm value under the no-manipulation contract and $V_p(\hat{c})$ firm value under the partial-manipulation contract. The result that $V_p(\hat{c}_p) \geq V_n(\hat{c}_n)$ trivially holds if $\hat{c}_n = 0$ because $\max\{V_p(\hat{c}_p), V_p(0)\} \geq V_p(0) = V_n(0)$. Furthermore, since the right-hand side of (31) exceeds the right-hand side of (30), it follows that $\hat{c}_p > 0$ if (31) is not satisfied. But if $\hat{c}_p > 0$, the fact that $V_p(\hat{c}_p) < V_n(\hat{c}_n)$ implies that $\hat{c}_n > 0$ as well. Thus, we are left to show that $V_p(\hat{c}_p) \geq V_n(\hat{c}_n)$ if and only if (31) is satisfied in case $\hat{c}_p > 0$ and $\hat{c}_n > 0$. If $\hat{c}_n > 0$, it follows from (28) and (78) that

$$V_n(\hat{c}_n) = V_0 + \frac{1}{\bar{c}}\left[\left((1-\lambda)(v_h - v_\ell) - \frac{\lambda(1-\delta)\,\bar{c}}{\delta - \lambda(1-\delta)}\right)\hat{c}_n - \hat{c}_n^2\right] = V_0 + \frac{\hat{c}_n^2}{\bar{c}}. \tag{83}$$

Similarly, if $\hat{c}_p > 0$, from (29) and (81) we have

$$V_p(\hat{c}_p) = V_0 + \frac{1}{\bar{c}}\left[\left((1-\lambda)(v_h - v_\ell) - \frac{\lambda(1-\delta)\,\bar{c}}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)}\right)\hat{c}_p \right. \tag{84}$$

$$\left. - \left(\frac{\delta - (1-\delta)(\lambda - \phi)}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)}\right)\hat{c}_p^2\right] \tag{85}$$

$$= V_0 + \left(\frac{\delta - (1-\delta)(\lambda - \phi)}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)}\right)\frac{\hat{c}_p^2}{\bar{c}}. \tag{86}$$

Thus, $V_p(\hat{c}_p) \geq V_n(\hat{c}_n)$ if and only if

$$\frac{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)}{\delta - (1-\delta)(\lambda - \phi)}\left((1-\lambda)(v_h - v_\ell) - \frac{\lambda(1-\delta)\,\bar{c}}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)}\right)^2 \geq \left((1-\lambda)(v_h - v_\ell) - \frac{\lambda(1-\delta)\,\bar{c}}{\delta - \lambda(1-\delta)}\right)^2. \tag{87}$$

Since $\hat{c}_n$ and $\hat{c}_p$ are positive, we can rewrite this condition as

$$\left(1 - \sqrt{\frac{\delta - (1-\delta)(\lambda - \phi)}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)}}\right)\frac{(1-\lambda)(v_h - v_\ell)}{\bar{c}} \geq \frac{\lambda(1-\delta)}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)} - \frac{\lambda(1-\delta)}{\delta - \lambda(1-\delta)}\sqrt{\frac{\delta - (1-\delta)(\lambda - \phi)}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)}}, \tag{88}$$

or, since the term under the square root sign is greater than one, as

$$\frac{(1-\lambda)(v_h - v_\ell)}{\bar{c}} \leq \frac{\frac{\lambda(1-\delta)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)} - \frac{\lambda(1-\delta)}{\delta-\lambda(1-\delta)}\sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}}{1-\sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}}. \tag{89}$$

The term on the right-hand side of (89) can be rearranged as follows:

$$\frac{\frac{\lambda(1-\delta)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)} - \frac{\lambda(1-\delta)}{\delta-\lambda(1-\delta)}\sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}}{1-\sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}} \tag{90}$$

$$= \frac{1-\delta}{\delta-\lambda(1-\delta)} + \frac{\frac{\lambda(1-\delta)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)} - \frac{1-\delta}{\delta-\lambda(1-\delta)} + \left(\frac{1-\delta}{\delta-\lambda(1-\delta)} - \frac{\lambda(1-\delta)}{\delta-\lambda(1-\delta)}\right)\sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}}{1-\sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}} \tag{91}$$

$$= \frac{1-\delta}{\delta-\lambda(1-\delta)} + \frac{\frac{\lambda(1-\delta)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)} - \frac{1-\delta}{\delta-\lambda(1-\delta)} + \frac{(1-\delta)(1-\lambda)}{\delta-\lambda(1-\delta)}\sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}}{1-\sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}} \tag{92}$$

$$= \frac{1-\delta}{\delta-\lambda(1-\delta)} + \frac{(1-\delta)(1-\lambda)}{\delta-\lambda(1-\delta)}\left(\frac{\frac{\frac{\lambda(1-\delta)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)} - \frac{1-\delta}{\delta-\lambda(1-\delta)}}{\frac{(1-\delta)(1-\lambda)}{\delta-\lambda(1-\delta)}} + \sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}}{1-\sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}}\right) \tag{93}$$

$$= \frac{1-\delta}{\delta-\lambda(1-\delta)} + \frac{(1-\delta)(1-\lambda)}{\delta-\lambda(1-\delta)}\left(\frac{\frac{\lambda}{1-\lambda}\left(\frac{\delta-\lambda(1-\delta)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)} - \frac{1}{\lambda}\right) + \sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}}{1-\sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}}\right) \tag{94}$$

$$= \frac{1-\delta}{\delta-\lambda(1-\delta)} + \frac{(1-\delta)(1-\lambda)}{\delta-\lambda(1-\delta)}\left(\frac{-\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)} + \sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}}{1-\sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}}\right) \tag{95}$$

$$= \frac{1-\delta}{\delta-\lambda(1-\delta)} + \frac{(1-\delta)(1-\lambda)}{\delta-\lambda(1-\delta)}\sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}. \tag{96}$$

Thus, $V_p(\hat{c}_p) \geq V_n(\hat{c}_n)$ if and only if

$$\frac{v_h - v_\ell}{\bar{c}} \leq \frac{1-\delta}{\delta-\lambda(1-\delta)}\left(\frac{1}{1-\lambda} + \sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}\right). \tag{97}$$

■

Proof of Proposition 7. For a given threshold $\hat{c}$, it follows from (22) and (24) that

$$w^n(r_h, v_h) = \frac{\hat{c}}{\delta - \lambda(1-\delta)} > \frac{\hat{c}}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)} = w^p(r_h, v_h). \quad (98)$$

For the partial-manipulation contract $\mathcal{C}^p$ to be optimal, the condition in (31) has to be satisfied (Proposition 6). The optimal threshold under such a contract is $\hat{c}_p$, defined in (29). Since $\hat{c}_p$ is increasing in $v_h - v_\ell$, the *highest* compensation payment $w^p(r_h, v_h)$ that may be observed in equilibrium under a partial-manipulation contract is the payment being offered when (31) is binding.

If the condition in (31) is not satisfied, the no-manipulation contract $\mathcal{C}^n$ is optimal. In this case, the optimal threshold is $\hat{c}_n$, defined in (28), which is also increasing in $v_h - v_\ell$. Thus, the *lowest* compensation payment $w^n(r_h, v_h)$ that may be observed in equilibrium under a no-manipulation contract is the payment being offered when (31) is binding.

The result thus holds if we can show that $\hat{c}_p < \hat{c}_n$ when the condition in (31) is binding.

The functions $V_p(\hat{c})$ and $V_n(\hat{c})$ intersect in $\hat{c} = 0$ (both contracts then implement neither effort nor manipulation, so they generate the same outcome). Both $V_p(\hat{c})$ and $V_n(\hat{c})$ are quadratic, concave functions of the cost threshold $\hat{c}$ (see the proof of Proposition 5), so they intersect at some $\hat{c} \neq 0$. When (31) is binding, such that $V_p(\hat{c}_p) = V_n(\hat{c}_n)$, this second intersection must be at some $\hat{c}$ such that $\min\{\hat{c}_p, \hat{c}_n\} \leq \hat{c} \leq \max\{\hat{c}_p, \hat{c}_n\}$. From Proposition 4, this intersection is at $\hat{c} = \kappa\bar{c}$ (with $\kappa \in (0,1)$).

Proposition 4 implies that we have $V_p(\hat{c}) > V_n(\hat{c})$ for all $\hat{c} \in (0, \kappa\bar{c})$, and $V_p(\hat{c}) < V_n(\hat{c})$ for all $\hat{c} \in (\kappa\bar{c}, \bar{c}]$. The optimality of $\hat{c}_n$ and $\hat{c}_p$ therefore implies that if $\hat{c}_n \neq \hat{c}_p$, then $\hat{c}_n > \kappa\bar{c}$ and $\hat{c}_p < \kappa\bar{c}$ (because $V_p(\hat{c}_p) > V_n(\hat{c}_p)$ and $V_p(\hat{c}_n) < V_n(\hat{c}_n)$).

Hence, we have $\hat{c}_p < \hat{c}_n$ when the condition in (31) is binding, and the result follows: any payment $w^n(r_h, v_h)$ that might be observed in equilibrium must be higher than any payment $w^p(r_h, v_h)$ that might be observed in equilibrium. ■

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