Reflexivity in Credit Markets*

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Abstract

Reflexivity is the idea that investors' *biased* beliefs affect market outcomes, and that market outcomes in turn affect investors' biased beliefs. We develop a behavioral model of the credit cycle featuring such a two-way feedback loop. In our model, investors form beliefs about firms' creditworthiness, in part, by extrapolating past default rates. Investor beliefs influence firms' actual creditworthiness because firms that can refinance maturing debt on favorable terms are less likely to default in the short run—even if fundamentals do not justify investors' generosity. Our model is able to match many features of credit booms and busts, including the imperfect synchronization of credit cycles with the real economy, the negative relationship between past credit growth and the future returns on risky bonds, and "calm before the storm" periods in which firm fundamentals have deteriorated but the credit market has not yet turned.

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1 Introduction

> Current views about financing reflect the opinions bankers ... hold about the uncertainties they must face. These current views reflect ... the recent past ... A history of success will tend to diminish the margin of safety ... bankers require ...; a history of failure will do the opposite.—Hyman Minsky, *Stabilizing an Unstable Economy*, 1986.

Over the past decade, researchers in finance and economics have documented a number of new facts about the credit cycle. High credit growth is associated with both a higher probability of a future financial crisis and lower GDP growth (Schularick and Taylor [2012], López-Salido, Stein, and Zakrajšek [2017], Mian, Sufi, and Verner [2017]). Other research has shown that credit market returns are predictable, suggesting a role for investor sentiment in driving the credit cycle. Greenwood and Hanson (2013) show that periods of elevated corporate credit growth and low average borrower credit quality forecast low returns to credit. In a large panel of countries, Baron and Xiong (2017) find that high bank credit growth forecasts low returns to bank stocks.

Another underappreciated feature of the credit cycle is how disconnected it can be from the stock market or the broader macroeconomy in the short run. In post-war U.S. history, credit expansions and contractions have often followed a similar pattern. Credit grows slowly as the economy emerges from a recession, picks up steam, but continues to expand even as the overall economy cools. For example, in the upswing preceding the 2008 financial crisis, GDP growth peaked in March 2005, but credit growth peaked two years later in March 2007, a period when credit spreads were near historical lows. Put simply, the credit cycle seems to have some life of its own at short horizons. However, these disconnects pose a challenge for many well-known models of the credit cycle—e.g., Bernanke and Gertler (1989), Holmström and Tirole (1997), Bernanke, Gertler, Gilchrist (1999)—and even for more recent behavioral models like Bordalo, Gennaioli, and Shleifer (2018). Specifically, although credit market frictions amplify business cycle fluctuations in these models, the business cycle and the credit cycle are essentially one and the same.

In this paper, we present a new behavioral model of the credit cycle that is consistent with much of the accumulating evidence on credit cycles, but also speaks to periods of disconnect between credit markets and the fundamentals of the economy. A key feature of our model is "reflexivity," the idea that there is a feedback loop between investors' *biased* beliefs and market outcomes. In finance, the idea of reflexivity is most prominently associated with the investor George Soros, who argued that "distorted views can influence the situation to which they relate because false views lead to inappropriate actions." (*Financial Times*, October 26, 2009).[1] In credit markets, reflexivity arises because investors who overestimate the creditworthiness of a

[1] See Soros (1987, 2013) for an extensive discussion of reflexivity.

borrower are likely to refinance maturing debt on more favorable terms, thereby making the borrower less likely to default, at least in the short run.

In our baseline model, a representative firm invests in a sequence of one-period projects. Each project requires an upfront investment of capital, which the firm finances using short-term debt that it must refinance each period. Projects generate a random cash flow that varies exogenously according to the state of the economy. Debt financing is provided by investors whose beliefs are partly rational and forward-looking, but also partly extrapolative and backward-looking. To the extent that they are backward-looking, investors extrapolate the firm's recent repayment history to infer the probability that the firm will repay its debt in the next period. Following periods of low defaults, investors believe that debt is safe, and refinance maturing debt on attractive terms.

Because investors hold extrapolative beliefs based on defaults—which are endogenously determined in the model—and not on the exogenously given cash flow fundamentals, this leads to a dynamic feedback loop between biased investor beliefs and defaults. The feedback loop arises because current investor beliefs affect *future* defaults via the terms on which investors are willing to refinance debt today.

Figure 1 illustrates this feedback loop. During credit booms, default rates are low, so investors believe that future default rates will continue to be low. In the near term, these beliefs can be self-fulfilling: the perception of low future defaults leads to elevated bond prices, which in turn, makes it easier for firms to refinance their maturing debt. Holding constant firms' cash flows, cheaper debt financing leads to slower debt accumulation and a near-term decline in future defaults, which further reinforces investor beliefs. If cash flow fundamentals deteriorate, the backward-looking nature of investors' beliefs may allow firms to "skate by" for some amount of time, a phenomenon that we refer to as the "calm before the storm." Eventually, the reality of poor cash flow fundamentals catches up with firms, and defaults escalate. As a result, the disconnect between investors' beliefs and economic fundamentals is often the greatest just before such an spike in borrower defaults.

Conversely, suppose that the economy has just been through a wave of defaults. Since investors over-extrapolate these recent outcomes, investors believe that the likelihood of future defaults is high. Investor beliefs turn out to be partially self-fulfilling in the short run: bearish credit market sentiment makes it harder for firms to refinance existing debts, leading to an increase in defaults in the short run. In some circumstances, this can lead to "default spirals" in which a default leads to further investor pessimism and an extended spell of further defaults, much like what has been observed in instances of sovereign debt restructuring (Das, Papaioannou, and Trebesch [2012]).

In our model, transitions between credit booms and credit busts are ultimately caused by changes in cash flow fundamentals. However, because investors extrapolate past defaults and not cash flows, changes in credit markets are not fully synchronized with changes in fundamental cash flows, and can be highly path-dependent. For example, as noted, our model generates "calm before the storm" periods in which the fundamentals of the economy have turned, but credit markets are still buoyant. Such episodes are consistent with Krishnamurthy and Muir (2018), who show that credit spreads are typically "too low" in the years preceding financial crises. But, because investors are also partially forward-looking in our model, credit spreads will jump up on the eve of a crisis, just as documented by Krishnamurthy and Muir (2018).

The model is also useful for understanding how credit evolves following an exogenous shock to investor beliefs. For example, suppose that investors become more optimistic about firms' creditworthiness. In this case, firms are able to roll over debt at more attractive rates, which in turn makes default less likely in the near term. For an investor who extrapolates past defaults, lending to firms now appears to be even safer, leading investors to become even more optimistic, further reducing credit spreads. Over time, a shock to beliefs can be self-perpetuating. There is a limit to this self-perpetuation, however, because the firm can eventually become over-leveraged, triggering a default. Moreover, although this feedback loop between biased expectations and outcomes is always present, there are times when it is stronger. We describe the conditions under which changes in investors' biased expectations have the most powerful impact on market outcomes: these "highly reflexive" conditions often arise when the firm is near default.

While the credit market investors in our model are not fully rational, their beliefs are often similar to those of fully rational agents. In part, this is due to reflexivity: when investors believe that default probabilities are low, these optimistic beliefs *cause* default probabilities to be low. Thus, while the investors in our model do make predictable mistakes, those mistakes need not be large in order to generate rich and realistic credit market dynamics.

The model matches a number of facts about credit cycles that researchers have documented in recent years. First, rapid credit growth appears to be quite useful for predicting future financial crises and business cycle downturns (Schularick and Taylor [2012], Mian, Sufi, and Verner [2017], López-Salido, Stein, and Zakrajšek [2017]), a result that is consistent with our model because outstanding credit will grow rapidly when sentiment is high but cash flow fundamentals are poor. Relatedly, economies that have experienced high credit growth are more fragile, in the sense that they are vulnerable to shocks (Krishnamurthy and Muir [2017]). Second, high credit growth predicts low future returns on risky bonds in a univariate forecasting regression (Greenwood and Hanson [2013], Baron and Xiong [2017], and Muir [2019]), a result that obtains in our model because investors do not fully understand that when credit is growing rapidly they are often quickly heading towards a credit bust. Furthermore, in a multivariate forecasting specification, lower quality borrowing negatively predicts future returns and credit spreads positively predict future returns (Greenwood and Hanson [2013] and López-Salido, Stein, and Zakrajšek [2017]). In fact, in our model, credit spreads are typically too low just before the economy experiences a wave of defaults, consistent with the evidence in Krishnamurthy and Muir (2018) about the behavior of credit spreads before financial crises. Third, when credit markets become highly overheated, our model generates *negative* conditional expected excess returns on risky bonds. This result, which is consistent with the evidence in Greenwood and Hanson (2013) and Baron and Xiong (2017), is difficult to square with rational, risk-based models of credit cycles and is a powerful motivator for models like ours which prominently feature biased investor beliefs.

After developing our baseline model, we consider two extensions. First, we allow firms to opportunistically take on greater leverage by issuing additional debt when investors are underpricing credit risk. This opportunistic supply response means that credit booms have the potential to sow the seeds of their own destruction. Specifically, overly optimistic investor beliefs about future defaults can lead firms to issue more debt—especially since extrapolative investors underreact to this increase in firm leverage—raising the likelihood of a future credit market bust. Our second extension features multiple issuing firms who face idiosyncratic cash flow shocks. This extension addresses a limitation of the baseline model which is that, with a single representative firm, defaults are necessarily binary events. Allowing for multiple firms naturally yields a continuous default rate for the economy and leads to more realistic model-implied dynamics.

Our paper has much in common with Austrian theories of the credit cycle, including Mises (1924) and Hayek (1925), as well as the accounts of booms, panics, and crashes by Minksy (1986) and Kindleberger (1978). More recently, the idea that investors may neglect tail risk in credit markets was developed theoretically by Greenwood and Hanson (2013), Gennaioli, Shleifer, and Vishny (2012, 2015), and Bordalo, Gennaioli, and Shleifer (2018) and has been supported by numerous accounts of the 2007-2009 financial crisis (Coval, Jurek, and Stafford [2009] and Gennaioli and Shleifer [2018]). We also draw on growing evidence that investors extrapolate cash flows, past returns, or past crash occurrences (Barberis, Shleifer, and Vishny [1998], Greenwood and Shleifer [2014], Barberis, Greenwood, Jin, and Shleifer [2015, 2018], Jin [2015], Greenwood and Hanson [2015]). Most related here is Jin (2015), who presents a model in which investors' perceptions of crash risk depend on recent experience.

We have emphasized that our model of the credit cycle hinges on a two-way feedback loop between extrapolative investor beliefs and credit market outcomes. Of course, rational expectations models also feature two-way feedback between agents' beliefs and market outcomes (Muth [1961]). In a rational expectations equilibrium, agents' beliefs help determine market outcomes and agents' beliefs are rationally consistent with those outcomes, giving rise to a fixed-point problem. By contrast, our model emphasizes the feedback between market outcomes and beliefs that are not fully rational. Furthermore, this feedback is dynamic in nature: investors' current biased beliefs influence future market outcomes, and past market outcomes shape investors' current biased beliefs. In our model, extrapolative beliefs affect future credit market outcomes via the terms on which investors are willing to refinance maturing debt. In this way, our paper fuses extrapolative expectations with one of the key ideas in the sovereign default literature: the rationally, self-fulfilling debt crisis (Calvo [1988] and Cole and Kehoe [2000]). In these rational expectations models, if investors rationally believe that a default is likely (unlikely), they require high (low) interest rates, increasing (decreasing) the debt burden and hence the true probability of default, thus validating the initial belief. By contrast, our behavioral model is capable of generating self-fulfilling dynamics—including both the "calm before the storm" and "default spiral" phenomena—even when beliefs are completely extrapolative and backward-looking.

Bordalo, Gennaioli, and Shleifer (2018) also provide a model of credit cycles in which extrapolative investor expectations play an important role and in which bond returns are predictable. Their model is similar to ours in several respects, but extrapolative expectations in their model are perfectly tied to cash flow fundamentals, rather than to endogenous credit market outcomes. Extrapolative expectations amplify exogenous fluctuations in fundamentals, but in their model the credit cycle and the business cycle are fully synchronized. The fact that investors extrapolate an endogenous outcome in our model leads to episodes in which the credit market can become quite disconnected from the economy, acquiring a life of its own in the short run.[2]

In Section 2, we summarize a number of stylized facts about the credit cycle, drawing on the papers cited above but also presenting some novel observations about the synchronicity of the credit cycle and the business cycle. Section 3 develops the baseline model featuring a single representative firm, and explain the two-way feedback mechanism that is at the heart of our model. In Section 4, we then discuss how the model can match a number of features of credit cycles that researchers have documented in recent years, such as the predictability of returns and low credit spreads before crises. Section 5 briefly explores two extensions of the baseline model: one featuring an opportunistic supply response by firms when investors are mispricing risky bonds and a second featuring multiple heterogeneous firms. Section 6 concludes.

2 Motivating facts about the credit cycle

We begin by summarizing a set of stylized facts about credit cycles. The first four facts are drawn from previous work; the fifth is based on some new empirical work of our own.

Observation 1. *Rapid credit growth predicts financial crises and business cycle downturns.*

In a panel of 14 countries dating back to 1870, Schularick and Taylor (2012) show that rapid credit growth predicts financial crises. Schularick and Taylor (2012) interpret their evidence as suggesting that financial crises are episodes of "credit booms gone bust." More recently, Mian, Sufi, and Verner (2017) show that rapid credit growth—and especially growth in household credit—predicts future declines in GDP growth in an panel of 30 countries from 1960 to 2012.

[2]See also Coval, Pan, and Stafford (2014) who suggest that in derivatives markets, model misspecification only reveals itself in extreme circumstances, by which time it is too late. Bebchuk and Goldstein (2011) present a model in which self-fulfilling credit market freezes can arise because of interdependence between firms.

López-Salido, Stein, and Zakrajšek (2017) show that frothy credit market conditions—proxied using declines in the credit quality of corporate borrowers and low credit spreads—predict low GDP growth in U.S. data from 1929 to 2015. López-Salido, Stein, and Zakrajšek (2017) attribute their findings to predictable reversals in credit market sentiment. Consistent with this view, using an international panel of 38 countries, Kirti (2018) shows that rapid credit growth that is accompanied by a deterioration in lending standards—i.e., by declining borrower credit quality—is associated with low future GDP growth. By contrast, when rapid credit growth is accompanied by stable lending standards, there is no such decline in future GDP growth.

A corollary of Observation 1—i.e., that credit growth predicts financial crises—is that economies that have experienced high credit growth are more fragile. Krishnamurthy and Muir (2018) argue that the natural way to interpret Schularick and Taylor's (2012) findings about credit growth and financial crises is that credit growth creates financial fragility. When a more leveraged economy is exogenously hit by a negative fundamental shock, such as a large decline in house prices, this results in a financial crisis. And, as one would expect, Krishnamurthy and Muir (2018) find that credit spreads spike on the eve of a financial crisis. Alternately, crises may be triggered by predictable reversals in credit market sentiment as argued by López-Salido, Stein, and Zakrajšek (2017). Consistent with this view, Krishnamurthy and Muir (2018) show that credit spreads are typically "too low" in the years preceding financial crises. The model we develop reflects both of these ideas: as leverage grows, the probability of a default becomes more and more sensitive to both changes in underlying fundamentals and to changes in biased investor beliefs.

Observation 2. *Credit market overheating—signaled either by (i) a rapid growth in debt outstanding or (ii) by a decline in the credit quality of debt issuers set against the backdrop of (relatively) low credit spreads—predicts low future returns on risky bonds.*

A growing literature has demonstrated that credit market overheating predicts low future returns on risky bonds. Greenwood and Hanson (2013) find that rapid growth in outstanding corporate credit is associated with low future returns on risky bonds in U.S. data. Muir (2019) finds the same pattern in an panel of 17 developed economics from 1870 to 2016. Adopting a similar intuition, Baron and Xiong (2017) show that bank credit expansion also predicts low bank equity returns—which are naturally tied to the returns on risky debt—in a panel of 20 developed economies from 1920 to 2012.

Greenwood and Hanson (2013) develop an even more statistically powerful measure of credit market overheating based on the credit quality of corporate debt issuers. Their "high yield share" measure—the share of all corporate bond issuance in a given year that is from high-yield-rated firms—captures the intuition that when credit markets are overheated, low quality firms increase their borrowing to take advantage. Greenwood and Hanson (2013) show that declines in issuer credit quality predict low future corporate bond returns in a univariate sense. Furthermore, as emphasized by Greenwood and Hanson (2013) and López-Salido, Stein, and Zakrajšek (2017), issuer quality contains information about future bond returns beyond that contained in credit spreads. Specifically, in a multivariate regression specification, low-quality issuance negatively predicts future bond returns and credit spreads positively predict future returns.

Table 1 updates the data from Greenwood and Hanson (2013) and also considers a set of additional proxies for credit market overheating. The table shows annual return forecasting regressions of the form:

$$rx^{HY}_{t\to t+k} = a + b \cdot Overheating_t + e_{t\to t+k}, \tag{1}$$

where $rx^{HY}_{t\to t+k}$ denotes the log return on high yield bonds in excess of the log returns on like-maturity Treasuries over a $k = 2$- or 3-year horizon beginning in year t. Here $Overheating_t$ is a proxy for credit market overheating, measured using data through the end of year t. All of our data begin in 1983 and run through 2014.[3]

Columns (1) and (5) show that the log high yield share ($\log(HYS_t)$) significantly predicts low future excess bond returns. A one standard deviation in $\log(HYS_t)$ is associated with an 8.3 percentage point reduction in log excess bond returns over the next two years, and a 9.7 percentage point reduction over the next three years.

Columns (2) and (6) of Table 1 show that the same forecasting results hold when credit market overheating is measured using the growth in aggregate nonfinancial corporate credit outstanding, $Credit\ Growth_t$. Aggregate nonfinancial corporate credit is the sum of nonfinancial corporate debt securities and loans from Table L103 of the Federal Reserve's Financial Accounts of the U.S. A one standard deviation increase in $Credit\ Growth_t$ forecasts a 7.4 percentage point reduction in excess bond returns over the next two years, and a 9.3 percentage point reduction over the next three years.

Table 1 shows results for two additional measures of credit market overheating. The first, $Easy\ Credit_t$, is based on the Federal Reserve's Senior Loan Officer Opinion Survey (SLOOS), and the second, $-1 \times EBP_t$, is negative one times the Excess Bond Premium (EBP_t) from Gilchrist and Zakrajšek (2012).[4,5] Table 1 shows that both of these additional measures of credit market overheating forecast low future returns on corporate bonds. To summarize, Table 1

[3] For results over different time horizons and with additional controls, see Greenwood and Hanson (2013) who compute alternate proxies for issuer quality that extend back as far as 1926.

[4] Every quarter, the Federal Reserve surveys senior loan officers of major domestic banks concerning their lending standards. Officers report whether they have eased or tightened lending standards in the past quarter. We construct a measure of credit market overheating, $Easy\ Credit_t$, by taking the three-year average percentage of banks that have reported easing credit standards to firms in any given quarter. The idea behind this averaging procedure is that we want to capture the level of bankers' beliefs about future creditworthiness, whereas the quarterly survey tracks changes from the previous quarter. The SLOOS begins in the first quarter of 1990, so this measure of overheating begins in December 1992. $Easy\ Credit_t$ is 55% correlated with the high yield share (HYS_t) and 68% correlated with $Credit\ Growth_t$.

[5] Gilchrist and Zakrajšek's (2012) EBP_t variable is a measure of average corporate credit spreads after deducting an estimate of each bond's expected credit losses and, thus, can be interpreted as a proxy for expected future credit returns.

confirms that periods of credit market overheating—periods featuring low credit quality debt issuance, rapid growth in outstanding credit, loose credit standards, and tight credit spreads—are followed by low subsequent returns on risky corporate bonds.

Observation 3. *Significant credit market overheating is associated with* ***negative*** *expected future returns on risky bonds.*

Of course, the fact that corporate bond returns are predictable does not imply that corporate bonds are occasionally *mispriced*. For instance, if the rationally-required return on risky corporate bonds fluctuates over time—e.g., due to movements in investor risk aversion (Campbell and Cochrane [1999]) or in the quantity of aggregate risk (Bansal and Yaron [2004], Gabaix [2012], Wachter [2013]), then the level of credit spreads might forecast future returns on corporate bonds. And, combining such fluctuations in rationally-required returns with the neoclassical q-theory of investment, one might expect recent credit growth and declines in debt issuer quality to forecast low future returns on risky corporate bonds (Greenwood and Hanson [2013], Santos and Veronesi [2018], and Gomes, Grotteria, and Wachter [2019]).

However, Greenwood and Hanson (2013) and Baron and Xiong (2017) present evidence that conditional expected excess returns on risky corporate bonds and bank stocks become reliably *negative* when credit markets appear to be significantly overheated—i.e., when many low quality borrowers are able to obtain credit and when credit growth is rapid. Furthermore, these same authors find that future risk is high when credit markets appear to be most overheated (see Muir [2019] for further evidence on this point). These negative expected returns and the negative conditional relationship between expected future risk and return are quite difficult to square with rational risk-based models—even rational models with intermediation frictions—and are powerful motivations for the behavioral approach we adopt in this paper.[6] Specifically, these facts are most consistent with the idea that credit market investors make biased forecasts of future corporate bond defaults, arguably because they over-extrapolate recent outcomes.

Observation 4. *Variables that forecast returns on risky bonds often do not forecast returns on equities, and vice versa. Moreover, episodes of credit market overheating tend to follow periods of tranquility in credit markets, namely periods when defaults are low and when the returns on risky bonds are high.*

What outcomes are credit-market investors over-extrapolating? One view is that investors over-extrapolate some underlying set of economic fundamentals—e.g., firm cash flows or the state of broader macroeconomy. This view leads to behavioral version of the q-theory of investment (Greenwood and Hanson [2015], Gennaioli, Ma, and Shleifer [2016], Bordalo, Gennaioli, and

[6] In models with intermediation frictions, changes in the health of intermediary balance sheets and the resulting shifts in risk appetite play an important role in determining asset prices. See, for example, He and Krishnamurthy (2013), Adrian, Etula, and Muir (2014), Brunnermeier and Sannikov (2014), and He, Kelly, and Manela (2017).

Shleifer [2017]) and generally suggests that equity-market sentiment and credit-market sentiment should be tightly linked over time. However, in the data, many measures that predict credit returns are not strong predictors of equity returns and vice versa (Greenwood and Hanson [2013] and Ma [2018]). This disconnect between equity and credit market sentiment recommends a more nuanced behavioral view in which equity and credit markets are partially-segmented and investors in each market extrapolate past *market-specific outcomes.*

Consistent with the idea that credit market investors tend to over-extrapolate recent credit market outcomes, Greenwood and Hanson (2013) show that past defaults and credit returns play a dominant role in shaping credit-market sentiment. Specifically, Greenwood and Hanson (2013) find that debt issuer quality tends to deteriorate following periods with low realized corporate defaults and high realized returns on risky corporate bonds. However, after controlling for these recent credit market outcomes, recent equity returns and macro variables have relatively little impact on debt issuer quality. These findings motivate our model where credit investors extrapolate past bond defaults, which themselves are not perfectly tied to firm fundamentals.[7]

Table 2 presents additional evidence that periods of credit market overheating follow times when corporate defaults are low. Specifically, Table 2 shows the results from estimating time-series regressions of the form:

$$Overheating_t = a + b \cdot Def_t + c \cdot Def_{t-1} + e_t, \tag{2}$$

where Def_t denotes the default rate on high yield bonds in year t. We estimate this regression using the same four measures of credit market overheating from Table 1. Table 2 shows that there is a strong negative relationship between recent default rates and current credit market overheating. Some measures of overheating ($\log(HYS_t)$ and $-1 \times EBP_t$) are more highly correlated with most recent default rates, while others are also strongly correlated with lagged default rates ($Credit\ Growth_t$ and $Easy\ Credit_t$).

Observation 5. *The credit cycle and the business cycle can be quite disconnected in the short run.*

Consistent with the market-specific extrapolation view discussed above, the credit cycle can become quite disconnected from both the broader business cycle as well as equity market conditions in the short run.

Figure 2 plots the annual growth rate of U.S. GDP alongside the annual growth rate of outstanding debt at nonfinancial corporations, both expressed in real terms. In the upswing proceeding the 2008 financial crisis, GDP growth peaked in March 2005, but credit growth

[7] Similarly, Greenwood and Shleifer (2014) show that past equity returns play an outsized role in shaping equity-market sentiment, motivating the model in Barberis, Greenwood, Jin, and Shleifer (2015) where equity investors extrapolate past equity returns (as opposed to firm fundamentals).

peaked more than two years later. This pattern of credit expansion at the end of an economic expansion is also apparent in the late 1990s, with credit growth rising only at the end of the business cycle. During downturns, the economy often recovers well before credit growth returns to normal rates. In the most recent economic recovery, real credit growth first reached 3% in 2013, several years after the economy began its recovery. Overall, the correlation between credit growth and GDP growth is only 43%.

Figure 3 illustrates the disconnect between the credit cycle and the business cycle in U.S. data. Here we provide additional perspective on the lack of synchronicity between the credit cycle and the business cycle. In particular, we show that credit growth tends to increase towards the end of a business cycle boom. In Panel A of Figure 3, we plot real GDP growth from trough to peak of the business cycle, by business cycle expansion quarter. As can be seen, GDP growth is high in the beginning of business cycle expansions, but after quarter five, it stabilizes and if anything, declines slightly in later quarters. In contrast, Panel B shows credit growth over the same periods. As the figure makes clear, credit expansion is particularly high in the latter part of the business cycle.

3 A model of credit market sentiment

In this section, we consider an infinite-horizon model with a representative firm and a set of identical, risk-neutral bond investors. Our assumption of a representative firm is made purely for simplicity. In Section 5, we introduce a continuum of firms which are subject to heterogeneous cash flow shocks.

We first describe firm borrowing behavior and then explain investor beliefs, collecting several preliminary results along the way. We then present a series of formal results and numerical simulations that trace out the model's key implications for credit market dynamics.

3.1 Firm borrowing

In order to focus on the dynamic interplay between investor beliefs and firm defaults, we model firm borrowing behavior in a deliberately sparing fashion. Each period t, the representative firm invests in a one-period project that requires a fixed up-front cost of $c > 0$. The next period, the project generates a random cash flow, x_{t+1}, that follows an exogenously given $AR(1)$ process

$$x_{t+1} - \bar{x} = \rho(x_t - \bar{x}) + \varepsilon_{t+1}, \tag{3}$$

where $\rho \in (0, 1)$, $\bar{x} > c$, and the fundamental cash flow shock $\varepsilon_{t+1} \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ is *i.i.d.* over time.

The firm issues one-period bonds in order to finance these projects. Each bond is a promise to pay back one dollar to investors in one period. At time t, the price of each bond is denoted

p_t. The total face amount of debt outstanding at time t is F_t, meaning that the firm is obligated to repay F_t dollars to investors at time $t+1$.

At time t, the firm must repay the face amount of debt issued the prior period F_{t-1}. The firm also must pay the cost c to begin a new project and receives the cash flow x_t from the prior period's project. Finally, the firm can issue new bonds at a price of p_t. Assuming the firm does not default and does not pay dividends to equity holders at time t, the total face amount of bonds outstanding at time t is

$$F_t = (F_{t-1} + c - x_t)/p_t, \tag{4}$$

which is obtained by equating sources and uses.[8] This law of motion is consistent with the fact that nonfinancial leverage is typically counter-cyclical, falling in good times when x_t is high and rising in bad times when x_t is low (Korajczyk and Levy [2003] and Halling, Yu, and Zechner [2016]).

We assume a simple mechanistic default rule. One should interpret a default by the representative firm in our model as a "credit market bust" in which there is an economy-wide spike in corporate defaults. Specifically, if at any time t, $F_{t-1} + c - x_t$ rises above a threshold of $\bar{F}$, the representative firm defaults. The existence of this threshold $\bar{F}$ can be seen as a reduced form for informational or agency frictions that grow more severe as the amount of required external financing rises. Alternately, such a threshold may arise from the optimal exercise of the firm's default option by equity holders as in Leland (1994). Formally, letting D_t denote a binary variable indicating whether or not a default occurs at time t, we have

$$D_t = \mathbf{1}_{\left\{F_{t-1}+c-x_t \geq \bar{F}\right\}}. \tag{5}$$

The "default boundary" is the line in (F_{t-1}, x_t) space where this default indicator switches on or off—i.e., the line $F_{t-1} = \bar{F} - c + x_t$.

In the event of default, the firm continues to operate. However, the firms' existing equity-holders are wiped out and the firm writes off a fraction of its debt much like under Chapter 11 of the U.S. Bankruptcy Code. Specifically, if the firm defaults, a fraction $1-\eta$ of the firm's debt is written off, generating losses for existing bondholders, and the remaining fraction $\eta \in (0,1)$ is refinanced at current market prices. Thus, if the firm defaults at time t, the amount of debt outstanding becomes

$$F_t = \eta(F_{t-1} + c - x_t)/p_t. \tag{6}$$

[8] We assume that the firm always decides to invest, even when expected cash flows tomorrow do not cover the current cost—i.e., when $c > \bar{x} + \rho(x_t - \bar{x})$. There are two possible interpretations of this assumption. First, we could assume that the firm is operating a *long-run* technology that generates the infinite stream $\{x_t\}$, that c is the cost of continuation each period, and that continuation is always efficient. Under this interpretation, continuation is indeed always efficient in our setting since $\overline{x} > c$, $\rho \in (0,1)$, and investors' have zero rate of time preference. Alternately, we could assume that managers receive private benefits from running the firm and will always choose continuation even if continuation is value destroying.

Finally, we assume that if $F_{t-1} + c - x_t \leq \underline{F}$, the firm sets $F_t = \underline{F}/p_t$ and pays all residual cash flows to equity holders as a dividend. The idea underlying the lower barrier $\underline{F} > 0$ for debt outstanding can be motivated via the pecking order theory of capital structure (Myers and Majluf [1984]). Firms only raise external finance in the form of debt. And when there is available free cash flow, the firm first uses this cash flow to retire existing debts. However, once the face value of debt reaches a sufficiently low level, the firm chooses to pay out all available free cash flow to its equity holders.[9]

In summary, our assumptions imply that the firm defaults and its debt is written down when outstanding debt grows too large relative to the firm's current free cash flows $(x_t - c)$—i.e., when $F_{t-1} \geq \bar{F} + (x_t - c)$. Conversely, the firm stops paying down its debt and instead pays all residual cash flows to equity holders when outstanding debt grows small relative to the firm's current free cash flows—i.e., when $F_{t-1} \leq \underline{F} + (x_t - c)$. In between these upper and lower boundaries, all else equal, the firm's debt outstanding grows faster when refinancing conditions are less favorable—i.e., when the price of new bonds p_t is lower—and when the ratio of the firm's free cash flow to outstanding debt $(x_t - c)/F_{t-1}$ is lower.[10] While clearly simplistic, these dynamics can be seen as a shorthand for the kinds of behavior that emerge from more complex, dynamic models of firm investment and capital structure choice.

Before turning to investor beliefs, we note that, taking F_t as given, it is straightforward to compute the fully-rational, forward-looking probability of a default at time $t+1$, which we denote by λ_t^R. Given the cash flow process in equation (3) and the default rule in equation (5), a default will occur at time $t+1$ if and only if

$$\bar{F} \leq F_t + c - x_{t+1} = F_t + c - \rho x_t - (1-\rho)\bar{x} - \varepsilon_{t+1}.$$

Thus, at time t, the true probability of default on the promised bond payments at time $t+1$ is

$$\lambda_t^R = \Phi\left(\frac{F_t - \bar{F} + c - \rho x_t - (1-\rho)\bar{x}}{\sigma_\varepsilon}\right), \tag{7}$$

where $\Phi(\cdot)$ denotes the cumulative normal distribution function.

[9] To endogenize the upper threshold $\bar{F}$, we could assume that at any time, equity holders can default on the firm's outstanding debt and abscond with some fraction of the firm's total enterprise value. Equity holders decide whether or not to exercise this default option by comparing the present value of expected future dividends to the value of this outside default option. Since, all else equal, the present value of expected future dividends is decreasing in the amount of outstanding debt, this means that equity holders will choose to default once the face value of debt reaches a sufficiently high level.

Similarly, the lower threshold $\underline{F}$ could be endogenized by assuming that the firm's equity holders trade off the value of receiving dividends today versus the value of further debt reduction. Further debt reduction lowers the probability of default in future periods and hence raises the expected value of future dividends. Since the benefits of debt reduction decline with the level of debt, the firm chooses to pay out free cash flow to equity holders once the face value of debt reaches a sufficiently low level.

[10] These statements follow from the fact that $(F_t - F_{t-1})/F_{t-1} = ((1-p_t) - (x_t - c)/F_{t-1})/p_t$ if the firm does not default or pay dividends at time t.

3.2 Investor beliefs

There is a continuum of risk-neutral bond investors with zero rate of time preference. Investors' beliefs at time t about the probability of a default at time $t+1$ are denoted λ_t^C. We assume that investors' beliefs λ_t^C are a mixture of (i) an extrapolative and backward-looking component λ_t^B based on past default rates and (ii) the fully rational and forward-looking belief λ_t^R. We assume that fraction $\theta \in [0,1]$ of investors' beliefs are extrapolative and backward-looking and the remaining fraction $1-\theta$ are fully-rational and forward-looking. Thus, we have:

$$\lambda_t^C = \theta\lambda_t^B + (1-\theta)\lambda_t^R = \lambda_t^R - \theta(\lambda_t^R - \lambda_t^B). \tag{8}$$

This formulation of beliefs is in the spirit of Fuster, Laibson, and Mendel (2010) and Fuster, Hebert, and Laibson (2011) who argue that many agents have "natural expectations" which are a combination of fully-rational expectations and extrapolative expectations. Equation (8) covers the polar cases of fully-rational expectations ($\theta = 0$) and fully-extrapolative expectations ($\theta = 1$).

Our formulation of beliefs in equation (8) embeds two distinct notions of "credit market sentiment." First, one might say that credit market sentiment is elevated when λ_t^B is low—i.e., when future defaults are perceived as being unlikely according to the extrapolative component of investor beliefs. Alternately, one might say that credit market sentiment is elevated when $\left(\lambda_t^R - \lambda_t^B\right)$ is high—i.e., when investors are underestimating the true likelihood of a future default.

In a moment, we will detail precisely how λ_t^B is specified and how λ_t^R is pinned down in a equilibrium when $\theta < 1$. For now, we take λ_t^B and λ_t^R as given. Since investors are risk-neutral and have zero rate of time preference, the bond price at time t is simply

$$p_t = p(\lambda_t^B, \lambda_t^R) = (1-\lambda_t^C) + \lambda_t^C\eta = \left[1-(1-\eta)\lambda_t^R\right] + (1-\eta)\theta(\lambda_t^R - \lambda_t^B). \tag{9}$$

Thus, relative to the price of $1-(1-\eta)\lambda_t^R$ in a fully-rational economy where $\theta = 0$, bond prices are elevated when $\lambda_t^R - \lambda_t^B$ is high and investors are underestimating the true likelihood of a future default.

The default rule in equations (5) and (6) and the bond pricing equation (9) give rise to the following law of motion for the amount of debt outstanding:

$$F_t = F(F_{t-1}, \lambda_t^B, \lambda_t^R, x_t) = \begin{cases} \underline{F}/p(\lambda_t^B, \lambda_t^R) & \text{if } F_{t-1} + c - x_t \le \underline{F} \\ (F_{t-1} + c - x_t)/p(\lambda_t^B, \lambda_t^R) & \text{if } \underline{F} < F_{t-1} + c - x_t < \bar{F} \\ \eta(F_{t-1} + c - x_t)/p(\lambda_t^B, \lambda_t^R) & \text{if } \bar{F} \le F_{t-1} + c - x_t \end{cases}. \tag{10}$$

Since $p(\lambda_t^B, \lambda_t^R) \le 1$, it follows that we always have $F_t \ge \underline{F}$. Thus, $\underline{F}$ is indeed a lower barrier for the amount of debt outstanding.

The model is fully characterized by equations (3), (9), and (10), together with the specifications for λ_t^B in equation (11) and λ_t^R in equation (13) which will be introduced below.

The extrapolative component of investor beliefs λ_t^B. We now introduce our specification for λ_t^B, the extrapolative, backward-looking component of investors' time-t beliefs about the likelihood of a default at time $t+1$. We assume that λ_t^B depends solely on past default realizations and past "sentiment" shocks that are unrelated to cash flow fundamentals. Specifically, we assume that the law of motion for this backward-looking component of beliefs is

$$\lambda_t^B = \max\left\{0, \min\left\{1, \beta\lambda_{t-1}^B + \alpha D_t + \omega_t\right\}\right\}, \tag{11}$$

where $0 < \beta < 1$ is a memory decay parameter, $\alpha > 0$ measures the incremental impact of a default event on backward-looking beliefs, and $\omega_t \sim \mathcal{N}(0, \sigma_\omega^2)$ is a random "sentiment" shock that is independent of the fundamental cash flow shocks ε_t. The min and max functions in equation (11) ensure that $\lambda_t^B \in [0,1]$ for all t. Assuming that λ_t^B is always between 0 and 1, we have

$$\lambda_t^B = \textstyle\sum_{j=0}^{\infty}\beta^j\left(\alpha D_{t-j} + \omega_{t-j}\right). \tag{12}$$

In this case, the extrapolative component of beliefs is just a geometric moving average of past defaults and past sentiment shocks.

The specification for extrapolative beliefs in equation (11) is similar to specifications in Barberis, Greenwood, Jin, and Shleifer (2015, 2018), and Nagel and Xu (2018). Empirically, equation (11) is motivated by the findings in Greenwood and Hanson (2013) who present evidence that credit market investors tend to extrapolate recent credit market outcomes. They show that credit market sentiment rises following periods when default rates have fallen and the returns on high-yield bonds have been high. These results hold controlling for contemporaneous conditions in the macroeconomy and in the stock market, suggesting that it is credit market outcomes, not fundamentals, that are being extrapolated.[11]

The following lemma explains how this extrapolative component of beliefs evolves over time.

Lemma 1 *Assume there are no sentiment shocks (i.e., $\omega_t = 0$ for all t), so the law of motion for the extrapolative component of beliefs is simply $\lambda_t^B = \max\left\{0, \min\left\{1, \beta\lambda_{t-1}^B + \alpha D_t\right\}\right\}$.*

- *If there is no default at time t, then we always have $\lambda_t^B \leq \lambda_{t-1}^B$ and $\lambda_t^B < \lambda_{t-1}^B$ if $\lambda_{t-1}^B > 0$— i.e., extrapolative beliefs always become more optimistic when there is no default.*

- *If there is a default at time t, there are two cases:*

[11] We do not take a strong stance of the psychological underpinnings of overextrapolation. Overextrapolation could stem from the representativeness heuristic and a belief in the "law of small numbers" (Kahneman and Tversky [1972], Barberis, Shleifer, and Vishny [1998], Rabin [2002], Gennaioli and Shleifer [2010])) or from experience effects and reinforcement learning in the presence of fading memory (Malmendier and Nagel [2011, 2016], Nagel and Xu [2018], Malmedier, Pouzo, and Vanasco [2018]).

– *If* $\alpha \geq 1-\beta$, *then* $\lambda_t^B \geq \lambda_{t-1}^B$ *and* $\lambda_t^B > \lambda_{t-1}^B$ *if* $\lambda_{t-1}^B < 1$*—i.e., extrapolative beliefs always become more pessimistic following a default. As a result,* λ_t^B *will converge to* 1 *following a long sequence of defaults.*

– *If* $\alpha < 1-\beta$, *then* $\lambda_t^B \gtreqless \lambda_{t-1}^B$ *as* $\lambda_{t-1}^B \lesseqgtr \alpha/(1-\beta)$. *As a result,* λ_t^B *will converge to* $\alpha/(1-\beta) < 1$ *following a long sequence of defaults.*

Proof. See the Appendix for all proofs. ∎

Naturally, the dynamics of λ_t^B are governed by the incremental impact of a default on beliefs α and the rate of memory decay $(1-\beta)$. As we will see below, the potential for backward-looking beliefs to drive persistent default cycles is greatest when the incremental belief impact α is high and when β is close to 1 so memory fades slowly. In this case, a default episode makes investors much more pessimistic, which in turn makes it more difficult for firms to refinance maturing debt.

Solving for rational expectations equilibrium. We now close the model by explaining how λ_t^R is pinned down in a rational expectations equilibrium when $\theta < 1$. According to equation (7), λ_t^R depends on F_t. However, equations (9) and (10) imply that F_t depends on λ_t^R when $\theta < 1$. Thus, when $\theta < 1$, λ_t^R and F_t must be simultaneously determined in equilibrium.[12]

The simultaneous determination of F_t and λ_t^R when $\theta < 1$ introduces the potential for multiple equilibria. The potential for equilibrium multiplicity reflects a straightforward self-fulfilling-prophecy intuition. If the rational component of investor beliefs about future default probabilities is low (high), then current bond prices are high (low). As a result, the face value of debt that the firm must promise to repay tomorrow is low (high), leading to a true probability of default tomorrow that is indeed low (high). Unlike in classic bank run models (e.g., Diamond and Dybvig [1983] and Goldstein and Pauzner [2005]), multiple equilibria in our model do not arise from strategic complementarities between the financing decisions of individual short-run creditors: the investors in our model are nonstrategic price-takers. Instead, multiple equilibria arise for reasons similar to those in classic models of sovereign default (e.g., Calvo [1988] and Cole and Kehoe [2000]).

Formally, combining equations (7) and (10), we see that the equilibrium value of λ_t^R must solve the following fixed-point problem when $\theta < 1$:

$$\lambda_t^R = g(\lambda_t^R | F_{t-1}, \lambda_t^B, x_t) \equiv \Phi\left(\frac{F(F_{t-1}, \lambda_t^B, \lambda_t^R, x_t) + c - \bar{F} - \rho x_t - (1-\rho)\bar{x}}{\sigma_\varepsilon}\right). \tag{13}$$

Note from (10) that the bond price $p(\lambda_t^B, \lambda_t^R)$ does not determine whether the firm defaults

[12] Although the rational component of beliefs (λ_t^R) will be pinned down by a rational expectations fixed point condition, so long as $\theta > 0$, our model differs crucially from a standard rational expectations model because investors' overal beliefs (λ_t^C) do not satisfy this rationality condition.

or pays dividends at time t; only F_{t-1} and x_t determine these outcomes. This means that $g(\lambda_t^R|F_{t-1}, \lambda_t^B, x_t)$ is a continuous and increasing function of λ_t^R for given values of $\left(F_{t-1}, \lambda_t^B, x_t\right)$. Also note that $g(0|F_{t-1}, \lambda_t^B, x_t) > 0$ and $g(1|F_{t-1}, \lambda_t^B, x_t) < 1$. Therefore, $g(\lambda_t^R|F_{t-1}, \lambda_t^B, x_t)$ is a continuous function that maps the unit interval into itself, so a fixed point always exists by Brouwer's fixed-point theorem.

Multiple equilibria are more likely to arise (i) when investor beliefs are more rational and forward-looking (i.e., when θ is low); (ii) when the configuration of $(F_{t-1}, \lambda_t^B, x_t)$ means that the firm will be near the default boundary at time $t+1$; and (iii) when cash flow volatility σ_ε is low. First, rational beliefs have a larger impact on current bond prices and hence on the likelihood of future defaults when θ is low. Indeed, there is always a single unique equilibrium when $\theta = 1$ and beliefs are completely extrapolative. Second, multiple equilibria will only arise when the firm will be near the default boundary at time $t+1$. If the firm is very far from the default boundary, then $\partial g(\lambda_t^R|\cdot)/\partial\lambda_t^R$ is always small—there is no scope for self-fulfilling rational beliefs—and there is a unique equilibrium. Finally, when future cash flows are volatile (i.e., when σ_ε is high), the downside risk for the future cash flows is generally high which reduces the effect of self-fulfilling rational beliefs on future defaults. In this case, the model has a unique equilibrium. Conversely, when future cash flows are not very volatile, self-fulling rational beliefs have a bigger impact on future defaults and sometimes lead to multiple equilibria.[13]

How do we select amongst these equilibria when more than one exists? We focus on the smallest λ^R that solves $\lambda^R = g(\lambda^R|\cdot)$—i.e., the model's "best" stable equilibrium.[14] An equilibrium is "stable" if it is robust to a small perturbation in investors' beliefs regarding the likelihood of a default tomorrow. In our setting, if $\partial g(\lambda^{R*}|\cdot)/\partial\lambda^R < 1$, then λ^{R*} is stable; if $\partial g(\lambda^{R*}|\cdot)/\partial\lambda^R > 1$, then λ_t^{R*} is unstable. Since $g(0|\cdot) > 0$ and $g(1|\cdot) < 1$, our model always has at least one stable equilibrium. Following the correspondence principle of Samuelson (1947), stable equilibria have local comparative statics that accord with common sense. For example, at a stable equilibrium, λ_t^R is locally increasing in F_{t-1} and decreasing in x_t.[15]

The following lemma explains how the true probability of default λ_t^R is influenced by movements in F_{t-1}, λ_t^B, and x_t.

Lemma 2 *First, assume that the economy is not near the default boundary $F_{t-1} = \bar{F} - c + x_t$ at time t, so small changes in F_{t-1} and x_t do not affect whether or not there is a default at time t. Then a small increase in F_{t-1} raises λ_t^R when $\underline{F} < F_{t-1} + c - x_t$, a small increase in λ_t^B*

[13]Formally, $g(\lambda_t^R|F_{t-1}, \lambda_t^B, x_t)$ is an S-shaped function of λ_t^R, a property that it inherits from the normal cumulative density function $\Phi(\cdot)$. As we increase σ_ε, $g(\lambda_t^R|F_{t-1}, \lambda_t^B, x_t)$ becomes closer to a linear function of λ_t^R—i.e., $\partial^2 g(\lambda_t^R|\cdot)/\partial(\lambda_t^R)^2$ approaches zero—so it is harder to have multiple equilibria. As $\sigma_\varepsilon \to 0$, $g(\lambda_t^R|F_{t-1}, \lambda_t^B, x_t)$ converges to a step-function and therefore it is easier to have multiple equilibria.

[14]We obtain very similar simulation results if we instead focus on the the largest λ^R that solves $\lambda^R = g(\lambda^R|\cdot)$—i.e., the model's "worst" stable equilibrium.

[15]By contrast, unstable equilibria have local comparative statics with the opposite signs, which run contrary to common sense. For example, at an unstable equilibrium, λ_t^R is locally decreasing in F_{t-1} and increasing in x_t.

always raises λ_t^R, and a small increase in x_t always reduces λ_t^R. When $\theta = 1$, λ_t^R is everywhere a continuous function of F_{t-1}, λ_t^B, and x_t. By contrast, when $\theta < 1$, λ_t^R can be discontinuous in F_{t-1}, λ_t^B, and x_t, jumping discretely in response to small changes in these variables when the smallest solution to equation (13) jumps—we call these jumps "equilibrium discontinuity points." However, λ_t^R is continuous and differentiable in these variables almost everywhere when $\theta < 1$.

Next, assume that the economy is near the default boundary at time t, so small changes in F_{t-1} and x_t can affect whether or not there is a default at time t. Near the default boundary, a small increase in F_{t-1} can trigger a default at time t, resulting in a discrete downward jump in the true probability of a default at $t+1$, λ_t^R. Similarly, near the default boundary, a small increase in x_t can avert a default at time t, resulting in a discrete upward jump in λ_t^R. However, it is still the case that a small increase in λ_t^B always raises λ_t^R.

4 Understanding Reflexivity

Our model captures the idea that, in credit markets, investors' biased beliefs can impact financial reality. Past defaults and sentiment shocks affect investors' beliefs about future defaults via equation (11). These biased beliefs then impact bond prices via equation (9). And, since bond prices influence the ease with which the firm can refinance its existing debt, they in turn affect the evolution of debt outstanding via equation (10) and hence the true probability of future defaults in equation (7). As a result, biased investor beliefs have the potential to become partially self-fulfilling.

In this section, we provide a set of analytical results and numerical simulations to illustrate the key implications of the model. We first introduce a baseline set of model parameters and demonstrate that the impact of biased investor beliefs on market outcomes is not constant: there are states where biased beliefs have a large impact on market outcomes and states where the impact is much smaller. We then lay out three main implications of the model: the "calm before the storm" phenomenon, the "default spiral" phenomenon, and the predictability of corporate bond returns. As we emphasize, these three novel implications reflect the interaction between default extrapolation and the reflexive nature of credit markets. In other words, these three results arise because (i) investors hold beliefs that are (partially) backward-looking—i.e., they extrapolate past defaults when forming beliefs about future defaults—and (ii) beliefs about future defaults are (partially) self-fulfilling. We also use the model to draw impulse-response functions which show how shocks to cash flow fundamentals and investor beliefs impact credit markets.

4.1 Model parameters and simulated data

When using the model to generate simulated data, we use the following set of baseline parameters:

- **Cash flow dynamics:** $\bar{x} = 2.4$, $\rho = 0.8$, $\sigma_\varepsilon = 0.5$.
- **Investment cost:** $c = 2$.
- **Default and dividend barriers:** $\bar{F} = 5$, $\underline{F} = 1.5$.
- **Write-off parameter:** $\eta = 0.5$.
- **Belief dynamics:** $\beta = 0.8$, $\alpha = 0.2$, $\sigma_\omega = 0.05$.
- **Belief mix:** $\theta = 0.5$.

While these parameters are only illustrative, they have a number of desirable properties based on simulating the model for $100,000$ periods (each period is one year):

1. **The unconditional default probability is realistic.** Here the unconditional probability of default is 12%. As noted above, one should interpret a default by our representative firm as a "credit market bust" in which there is an economy-wide spike in corporate defaults. Thus, these parameters imply that roughly one in ten years corresponds to such a bust.

2. **The unconditional means of λ_t^B and λ_t^R are similar.** Here the average of λ_t^R is 12% and λ_t^B is 15%. Thus, the behavioral component of beliefs is reasonable on average. As a result, the means of $\left(\lambda_t^R - \lambda_t^B\right)$ and r_{t+1}, the realized rate of return on bonds from t to $t+1$, are small. Here the mean of $\left(\lambda_t^R - \lambda_t^B\right)$ is -3% and the average annual bond return is 0.3%.

3. **The time-series correlation between λ_t^B and λ_t^R is strong.** While clearly imperfect, investors' beliefs are reasonable over time. Specifically, we have $Corr\left(\lambda_t^B, \lambda_t^R\right) = 0.58$. That is, the backward-looking component of investors' beliefs is strongly correlated with fully-rational beliefs over time. As a result, investors' combined beliefs $\lambda_t^C = \theta\lambda_t^B + (1-\theta)\lambda_t^R$ are close to the fully-rational ideal: $Corr\left(\lambda_t^C, \lambda_t^R\right) = 0.93$.

4. **Relation of α and β.** The strength of the default spiral mechanism is increasing in both α and β. Specifically, if $\alpha > (1-\beta)$ then λ_t^B always rises when $D_t = 1$ and $\omega_t = 0$. However, if $\alpha \leq 1-\beta$ then λ_t^B can actually fall when $D_t = 1$ and $\omega_t = 0$. Since $\alpha = 1-\beta$ in this calibration, default spirals are possible.

Figure 4 shows a typical sample path of simulated data using these parameters. Notice that the time-series distribution of λ_t^R is bimodal: λ_t^R is typically either close to zero or 1. This bimodal distribution is largely a function of the short-term nature of debt in our model: short-term debt is extremely safe until it suddenly becomes risky.[16]

[16] However, the partially forward-looking nature of investor beliefs also contributes to the bimodal distribution of λ_t^R. Specifically, when $\theta < 1$, the model admits multiple equilibria and the smallest stable equilibrium will often discretely jump from $\lambda_t^R \approx 0$ to $\lambda_t^R \approx 1$ as the economy approaches the default boundary. This effect is diminished when we increase θ, so the distribution of λ_t^R becomes less bimodal as beliefs become more backward-looking.

4.2 The degree of reflexivity varies over time

As noted above, our model captures the idea that investors' biased beliefs can impact reality because refinancing terms are a first-order determinant of firm creditworthiness. While the feedback loop between biased beliefs and credit market outcomes is always present, there are times when this reflexive feedback loop is particularly strong. Specifically, we say that the economy is in a "highly reflexive" state when the true probability of default is highly dependent on the extrapolative component of investor beliefs λ_t^B—i.e., when $\partial\lambda_t^R/\partial\lambda_t^B$ is large. While λ_t^R always depends positively on λ_t^B, there are "non-reflexive" regions in which $\partial\lambda_t^R/\partial\lambda_t^B$ is quite small. For example, states where debt is low and the economy has not experienced defaults for a long time are typically non-reflexive. However, there are also "highly reflexive" regions where $\partial\lambda_t^R/\partial\lambda_t^B$ is large: here a change in the extrapolative component of beliefs λ_t^B—whether due to a current default or a sentiment shock ω_t—will have a large impact on the true probability of default λ_t^R.

When will $\partial\lambda_t^R/\partial\lambda_t^B$ be large? Using equations (9), (10), and (13), we have

$$\frac{\partial\lambda_t^R}{\partial\lambda_t^B} = \frac{\frac{\partial g(\lambda_t^R|F_{t-1},\lambda_t^B,x_t)}{\partial\lambda_t^B}}{1-\frac{\partial g(\lambda_t^R|F_{t-1},\lambda_t^B,x_t)}{\partial\lambda_t^R}} = \frac{\frac{\phi(dist_t)}{\sigma_\varepsilon}\frac{F(F_{t-1},\lambda_t^B,\lambda_t^R,x_t)}{p(\lambda_t^R,\lambda_t^B)}(1-\eta)\,\theta}{1-\frac{\phi(dist_t)}{\sigma_\varepsilon}\frac{F(F_{t-1},\lambda_t^B,\lambda_t^R,x_t)}{p(\lambda_t^R,\lambda_t^B)}(1-\eta)\,(1-\theta)}, \tag{14}$$

where $\phi(\cdot)$ is the standard normal density and

$$dist_t = \frac{F(F_{t-1},\lambda_t^B,\lambda_t^R,x_t)+c-\bar{F}-\rho x_t-(1-\rho)\bar{x}}{\sigma_\varepsilon} \tag{15}$$

is the expected distance-to-default at time $t+1$. Thus, $\partial\lambda_t^R/\partial\lambda_t^B$ is likely to be large at time t when $|dist_t|$ is small so the economy is expected to be close to the default boundary at time $t+1$, when $F_t = F(F_{t-1},\lambda_t^B,\lambda_t^R,x_t)$ is large, and when $p(\lambda_t^R,\lambda_t^B)$ is small. Furthermore, there is a greater potential to have a larger value of $\partial\lambda_t^R/\partial\lambda_t^B$ when θ is high and when σ_ε is low. These highly reflexive regions play an important role in driving credit market dynamics in our model.

Figure 5 illustrates the existence of these highly reflexive regions. Using our baseline set of parameters, the heatmap in Panel A shows how λ_t^R varies as a function of (x_t, F_{t-1}) when $\lambda_t^B = 0.2$. The dashed white line shows the default boundary at time t, namely $F_{t-1} = \bar{F}-c+x_t$, so the default region is to the northwest of this boundary. When the economy crosses the default boundary, there is a default today $(D_t = 1)$ and a fraction $(1-\eta)$ of debt is written down, leading the probability of another default tomorrow to jump downward. As shown, there are two regions where λ_t^R is sensitive to movements in (x_t, F_{t-1}). The first of these regions—the one towards the southeast—is where there is no default today $(D_t = 0)$, but where changes in current cashflows (x_t) and past debt (F_{t-1}) have a large impact on the default probability tomorrow. The second of these regions—the one towards the northwest—is where there is a default today

$(D_t = 1)$ and where changes in (x_t, F_{t-1}) have a large impact on λ_t^R.

The heatmap in Panel B shows how $\partial\lambda_t^R/\partial\lambda_t^B$ varies as a function of (x_t, F_{t-1}) when $\lambda_t^B = 0.2$. There are two highly reflexive regions where $\partial\lambda_t^R/\partial\lambda_t^B$ is large. The highly reflexive region in the southeast is where there is no default today $(D_t = 0)$ and where a small increase in λ_t^B has a large impact on the likelihood of a default tomorrow λ_t^R. Intuitively, in this region, a small increase in the backward-looking component of investors' beliefs (λ_t^B) triggers an increase in the firm's equilibrium debt burden, pushing the firm toward the brink of default. The second reflexive region in the northwest is where there is a default today $(D_t = 1)$ and where a small increase in λ_t^B has a large impact on the likelihood of another default tomorrow. Panel C shows these two results together, plotting both λ_t^R and $\partial\lambda_t^R/\partial\lambda_t^B$ versus F_{t-1} when $x_t = 1$ and $\lambda_t^B = 0.2$. Here the firm defaults today whenever $F_{t-1} \geq 4$. The economy is a highly reflexive state when F_{t-1} is near 2.5 (corresponding to the first reflexive region mentioned above) or near 6 (corresponding to the second region).

In summary, highly reflexive states are likely to arise near the end of a long credit boom where investors are still bullish, but default is not yet imminent. In this case, a credit crisis can suddently become far more likely if investors become slightly more bearish on credit (i.e., if λ_t^B rises slightly). Highly reflexive states can also arise in the wake of a credit bust where investors are still bearish. Here the likelihood that the credit crisis persists can drop dramatically if investors become slightly more bullish on credit (i.e., if λ_t^B falls slightly).

4.3 The "calm before the storm" phenomenon

An elevated level of credit market sentiment—here in the sense of a lower level of λ_t^B—slows down the accumulation of debt in the face of deteriorating cash flows fundamentals, thereby delaying or even preventing future defaults altogether. We term this phenomenon the "calm before the storm." Below we provide a formal result regarding this phenomenon.

Proposition 1 Calm before the storm. *Assume that $\theta > 0$. For any initial level of debt outstanding F_{t-1} and cash flow x_t, lowering the initial extrapolative component of investor beliefs λ_t^B weakly delays the next default path by path—i.e., for any given time series of future cash flow and sentiment shocks—and strictly delays the next default in expectation.*

To illustrate the "calm before the storm" phenomenon, Panel A of Figure 6 depicts a sample path of the model using our baseline set of parameters where $\theta = 0.5$. The cash flow fundamental x_t is initially set to $x_0 = 1.5 < 2 = c$ and debt is set to $F_0 = 3.5$. We assume that all of the subsequent shocks are zero $(\varepsilon_t = \omega_t = 0)$. Figure 6 plots cash flow x_t, debt outstanding F_t, the default indicator D_t, bond prices p_t, rational beliefs λ_t^R, and backward-looking behavioral beliefs λ_t^B. We compare the model dynamics starting from a low initial value $\lambda_0^B\,(Low) = 0.15$ and a high initial value $\lambda_0^B\,(High) = 0.30$ of the backward-looking component of beliefs. As can be seen, the firm defaults at time 3 when $\lambda_0^B = \lambda_0^B\,(High)$ and at time 4 and $\lambda_0^B = \lambda_0^B\,(Low)$. Consistent with Proposition 1, more optimistic initial beliefs have the potential to delay default in the face of poor fundamental cash flows.

Naturally, this effect becomes stronger for higher values of θ—i.e., as beliefs become more backward-looking. For example, Panel B of Figure 6 shows that, if we instead set $\theta = 1$, then the firm defaults at $t = 3$ when $\lambda_0^B = \lambda_0^B\,(High)$, but is able to skate by when $\lambda_0^B = \lambda_0^B\,(Low)$, narrowly averting default altogether. This happens because, in this case, bond prices stay high for long enough that the firm is able to refinance its debt until fundamentals rise back above c.

This "calm before the storm" behavior is consistent with the evidence in Krishnamurthy and Muir (2018) who examine the behavior of credit spreads around a large sample of financial crises in developed countries. Specifically, in our model, credit spreads are typically "too low" in the run-up to a default, but jump up on the eve of a default, just as Krishnamurthy and Muir (2018) find in the data. We can draw the link to Krishnamurthy and Muir's (2018) results more formally by tracing out the model-implied expected path of credit spreads conditional on a financial crisis at time $\tau = 0$. Specifically, we take simulated data from the model and estimate regression specifications of the form:

$$(1-\eta)\,\lambda_t^C = a + \textstyle\sum_{\tau=-T}^{T} b_\tau \mathbf{1}_{\{D_{t+\tau}=1\}} + e_t. \tag{16}$$

Here $(1-\eta)\,\lambda_t^C = 1 - p_t$ is the analog to the credit spread in our model. We then plot the b_τ regression coefficients versus event time τ for this regression in Figure 7, effectively tracing out the model-implied expected path of credit spreads in event time conditional on a financial crisis at time $\tau = 0$. For purposes of comparison, we repeat the exercise separately for the rational, forward-looking component of spreads, $(1-\eta)\,\lambda_t^R$, and the extrapolative, backward-looking component, $(1-\eta)\,\lambda_t^B$.

As shown in Figure 7, credit spreads $(1-\eta)\,\lambda_t^C = (1-\eta)\left[\theta\lambda_t^B + (1-\theta)\,\lambda_t^R\right]$ jump up on the eve of a crisis at $\tau = -1$ due to their rational forward-looking component $(1-\eta)\,\lambda_t^R$. However, comparing the coefficients for λ_t^R and λ_t^C, we see that credit spreads are typically "too narrow" prior to financial crises as argued in Krishnamurthy and Muir (2018). On the other hand, Figure 7 also shows that credit spreads are usually "too wide" in the aftermath of crises.

The calm before the storm phenomenon also helps make sense of what Gennaioli and Shleifer (2018) have dubbed the "quiet period" of the 2008 global financial crisis—the period between the initial disruptions in housing and credit markets in the summer of 2007 and onset of a full-blown financial crisis in the fall of 2008. Indeed, as Gennaioli and Shleifer (2018) argue, if investors were fully forward-looking ($\theta = 0$), one should have expected a more rapid deterioration of financial conditions in late 2007 rather than the slow slide into crisis that was witnessed.

4.4 The "default spiral" phenomenon

Once the storm hits the credit market, default extrapolation can generate a "default spiral": extrapolative, backward-looking beliefs lead to a form of default persistence that is absent when beliefs are fully rational and forward-looking (i.e., when $\theta = 0$). Specifically, investor beliefs typically become more pessimistic following a default according to equation (11). This pushes down bond prices, raising debt outstanding, and increasing the likelihood of future defaults.

Persistent default spirals can arise even when fundamental cash flows are strong ($x_t > c$) if (i) θ is sufficiently large, (ii) the increment α is large relative to the decay rate of extrapolative beliefs $(1 - \beta)$, (iii) the initial debt level is sufficiently high, and (iv) the initial backward-looking component of beliefs is sufficiently pessimistic. We formalize this observation in the following proposition.

Proposition 2 Default spirals. *Assume that $\theta > 0$. Suppose that (i) $F_{t-1}+c-x_t \geq \bar{F}$, so there is a default at time t ($D_t = 1$); (ii) $\alpha > (1-\beta)$ and $\omega_t = 0$, so extrapolative beliefs necessarily become more pessimistic following this default; (iii) extrapolative beliefs are initially relatively pessimistic ($\lambda_{t-1}^B \geq \lambda_{t-1}^R$); and (iv) $x_t = x_{t-1} = x > c$. Let $p_t(\theta)$, $F_t(\theta)$, and $\lambda_t^R(\theta)$ denote the time-t price, amount of outstanding debt, and true probability of default when a fraction θ of beliefs is backward-looking. Although default leads to a reduction in debt—i.e., $F_t(\theta) < F_{t-1}$ for any θ, $p_t(\theta)$ is decreasing in θ. And $F_t(\theta)$ and $\lambda_t^R(\theta)$ are increasing in θ. Thus, a larger extrapolative component of beliefs lowers prices and slows the process of debt discharge in the event of default, increasing the likelihood of a future default.*

Proposition 2 says that, even when current fundamentals are strong ($x_t > c$), a large accumulated debt balance can lead to a string of multiple defaults due to the negative feedback loop induced by default extrapolation. Specifically, if investor beliefs are highly backward-looking (i.e., if θ is high), then default extrapolation keeps bond prices low and the debt level high for many periods. As a result, there is a lengthy sequence of defaults, especially when investors are initially highly pessimistic (i.e., when λ_t^B is initially elevated). By contrast, if investor beliefs are largely rational and forward-looking (i.e., when θ is low), the debt writedown that occurs upon default leads to an immediate decrease in the rationally-expected default rate λ_t^R. As a result, bond prices quickly recover following the default and the firm rapidly repays its debts.

This default spiral dynamic highlights the potential disconnect between the endogenous credit cycle and the underlying business cycle that is at the heart of our model. In particular, the extrapolative nature of investor beliefs can make the financial recovery from a crisis slower and more protracted than in a world with fully forward-looking investors. This dynamic means that a moderate improvement in cash flows can be insufficient to "rescue" credit markets from a depressed state. Moreover, the likely timing of the recovery is influenced by the extent of backward-looking extrapolation (θ) and the initial pessimism of investor beliefs (λ_0^B): one needs a large improvement in cash flows to ensure a recovery when both θ and λ_0^B are high. The crucial role that investor beliefs play in driving default spirals suggests that a favorable sentiment shock (i.e., a large negative draw of ω_t) coming from a policy intervention may also be an effective way to help credit markets recover.

4.5 Bond return predictability

Bond returns are predictable in our model whenever $\theta > 0$—i.e., whenever beliefs are partially extrapolative and backward-looking. To see this, note that an investor who buy bonds for a price of $p_t = p(\lambda_t^B, \lambda_t^R)$ at time t will receive a payment of $1 - (1-\eta)D_{t+1}$ at time $t+1$. Thus, the realized return on risky bonds from time t to $t+1$ is

$$r_{t+1} = \frac{1-(1-\eta)D_{t+1}}{p(\lambda_t^B, \lambda_t^R)} - 1. \tag{17}$$

At any time t, investors in our model believe that $\mathbb{E}_t^C[D_{t+1}] = \lambda_t^C$ and bond prices are $p(\lambda_t^B, \lambda_t^R) = 1-(1-\eta)\mathbb{E}_t^C[D_{t+1}] = 1-(1-\eta)\lambda_t^C$. Thus, by construction, investors always perceive a zero expected return on bonds from time t to $t+1$—i.e., $\mathbb{E}_t^C[r_{t+1}] = 0$. However, since $\mathbb{E}_t^R[D_{t+1}] = \lambda_t^R$ from the vantage point of a rational econometrician, the rationally-expected return on bonds is

$$\mathbb{E}_t^R[r_{t+1}] = \frac{1-(1-\eta)\lambda_t^R}{p(\lambda_t^B, \lambda_t^R)} - 1 = \frac{-(1-\eta)\theta(\lambda_t^R - \lambda_t^B)}{1-(1-\eta)\lambda_t^R + (1-\eta)\theta(\lambda_t^R - \lambda_t^B)}. \tag{18}$$

Thus, when $\theta > 0$, expected bond returns are negative (positive) when investors are overly bullish (bearish) about default probabilities—i.e., $\mathbb{E}_t^R[r_{t+1}] \gtreqless 0$ as $\lambda_t^R \lesseqgtr \lambda_t^B$. (Obviously, we always have $\mathcal{E}_t^R[r_{t+1}] = 0$ when $\theta = 0$.) For instance, in a "calm before the storm" scenario where firm fundamentals have deteriorated but extrapolative investors remain bullish on credit, we have $\lambda_t^R > \lambda_t^B$ and $\mathbb{E}_t^R[r_{t+1}] < 0$. Conversely, in a "default spiral" scenario where investors are over-estimating the likelihood of future defaults because they have just witnessed a default, $\lambda_t^R < \lambda_t^B$ and $\mathbb{E}_t^R[r_{t+1}] > 0$.

The fact bond returns are predictable is not surprising given our assumption that investor beliefs are not fully rational. However, the comparative statics of expected bond returns $\mathbb{E}_t^R[r_{t+1}]$ are informative and are consistent with much recent research on credit cycles. First, using equation (18) we can ask how a small change in λ_t^R impacts expected bond returns. When $\theta > 0$, holding fixed λ_t^B, an increase in λ_t^R is associated with a decline in expected returns:

$$\frac{\partial \mathbb{E}_t^R[r_{t+1}]}{\partial \lambda_t^R} = -\frac{\theta(1-\eta)\left(1-\lambda_t^B(1-\eta)\right)}{\left[p(\lambda_t^B, \lambda_t^R)\right]^2} < 0. \tag{19}$$

If we interpret λ_t^R as an inverse measure of issuing firms' creditworthiness, then equation (19) is consistent with the findings in Greenwood and Hanson (2013) who find that a deterioration the credit quality of issuing firms negatively predicts future debt returns in a univariate sense. Going further, and assuming we are not at the default boundary, Lemma 1 implies that, all else equal, a small increase in F_{t-1} leads to decline in $\mathbb{E}_t^R[r_{t+1}]$ and a small increase in x_t leads to an increase in $\mathbb{E}_t^R[r_{t+1}]$. Intuitively, when investors are extrapolative, holding fixed the extrapolative component of beliefs λ_t^B, worse cash flow fundamentals and higher levels of leverage predict lower future bond returns.

What is more interesting is that changes in investor sentiment—i.e., movements in λ_t^B—have an ambiguous impact on expected bond returns due to the reflexive nature of credit markets. Holding fixed expected future debt repayments, more bearish investor beliefs (higher values of λ_t^B) lower bond prices, raising expected bond returns. This is the intuition we have from standard settings where beliefs do not impact security payoffs. However, there is a competing effect that arises in our model because investor beliefs about future defaults are partially self-fulfilling. Specifically, more bearish investor sentiment makes it more difficult for firms to refinance maturing debt, raising the true probability of default and lowering expected future debt repayments. And, in highly reflexive states where investor beliefs have a large impact on the true likelihood of default—i.e., where $\partial\lambda_t^R/\partial\lambda_t^B > 0$ is large—the latter effect can outweigh the former. As a result, the total impact of a shift in λ_t^B on expected returns is ambiguous: depending on which effect dominates, a small increase in λ_t^B can either lead $\mathbb{E}_t^R[r_{t+1}]$ to rise or fall. Formally, assuming we are at an equilibrium continuity point where $\partial\lambda_t^R/\partial\lambda_t^B$ exists,[17] we have

$$\begin{aligned}\frac{\partial\mathbb{E}_t^R[r_{t+1}]}{\partial\lambda_t^B} &= \overbrace{\frac{\partial\mathbb{E}_t^R[r_{t+1}]}{\partial\lambda_t^B}\Big|_{\lambda_t^R=\text{Constant}}}^{>0} + \overbrace{\frac{\partial\mathbb{E}_t^R[r_{t+1}]}{\partial\lambda_t^R}}^{<0} \times \overbrace{\frac{\partial\lambda_t^R}{\partial\lambda_t^B}}^{>0} \\ &= \frac{\theta(1-\eta)}{\left[p(\lambda_t^B,\lambda_t^R)\right]^2}\left[\left(1-\lambda_t^R(1-\eta)\right)-\left(1-\lambda_t^B(1-\eta)\right)\frac{\partial\lambda_t^R}{\partial\lambda_t^B}\right],\end{aligned} \tag{20}$$

which is ambiguous. And, we are more likely to have $\partial\mathbb{E}_t^R[r_{t+1}]/\partial\lambda_t^B < 0$ in highly reflexive states where $\partial\lambda_t^R/\partial\lambda_t^B > 0$ is large—e.g., near the end of a long credit boom where investors are still bullish, but default is not yet imminent.

Figure 8 shows the potentially ambiguous relationship between $\mathbb{E}_t^R[r_{t+1}]$ and λ_t^B. The figure plots $\mathbb{E}_t^R[r_{t+1}]$ and λ_t^R versus λ_t^B using our baseline parameter values for $(x_t, F_{t-1}) = (1.6, 3.4)$, which is a highly reflexive state. For λ_t^B less than 0.26, λ_t^R rises gradually with λ_t^B and $\mathbb{E}_t^R[r_{t+1}]$ is increasing in λ_t^B. In this range, the negative effect of λ_t^B on bond price outweighs the positive effect on λ_t^R. For λ_t^B between 0.26 and 0.32, λ_t^R rises more rapidly with λ_t^B and $\mathbb{E}_t^R[r_{t+1}]$ is decreasing in λ_t^B: here the positive effect on λ_t^R outweighs the negative effect on price. At

[17] As explained in Lemma 1, the derivative $\partial\lambda_t^R/\partial\lambda_t^B > 0$ only exists at equilibrium continuity points. At equilibrium discontinuity points, λ_t^R jumps up discretely in response to a small increase in λ_t^B.

$\lambda_t^B = 0.33$, λ_t^R jumps up discretely—the low default probability equilibrium disappears—and expected returns fall significantly. Returns continue falling until λ_t^B reaches 0.35 after which they are again increasing.

Our model is also consistent with the multivariate return forecasting regressions emphasized in Greenwood and Hanson (2013) and López-Salido, Stein, and Zakrajšek (2017). These authors estimate forecasting regressions of the form:

$$r_{t+1} = \alpha + \beta_1 \cdot \textit{credit-spread}_t + \beta_2 \cdot \textit{low-quality-issuance}_t + \xi_{t+1}, \tag{21}$$

where $\textit{credit-spread}_t$ is a measure of credit spreads and $\textit{low-quality-issuance}_t$ is an inverse measure of the creditworthiness of issuing firms. These authors find that $\beta_1 > 0$ and $\beta_2 < 0$: all else equal, future bond returns are high when current credit spreads are wide and are low when current debt issuers are less creditworthy. While credit spreads, borrower credit quality, and expected debt returns are endogenous equilibrium outcomes in our model, our model implies a strong multivariate forecasting relationship of the form found in the data. Specifically, the model analog of $\textit{credit-spread}_t$ is $1 - p_t = (1-\eta)\left[\theta\lambda_t^B + (1-\theta)\,\lambda_t^R\right]$, and the model analog of $\textit{low-quality-issuance}_t$ is λ_t^R. Since

$$\mathbb{E}_t^R[r_{t+1}] \approx -k \cdot (1-\eta)\theta(\lambda_t^R - \lambda_t^B) = k \cdot [\textit{credit-spread}_t - (1-\eta) \cdot \textit{low-quality-issuance}_t]\,,$$

by equation (18) for some constant $k > 1$, we have $\beta_1 \approx k > 0$ and $\beta_2 \approx -(1-\eta)\,k < 0$. Intuitively, holding fixed $\textit{low-quality-issuance}_t = \lambda_t^R$, higher credit spreads signal higher values of λ_t^B—i.e., more bearish investor sentiment—and, thus, higher expected bond returns. Conversely, holding fixed credit spreads, more $\textit{low-quality-issuance}_t$ signals a larger gap between true credit risk and the risk perceived by investors—i.e., $\lambda_t^R - \lambda_t^C = \theta(\lambda_t^R - \lambda_t^B)$—and, therefore, lower expected returns.

We collect these observations in Proposition 3.

Proposition 3 Return predictability. *If investor beliefs are fully rational, then bond returns cannot be predicted. Formally, if $\theta = 0$, then $\mathbb{E}_t^R[r_{t+1}] = 0$.*

If investor beliefs are partially extrapolative, then bond returns are predictable. Specifically, when $\theta > 0$, $\mathbb{E}_t^R[r_{t+1}]$ is decreasing in $\lambda_t^R - \lambda_t^B$ and is equal to zero when $\lambda_t^R - \lambda_t^B = 0$.

- *Holding fixed λ_t^B, a small increase in λ_t^R is associated with a decrease in $\mathbb{E}_t^R[r_{t+1}]$. As a result, if the economy is not at the default boundary at time t, then, all else equal, $\mathbb{E}_t^R[r_{t+1}]$ is increasing in x_t and is decreasing in F_{t-1}. However, these relationships flip signs when the economy is at the default boundary.*

- *Holding fixed x_t and F_{t-1}, a small increase in λ_t^B has an ambiguous effect on $\mathbb{E}_t^R[r_{t+1}]$:*

– *In non-reflexive states—where a small increase in* λ_t^B *has a small effect on* λ_t^R*—a small increase in* λ_t^B *leads to an increase in* $\mathbb{E}_t^R[r_{t+1}]$*.*

– *In highly reflexive states—where a small increase in* λ_t^B *has a large effect on* λ_t^R*—a small increase in* λ_t^B *leads to a decline in* $\mathbb{E}_t^R[r_{t+1}]$*.*

- *We have* $\beta_1 > 0$ *and* $\beta_2 < 0$ *for the following multivariate regression:*

$$r_{t+1} = \alpha + \beta_1 \cdot \textit{credit-spread}_t + \beta_2 \cdot \textit{low-quality-issuance}_t + \xi_{t+1},$$

where $\textit{credit-spread}_t \equiv 1 - p_t$ *and* $\textit{low-quality-issuance}_t \equiv \lambda_t^R$*.*

Another question we can address is whether rapid credit growth negatively forecasts the returns on risky bonds and other credit-sensitive instruments documented in Greenwood and Hanson (2013) and Baron and Xiong (2017) and suggested in Schularick and Taylor (2012). Assuming that $\underline{F} < F_{t-1} + c - x_t < \bar{F}$, the change is debt outstanding at time t is

$$\Delta F_t = \frac{F_{t-1} + c - x_t}{p_t} - F_{t-1}.$$

Using Lemma 2, it is straightforward to show that $\partial \Delta F_t / \partial x_t < 0$, $\partial \Delta F_t / \partial F_{t-1} > 0$, and $\partial \Delta F_t / \partial \lambda_t^B > 0$ when the economy is not near the default boundary. Combining these results with those in Proposition 3, one would expect large values of ΔF_t to predict low future values of r_{t+1}. This occurs because changes in x_t and F_{t-1} have opposing effects on $\mathbb{E}_t^R[r_{t+1}]$ and ΔF_t. And, changes in λ_t^B will have opposing effects on ΔF_t and $\mathbb{E}_t^R[r_{t+1}]$ in reflexive states where $\partial \lambda_t^R / \partial \lambda_t^B$ is large. Thus, one expects the model to yield a negative relationship between ΔF_t and $\mathbb{E}_t^R[r_{t+1}]$ and, indeed, we confirm this in numerical simulations below.

Forecasting returns and defaults in model simulations. To further explore the model's implications for return and default predictability, we first simulate the model for 100,000 periods using the baseline parameters introduced above; each period represents one year. We then examine the return and default forecasting regressions using current variables such as credit growth and sentiment.

Table 3 shows that the model is able to match a number of facts that researchers have documented about the credit cycle. First, consistent with Proposition 3, a higher level of firm cash flow (x_t) and a lower level of firm debt each positively forecast future bond returns. And, a lower level of cash flow (x_t) and a higher level of debt both positively forecast future defaults.

Second, consistent with the findings in Greenwood and Hanson (2013) and Barron and Xiong (2017), recent credit growth negatively forecasts future bond returns: regressing returns over the next year (r_{t+1}) on the debt growth over the prior four years ($F_t - F_{t-4}$) yields a coefficient of

-0.04 with an R-squared of 33%. Furthermore, high recent credit growth also positively forecasts defaults.

Third, credit spreads forecast future bond returns and defaults. Somewhat surprisingly, in simulations using our baseline parameters, high credit spreads ($\textit{credit-spread}_t = 1-p_t$) negatively forecast future bond returns in a univariate regression—i.e., high bond prices predict high future returns. However, this is not a robust implication of our model. Indeed, if we increase θ or σ_ω^2, thereby raising the extrapolative component of beliefs or the volatility of sentiment shocks, this raises the coefficient on $\textit{credit-spread}_t$, enabling the model to match the positive univariate forecasting relationship between credit credits and bond returns that we see in the data.[18]

Fourth, elevated credit market sentiment—here in the sense that $\lambda_t^R - \lambda_t^B$ is large—strongly predicts future returns and future defaults. Since $\mathbb{E}_t^R[r_{t+1}] \approx -k \cdot (1-\eta)\theta(\lambda_t^R - \lambda_t^B)$, regressing bond returns over the next year on the current level of sentiment $(\lambda_t^R - \lambda_t^B)$, the model generates a coefficient of -0.34 with an R-squared of 82%. Regressing one-period ahead defaults on the current level of sentiment, the coefficient is 0.90, with an R-squared of 52%. Intuitively, towards the end of the "calm before the storm" period, sentiment rises because the true default likelihood increases while the backward-looking extrapolative expectations does not keep pace.

Fifth, the rationally-expected default rate λ_t^R, which can be interpreted as a measure of low-quality debt issuance (i.e., $\textit{low-quality-issuance}_t \equiv \lambda_t^R$), is a strong negative predictor of future returns. This is because the backward-looking component of beliefs, λ_t^B, does not move one-for-one with λ_t^R: the coefficient from a regression of λ_t^B on λ_t^R is just 0.39. As a result, the univariate relationship between expected returns and λ_t^R is strong. By contrast, λ_t^B is a not a strong univariate predictor of future returns. This is because λ_t^R moves nearly one-for-one with λ_t^B: the coefficient from a regression of λ_t^R on λ_t^B is 0.86, reflecting the reflexivity effect $(\partial\lambda_t^R/\partial\lambda_t^B > 0)$ that is at the heart of our model. As a result, the univariate relationship between λ_t^B and future returns r_{t+1} is weak.[19] Furthermore, the large coefficient that one obtains from

[18] The univariate regression coefficient of r_{t+1} on $\textit{credit-spread}_t$ is:

$$\frac{Cov\left[r_{t+1}, \textit{credit-spread}_t\right]}{Var\left[\textit{credit-spread}_t\right]} \approx k \cdot \frac{\theta^2 Var[\lambda_t^B - \lambda_t^R] + \theta Cov[\lambda_t^B - \lambda_t^R, \lambda_t^R]}{\theta^2 Var[\lambda_t^B - \lambda_t^R] + 2\theta Cov[\lambda_t^B - \lambda_t^R, \lambda_t^R] + Var[\lambda_t^R]}.$$

In our baseline calibration, the coefficient is negative because $\theta Cov[\lambda_t^B - \lambda_t^R, \lambda_t^R] < -\theta^2 Var[\lambda_t^B - \lambda_t^R] < 0$. Furthermore, since

$$\frac{1}{k}\frac{Cov[r_{t+1}, \textit{credit-spread}_t]}{Var[\textit{credit-spread}_t]} + (1-\eta) \cdot \frac{Cov[D_{t+1}, \textit{credit-spread}_t]}{Var[\textit{credit-spread}_t]} \approx 1,$$

—i.e., high credit spreads must either predict high returns on bond or high future defaults—the negative forecasting relationship between $\textit{credit-spread}_t$ and r_{t+1} is the mirror image of very strong positive relatiohsip between $\textit{credit-spread}_t$ and D_{t+1}. However, if we increase θ or σ_ω^2, this tends to raise $Cov\left[r_{t+1}, \textit{credit-spread}_t\right]/Var\left[\textit{credit-spread}_t\right]$ and lower $Cov\left[D_{t+1}, \textit{credit-spread}_t\right]/Var\left[\textit{credit-spread}_t\right]$, matching the univariate forecasting relationships like the ones we see in the data.

[19] Formally, we have $Cov[\mathbb{E}_t^R[r_{t+1}], \lambda_t^R]/Var[\lambda_t^R] = -k \cdot (1-\eta)\theta(1 - Cov[\lambda_t^B, \lambda_t^R]/Var[\lambda_t^R])$ and $Cov[\mathbb{E}_t^R[r_{t+1}], \lambda_t^B]/Var[\lambda_t^B] = -k \cdot (1-\eta)\theta(Cov[\lambda_t^B, \lambda_t^R]/Var[\lambda_t^B] - 1)$.

a regression of λ_t^R on λ_t^B means that backward-looking beliefs are often reasonable, in the sense that they often correlate with forward-looking beliefs.

Finally, we report the results from multivariate regressions of returns and defaults on both $\textit{credit-spread}_t \equiv 1 - p_t$ and $\textit{low-quality-issuance}_t \equiv \lambda_t^R$. As explained in Proposition 3 and consistent with Greenwood and Hanson (2013) and López-Salido, Stein, and Zakrajšek (2017), credit spreads positively forecast returns and low-quality issuance negatively predicts returns in this multivariate specification. Indeed, these two observable variables contain all the information that is contained in sentiment, $\lambda_t^R - \lambda_t^B$. Furthermore, both spreads and $\textit{low-quality-issuance}_t$ positively predict future default rates.

4.6 Model-implied impulse-response functions

To further explore the dynamic behavior implied by the model, we report impulse-response functions which trace out the dynamic impact of shocks to underlying cash flow fundamentals and investor sentiment. Let $\mathbf{z}_t = (x_t, F_{t-1}, \lambda_t^B)$ denote the model's state vector and consider some model-implied quantity y_t. The response of y_{t+j} following an impulse $\varepsilon_t = s_\varepsilon$ to cash flow fundamentals x_t at time t is:

$$\Phi_y\left(j, \mathbf{z}_{t-1}, \varepsilon_t = s_\varepsilon\right) = \mathbb{E}^R[y_{t+j}|\mathbf{z}_{t-1}, \varepsilon_t = s_\varepsilon] - \mathbb{E}^R[y_{t+j}|\mathbf{z}_{t-1}, \varepsilon_t = 0].$$

Similarly, the response of y_{t+j} following an impulse $\omega_t = s_\omega$ to investor sentiment λ_t^B at time t is:

$$\Phi_y\left(j, \mathbf{z}_{t-1}, \omega_t = s_\omega\right) = \mathbb{E}^R[y_{t+j}|\mathbf{z}_{t-1}, \omega_t = s_\omega] - \mathbb{E}^R[y_{t+j}|\mathbf{z}_{t-1}, \omega_t = 0].$$

Due to the nonlinear nature of the model, these impulse response functions (IRFs) can be asymmetric in the sense that, for example, $\Phi_y\left(j, \mathbf{z}_{t-1}, \omega_t = -s_\omega\right) \neq -\Phi_y\left(j, \mathbf{z}_{t-1}, \omega_t = s_\omega\right)$. The IRFs are also state-contingent in the sense that both $\Phi_y\left(j, \mathbf{z}_{t-1}, \varepsilon_t = s_\varepsilon\right)$ and $\Phi_y\left(j, \mathbf{z}_{t-1}, \omega_t = s_\omega\right)$ depend on the initial condition $\mathbf{z}_{t-1}$.

Figure 9 shows the IRFs for an impulse to cash flows x_t and to beliefs λ_t^B at time 1.[20] The initial condition in Figure 9 is $x_0 = 2.25$, $F_{-1} = 2.25$, and $\lambda_0^B = 0.30$. This is a non-reflexive region of the parameter space: firm leverage is low and cash flows are strong, so changes in investor beliefs are unlikely to have a big impact on the true likelihood of default. Nonetheless, the impulse-responses in Figure 9 are highly asymmetric. Starting from this initial condition, bad shocks to either fundamentals (a downward shock to firm cash flows x_t) or investor beliefs (an upward shock to the perceived default likelihood λ_t^B) have a much larger and more persistent

[20] To compute these IRFs, we shock x_t or λ_t^B up or down at $t = 1$ and then generate $10,000$ random paths following this shock. We also generate $10,000$ random paths in the absence of a shock at $t = 1$. The IRF is just the difference in outcomes between the average path following this shock and the average path in the absence of a shock.

effects on credit market outcomes than good shocks.[21]

Figure 10 shows the same impulse-response functions starting from an initial condition of $x_0 = 1.6$, $F_{-1} = 3.4$, and $\lambda_0^B = 0.33$. This is a highly reflexive region of the parameter space: the firm is entering financial distress, so changes in investor beliefs can have a large impact on the true probability of default. First, consider an impulse to cash flows x_t at time 1. Compared to the responses in Figure 9, the same impulse to fundamentals now has a far larger impact on other model-implied quantities in Figure 10. In this way, our model naturally captures the fact that the build up of debt creates fragility as emphasized by Krishnamurthy and Muir (2018). In Figure 10, a positive shock to fundamentals at time 1 often helps avert what would otherwise be a likely default at time 2. Next, consider a shock to investor beliefs at time 1. In this highly reflexive region, a shock to investor beliefs has far larger impact on debt accumulation and defaults than in the less reflexive region shown in Figure 9. (Note the difference in the y-axis scales in Figures 9 and 10.)

5 Model Extensions

In this section, we briefly consider two extensions of the baseline model. The first model extension allows for opportunistic debt issuance by firms to exploit the mispricing of debt. Under this extension, credit booms can naturally sow the seeds of their own destruction. The second model extension features multiple firms who face idiosyncratic cash flow shocks. This extension delivers more empirically realistic dynamics of the aggregate default rate.

5.1 Opportunistic debt issuance

The first model extension allows for opportunistic debt issuance by firms to exploit the mispricing of debt. This extension addresses a first limitation of our baseline model: there is no sense in which a credit boom that is triggered by elevated levels of credit market sentiment naturally sows the seeds of its own destruction. Instead, all else equal, high levels of credit sentiment—i.e., lower levels of λ_t^B—always lead to a slower accumulation of debt, reducing the likelihood of a future crisis.[22] However, the boom-bust narratives in Kindleberger (1978) and Minsky (1986) suggest that one might instead think a large over-valuation of risky debt (i.e., a large gap between λ_t^R and λ_t^B) could lead to an opportunistic increase in bond supply from firms—with extrapolative investors underreacting to the resulting increase in firm leverage, thereby raising the risk of a

[21] Following shocks to sentiment, the saw-tooth patterns arise, even in expectation, because of the jaggedness of debt outstanding in individual sample paths due to our mechanistic default rule.

[22] To see this recall that, assuming that $\underline{F} < F_{t-1} + c - x_t < \bar{F}$, the change is debt at time t is $\Delta F_t = (F_{t-1} + c - x_t) / p_t - F_{t-1}$ which is decreasing in the bond price p_t and, thus, increasing in λ_t^B.

future credit crisis.[23]

As explained in the Appendix, this Minskian dynamic does not occur in our baseline model because both the demand and supply of bonds are downward-sloping: the non-standard downward-sloping supply curve arises because the supply of bonds is determined by firms' binding sources-and-used constraint. Of course, it is precisely the fact that both demand and supply slope downwards that makes investor beliefs potentially self-fulfilling. However, the fact that supply cannot be upward-sloping precludes the kind of opportunistic supply response that might allow a credit boom to sow the seeds of its own destruction.

To allow for an opportunistic bond supply by firms, we instead assume that:

$$\begin{aligned} F_t &= \frac{F_{t-1}+c-x_t}{p_t} + M \times \left[p_t - \left(1-(1-\eta)\,\lambda_t^R\right)\right] \\ &= \frac{F_{t-1}+c-x_t}{1-\textit{credit-spread}_t} + M \times \left[\textit{credit-spread}_t^R - \textit{credit-spread}_t\right] \\ &\approx \frac{F_{t-1}+c-x_t}{1-\textit{credit-spread}_t} - (M/k)\,\mathbb{E}_t^R[r_{t+1}]. \end{aligned} \tag{22}$$

Here $M \geq 0$ controls the aggressiveness of firms' opportunistic supply response in response to debt mispricing, $\textit{credit-spread}_t = 1 - p_t = (1-\eta)\,\lambda_t^C$, $\textit{credit-spread}_t^R = (1-\eta)\,\lambda_t^R$, and the final line follows from the fact that $\mathbb{E}_t^R[r_{t+1}] \approx -k\,(1-\eta)\,\theta\left(\lambda_t^R - \lambda_t^B\right) = -k \times \left[\textit{credit-spread}_t^R - \textit{credit-spread}_t\right]$. In this extension, opportunistic debt issuance raises firm leverage, but does not affect the quantity and quality of firm investment projects which are assumed to be fixed. Specifically, equation (22) implies that, all else equal, firms opportunistically take on additional leverage when credit spreads are too low—i.e., when rationally-expected returns are low.[24] As discussed in the Appendix, we can solve for the equilibrium level of λ_t^R as before using equation (13), but now setting $F\left(F_{t-1}, \lambda_t^R, \lambda_t^B, x_t\right)$ to $\left(F_{t-1}+c-x_t\right)/p\left(\lambda_t^R, \lambda_t^B\right) + M \times \left[p\left(\lambda_t^R, \lambda_t^B\right) - \left(1-(1-\eta)\,\lambda_t^R\right)\right]$ in equation (10) when $\underline{F} < F_{t-1} + c - x_t < \bar{F}$.

With this modification, the impact on F_t of a change in λ_t^B has an ambiguous sign. Specifically, when the firm is nearing default and $\left(F_{t-1}+c-x_t\right)/p\left(\lambda_t^R, \lambda_t^B\right)$ is large, a decline in λ_t^B will lower F_t just as in our baseline model. However, if the opportunistic supply response is sufficiently large, then when the firm is far from default and $\left(F_{t-1}+c-x_t\right)/p\left(\lambda_t^R, \lambda_t^B\right)$ is small, a decline in λ_t^B can raise F_t. In other words, favorable credit market sentiment can lead to a boom—rising debt issuance and a decline in credit quality—that sows the seeds of its own destruction by increasing future default probabilities.

Using this modified model with $M = 5$, Figure 11 shows the IRFs for an impulse to cash flows

[23] Greenwood and Hanson (2013) introduce an earlier model along these lines, although their model does not feature reflexivity.

[24] Going further, we could also allow the quantity and quality (e.g., as parameterized by $\overline{x}$ and c) of firm investment projects to depend on debt mispricing. Doing so would likely amplify the real effects of credit booms and busts even further.

x_t and to beliefs λ_t^B. The initial condition is the same as in Figure 9: $x_0 = 2.25$, $F_{-1} = 2.25$, and $\lambda_0^B = 0.30$, which is a non-reflexive region. The responses following an impulse to cash flows are similar in Figures 9 and 11. However, in Figure 11, firms' opportunistic supply response means that—in contrast to the baseline IRFs shown in Figure 9—a downward shock to λ_t^B now triggers an increase in outstanding debt F_t and, hence, a rise in firm leverage and the likelihood of a future default crisis. In summary, firms' opportunistic response to credit market sentiment means that credit booms have the potential to sow the seeds of their own destruction.

5.2 Multiple firms

The second model extension features multiple firms who face idiosyncratic cash flow shocks. This extension addresses a limitation of baseline model which is that, with a single representative firm, defaults are necessarily binary events. Allowing for multiple firms naturally yields a continuous default rate for the economy and leads to more realistic model-implied dynamics.

We assume that there are N firms, $i = 1, 2, \ldots, N$. The exogenous cash flow of firm i, x_{it}, consists of two components:

$$x_{it} = x_t + z_{it}, \tag{23}$$

where the systematic component x_t evolves according to equation (3) and the mean-zero, firm-specific component z_{it} follows

$$z_{it} = \psi z_{it-1} + \xi_{it}, \tag{24}$$

where $\xi_{it} \sim \mathcal{N}(0, \sigma_\xi^2)$ is $i.i.d.$ over time, independent across firms, and independent of the systematic cash flow shock (ε_t) and the aggregate sentiment shock (ω_t).

We need to make an assumption about how investors price firms' bonds. For simplicity, we assume that all firms' bonds are priced identically even though firms have heterogeneous cash flows and debt levels. This assumption can be seen as a short-hand for the idea that investors cannot perfectly observe each firm's cash flow x_{it} and leverage F_{it-1} and treat some class of firms as a homogeneous category. The rule for firm default is similar to that in the baseline model: if at any time t, $F_{it-1} + c - x_{it}$ rises above $\bar{F}$, then firm i defaults at time t. Thus, the law of motion for each firm's outstanding bonds F_{it} is similar to the baseline model. Specifically, we have

$$F_{it} = F\left(F_{it-1}, \lambda_t^B, \lambda_t^R, x_{it}\right) = \begin{cases} \underline{F}/p(\lambda_t^B, \lambda_t^R) & \text{if } F_{it-1} + c - x_{it} \leq \underline{F} \\ (F_{it-1} + c - x_{it})/p(\lambda_t^B, \lambda_t^R) & \text{if } \underline{F} < F_{it-1} + c - x_{it} < \bar{F} \\ \eta(F_{it-1} + c - x_{it})/p(\lambda_t^B, \lambda_t^R) & \text{if } \bar{F} \leq F_{it-1} + c - x_{it} \end{cases}, \tag{25}$$

where

$$p(\lambda_t^B, \lambda_t^R) = \left[1 - (1-\eta)\lambda_t^R\right] + (1-\eta)\theta(\lambda_t^R - \lambda_t^B) \tag{26}$$

is the price of corporate bonds.

With multiple firms, the two components of investor beliefs λ_t^B and λ_t^R are specified as follows. Let $D_{it} = \mathbf{1}_{\left\{F_{it-1}+c-x_{it}\geq\bar{F}\right\}}$ be an binary variable indicating whether firm i defaults at time t and let $\overline{D}_t = N^{-1}\sum_{i=1}^{N} D_{it}$ denote the economy-wide default rate at time t. We then assume that

$$\lambda_t^B = \max\left\{0, \min\left\{1, \beta\lambda_{t-1}^B + \alpha\overline{D}_t + \omega_t\right\}\right\}, \tag{27}$$

and

$$\begin{aligned}\lambda_t^R &= g(\lambda_t^R|\{F_{it-1}\}_{i=1}^N, \{z_{it}\}_{i=1}^N, x_t, \lambda_t^B) \\ &\equiv \frac{1}{N}\sum_{i=1}^{N}\Phi\left(\frac{F(F_{it-1}, \lambda_t^B, \lambda_t^R, x_{it}) - \bar{F} + c - \rho x_t - (1-\rho)\bar{x} - \psi z_{it}}{\sqrt{\sigma_\varepsilon^2 + \sigma_\xi^2}}\right).\end{aligned} \tag{28}$$

Thus, the law of motion for λ_t^B with multiple firms in equation (27) is analogous to that with a representative firm in (11) with the continuous economy-wide default rate $\overline{D}_t \in [0,1]$ replacing the binary default indicator $D_t \in \{0,1\}$ for the representative firm. For instance, assuming $\omega_t = 0$ and $\lambda_t^B \in (0,1)$, we have $\lambda_t^B - \lambda_{t-1}^B = \alpha\overline{D}_t - (1-\beta)\,\lambda_{t-1}^B$, so $\lambda_t^B \gtreqless \lambda_{t-1}^B$ as $\overline{D}_t \gtreqless [(1-\beta)/\alpha]\cdot\lambda_{t-1}^B$. And, by equation (28), λ_t^R is the rationally-expected economy-wide default rate at at time $t+1$—i.e., $\lambda_t^R = \mathbb{E}_t^R\left[\overline{D}_{t+1}\right]$.

We make three observations regarding the setup with multiple firms. First, just as in the baseline model, equations (25) and (28) imply that the right hand side of (28) can be viewed as a continuous function of λ_t^R that maps the unit interval into itself. Therefore, by Brouwer's fixed-point theorem, a solution exists. But, in addition to x_t and λ_t^B, the distributions of $\{F_{it-1}\}_{i=1}^N$ and $\{z_{it}\}_{i=1}^N$ now impact λ_t^R. Nonetheless, the behavior of λ_t^R with multiple firms is qualitatively similar to that described in Lemma 2 for a single representative firm. Second, equations (27) and (28) imply that there is belief contagion: the past defaults and likely future defaults of each firm affect the bond price that applies to all firms. Third, one consequence of having firms with different cash flow and debt levels is that, at each point in time, only a fraction of firms is close to default. This makes it more difficult for the multiple equilibria described in Section 3 to arise.

The model with multiple firms yields similar qualitative implications to the baseline model featuring a single representative firm, albeit with more realistic time-series dynamics. For instance, the model with multiple firms still features the "calm before the storm" and "default spiral" phenomena. Similarly, since the realized return on an equal-weighted portfolio of risky bonds from time t to $t+1$ is

$$\overline{r}_{t+1} = \frac{1-(1-\eta)\overline{D}_{t+1}}{p(\lambda_t^B, \lambda_t^R)} - 1, \tag{29}$$

and, since $\mathbb{E}_t^R\left[\overline{D}_{t+1}\right]=\lambda_t^R$, we have

$$\mathbb{E}_t^R\left[\overline{r}_{t+1}\right]=\frac{1-(1-\eta)\lambda_t^R}{p(\lambda_t^B,\lambda_t^R)}-1=\frac{-(1-\eta)\theta(\lambda_t^R-\lambda_t^B)}{1-(1-\eta)\lambda_t^R+(1-\eta)\theta(\lambda_t^R-\lambda_t^B)}, \tag{30}$$

just as in the baseline model. Thus, the model with multiple heterogeneous firms has similar qualitative implications for bond return predictability as the baseline model with a representative firm.

Quantitatively, however, the model with multiple heterogeneous firms leads to more realistic time-series dynamics. To illustrate, Panel A of Figure 12 reports a sample path of the model with $N=100$ firms. As a comparison, Panel B of Figure 12 plots the sample path using the same aggregate cash flow shocks and sentiment shocks but with a single representative firm. As can be seen, the dynamics of the economy-wide aggregates from the model with multiple heterogeneous firms are more empirically realistic than their counterparts from the model with a single representative firms.

In the example shown in Panel A of Figure 12, after staying above the long-run mean $\bar{x}$ for many periods, cash flow fundamentals x_t begin to deteriorate in period 32. The actual default rate stays lows for one more period—a "calm before the storm" period—and then starts to rise in period 34. Furthermore, there is a clear lead-lag structure between the rational and the behavioral components of investor beliefs: λ_t^R responds to deteriorating market fundamentals in period 34 while λ_t^B only responds several periods later. Similarly, λ_t^R responds to improving market fundamentals in period 41 while λ_t^B stays high for several more periods. Overall, the presence of multiple firms makes the rational and the behavioral components of investor beliefs more synchronized: in this example, the time-series correlation between λ_t^R and λ_t^B increases from 30% in the single firm case (Panel B) to 69% in the multiple firm case (Panel A).

6 Conclusion

We present a model of credit market cycles in which investors extrapolate past defaults. Our key contribution is to model reflexivity in credit markets, an endogenous two-way feedback between biased investor beliefs and credit market outcomes. This feedback mechanism is particularly germane in credit markets, because firms must return to the market to refinance maturing debts, and the terms on which debt is refinanced will impact the likelihood of future defaults.

As we have shown, the combination of extrapolative beliefs and reflexive dynamics can lead to large short-run disconnects between cash flow fundamentals and credit market outcomes, including "calm before the storm" and "default spiral" episodes. Extrapolative beliefs also naturally lead to bond return predictability. But what is most striking here is that changes in investor sentiment can have an ambiguous impact on expected bond returns due to the reflexive nature of credit markets. When investors become more bullish, in the short run this can predict positive returns, even if at longer horizons expected returns become more negative.

Our analysis leaves open at least three areas for future research. First, we have not allowed conditions in credit markets to explicitly affect the underlying cash flow fundamentals of the economy. However, as demonstrated by a growing macro-finance literature, the inability to access credit on reasonable terms following a credit market bust may exacerbate an incipient economic downturn. Relatedly, according to Austrian accounts of the credit cycles, as the credit boom grows, increasing amounts of capital are devoted to poor quality projects to the detriment of future macroeconomic fundamentals. Indeed, López-Salido, Stein, and Zakrajšek (2017) and Mian, Sufi, and Verner (2017) show that periods of credit market overheating forecast low economic growth. Incorporating these features into our model would likely further strengthen the feedback loop between investor sentiment and credit market outcomes.

Second, we have been silent on issues of welfare and optimal policy, even though our model suggests a potential role for policy. During credit booms, high sentiment can prevent defaults from occurring in the near future, which can be welfare-improving if fundamentals recover soon enough. Nonetheless, self-fulfilling beliefs during default spirals can be welfare-reducing, both because these deteriorating beliefs accelerate future default realizations and because they lead to a slow recovery in the presence of improving fundamentals. Accepting these take-aways at face value, our model suggests a role for policy in moderating investor beliefs.

Finally, our model has relevance for the literature on sovereign debt crises, suggesting how one might incorporate extrapolative expectations into standard models of sovereign crises (Calvo [1988] and Cole and Kehoe [2000]). Specifically, the introduction of extrapolative expectations may help explain the kinds of "slow-moving debt crises" studied in Lorenzoni and Werning (2019). And, our extension with multiple firms may help capture the idea of belief-driven market contagion across sovereign borrowers, which may prove useful in understanding events like the 1997 Asian financial crisis and the post-2010 European debt crisis.

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A Proofs

Proof of Lemma 1: Since $\beta < 1$, λ_t^B weakly declines if there is no default at time t and the decline is strict if $\lambda_{t-1}^B > 0$.

How do extrapolative beliefs typically react to a default at time t—i.e., if $D_t = 1$ and $\omega_t = 0$? In this case, $\lambda_t^B = \min\left\{1, \beta\lambda_{t-1}^B + \alpha\right\} > 0$. If $\alpha \geq (1-\beta)$, λ_t^B weakly increases following a default and the increase is strict if $\lambda_{t-1}^B < 1$. Specifically, if $\lambda_t^B < 1$, then $\lambda_t^B - \lambda_{t-1}^B = \alpha - (1-\beta)\,\lambda_{t-1}^B > 0$ for all $\lambda_{t-1}^B \in [0,1)$ since $\alpha \geq (1-\beta)$. By contrast, if $\lambda_t^B = 1$, then we trivially have $\lambda_t^B - \lambda_{t-1}^B > 0$ for all $\lambda_{t-1}^B \in [0,1)$. Thus, if $\alpha \geq (1-\beta)$, extrapolative beliefs will converge to $\lambda_t^B = 1$ following a long sequence of defaults.

By contrast, if $\alpha < (1-\beta)$, extrapolative beliefs will not always become more pessimistic following a default. Specifically, if $D_t = 1$ and $\omega_t = 0$, then we have $\lambda_t^B \gtreqless \lambda_{t-1}^B$ as $\lambda_{t-1}^B \lesseqgtr \alpha/(1-\beta)$ and extrapolative beliefs will converge to $\lambda_t^B = \alpha/(1-\beta) < 1$ following a long sequence of defaults.

Proof of Lemma 2: First, assume that the economy is not near the default boundary $F_{t-1} + c - x_t = \bar{F}$, so small changes in F_{t-1} and x_t do not affect whether there is a default or the firm pays dividends at time t. Suppose that we are at an equilibrium continuity point where the smallest solution to $\lambda_t^R = g(\lambda_t^R | F_{t-1}, \lambda_t^B, x_t)$ is a continuous and differentiable function of $F_{t-1}, \lambda_t^B, x_t$. ($g(\lambda_t^R | F_{t-1}, \lambda_t^B, x_t)$ is continuous, but not differentiable in F_{t-1} at the dividend payout boundary $\underline{F} = F_{t-1} + c - x_t$.) At such a continuity point, for any $z_t \in \left\{F_{t-1}, \lambda_t^B, x_t\right\}$, we have $\lambda_t^R/\partial z_t = [\partial g(\cdot)/\partial z_t]/[1 - \partial g(\cdot)/\partial \lambda_t^R]$. At a stable equilibrium we have $\partial g(\cdot)/\partial \lambda_t^R < 1$, so this has the same sign as $\partial g(\cdot)/\partial z_t$. This argument shows that $\partial \lambda_t^R/\partial F_{t-1} > 0$, $\partial \lambda_t^R/\partial \lambda_t^B > 0$, and $\partial \lambda_t^R/\partial x_t < 0$. There are also equilibrium discontinuity points where the number of solutions to the fixed-point problem changes and the smallest solution discretely jumps. Although λ_t^R is not a continuous function of F_{t-1}, λ_t^B, x_t at these equilibrium discontinuity points, the signs of discrete jumps in λ_t^R at these points will have the same signs as the partial derivatives at equilibrium continuity points. For instance, an small increase in x_t shifts the $g(\lambda_t^R|\cdot)$ function down for all λ_t^R. At an equilibrium continuity point where the relevant partial derivative is well-defined, this results in a small decline in λ_t^R. At an equilibrium discontinuity point where the relevant partial derivative is not well-defined, this results in a discrete downward jump in λ_t^R.

Second, assume that we are near the default boundary. At the default boundary the derivatives with respect to F_{t-1} and x_t are undefined. Near the default boundary, a small increase in F_{t-1} can trigger a default at time t, resulting in a discrete downward jump in λ_t^R. Similarly, a small increase in x_t can avert a default at time t, resulting in a discrete upward jump in λ_t^R.

Proof of Proposition 1 (Calm Before the Storm): We compare two sample paths, denoted L and H, that differ only in their initial levels of λ_t^B. Specifically, suppose that $\lambda_t^B(L) < \lambda_t^B(H)$. Because shocks to cash flows and sentiment are exogenous, we have $x_{t+j}(L) = x_{t+j}(H)$ and $\omega_{t+j}(L) = \omega_{t+j}(H)$ for all $j \geq 0$. Because λ_t^R and F_t are always increasing in λ_t^B, we have $\lambda_t^R(L) < \lambda_t^R(H)$ and $F_t(L) < F_t(H)$. Since $F_t(L) < F_t(H)$, if there is a default at time $t+1$ in the L path, then there is also a default at time $t+1$ in the H path. However, we can have default in the H path, but not in the L path at time $t+1$.

Assume that there is no default at time $t+1$ along either the L or H paths. Then we have $\lambda_{t+1}^B(L) \leq \lambda_{t+1}^B(H)$ by equation (11) and the equality is strict so long as $0 < \lambda_{t+1}^B(H)$. Since λ_{t+1}^R and F_{t+1} are increasing in λ_{t+1}^B and F_t, it also follows that $\lambda_{t+1}^R(L) \leq \lambda_{t+1}^R(H)$ and $F_{t+1}(L) \leq F_{t+1}(H)$ and these inequalities are strict when $0 < \lambda_{t+1}^B(H)$. Since $F_{t+1}(L) \leq F_{t+1}(H)$, if the first default occurs at time $t+2$ in the L path, then first default also occurs at time $t+2$ in the H path. However, we can have default in the H path, but not in the L path at $t+2$.

Proceeding inductively in this fashion, we see that, so long as there is no default along either path by time $t+j$, we have $\lambda^B_{t+j}(L) \leq \lambda^B_{t+j}(H)$ and $F_{t+j}(L) \leq F_{t+j}(H)$ and these inequalities are strict when $\lambda^B_{t+j}(H) > 0$. Thus, lowering the default rate λ^B_t weakly delays the next future default stochastic path by stochastic path. And, averaging across these paths, lowering the default rate λ^B_t strictly delays the next default in expectation.

Proof of Proposition 2 (Default Spiral): Since $p_t \geq \eta$, if there is a default at time t (i.e., $D_t = 1$) we have $F_t = \eta(F_{t-1} + c - x_t)/p_t \leq F_{t-1} + c - x_t$. Thus, if $D_t = 1$ and $x_t > c$, we always have $F_t < F_{t-1}$. By contrast, if $D_t = 1$ and $x_t < c$, we have $F_t > F_{t-1}$ if $(c - x_t)/(p_t/\eta - 1) > F_{t-1}$ and $F_t < F_{t-1}$ if $(c - x_t)/(p_t/\eta - 1) < F_{t-1}$.

Next, note that

$$\lambda^R_t = \Phi\left(\frac{\frac{O_t \underline{F} + (1-O_t)(1-D_t(1-\eta))(F_{t-1}+c-x_t)}{1-(1-\eta)\theta\lambda^B_t - (1-\eta)(1-\theta)\lambda^R_t} + c - \bar{F} - \rho x_t - (1-\rho)\bar{x}}{\sigma_\varepsilon}\right)$$

where $D_t = \mathbf{1}_{\left\{F_{t-1}+c-x_t \geq \bar{F}\right\}}$ and $O_t = \mathbf{1}_{\{F_{t-1}+c-x_t \leq \underline{F}\}}$. Thus, we have

$$\frac{\partial \lambda^R_t}{\partial \theta} = \frac{\partial g\left(\lambda^R_t|\cdot\right)/\partial\theta}{1 - \partial g\left(\lambda^R_t|\cdot\right)/\partial\lambda^R_t} = \phi(\cdot)\frac{\frac{O_t \underline{F} + (1-O_t)(1-D_t(1-\eta))(F_{t-1}+c-x_t)}{\left[1-(1-\eta)\theta\lambda^B_t - (1-\eta)(1-\theta)\lambda^R_t\right]^2}}{1 - \partial g\left(\lambda^R_t|\cdot\right)/\partial\lambda^R_t}\frac{(1-\eta)\left(\lambda^B_t - \lambda^R_t\right)}{\sigma_\varepsilon} \propto \left(\lambda^B_t - \lambda^R_t\right)$$

where $\phi(\cdot)$ is the standard normal density evaluated at the argument given in the previous equation. Thus, λ^R_t is increasing in θ when $\lambda^B_t - \lambda^R_t > 0$.

We have assumed that (i) $F_{t-1} + c - x_t \geq \bar{F}$, so $D_t = 1$; (ii) $\alpha > (1-\beta)$ and $\omega_t = 0$; (iii) $\lambda^B_{t-1} \geq \lambda^R_{t-1}$; and (iv) $x_t = x_{t-1} = x > c$. Since $\alpha > (1-\beta)$, $D_t = 1$, and $\omega_t = 0$, we have $\lambda^B_t \geq \lambda^B_{t-1}$. Since $p_t(\theta) \geq \eta$ and $x_t > c$ we have $F_t(\theta) = \eta(F_{t-1} + c - x_t)/p_t(\theta) \leq F_{t-1} + c - x_t < F_{t-1}$. Thus, since $x_t = x_{t-1}$, we have

$$\lambda^R_t(\theta) = \Phi\left(\frac{F_t(\theta) - \bar{F} + c - \rho x_t - (1-\rho)\bar{x}}{\sigma_\varepsilon}\right) < \Phi\left(\frac{F_{t-1} - \bar{F} + c - \rho x_{t-1} - (1-\rho)\bar{x}}{\sigma_\varepsilon}\right) = \lambda^R_{t-1}$$

irrespective of the value of $\theta \in [0,1]$. Thus, we have $\lambda^B_t \geq \lambda^B_{t-1} \geq \lambda^R_{t-1} > \lambda^R_t(\theta)$ irrespective of θ, so we have $\partial\lambda^R_t(\theta)/\partial\theta > 0$ and

$$\frac{\partial\lambda^C_t(\theta)}{\partial\theta} = \left(\lambda^B_t - \lambda^R_t(\theta)\right) + (1-\theta)\frac{\partial\lambda^R_t(\theta)}{\partial\theta} > 0.$$

In other words, both the rational component and the combined belief become more pessimistic as the fraction of backward-looking beliefs rises.

Since $\partial\lambda^C_t(\theta)/\partial\theta > 0$, it then follows that $\partial p_t(\theta)/\partial\theta < 0$ and $\partial F_t(\theta)/\partial\theta > 0$. Thus, a larger extrapolative component of beliefs lowers prices and slows the process of debt discharge in the event of default, increasing the chances of subsequent defaults. Indeed, for any $\theta > 0$, we have $\lambda^R_t(\theta) > \lambda^R_t(0)$.

Model with opportunistic supply response: Assuming the firm does not default or pay dividends at time t, one can think of the baseline model as reflecting the interplay between the demand and supply for risk bonds:

$$\text{Demand for bonds} \; : \; p_t^D = 1-(1-\eta)\theta\lambda_t^B - (1-\eta)(1-\theta)\,\Phi\left(\frac{F_t + c - \bar{F} - \rho x_t - (1-\rho)\bar{x}}{\sigma_\varepsilon}\right)$$

$$\text{Supply of bonds:} \quad p_t^S = \frac{F_{t-1}+c-x_t}{F_t},$$

where F_t is the quantity of risky bonds issued at time t. We have $\partial p_t^D/\partial F_t < 0$, so the demand for bonds is downward sloping as is standard. (Here this works through a change in fundamentals: the default probability increases as debt outstanding rises.) However, we also have $\partial p_t^S/\partial F_t < 0$: supply is also downward sloping, which is non-standard. This is because the supply of bonds is determined by firms' binding sources-and-used constraint. Of course, it is the fact that both demand and supply slope downwards that makes investor beliefs potentially self-fulfilling. However, the fact that supply cannot be upward-sloping precludes the kind of opportunistic supply response that might might allow a credit boom to sow the seeds of its own destruction.

Once we introduce a opportunistic debt supply response, the equilibrium value of λ_t^R must solve the following fixed-point problem:

$$\lambda_t^R = g_{opp}(\lambda_t^R | F_{t-1}, \lambda_t^B, x_t) \equiv \Phi\left(\frac{F_{opp}(F_{t-1}, \lambda_t^B, \lambda_t^R, x_t) + c - \bar{F} - \rho x_t - (1-\rho)\bar{x}}{\sigma_\varepsilon}\right).$$

Here

$$F_{opp}(F_{t-1}, \lambda_t^B, \lambda_t^R, x_t) = \begin{cases} \underline{F}/p(\lambda_t^B, \lambda_t^R) & \text{if } F_{t-1}+c-x_t \leq \underline{F} \\ (F_{t-1}+c-x_t)/p(\lambda_t^B, \lambda_t^R) & \\ \quad +M \times \left[p(\lambda_t^B, \lambda_t^R) - \left(1-(1-\eta)\,\lambda_t^R\right)\right] & \text{if } \underline{F} < F_{t-1}+c-x_t < \bar{F} \\ \eta(F_{t-1}+c-x_t)/p(\lambda_t^B, \lambda_t^R) & \text{if } \bar{F} \leq F_{t-1}+c-x_t \end{cases},$$

where $M \geq 0$ controls the aggressiveness of the corporate supply response to debt mispricing.

Unlike in the baseline model where $M = 0$, $g_{opp}(\lambda_t^R | F_{t-1}, \lambda_t^B, x_t)$ need not be monotonically increasing in λ_t^R when $M > 0$. However, for a given value of $\left(F_{t-1}, \lambda_t^B, x_t\right)$, $g(\lambda_t^R | F_{t-1}, \lambda_t^B, x_t)$ is still a continuous function that maps the unit interval into itself—i.e., $g(\lambda_t^R | F_{t-1}, \lambda_t^B, x_t) \in [0,1]$ for any $\lambda_t^R \in [0,1]$—so a fixed point always exists by Brouwer's fixed-point theorem. As in the baseline model, we select the smallest λ_t^R that solves $\lambda_t^R = g_{opp}(\lambda_t^R | F_{t-1}, \lambda_t^B, x_t)$.

Let F_t denote the equilibrium level of debt at time t. Holding λ_t^R fixed, we now have

$$\frac{\partial F_t}{\partial \lambda_t^B} = (1-\eta)\,\theta\left(\frac{F_{t-1}+c-x_t}{\left[1-(1-\eta)\,\lambda_t^R + (1-\eta)\,\theta\left(\lambda_t^R - \lambda_t^B\right)\right]^2} - M\right).$$

The size of $\partial F_t/\partial \lambda_t^B$ is ambiguous when $M > 0$. And, holding fixed λ_t^B, we have

$$\frac{\partial F_t}{\partial \lambda_t^R} = (1-\eta)\left((1-\theta)\frac{F_{t-1}+c-x_t}{\left[1-(1-\eta)\,\lambda_t^R + (1-\eta)\,\theta\left(\lambda_t^R - \lambda_t^B\right)\right]^2} + \theta M\right) > 0.$$

Thus, we have

$$
\begin{aligned}
\frac{\delta F_t}{\delta \lambda_t^B} &= \overbrace{\frac{\partial F_t}{\partial \lambda_t^R}}^{+} \times \overbrace{\frac{\partial \lambda_t^R}{\partial \lambda_t^B}}^{+} + \overbrace{\frac{\partial F_t}{\partial \lambda_t^B}}^{??} \\
&= (1-\eta)\,\frac{F_{t-1}+c-x_t}{\left[1-(1-\eta)\,\lambda_t^R+(1-\eta)\,\theta\left(\lambda_t^R-\lambda_t^B\right)\right]^2}\overbrace{\left[(1-\theta)\,\frac{\partial \lambda_t^R}{\partial \lambda_t^B}+\theta\right]}^{>0} \\
&\quad +(1-\eta)\,\theta M\overbrace{\left[\frac{\partial \lambda_t^R}{\partial \lambda_t^B}-1\right]}^{??}.
\end{aligned}
$$

Thus, the sign of $\delta F_t/\delta\lambda_t^B$ can vary across the parameter space when $M>0$. Specifically, for sufficiently large M, we will have $\delta F_t/\delta\lambda_t^B<0$ in good times when $F_{t-1}+c-x_t$ is small and $\partial\lambda_t^R/\partial\lambda_t^B$ is small. Here the opportunistic respond dominates and supply is upward sloping. By contrast, we will have $\delta F_t/\delta\lambda_t^B>0$ in bad times when $F_{t-1}+c-x_t$ is large and $\partial\lambda_t^R/\partial\lambda_t^B$ is large. Here supply is downward sloping.

Figure 1. The credit cycle.

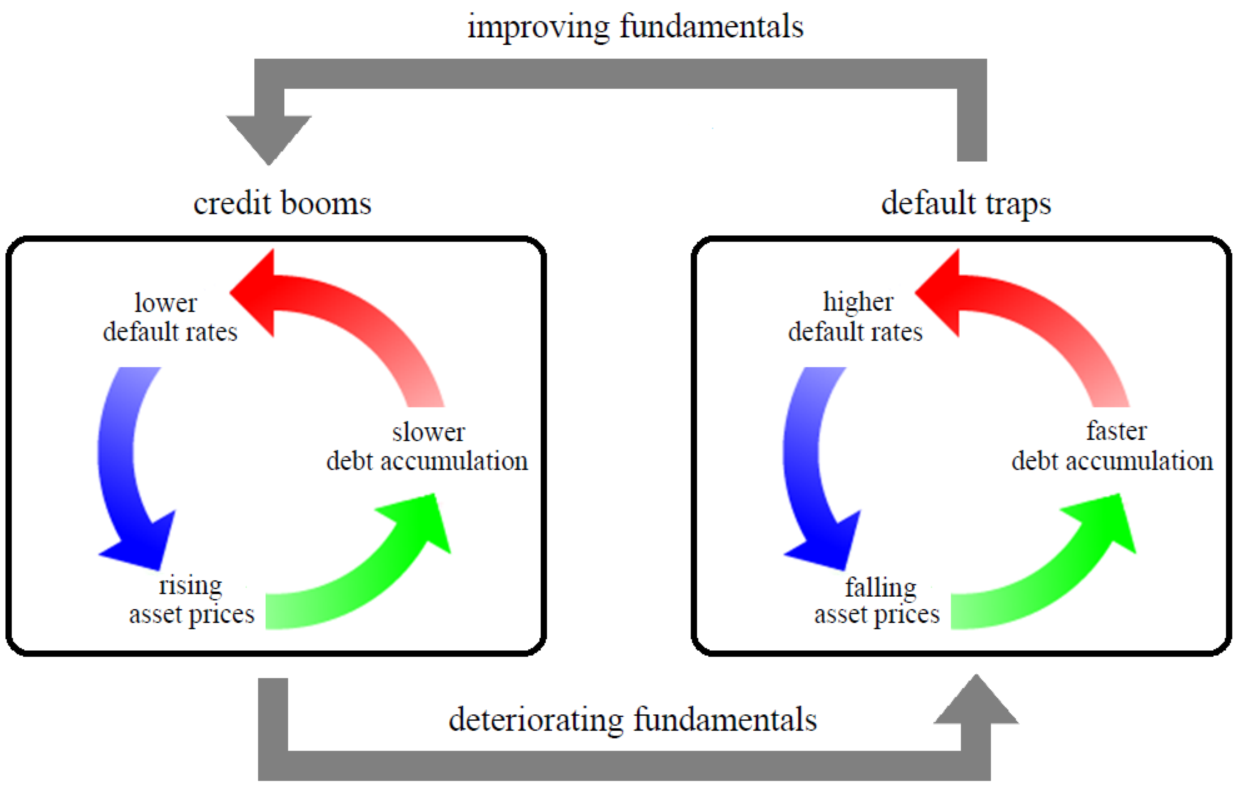
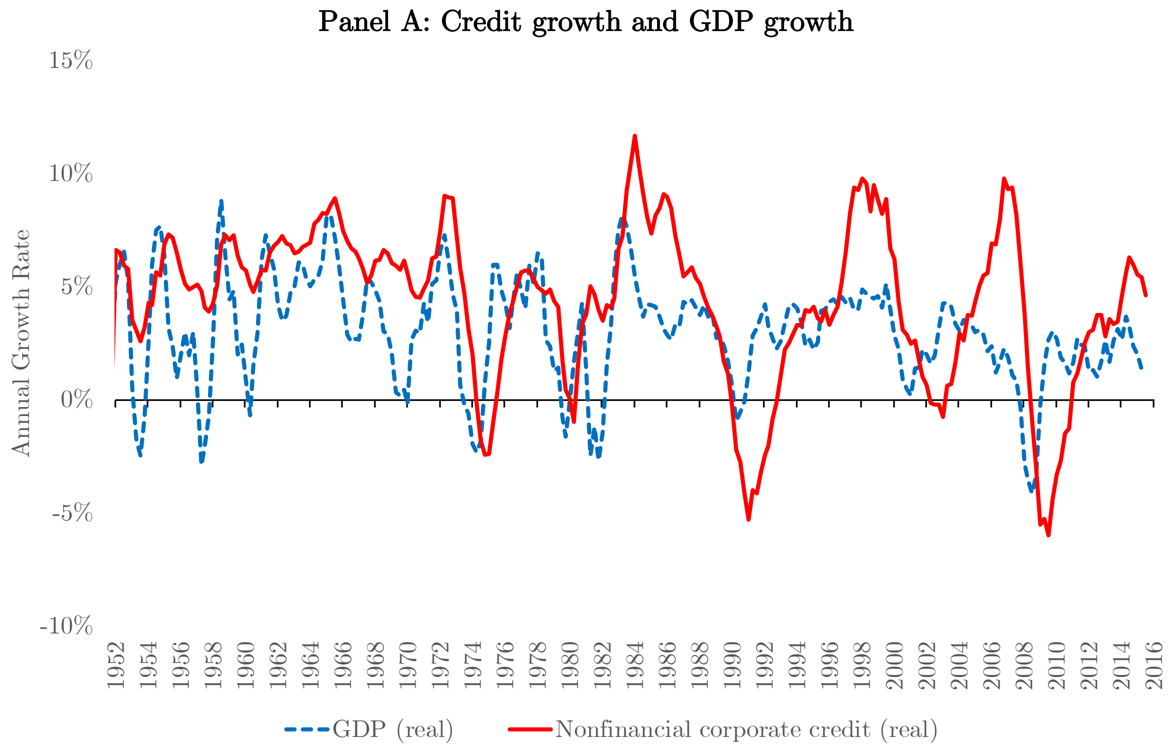


Figure 2. The credit market cycle. Panel A plots the year-over-year growth in real GDP and the year-over-year growth in real credit outstanding (defined as the sum of loans and bonds) to nonfinancial corporate businesses from the Federal Reserve's Financial Accounts of the United States. Panel B plots real year-over-year credit growth versus the corporate credit spread, measured as the yields on Moody's seasoned Baa corporate bond yield minus the 10-year constant maturity Treasury yield.



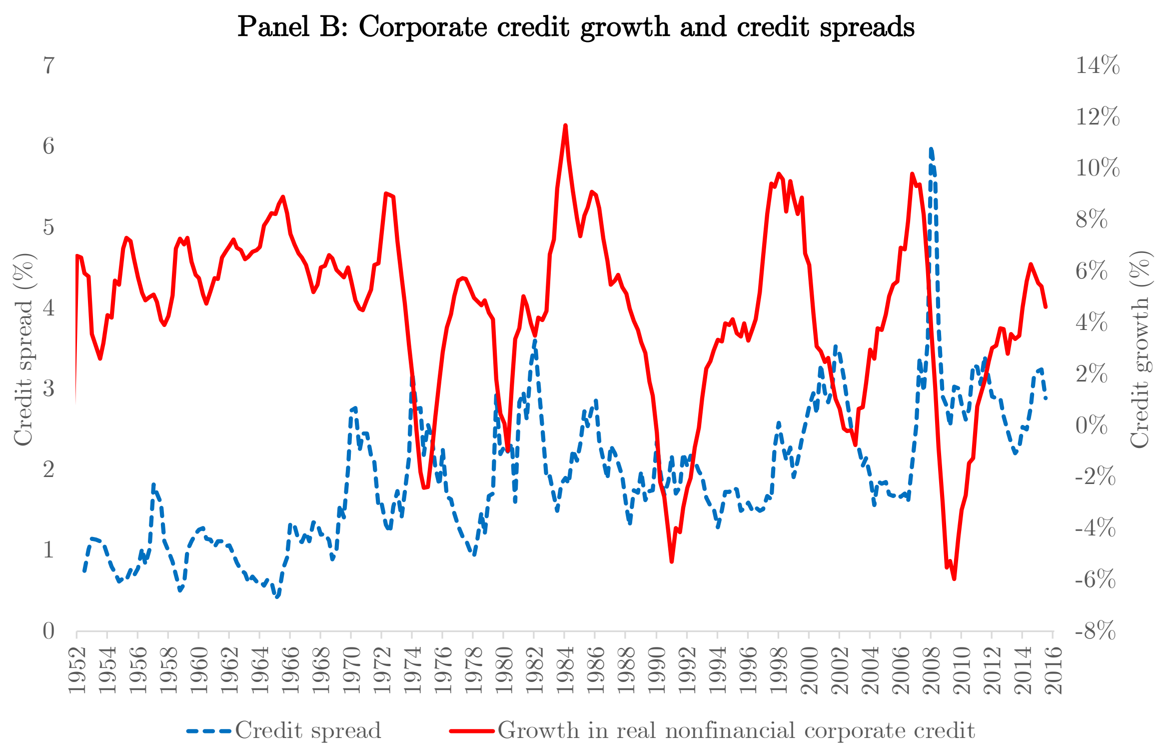
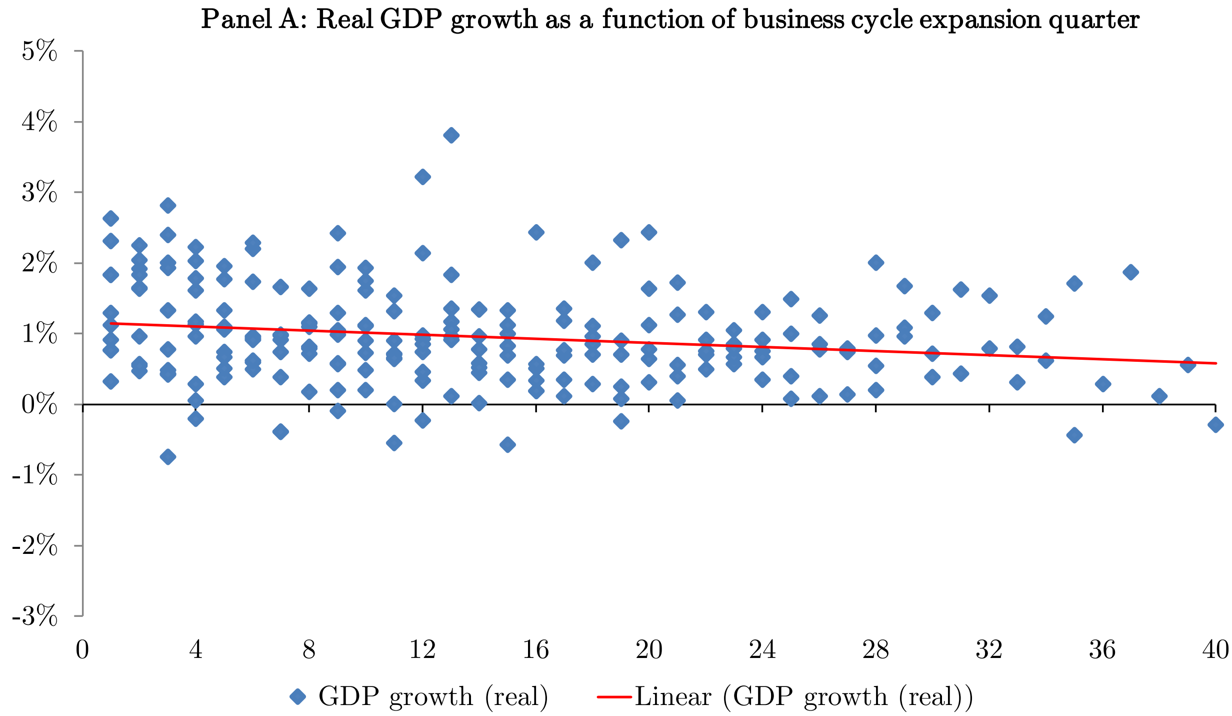


Figure 3. Real GDP growth and credit growth as a function of business cycle expansion quarter. This figure plots real GDP growth and real credit growth—the growth in real nonfinancial corporate loans and bonds from the Financial Accounts of the United States—as a function of NBER business cycle expansion quarter. Data are 1952-2016.



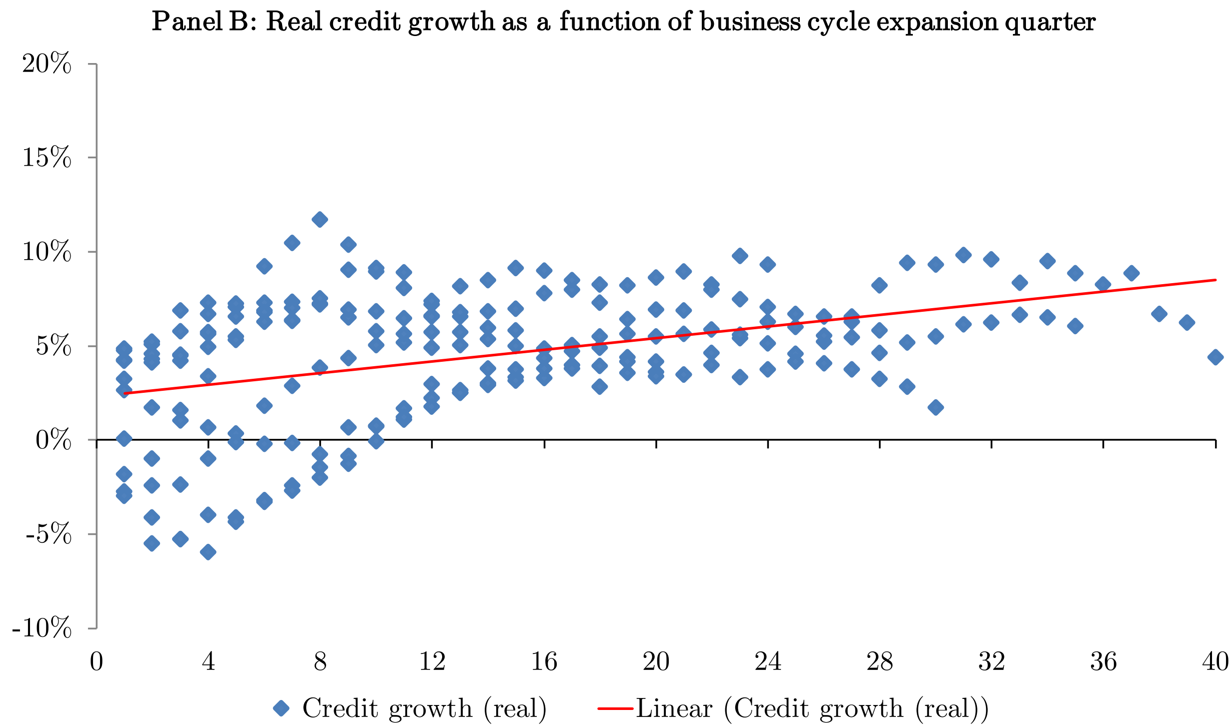


Figure 4. Simulated data using baseline parameter values. This figure shows a typical path of simulated data using our baseline set of parameter values in which beliefs are partially backward-looking and partially forward-looking (θ = 0.5). Specifically, the baseline parameters are $\bar{x}$ = 2.4, ρ = 0.8, σ_ε = 0.5, c = 2, $\underline{F}$ = 1.5, $\overline{F}$ = 5, η = 0.5, β = 0.8, α = 0.2, σ_ω = 0.05, and θ = 0.5. We plot the evolution of cash flow (x_t), debt outstanding (F_t), the default indicator (D_t), bond prices (p_t), rational forward-looking beliefs about future defaults (λ_t^R), and extrapolative backward-looking beliefs about future defaults (λ_t^B). Each period represents one year.

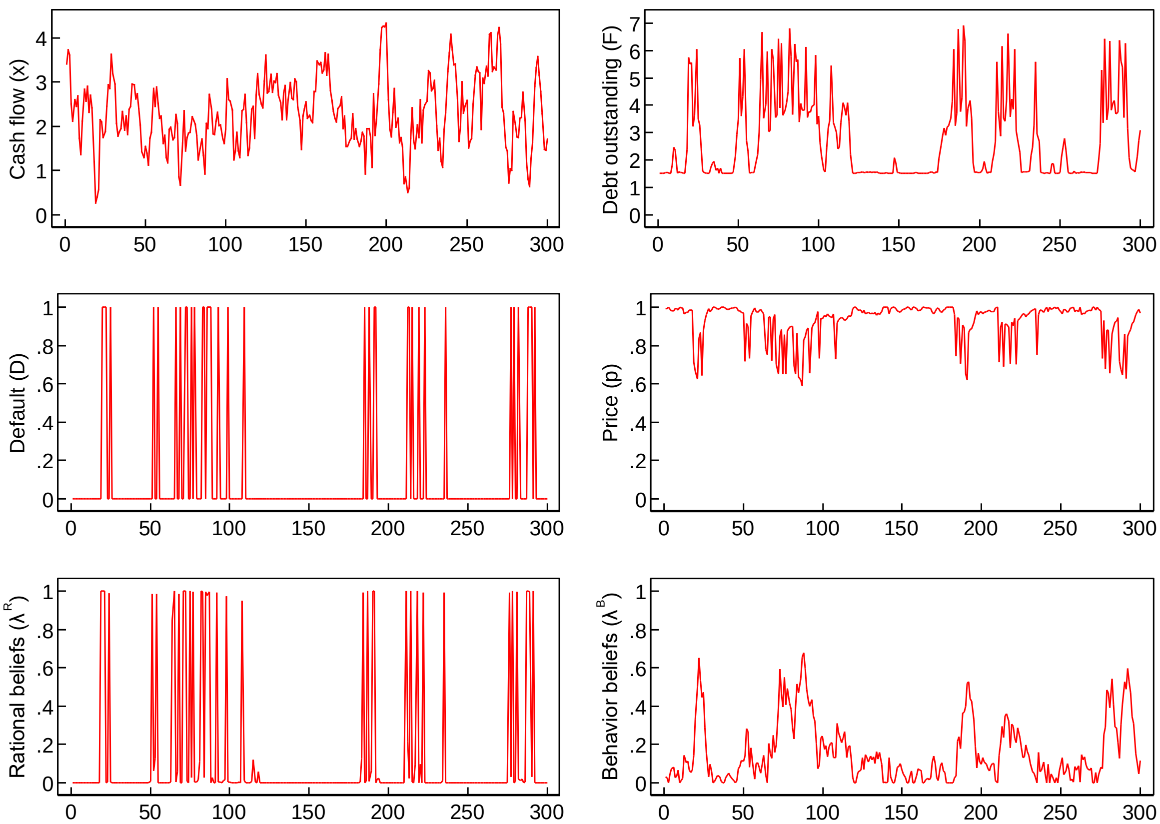
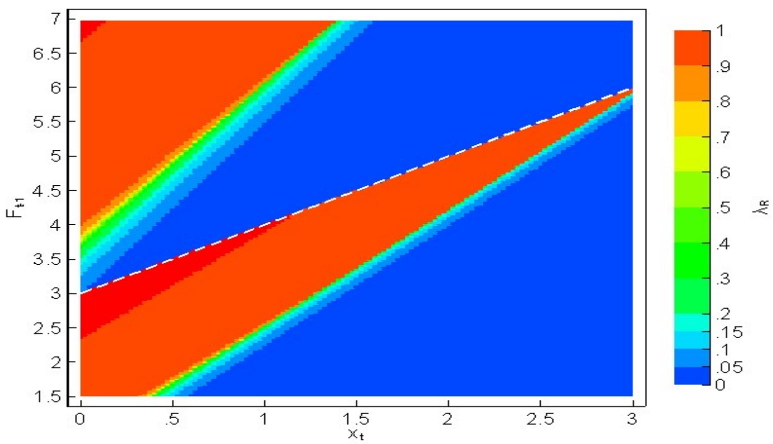


Figure 5. Reflexive regions. This figure illustrates the existence of reflexive regions in our model. The heatmap in Panel A plots λ_t^R vs. (x_t, F_{t-1}) for $\lambda_t^B = 0.2$. The heatmap in Panel B plots $\partial\lambda_t^R/\partial\lambda_t^B$ vs. (x_t, F_{t-1}) for $\lambda_t^B = 0.2$. (The dashed white line in Panels A and B is the default boundary at time t.) Finally, Panel C plots λ_t^R and $\partial\lambda_t^R/\partial\lambda_t^B$ vs. F_{t-1} for $\lambda_t^B = 0.2$ and $x_t = 1$. (The vertical black line in the default boundary at time t.) The model parameters are the same as those in Figure 4.

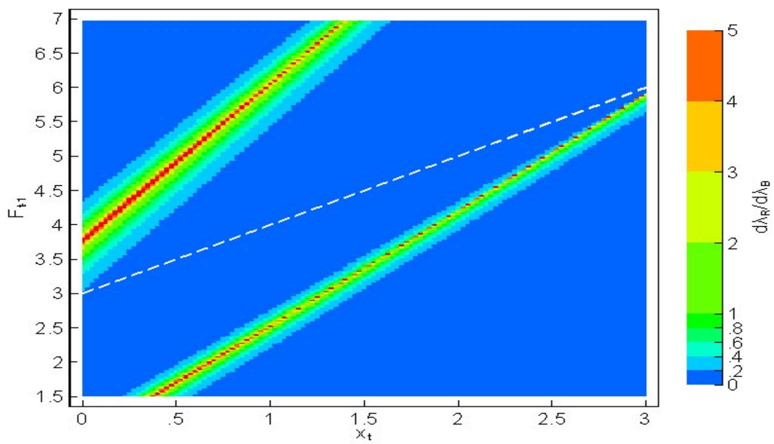
Panel A:

λ_t^R vs. (x_t, F_{t-1}) for $\lambda_t^B = 0.2$



Panel B:

$\partial\lambda_t^R/\partial\lambda_t^B$ vs. (x_t, F_{t-1}) for $\lambda_t^B = 0.2$



Panel C:

λ_t^R and $\partial\lambda_t^R/\partial\lambda_t^B$ vs. F_{t-1} for $\lambda_t^B = 0.2$ and $x_t = 1$

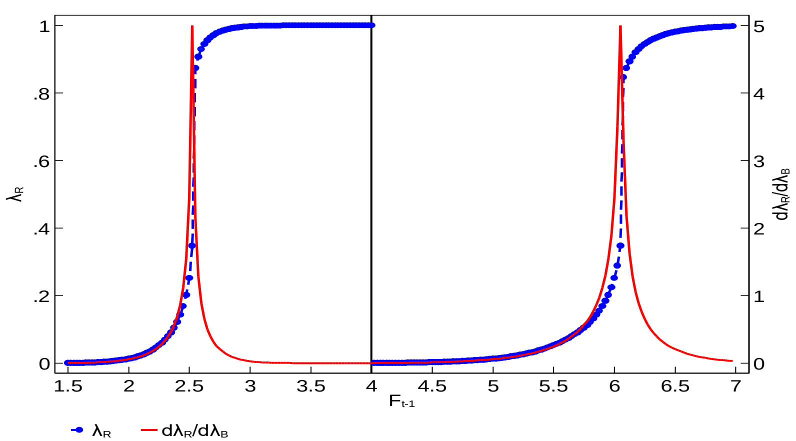
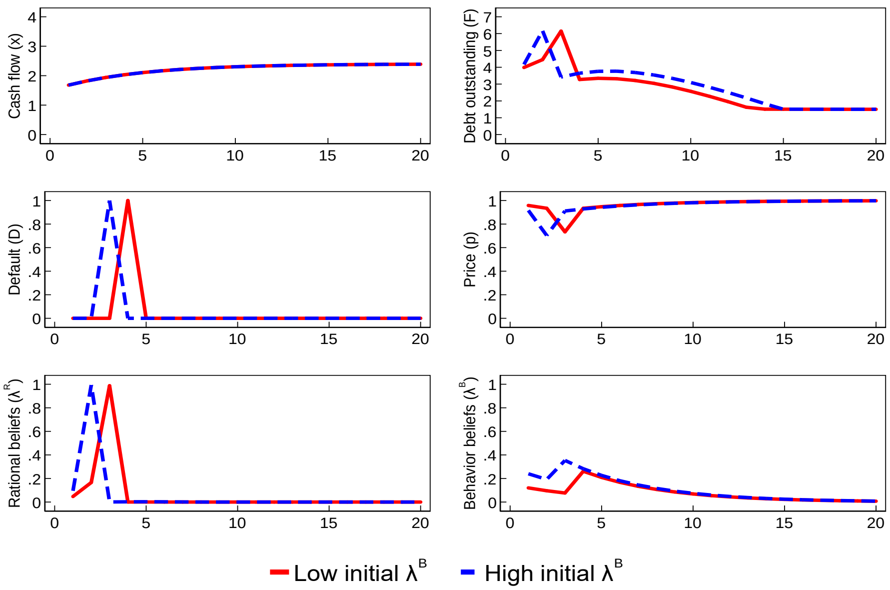


Figure 6. Calm before the storm. This figure illustrates the "calm before the storm" phenomenon. The figure depicts sample paths of the model with cash flows initially set to $x_0 = 1.5 < 2 = c$ and debt initially set to $F_0 = 3.5$. We compare the model dynamics starting from a low initial value of $\lambda_0^B(L) = 0.15$ and a high initial value $\lambda_0^B(H) = 0.3$. We assume all subsequent shocks are zero ($\varepsilon_t = \omega_t = 0$). We separately plot the dynamics for various values of $\theta = 0.5$ and $\theta = 1$. Otherwise, the model parameters are the same as those in Figure 4. Specifically, we set $\overline{x} = 2.4$, $\rho = 0.8$, $c = 2$, $\underline{F} = 1.5$, $\overline{F} = 5$, $\eta = 0.5$, $\beta = 0.8$, and $\alpha = 0.2$.

Panel A: $\theta = 0.5$



Panel B: $\theta = 1$

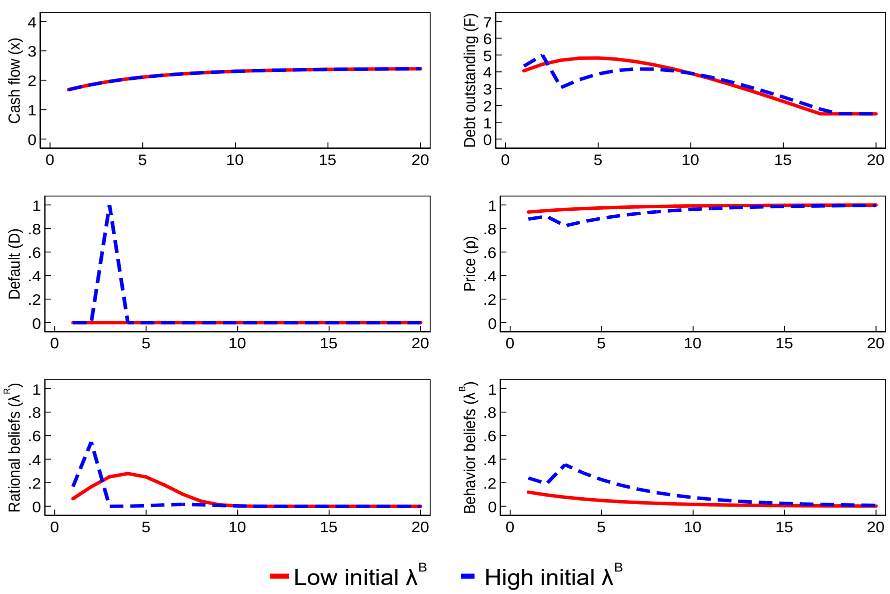


Figure 7. Model-implied path of credit spreads around a financial crisis. This figure shows the model-implied expected path of credit spreads in "event time" conditional on the onset of a crisis at time $\tau = 0$. Specifically, using 100,000 periods of simulated data assuming our baseline set of parameters, we estimate regressions of the form:

$$(1-\eta)\lambda_t^Y = a + \sum_{\tau=-T}^{T} b_\tau \mathbf{1}_{\{D_{t+\tau}=1\}} + e_t,$$

for $Y \in \{B, R, C\}$. We plot the b_τ coefficients versus event time τ below. Since $\theta = 0.5$ in our baseline parameters, the coefficients for $(1-\eta)\lambda_t^C$ are a 50:50 mixture of the coefficients for $(1-\eta)\lambda_t^R$ and $(1-\eta)\lambda_t^B$.

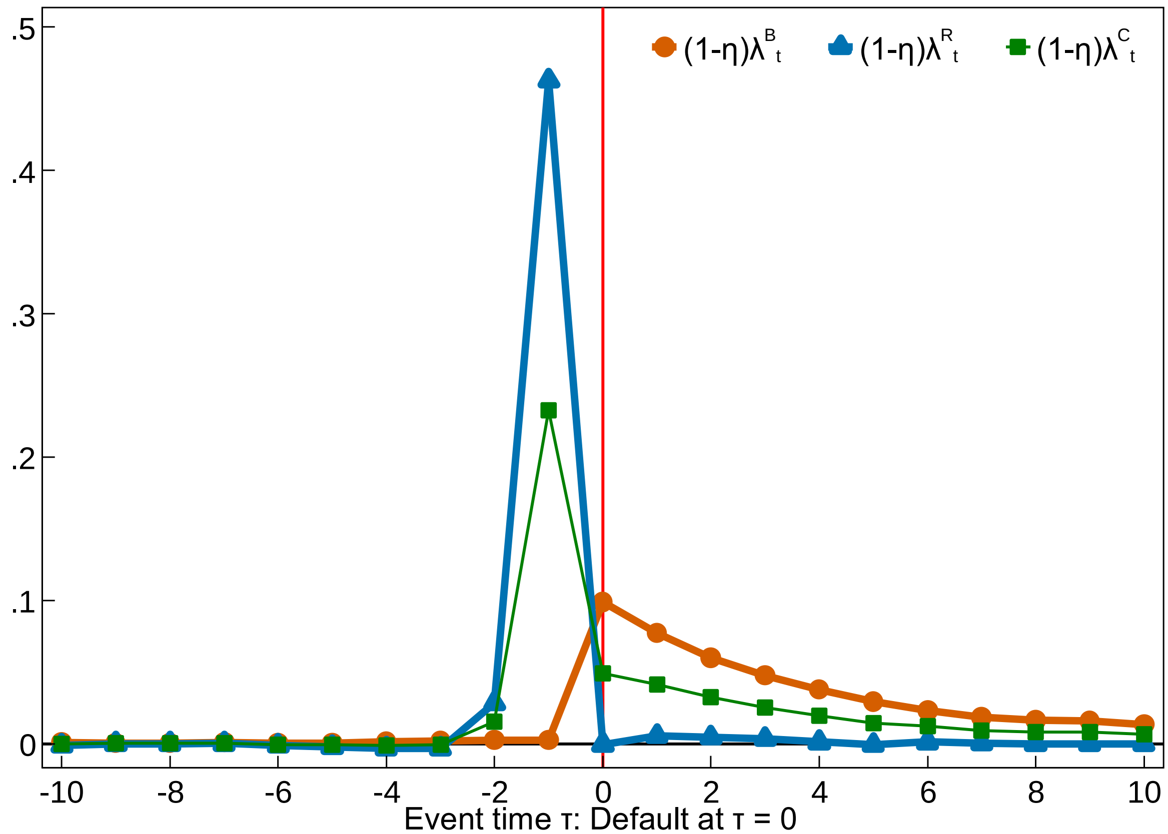


Figure 8. Impact of backward-looking beliefs on the true default probability and expected returns. This figure plots the true default probability λ_t^R (blue) and rationally-expected returns $\mathbb{E}_t^R[r_{t+1}]$ (red) against backward-looking beliefs λ_t^B in a highly reflexive region of the state space—i.e., a region where $\partial\lambda_t^R/\partial\lambda_t^B$ is large so changes in beliefs have a large impact on future defaults. Specifically, we set $x_t = 1.6 < 2 = c$ and $F_{t-1} = 3.4$. The other model parameters are the same as those in Figure 4: $\bar{x} = 2.4$, $\rho = 0.8$, $\sigma_\varepsilon = 0.5$, $c = 2$, $\underline{F} = 1.5$, $\overline{F} = 5$, $\eta = 0.5$, $\beta = 0.8$, $\alpha = 0.2$, $\sigma_\omega = 0.05$, and $\theta = 0.5$.

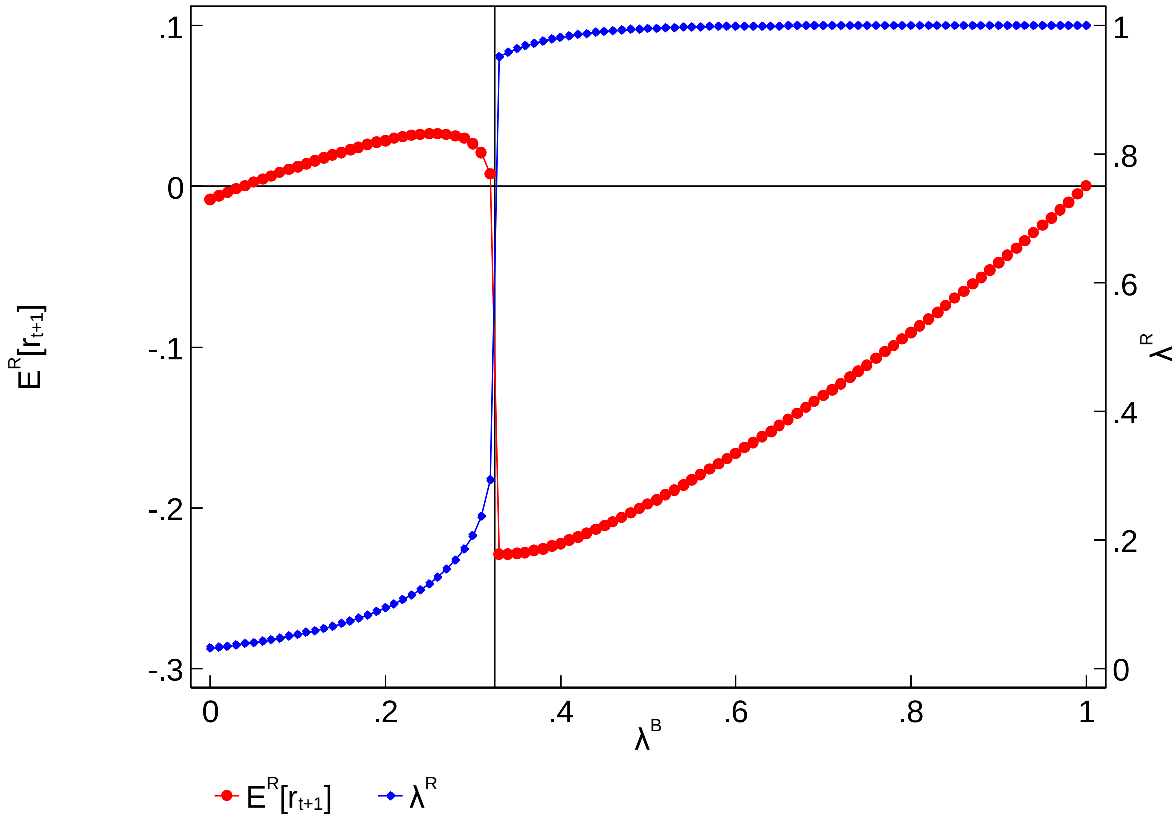


Figure 9. Model-implied impulse response functions in a non-reflexive region. The top panel shows the responses following a 0.5 up or down impulse to cash flows (x_t) at $t = 1$. The bottom panel shows the responses following a 0.25 up or down impulse to backward-looking beliefs (λ_t^B) at $t = 1$. The initial condition in both cases is $x_0 = 2.25$, $F_{-1} = 2.25$, and $\lambda_0^B = 0.30$. The model parameters are the same as those in Figure 4. Specifically, the model parameters are $\bar{x} = 2.4$, $\rho = 0.8$, $\sigma_\varepsilon = 0.5$, $c = 2$, $\underline{F} = 1.5$, $\bar{F} = 5$, $\eta = 0.5$, $\beta = 0.8$, $\alpha = 0.2$, $\sigma_\omega = 0.05$, and $\theta = 0.5$.

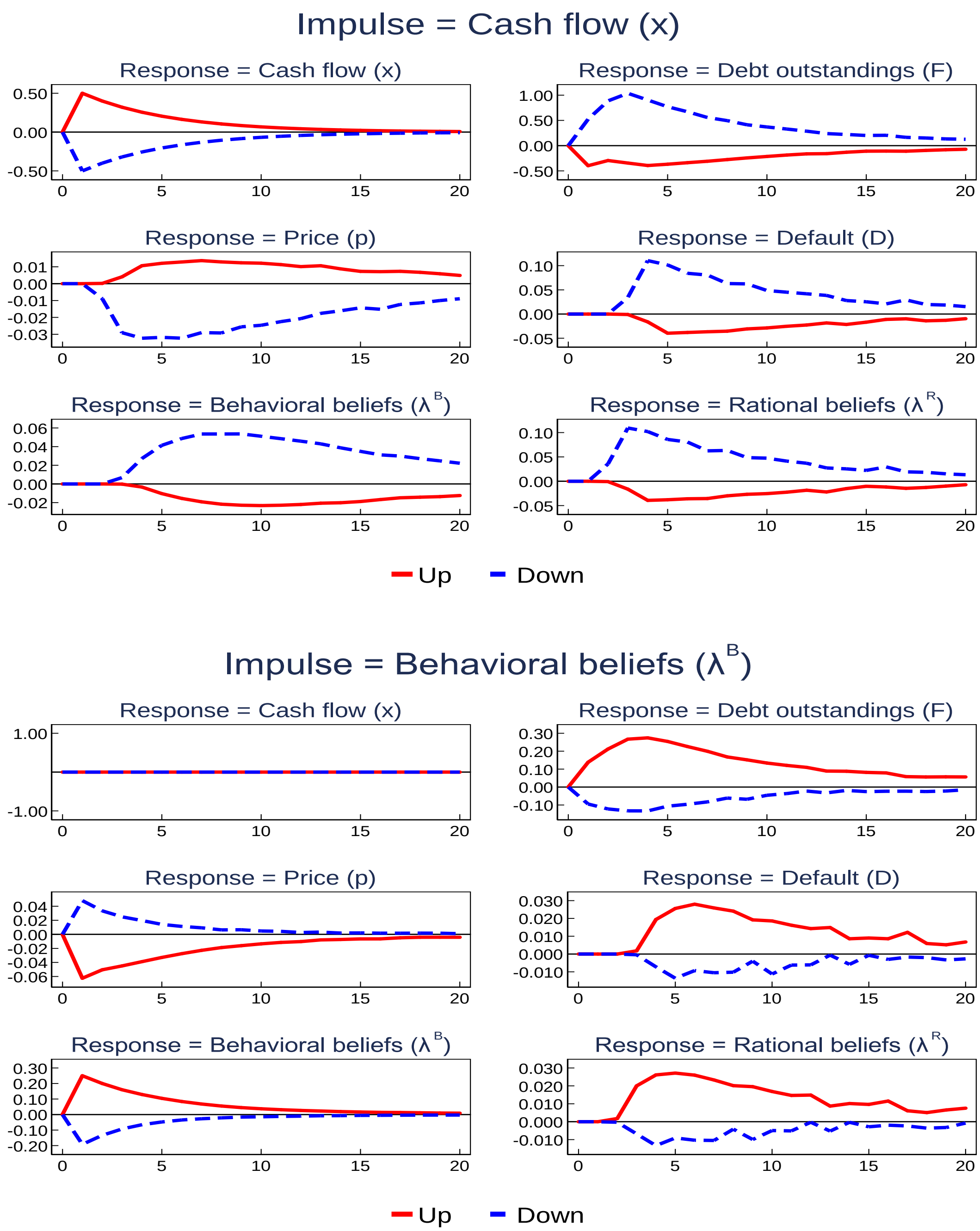


Figure 10. Model-implied impulse response functions in a reflexive region. The top panel shows the responses following a 0.5 up or down impulse to cash flows (x_t) at $t = 1$. The bottom panel shows the responses following a 0.25 up or down impulse to backward-looking beliefs (λ_t^B) at $t = 1$. The initial condition in both cases is $x_0 = 1.6$, $F_{-1} = 3.4$, and $\lambda_0^B = 0.33$. The model parameters are the same as those in Figure 4. Specifically, the model parameters are $\overline{x} = 2.4$, $\rho = 0.8$, $\sigma_\varepsilon = 0.5$, $c = 2$, $\underline{F} = 1.5$, $\overline{F} = 5$, $\eta = 0.5$, $\beta = 0.8$, $\alpha = 0.2$, $\sigma_\omega = 0.05$, and $\theta = 0.5$.

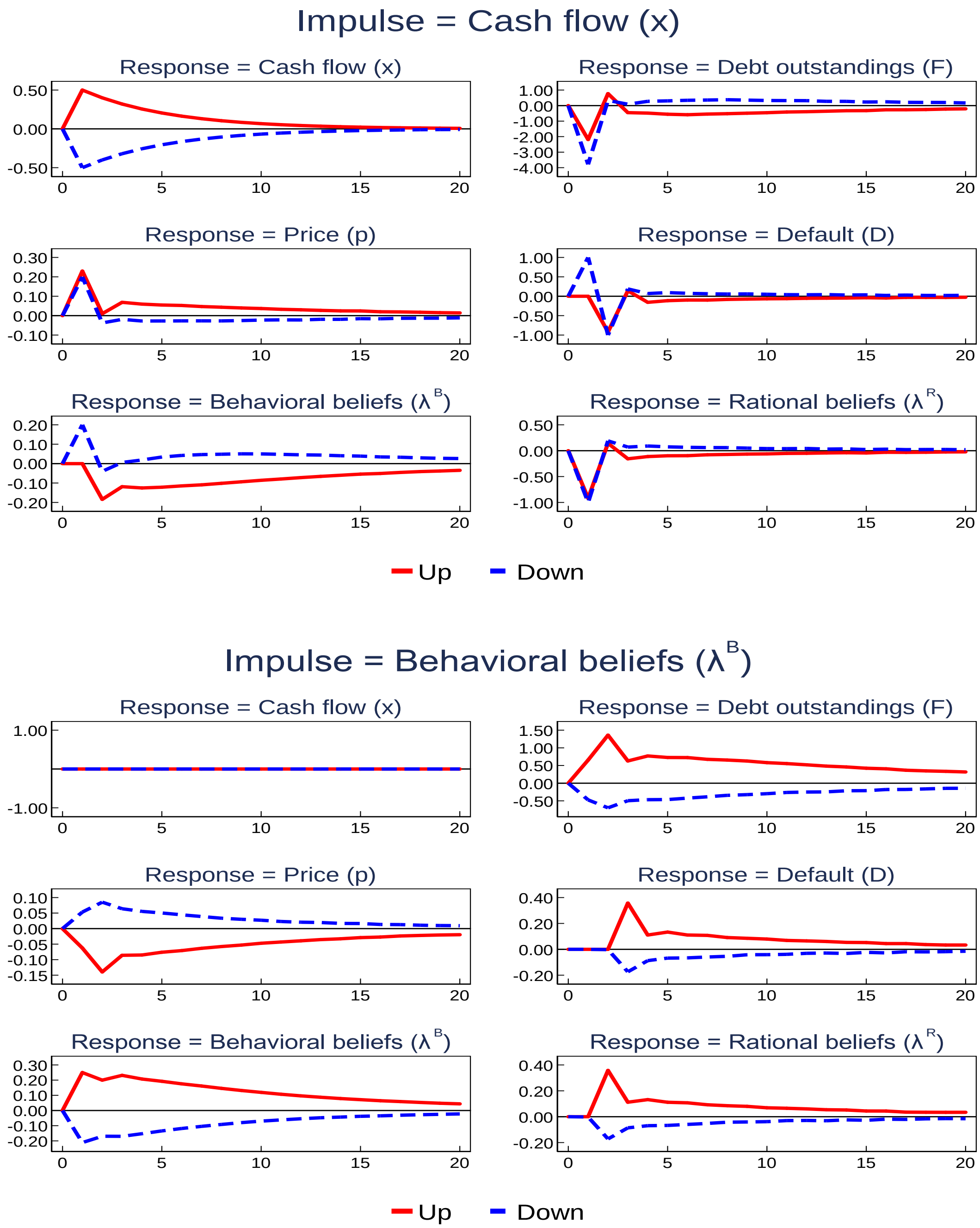


Figure 11. Allowing for an opportunistic supply response: Model-implied impulse response functions in a non-reflexive region. The top panel shows the responses following a 0.5 up or down impulse to cash flows (x_t) at $t = 1$. The bottom panel shows the responses following a 0.25 up or down impulse to backward-looking beliefs (λ_t^B) at $t = 1$. The initial condition in both cases is $x_0 = 2.25$, $F_{-1} = 2.25$, and $\lambda_0^B = 0.30$. The market timing parameter is set to $M = 5$. Otherwise, the model parameters are the same as those in Figure 4. Specifically, the model parameters are $\overline{x} = 2.4$, $\rho = 0.8$, $\sigma_\varepsilon = 0.5$, $c = 2$, $\underline{F} = 1.5$, $\overline{F} = 5$, $\eta = 0.5$, $\beta = 0.8$, $\alpha = 0.2$, $\sigma_\omega = 0.05$, and $\theta = 0.5$.

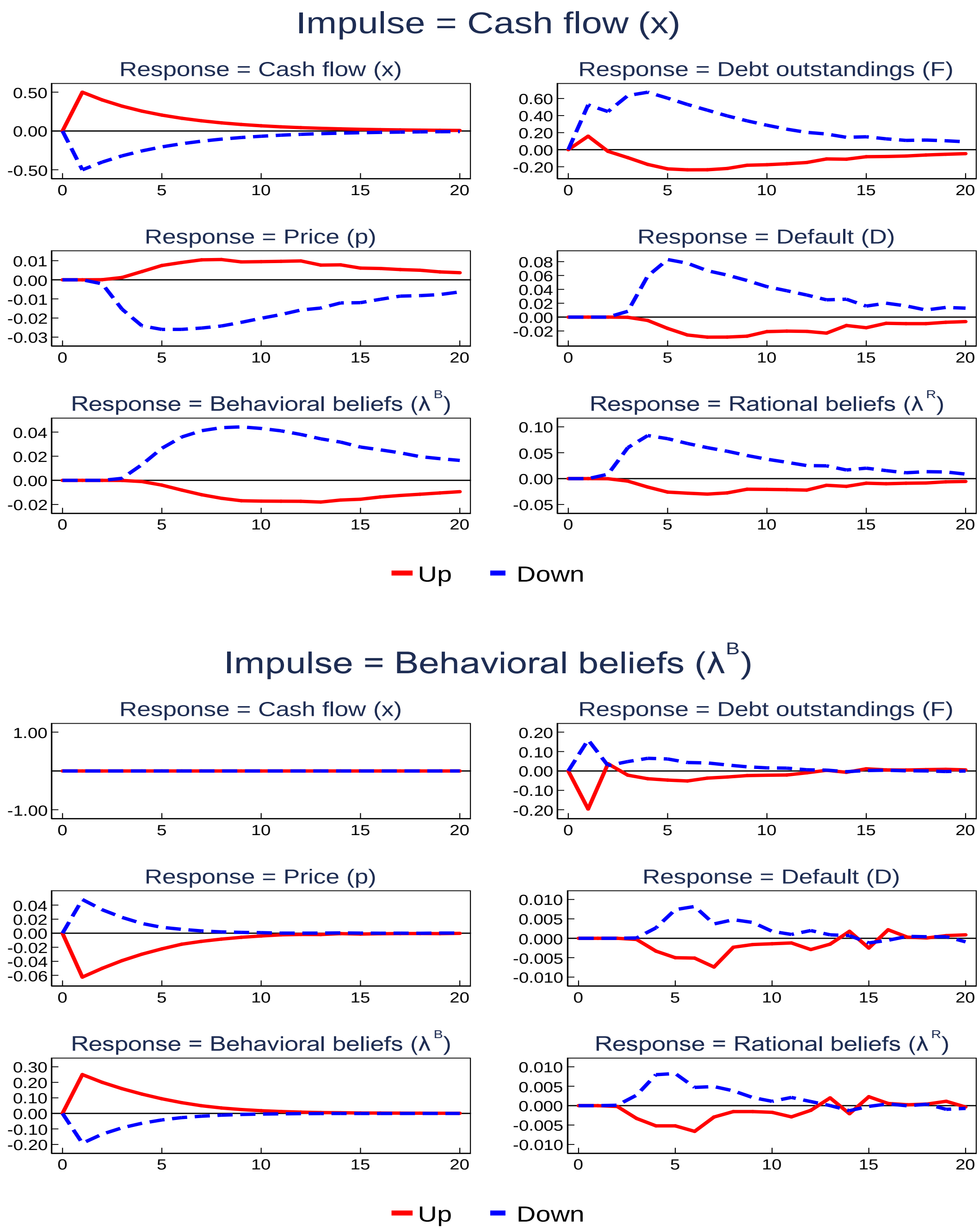
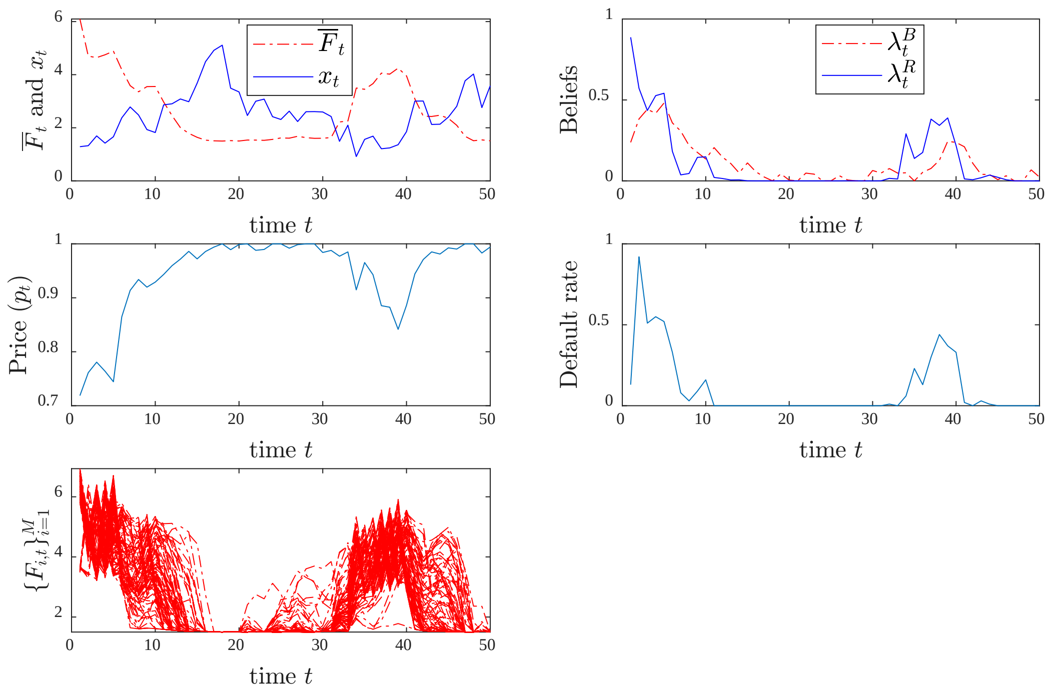


Figure 12. Simulated data with multiple firms. Panel A shows a typical path of simulated data for multiple firms ($N = 100$). Panel B shows the analogous simulation for a representative firm using the exact same time series of aggregate cash flow shocks and sentiment shocks as those in Panel A. For Panel A, the initial state of the economy is $\lambda_0^B = 0.2$, $x_0 = 2$, $z_0 = 0$, and $F_{i0} = 4$ for all i. For Panel B, the initial state of the economy is $\lambda_0^B = 0.2$, $x_0 = 2$, and $F_0 = 4$. In both panels, the model parameters are $\overline{x} = 2.4$, $\rho = 0.8$, $\sigma_\varepsilon = 0.5$, $c = 2$, $\underline{F} = 1.5$, $\overline{F} = 5$, $\eta = 0.5$, $\beta = 0.8$, $\alpha = 0.2$, $\sigma_\omega = 0.05$, and $\theta = 0.5$. In the Panel A, we also assume that $\psi = 0.8$ and $\sigma_\xi = 0.25$.

Panel A: Multiple firms ($N = 100$)



Panel B: Representative firm

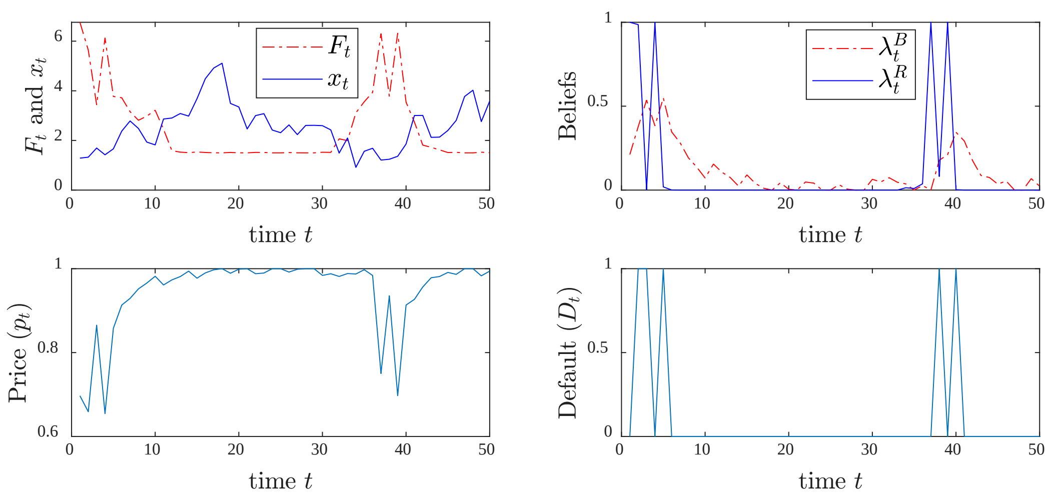


Table 1. Credit market overheating and future corporate bond returns. This table presents time-series regressions of the form:

$$rx^{HY}_{t\to t+k} = a + b \cdot Overheating_t + e_{t\to t+k},$$

where $Overheating_t$ is a proxy for credit market overheating in year t. The data begins in 1983 and ends in 2014. The dependent variable is the cumulative k = 2- or 3-year excess return on high-yield bonds over like-maturity Treasuries. HYS_t is the fraction of nonfinancial corporate bond issuance with a high-yield rating from Moody's, as defined in Greenwood and Hanson (2013). $Credit\ Growth_t$ is the percentage change in outstanding corporate credit and is computed using Table L103 from the Flow of Funds. $Easy\ Credit_t$ is the three-year average of the percentage of bank loan officers reporting a loosening of commercial lending standards from the Federal Reserve's Senior Loan Office Opinion Survey. $-1 \times EBP_t$ is negative one times the excess bond premium from Gilchrist and Zakrajšek (2012). t-statistics for k-period forecasting regressions (in brackets) are based on Newey-West (1987) standard errors, allowing for serial correlation up to k-lags.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	2-year future excess returns: $rx^{HY}_{t\to t+2}$				3-year future excess returns: $rx^{HY}_{t\to t+3}$			
$\log(HYS_t)$	−15.95				−18.63			
	[−3.06]				[−3.78]			
$Credit\ Growth_t$		−126.50				−158.02		
		[−2.28]				[−2.86]		
$Easy\ Credit_t$			−0.57				−0.80	
			[−2.12]				[−4.03]	
$-1 \times EBP_t$				−19.29				−24.72
				[−2.43]				[−4.79]
Constant	−15.67	11.46	−0.27	3.25	−17.54	14.93	−1.26	4.51
	[−2.18]	[3.85]	[−0.06]	[0.94]	[−2.29]	[4.42]	[−0.21]	[1.15]
N	29	29	20	29	28	28	19	28
R-squared	0.20	0.11	0.20	0.20	0.25	0.16	0.38	0.32

Table 2. Credit market overheating and current and past default rates. This table presents the results from estimating time-series regressions of the form:

$$Overheating_t = a + b \cdot Def_t + c \cdot Def_{t-1} + e_t,$$

where *Def* denotes the default rate on speculative grade bonds and *Overheating* is a measure of credit market overheating. The data begins in 1983 and ends in 2014. HYS_t is the fraction of nonfinancial corporate bond issuance with a high-yield rating from Moody's, as defined in Greenwood and Hanson (2013). $Credit\ Growth_t$ is the percentage change in outstanding corporate credit and is computed using Table L103 from the Flow of Funds. $Easy\ Credit_t$ is the three-year average of the percentage of bank loan officers reporting a loosening of commercial lending standards from the Federal Reserve's Senior Loan Office Opinion Survey. $-1 \times EBP_t$ is negative one times the excess bond premium from Gilchrist and Zakrajšek (2012). *t*-statistics (in brackets) are based on Newey-West (1987) standard errors, allowing for serial correlation up to 3-lags.

	Dependent variable:			
	(1)	(2)	(3)	(4)
	$\log(HYS_t)$	$Credit\ Growth_t$	$Easy\ Credit_t$	$-1 \times EBP_t$
Def_t	–0.113	–0.005	–3.425	–0.104
	[–2.21]	[–1.22]	[–11.19]	[–4.98]
Def_{t-1}	0.009	–0.008	–2.152	0.021
	[0.44]	[–3.47]	[–4.24]	[0.96]
Constant	–0.734	0.118	18.076	–0.36
	[–3.47]	[7.92]	[4.58]	[–2.21]
N	31	31	22	30
R-squared	0.400	0.436	0.813	0.426

Table 3. Return and default forecasting results via model simulations. This table reports univariate and multivariate forecasting regressions for cumulative returns (1 through 5 years) and cumulative number of defaults (1 through 5 years). These regressions are estimated using 100,000 periods of simulated model data; each period represents one year. Numbers in percentage are the adjusted R-squared. The model parameters are the same as those in Figure 4. Specifically, the model parameters are $\overline{x} = 2.4$, $\rho = 0.8$, $\sigma_\varepsilon = 0.5$, $c = 2$, $\underline{F} = 1.5$, $\overline{F} = 5$, $\eta = 0.5$, $\beta = 0.8$, $\alpha = 0.2$, $\sigma_\omega = 0.05$, and $\theta = 0.5$.

		Panel A: Return forecasting					Panel B: Default forecasting				
	Univariate forecasting	**1-yr**	**2-yr**	**3-yr**	**4-yr**	**5-yr**	**1-yr**	**2-yr**	**3-yr**	**4-yr**	**5-yr**
(1)	Cashflow (x_t)	0.04	0.07	0.09	0.11	0.12	–0.19	–0.37	–0.53	–0.69	–0.84
		12%	18%	19%	18%	16%	24%	30%	32%	32%	32%
(2)	Debt face value (F_t)	–0.02	–0.02	–0.03	–0.03	–0.03	0.13	0.23	0.32	0.41	0.49
		15%	13%	10%	7%	5%	65%	69%	68%	65%	63%
(3)	Debt growth ($F_t - F_{t-4}$)	–0.04	–0.06	–0.07	–0.08	–0.09	0.11	0.17	0.22	0.28	0.33
		33%	32%	30%	27%	24%	22%	17%	16%	14%	13%
(4)	Credit spreads ($1 - p_t$)	–0.37	–0.38	–0.39	–0.35	–0.29	2.49	4.06	5.61	7.01	8.29
		21%	11%	7%	4%	2%	83%	76%	72%	68%	64%
(5)	Sentiment ($\lambda_t^R - \lambda_t^B$)	–0.34	–0.37	–0.42	–0.46	–0.48	0.90	1.10	1.37	1.61	1.84
		82%	49%	40%	33%	27%	52%	26%	20%	17%	15%
(6)	Rational beliefs (λ_t^R)	–0.22	–0.23	–0.25	–0.25	–0.25	1.00	1.51	2.03	2.51	2.94
		51%	29%	21%	15%	10%	96%	74%	67%	62%	57%
(7)	Extrapolative beliefs (λ_t^B)	0.02	0.04	0.07	0.12	0.18	0.86	1.68	2.43	3.11	3.73
		0%	0%	1%	2%	2%	32%	42%	44%	43%	42%
	Multivariate forecasting	**1-yr**	**2-yr**	**3-yr**	**4-yr**	**5-yr**	**1-yr**	**2-yr**	**3-yr**	**4-yr**	**5-yr**
(1)	Credit spreads ($1 - p_t$)	1.24	1.41	1.72	2.02	2.32	0.00	2.36	4.15	5.79	7.25
	Low-quality-issuance (λ_t^R)	–0.65	–0.72	–0.85	–0.95	–1.05	1.00	0.69	0.59	0.49	0.42
		82%	49%	40%	33%	27%	96%	78%	73%	68%	64%